Tutorial 12 - 28.01.2020

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Sorting Algorithms

Today's Agenda

- Bubble Sort
- **Exercise 12.1/2: Bubble Sort**
- Merge Sort
- **Exercise 12.3/4: Merge Sort**
- Comparing Algorithms: Big-O
- Application Example: Element Lookup
- **Exercise 12.5 (optional): Implement Binary Search**

Interesting Sources:

https://www.geeksforgeeks.org/sorting-algorithms/

Important**: Most of the code-examples used (e.g. sorting steps) are **not valid C-code!

General

Input: An array of elements which appear in a random order.

Output: The same array but with all of its element in an ascending/descending order regarding some logic.

The input and output data structures do not have to be C arrays. Good Options:

- Singly-/Doubly-Linked List: Very easy to move elements around
- (Binary) Tree: Common for searching

However in this tutorial we will only use sequentially stored arrays.

Bubble Sort

This is probably the **simplest sorting algorithm** to implement. However it is also **one of the least performant** ones.

Basic procedure:

1. Interate through all successive pairs of elements (`1` / `2`, `2` / `3`, `3` / `4`, ...)
2. For every pair: Determine if the order has to be changed - if so, swap the elements
3. When the end of the array has been reached and at least one pair has been swapped, jump to step 1
4. If no pair has been swapped through the whole iteration, the array is sorted

Bubble Sort - Example

```
[5, 1, 4, 2, 8];  // input
```

Iteration 1:

```
[5, 1, 4, 2, 8];  // Looking at 5/1
[1, 5, 4, 2, 8];

[1, 5, 4, 2, 8];  // Looking at 5/4
[1, 4, 5, 2, 8];

[1, 4, 5, 2, 8];  // Looking at 5/2
[1, 4, 2, 5, 8];

[1, 4, 2, 5, 8];  // Looking at 5/8
[1, 4, 2, 5, 8];
```

Iteration 2:

```
[1, 4, 2, 5, 8];  // Looking at 1/4
[1, 4, 2, 5, 8];

[1, 4, 2, 5, 8]; // Looking at 4/2
[1, 2, 4, 5, 8];

[1, 2, 4, 5, 8];  // Looking at 4/5
[1, 2, 4, 5, 8];

[1, 2, 4, 5, 8];  // Looking at 5/8
[1, 2, 4, 5, 8];
```

Iteration 3:

```
[1, 2, 4, 5, 8];  // Looking at 1/4
[1, 2, 4, 5, 8];

[1, 2, 4, 5, 8];  // Looking at 4/2
[1, 2, 4, 5, 8];

[1, 2, 4, 5, 8];  // Looking at 4/5
[1, 2, 4, 5, 8];

[1, 2, 4, 5, 8];  // Looking at 5/8
[1, 2, 4, 5, 8];
```

Not a single swap in this iteration: **The array is sorted**.

Bubble Sort in action:

https://www.youtube.com/watch?v=nmhjrI-aW5o

Exercise 12.1: Bubble Sort on Paper

```
[G, L, A, C, I, E, R, S];  // input
```

(a) Sort the array on paper.

(b) How many full passes through the array did it take?

(c) How many comparison operations did it take?

Exercise 12.2: Implement Bubble Sort

Implement a bubble sort algorithm in C.

I do supply a template `exercise_12_02_bubble_sort_boilerplate.c` **but** you are free to use whichever program structure you prefer.

Merge Sort

Algorithms like Merge Sort are also called **divide-and-conquer algorithms**. **Binary search** on a tree data structure is another good example.

Prerequisites:

- A list of length `0` or `1` is already sorted
- The process of combining two already-sorted lists into one sorted list is called **"merging"**

Merging Two Sorted Lists

The new list is empty at the beginning.

Repeat until both old lists are empty

1. Look at the first elements of each list.
2. Append* the smaller element to the newly created list
3. Remove that element from its list (converting this lists second element into the first one)

**append = add at the end*

Merging - Example

Two already-sorted lists:

```
[3, 27, 38, 43], [9, 10, 82]
```

New list:

```
[]
```

Step 1: `3 < 9` so append `3` to the new list.

Old list's:

```
[27, 38, 43], [9, 10, 82]
```

New list:

```
[3]
```

Step 2: `27 > 9` so append `9` to the new list.

