CMT Chapter 4Generalized Theory of Mode Excitation For Space-Dispersive Media Waveguides

1 Modal Expansion Fields With Separating Potential Fields

Any eigenmode basis for any waveguide structure is incomplete inside the region of any sources of EM excitation. Classic CMT ignores this fact.The expansion of the modal fields must be supplemented with orthogonal complementary fields Eb and Hb. The complentary fields are longitudinal fields, related to the longitudinal components of the bulk exciting currents Jbzm and Jbze. [XREF: 3.4.24, 3.4.25]

(% i2)
```
kill(all);
load(vect);
load(eigen);
```

done (% o0)

(% o1)

"/usr/share/maxima/5.45.1/share/vector/vect.mac"

(% o2)

"/usr/share/maxima/5.45.1/share/matrix/eigen.mac"

(% i3)
```
cmt_ch4_101:[E[a]=sum(A[k]*E[k],k,-N,N-1 ),H[a]=sum(A[k]*H[k],k,-N,N-1 )
];
```

$$\left[E_a = \sum_{k=-N}^{N-1} A_k E_k \; , H_a = \sum_{k=-N}^{N-1} A_k H_k\right] \qquad (\% \text{ o3})$$

An unknown longitudinal dependence of Ak(z) in the 2 equations below is due to the external sources and the orthognal complementary fields Eb and Hb.The complete electromagnetic fields inside the source region have the following form: [XREF: 3.4.9, 3.4.10]

(% i4)
```
cmt_ch4_421:[E(r[t],z)=E[a](r[t],z)+E[b](r[t],z), E(r[t],z)=sum(A[k](z)*E[k](r[t],z)+E[b](r[t],z),k,-
N,N-1) ];
```

$$\left[\mathrm{E}\left(r_t\,, z\right) = E_b\left(r_t\,, z\right) + E_a\left(r_t\,, z\right), \mathrm{E}\left(r_t\,, z\right) = \sum_{k=-N}^{N-1} E_k\left(r_t\,, z\right) A_k(z) + E_b\left(r_t\,, z\right)\right]$$

(% o4)

(% i5) cmt_ch4_422:[H(r[t],z)=H[a](r[t],z)+H[b](r[t],z), H(r[t],z)=sum(A[k](z)*H[k](r[t],z)+H[b](r[t],z),k,-N,N-1)];

$$\left[\mathrm{H}\left(r_t\,,z\right)=H_b\left(r_t\,,z\right)+H_a\left(r_t\,,z\right),\mathrm{H}\left(r_t\,,z\right)=\sum_{k=-N}^{N-1}H_k\left(r_t\,,z\right)A_k(z)+H_b\left(r_t\,,z\right)\right]$$

(% o5)

Total fields H and E can be represented asthe sum of their curl (Ec,Hc) and potential (Ep,Hp) components, based on the Helmholtz decomposition theorem.

(% i6) cmt_ch4_422_A:[Ep=-grad(phi),Hp=-grad(psi)];

$$[\mathrm{Ep}=-\,\mathrm{grad}(phi)\,,\mathrm{Hp}=-\,\mathrm{grad}(psi)]$$

(% o6)

Then the eigenfields of every kth mode can be represented as:

(% i7) cmt_ch4_423:[div(Ec[k])=0, div(Hc[k])=0,E[k]=Ec[k]+Ep[k],E[k]=Ec[k]-grad(phi[k]),H[k]=Hc[k]+Hp[k],H[k]=Hc[k]-grad(psi[k])];

$$[\mathrm{div}\left(\mathrm{Ec}_k\right)=0\,,\mathrm{div}\left(\mathrm{Hc}_k\right)=0\,,E_k=\mathrm{Ep}_k+\mathrm{Ec}_k\,,E_k=\mathrm{Ec}_k-\mathrm{grad}\left(phi_k\right),H_k=\mathrm{Hp}_k+\mathrm{Hc}_k\,,H_k=\mathrm{Hc}_k-\mathrm{grad}\,($$

(% o7)

Now two sets of total eigenfields form the basis of a resulting Hilbert space, instead of just the (E[k],H[k]) eigenfields. The second setconsists of the quasi-static eigenpotentials (psi[k] and phi[k]) and the curl eigenfields (Ec[k],Hc[k]). This increases the numberof dimensions of the Hilbert space. This makes it possible to expand the complementary fields Eb and Hb in terms of the thescalar basis potentials (psi[k], phi[k]).Apply the vector curl-field basis (Ec[k],Hc[k]) to expand the curl fields

(% i8) cmt_ch4_424:[Ec(r[t],z)=sum(A[k](z)*Ec[k](r[t],z),k,-N,N-1), Hc(r[t],z)=sum(A[k](z)*Hc[k](r[t],z),k,-N,N-1)];

$$\left[\mathrm{Ec}\left(r_t\,,z\right)=\sum_{k=-N}^{N-1}\mathrm{Ec}_k\left(r_t\,,z\right)A_k(z)\,,\mathrm{Hc}\left(r_t\,,z\right)=\sum_{k=-N}^{N-1}\mathrm{Hc}_k\left(r_t\,,z\right)A_k(z)\right]$$

(% o8)

Apply the scalar-potential basis (phi(k),psi(k)) to expand the quasi-static potentialsHave to use fpsi and fphi because psi is part of the Gamma Functions package.

(%i9) cmt_ch4_425:[fphi(r[t],z)=sum(A[k](z)*fphi[k](r[t],z),k,-N,N-1), fpsi(r[t],z)=sum(A[k](z)*fpsi[k](r[t],z),k,-N,N-1)];

$$\left[\mathrm{fphi}\left(r_t\,,z\right)=\sum_{k=-N}^{N-1}\mathrm{fphi}_k\left(r_t\,,z\right)A_k(z)\,,\mathrm{fpsi}\left(r_t\,,z\right)=\sum_{k=-N}^{N-1}\mathrm{fpsi}_k\left(r_t\,,z\right)A_k(z)\right] \tag{\%o9}$$

In this case the expressions for complete fields inside the source region have the following form:

(%i10) cmt_ch4_426:[E=Ec-express(grad(phi)), E=sum(A[k]*(Ec[k]-grad(phi[k])) , k,-N,N-1) -e[z]*sum('diff(A[k],z)*phi[k],k,-N,N-1), E=sum(A[k]*E[k],k,-N,N-1)-e[z]*sum('diff(A[k],z),k,-N,N-1)*psi[k]];

$$[\,E=\left[\mathrm{Ec}-\frac{d}{dx}phi\,,\mathrm{Ec}-\frac{d}{dy}phi\,,\mathrm{Ec}-\frac{d}{dz}phi\right],E=\left(\sum_{k=-N}^{N-1}A_k\left(\mathrm{Ec}_k-\mathrm{grad}\left(phi_k\right)\right)\right)-\left(\sum_{k=-N}^{N-1}phi_k\left(\frac{d}{dz}A_k\right)\right) \tag{\%o10}$$

(%i11) cmt_ch4_427:[H=Hc-express(grad(psi)) ,H=sum(A[k]*(Hc[k]-grad(psi[k])) , k,-N,N-1) -e[z]*sum('diff(A[k],z)*psi[k],k,-N,N-1), H=sum(A[k]*H[k],k,-N,N-1)-e[z]*sum('diff(A[k],z),k,-N,N-1)*psi[k]];

$$[\,H=\left[\mathrm{Hc}-\frac{d}{dx}psi\,,\mathrm{Hc}-\frac{d}{dy}psi\,,\mathrm{Hc}-\frac{d}{dz}psi\right],H=\left(\sum_{k=-N}^{N-1}A_k\left(\mathrm{Hc}_k-\mathrm{grad}\left(psi_k\right)\right)\right)-\left(\sum_{k=-N}^{N-1}psi_k\left(\frac{d}{dz}A_k\right)\right) \tag{\%o11}$$

(%i12) cmt_ch4_428:[Eb=-e[z]*sum('diff(A[k],z)*phi[k],k,-N,N-1)];

$$\left[\mathrm{Eb}=-\left(\sum_{k=-N}^{N-1}phi_k\left(\frac{d}{dz}A_k\right)\right)e_z\right] \tag{\%o12}$$

(%i13) cmt_ch4_429:[Hb=-e[z]*sum('diff(A[k],z)*psi[k],k,-N,N-1)];

$$\left[\mathrm{Hb}=-\left(\sum_{k=-N}^{N-1}psi_k\left(\frac{d}{dz}A_k\right)\right)e_z\right] \tag{\%o13}$$

A[k] is replaced by ∂A[k]/∂z, which vanishes outside the source region. Outside the source region, A[k] is constant, making it's derivative zero.The Hilbert space spanned by the 2 sets of basis functions is closed w.r.t. any function that corresponds to any external source.The expressions for the curl fields and the quasi-static potentials do not contain orthogonal complements.Then if the potential fields Eb and Hb are excluded, we can build a eigenmode basis that produces a modal expansion with no orthogonal complements.Use the scalar potentials instead of the potential field expressions, together with the curl fields Ec and Hc.The appropriate sets of mode quantities phi[k] , psi[k] will then be a complete basis with no orthogonal complements.This does not apply to the polarization vector and magnetic field vector.Equations of motion and constitutive relations (tensors) contain the complete fields, not the curl and quasi-static parts.Polarization P and Magnetization M are derived from the tensors and equations of motionTherefore P and M are required to have orthogonal complements Pb, Mb generated by Eb and Hb.fpsi and fphi are maxima placeholders for psi and phi Total fields are given here:

(% i14) cmt_ch4_429_A:[E=Ec-grad(fphi),E=Ea+Eb, H=Hc-grad(fpsi), H=Ha+Hb, P=Pa+Pb, M=Ma+Mb, Ecb=0, Hcb=0, fphi_b=0, fpsi_b=0];

$$[\,E = \mathrm{Ec}-\mathrm{grad}\,(\mathrm{fphi})\,, E = \mathrm{Eb}+\mathrm{Ea}\,, H = \mathrm{Hc}-\mathrm{grad}\,(\mathrm{fpsi})\,, H = \mathrm{Hb}+\mathrm{Ha}\,, P = \mathrm{Pb}+\mathrm{Pa}\,, M = \mathrm{Mb}+\mathrm{Ma}\,, \mathrm{Ecb} = 0\,, \quad (\%\ o14)$$

The Quasi-Static Approximation x E ≈ 0 and E ≈ Ep = - ϕ, but H = Hc beause . H = 0 and x H ≠ 0For this purpose I will introduce a new assertion using function notation.approx(e1, e2, [tol]) means e1 is close to e2 but may vary from e2 by tol (tolerance)Maxima doesn't recognize the wxmaxima approximation symbol, so I defined something that works.

(% i15) approx(curl(E),0);

$$\mathrm{approx}\,(\mathrm{curl}(E)\,, 0) \quad (\%\ o15)$$

The Quasi-Magnetostatic ApproximationUseful for MSW in magnetized Ferrites x H ≈ 0 and H ≈ Hp = - ψ but E=Ec because . E = 0 and x E ≠ 0The consequence is that there are slow waves due to the existence of a specific potential field.The curl component (fast waves) is eliminated from the approximation expression.The basic task of Coupled Mode Theory is to find the modal excitation amplitudes Ak(z) inside the region of influence of external sources.The external sources are assumed to be given when starting the first stage of the analysis. 4.3 Constitutive Relations and Dynamic Equations For Space-Dispersive Active Media4.3.1 Piezoelastic Properties of a Medium

(% i16) cmt_ch04_431:[S_ij = (1/2) *('diff(μ_i, r_j) + 'diff(μ_j,r_i))];

$$\left[S_{\mathrm{ij}} = \frac{\frac{d}{d r_i}\mu_j + \frac{d}{d r_j}\mu_i}{2}\right] \quad (\%\ o16)$$

(% i18) T_bar:[T_x, T_y, T_z];T_bar_Σ:[T_Σ_x,T_Σ_y,T_Σ_z];

$$[T_x\,, T_y\,, T_z] \quad (\%\ o17)$$

$$[\mathrm{T\_}_x, \mathrm{T\_}_y, \mathrm{T\_}_z] \quad (\%\ o18)$$

(% i19) cmt_ch04_432: [p_m * 'diff(U_i ,t) = 'diff(T_ij,r_j), p_m * 'diff(U,t) = express(div(T_bar))];

$$\left[\left(\frac{d}{dt}U_i\right)p_m = \frac{d}{dr_j}T_{\mathrm{ij}}, \left(\frac{d}{dt}U\right)p_m = \frac{d}{dz}T_z + \frac{d}{dy}T_y + \frac{d}{dx}T_x\right] \quad (\%\ o19)$$

Values and Tensors Used For Piezoelectric AnalysisTotal Stress Tensor T_bar[Σ] = T_bar + T_bar_fr2nd rank stress tensor T_bar2nd rank susceptibility X_bar[S]2nd rank permittivity ϵ_bar[S]3rd rank piezoelectric stress e_bar_bar4th rank elastic stiffness c_bar_bar[E]internal friction stress T_bar[fr]viscosity tensor η_barinverse relaxation time τ(-1)

(% i20) cmt_ch04_432_A:[T_bar_Σ,X_bar[s] , ϵ_bar[s] = ϵ_0 * (I_bar + X_bar[s]) , e_bar_bar, c_bar_bar[E],T_bar_fr] ;

$$[[\mathrm{T\_}_x, \mathrm{T\_}_y, \mathrm{T\_}_z], \mathrm{X\_bar}_s, \epsilon\mathrm{\_bar}_s = (\mathrm{X\_bar}_s + I_{\mathrm{bar}})\,\epsilon_0, \mathrm{e\_bar\_bar}, \mathrm{c\_bar\_bar}_E, \mathrm{T\_bar\_fr}] \quad (\%\ o20)$$

(% i21) cmt_ch04_433:[P[k] = e[k,i,j] * S[i,j] + ϵ_0 *X[i,k,s] *E[i], P = e_bar_bar . S_bar + ϵ_0 * X_bar[s] . E];

$$[P_k = E_i X_{i,k,s}\epsilon_0 + S_{i,j}e_{k,i,j}, P = (\mathrm{X\_bar}_s.E)\,\epsilon_0 + \mathrm{e\_bar\_bar}.S_{\mathrm{bar}}] \quad (\%\ o21)$$

(% i22) cmt_ch04_434:[T[i,j] = c[i,j,k,l,E] * S[i,j] + ϵ[i,k] * E[i] , T_bar = c_bar_bar[E] . S_bar + e_bar_bar . E];

$$[T_{i,j} = E_i\epsilon_{i,k} + S_{i,j}c_{i,j,k,l,E}, [T_x, T_y, T_z] = \mathrm{e\_bar\_bar}.E + \mathrm{c\_bar\_bar}_E.S_{\mathrm{bar}}] \quad (\%\ o22)$$

(% i23) cmt_ch04_435:[D[k] = e[ik,i,j] * S[i,j] + ϵ[i,k,s] * E[i], D = e_bar_bar . S_bar + ϵ_bar[s] . E];

$$[D_k = S_{i,j}e_{\mathrm{ik},i,j} + E_i\epsilon_{i,k,s}, D = \epsilon\mathrm{\_bar}_s.E + \mathrm{e\_bar\_bar}.S_{\mathrm{bar}}] \quad (\%\ o23)$$

(% i24) cmt_ch04_436:[T_fr[i,j] =η_[i,j,k,l] * 'diff(S[k,l] , t), T_bar_fr = η_bar_bar . S_bar_dot];

$$\left[T_{\mathrm{fr}\,i,j} = \eta_{\_i,j,k,l}\left(\frac{d}{dt}S_{k,l}\right), \mathrm{T\_bar\_fr} = \eta\mathrm{\_bar\_bar.S\_bar\_dot}\right] \quad (\%\ o24)$$

(%i25) cmt_ch04_437:[F_fr[i] = -τ[i,j]^-1 * ρ_m * U[i], F_fr = -τ^-1 . ρ_m * U];

$$\left[F_{\mathrm{fr}\,i} = -\frac{U_i \rho_m}{\tau_{i,j}} \,, F_{\mathrm{fr}} = -U \left(\frac{1}{\tau} . \rho_m \right) \right] \quad (\%o25)$$

