Calculation Of Coupling Coefficents andCross Norms for Two-Waveguide Systems

From:Modern Electrodynamics and Coupled Mode TheoryBarybin and DmitrievTODO:~FIX UP LATEX CODEADD EQUATIONS FROM EARLIER CHAPTERS REFERENCED BY APPENDIX LTRANSCRIBE ANY GREEK LETTERS TO CLOSEST ENGLISH EQUIVALENT FOR MAXIMA

1 Appendix L

1.1 Coupling Coefficients for the TE~Modes

The TE modes of planar waveguides have the following field components (From Section 5.6.1) Tex: $ E_z \neq 0\, H_y \neq 0\, H_z \neq 0,\, and E+y = E_z = H_z = 0 From (7.8.64)~to (7.8.65) it follows that the surface contribution to the coupling coeffcients for the TE~modes in planar dielectric waveguides is absent. Tex:~c_11ŝ~=c_22ŝ = c_12ŝ~= c_21ŝ~= 0 Equations (7.8.48)~to (7.8.51) and (7.8.62)~to (7.8.63)~yield these expressions~for the self- and cross-coupling coefficients: The dielectric profiles Δϵâ(y) and Δϵb̂(y) are given in equations~(7.8.56)~and (7.8.57).

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-->  L1:~ [c_11=c_mm_aa_bulk, c_11 = ((%i*ω*w)/(N_m_a))*integrate(Δϵ
     ^a(y)*abs(E_mx_a(y))^2,y) ];
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$$[c_{11} = \mathrm{c\_mm\_aa\_bulk}, c_{11} = \frac{\%iw \int \mathrm{E\_mx}_a(y)^2\ \epsilon^{\mathrm{a}(y)}\, dy\, \omega}{\mathrm{N\_m}_a}, c_{22} = \mathrm{c\_nn\_bb\_bulk}, c_{22} = \frac{\%iw \int \mathrm{E\_nx}_b(y)^2\ \epsilon^{\mathrm{b}(y}}{\mathrm{N\_n}_b} \tag{L11}$$

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-->  L2:[c_22 = c_nn_bb_bulk , c_22= ((%i*ω*w)/(N_n_b))*integrate(Δϵ
     ^b(y)*abs(E_nx_b(y))^2,y)];
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$$[c_{22} = \mathrm{c\_nn\_bb\_bulk}, c_{22} = \frac{\%iw \int \mathrm{E\_nx}_b(y)^2\ \epsilon^{\mathrm{b}(y)}\, dy\, \omega}{\mathrm{N\_n}_b}] \tag{L12}$$

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-->  L3:[c_12=c_mn_ab_bulk, c_11 = ((%i*ω*w)/(N_m_a))*integrate(Δϵ
     ^a(y)*conjugate(E_mx_a(y))*E_nx_b(y) ,y) ];
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$$[c_{12} = \mathrm{c\_mn\_ab\_bulk}, c_{11} = \frac{\%iw \int \mathrm{E\_nx}_b(y)\overline{\mathrm{E\_mx}_a(y)}\ \epsilon^{\mathrm{a}(y)}\, dy\, \omega}{\mathrm{N\_m}_a}] \tag{L13}$$

–> L4:∼ [c_22 = c_nm_ba_bulk , c_22= ((%i*ω*w)/(N_n_b))*integrate($\Delta\epsilon$ ^b(y)*conjugate(E_nx_b(y))*E_mx_a(y),y)];

$$[c_{22} = \text{c\_nm\_ba\_bulk}, c_{22} = \frac{\%iw \int \text{E\_mx}_a(y)\overline{\text{E\_nx}_b(y)}\ \epsilon^{\text{b}(y)} dy\,\omega}{\text{N\_n}_b}] \quad \text{(L14)}$$

Prior information is needed in order to make use of L1 through L4. It is necessary to know the cross-sectional field profiles (E_mx_a(y), E_nx_b)) and the norms (N_n_a,N_n_b)∼ for the mth mode in waveguide a and the nth mode in waveguide b. The expressions for the fields and norms are given in sections 5.6.1 and 5.7.1 for reference.The unperturbed permittivity profiles â(y)∼and ϵb̂(y) shown in figures 7.2 (c) and (d) are symmetric relative to the centre (y = h +∼a and y = -(h+b)) in both waveguides. We can then apply the expressions for even modes of symmetric waveguides. Also see equations (5.7.6),(5.7.19),(5.7.20),(5.7.22).