Old list's:

```
[27, 38, 43], [10, 82]
```

New list:

```
[3, 9]
```

Step 3: `27 > 10` so append `10` to the new list.

Old list's:

```
[27, 38, 43], [82]
```

New list:

```
[3, 9, 10]
```

Step 4: `27 < 82` so append `27` to the new list.

Old list's:

```
[38, 43], [82]
```

New list:

```
[3, 9, 10, 27]
```

Step 5: `38 < 82` so append `38` to the new list.

Old list's:

```
[43], [82]
```

New list:

```
[3, 9, 10, 27, 38]
```

Step 6: `43 < 82` so append `43` to the new list.

Old list's:

```
[], [82]
```

New list:

```
[3, 9, 10, 27, 38, 43]
```

Step 7: `list_1` is empty so append `list_2` to the new list.

Old list's:

```
[], []
```

New list:

```
[3, 9, 10, 27, 38, 43, 82]
```

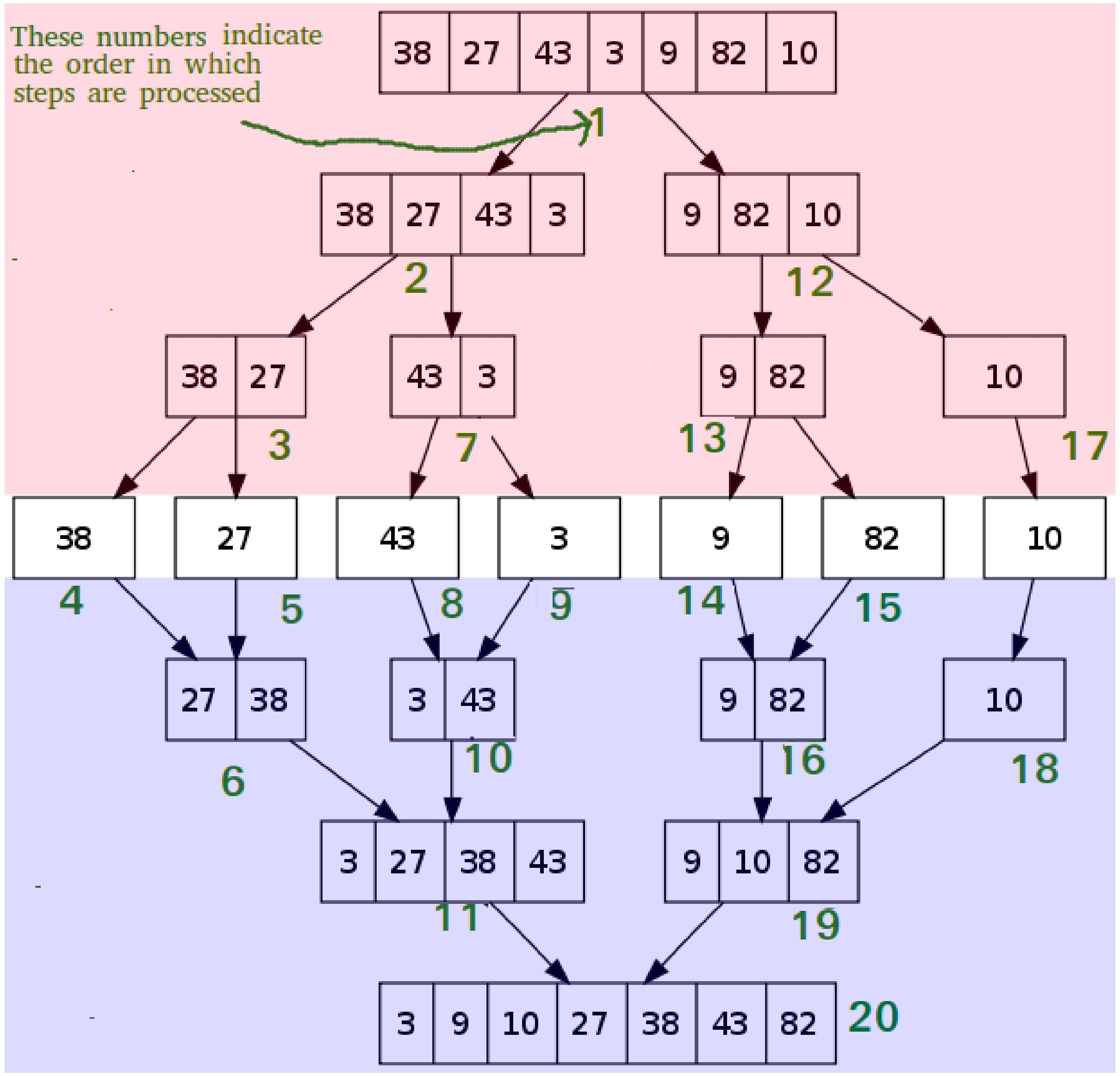
Merge Sort - Process

If a list has two or more elements split it in half:

1. Merge-Sort each list separately
2. Merge the two sorted lists into one big list

As you might recall, a list with less than 2 elements is already sorted so "Merge-Sorting" a list with `1` or `0` elements will just return the same list.

This is a prime example of a **recursive definition**.



1. Splitting
2. Merging

Merge Sort in action:

https://www.youtube.com/watch?v=JSceec-wEyw

Exercise 12.3: Merge Sort on Paper

```
[G, L, A, C, I, E, R, S];  // input
```

(a) Sort the array on paper.

(b) How many split operation and how many merge-operations (two lists -> one list) did it take?

(c) How many comparison operations did it take?

Exercise 12.4: Implement Merge Sort

Implement a merge sort algorithm in C.

I do supply a template `exercise_12_04_merge_sort_boilerplate.c` **but** you are free to use whichever program structure you prefer.

Comparing Algorithms

We need some way to **measure how our algorithm performs** with an increasing number of input arguments (here: Length of the array to be sorted).

Question: Any ideas on how we might do that?

Comparing Algorithms - Two Ways

1. Measuring how much time the algorithm takes - rather inelegent way
2. Counting the number of operations the algorithm takes to complete - e.g. how many comparisons do we have to make.

No claim of completeness.

However we need some way to quantify that.