Allowing for (4.3.6) and (4.3.7) ,re-write (4.3.1) and (4.3.2) as

(%i26) cmt_ch04_438:['diff(S[i,j] ,t) = (1/2) * ('diff(U[i],r[j]) + 'diff(U[j],r[i]))];

$$\left[\frac{d}{dt} S_{i,j} = \frac{\frac{d}{dr_i} U_j + \frac{d}{dr_j} U_i}{2} \right] \quad (\%o26)$$

(%i27) cmt_ch04_439:[ρ_m * 'diff(U,t) = express(div(T_bar_Σ)) + F_fr];

$$\left[\left(\frac{d}{dt} U \right) \rho_m = \frac{d}{dz} \mathrm{T\_}_z + \frac{d}{dy} \mathrm{T\_}_y + \frac{d}{dx} \mathrm{T\_}_x + F_{\mathrm{fr}} \right] \quad (\%o27)$$

For Pure Harmonic Processes

(%i28) cmt_ch04_4310:[U = U[1], U[1] = 'diff(u[1], t) , ρ_m =ρ_m[0] + ρ_m[1]];

$$\left[U = U_1 \,, U_1 = \frac{d}{dt} u_1 \,, \rho_m = \rho_{m\,1} + \rho_{m\,0} \right] \quad (\%o28)$$

4.3.2 Ferrimagnetic Properties of a MediumExternal static magnetic field H[0,e]Saturation Magnetization M[0]Total magnetization vector MEffective Magnetic Field H_effExternal DC Field H_0_eMaxwellian Field HCrystal anisotropy H_c = -N_bar[c] . MDemagnetizing Field H[d] = N_bar[d] . MExchange Field H[ex] = λ[ez] * 2 MNet Tensor N_bar = N_bar[c] + N_bar[d]Relaxation Term R

(%i30) H_eff:[Heff_x, Heff_y, Heff_z];M:[M_x, M_y, M_z];

$$[\mathrm{Heff}_x \,, \mathrm{Heff}_y \,, \mathrm{Heff}_z] \quad (\%o29)$$

$$[M_x \,, M_y \,, M_z] \quad (\%o30)$$

(%i31) cmt_ch04_4311:[γ = abs(e) / m[0],'diff(M, t) = -γ*μ_0 * (express(M ~ H_eff)) + R];

$$[\gamma = \frac{|e|}{m_0} \,, \frac{d}{dt} [M_x \,, M_y \,, M_z] = [R - (\mathrm{Heff}_z M_y - \mathrm{Heff}_y M_z)\, \gamma \mu_0 \,, R - (\mathrm{Heff}_x M_z - \mathrm{Heff}_z M_x)\, \gamma \mu_0 \,, R - (\mathrm{Heff}_y M_x -$$

(%o31)

(% i32) cmt_ch04_4312:[H_eff = H[0,e] + H - N_bar . M + λ[ez] * express(laplacian(M))];

$$[[\mathrm{Heff}_x, \mathrm{Heff}_y, \mathrm{Heff}_z] = \left(\frac{d^2}{dz^2}[M_x, M_y, M_z] + \frac{d^2}{dy^2}[M_x, M_y, M_z] + \frac{d^2}{dx^2}[M_x, M_y, M_z]\right)\lambda_{\mathrm{ez}} - N_{\mathrm{bar}}.[M_x, M_y, M \tag{\% o32}$$

(% i33) cmt_ch04_4313:[α=ΔH/H[0],R = α*((M/M_0) + 'diff(M,t))];

$$\left[\alpha = \frac{\mathrm{H}}{H_0}, R = \left(\frac{d}{dt}[M_x, M_y, M_z] + \left[\frac{M_x}{M_0}, \frac{M_y}{M_0}, \frac{M_z}{M_0}\right]\right)\alpha\right] \tag{\% o33}$$

(% i34) cmt_ch04_4314:[M = M_0 + M_1, abs(M_1) < abs(M_0)];

$$[[M_x, M_y, M_z] = M_1 + M_0, |M_1| < |M_0|] \tag{\% o34}$$

(% i35) cmt_ch04_4315:[H_0 = H[0,e] - N_bar . M_0, H_eff = H_0 + H_1 - N_bar . M_1 + λ[ez] * express(laplacian(M_1))];

$$\left[H_0 = H_{0,e} - \mathrm{N\_bar}.M_0, [\mathrm{Heff}_x, \mathrm{Heff}_y, \mathrm{Heff}_z] = \left(\frac{d^2}{dz^2}M_1 + \frac{d^2}{dy^2}M_1 + \frac{d^2}{dx^2}M_1\right)\lambda_{\mathrm{ez}} - N_{\mathrm{bar}}.M_1 + H_1 + H_0\right] \tag{\% o35}$$

(4.3.3) Drifting Charge Carriers in a Medium (Plasmas)Hydrodynamic Force EquationEffective electron mass mFree electron mass m_0τ_e energy relaxation timeT_M = ϵ/σ = ϵ*m/e2*n*τ Maxwellian relaxation timeτ_e determines rate of electron perturbationsT_M determines time scale of signal changes in the electric field and charge distributionτ_e $\ll$ T_MThis condition means the temperature keeps pace with signal changes in the electric fieldThis provides a local relationship between T and E.This allows the momentum relaxation time τ to be considered a function of EE is found from measuring the field dependence ofmobility μ(E) = (e/m)*τ(E)diffusion D(E) = v_T2 *τ(E)

(% i41) B:[B_x,B_y,B_z];v:[v_x, v_y, v_z];r_1:[r1_x, r1_y, r1_z]; r_0:[r0_x, r0_y, r0_z];
J_b1_e:[J_b1e_x, J_b1e_y, J_b1e_z]; J_b1_m:[J_b1m_x, J_b1m_y, J_b1m_z];

$$[B_x, B_y, B_z] \tag{\% o36}$$

$$[v_x, v_y, v_z] \tag{\% o37}$$

$$[\mathrm{r1}_x\,,\mathrm{r1}_y\,,\mathrm{r1}_z] \tag{\% o38}$$

$$[\mathrm{r0}_x\,,\mathrm{r0}_y\,,\mathrm{r0}_z] \tag{\% o39}$$

$$\left[\mathrm{J\_b1e}_x\,,\mathrm{J\_b1e}_y\,,\mathrm{J\_b1e}_z\right] \tag{\% o40}$$

$$\left[\mathrm{J\_b1m}_x\,,\mathrm{J\_b1m}_y\,,\mathrm{J\_b1m}_z\right] \tag{\% o41}$$

(% i42) cmt_ch04_4316:['diff(v,t) + (v . express(div(v))) = (e/m) * (E + express(v ~ B) - express(grad((n*k[B] * T)/(m*n))))- (v/τ)];

$$[\,[v_x\,,v_y\,,v_z]\,.\left(\frac{d}{dz}v_z+\frac{d}{dy}v_y+\frac{d}{dx}v_x\right)+\frac{d}{dt}\,[v_x\,,v_y\,,v_z]=[\,\frac{e\left(-B_y v_z+B_z v_y-\frac{d}{dx}\frac{k_{[B_x\,,\,B_y\,,\,B_z]}T}{m}+E\right)}{m}-\frac{v_x}{\tau}\,,\,\frac{e\left(\right.}{} \tag{\% o42}$$

(% i43) cmt_ch04_4316_A:[express(grad(p)) = (m*v_T^2)*express(grad(n))];

$$\left[\left[\frac{d}{dx}p\,,\frac{d}{dy}p\,,\frac{d}{dz}p\right]=\left[m\left(\frac{d}{dx}n\right)v_T{}^2\,,m\left(\frac{d}{dy}n\right)v_T{}^2\,,m\left(\frac{d}{dz}n\right)v_T{}^2\right]\right] \tag{\% o43}$$

(% i44) μ(E):= (e/m)*τ(E);

$$\mu(E):=\frac{e}{m}\,\tau(E) \tag{\% o44}$$

(% i45) D(E) := v_T^2 *τ(E);

$$\mathrm{D}(E):=v_T{}^2\,\tau(E) \tag{\% o45}$$

For Plasmas in a magnetic field B the electron heating is the result of anelectric field called the effective heating field (Appendix D.5)b takes into account an influence of the magnetic fields on the heating effectτ,μ, D now depend on E_h

(%i46) cmt_ch04_4317:[b = μ*B,E_h = sqrt(E^2 + ((b_vec. E)^2)/(1+b^2))] ;

$$\left[b=\left[B_x \mu\,,B_y \mu\,,B_z \mu\right],E_h=\sqrt{\frac{\left(b_{\mathrm{vec}}.E\right)^2}{b^2+1}+E^2}\right] \tag{\%o46}$$

Small Signal Analysis: All signal values with subscript 1 << those with subscript 0

(%i47) cmt_ch04_4317_B:[E=E_0 + E_1, B= B_0+B_1, E_h = E_h0 + E_h1, τ(E_h) = τ(E_h0) + 'diff(τ,E), E_h1= τ_0 + τ_1, τ_0 = τ(E_h0)];

$$\left[E=E_1+E_0\,,\left[B_x\,,B_y\,,B_z\right]=B_1+B_0\,,E_h=E_{\mathrm{h1}}+E_{\mathrm{h0}}\,,\tau\left(E_h\right)=\frac{d}{dE}\tau+\tau\left(E_{\mathrm{h0}}\right),E_{\mathrm{h1}}=\tau_1+\tau_0\,,\tau_0=\tau\left(E_{\mathrm{h0}}\right)\right] \tag{\%o47}$$

(%i48) cmt_ch04_4317_C:[(τ-1/τ_0) = 'diff(log(τ), log(E)) *(E_h1/E_h0), τ_1/τ_0 =(κ_0-1)*(E_h1/E_h0)];

$$\left[\tau-\frac{1}{\tau_0}=\frac{E_{\mathrm{h1}}\left(\frac{d}{d\log\left(E\right)}\log\left(\tau\right)\right)}{E_{\mathrm{h0}}}\,,\frac{\tau_1}{\tau_0}=\frac{E_{\mathrm{h1}}\left(\kappa_0-1\right)}{E_{\mathrm{h0}}}\right] \tag{\%o48}$$

(%i49) cmt_ch04_4318:[τ_1/τ_0 = (κ_0-1) * (F_0/E_0) . ((E_1 + v_0~ B_1)/E_0)];

$$\left[\frac{\tau_1}{\tau_0}=\left(\frac{F_0}{E_0}.\frac{E_1-B_1\sim v_0}{E_0}\right)\left(\kappa_0-1\right)\right] \tag{\%o49}$$

(%i50) cmt_ch04_4319:[b_0 = μ_e*B_0, F_0= ((1+ b_0^2) * (E_0 + (b_0 . E_0) * b_0))/ ((1+κ_0*b_0^2) + ((1+b_0^2) + (1-κ_0)) * (b_0 . E_0)^2/E_0 ^2)];

$$\left[b_0=B_0\mu_e\,,F_0=\frac{\left({b_0}^2+1\right)\left(b_0\left(b_0.E_0\right)+E_0\right)}{{b_0}^2\kappa_0+\frac{\left(b_0.E_0\right)^2\left(-\kappa_0+{b_0}^2+2\right)}{{E_0}^2}+1}\right] \tag{\%o50}$$

(%i51) cmt_ch04_4320:[E=E[k,0], μ_d = 'diff(μ(E)*E,E) , μ_e = μ(E[h,0]), μ_e = (e/m)*τ(E[h,0]), μ_e = (e/m)*τ_0];

$$\left[E=E_{k,0}\,,\mu_d=\frac{d}{dE}\frac{E\,\tau(E)e}{m}\,,\mu_e=\frac{e\,\tau\left(E_{h,0}\right)}{m}\,,\mu_e=\frac{e\,\tau\left(E_{h,0}\right)}{m}\,,\mu_e=\frac{e\tau_0}{m}\right] \tag{\%o51}$$

Electron displacement vector, a function of the unperturbed position vector r_0The trajectory of liquid (virtual) particle motion is replaced by the E field description.Now deal with vector field of electron displacement r_1(r_0,t)r_1 is identical to the field of the lattice particle displacement μ(r_0,t)μ(r_0,t) is a term from elasticity theory

(% i52) cmt_ch04_4321:[r_1(r_0,t) = r(t) - r_0(t)];

$$[[\mathrm{r1}_x, \mathrm{r1}_y, \mathrm{r1}_z]([\mathrm{r0}_x, \mathrm{r0}_y, \mathrm{r0}_z], t) = \mathrm{r}(t) - [\mathrm{r0}_x, \mathrm{r0}_y, \mathrm{r0}_z](t)] \quad (\%\ \mathrm{o52})$$

Total instaneous velocity v(r,t) of a group of charges satisfying (4.3.16) is:

(% i53) cmt_ch04_4322:[v(r_vec,t) = v_0(r_vec) + u_1(r_vec,t) , v(r_vec,t) = v_0(r_0) + v_1(r_0,t)];

$$[[v_x, v_y, v_z](r_{\mathrm{vec}}, t) = u_1(r_{\mathrm{vec}}, t) + v_0(r_{\mathrm{vec}}), [v_x, v_y, v_z](r_{\mathrm{vec}}, t) = v_1([\mathrm{r0}_x, \mathrm{r0}_y, \mathrm{r0}_z], t) + v_0([\mathrm{r0}_x, \mathrm{r0}_y, \mathrm{r0}_z])] \quad (\%\ \mathrm{o53})$$

Barybin uses (r .) v_0 but maxima won't accept that operator

(% i54) cmt_ch04_4322_A:[v_1 = u_1 + (r_1 . express(grad(v_0)))];

$$\left[v_1 = \mathrm{r1}_z\left(\frac{d}{dz}v_0\right) + \mathrm{r1}_y\left(\frac{d}{dy}v_0\right) + \mathrm{r1}_x\left(\frac{d}{dx}v_0\right) + u_1\right] \quad (\%\ \mathrm{o54})$$

Polarization vector v_1 obeys the equation of motion obtained from (4.3.22) as follows:Again Barybin uses (r_1 .) v_0 for example, but maxima won't accept this directlyWill have to use apply/define/makefun/buildq and macros to make it work.Using E_1 as a small signal field vectorUsing E[1,p] as a symbolic placeholder for E[1,+] for entry into maxima

(% i55) cmt_ch04_4323:[τ_0 = τ(E_h0), 'diff(v1,t) + (v_0 . express(grad(v1))) = (e/m)*(E_1 + r_1 . express(grad(E_0)) + v_1 ~ B_0 + v_0 ~ B_1 + v_0 ~ (r_1 . express(grad(B_0)))) + (v_T^2/ρ_0) * (ρ_0* express(grad(express(div(r_1)))) + express(grad(r_1)) . express(grad(ρ_0))) - (v_1/τ_0) + (v_0/τ_0) * ((τ_1 + (r_1 . express(grad(τ_0)))) / τ_0)];

$$[\tau_0 = \tau(E_{\mathrm{h0}}), v_0 \cdot \left[\frac{d}{dx}\mathrm{v1}, \frac{d}{dy}\mathrm{v1}, \frac{d}{dz}\mathrm{v1}\right] + \frac{d}{dt}\mathrm{v1} = \frac{v_0\left(\tau_1 + \mathrm{r1}_z\left(\frac{d}{dz}\tau_0\right) + \mathrm{r1}_y\left(\frac{d}{dy}\tau_0\right) + \mathrm{r1}_x\left(\frac{d}{dx}\tau_0\right)\right)}{\tau_0^2} - \frac{v_1}{\tau_0} + (v_T^2((\quad (\%\ \mathrm{o55})$$

(% i56) cmt_ch04_4324:[v_1 = 'diff(r_1,t) + (v_0 . express(grad(r_1)))];

$$\left[v_1 = v_0 \cdot \left[\frac{d}{dx}[\mathrm{r1}_x, \mathrm{r1}_y, \mathrm{r1}_z], \frac{d}{dy}[\mathrm{r1}_x, \mathrm{r1}_y, \mathrm{r1}_z], \frac{d}{dz}[\mathrm{r1}_x, \mathrm{r1}_y, \mathrm{r1}_z]\right] + \frac{d}{dt}[\mathrm{r1}_x, \mathrm{r1}_y, \mathrm{r1}_z]\right]$$

(%o56)