1.2 TE_m mode of waveguide a

–> L5A:[E_mx_a(y)=E_m_a*cos(κ_m_a)*a*exp(-ζ_m_a*(y-(h+2*a))), y >= (h+2*a)];

$$\left[\text{E\_mx}_a(y) = \text{E\_m}_a a \% e^{-(y-h-2a)\zeta\_\text{m}_a} \cos\left(\kappa\_\text{m}_a\right), y \geq h + 2a\right] \quad \text{(L5A)}$$

–> L5B:[E_mx_a(y)=E_m_a*cos(κ_m_a)*(y-(h+a)), h <=y, y <= (h+2*a)];

$$[\text{E\_mx}_a(y) = \text{E\_m}_a\,(y - h - a)\cos\left(\kappa\_\text{m}_a\right), h \leq y\,, y \leq h + 2a] \quad \text{(L5B)}$$

–> L5C:[E_mx_a(y)=E_m_a*cos(κ_m_a)*exp(-ζ_m_a*(y-h)), y <= h];

–> L6:[N_m_a=w*((β_m_a*d_m_a)/(ω*μ_0)) *(E_m_a) ^2,d_m_a=2*(a+(1/(ζ_m_a))∼)];

$$[\text{N\_m}_a = \frac{\text{E\_m}_a{}^2 \text{d\_m}_a w \beta\_\text{m}_a}{\mu_0 \omega}, \text{d\_m}_a = 2\left(\frac{1}{\zeta\_\text{m}_a} + a\right)] \quad \text{(L6)}$$

–> L7:[κ_m_a=sqrt(ω ^2*ϵ_0*μ_0-(β_m_a) ^2),ζ_m_a=sqrt((β_m_a) ^2-ω ^2*ϵ_0*μ_0)];

$$[\kappa\_\text{m}_a = \sqrt{\epsilon_0\mu_0\omega^2 - \beta\_\text{m}_a{}^2}, \zeta\_\text{m}_a = \sqrt{\beta\_\text{m}_a{}^2 - \epsilon_0\mu_0\omega^2}] \quad \text{(L7)}$$

–> L8:[cos(κ_m_a*a=(κ_m_a/sqrt((κ_m_a ^2) +~ (ζ_m_a) ^2))) , cos(κ_m_a)*a=κ_m_a/(ω*sqrt((ϵ_a-ϵ_0)*μ_0) , sin(κ_m_a)*a=(ζ_m_a/sqrt((κ_m_a ^2) + (ζ_m_a) ^2))), sin(κ_m_a)*a=ζ_m_a/(ω*sqrt(ϵ_a-ϵ_0)*μ_0)];

$$\left[\cos\left(a\kappa\_\mathrm{m}_a\right)=\cos\left(\frac{\kappa\_\mathrm{m}_a}{\sqrt{\kappa\_\mathrm{m}_a{}^2+\zeta\_\mathrm{m}_a{}^2}}\right),a\cos\left(\kappa\_\mathrm{m}_a\right)=\left(\frac{\kappa\_\mathrm{m}_a}{a\sin\left(\kappa\_\mathrm{m}_a\right)}=\frac{\kappa\_\mathrm{m}_a\sqrt{\kappa\_\mathrm{m}_a{}^2+\zeta\_\mathrm{m}_a{}^2}}{\zeta\_\mathrm{m}_a}\right),a \tag{L8}$$

where L8 are a result of he dispersion relation (5.6.15)~for even modes. Replace i*κ_y2_μ → ζ_m_a and κ_y1 → κ_m_a in ζ_m_a =κ_m_a*tan(κ_m_a)*a

1.3 TEn Mode Of Waveguide b

–> L9A:[E_nx_b(y)=E_n_b*cos(κ_n_b)*b*exp(-ζ_n_b*(y+h)), y >= -h];

$$\left[\mathrm{E\_nx}_b(y)=\mathrm{E\_n}_b b\%e^{-(y+h)\zeta\_\mathrm{n}_b}\cos\left(\kappa\_\mathrm{n}_b\right),y\geq -h\right] \tag{L9A}$$

–> L9B:[E_nx_b(y)=E_n_b*cos(κ_n_b)*(y+(h+b)), -(h+2*b)~ <=y, y <=-h];

$$\left[\mathrm{E\_nx}_b(y)=\mathrm{E\_n}_b\left(y+h+b\right)\cos\left(\kappa\_\mathrm{n}_b\right),-h-2b\leq y,y\leq -h\right] \tag{L9B}$$

