```
(% i1) values;
(\% \text{ o1})
                                       (% i3) usefastarrays:true; use_fast_arrays:true;
(usefastarrays)
                                      true
(use_fast_arrays)
                                      true
DEFINING INFO STRUCTURES AND TABLES [HASH ARRAYS]
(% i4) defstruct(var_info(name, value, latex));
(\% \text{ o4})
                         [var_info (name, value, latex)]
(% i5) defstruct(fun_info(ftyp,name,fargs,body,latex));
(\% \text{ o5})
                    [fun\_info(ftyp, name, fargs, body, latex)]
(% i6) usefastarrays:true;
(usefastarrays)
                                      true
(% i7) defstruct(var_table(count,vars));
(\% \text{ o}7)
                            [var_table (count, vars)]
(% i8) defstruct(fun_table(count,funs));
(% o8)
                            [fun\_table(count, funs)]
MANUAL OPERATIONS USING STRUCTURES AND HASH ARRAYS
(% i9) vinfo_ch01:new(var_info);
(vinfo_- ch01)
                                var_info (name, value, latex)
(%
        vinfo_ch01@name:P_p1_s;
i10)
(\% \text{ o}10)
                                     P_-p1_s
```

```
(%
          vinfo_ch01@value:0.0531;
i11)
(% o11)
                                           0.0531
(%
          vinfo\_ch01@latex:[['math,"P\_\backslash scriptscriptstyle[+1,s]"],['text,"P\_[+1,s]"]];\\
i12)
(% o12)
                                          [[math,
P_{-} \left\{ \left\langle scriptscriptstyle\{[+1,s]\}\right\rangle \right], [text, P_{-} \left\{[+1,s]\right\}]
                                              ]]
(%
          assoc('math,vinfo_ch01@latex);
i13)
(% o13)
P_{-}\left\{ \backslash scriptscriptstyle\{[+1,s]\}\right\}
(%
          assoc('text,vinfo_ch01@latex);
i14)
(% o14)
P_{-}\{[+1,s]\}
(%
          vch01_ltx:assoc(math,vinfo_ch01@latex);
i15)
(vch01_ltx)
P_{-}\{\scriptscriptstyle\{[+1,s]\}\}
(%
          vch01_ltx;
i16)
(% o16)
P_{-}\{\scriptscriptstyle\{[+1,s]\}\}
(%
          vtab_ch01:new(var_table);
i17)
(vtab\_ch01)
                                 var_table (count, vars)
```

```
(%
                                         vtab_ch01@vars:[vinfo_ch01];
i18)
(\% \text{ o}18)
[\text{var\_info} \left(name = P\_p1_s, value = 0.0531, latex = [[math, P\_\{\setminus scriptscriptstyle\{[+1, s]\}\}], [text, P\_\{[+1, s]\}]])]
(%
                                          vtab_ch01@count:1;
i19)
(% o19)
                                                                                                                                                                                                  1
(%
                                          finfo_Pv_z:new(fun_info);
i20)
(finfo_Pv_z)
                                                                                                                                       fun_info (ftyp, name, fargs, body, latex)
                                          finfo_Pv_z@ftyp:"FUN";
(%
i21)
(% o21)
FUN
(%
                                          finfo_Pv_z@name:P_v;
i22)
(\% \text{ o}22)
                                                                                                                                                                                                P_v
(%
                                          finfo_Pv_z@fargs:[z];
i23)
(\% \text{ o}23)
                                                                                                                                                                                                [z]
(%
                                          finfo\_Pv\_z@body:"abs(w\_v)*sum(p\_m*abs(a\_vm(z))^2,m,minf,inf)";
i24)
(\% \text{ o}24)
abs(w_v) \cdot sum(p_m \cdot abs(a_v m(z))^2, m, minf, inf)
(%
                                          i25)
                                          \left\langle \frac{1}{n} \right\rangle = -\frac{2}{n} - \frac{2}{n} = \frac{2}{n} - \frac{2}{n} = \frac{2}{n} - \frac{2}{n} = \frac{2}{n}
(\% \text{ o}25)
```

```
P_- \setminus nu(z) \sim = \sim \setminus left \mid \setminus omega_- \setminus nu \mid right \mid \sim \setminus sum_- \{- \mid infty\} \{ \mid infty \} \sim p_- \{m\} \sim \setminus left \mid \sim a_- \{- \mid nu \} \sim (z) \setminus right \mid \sim
   (%
                                                                                    ftab_ch01:new(fun_table);
   i26)
   (ftab_- ch01)