(%i67) J_1: [J1_x, J1_y,J1_z];p_1:[p1_x, p1_y, p1_z];v_0:[v0_x,v0_y,v0_z];E_1: [E1_x, E1_y,E1_z];D_1: [D1_x, D1_y,D1_z];
H_1: [H1_x, H1_y,H1_z]; B_1: [B1_x, B1_y,B1_z]; M_1: [M1_x, M1_y,M1_z]; P_1: [P1_x, P1_y,P1_z];
H_2_p:[H2p_x, H2p_y, H2p_z];
E_2:[E2_x, E2_y, E2_z];

$$[\mathrm{J1}_x, \mathrm{J1}_y, \mathrm{J1}_z] \tag{\%o57}$$

$$[\mathrm{p1}_x, \mathrm{p1}_y, \mathrm{p1}_z] \tag{\%o58}$$

$$[\mathrm{v0}_x, \mathrm{v0}_y, \mathrm{v0}_z] \tag{\%o59}$$

$$[\mathrm{E1}_x, \mathrm{E1}_y, \mathrm{E1}_z] \tag{\%o60}$$

$$[\mathrm{D1}_x, \mathrm{D1}_y, \mathrm{D1}_z] \tag{\%o61}$$

$$[\mathrm{H1}_x, \mathrm{H1}_y, \mathrm{H1}_z] \tag{\%o62}$$

$$[\mathrm{B1}_x, \mathrm{B1}_y, \mathrm{B1}_z] \tag{\%o63}$$

$$[\mathrm{M1}_x, \mathrm{M1}_y, \mathrm{M1}_z] \tag{\%o64}$$

$$[\mathrm{P1}_x, \mathrm{P1}_y, \mathrm{P1}_z] \tag{\%o65}$$

$$\left[\mathrm{H2p}_x\,,\mathrm{H2p}_y\,,\mathrm{H2p}_z\right] \tag{\% o66}$$

$$[\mathrm{E2}_x\,,\mathrm{E2}_y\,,\mathrm{E2}_z] \tag{\% o67}$$

(% i68) cmt_ch04_4325:['diff(ρ_1,t) + express(div(J_1) = 0)];

$$\left[\frac{d}{d t}\rho_1+\frac{d}{d z}\mathrm{J1}_z+\frac{d}{d y}\mathrm{J1}_y+\frac{d}{d x}\mathrm{J1}_x=\frac{d}{d t}\rho_1\right] \tag{\% o68}$$

(% i69) cmt_ch04_4326:[ρ_1 = -express(div(p_1))];

$$\left[\rho_1=-\frac{d}{d z}\mathrm{p1}_z-\frac{d}{d y}\mathrm{p1}_y-\frac{d}{d x}\mathrm{p1}_x\right] \tag{\% o69}$$

(% i70) cmt_ch04_4327:[J_1 = 'diff(p_1, t) + express(curl(p_1 ~ v_0))];

$$[\,[\mathrm{J1}_x\,,\mathrm{J1}_y\,,\mathrm{J1}_z]=[\,\frac{d}{d y}\left(\mathrm{p1}_x\mathrm{v0}_y-\mathrm{p1}_y\mathrm{v0}_x\right)-\frac{d}{d z}\left(\mathrm{p1}_z\mathrm{v0}_x-\mathrm{p1}_x\mathrm{v0}_z\right),\frac{d}{d z}\left(\mathrm{p1}_y\mathrm{v0}_z-\mathrm{p1}_z\mathrm{v0}_y\right)-\frac{d}{d x}\left(\mathrm{p1}_x\mathrm{v0}_y-\mathrm{p1}_y\mathrm{v}\right. \tag{\% o70}$$

4.3.4 Electrodynamic Formulations For Active Polarized Media

(% i71) cmt_ch04_4328:[express(curl(E_1)) = -'diff(B_1,t)];

$$\left[\left[\frac{d}{d y}\mathrm{E1}_z-\frac{d}{d z}\mathrm{E1}_y\,,\frac{d}{d z}\mathrm{E1}_x-\frac{d}{d x}\mathrm{E1}_z\,,\frac{d}{d x}\mathrm{E1}_y-\frac{d}{d y}\mathrm{E1}_x\right]=-\frac{d}{d t}\left[\mathrm{B1}_x\,,\mathrm{B1}_y\,,\mathrm{B1}_z\right]\right] \tag{\% o71}$$

(% i72) cmt_ch04_4329:[express(curl(H_1)) = 'diff(D_1,t) + J1];

$$\left[\left[\frac{d}{d y}\mathrm{H1}_z-\frac{d}{d z}\mathrm{H1}_y\,,\frac{d}{d z}\mathrm{H1}_x-\frac{d}{d x}\mathrm{H1}_z\,,\frac{d}{d x}\mathrm{H1}_y-\frac{d}{d y}\mathrm{H1}_x\right]=\mathrm{J1}+\frac{d}{d t}\left[\mathrm{D1}_x\,,\mathrm{D1}_y\,,\mathrm{D1}_z\right]\right] \tag{\% o72}$$

(% i73) cmt_ch04_4330:[express(div(D_1)) = ρ_1];

$$\left[\frac{d}{d z}\mathrm{D1}_z+\frac{d}{d y}\mathrm{D1}_y+\frac{d}{d x}\mathrm{D1}_x=\rho_1\right] \tag{\% o73}$$

(%i74) cmt_ch04_4331:express(div(B_1)) = 0;

$$\frac{d}{dz}B1_z + \frac{d}{dy}B1_y + \frac{d}{dx}B1_x = 0 \qquad (\%o74)$$

(%i75) cmt_ch04_4332:[D_1 = (ϵ_0 * E_1) + P_1];

$$[[D1_x, D1_y, D1_z] = [E1_x\epsilon_0 + P1_x, E1_y\epsilon_0 + P1_y, E1_z\epsilon_0 + P1_z]] \qquad (\%o75)$$

(%i76) cmt_ch04_4333:[H_1 = ((1/μ_0) * B_1) - M_1];

$$\left[[H1_x, H1_y, H1_z] = \left[\frac{B1_x}{\mu_0} - M1_x, \frac{B1_y}{\mu_0} - M1_y, \frac{B1_z}{\mu_0} - M1_z\right]\right] \qquad (\%o76)$$

(%i78) n[s,p]:[n_sp_x, n_sp_y, n_sp_z];n[s,m]:[n_sm_x, n_sm_y, n_sm_z];

$$[n\_sp_x, n\_sp_y, n\_sp_z] \qquad (\%o77)$$

$$[n\_sm_x, n\_sm_y, n\_sm_z] \qquad (\%o78)$$

The field vectors at the boundaries are also plus or minus (outward or inward)

(%i80) E_1_p: [E1_p_x, E1_p_y,E1_p_z];E_1_m: [E1_m_x, E1_m_y,E1_m_z];

$$[E1\_p_x, E1\_p_y, E1\_p_z] \qquad (\%o79)$$

$$[E1\_m_x, E1\_m_y, E1\_m_z] \qquad (\%o80)$$

(%i82) H_1_p: [H1_p_x, H1_p_y,H1_p_z];H_1_m: [H1_m_x, H1_m_y,H1_m_z];

$$[H1\_p_x, H1\_p_y, H1\_p_z] \qquad (\%o81)$$

$$[H1\_m_x, H1\_m_y, H1\_m_z] \qquad (\%o82)$$

(%i86) D_1_p: [D1_p_x, D1_p_y,D1_p_z];D_1_m: [D1_m_x, D1_m_y,D1_m_z];H_1P:[H_1P_x, H_1P_y, H_1P_z];D_1P:[D_1P_x, D_1P_y, D_1P_z];

$$[D1\_p_x, D1\_p_y, D1\_p_z] \qquad (\%o83)$$

$$[\mathrm{D1\_m}_x, \mathrm{D1\_m}_y, \mathrm{D1\_m}_z] \tag{\% o84}$$

$$[\mathrm{H\_1P}_x, \mathrm{H\_1P}_y, \mathrm{H\_1P}_z] \tag{\% o85}$$

$$[\mathrm{D\_1P}_x, \mathrm{D\_1P}_y, \mathrm{D\_1P}_z] \tag{\% o86}$$

(% i93) B_1_p: [B1_p_x, B1_p_y,B1_p_z];B_1_m: [B1_m_x, B1_m_y,B1_m_z];M_net:[Mnet_x, Mnet_y, Mnet_z];P_net:[Pnet_x, Pnet_y, Pnet_z];v_o:[v0_x , v0_y, v0_z]; B_0:[B0_x, B0_y, B0_z];r:[r_x, r_y, r_z];

$$[\mathrm{B1\_p}_x, \mathrm{B1\_p}_y, \mathrm{B1\_p}_z] \tag{\% o87}$$

$$[\mathrm{B1\_m}_x, \mathrm{B1\_m}_y, \mathrm{B1\_m}_z] \tag{\% o88}$$

$$[\mathrm{Mnet}_x, \mathrm{Mnet}_y, \mathrm{Mnet}_z] \tag{\% o89}$$

$$[\mathrm{Pnet}_x, \mathrm{Pnet}_y, \mathrm{Pnet}_z] \tag{\% o90}$$

$$[\mathrm{v0}_x, \mathrm{v0}_y, \mathrm{v0}_z] \tag{\% o91}$$

$$[\mathrm{B0}_x, \mathrm{B0}_y, \mathrm{B0}_z] \tag{\% o92}$$

$$[r_x, r_y, r_z] \tag{\% o93}$$

(% i94) cmt_ch04_4334:[express(n[s,p] ~ E_1_p) + express(n[s,m] ~ E_1_m) = 0];

$$[[-\mathrm{E1\_p}_y \mathrm{n\_sp}_z + \mathrm{E1\_p}_z \mathrm{n\_sp}_y - \mathrm{E1\_m}_y \mathrm{n\_sm}_z + \mathrm{E1\_m}_z \mathrm{n\_sm}_y, \mathrm{E1\_p}_x \mathrm{n\_sp}_z - \mathrm{E1\_p}_z \mathrm{n\_sp}_x + \mathrm{E1\_m}_x \mathrm{n\_sm}_z -$$

(%o94)

(%i95) cmt_ch04_4335:[express(n[s,p] ~ H_1_p) + express(n[s,m] ~ H_1_m) = J[s,eq]];

$$[[-\mathrm{H1\_p}_y \mathrm{n\_sp}_z+\mathrm{H1\_p}_z \mathrm{n\_sp}_y-\mathrm{H1\_m}_y \mathrm{n\_sm}_z+\mathrm{H1\_m}_z \mathrm{n\_sm}_y, \mathrm{H1\_p}_x \mathrm{n\_sp}_z-\mathrm{H1\_p}_z \mathrm{n\_sp}_x+\mathrm{H1\_m}_x \mathrm{n\_sm}_z \tag{\% o95}$$

(%i96) cmt_ch04_4336:[express(n[s,p] ~ D_1_p) + express(n[s,m] ~ D_1_m) =ρ[s,eq]];

$$[[-\mathrm{D1\_p}_y \mathrm{n\_sp}_z+\mathrm{D1\_p}_z \mathrm{n\_sp}_y-\mathrm{D1\_m}_y \mathrm{n\_sm}_z+\mathrm{D1\_m}_z \mathrm{n\_sm}_y, \mathrm{D1\_p}_x \mathrm{n\_sp}_z-\mathrm{D1\_p}_z \mathrm{n\_sp}_x+\mathrm{D1\_m}_x \mathrm{n\_sm}_z \tag{\% o96}$$

(%i97) cmt_ch04_4337:[express(n[s,p] ~ B_1_p) + express(n[s,m] ~ B_1_m) = 0];

$$[[-\mathrm{B1\_p}_y \mathrm{n\_sp}_z+\mathrm{B1\_p}_z \mathrm{n\_sp}_y-\mathrm{B1\_m}_y \mathrm{n\_sm}_z+\mathrm{B1\_m}_z \mathrm{n\_sm}_y, \mathrm{B1\_p}_x \mathrm{n\_sp}_z-\mathrm{B1\_p}_z \mathrm{n\_sp}_x+\mathrm{B1\_m}_x \mathrm{n\_sm}_z \tag{\% o97}$$

(%i98) cmt_ch04_4338:[p_eq[s] = p_0*(n . r_1) , p_eq[s]= n . p_1];

$$\left[p_{\mathrm{eq}_s} = (n.\left[\mathrm{r1}_x, \mathrm{r1}_y, \mathrm{r1}_z\right])\, p_0, p_{\mathrm{eq}_s} = n.\left[\mathrm{p1}_x, \mathrm{p1}_y, \mathrm{p1}_z\right]\right] \tag{\% o98}$$

(%i99) cmt_ch04_4339:[J_eq[s] = p_eq[s] * v_0 , J_eq[s] = (n . p-1) * v_0];

$$\left[J_{\mathrm{eq}_s} = \left[p_{\mathrm{eq}_s}\mathrm{v0}_x, p_{\mathrm{eq}_s}\mathrm{v0}_y, p_{\mathrm{eq}_s}\mathrm{v0}_z\right], J_{\mathrm{eq}_s} = (n.p-1)\left[\mathrm{v0}_x, \mathrm{v0}_y, \mathrm{v0}_z\right]\right] \tag{\% o99}$$

(%i100) cmt_ch04_4340:[P_net = P_1 + p_1, M_net = M_1 + m_1, M___net = approx(M_1 + (P_1 ~ v_0))];

$$[[\mathrm{Pnet}_x, \mathrm{Pnet}_y, \mathrm{Pnet}_z] = \left[\mathrm{p1}_x+\mathrm{P1}_x, \mathrm{p1}_y+\mathrm{P1}_y, \mathrm{p1}_z+\mathrm{P1}_z\right], [\mathrm{Mnet}_x, \mathrm{Mnet}_y, \mathrm{Mnet}_z] = [m_1+\mathrm{M1}_x, m_1+\mathrm{M1} \tag{\% o100}$$

(%i101) cmt_ch04_4341:ρ_P = -express(div(P_net)) , J+P = 'diff(P_net,t), J_M = express(curl(M_net));

$$\rho_P = -\frac{d}{dz}\mathrm{Pnet}_z - \frac{d}{dy}\mathrm{Pnet}_y - \frac{d}{dx}\mathrm{Pnet}_x \tag{\% o101}$$

Chu Formulation Of Maxwell's Equations

(%i102) cmt_ch04_4342:[express(curl(E_1)) = 'diff(B_1,t)];

$$\left[\left[\frac{d}{dy}\mathrm{E1}_z-\frac{d}{dz}\mathrm{E1}_y\,,\frac{d}{dz}\mathrm{E1}_x-\frac{d}{dx}\mathrm{E1}_z\,,\frac{d}{dx}\mathrm{E1}_y-\frac{d}{dy}\mathrm{E1}_x\right]=\frac{d}{dt}\left[\mathrm{B1}_x\,,\mathrm{B1}_y\,,\mathrm{B1}_z\right]\right] \tag{\% o102}$$

(%i103) cmt_ch04_4343:[express(curl((1/μ_0) * B_1)) = ϵ_0 * 'diff(E_1,t) + (J_P + J_M)];

$$[\left[\frac{d}{dy}\frac{\mathrm{B1}_z}{\mu_0}-\frac{d}{dz}\frac{\mathrm{B1}_y}{\mu_0}\,,\frac{d}{dz}\frac{\mathrm{B1}_x}{\mu_0}-\frac{d}{dx}\frac{\mathrm{B1}_z}{\mu_0}\,,\frac{d}{dx}\frac{\mathrm{B1}_y}{\mu_0}-\frac{d}{dy}\frac{\mathrm{B1}_x}{\mu_0}\right]=\left(\frac{d}{dt}\left[\mathrm{E1}_x\,,\mathrm{E1}_y\,,\mathrm{E1}_z\right]\right)\epsilon_0+J_P+J_M\,] \tag{\% o103}$$

(%i104) cmt_ch04_4344:[express(div(E_1)) = (1/ϵ_0) * ρ_P];

$$\left[\frac{d}{dz}\mathrm{E1}_z+\frac{d}{dy}\mathrm{E1}_y+\frac{d}{dx}\mathrm{E1}_x=\frac{\rho_P}{\epsilon_0}\right] \tag{\% o104}$$