Big-O

With the **Big-O-Notation** we are given a way to describe this **"scalability-performance" of an algorithm**.

This is called **complexity-analysis**.

Every algorithm that is executed on input data of variable length `n` has a Big-O-Complexity.

Big-O - Bubble Sort

Worst and Average Case Time Complexity: $O(n^2)$.

Best Case Time Complexity: $O(n)$.

Worst case occurs when array is reverse sorted.

Best case occurs when array is already sorted.

This means that in most cases when **doubling the input length**, the **execution time is quadrupled**.

Big-O - Merge Sort

Worst, Average and Best Case Time Complexity: $O(n \cdot \log{(n)})$.

This means that in most cases when **doubling the input length**, the **execution time is** only a bit more than **doubled**.

Different Types of Complexities

Time Complexity = How much time does it take to execute the algorithm?

Space Complexity = How much memory space does the algorithm take up?

If you want to learn more about Big-O notation, you can check out a video (~20 min) of one of my favorite JS-Developer-Youtubers:

https://www.youtube.com/watch?v=kS_gr2_-ws8

The CS-lecture at TUM where this will be covered best is:
IN0007 Grundlagen: Algorithmen und Datenstrukturen

https://campus.tum.de/tumonline/WBMODHB.wbShowMHBReadOnly?pKnotenNr=452818&pOrgNr=14189

*You can take this course in your 5th/6th semester in **Wahlbereich 2**.*

Application Example: Element Search

Task: Check if a specific number occurs in a given list of integers.

Brute-Force Search = Iterating through the whole list and check every single element.

Element Search - Brute-Force

In general a **Brute-Force Approach** is an solution-finding approach where you don't use any logically determined guesses but just try any possible combination.

For example if you were to guess an **alpha-numeric password of length 8**.

- Brute-Forcing: Try out every combination from `00000000` to `ZZZZZZZZ`.
- Better Approach: Try to make educated guesses - names of pets/relatives in combination with numbers, etc.

If the **list were to be already sorted**:

What better approach than brute-force can we use for our search?