                                                                                                                                                                                                                                                                                       fun_table (count, funs)
     (%
                                                                                     ftab_ch01@count:1;
   i27)
   (\% \text{ o}27)
                                                                                                                                                                                                                                                                                                                                                                                             1
   (%
                                                                                     ftab_ch01@funs:[finfo_Pv_z];
   i28)
   (\% \text{ o}28)
                                                                                                                                                                                                                                                                                                                     [fun\_info(ftyp =
   FUN
                                                                                                                                                                                                                                        , name = P_v, fargs = [z], body =
   abs(w_v) \cdot sum(p_m \cdot abs(a_v m(z))2, m, minf, inf), latex =
 P_- \setminus nu(z) \sim = \sim |left| \setminus omega_- \setminus nu \setminus right| \sim |sum_{-}(-\ln fty)| \setminus |sum_{-}(-\ln fty)| \sim |sum_{-}(-\ln f
     (%
                                                                                     finfo_Pdz_z:new(fun_info);
   i29)
   (finfo_Pdz_z)
                                                                                                                                                                                                                                                                              fun_info (ftyp, name, fargs, body, latex)
   (%
                                                                                     finfo_Pdz_z@ftyp:"FUN";
   i30)
   (\% \text{ o}30)
   FUN
     (%
                                                                                     finfo_Pdz_z@name:P_dz;
   i31)
   (% o31)
                                                                                                                                                                                                                                                                                                                                                                                   P_{dz}
     (%
                                                                                    finfo_Pdz_z@fargs:[z];
   i32)
   (\% \text{ o}32)
                                                                                                                                                                                                                                                                                                                                                                                        [z]
```

```
(%
                                    finfo_Pdz_z@body:"diff(sum(q_v*p_m*abs(a_vm(z))^2,m,minf,inf),z)";
i33)
(\% \text{ o}33)
diff(sum(q_v \cdot p_m \cdot abs(a_v m(z)))^2, m, minf, inf), z)
                                   (%
                                    \left\langle \frac{1}{n} \right\rangle = -\left\langle \frac{2}{r} \right\rangle
i34)
(\% \text{ o}34)
P_{-}\{dz\}(z) \sim = \sim |left| \sim 
(%
                                    finfo_S_w0:new(fun_info); finfo_S_w0@name:'finfo_S_w0;
i36)
(finfo_S_w0)
                                                                                                                   fun_info (ftyp, name, fargs, body, latex)
(\% \text{ o}36)
                                                                                                                                                 finfo\_S\_w\theta
(%
                                    finfo_S_w0;
i37)
(\% \text{ o}37)
                                                                         fun_info(ftyp, name = finfo_S_w0, fargs, body, latex)
 (%
                                    finfo_S = w0@body: "A* sin(w_0*t+phi)";
i38)