(%i105) cmt_ch04_4345:[express(curl(B_1)) = 0];

$$\left[\left[\frac{d}{dy}\mathrm{B1}_z-\frac{d}{dz}\mathrm{B1}_y\,,\frac{d}{dz}\mathrm{B1}_x-\frac{d}{dx}\mathrm{B1}_z\,,\frac{d}{dx}\mathrm{B1}_y-\frac{d}{dy}\mathrm{B1}_x\right]=0\right] \tag{\% o105}$$

Minkowski Formulation Of Maxwell's Equations

(%i106) cmt_ch04_4346:[D_1_P = ϵ_0 * E_1 + P_net];

$$[\mathrm{D\_1}_P=[\mathrm{E1}_x\epsilon_0+\mathrm{Pnet}_x\,,\mathrm{E1}_y\epsilon_0+\mathrm{Pnet}_y\,,\mathrm{E1}_z\epsilon_0+\mathrm{Pnet}_z]] \tag{\% o106}$$

(%i107) cmt_ch04_4347:[H_1_P = (1/μ_0) * B_1 - M_net];

$$\left[\mathrm{H\_1}_P=\left[\frac{\mathrm{B1}_x}{\mu_0}-\mathrm{Mnet}_x\,,\frac{\mathrm{B1}_y}{\mu_0}-\mathrm{Mnet}_y\,,\frac{\mathrm{B1}_z}{\mu_0}-\mathrm{Mnet}_z\right]\right] \tag{\% o107}$$

(%i108) cmt_ch04_4348:[express(curl(E_1)) = 'diff(B_1,t)];

$$\left[\left[\frac{d}{dy}\mathrm{E1}_z-\frac{d}{dz}\mathrm{E1}_y\,,\frac{d}{dz}\mathrm{E1}_x-\frac{d}{dx}\mathrm{E1}_z\,,\frac{d}{dx}\mathrm{E1}_y-\frac{d}{dy}\mathrm{E1}_x\right]=\frac{d}{dt}\left[\mathrm{B1}_x\,,\mathrm{B1}_y\,,\mathrm{B1}_z\right]\right]$$

(%o108)

(%i109) cmt_ch04_4349:[express(curl(H_1P)) = 'diff(D_1_P ,t)];

$$\left[\left[\frac{d}{dy}\mathrm{H\_1P}_z - \frac{d}{dz}\mathrm{H\_1P}_y , \frac{d}{dz}\mathrm{H\_1P}_x - \frac{d}{dx}\mathrm{H\_1P}_z , \frac{d}{dx}\mathrm{H\_1P}_y - \frac{d}{dy}\mathrm{H\_1P}_x\right] = \frac{d}{dt}\mathrm{D\_1}_P\right] \quad (\%o109)$$

(%i110) cmt_ch04_4350:[express[div(D_1P)]= 0];

$$\left[\frac{d}{dz}\mathrm{D\_1P}_z + \frac{d}{dy}\mathrm{D\_1P}_y + \frac{d}{dx}\mathrm{D\_1P}_x = 0\right] \quad (\%o110)$$

(%i111) cmt_ch04_4351:[express(div(B_1))=0];

$$\left[\frac{d}{dz}\mathrm{B1}_z + \frac{d}{dy}\mathrm{B1}_y + \frac{d}{dx}\mathrm{B1}_x = 0\right] \quad (\%o111)$$

(%i112) cmt_ch04_4352:[D_1P = D_1 + p_1, H_1P = H_1 + express(v_0 ~ p_1)];

$$[\left[\mathrm{D\_1P}_x , \mathrm{D\_1P}_y , \mathrm{D\_1P}_z\right] = \left[\mathrm{p1}_x + \mathrm{D1}_x , \mathrm{p1}_y + \mathrm{D1}_y , \mathrm{p1}_z + \mathrm{D1}_z\right] , \left[\mathrm{H\_1P}_x , \mathrm{H\_1P}_y , \mathrm{H\_1P}_z\right] = \left[-\mathrm{p1}_y \mathrm{v0}_z + \right. \quad (\%o112)$$

Use express() here once the E and n vectors are defined properly

(%i113) cmt_ch04_4353:[n[s,p] ~ E[1,p] + n[s,m] ~ E[1,m] = 0] ;

$$\left[-E_{1,p} \sim \left[\mathrm{n\_sp}_x , \mathrm{n\_sp}_y , \mathrm{n\_sp}_z\right] - E_{1,m} \sim \left[\mathrm{n\_sm}_x , \mathrm{n\_sm}_y , \mathrm{n\_sm}_z\right] = 0\right] \quad (\%o113)$$

(%i114) cmt_ch04_4354:[n[s,p] ~ H[1, p,p] + n[s,m] ~ H[1,p,m] = 0] ;

$$\left[-H_{1,p,p} \sim \left[\mathrm{n\_sp}_x , \mathrm{n\_sp}_y , \mathrm{n\_sp}_z\right] - H_{1,p,m} \sim \left[\mathrm{n\_sm}_x , \mathrm{n\_sm}_y , \mathrm{n\_sm}_z\right] = 0\right] \quad (\%o114)$$

(%i115) cmt_ch04_4355:[n[s,p] ~ D[1,p,p] + n[s,m] ~ D[1,p,m] = 0] ;

$$\left[-D_{1,p,p} \sim \left[\mathrm{n\_sp}_x, \mathrm{n\_sp}_y, \mathrm{n\_sp}_z\right] - D_{1,p,m} \sim \left[\mathrm{n\_sm}_x, \mathrm{n\_sm}_y, \mathrm{n\_sm}_z\right] = 0\right] \quad (\%o115)$$

Removing B to allow for use of symbolic placeholder

(%i116) kill(B);

done (%o116)

(%i117) cmt_ch04_4356:[n[s,p] ~ B[1,p,p] + n[s,m] ~ B[1,p,m] = 0] ;

$$\left[-B_{1,p,p} \sim \left[\mathrm{n\_sp}_x, \mathrm{n\_sp}_y, \mathrm{n\_sp}_z\right] - B_{1,p,m} \sim \left[\mathrm{n\_sm}_x, \mathrm{n\_sm}_y, \mathrm{n\_sm}_z\right] = 0\right] \quad (\%o117)$$

(%i118) B:[B_x, B_y, B_z];

$$[B_x, B_y, B_z] \quad (\%o118)$$

4.4 General Power-Energy Relations for Space-Dispersive Active Media4.4.1 Generalized Poynting's Theorem For SDAM

(%i123) E:[E_x,E_y, E_z];H:[H_x,H_y,H_z];D:[D_x,D_y, D_z];S:[X_x,S_y, S_z];p_1:[p1_x, p1_y, p1_z];

$$[E_x, E_y, E_z] \quad (\%o119)$$

$$[H_x, H_y, H_z] \quad (\%o120)$$

$$[D_x, D_y, D_z] \quad (\%o121)$$

$$[X_x, S_y, S_z] \quad (\%o122)$$

$$\left[\mathrm{p1}_x\,,\mathrm{p1}_y\,,\mathrm{p1}_z\right] \tag{\% o123}$$

(% i124) cmt_ch04_441:[express(curl(E) = - 'diff(B,t))];

$$\left[\left[\frac{d}{dy}E_z-\frac{d}{dz}E_y\,,\frac{d}{dz}E_x-\frac{d}{dx}E_z\,,\frac{d}{dx}E_y-\frac{d}{dy}E_x\right]=-\frac{d}{dt}\left[B_x\,,B_y\,,B_z\right]\right] \tag{\% o124}$$

(% i125) cmt_ch04_442:[express(curl(H)) = 'diff(D,t) + J];

$$\left[\left[\frac{d}{dy}H_z-\frac{d}{dz}H_y\,,\frac{d}{dz}H_x-\frac{d}{dx}H_z\,,\frac{d}{dx}H_y-\frac{d}{dy}H_x\right]=J+\frac{d}{dt}\left[D_x\,,D_y\,,D_z\right]\right] \tag{\% o125}$$

Scalar Multiply 4.4.1 by H, 4.4.2 by -E and add results.This gives the instanteous Poynting theorem

(% i126) cmt_ch04_443:['diff(((E . D)/2) + ((H.B)/2),t) + express(div(E ~ H)) = -I_P - I_m - I_J];

$$[\frac{d}{dt}\left(\frac{B_zH_z+B_yH_y+B_xH_x}{2}+\frac{D_zE_z+D_yE_y+D_xE_x}{2}\right)+\frac{d}{dx}\left(E_yH_z-E_zH_y\right)+\frac{d}{dy}\left(E_zH_x-E_xH_z\right)+\frac{d}{dz}\left(E_xH \tag{\% o126}$$

where

(% i127) cmt_ch04_444:[(1/2) * ((E . 'diff(D,t)) - (D . 'diff(E,t))) = (1/2) * ((E . 'diff(P,t)) - (P . 'diff(E,t)))];

$$\left[\frac{\left[E_x\,,E_y\,,E_z\right].\frac{d}{dt}\left[D_x\,,D_y\,,D_z\right]-\left[D_x\,,D_y\,,D_z\right].\frac{d}{dt}\left[E_x\,,E_y\,,E_z\right]}{2}=\frac{\left[E_x\,,E_y\,,E_z\right].\frac{d}{dt}P-P.\frac{d}{dt}\left[E_x\,,E_y\,,E_z\right]}{2}\right] \tag{\% o127}$$

(% i128) cmt_ch04_445:[I_M = (1/2)*((H . 'diff(B,t)) - (B . 'diff(H,t)), I_M = (1/2) * (H . 'diff(μ_0*M.t)) - μ_0*M . 'diff(H,t))];

$$\left[I_M=\left(\frac{I_M}{2}=\frac{\frac{\left[H_x\,,H_y\,,H_z\right].\,\mathrm{del}(([M_x\,,M_y\,,M_z].t)\mu_0)}{2}-\left(\left[M_x\,,M_y\,,M_z\right].\frac{d}{dt}\left[H_x\,,H_y\,,H_z\right]\right)\mu_0}{2}\right)\right]$$

(% o128)

(% i129) cmt_ch04_446:[I_J = J . E];

$$[I_J = J.[E_x, E_y, E_z]] \tag{\% o129}$$

(% i130) cmt_ch04_4477:[D = ϵ_0 * E + P , B = μ_0 * (H+M)];

$$[[D_x, D_y, D_z] = [E_x \epsilon_0 + P, E_y \epsilon_0 + P, E_z \epsilon_0 + P], [B_x, B_y, B_z] = [(M_x + H_x)\mu_0, (M_y + H_y)\mu_0, (M_z + H_z)\mu_0 \tag{\% o130}$$

(% i131) cmt_ch04_448:[w_em = (E . D) / 2 + (H . B) /2];

$$\left[w_{\mathrm{em}} = \frac{B_z H_z + B_y H_y + B_x H_x}{2} + \frac{D_z E_z + D_y E_y + D_x E_x}{2}\right] \tag{\% o131}$$

(% i132) cmt_ch04_449:[S_em = express(E ~ H)];

$$[S_{\mathrm{em}} = [E_y H_z - E_z H_y, E_z H_x - E_x H_z, E_x H_y - E_y H_x]] \tag{\% o132}$$

(% i133) cmt_ch04_4410:['diff(w,t) + express(div(S) + q = 0)];

$$\left[\frac{d}{dt} w + q + \frac{d}{dx} X_x + \frac{d}{dz} S_z + \frac{d}{dy} S_y = \frac{d}{dt} w\right] \tag{\% o133}$$

(% i134) cmt_ch04_4411:[div(avg_t(S) + avg_t(q) = 0)];

$$[\mathrm{div}\left(\mathrm{avg}_t(q) + \mathrm{avg}_t([X_x, S_y, S_z]) = 0\right)] \tag{\% o134}$$

(% i135) cmt_ch04_4412:[w_add = w_el + w_fm + w_pl];

$$[w_{\mathrm{add}} = w_{\mathrm{pl}} + w_{\mathrm{fm}} + w_{\mathrm{el}}] \tag{\% o135}$$

(% i136) cmt_ch04_4413:[S_add = S_el + S_fm + S_pl];

$$[S_{\mathrm{add}} = S_{\mathrm{pl}} + S_{\mathrm{fm}} + S_{\mathrm{el}}] \tag{\% o136}$$

(%i137) cmt_ch04_4414:[q_add = q_el + q___fm + q_pl];

$$[q_{\mathrm{add}} = q_{\mathrm{pl}} + q_{\mathrm{el}} + \mathrm{q\_\_\_fm}] \quad (\%\ o137)$$

4.4.1.1 Contribution From Piezoelastic Properties of a Medium

(%i138) cmt_ch04_4415:[I_P=(1/2) * (E . 'diff(P,t) - P . 'diff(E,t)), I_P = 'diff(w_el.t) + div(S_el) + q_el];

$$\left[I_P = \frac{[E_x\,, E_y\,, E_z]\,.\frac{d}{dt}P - P.\frac{d}{dt}\,[E_x\,, E_y\,, E_z]}{2}\,, I_P = \mathrm{del}\,(w_{\mathrm{el}}.t) + q_{\mathrm{el}} + \mathrm{div}\,(S_{\mathrm{el}})\right] \quad (\%\ o138)$$

The author uses a colon : instead of dot between T_bar and S_barT_bar and S_bar are tensors in this contextThe : operator is a double dot product given byΣ_j Σ_i (a_i . d_j) * (b_i . c_j)orΣ_j Σ_i (a_i . c_j) * (b_i . d_j)Load the following package to get mattrace (trace) of a matrixdd(A,B) gives the proper result for a double dot product (:)TODO: RETROFIT CHAPTERS 2-3 WITH THIS DEFINITION

(%i139) load ("nchrpl");

(% o139)

"/usr/share/maxima/5.45.1/share/matrix/nchrpl.mac"

(%i140) dd(A,B):=mattrace(A * transpose(B));

$$\mathrm{dd}\,(A\,, B) := \mathrm{mattrace}\,(A\,\mathrm{transpose}(B)) \quad (\%\ o140)$$

(%i141) T_bar:matrix([T_11, T_12, T_13],[T_21,T_22, T_23],[T_31, T_32, T_33]);

$$\begin{pmatrix} T_{11} & T_{12} & T_{13} \\ T_{21} & T_{22} & T_{23} \\ T_{31} & T_{32} & T_{33} \end{pmatrix} \quad (\%\ o141)$$

(%i142) S_bar:matrix([S_11, S_12, S_13],[S_21,S_22, S_23],[S_31, S_32, S_33]);

$$\begin{pmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{pmatrix} \quad (\%\ o142)$$

(%i143) η_bar:matrix([η_11, η_12, η_13],[η_21,η_22, η_23],[η_31, η_32, η_33]);

$$\begin{pmatrix}\eta_{11} & \eta_{12} & \eta_{13}\\ \eta_{21} & \eta_{22} & \eta_{23}\\ \eta_{31} & \eta_{32} & \eta_{33}\end{pmatrix} \quad (\%o143)$$

(%i144) S_bar_dot:matrix(['diff(S_11,t), 'diff(S_12,t), 'diff(S_13,t)] ,['diff(S_21,t), 'diff(S_22,t), 'diff(S_23,t)] ,['diff(S_31,t), 'diff(S_32,t), 'diff(S_33,t)]);

$$\begin{pmatrix}\frac{d}{dt}S_{11} & \frac{d}{dt}S_{12} & \frac{d}{dt}S_{13}\\ \frac{d}{dt}S_{21} & \frac{d}{dt}S_{22} & \frac{d}{dt}S_{23}\\ \frac{d}{dt}S_{31} & \frac{d}{dt}S_{32} & \frac{d}{dt}S_{33}\end{pmatrix} \quad (\%o144)$$

(%i145) cmt_ch04_4416:[w_el = (1/2) * p_m * U^2 + (1/2) * dd(T_bar, S_bar)];

$$\left[w_{\mathrm{el}} = \frac{U^2 p_m}{2} + \frac{S_{33}T_{33} + S_{22}T_{22} + S_{11}T_{11}}{2}\right] \quad (\%o145)$$

Assuming dot over bar means ordinary derivative of matrix elements with timeTODO: Check on matrix differentiation rules for this

(%i146) cmt_ch04_4417:[S_el = (1/2) * p_m * U^2 *Y - T_bar^2 . U];

$$\left[S_{\mathrm{el}} = \frac{U^2 Y p_m}{2} - \begin{pmatrix}{T_{11}}^2 & {T_{12}}^2 & {T_{13}}^2\\ {T_{21}}^2 & {T_{22}}^2 & {T_{23}}^2\\ {T_{31}}^2 & {T_{32}}^2 & {T_{33}}^2\end{pmatrix}.U\right] \quad (\%o146)$$