–> L9C:[E_nx_b(y)=E_n_b*cos(κ_n_b)*exp(abs(ζ_n_b)*y +~ (h+2*b)), y <= -(h+2*b)];

$$\left[\mathrm{E\_nx}_b(y)=\mathrm{E\_n}_b\%e^{y|\zeta\_\mathrm{n}_b|+h+2b}\cos\left(\kappa\_\mathrm{n}_b\right),y\leq -h-2b\right] \tag{L9C}$$

–> L10:[N_n_b=w*((β_n_b*d_n_b)/(ω*μ_0)) *L15:[c_22=-((i*ω*w)/N_n_b)*(integrate((ϵ_a-ϵ_0)*(E_n_b*cos(κ_n_b)*b) ^2 *exp(2*ζ_n_b*(y-h)),y,h,(h+2*a))+integrate((ϵ_a-ϵ_0)*(E_n_b*cos(κ_n_b)*b) ^2*exp(2*ζ_n_b*(y-h)),y,-h,h)) ,c_22=((%i*ω*w)/N_n_b)*(E_n_b) ^2*(cos(κ_n_b)*b) ^2*((ϵ_a-ϵ_0)*J_3+(ϵ_2-ϵ_0)*J_4),c_22=-%i*((κ_n_b) ^2/(β_n_b*d_n_b))*(((ϵ_a-ϵ_0)/(ϵ_b-ϵ_0))*((1-exp(-4*ζ_n_b*a))/(2*ζ_n_b))-((ϵ_2-ϵ_0)/(ϵ_b-ϵ_0))*((1-exp(-4*ζ_n_b*h))/(2*ζ_n_b))*exp(-4*ζ_n_b*h))] ;(E_n_b) ^2,d_b_n=2*(b+(1/(ζ_n_b)))];

$$\left[\mathrm{N\_n}_b=\frac{\mathrm{E\_n}_b{}^2\mathrm{d\_n}_b w\beta\_\mathrm{n}_b}{\mu_0\omega},\mathrm{d\_b}_n=2\left(\frac{1}{\zeta\_\mathrm{n}_b}+b\right)\right] \tag{L10}$$

−> L11:[κ_n_b=sqrt(ω ^2*ϵ_0*μ_0-(β_n_b) ^2),ζ_n_b=sqrt((β_n_b) ^2-ω ^2*ϵ_0*μ_0)];

$$\left[\kappa\_\mathrm{n}_b = \sqrt{\epsilon_0 \mu_0 \omega^2 - \beta\_\mathrm{n}_b{}^2}\,, \zeta\_\mathrm{n}_b = \sqrt{\beta\_\mathrm{n}_b{}^2 - \epsilon_0 \mu_0 \omega^2}\right] \tag{L11}$$

−> L12:[cos(κ_n_b*b=(κ_n_b/sqrt((κ_n_b ^2) + (ζ_n_b) ^2))) , cos(κ_n_b)*b=κ_n_b/(ω*sqrt((ϵ_b-ϵ_0)*μ_0) , sin(κ_n_b)*b=(ζ_n_b/sqrt((κ_n_b ^2) + (ζ_m_a) ^2))), sin(κ_n_b)*b=ζ_n_b/(ω*sqrt(ϵ_b-ϵ_0)*μ_0)];

$$\left[\cos\left(b\kappa\_\mathrm{n}_b\right) = \cos\left(\frac{\kappa\_\mathrm{n}_b}{\sqrt{\kappa\_\mathrm{n}_b{}^2 + \zeta\_\mathrm{n}_b{}^2}}\right), b\cos\left(\kappa\_\mathrm{n}_b\right) = \left(\frac{\kappa\_\mathrm{n}_b}{b\sin\left(\kappa\_\mathrm{n}_b\right)} = \frac{\kappa\_\mathrm{n}_b\sqrt{\kappa\_\mathrm{n}_b{}^2 + \zeta\_\mathrm{m}_a{}^2}}{\zeta\_\mathrm{n}_b}\right), b\sin\left(\kappa\_\right. \tag{L12}$$

$$\frac{\zeta\_\mathrm{n}_b}{\sqrt{\epsilon_b - \epsilon_0}\mu_0\omega}$$

where L12 are a result of he dispersion relation (5.6.15)~for even modes. where L8 are a result of he dispersion relation (5.6.15)~for even modes. Replace i*κ_y2_μ $\rightarrow$ ζ_n_b and κ_y1 $\rightarrow$ κ_n_b in ζ_m_a =κ_m_a*tan(κ_m_a)*a. Replace i*κ_y2_μ $\rightarrow$ ζ_m_a and κ_y1 $\rightarrow$ κ_m_a in ζ_n_b =κ_n_b*tan(κ_n_b)*b