(\% \text{ o}38)
A \cdot sin(w_0 \cdot t + phi)
(%
                                    finfo_S_w0@fargs: [A,w_0,t,phi];
i39)
                                                                                                                                             [A, w_0, t, phi]
(\% \text{ o}39)
                                    finfo\_S\_w0@ftyp:"SYMBOLIC";
(%
i40)
(\% \text{ o}40)
```

SYMBOLIC

```
(%
                               finfo_S_w0;
i41)
(% o41)
fun\_info (ftyp = SYMBOLIC, name = finfo\_S\_w0, fargs = [A, w_0, t, phi], body = A \cdot sin(w\_0 \cdot t + phi), latex)
(%
                               finfo\_S\_w0@latex: "S(A, \omega\_0, t, \phi)=A \cdot sin(\omega\_0 \cdot sin(\omega\_0, 
                               t + \phi)";
i42)
(\% \text{ o}42)
S(A, \omega_0, t, \phi_i) = A \cdot cdotsin(\omega_0, t, \phi_i)
(%
                               arr_funs[finfo_S_w0@name]:finfo_S_w0;
i43)
ARRSTORE: use_fast_arrays=true; allocate a new value hash table for $ ARR_ FUNS
(\% \text{ o}43)
                                                                                                                     fun_info(ftyp =
SYMBOLIC
                                                   , name = finfo\_S\_w0, fargs = [A, w_0, t, phi], body =
A \cdot sin(w_0 \cdot t + phi), latex =
S(A, \lozenge omega\_0, t, \lozenge phi) = A \lang cdotsin(\lozenge omega\_0 \lang cdott + \lang phi))
(%
                              arr_funs['finfo_S_w0]@latex;
i44)
(\% \text{ o}44)
S(A, \omega_0, t, phi) = A \cdot cdotsin(\omega_0 - \omega_0 \cdot cdott + phi)
(%
                               tf01@body;
i45)
(\% \text{ o}45)
                                                                                                                               tf01@body
(%
                               finfo_S_w0@body;
i46)
(\% \text{ o}46)
```

```
A \cdot sin(w_0 \cdot t + phi)
(%
                    fp:[['ftyp,
                                                                                                          "SYMBOLIC"],['name,'Ak_A0_kA_z],['fargs,
                    [A_0,k_A,z], ['body,
                                                                                               (A_0/z)*\sin(k_a*z)*\exp(-\%i*z/4*\%pi)],['latex,
i47)
                    "\fracA_0z\,\sin(k_a\sim z)\sim e^{\left(\frac{-i^2}{i^2}\right)} ];
(fp)
                                                                                         [[ftyp,
SYMBOLIC
                                       [], [name, Ak_A 0_k A_z], [fargs, [A_0, k_A, z]], [body, A_0, k_A, z]]
(A_- 0/z) \cdot sin(k_-a \cdot z) \cdot exp(-\%i \cdot z/4 \cdot \%pi), [latex,
frac\{A_{0}\}\{z\}\, sin(k_a \sim z) \sim \hat{e}\{\left\{ \int ft(\int rac\{-i \sim pi \sim z\}\{4\} \right\})\}
(%
                    assoc('latex,fp);
i48)
(\% \text{ o}48)
\frac{A_{0}}{z} \  \  \sin(k_a \sim z) \sim \hat{e} \left\{ \left( \frac{-i}{z} \sim pi \sim z \right) \right\} 
(%
                    arr_funs['finfo_S_w0]@latex;
i49)
(\% \text{ o}49)
S(A, \omega_0, t, phi) = A \cdot cdotsin(\omega_0, t, phi)
(%
                    arr_funs['Ak_A0_kAz];
i50)
(\% \text{ o}50)
                                                                                           false
DEFINING FUNCTIONS TO ADD STRUCTURES TO HASH ARRAYS US-
ING FUNCTION NAME AS HASH KEY FOR LOOKUP. WORKS WELL
AND SEEMS FAST ENOUGH FOR TESTING COMPILE ALL CODE TO
LISP ONCE IT WORKS - BETTER PERFORMANCE RESULTS ftyp FIELD
- FOR NOW USE IT FOR NUMERIC OR SYMBOLIC
(%
                    make_finfo(fl):=new(fun_info("assoc('ftyp, fl), "assoc('name,fl), "assoc('fargs,fl), "assoc('body,fl), "assoc('body,fl),
i51)
(\% \text{ o}51)
make_finfo (fl) := \text{new}(\text{fun\_info}(ftyp = \neq \{\text{Lispfunction}\} ('ftyp, fl), name = \neq \{\text{Lispfunction}\} ('name, fl), fargs
(%
                    add_fun(fl):=arr_funs["assoc('name,fl)]:make_finfo(fl);
i52)
(\% \text{ o}52)
                              add\_fun(fl) := arr\_funs_{\neq \{Lispfunction\}('name,fl)} : make\_finfo(fl)
```

ADDING ITEMS WITH add_fun

```
add\_fun([['ftyp, "SYMBOLIC"] \ , \ ['name, 'E\_r\_t\_z\_t] \ , \ ['fargs, \ [r\_t,z,t]] \ , \\
(%
                              ['body, "sum(A_k(A_0,k_A,z)*E_k(r_t)*exp(%i*\omega*t-\gamma_k*z),k,minf,inf)"] , ['la-
i53)
                              tex, "E(r_t,z,t)\sim=\sim\\sum_k=-\\infty^ \\inftyA_k(A_0,k_A,z)\simE_k(r_t)\sime^
                              \\left( i \\omega t-\\gamma_k z\\right) "]]);