(%i147) cmt_ch04_4418:[q_el = dd(dd(S_bar_dot, η_bar),S_bar_dot)];

$$[q_{\mathrm{el}} = \left(\frac{d}{dt}S_{33}\right)\left(\left(\frac{d}{dt}S_{33}\right)\eta_{33} + \left(\frac{d}{dt}S_{22}\right)\eta_{22} + \left(\frac{d}{dt}S_{11}\right)\eta_{11}\right) + \left(\frac{d}{dt}S_{22}\right)\left(\left(\frac{d}{dt}S_{33}\right)\eta_{33} + \left(\frac{d}{dt}S_{22}\right)\eta_{22} + \left(\right.\right.$$

(%o147)

(%i148) cmt_ch04_4419:[w_el = (1/2) * p_m0 * U_1^2 + (1/2) * dd(T_bar[1] , S_bar[1])];

$$\left[w_{\mathrm{el}} = \frac{{U_1}^2 p_{\mathrm{m0}}}{2} + \frac{S_{11}T_{11}}{2}\right] \quad (\%o148)$$

(%i149) cmt_ch04_4420:[S_el = -T[1,Σ] . U_1];

$$[S_{\mathrm{el}} = -T_{1,\Sigma}.U_1] \quad (\%o149)$$

(%i150) cmt_ch04_4421:[q_el = dd(dd(S_bar_dot[1] , η_bar),S_bar_dot) + p_m0 * U_1 . τ_bar[-1] . U_1];

$$[q_{\mathrm{el}} = \left(\frac{d}{dt}S_{33}\right)\left(\left(\frac{d}{dt}S_{13}\right)\eta_{33} + \left(\frac{d}{dt}S_{12}\right)\eta_{22} + \left(\frac{d}{dt}S_{11}\right)\eta_{11}\right)+\left(\frac{d}{dt}S_{22}\right)\left(\left(\frac{d}{dt}S_{13}\right)\eta_{33} + \left(\frac{d}{dt}S_{12}\right)\eta_{22} + \left(\right.\right. \quad (\% o150)$$

4.4.1.2 Contribution From Ferrimagnetic Properties of a Medium

(%i151) S_fm:[S_fm_x, S_fm_y, S_fm_z];

$$[\mathrm{S\_fm}_x, \mathrm{S\_fm}_y, \mathrm{S\_fm}_z] \quad (\% o151)$$

(%i152) cmt_ch04_4422:[I_M = (1/2) * (H . 'diff(μ_0*M,t) - μ_0*M . 'ddiff(H,t)), I_M = 'diff(w_fm,t) + express(div(S_fm)) + q_fm];

$$[I_M = \frac{[H_x, H_y, H_z] . \frac{d}{dt}[M_x\mu_0, M_y\mu_0, M_z\mu_0] - ([M_x, M_y, M_z] . \mathrm{ddiff}([H_x, H_y, H_z], t))\,\mu_0}{2}, I_M = \frac{d}{dt}w_{\mathrm{fm}}+q_{\mathrm{fm}} \quad (\% o152)$$

(%i153) cmt_ch04_4423:[w_fm = w_z + w_md + w_an+ w_ex];

$$[w_{\mathrm{fm}} = w_z + w_{\mathrm{md}} + w_{\mathrm{ex}} + w_{\mathrm{an}}] \quad (\% o153)$$

(%i154) cmt_ch04_4424:[w_z=-μ_0*M . H_0[e]];

$$[w_z = -([M_x, M_y, M_z] . H_{0e})\,\mu_0] \quad (\% o154)$$

(%i155) cmt_ch04_4435:[w_md=(-μ_0/2) * M . H];

$$\left[w_{\mathrm{md}} = -\frac{(H_zM_z + H_yM_y + H_xM_x)\,\mu_0}{2}\right] \quad (\% o155)$$

(%i156) cmt_ch04_4426:[w_an(-μ_0/2) * M . H_an, w_an = (μ_0/2)*M . N_bar . M];

$$\left[([M_x, M_y, M_z] . H_{\mathrm{an}})\, w_{\mathrm{an}}\left(-\frac{\mu_0}{2}\right), w_{\mathrm{an}} = \frac{([M_x, M_y, M_z] . N_{\mathrm{bar}} . [M_x, M_y, M_z])\,\mu_0}{2}\right]$$

(% o156)

(% i157) cmt_ch04_4427:[w_ez=(-μ_0/2) * M . H_ez, H_ex = λ_ex * express(laplacian(M)),w_an = -λ_ez * (μ_0/2) * express(laplacian(M))];

$$[w_{ez} = -\frac{([M_x, M_y, M_z].H_{ez})\mu_0}{2}, H_{ex} = \left(\frac{d^2}{dz^2}[M_x, M_y, M_z] + \frac{d^2}{dy^2}[M_x, M_y, M_z] + \frac{d^2}{dx^2}[M_x, M_y, M_z]\right)\lambda\_e \quad (\% o157)$$

(% i158) cmt_ch04_4428:[S_fm= λ_ex * (μ_0/2) * (M . 'diff(express(grad(M)) ,t) - express(grad(M)) . 'diff(M,t))] ;

$$[[\text{S\_fm}_x, \text{S\_fm}_y, \text{S\_fm}_z] = (([M_x, M_y, M_z].\frac{d}{dt}\left[\frac{d}{dx}[M_x, M_y, M_z], \frac{d}{dy}[M_x, M_y, M_z], \frac{d}{dz}[M_x, M_y, M_z]\right] - [\quad (\% o158)$$

(% i159) cmt_ch04_4429:[q_fm = α* (μ_0/w_M) * ('diff(M,t) . 'diff(M,t))] ;

$$\left[q_{fm} = \frac{\left(\frac{d}{dt}[M_x, M_y, M_z].\frac{d}{dt}[M_x, M_y, M_z]\right)\alpha\mu_0}{w_M}\right] \quad (\% o159)$$

(% i160) cmt_ch04_30:[w_z = μ_0*M . H_0, w_z=-μ_0*H_0*M_z, w_z=-μ_0*H_0*sqrt(M_0 ^2 - M_1 ^2), w_z=-μ_0*H_0*M_0 + (μ_0/2) *(H_0/M_0) * (M_1 . M_2)];

$$[w_z = ([M_x, M_y, M_z].H_0)\mu_0, w_z = -H_0 M_z \mu_0, w_z = \left[-H_0\sqrt{{M_0}^2 - {\text{M1}_x}^2}\mu_0, -H_0\sqrt{{M_0}^2 - {\text{M1}_y}^2}\mu_0, -H_0\sqrt{\text{M} \quad (\% o160)$$

(% i161) cmt_ch04_4431:[w_fm = (μ_0/2) * (((H_0/M_0) * M_1 - H_1) . M_1 + M_1 . N_bar . M_1 - λ_ex * express(laplacian(M_1) . M_1))];

$$[w_{fm} = ((-\left(\left(\frac{d^2}{dz^2}[\text{M1}_x, \text{M1}_y, \text{M1}_z] + \frac{d^2}{dy^2}[\text{M1}_x, \text{M1}_y, \text{M1}_z] + \frac{d^2}{dx^2}[\text{M1}_x, \text{M1}_y, \text{M1}_z]\right).[\text{M1}_x, \text{M1}_y, \text{M1}_z]\right)\lambda \quad (\% o161)$$

(% i162) cmt_ch04_4432:[S_fm = λ_ex * (μ_0/2) * (M_1 . diff(express(grad(M1)),t) - express(grad(M_1)) . diff(M_1,t))];

$$[[\text{S\_fm}_x, \text{S\_fm}_y, \text{S\_fm}_z] = 0] \quad (\% o162)$$

(%i163) cmt_ch04_4433:[q_fm = α*(μ_0/w_M) * ('diff(M_1,t) . 'diff(M_1,t))];

$$\left[q_{\text{fm}}=\frac{\left(\frac{d}{dt}\left[\mathrm{M1}_x,\mathrm{M1}_y,\mathrm{M1}_z\right].\frac{d}{dt}\left[\mathrm{M1}_x,\mathrm{M1}_y,\mathrm{M1}_z\right]\right)\alpha\mu_0}{w_M}\right] \quad (\%\ o163)$$

4.4.1.3 Contribution Of Drifting Charge Carriers in a Medium

(%i164) cmt_ch04_4434:[I_J= J_1 . E_1 , I_J = (1/2) * 'diff((E_1 . p_1 - B_1 . express(p_1 ~ v_0)) -express(div(express(E_1 ~ express(p_1 ~ v_0)))) ,t)] ;

$$[I_J=\mathrm{E1}_z\mathrm{J1}_z+\mathrm{E1}_y\mathrm{J1}_y+\mathrm{E1}_x\mathrm{J1}_x\,,I_J=\mathrm{diff}\backslash((-\frac{d}{dy}\left(\mathrm{E1}_z\left(\mathrm{p1}_y\mathrm{v0}_z-\mathrm{p1}_z\mathrm{v0}_y\right)-\mathrm{E1}_x\left(\mathrm{p1}_x\mathrm{v0}_y-\mathrm{p1}_y\mathrm{v0}_x\right)\right)-\frac{d}{dz}\left(\mathrm{E1}\right.$$
(% o164)

(%i165) cmt_ch04_4435: ['diff(((E_1. D_1) / 2) + ((H_1 . B_1) / 2),t), 'diff(((E_1 . D_1_p)/2) + ((H_1_p . B_1) / 2) , t)];

$$[\frac{d}{dt}\left(\frac{\mathrm{B1}_z\mathrm{H1}_z+\mathrm{B1}_y\mathrm{H1}_y+\mathrm{B1}_x\mathrm{H1}_x}{2}+\frac{\mathrm{D1}_z\mathrm{E1}_z+\mathrm{D1}_y\mathrm{E1}_y+\mathrm{D1}_x\mathrm{E1}_x}{2}\right),\frac{d}{dt}\left(\frac{\mathrm{B1}_z\mathrm{H1\_p}_z+\mathrm{B1}_y\mathrm{H1\_p}_y+\mathrm{B1}_x\mathrm{H1}}{2}\right.$$
(% o165)

(%i166) cmt_ch04_4336:[express(div(express(E_1 ~ H_1))), express(div(express(E_1 ~ H_1)))];

$$[\frac{d}{dx}\left(\mathrm{E1}_y\mathrm{H1}_z-\mathrm{E1}_z\mathrm{H1}_y\right)+\frac{d}{dy}\left(\mathrm{E1}_z\mathrm{H1}_x-\mathrm{E1}_x\mathrm{H1}_z\right)+\frac{d}{dz}\left(\mathrm{E1}_x\mathrm{H1}_y-\mathrm{E1}_y\mathrm{H1}_x\right),\frac{d}{dx}\left(\mathrm{E1}_y\mathrm{H1}_z-\mathrm{E1}_z\mathrm{H1}_y\right)+\frac{d}{dy}\left(\mathrm{E1}_z\right.$$
(% o166)

(%i167) cmt_ch04_4337:[w_pl = W_ek, w_pl = (m/(2*c)) * (v_1_p . 'diff(p_1,t) - p_1 . 'diff(v_1_p,t))];

$$\left[w_{\text{pl}}=W_{\text{ek}}\,,w_{\text{pl}}=\frac{m\left(\mathrm{v\_1}_p.\frac{d}{dt}\left[\mathrm{p1}_x,\mathrm{p1}_y,\mathrm{p1}_z\right]-\left[\mathrm{p1}_x,\mathrm{p1}_y,\mathrm{p1}_z\right].\frac{d}{dt}\mathrm{v\_1}_p\right)}{2c}\right]$$
(% o167)

(%i168) cmt_ch04_4338:[S_pl = S_ek + Sth, S_pl = (m/(2*e)) * v_0 * (v_1_p . 'diff(p_1,t) - p_1 . 'diff(v_1_p,t))+ (m/(2*e)) * (v_τ^2/p_0) * p_1 * 'diff(express(div(p_1)),t) - express(div(p_1))*'diff(p_1,t)];

$$[S_{\text{pl}}=\mathrm{Sth}+S_{\text{ek}}\,,S_{\text{pl}}=\left[\frac{m\mathrm{v0}_x}{2e},\frac{m\mathrm{v0}_y}{2e},\frac{m\mathrm{v0}_z}{2e}\right]\left(\mathrm{v\_1}_p.\frac{d}{dt}\left[\mathrm{p1}_x,\mathrm{p1}_y,\mathrm{p1}_z\right]-\left[\mathrm{p1}_x,\mathrm{p1}_y,\mathrm{p1}_z\right].\frac{d}{dt}\mathrm{v\_1}_p\right)+\left(\frac{d}{dt}\left[\mathrm{p}\right.\right.$$

(% o168)

(% i169) cmt_ch04_4339:[q_pl = (1/τ_0) * (m/(2*e)) * ((v_1 . 'diff(p_1,t)) - p_1 . 'diff(v_1,t)) - (f_1 *'diff(p_1,t) - p_1 * 'diff(f_1,t)) . v_0, f_1 = (τ_1 + r_1 . express(grad(τ_0))) /τ_0];

$$[q_{\mathrm{pl}} = \frac{m\left(v_1 . \frac{d}{dt}\left[\mathrm{p1}_x, \mathrm{p1}_y, \mathrm{p1}_z\right] - \left[\mathrm{p1}_x, \mathrm{p1}_y, \mathrm{p1}_z\right] . \frac{d}{dt} v_1\right)}{2 e \tau_0} - \left(f_1 \left(\frac{d}{dt}\left[\mathrm{p1}_x, \mathrm{p1}_y, \mathrm{p1}_z\right]\right) + \left[-\left(\frac{d}{dt} f_1\right) \mathrm{p1}_x, -\left(\frac{d}{dt}\right.\right.\right.$$

(% o169)

(% i170) cmt_ch04_4440:[w_L = -(e/(2*m)) * B_0,v_1_p = v_1 - (e/(2*m)) * express(r ~ B_0) , v_1_p = v_1 + express(r_1 ~ ((e/(2*m)) * B_0))];

$$[w_L = \left[-\frac{\mathrm{B0}_x e}{2m}, -\frac{\mathrm{B0}_y e}{2m}, -\frac{\mathrm{B0}_z e}{2m}\right], \mathrm{v\_1}_p = \left[v_1 - \frac{e\left(\mathrm{B0}_z r_y - \mathrm{B0}_y r_z\right)}{2m}, v_1 - \frac{e\left(\mathrm{B0}_x r_z - \mathrm{B0}_z r_x\right)}{2m}, v_1 - \frac{e\left(\mathrm{B0}_y r_x\right.}{2r}\right.$$

(% o170)

4.4.1.4 Small-signal power theorem for generalized space-dispersive active media

(% i171) cmt_ch04_4441:['diff(w_em + w_el + efm + w_pl,t) + express(div(S_em + S_el + S_fm + S_pl)) + (q_el + q_fm + q_pl) = 0] ;

$$[\frac{d}{dt}\left(w_{\mathrm{pl}} + w_{\mathrm{em}} + w_{\mathrm{el}} + \mathrm{efm}\right) + q_{\mathrm{pl}} + q_{\mathrm{fm}} + q_{\mathrm{el}} + \frac{d}{dz}\left(S_{\mathrm{pl}} + \mathrm{S\_fm}_z + S_{\mathrm{em}} + S_{\mathrm{el}}\right) + \frac{d}{dy}\left(S_{\mathrm{pl}} + \mathrm{S\_fm}_y + S_{\mathrm{em}} + S_{\mathrm{el}}\right) + \frac{d}{dx}$$

(% o171)

(% i172) cmt_ch04_4442:[w_em = ((E_1 . D_1_p) / 2) + ((H_1_p . B_1) /2)];

$$\left[w_{\mathrm{em}} = \frac{\mathrm{B1}_z \mathrm{H1\_p}_z + \mathrm{B1}_y \mathrm{H1\_p}_y + \mathrm{B1}_x \mathrm{H1\_p}_x}{2} + \frac{\mathrm{D1\_p}_z \mathrm{E1}_z + \mathrm{D1\_p}_y \mathrm{E1}_y + \mathrm{D1\_p}_x \mathrm{E1}_x}{2}\right]$$