2 Self-Coupling Coefficients

(% i27) L14:[c_11=-((i*ω*w)/N_m_a)*('integrate((ϵ_b-ϵ_0)*(E_m_a*cos(κ_m_a)*a) ^2 *exp(2*ζ_m_a*(y-h)),y,-(h+2*b),-h))+ 'integrate((ϵ_2-ϵ_0)*(E_m_a*cos(κ_m_a)*a) ^2*exp(2*ζ_m_a*(y-h)),y,-h,h) ,c1_11=((%i*ω*w)/N_m_a)*(E_m_a) ^2*(cos(κ_m_a)*a) ^2,((_b-ϵ_0)*J_1+(ϵ_2-ϵ_0)*J_2),c_11=-%i*((κ_m_a) ^2/(β_m_a*d_m_a))*(((ϵ_b-ϵ_0)/(ϵ_a-ϵ_0))*((1-exp(-4*ζ_m_a*b))/(2*ζ_m_a))-((ϵ_2-ϵ_0)/(ϵ_a-ϵ_0))*((1-exp(-4*ζ_m_a*h))/(2*ζ_m_a))*exp(-4*ζ_m_a*h))] ;

$$\left[c_{11} = \mathrm{E\_m}_a{}^2 a^2 \left(\epsilon_2 - \epsilon_0\right) \int_{-h}^{h} \%e^{2(y-h)\zeta\_\mathrm{m}_a} dy \cos\left(\kappa\_\mathrm{m}_a\right)^2 - \frac{\mathrm{E\_m}_a{}^2 a^2 i w \left(\epsilon_b - \epsilon_0\right) \int_{-h-2b}^{-h} \%e^{2(y-h)\zeta\_\mathrm{m}_a} dy \cos}{\mathrm{N\_m}_a}\right. \tag{L14}$$

(% i28)

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L15:[c_22=-((i*ω*w)/N_n_b)*( integrate((ϵ_a-ϵ_0)*(E_n_b*cos(κ_n_b)*b) ^2 *exp(2*ζ_n_b*(y-h)),y,h,(h+2*a))+integrate((ϵ_a-ϵ_0)*(E_n_b*cos(κ_n_b)*b)
^2*exp(2*ζ_n_b*(y-h)),y,-h,h)) ,c_22=((%i*ω*w)/N_n_b)*(E_n_b)
^2*(cos(κ_n_b)*b) ^2*((ϵ_a-ϵ_0)*J_3+(ϵ_2-ϵ_0)*J_4),c_22=-%i*((κ_n_b) ^2/(β_n_b*d_n_b))*(((ϵ_a-ϵ_0)/(ϵ_b-ϵ_0))*((1-exp(-4*ζ_n_b*a))/(2*ζ_n_b))-((ϵ_2-ϵ_0)/(ϵ_b-ϵ_0))*((1-exp(-4*ζ_n_b*h))/(2*ζ_n_b))*exp(-4*ζ_n_b*h)) ] ;
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$$[\, c_{22} = -\frac{iw\left(\mathrm{E\_n}_b{}^2 b^2\left(\epsilon_a-\epsilon_0\right)\left(\frac{1}{2\zeta\_\mathrm{n}_b}-\frac{\%e^{-4h\zeta\_\mathrm{n}_b}}{2\zeta\_\mathrm{n}_b}\right)\cos\left(\kappa\_\mathrm{n}_b\right)^2+\mathrm{E\_n}_b{}^2 b^2\left(\epsilon_a-\epsilon_0\right)\left(\frac{\%e^{4a\zeta\_\mathrm{n}_b}}{2\zeta\_\mathrm{n}_b}-\frac{1}{2\zeta\_\mathrm{n}_b}\right)\cos\left(\kappa \cdots\right.\right.}{\mathrm{N\_n}_b} \tag{L15}$$

(% i29)