(\% \text{ o}53)
                                                                                                                   fun_info(ftyp =
SYMBOLIC
                                                                  , name = E_{-}r_{-}t_{-}z_{t}, fargs = [r_{t}, z, t], body =
\mathrm{sum}(\mathbf{A}_{-}\,\mathbf{k}(\mathbf{A}_{-}\,\mathbf{0},\mathbf{k}_{-}\,\mathbf{A},\mathbf{z})\cdot E_{-}k(r_{-}t)\cdot \exp(\%i\cdot\omega\cdot t - \gamma_{-}k\cdot z),k,minf,inf),latex = \mathbf{E}(\mathbf{r}_{-}\,\mathbf{k}\cdot\mathbf{z})\cdot \mathbf{k}\cdot \mathbf{k}
t,z,t) \sim = \sim \sum_{k=-\infty} \{k = -\inf\{y\}\} \{ \inf\{y\}\} A - k(A - 0, k - A, z) \sim E - k(r - t) \sim \hat{e}\{ \{t\}\} \{ t \in A, z\} \} 
(%
                              arr_funs[E_r_t_z_t];
i54)
(\% \text{ o}54)
                                                                                                                   fun\_info(ftyp =
SYMBOLIC
                                                                  , name = E_r_t_z_t, fargs = [r_t, z, t], body =
t,z,t) \sim = \sim \sum_{k=-\infty} \{k = -\inf\{y\} \{ \inf\{y\}\} A_{-k}(A_{-0},k_{-}A,z) \sim E_{-k}(r_{-}t) \sim e^{i\{t\}} \{ \inf\{y\}\} A_{-k}(A_{-0},k_{-}A,z) \sim 
(%
                              arr_funs[E_r_t_z_t]@name;
i55)
(\% \text{ o}55)
                                                                                                                                E_{-}r_{-}t_{-}z_{t}
(%
                              arr_funs[E_r_t_z_t]@ftyp;
i56)
(\% \text{ o}56)
SYMBOLIC
(%
                              arr_funs[E_r_t_z_t]@fargs;
i57)
                                                                                                                                  [r_t, z, t]
(\% \text{ o}57)
(%
                              arr_funs[E_r_t_z_t]@body;
i58)
(\% \text{ o}58)
```

```
\operatorname{sum}(A_- k(A_- 0, k_- A, z) \cdot E_- k(r_- t) \cdot exp(\% i \cdot \omega \cdot t - \gamma_- k \cdot z), k, minf, inf)
(%
                                                 arr_funs[E_r_t_z_t]@latex;
i59)
(\% \text{ o}59)
(%
                                                 add_fun([['ftyp, "SYMBOLIC"] , ['name,'EQ_1242_B] , ['fargs, [z]]
                                                                                                                                      "abs(w_v)*sum(p_m*abs(a_vm(z))^2,m,minf,inf)"] , ['latex,
i60)
                                                "P^\\left(
(\% 060)
                                                                                                                                                                                        fun_info(ftyp =
SYMBOLIC
                                                                                                                , name = EQ_{-}1242_{B}, fargs = [z], body =
abs(w_v) \cdot sum(p_m \cdot abs(a_v m(z))), m, minf, inf), latex =
P^{\left(\ln t(\ln u + ight)\right)}(z) = p_m \left(\ln t(\ln u + ight)\right) = p_m \left(\ln u + ight)
(%
                                                 arr_funs[EQ_1242_B];
i61)
(% o61)
                                                                                                                                                                                        fun_info(ftyp =
SYMBOLIC
                                                                                                               , name = EQ_{-}1242_{B}, fargs = [z], body =
abs(w_v) \cdot sum(p_m \cdot abs(a_v m(z))), m, minf, inf), latex =
 P^{\left( \ln t \leq n \right)}(z) = p_m \left( \ln t \leq n \right) = 
 (%
                                                 arr_funs[EQ_1242_B]@ftyp;
i62)
(\% \text{ o}62)
SYMBOLIC
 (%
                                                 arr_funs[EQ_1242_B]@name;
i63)
(% o63)
                                                                                                                                                                                                      EQ_{-}1242_{B}
 (%
                                                 arr_funs[EQ_1242_B]@fargs;
i64)
(\% \text{ o}64)
                                                                                                                                                                                                                               [z]
```

```
(%
                     arr_funs[EQ_1242_B]@body;
i65)
(\% 065)
abs(w_v) \cdot sum(p_m \cdot abs(a_v m(z))) = m, minf, inf)