(% o172)

(% i173) cmt_ch04_4443:[S_em = express(E ~ H_1_p)];

$$\left[S_{\mathrm{em}} = \left[E_y \mathrm{H1\_p}_z - E_z \mathrm{H1\_p}_y, E_z \mathrm{H1\_p}_x - E_x \mathrm{H1\_p}_z, E_x \mathrm{H1\_p}_y - E_y \mathrm{H1\_p}_x\right]\right]$$

(% o173)

Left out express(div()) here because avg_t is not yet definedavg_t(F, n) will be a time-averaged value of F over n time intervals)

(%i174) cmt_ch04_4444:[(div((avg_t(S_em) + avg_t(S_el) + avg_t(S_fm) + avg_t(S_pl)) + (avg_t(q_el) + avg_t(q_fm) + avg_t(q_pl)))) = 0];

$$[\,\mathrm{div}\left(\mathrm{avg}_t\left(q_{\mathrm{pl}}\right)+\mathrm{avg}_t\left(q_{\mathrm{fm}}\right)+\mathrm{avg}_t\left(q_{\mathrm{el}}\right)+\mathrm{avg}_t\left(S_{\mathrm{pl}}\right)+\mathrm{avg}_t\left(\left[\mathrm{S\_fm}_x\,,\mathrm{S\_fm}_y\,,\mathrm{S\_fm}_z\right]\right)+\mathrm{avg}_t\left(S_{\mathrm{em}}\right)+\mathrm{avg}_t\left(S_{\mathrm{el}}\right.\right. \tag{\% o174}$$

Elastic Properties of a MediumElastic properties of a medium contribute these terms to the time-averaged power relation (4.4.4.4)The average elastic energy flux density

(%i175) cmt_ch04_4445:[avg_t(S_el) = (1/2) * Re(-E_1_Σ . conjugate(U_1)), avg_t(S_el) = (1/2) * Re(V_1_a . conjugate(J_1_a))];

$$\left[\mathrm{avg}_t\left(S_{\mathrm{el}}\right)=\frac{\mathrm{Re}\left(-\mathrm{E\_1\_}.U_1\right)}{2}\,,\mathrm{avg}_t\left(S_{\mathrm{el}}\right)=\frac{\mathrm{Re}\left(\mathrm{V\_1}_a.\mathrm{J\_1}_a\right)}{2}\right] \tag{\% o175}$$

The average elastic power loss densityAuthor uses : for double-dot or tensor (term -by term) multiplicationNOTE: For now I use dot product but this needs to be addressed properlyRe can be defined to call realpart()

(%i176) Re(ev):=”realpart(ev);

$$\mathrm{Re}\left(\mathrm{ev}\right):=\mathrm{realpart}\left(\mathrm{ev}\right) \tag{\% o176}$$

(%i177) cmt_ch04_4446:[
avg_t(q_el,n) = (1/2) * Re(conjugate(S_1_dot_bar) . η_bar . S_1_dot_bar + p_m0 * conjugate(U1) . τˆ(-1) . U_1),
avg_t(q_el,n) = (ωˆ2/2) * conjugate(S_1) . η_bar . S_1 + (p_m0/2) * conjugate(U_1) . τˆ(-1) . U_1
];

$$[\,\mathrm{avg}_t\left(q_{\mathrm{el}}\,,n\right)=\frac{\mathrm{realpart}\left(\mathrm{U1}.\frac{1}{\tau}.U_1\right)p_{\mathrm{m0}}+\mathrm{realpart}\left(\mathrm{S\_1\_dot\_bar}.\begin{pmatrix}\eta_{11}&\eta_{12}&\eta_{13}\\\eta_{21}&\eta_{22}&\eta_{23}\\\eta_{31}&\eta_{32}&\eta_{33}\end{pmatrix}.\mathrm{S\_1\_dot\_bar}\right)}{2}\,,\mathrm{avg}_t\left(q\right. \tag{\% o177}$$

(%i178) cmt_ch04_4447:[J_1_a = %i*ω*u_1, J_1_a = U_1];

$$[\mathrm{J\_1}_a=\%i u_1\omega\,,\mathrm{J\_1}_a=U_1] \tag{\% o178}$$

(%i179) cmt_ch04_4448:[V_1_a = -T_1_Σ];

$$[\mathrm{V\_1}_a=-\mathrm{T\_1\_}\,] \tag{\% o179}$$

(%i180) cmt_ch04_4449:[avg_t(S_fm,n) = (1/2) * Re(%i*ω*μ_0*λ_ex * express(grad(M_1)) . conjugate(M_1)) , avg_t(S_fm,n) = (1/2) * Re(V_1_m . conjugate(J_1_m))];

$$[\mathrm{avg}_t\left(\left[\mathrm{S\_fm}_x , \mathrm{S\_fm}_y , \mathrm{S\_fm}_z\right] , n\right) = \frac{\left(-\left(\frac{d}{dz}\left[0 , 0 , 0\right]\right)\mathrm{M1}_z - \left(\frac{d}{dy}\left[0 , 0 , 0\right]\right)\mathrm{M1}_y - \left(\frac{d}{dx}\left[0 , 0 , 0\right]\right)\mathrm{M1}_x\right)\lambda\_\mathrm{ex}\mu_0}{2} \quad (\%\ \mathrm{o}180)$$

(%i181) cmt_ch04_4450:[avg_t(q_fm,n) = (1/2) * Re(α * (μ_0/ω_M) * ('diff(M_1,t) . 'diff(conjugate(M_1),t))) , avg_t(q_fm,n) = ν_M * (μ_0/2) * (ω/ω_M)^2 * abs(M_1)^2];

$$[\mathrm{avg}_t\left(q_{\mathrm{fm}} , n\right) = \frac{\mathrm{realpart}\left(\frac{d}{dt}\left[\mathrm{M1}_x , \mathrm{M1}_y , \mathrm{M1}_z\right] . \frac{d}{dt}\left[\mathrm{M1}_x , \mathrm{M1}_y , \mathrm{M1}_z\right]\right)\alpha\mu_0}{2\omega_M} , \mathrm{avg}_t\left(q_{\mathrm{fm}} , n\right) = \left[\frac{{\mathrm{M1}_x}^2\mu_0\nu_M\omega^2}{2{\omega_M}^2} , \mathrm{M1}_y \quad (\%\ \mathrm{o}181)$$

(%i182) cmt_ch04_4451:[J_1_m = %i*ω*μ_00*M_1];

$$[\mathrm{J\_1}_m = [\%i\mathrm{M1}_x\mu_{00}\omega , \%i\mathrm{M1}_y\mu_{00}\omega , \%i\mathrm{M1}_z\mu_{00}\omega]] \quad (\%\ \mathrm{o}182)$$

(%i183) cmt_ch04_4452:[V_1_m = -λ_ex * express(grad(M_1))];

$$[\mathrm{V\_1}_m = \left[-\left(\frac{d}{dx}\left[\mathrm{M1}_x , \mathrm{M1}_y , \mathrm{M1}_z\right]\right)\lambda\_\mathrm{ex} , -\left(\frac{d}{dy}\left[\mathrm{M1}_x , \mathrm{M1}_y , \mathrm{M1}_z\right]\right)\lambda\_\mathrm{ex} , -\left(\frac{d}{dz}\left[\mathrm{M1}_x , \mathrm{M1}_y , \mathrm{M1}_z\right]\right)\lambda\_\mathrm{ex}\right] \quad (\%\ \mathrm{o}183)$$

Plasma Properties of a MediumPlasma properties contribute these terms to the time-average power relation (4.4.44)(i) The average plasma energy flow density

(%i184) cmt_ch04_4453:[avg_t(S_pl,n) = avg_t(S_ek,n) + avg_t(S_th,n), avg_t(S_pl,n) = (1/2)* Re((m/e) * v_0*v_1_p . conjugate(%i*ω*p_1)) + (1/2) * Re(-(m/e) * (v_τ^2/p_0) * express(div(p_1)) * conjugate(i%ω*p-1)) ,
avg_t(S_pl,n) = (1/2) * Re(V_1_ek . conjugate(J_1_e)) + (1/2) * Re(V_1_th * conjugate(J_1_e)) ,
avg_t(S_pl,n) = (1/2) * Re(V_1_e . conjugate(J_1_e))];

$$[\mathrm{avg}_t\left(S_{\mathrm{pl}} , n\right) = \mathrm{avg}_t\left(S_{\mathrm{th}} , n\right) + \mathrm{avg}_t\left(S_{\mathrm{ek}} , n\right) , \mathrm{avg}_t\left(S_{\mathrm{pl}} , n\right) = \left(\mathrm{realpart}\left(\mathrm{v\_1}_p . \left[-\%i\mathrm{p1}_x\omega , -\%i\mathrm{p1}_y\omega , -\%i\mathrm{p1}_z\omega\right]\right.\right.$$

(% o184)

(ii) The average plasma power loss densitywhere μ_e = (e/m) is the static electron mobility andj_1 = (τ_1 + r_1 . express(grad(τ_0))) / τ_0andj_1 = (κ_0 - 1) * (E_0_v / E_0) . ((E_1 + express(v_0 ~ B-1)) / E_0)

(% i185) cmt_ch04_4454:[avg_t(q_pl,n) = (1/2) * Re((1/τ_0) * (m/e) * (v_1 - j_1 * v_0) . conjugate(%i*ω*p_1)), avg_t(q_pl,n) = (1/2) * Re((1/μ_e) * (v_1 - j_1 * v_0) . conjugate(J_1_e))];

$$\left[\mathrm{avg}_t\left(q_{\mathrm{pl}}, n\right) = 0, \mathrm{avg}_t\left(q_{\mathrm{pl}}, n\right) = \frac{\mathrm{realpart}\left(\left[v_1 - j_1 \mathrm{v0}_x, v_1 - j_1 \mathrm{v0}_y, v_1 - j_1 \mathrm{v0}_z\right] . \mathrm{J\_1}_e\right)}{2\mu_e}\right] \quad (\% \ o185)$$

Two new quantities in (4.4.53)A vector of the electronic polarization current densityj_1_e = %i*ω*p_1

(% i186) cmt_ch04_4455:[j_1_e = %i*ω*p_1];

$$\left[\mathrm{j\_1}_e = \left[\%i\mathrm{p1}_x\omega, \%i\mathrm{p1}_y\omega, \%i\mathrm{p1}_z\omega\right]\right] \quad (\% \ o186)$$

A tensor of the effective electronic potentialV_1_e = V_1_ek + V_1_th * I_bar = (m/e) * (v_0*v_1_p + (v_r2/p_0) * p_1 * I_bar)

(% i187) cmt_ch04_4456:[V_1_e = V_1_ek + V_1_th * I_bar, V_1_e = (m/e) * (v_0*v_1_p + (v_r^2/p_0) * p_1 * I_bar)];

$$\left[\mathrm{V\_1}_e = I_{\mathrm{bar}}\mathrm{V\_1\_th} + \mathrm{V\_1\_ek}, \mathrm{V\_1}_e = \left[\frac{m\left(\frac{I_{\mathrm{bar}}\mathrm{p1}_x {v_r}^2}{p_0} + \mathrm{v0}_x \mathrm{v\_1}_p\right)}{e}, \frac{m\left(\frac{I_{\mathrm{bar}}\mathrm{p1}_y {v_r}^2}{p_0} + \mathrm{v0}_y \mathrm{v\_1}_p\right)}{e}, \frac{m\left(\frac{I_{\mathrm{bar}}\mathrm{p1}}{p_0}\right.}{}\right.\right.$$

(% o187)

which involves the tensor of electrostatic potential

(% i188) cmt_ch04_4457:[V_1_ek = (m/e) * v_0 * v_1_p];

$$\left[\mathrm{V\_1\_ek} = \left[\frac{m\mathrm{v0}_x \mathrm{v\_1}_p}{e}, \frac{m\mathrm{v0}_y \mathrm{v\_1}_p}{e}, \frac{m\mathrm{v0}_z \mathrm{v\_1}_p}{e}\right]\right] \quad (\% \ o188)$$

and a scalar of the thermal potential (with unit dyadic I_bar)

(% i189) cmt_ch04_4458:[V_1_th = -(m/e) * (v_τ^2/p_0) * (express(div(p_1)) , V_1_th = ((k_B * T)/e)) * (p_1/p_0)];

$$\left[\mathrm{V\_1\_th} = \left(\left[-\frac{\mathrm{V\_1\_th}m\mathrm{p1}_x {v_\tau}^2}{e{p_0}^2}, -\frac{\mathrm{V\_1\_th}m\mathrm{p1}_y {v_\tau}^2}{e{p_0}^2}, -\frac{\mathrm{V\_1\_th}m\mathrm{p1}_z {v_\tau}^2}{e{p_0}^2}\right] = \left[-\frac{Tk_B m\mathrm{p1}_x {v_\tau}^2}{e^2{p_0}^2}, -\frac{Tk_B m\mathrm{p1}}{e^2 p_0}\right.\right.\right.$$

(%o189)

EM contribution to time-average power relation (4.4.44) according to generalized Poynting vecdtor (4.4.43) is:

(%i190) cmt_ch04_4459:[avg_t(S_em,n) = (1/2) * Re(express(E_1 ~ conjugate(H_1_p)))];

$$\left[\mathrm{avg}_t\left(S_{\mathrm{em}}, n\right)=\left[\frac{\mathrm{E1}_y \mathrm{H1\_p}_z-\mathrm{E1}_z \mathrm{H1\_p}_y}{2}, \frac{\mathrm{E1}_z \mathrm{H1\_p}_x-\mathrm{E1}_x \mathrm{H1\_p}_z}{2}, \frac{\mathrm{E1}_x \mathrm{H1\_p}_y-\mathrm{E1}_y \mathrm{H1\_p}_x}{2}\right]\right] \quad (\%o190)$$

Use the Helmholtz Decomposition Theorem to represent the EM field intensity vectorsas sum of curl and potential components

(%i191) cmt_ch04_4460:[E_1 = E_c1 - express(grad(ϕ_1)), H_1 = H_c1 - express(grad(ψ_1))];

$$\left[\left[\mathrm{E1}_x, \mathrm{E1}_y, \mathrm{E1}_z\right]=\left[E_{\mathrm{c1}}-\frac{d}{dx}\phi_1, E_{\mathrm{c1}}-\frac{d}{dy}\phi_1, E_{\mathrm{c1}}-\frac{d}{dz}\phi_1\right], \left[\mathrm{H1}_x, \mathrm{H1}_y, \mathrm{H1}_z\right]=\left[H_{\mathrm{c1}}-\frac{d}{dx}\psi_1, H_{\mathrm{c1}}-\frac{d}{dy}\psi_1, H\right. \quad (\%o191)$$

Modify S_em by replacing H_1 with H-1_p to get S_em = express(E_1 ~ H_1_p)From (4.3.48 and 4.3.61) it follows that:

(%i192) cmt_ch04_4461:[H_1_p = H_e1_p - express(grad(ψ_1)), H_c1_p = H_c1 - express(p_1 ~ v_0)] ;

$$\left[\left[\mathrm{H1\_p}_x, \mathrm{H1\_p}_y, \mathrm{H1\_p}_z\right]=\left[\mathrm{H\_e1}_p-\frac{d}{dx}\psi_1, \mathrm{H\_e1}_p-\frac{d}{dy}\psi_1, \mathrm{H\_e1}_p-\frac{d}{dz}\psi_1\right], \mathrm{H\_c1}_p=\left[-\mathrm{p1}_y \mathrm{v0}_z+\mathrm{p1}_z \mathrm{v0}_y\right. \quad (\%o192)$$

Use Maxwell's equations in dielectric form (4.43.48) and (4.3.61) along with (4.4.60) and (4.4.61)Also use the following vector identities

(%i194) A:[A_x,A_y,A_z];Φ:[Φ-x, Φ_y, Φ_z];

$$\left[A_x, A_y, A_z\right] \quad (\%o193)$$

$$\left[\Phi-x, \Phi_y, \Phi_z\right] \quad (\%o194)$$

(%i195) cmt_ch04_4461_A:[
express(curl(Φ*A)) = express(Φ*A ~ A) + express(express(grad(Φ)) ~ A) ,
express(div(express(curl(Φ*A)))) = 0,
express(curl(curl(Φ))) = 0
] ;

$$[\left[\frac{d}{dy}(A_z\Phi_z)-\frac{d}{dz}(A_y\Phi_y),\frac{d}{dz}(A_x(\Phi-x))-\frac{d}{dx}(A_z\Phi_z),\frac{d}{dx}(A_y\Phi_y)-\frac{d}{dy}(A_x(\Phi-x))\right]=[A_z\left(\frac{d}{dy}[\Phi-x,\Phi\right.$$ (%o195)