Element Search - Binary Search

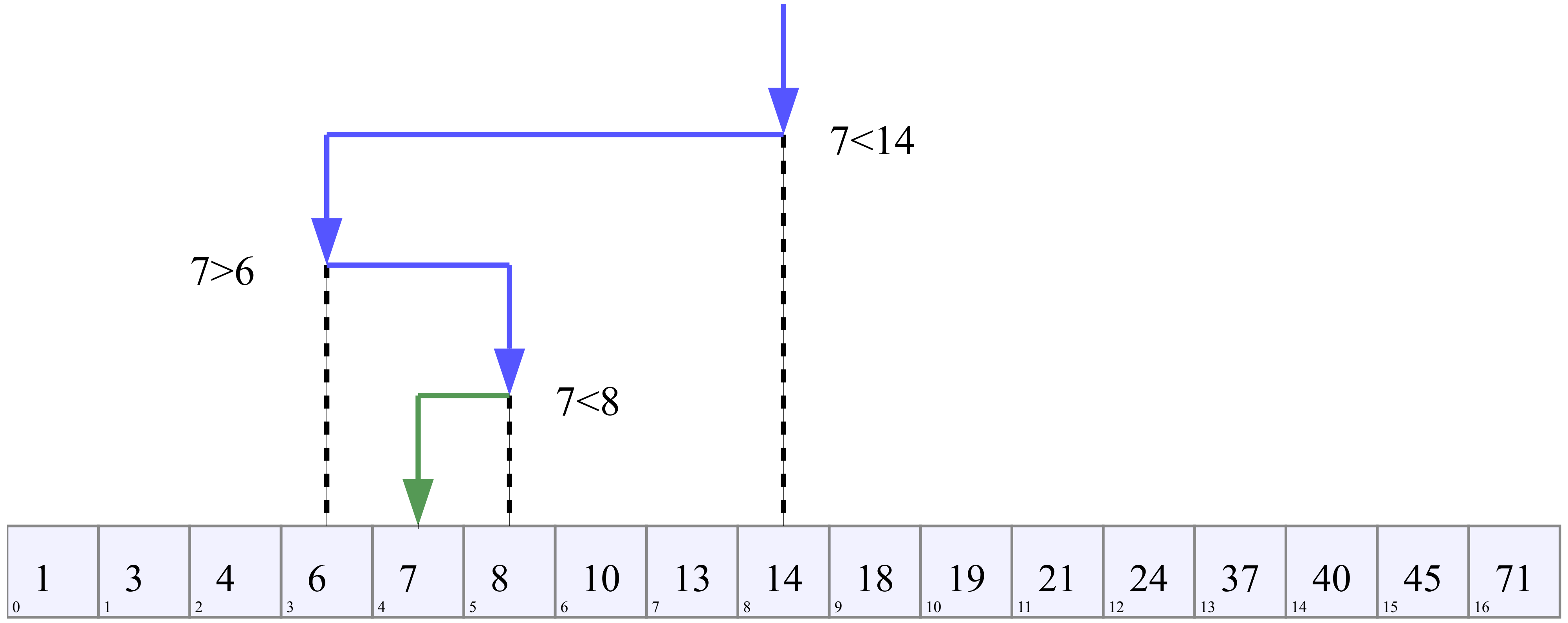
Looking for element `x` in a given list with length `n`.

Element in the middle = Element at index `n/2` (rounded up or down)

Iterative process:

1. If the **element in the middle is equal to** **`x`** return `true`
2. If the **element in the middle is greater than** **`x`** run the **binary search on the left half** of the array (excluding the element in the middle)
3. If the **element in the middle is smaller than** **`x`** run the **binary search on the right half** of the array (excluding the element in the middle)

Looking for the integer `7`:



https://www.youtube.com/watch?v=9FzT2I21F3k

Exercise 12.5 (optional): Implement Binary Search

Implement a binary search on a standard already-sorted C-array.

I do supply a template `exercise_12_05_binary_search_boilerplate.c` **but** you are free to use whichever program structure you prefer.

See You Next Week!

All **code examples** and **exercise solutions** (available right after my tutorial) on **GitHub**.
https://github.com/dostuffthatmatters/Engineering-Informatics-1-MSE-WS1920 .

you vs. the guy she tells you not to worry about

$O(n^2)$	$O(n \log n)$