(%
                     arr_funs[EQ_1242\_B]@latex:"P^{\left( \nu \right)(z)=\left-w_v \right)}
i66)
                     \sum_{m=-\infty}\left(\frac{nu}{nu}\right)(z)\right)
(\% \text{ o}66)
add_fun([['ftyp, "SYMBOLIC"] , ['name,'E_r_t_z_t] , ['fargs, [r_t,z,t]] ,
(%
i67)
                     ['body, "sum(A_k(A_0,k_A,z)*E_k(r_t)*exp(%i*\omega*t-\gamma_k*z),k,minf,inf)"], ['la-
                     tex, "E(r_t,z,t) \sim = \sim \sum_{k=-\infty} (-1)^{-1} \sin(tyA_k(A_0,k_A,z) \sim E_k(r_t) \sim e^{-1}
                     \left(i\sim \right) = \left(i\sim \right) "]]);
(\% \text{ o}67)
                                                                                 fun_info(ftyp =
SYMBOLIC
                                              , name = E_r_t_z_t, fargs = [r_t, z, t], body =
t,z,t\rangle \sim = \sim \sum_{k=-\infty} \{k = -\inf\{t\}\} \{\inf\{t\}\} A - k(A - 0, k - A, z) \sim E - k(r - t) \sim e\{\{t\}\} \{\inf\{t\}\} - k(r - t)\} = e\{\{t\}\} = e\{\{t\}\} = e\{\{t\}\} - k(r - t)\} = e\{\{t\}\} 
(%
                     add_{fun}([['ftyp, "SYMBOLIC"] , ['name,'H_r_t_z_t] , ['fargs, [r_t,z,t]] ,
i68)
                     ['body, "sum(A_k(A_0,k_A,z)*H_k(r_t)*exp(%i*\omega*t-\gamma_k*z),k,minf,inf)"], ['la-
                     tex, "H(r_t,z,t)\sim=\sim\\sum_k=-\\infty^ \\inftyA_k(A_0,k_A,z)\simH_k(r_t)\sime^
                     \\left( i \\omega~t-\\gamma_k z\\right) "]]);
(\% 068)
                                                                                 fun_info(ftyp =
SYMBOLIC
                                              , name = H_{r_t}, fargs = [r_t, z, t], body = [r_t, z, t]
sum(A_-k(A_-0,k_-A,z)\cdot H_-k(r_-t)\cdot exp(\%i\cdot\omega\cdot t-\gamma_-k\cdot z),k,minf,inf),latex=H(r_-k(A_-0,k_-A,z)\cdot H_-k(r_-t)\cdot exp(\%i\cdot\omega\cdot t-\gamma_-k\cdot z),k,minf,inf)
(%
                     {\rm add\_fun}([['ftyp,\ "SYMBOLIC"]\ ,\ ['name,'EQ\_3217\_A]\ ,\ ['fargs,\ [L\_P,N]]\ ,
i69)
                     ['body, "(1/2)*avg_t(E.'diff(P,t)"], ['latex, "avg_t \\left( I_P,N \\right) =
                     \frac{12\sim avg_t \left( E \cdot \left( E \cdot \left( E \cdot \right) \right)}{12\sim avg_t \left( E \cdot \left( E \cdot \left( E \cdot \right) \right)}\right)};
(\% 069)
                                                                                 fun_info(ftyp =
```

, $name = EQ_{-3217_A}$, $fargs = [I_P, N]$, body =

SYMBOLIC

```
(1/2) \cdot avg_{-}t(E.'diff(P,t), latex =
avg_t \setminus left(I_P, N \mid flat(1) \} \{1\} \{2\} \sim avg_t \setminus left(E \mid flat(1) \} \{partial\{t\}\} \} \{1\} \}
   (%
                                                                             arr_funs[EQ_3217_A]@latex:
                                                                                                                                                                                                                                                                                                                                                                                                                "avg_t
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             \left| \cdot \right|
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            LP,N
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     \\right)
i70)
                                                                             \\frac\\partialE\\partialt\\,N\\right)";
(\% \text{ o}70)
avg_t \setminus left(I_P, N \cap \{1\}) = \int rac\{1\}\{2\} \sim avg_t \setminus left(E \setminus cdot \cap \{1\})\}\{ \setminus partial\{t\}\} - avg_t \setminus left(E \setminus cdot \cap \{1\})\} = \int rac\{1\}\{2\} \sim avg_t \setminus left(E \setminus cdot \cap \{1\})\} = \int rac\{1\}\{2\} \sim avg_t \setminus left(E \setminus cdot \cap \{1\})\} = \int rac\{1\}\{2\} \sim avg_t \setminus left(E \setminus cdot \cap \{1\})\} = \int rac\{1\}\{2\} \sim avg_t \setminus left(E \setminus cdot \cap \{1\})\} = \int rac\{1\}\{2\} \sim avg_t \setminus left(E \setminus cdot \cap \{1\})\} = \int rac\{1\}\{2\} \sim avg_t \setminus left(E \setminus cdot \cap \{1\})\} = \int rac\{1\}\{2\} \sim avg_t \setminus left(E \setminus cdot \cap \{1\})\} = \int rac\{1\}\{2\} \sim avg_t \setminus left(E \setminus cdot \cap \{1\})\} = \int rac\{1\}\{2\} \sim avg_t \setminus left(E \setminus cdot \cap \{1\})\} = \int rac\{1\}\{2\} \sim avg_t \setminus left(E \setminus cdot \cap \{1\})\} = \int rac\{1\}\{2\} \sim avg_t \setminus left(E \setminus cdot \cap \{1\})\} = \int rac\{1\}\{avg_t \cap \{1\}\} = 