This gives rise to the relation:TODO: CREATE A VECTOR-VALUED VERSION OF avg_tTODO: USE express to evaluate curls, divergence, grad and cross-product

(%i196) cmt_ch04_4462:[(div(avg_t(S_em,n))) = avg_t(S_add,n) and avg_t(q_b,n) = (div((E_1 ~ h_1_P))) ,
div(avg_t(S_em,n)) = div(avg_t((E_c1 ~ H_c1_p) + ϕ-1 *'diff(D_1_p,t) + ψ_1 * 'diff(B_1,t) ,n))] ;

$$[\text{false}, \mathrm{div}(\mathrm{avg}_t(S_{\mathrm{em}},n))=\mathrm{div}\left(\mathrm{avg}_t\left(\left(\left(\frac{d}{dt}[\mathrm{B1}_x,\mathrm{B1}_y,\mathrm{B1}_z]\right)\psi_1+\phi+E_{\mathrm{c1}}\sim\mathrm{H\_c1}_p-\frac{d}{dt}[\mathrm{D1\_p}_x,\mathrm{D1\_p}_y,\mathrm{D}\right.\right.\right.$$ (%o196)

Substitute (4.4.62) into (44.44) to give:

(%i197) cmt_ch04_4463:[div(avg_t(S_el,n) + avg_t(S_add)) + avg_t(q_b,n) = 0] ;

$$[\mathrm{avg}_t(q_b,n)+\mathrm{div}(\mathrm{avg}_t(S_{\mathrm{el}},n)+\mathrm{avg}_t(S_{\mathrm{add}}))=0] \qquad (\%\text{o197})$$

where avg_t(S_add,n) and avg_t(q_b,n) are time-averaged values of (4.4.13) and (4.4.14)The constituents of avg_t(S_add,n) and avg_t(q_b,n) are obtained fromeqns(4.4.45),(4.4.46), (4.4.49),(4.4.50),(4.4.53),(4.4.54)Now avg_t(S_add,n) and avg_t(q_b,n) can be given as follows:Not using express for div curl grad cross-product (~) for nowUse express when vector quantities are properly defined

(%i198) cmt_ch04_4464:[
avg_t(S_add, n) = avg_t(S_el,n) + avg_t(S_fm,n) + avg_t(S_pl,n) ,
avg_t(S_add, n) = (1/2) * Re(V_1_a . conjugate(J_1_a) + V_1_m . conjugate(J_1_m) + V_1_e . conjugate(J_1_e))
];

$$[\mathrm{avg}_t(S_{\mathrm{add}},n)=\mathrm{avg}_t(S_{\mathrm{pl}},n)+\mathrm{avg}_t([\mathrm{S\_fm}_x,\mathrm{S\_fm}_y,\mathrm{S\_fm}_z],n)+\mathrm{avg}_t(S_{\mathrm{el}},n),\mathrm{avg}_t(S_{\mathrm{add}},n)=\frac{\mathrm{realpart}(\mathrm{V\_}}{}$$

(%o198)

(%i199) cmt_ch04_4465:[
avg_t(q_b,n) = avg_t(q_add,n),
avg_t(q_b,n) = avg_t(q_el,n) + avg_t(q_fm,n) + avg_t(q_pl,n),
avg_t(q_b,n) = (ω^2/2 * conjugate(S_1_bar) . η_bar . S_1_bar)
+ (p___m0/2) * conjugate(U_1) . τ^(-1) . U_1
+ v_M * (μ_0/2) * (ω/ω_M)^2 * abs(M_1)^2 + (1/2) * Re((1/μ_e) * (v_1 - j_1*v_0) . conjugate(J_1_e))
];

$$[\mathrm{avg}_t\left(q_b\,,n\right)=\mathrm{avg}_t\left(q_{\mathrm{add}}\,,n\right),\mathrm{avg}_t\left(q_b\,,n\right)=\mathrm{avg}_t\left(q_{\mathrm{pl}}\,,n\right)+\mathrm{avg}_t\left(q_{\mathrm{fm}}\,,n\right)+\mathrm{avg}_t\left(q_{\mathrm{el}}\,,n\right),\mathrm{avg}_t\left(q_b\,,n\right)=\frac{\Big(\mathrm{S\_1\_ba}}{}$$

(%o199)

(%i200) cmt_ch04_4466:[avg_t(S_em,n) = (1/2) * Re((E_c1 ~ conjugate(H_c1_p) + ϕ_1 * conjugate(%i*ω*D_1_p)) + ψ_1 * conjugate(%i*ω*B_1)),
avg_t(S_em,n) = avg_t(S_em_c,n) + avg_t(S_es,n) + avg_t(S_ms,n)];

$$[\mathrm{avg}_t\left(S_{\mathrm{em}}\,,n\right)=\left[\frac{\mathrm{realpart}\left(E_{\mathrm{c1}}\sim\mathrm{H\_c1}_p\right)}{2}\,,\frac{\mathrm{realpart}\left(E_{\mathrm{c1}}\sim\mathrm{H\_c1}_p\right)}{2}\,,\frac{\mathrm{realpart}\left(E_{\mathrm{c1}}\sim\mathrm{H\_c1}_p\right)}{2}\right],\mathrm{avg}_t\left(S_{\mathrm{em}}\,,n\right)=$$

(%o200)

(%i201) cmt_ch04_4467:[avg_t(S_em_c,n) = (1/2) * Re(E_c1 ~ conjugate(H_c1_p))];

$$\left[\mathrm{avg}_t\left(\mathrm{S\_em}_c\,,n\right)=\frac{\mathrm{realpart}\left(E_{\mathrm{c1}}\sim\mathrm{H\_c1}_p\right)}{2}\right] \quad (\%o201)$$

(%i202) cmt_ch04_4468:[avg_t(S_es,n) = (1/2) * Re(ϕ_1* conjugate(%i*ω*D_1_p))];

$$[\mathrm{avg}_t\left(S_{\mathrm{es}}\,,n\right)=[0\,,0\,,0]] \quad (\%o202)$$

(%i203) cmt_ch04_4469:[avg_t(S_ms,n) = (1/2) * Re(ψ_1*conjugate(%i*ω*B_1))] ;

$$[\mathrm{avg}_t\left(S_{\mathrm{ms}}\,,n\right)=[0\,,0\,,0]] \quad (\%o203)$$

4.4.2 Modal Transmission and Dissipation Of Power4.4.2.1 General Power Relations

(%i204) kill(S);

done (%o204)

(%i205) cmt_ch04_4469A:[div(avg_t(S_em,n) + avg_t(S_add,n))] ;

$$[\operatorname{div}(\operatorname{avg}_t(S_{\mathrm{em}}, n) + \operatorname{avg}_t(S_{\mathrm{add}}, n))] \quad (\%o205)$$

(%i206) cmt_ch04_4470:['integrate(div(avg_t(S_em,n) + avg_t(S_add,n)) ,S) = 'diff('integrate(e_z . avg_t(S_em,n),S) ,z) + 'diff('integrate(e_z . avg_t(S_add,n),S),z)
- 'sum('integrate(n_i_p . avg_t(S_em_p,n) + n_i_m . avg_t(S_em_m,n) ,L_i),i,1,N)
- 'sum('integrate(n_i_p . avg_t(S_add_p,n) + n_i_m . avg_t(S_add_m,n) ,L_i),i,1,N)
];

$$[S\operatorname{div}(\operatorname{avg}_t(S_{\mathrm{em}}, n) + \operatorname{avg}_t(S_{\mathrm{add}}, n)) = -L_i N\left(\mathrm{n\_i}_p.\operatorname{avg}_t\left(\mathrm{S\_em}_p, n\right) + \mathrm{n\_i}_m.\operatorname{avg}_t(\mathrm{S\_em}_m, n)\right) - L_i N\left(\mathrm{n\_i}_p\right. \quad (\%o206)$$

(%i207) cmt_ch04_4471:[Q_em_s = 'sum('integrate(n_i_p . avg_t(S_em_p,n) + n_i_m . avgs_t(S_em_m,n),S),i,1,N)];

$$\left[\mathrm{Q\_em}_s = NS\left(\mathrm{n\_i}_p.\operatorname{avg}_t\left(\mathrm{S\_em}_p, n\right) + \mathrm{n\_i}_m.\operatorname{avgs}_t(\mathrm{S\_em}_m, n)\right)\right] \quad (\%o207)$$

(%i208) cmt_ch04_4472:[Q_add_s = 'sum('integrate(n_i_p . avg_t(S_add_p,n) + n_i_m . avgs_t(S_add_m,n),S),i,1,N)];

$$\left[\mathrm{Q\_add}_s = NS\left(\mathrm{n\_i}_p.\operatorname{avg}_t\left(\mathrm{S\_add}_p, n\right) + \mathrm{n\_i}_m.\operatorname{avgs}_t(\mathrm{S\_add}_m, n)\right)\right] \quad (\%o208)$$

Contribution by a metallic boundary

(%i209) cmt_ch04_4473:[Q_em_s = (1/2) * 'integrate(R_s*(H_τ . conjugate(H_τ)), L)];

$$\left[\mathrm{Q\_em}_s = \frac{(H_\tau.H_\tau)\, LR_s}{2}\right] \quad (\%o209)$$

(%i210) cmt_ch04_4474:['diff(P(z),z)~ +~ Q(z)~ = 0];

$$\left[\frac{d}{dz}\,\mathrm{P}(z)+\mathrm{Q}(z)=0\right] \tag{\%o210}$$

(%i211) cmt_ch04_4475:[P~ = P_EM +~ P_PM, P~ = 'integrate(avg_t(S_em,n)~ . e_z, S) +~ 'integrate(avg_t(S_add,n)~ . e_z, S)];

$$[P=P_{\mathrm{PM}}+P_{\mathrm{EM}}\,,P=S\,\left(\mathrm{avg}_t\left(S_{\mathrm{em}}\,,n\right).e_z\right)+S\,\left(\mathrm{avg}_t\left(S_{\mathrm{add}}\,,n\right).e_z\right)] \tag{\%o211}$$

(%i212) cmt_ch04_4476:[
P_EM = 'integrate(avg_t(S_EM,n)~ .e_z ,S) ,
P_EM = 'integrate(avg_t(S_em,n)~ . e_z, S),
P_EM = (1/2)~ *~ Re('integrate((E_c1 ~ conjugate(H_c1_p) +~ ϕ_1 *~ conjugate(%i*ω*D_1_P) +~ ψ_1 *~ conjugate(%i*ω*B_1)). e_z,S))
];

$$[\,P_{\mathrm{EM}}=S\,\left(\mathrm{avg}_t\left(S_{\mathrm{EM}}\,,n\right).e_z\right),P_{\mathrm{EM}}=S\,\left(\mathrm{avg}_t\left(S_{\mathrm{em}}\,,n\right).e_z\right),P_{\mathrm{EM}}=\left(S\right)/2\,] \tag{\%o212}$$

(%i214) E_c1:[Ec1_x, Ec1_y, E_c1_z];
H_c1_p:[Hc1p_x, Hc1p_y, Hc1p_z];

$$[\mathrm{Ec1}_x\,,\mathrm{Ec1}_y\,,\mathrm{E\_c1}_z] \tag{\%o213}$$

$$\left[\mathrm{Hc1p}_x\,,\mathrm{Hc1p}_y\,,\mathrm{Hc1p}_z\right] \tag{\%o214}$$

(%i215) cmt_ch04_4477:[P_em = 'integrate(~ avt_t(S_em_c, n)~ . e_z, S), P_em = (1/2)~ *~ Re('integrate(express(E_c1 ~ conjugate(H_c1_p))~ . e_z, S))
];

$$[\,P_{\mathrm{em}}=S\,\left(\mathrm{avt}_t\left(\mathrm{S\_em}_c\,,n\right).e_z\right),P_{\mathrm{em}}=\frac{\mathrm{realpart}\left(\left[\mathrm{Ec1}_y\mathrm{Hc1p}_z-\mathrm{E\_c1}_z\mathrm{Hc1p}_y\,,\mathrm{E\_c1}_z\mathrm{Hc1p}_x-\mathrm{Ec1}_x\mathrm{Hc1p}_z\,,\mathrm{Ec1}\right.\right.}{2} \tag{\%o215}$$

(%i216) cmt_ch04_4478:[P_es = 'integrate(avg_t(S_es,n)~ . ez, S) , P_es = (1/2)~ *~ Re('integrate(conjugate(ψ_1(%i*ω*D_1_P)) . e_z, S))];

$$\left[P_{\mathrm{es}}=S\,\left(\mathrm{avg}_t\left(S_{\mathrm{es}}\,,n\right).\mathrm{ez}\right),P_{\mathrm{es}}=\frac{S\,\mathrm{realpart}\left(\overline{\psi_1\left(\%i\mathrm{D\_1}_P\omega\right)}.e_z\right)}{2}\right] \tag{\%o216}$$

(%i217) cmt_ch04_4479:[P_ms = 'integrate(avg_t(S_ms,n)~ . e_z, S), P_ms = (1/2)~ *~ Re(~ 'integrate(conjugate(ψ_1(%i*ω*B_1)) . e_z,S))] ;

$$\left[P_{\mathrm{ms}} = S\ \left(\mathrm{avg}_t\left(S_{\mathrm{ms}}\,,n\right).e_z\right), P_{\mathrm{ms}} = \frac{S\,\mathrm{realpart}\left(\overline{\psi_1\left([\%i\mathrm{B1}_x\omega\,,\%i\mathrm{B1}_y\omega\,,\%i\mathrm{B1}_z\omega]\right)}.e_z\right)}{2}\right]$$

(%o217)

(%i218) cmt_ch04_4480:[
P_PM = 'integrate(avg_t(S_add,n)~ . e_z, S),
P_PM= (1/2)~ *~ Re(~ 'integrate((V_1_a . conjugate(J_1_a)~ +~ V_1_m . conjugate(J_1_m)~ +~ V_1_e . conjugate(J_1_e)) . e_z,S))
];

$$\left[P_{\mathrm{PM}} = S\ \left(\mathrm{avg}_t\left(S_{\mathrm{add}}\,,n\right).e_z\right), P_{\mathrm{PM}} = \frac{S\,\mathrm{realpart}\left(\left(\mathrm{V\_1}_m.\mathrm{J\_1}_m + \mathrm{V\_1}_e.\mathrm{J\_1}_e + \mathrm{V\_1}_a.\mathrm{J\_1}_a\right).e_z\right)}{2}\right]$$

(%o218)

4480 involves 3 contributions

(%i219) cmt_ch04_4481:[P_el = 'ntegrate(avg_t(S_el,n)~ . e_z, s), P_el = (1/2)~ *~ Re(~ 'integrate((V_1_a . conjugate(J_1_a))~ . e_z,S))];

$$\left[P_{\mathrm{el}} = \mathrm{ntegrate}\left(\mathrm{avg}_t\left(S_{\mathrm{el}}\,,n\right).e_z\,,s\right), P_{\mathrm{el}} = \frac{S\,\mathrm{realpart}\left(\left(\mathrm{V\_1}_a.\mathrm{J\_1}_a\right).e_z\right)}{2}\right]$$

(%o219)

(%i220) cmt_ch04_4482:[P_fm = 'integrate(avg_t(~ S_fm,n)~ . e-z, S), P_fm = (1/2)~ *~ Re(~ 'integrate ((V_1_m . conjugate(J_1_m))~ . e_z, S))];

$$\left[P_{\mathrm{fm}} = S\ \left(\mathrm{avg}_t\left(\left[\mathrm{S\_fm}_x\,,\mathrm{S\_fm}_y\,,\mathrm{S\_fm}_z\right],n\right).e-z\right), P_{\mathrm{fm}} = \frac{S\,\mathrm{realpart}\left(\left(\mathrm{V\_1}_m.\mathrm{J\_1}_m\right).e_z\right)}{2}\right]$$