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L16:[c_12=-((i*ω*w)/N_m_a)*integrate((ϵ_b-ϵ_0)*(E_m_a*cos(κ_m_a)*a)
*exp(ζ_m_a*(y-h))*E_n_b*cos(κ_n_b)*(y+(h+b)) ,y,-(h+2*b),-h) + integrate((ϵ_2-ϵ_0)*(E_m_a*cos(κ_m_a*a))*exp(ζ_m_a*(y-h))*(E_n_b*cos(κ_n_b*b))*exp(-ζ_n_b*(y+h)),y,-h,h),c_22=((%i*ω*w)/N_m_a)*(E_m_a*E_n_b)
^2*(cos(κ_m_a)*a)*((ϵ_b-ϵ_0)*J_5+(ϵ_2-ϵ_0)*cos(κ_n_b)*b*J_6),c_22=-%i*((κ_m_a*κ_n_b)/sqrt(β_m_a*d_m_a*β_n_b*d_n_b))*(sqrt((ϵ_b-ϵ_0)/(ϵ_a-ϵ_0))*(((1-exp( -2*ζ_m_a*b))*ζ_m_a~ +~ (1+exp(-2*ζ_m_a*b))*ζ_m_a)/((ζ_m_a) ^2+(κ_n_b) ^2))-((ϵ_2-ϵ_0)/(sqrt((ϵ_a-ϵ_0)*(ϵ_b-ϵ_0)))) *~ ((1-exp(2*(ζ_m_a-ζ_n_b)*h)/(ζ_m_a-ζ_n_b))))
*exp(-2*ζ_m_a*h) ] ;
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$$[\, c_{12} = \mathrm{E\_m}_a \mathrm{E\_n}_b\left(\epsilon_2-\epsilon_0\right)\left(\frac{1}{\%e^{2h\zeta\_\mathrm{m}_a}\zeta\_\mathrm{n}_b-\zeta\_\mathrm{m}_a\%e^{2h\zeta\_\mathrm{m}_a}}-\frac{\%e^{-2h\zeta\_\mathrm{n}_b}}{\zeta\_\mathrm{n}_b-\zeta\_\mathrm{m}_a}\right)\cos\left(a\kappa\_\mathrm{m}_a\right)\cos\left(b\kappa\_\mathrm{n}_b\right) - \cdots \tag{L16}$$

L17 IS NOT CORRECT YET - EDIT TO MATCH THE TEXT

(% i30) L17:[c__21=-((i*ω*w)/N__n__b)*(integrate((ϵ__a-ϵ__0)*(E__n__b*cos(κ__n__b)*b) ^2 *exp(2*ζ__n__b*(y-h)),y,h,(h+2*a))+integrate((ϵ__a-ϵ__0)*(E__n__b*cos(κ__n__b)*b) ^2*exp(2*ζ__n__b*(y-h)),y,-h,h)) ,c__21=((%i*ω*w)/N__n__b)*(E__n__b) ^2*(cos(κ__n__b)*b) ^2*((ϵ__a-ϵ__0)*J__3+(ϵ__2-ϵ__0)*J__4),c__21=-%i*((κ__n__b) ^2/(β__n__b*d__n__b))*(((ϵ__a-ϵ__0)/(ϵ__b-ϵ__0))*((1-exp(-4*ζ__n__b*a))/(2*ζ__n__b))-((ϵ__2-ϵ__0)/(ϵ__b-ϵ__0))*((1-exp(-4*ζ__n__b*h))/(2*ζ__n__b))*exp(-4*ζ__n__b*h))] ;

$$[\, c_{21} = -\frac{iw\,\left(\mathrm{E\_n}_b{}^2 b^2\left(\epsilon_a-\epsilon_0\right)\left(\frac{1}{2\zeta\_\mathrm{n}_b}-\frac{\%e^{-4h\zeta\_\mathrm{n}_b}}{2\zeta\_\mathrm{n}_b}\right)\cos\left(\kappa\_\mathrm{n}_b\right)^2+\mathrm{E\_n}_b{}^2 b^2\left(\epsilon_a-\epsilon_0\right)\left(\frac{\%e^{4a\zeta\_\mathrm{n}_b}}{2\zeta\_\mathrm{n}_b}-\frac{1}{2\zeta\_\mathrm{n}_b}\right)\cos\left(\kappa\right.}{\mathrm{N\_n}_b} \tag{L17}$$