P \cdot dot \cdot frac \{ \cdot partial \{ E \} \} \{ \cdot partial \{ t \} \} \setminus, N \cdot right \}
(%
                                                                             add_fun([['ftyp, "SYMBOLIC"], ['name,'EQ_3217_B], ['fargs, [LP,N]], ['body,
i71)
                                                                             "avg_t(I_P,N)=(1/2)*avg_t(E.'diff(D,t)-D.'diff(E,t),N)"], ['latex, "avg_t \\left(
                                                                             LP,N \wedge = \frac{2}{e^{t} \cdot E \cdot \frac{E \cdot \cdot E}{E \cdot E}} - \frac{12}{e^{t} \cdot E}
                                                                             D \\cdot \\frac\\partialE\\partialt,N \\right)"]]);
(\% \text{ o}71)
                                                                                                                                                                                                                                                                                                    fun_info(ftyp =
SYMBOLIC
                                                                                                                                                              , name = EQ_{-3217_B}, fargs = [I_P, N], body =
avg_{-}t(I_{-}P,N)=(1/2)\cdot avg_{-}t(E'diff(D,t)-D'diff(E,t),N), latex =
avg_t \setminus left(I_P, N \cap \{1\}) = \int rac\{1\}\{2\} \sim avg_t \setminus left(E \setminus cdot \cap \{1\})\}\{ \setminus partial\{t\}\} - avg_t \setminus left(E \setminus cdot \cap \{1\})\} = \int rac\{1\}\{2\} \sim avg_t \setminus left(E \setminus cdot \cap \{1\})\} = \int rac\{1\}\{2\} \sim avg_t \setminus left(E \setminus cdot \cap \{1\})\} = \int rac\{1\}\{2\} \sim avg_t \setminus left(E \setminus cdot \cap \{1\})\} = \int rac\{1\}\{2\} \sim avg_t \setminus left(E \setminus cdot \cap \{1\})\} = \int rac\{1\}\{2\} \sim avg_t \setminus left(E \setminus cdot \cap \{1\})\} = \int rac\{1\}\{2\} \sim avg_t \setminus left(E \setminus cdot \cap \{1\})\} = \int rac\{1\}\{2\} \sim avg_t \setminus left(E \setminus cdot \cap \{1\})\} = \int rac\{1\}\{2\} \sim avg_t \setminus left(E \setminus cdot \cap \{1\})\} = \int rac\{1\}\{2\} \sim avg_t \setminus left(E \setminus cdot \cap \{1\})\} = \int rac\{1\}\{2\} \sim avg_t \setminus left(E \setminus cdot \cap \{1\})\} = \int rac\{1\}\{2\} \sim avg_t \setminus left(E \setminus cdot \cap \{1\})\} = \int rac\{1\}\{avg_t \cap \{1\}\} = 
D \cdot cdot \cdot frac \{ \cdot partial \{ E \} \} \{ \cdot partial \{ t \} \}, N \cdot right ) \}
   (%
                                                                             add_fun([['ftyp, "SYMBOLIC"] , ['name,'EQ_3217_C] , ['fargs, [I_P,N]]
                                                                                                                                                                                                                                                                                                  "avg_t(I_P,N)=(1/4)*'realpart('conjugate(E).(%i*\omega*D)-
i72)
                                                                                "conjugate(D).(\%i*\omega*E))"], ['latex, "avg_t \land left( I_P,N \land left) = left( I_P,N \land left( I_P,N \land left) = left( I_P,N \land left( I
                                                                             \  \ \\left(\ \overlineE \\cdot \\left(\ \overlineB \\ \- \\) -
                                                                             \\overlineD \\cdot \\left( i \\omega E \\right) \\right)"]]);
(\% \text{ o}72)
                                                                                                                                                                                                                                                                                                    fun_info(ftyp =
SYMBOLIC
                                                                                                                                                           , name = EQ_{-3217_C}, fargs = [I_P, N], body =
avg_{-}t(I_{-}P,N)=(1/4)\cdot'realpart('conjugate(E).(\%i\cdot\omega\cdot D)-'conjugate(D).(\%i\cdot\omega\cdot D)
\omega \cdot E), latex =
\operatorname{avg\_t} \setminus \operatorname{left}(I\_P, N \setminus right) = \operatorname{frac}\{1\}\{4\} \sim \operatorname{Re} \setminus \operatorname{left}(\operatorname{left}(i \setminus right) - \operatorname{left}(i \setminus right)) = \operatorname{left}(\operatorname{left}(i \setminus right) - \operatorname{left}(i \setminus right)) = \operatorname{left}(\operatorname{left}(i \setminus r
\langle overline\{D\} \langle cdot \rangle left(i \langle omegaE \rangle right) \langle right \rangle
   (%
                                                                             arr_funs[EQ_3217_C@latex]:"P_dz
                                                                                                                                                                                                                                                                                                                                                                                                                                         \left| \cdot \right|
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     \mathbf{z}
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                \\right)
i73)
                                                                             \\partial\\partialz
                                                                                                                                                                                                                                                                                                        \sum_{m=-\infty} \left( \frac{y^{-y_m}}{\left( \frac{y^{-y_m}}}{\left( \frac{y^{-y_m}}{\left( \frac{y^{-y_m}}{\left( \frac{y^{-y_m}}{\left( \frac{y^{-y_m}}}{\left( \frac{y^{-y_m}}{\left( \frac{y^{-y_m}}{\left( \frac{y^{-y_m}}{\left( \frac{y^{-y_m}}}{\left( \frac{y^{-y_m}}{\left( \frac{y^{-y_m}}}{\left( \frac{y^{-y_m}}{\left( \frac{y^{-y_m}}}{\left( \frac{y^{-y_m}}}{\left( \frac{y^{-y_m}}}{\left( \frac{y^{-y_m}}}{\left( \frac{y^{-y_m}}}{\left( \frac{y^{-y_m}}{\left( \frac{y^{-y_m}}{\left( \frac{y^{-y_m}}{\left( \frac{y^{-y_m}}}{\left( \frac{y^{-y_m}}}{\left( \frac{y^{-y_m}}}{\left( \frac{y^{-y_m}}}{\left( \frac{y^{-y_m}}}{\left( \frac{y^{-y_m}}}{\left( \frac{y^{-y_m}}}{\left( \frac{y^{-y_m}}}{\left( \frac{y^{-y_m}}}{\left( \frac{y^{-y_m}}}}{\left( \frac{y^{-y_m}}}{\left( \frac{y^{-y_m}}}{\left( \frac{y^{-y_m}}}{\left( \frac{y^{-y_m}}}{\left( \frac{y^{-y_m}}}{\left( \frac{y^{-y_m}}}{\left( \frac{y^{-y_m}}}}{\left( \frac{y^{-y_m}}}}{\left( \frac{y^{-y_m}}}{\left( \frac{y^{-y_m}}}}{\left( \frac{y^{-y_m}}}{\left( \frac{y^{-y_m}}}}{\left( \frac{y^{-y_m}}}{\left( \frac{y^{-y_m}}}}{\left( \frac{y^{-y_m}}}{\left( \frac{y^{-y_m}}}{\left( \frac{y^{-y_m}}}}{\left( \frac{y^{-y_m}}}}{\left( \frac{y^{-y_m}}}{\left( \frac{y^{-y_m}}}}{\left( \frac{y^{-y_m}}}}{\left( \frac{y^{-y_m}}}}{\left( \frac{y^{-y_m}}}}{\left
                                                                             (\% \text{ o}73)
```

```
P_{dz} \left\{ \frac{1}{z} \right\} \left\{ \frac{1}
 (%
                                                            arr_funs[EQ_3217_C@latex]:"avg_t \land (I_P,N \land ) = \land (A_Re \land )
i74)
                                                              \\omega E \\right) \\right)";
(\% \text{ o} 74)
\operatorname{avg\_t} \setminus \operatorname{left}(I\_P, N \setminus right) = \operatorname{frac}\{1\}\{4\} \sim \operatorname{Re} \setminus \operatorname{left}(\operatorname{left}(i \setminus right) - \operatorname{left}(i \setminus right)) = \operatorname{left}(\operatorname{left}(i \setminus right) - \operatorname{left}(i \setminus right)) = \operatorname{left}(\operatorname{left}(i \setminus r
\langle overline\{D\} \langle cdot \rangle left(i \langle omegaE \rangle right) \langle right \rangle
  (%
                                                            add_fun([['ftyp, "SYMBOLIC"], ['name, 'EQ_3217_D], ['fargs, [LP,N]], ['body,
i75)
                                                            "avg_t(I_P,N)=(1/4)*'realpart(%i*'\omega*('conjugate(E).\epsilon_bar.E+'conjugate(E)*\xi_bar.H-
                                                            tex, "avg_t \left( I_P, N \right) = \frac{14}{Re} \left( \operatorname{l.p.N} \right) = \frac{14}{Re} \left( \operatorname{l.p.N} \right)
                                                            \\left(i \\omega D \\right) - \\overlineD \\cdot \\left( i \\omega E \\right)
                                                            \\right)"]]);
(\% \text{ o}75)
                                                                                                                                                                                                                                    fun_info(ftyp =
SYMBOLIC
                                                                                                                          , name = EQ_{-}3217_{D}, fargs = [I_{P}, N], body =
avg_{-}t(I_{-}P,N)=(1/4)\cdot'realpart(\%i\cdot'\omega\cdot('conjugate(E).\epsilon\_bar.E+'conjugate(E)\cdot
\xi-bar.H-E.'conjugate(\epsilon-bar).'conjugate(E)-E.'conjugate(E-bar).'conjugate(E)), latex =
\operatorname{avg\_t} \setminus \operatorname{left}(I\_P, N \setminus right) = \operatorname{frac}\{1\}\{4\} \sim \operatorname{Re} \setminus \operatorname{left}(\operatorname{left}(i \setminus right) - \operatorname{left}(i \setminus right)) = \operatorname{left}(\operatorname{left}(i \setminus right) - \operatorname{left}(i \setminus right)) = \operatorname{left}(\operatorname{left}(i \setminus r
\langle overline\{D\} \langle cdot \rangle left(i \langle omegaE \rangle right) \langle right \rangle)
 (%
                                                            arr_funs[EQ_3217_D@latex]:"avg_t \\left( I_P,N \right) = \\frac14~Re \\left( i
i76)
                                                            (\% \text{ o} 76)
avg_t \setminus left(I\_P, Nright) = \int frac\{1\}\{4\} \sim Re \setminus left(i \setminus omegaE\{\cdot\} \setminus cdot \setminus left(bar\{\setminus epsilon\} - \setminus bar\{\setminus epsilon\}\})
                                                            {\rm add\_fun}([['ftyp,"SYMBOLIC"]\ ,\ ['name,'EQ\_3217\_F]\ ,\ ['fargs,\ [I\_P,N]]\ ,\ ['body,"])
 (%
i77)
                                                            "avg_t(I_P,N)=(1/2)*'realpart(%i*\omega*('conjugate(\epsilon_bar).E.'conjugate(E))+\epsilon_bar.H.'conjugate(E))"]
                                                              , ['latex, "avg_t \\left( I_P,N \\right) = \\frac12 Re \\big\\ i \\omega \\left(
                                                              (\% \text{ o}77)
                                                                                                                                                                                                                                    fun_info(ftyp =
```

```
SYMBOLIC
                                                                                                                                                                                                                                                                                , name = EQ_{-}3217_{F}, fargs = [I_{P}, N], body =
avg_- t(I_- P,N) = (1/2) \cdot realpart(\%i \cdot \omega \cdot ('conjugate(\epsilon\_bar).E.'conjugate(E)) + (1/2) \cdot realpart(\%i \cdot \omega \cdot ('conjugate(E))) + (1/2) \cdot realpart(E) \cdot re
\epsilon_bar.H.'conjugate(E)), latex =
\operatorname{avg\_t} \setminus left(I\_P, N \setminus right) = \setminus frac\{1\}\{2\}Re \setminus big \setminus \{i \setminus bar\{\setminus epsilon\}\}\{ \sim a \} : EE\{\cdot\} + \setminus xi : HE\{\cdot\}\} = \sum_{i=1}^{n} \left( \sum_{i=1}^{n} \left
(%
                                                                                                                                    arr_funs[EQ_3217_C@latex];
i79)
(\% \text{ o} 79)
\operatorname{avg\_t} \setminus \operatorname{left}(I\_P, N \setminus right) = \operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}(\operatorname{left}
\langle overline\{D\} \langle cdot \rangle left(i \langle omegaE \rangle right) \langle right \rangle
(%
                                                                                                                                      arr_funs[EQ_3217_D@latex];
i78)
(\% \text{ o} 78)
avg_{t} \setminus left(I\_P, Nright) = \setminus frac\{1\}\{4\} \sim Re \setminus left(i \setminus omegaE\{\cdot\} \setminus cdot \setminus left(bar\{\setminus epsilon\} - \setminus bar\{\setminus epsilon\})\}
(%
                                                                                                                                    arr_funs[EQ_3217_E@latex];
i80)
(\% 080)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    false
(%
                                                                                                                                    arr_funs[EQ_3217_F@latex];
i81)
```

false

(% o81)