(%o220)

(%i221) cmt_ch04_4483: [
P_pl = 'integrate(avg_t(S_pl,n)~ . e_z, S),
P_pl = (1/2)~ *~ Re(~ 'integrate((V_1_e . conjguate(J_1_e)) . e_z,S))
];

$$\left[P_{\mathrm{pl}} = S\ \left(\mathrm{avg}_t\left(S_{\mathrm{pl}}\,,n\right).e_z\right), P_{\mathrm{pl}} = \frac{S\,\mathrm{realpart}\left(\left(\mathrm{V\_1}_e.\,\mathrm{conjguate}\left(\mathrm{J\_1}_e\right)\right).e_z\right)}{2}\right]$$

(% o221)

(% i222) cmt_ch04_4484:[Q = Q_b +~ Q_s , Q = integrate(avg_t(q_b,n)~ ,S) +~ integrate(avg_t(q_add_s,n),L)];

$$[Q = Q_s + Q_b \,, Q = S\mathrm{avg}_t\,(q_b\,,n) + L\mathrm{avg}_t\,(\mathrm{q\_add}_s\,,n)] \qquad (\%\ \mathrm{o222})$$

(% i223) cmt_ch04_4485:[
Q_b = 'integrate(avg_t(q_b,n),S)~ ,
Q_b = 'integrate(avg_t(q_add,n),S) ,
Q_b = 'integrate(avg_t(q_el,n)~ +~ avg_t(q_fm,n)~ +~ avg_t(q_pl,n),S),
Q_b =Q_el_b +~ Q_fm_b +~ Q_pl_b
];

$$[\,Q_b = S\mathrm{avg}_t\,(q_b\,,n)\,, Q_b = S\mathrm{avg}_t\,(q_{\mathrm{add}}\,,n)\,, Q_b = S\,(\mathrm{avg}_t\,(q_{\mathrm{pl}}\,,n) + \mathrm{avg}_t\,(q_{\mathrm{fm}}\,,n) + \mathrm{avg}_t\,(q_{\mathrm{el}}\,,n))\,, Q_b = \mathrm{Q\_pl}_b +$$

(% o223)

Note : (colon) double-dot notation which is a term-by-term tensor productis being replaced temporarily by standard multiplication. This is easyto implment in octave and matlab with the dotted operationsBulk losses from 4485 include contributions from:

(% i224) cmt_ch04_4486:[
Q_el_b = 'integrate(avg_t(q_el,n),s),
Q_el_b = (ω^2/2)~ *~ 'integrate(conjugate(S_1_bar)*~ η_bar *~ S_1_bar ,S)
+~
(ρ_m0~ /2) *~ 'integrate(conjugate(U_1)~ . (1/τ_bar). U_1,S)] ;

$$[\,\mathrm{Q\_el}_b = \mathrm{avg}_t\,(q_{\mathrm{el}}\,,n)\,s\,, \mathrm{Q\_el}_b = \begin{pmatrix} \frac{S\mathrm{S\_1\_bar}^2\eta_{11}\omega^2}{2} + \frac{S\left(U_1\cdot\frac{1}{\tau\_\mathrm{bar}}\cdot U_1\right)\rho\_\mathrm{m0}}{2} & \frac{S\mathrm{S\_1\_bar}^2\eta_{12}\omega^2}{2} + \frac{S\left(U_1\cdot\frac{1}{\tau\_\mathrm{bar}}\cdot U_1\right)\rho\_\mathrm{m0}}{2} \\ \frac{S\mathrm{S\_1\_bar}^2\eta_{21}\omega^2}{2} + \frac{S\left(U_1\cdot\frac{1}{\tau\_\mathrm{bar}}\cdot U_1\right)\rho\_\mathrm{m0}}{2} & \frac{S\mathrm{S\_1\_bar}^2\eta_{22}\omega^2}{2} + \frac{S\left(U_1\cdot\frac{1}{\tau\_\mathrm{bar}}\cdot U_1\right)\rho\_\mathrm{m0}}{2} \\ \frac{S\mathrm{S\_1\_bar}^2\eta_{31}\omega^2}{2} + \frac{S\left(U_1\cdot\frac{1}{\tau\_\mathrm{bar}}\cdot U_1\right)\rho\_\mathrm{m0}}{2} & \frac{S\mathrm{S\_1\_bar}^2\eta_{32}\omega^2}{2} + \frac{S\left(U_1\cdot\frac{1}{\tau\_\mathrm{bar}}\cdot U_1\right)\rho\_\mathrm{m0}}{2} \end{pmatrix}$$

(% o224)

(% i225) cmt_ch04_4487:[
Q_fm_b = 'integrate(avg_t(q_fm,n),S) , Q_fm_b = ν_M~ *~ (μ_0/2)~ *~ (ω/ω_M)^2~ *~ 'integrate(abs(M_1)^2,S)

];

$$\left[\mathrm{Q\_fm}_b = S\mathrm{avg}_t\,(q_{\mathrm{fm}}\,,n)\,, \mathrm{Q\_fm}_b = \left[\frac{\mathrm{M1}_x{}^2 S\mu_0\nu_M\omega^2}{2\omega_M{}^2}, \frac{\mathrm{M1}_y{}^2 S\mu_0\nu_M\omega^2}{2\omega_M{}^2}, \frac{\mathrm{M1}_z{}^2 S\mu_0\nu_M\omega^2}{2\omega_M{}^2}\right]\right]$$

(% o225)

(% i226) cmt_ch04_4488:[
Q_pl_b = 'integrate(avg_t(q_pl, n)~ , S),
Q_pl_b = (1/2)~ *~ Re('integrate((1/μ_e)~ *~ (v_1 - j_1 *~ v_0)~ . conjugate(J_1) ,S))

];

$$\left[\mathrm{Q\_pl}_b = S\mathrm{avg}_t\left(q_{\mathrm{pl}}, n\right), \mathrm{Q\_pl}_b = \frac{S\,\left(\mathrm{J1}_z\left(v_1 - j_1\mathrm{v0}_z\right) + \mathrm{J1}_y\left(v_1 - j_1\mathrm{v0}_y\right) + \mathrm{J1}_x\left(v_1 - j_1\mathrm{v0}_x\right)\right)}{2\mu_e}\right]$$

(% o226)

(% i227) cmt_ch04_4489:[

Q_s = 'integrate(avg_t(q_add_s, n)~ ,L),
Q_s = 'integrate(n . avg_t(S_add, n), L),
Q_s = integrate(n . (~ avg_t(S_el,n)~ +~ avg_t(S_fm,n)~ +~ avg_t(S_pl,n)) ,L) ,
Q_s = Q_el_s +~ Q_fm_s +~ Q_pl_s
];

$$\left[Q_s = L\mathrm{avg}_t\left(\mathrm{q\_add}_s, n\right), Q_s = L\,\left(n.\mathrm{avg}_t\left(S_{\mathrm{add}}, n\right)\right), Q_s = L\left(n.\left(\mathrm{avg}_t\left(S_{\mathrm{pl}}, n\right) + \mathrm{avg}_t\left(\left[\mathrm{S\_fm}_x, \mathrm{S\_fm}_y, \mathrm{S\_fm}\right.\right.\right.\right.$$

(% o227)

4489 Includes 3 contributionsQ_el_S from elastic mediia,Q_fm_s from surfaces of ferrimagnetic media,Q_pl_s from surfaces of charge carrier streams in plasmas

(% i228) cmt_ch04_4490:[
Q_el_s = integrate(avg_t(q_el_s,n),L),
Q_el_s = integrate(n . avg_t(S_el,n)~ ,L) ,
Q_el_s = (1/2)~ *~ Re(integrate((n . V_bar_1_a) . conjugate(J_1_a) ,L)) ,
Q_el_s = (1/2)~ *~ Re(integrate(- (n . T_1_Σ) . conjugate(U_1) ,L))
];

$$\left[\mathrm{Q\_el}_s = L\mathrm{avg}_t\left(\mathrm{q\_el}_s, n\right), \mathrm{Q\_el}_s = L\,\left(n.\mathrm{avg}_t\left(S_{\mathrm{el}}, n\right)\right), \mathrm{Q\_el}_s = \frac{L\,\mathrm{realpart}\left(\left(n.\mathrm{V\_bar\_1}_a\right).\mathrm{J\_1}_a\right)}{2}, \mathrm{Q\_el}_s =\right.$$

(% o228)

(%i229) cmt_ch04_4491:[
Q_fm_s = integrate(avg_t(q_fm_s,n),L),
Q_fm_s = integrate(n . avg_t(S_fm,n) ,L) ,
Q_fm_s = (1/2) * Re(integrate((n . V_bar_1_m) . conjugate(J_1_m) ,L)) ,
Q_fm_s = (1/2) * Re(integrate(%i*ω*μ_0*λ_ex(n . express(grad(M_1))) . conjugate(M_1)~ ,L))
];

$$[\,\mathrm{Q\_fm}_s = L\mathrm{avg}_t\left(\mathrm{q\_fm}_s\,, n\right), \mathrm{Q\_fm}_s = L\,\left(n.\mathrm{avg}_t\left(\left[\mathrm{S\_fm}_x\,, \mathrm{S\_fm}_y\,, \mathrm{S\_fm}_z\right], n\right)\right), \mathrm{Q\_fm}_s = \frac{L\,\mathrm{realpart}\left((n.\mathrm{V\_}\right.}{2} \quad (\%\,o229)$$

(%i230) cmt_ch04_4492:[
Q_pl_s = integrate(avg_t(q_pl_s,n),L),
Q_pl_s = integrate(n . avg_t(S_pl,n) ,L) ,
Q_pl_s = (1/2) * Re(integrate((n . V_bar_1_e) . conjugate(J_1_e) ,L))
,
Q_pl_s = (1/2) * Re(integrate(%i*ω*(m/e)~ *~ (v_r^2/p_0) *~ (n . p_v_1)~ *~ conjugate(p_1)~ ,L))
];

$$[\,\mathrm{Q\_pl}_s = L\mathrm{avg}_t\left(\mathrm{q\_pl}_s\,, n\right), \mathrm{Q\_pl}_s = L\,\left(n.\mathrm{avg}_t\left(S_{\mathrm{pl}}\,, n\right)\right), \mathrm{Q\_pl}_s = \frac{L\,\mathrm{realpart}\left(\left(n.\mathrm{V\_bar\_1}_e\right).\mathrm{J\_1}_e\right)}{2}, \mathrm{Q\_pl}_s = \quad (\%\,o230)$$

For free solid surface , For rigid solid surface

(%i232) cmt_ch04_4493: n . T_bar_1_Σ = 0;
cmt_ch04_4494: U_1 = 0
;

$$n.\mathrm{T\_bar\_1\_} = 0 \quad (\%\,o231)$$

$$U_1 = 0 \quad (\%\,o232)$$

(%i233) cmt_ch04_4494_A:~ [
n . V_bar_1_a = 0 ,
J_1_a = 0
];

$$[n.\mathrm{V\_bar\_1}_a = 0\,, \mathrm{J\_1}_a = 0] \quad (\%\,o233)$$

Boundary Conditions On Ferrite Surface For Calculation of Ferrimagnetic LossesTextbook ⟶ λ_ex(n .)~. M_1What exactly does this mean?TODO:~I~have split it into n . div(M_1) but this needs to be verified

(%i234) cmt_ch04_4494_B:[
n . V_bar_1_m = -λ_ex(n . div(M_1)),
j_1_M = %I* ω*~ μ_0*~ m_1
];

$$[n.\mathrm{V\_bar\_1}_m = -\lambda\mathrm{\_ex}\left(n.\operatorname{div}\left([\mathrm{M1}_x, \mathrm{M1}_y, \mathrm{M1}_z]\right)\right), \mathrm{j\_1}_M = \%\mathrm{I}m_1\mu_0\omega] \quad (\%\ o234)$$

For free surface spins

(%i235) cmt_ch04_4495: ρ_1 = 0;

$$\rho_1 = 0 \quad (\%\ o235)$$

For rigid surface spins

(%i236) cmt_ch04_4496: n . p_v_1 = 0;

$$n.\mathrm{p\_v}_1 = 0 \quad (\%\ o236)$$

4.4.97 and 4.4.98 ensure the following conditions on the unperturbed carrier stream boundaryThe result is that the countour integral in (4.4.92) vanishes so Q_pl_s = 0This shows that there are no plasma surface losses

(%i237) cmt_ch04_4498_A:[
n . V_bar_1_e = n *~ V_1_th, n . V_bar_1_e = 0,
n . J_1_e = 0 , n . v_0 = 0
];

$$[n.\mathrm{V\_bar\_1}_e = \mathrm{V\_1\_th}n, n.\mathrm{V\_bar\_1}_e = 0, n.\mathrm{J\_1}_e = 0, n.[\mathrm{v0}_x, \mathrm{v0}_y, \mathrm{v0}_z] = 0] \quad (\%\ o237)$$

(%i238) cmt_ch04_4499:[
a_k(z)~ = A_k *~ %e^(-γ_k*z) , a_k(z) = A_k *~ %e^(-α_k*z) *~ %e ^(-%i*β_k *z)
];

$$\left[a_k(z) = A_k\%e^{-z\gamma_k}, a_k(z) = A_k\%e^{-\%iz\beta_k - z\alpha_k}\right] \quad (\%\ o238)$$

(%i239) cmt_ch04_44100:[
Φ_1(r_t,z) = 'sum(A_k *~ Φ_k(r_t,z) , k,1,N),
Φ_1(r_t,z)~ = 'sum (A_k *~ Φ_k_bar(r_t)~ *~ %e^(-γ_k*z), k,1,N),
Φ_1(r_t,z)~ = 'sum(a_k(z)~ *~ Φ_k_bar(r_t), k, 1,N)
];

$$\left[\Phi_1\left(r_t\,,z\right)=A_k N \Phi_k\left(r_t\,,z\right),\Phi_1\left(r_t\,,z\right)=A_k N\ \text{\_k\_bar}\left(r_t\right)\%e^{-z\gamma_k},\Phi_1\left(r_t\,,z\right)=N\ \text{\_k\_bar}\left(r_t\right)a_k(z)\right] \quad (\%\text{o}239)$$

For the total power flow P defined in (4.4.75)~

(%i240) cmt_ch04_44101:[
P(z) = P_EM(z) + P_PM(z) ,
P(z) = sum(sum(P_kl(z), L, 1,N),K,1,N) , (1/2)*Re(sum(sum(N_kl * conjugate(a_k(z)) * a_l(z),l, 1,m),k,1,n)) ,
P(z) = (1/4) * sum(sum(N_kl * conjugate(a_k(z)) * conjugate(a_l(z)), l,1,m), k,1,n) ,
P(z) = (1/4) * sum(N_k*abs(a_k(z))^2,k,1,n) + (1/2)*Re(sum(sum(N_kl * conjugate(a_k(z)) * a_l(z),l, 1,m),k,1,n)) ,
P(z) = sum(P_k(z),k,1,n) + sum(sum(Q_kl_pair(z) ,l,1,m),k,1,n)
];

$$[\,\mathrm{P}(z)=P_{\mathrm{PM}}(z)+P_{\mathrm{EM}}(z)\,,\mathrm{P}(z)=N^2P_{\mathrm{kl}}(z)\,,\frac{N_{\mathrm{kl}}mn\,\left(\mathrm{realpart}\left(a_k(z)\right)\mathrm{realpart}\left(a_l(z)\right)+\mathrm{imagpart}\left(a_k(z)\right)\mathrm{imagp}\right.}{2} \quad (\%\text{o}240)$$

For the total power loss Q defined in (4.4.84)~

(%i241) cmt_ch04_44102: [
Q(z)~ = Q_b(z)~ +~ Q_s(z)~ ,
Q(z)~ = sum(sum(P_kl(z)~ , l,1,m),k,1,n),
Q(z)~ = (1/4)~ *~ sum(sum(M_kl *~ conjugate(a__k(z))~ *~ conjugate(a_l(z)) , l,1,m),k,1,n) ,
Q(z)~ = (1/4)~ *~ sum(~ sum(~ M_k *~ abs(a_k(z)) ^2,l,1,m),k,1,n) +~ (1/2)~ *~ Re(sum(sum(~ M_kl *~ conjugate(a_k(z)~)~ *~ a_l(z), l,1,m),k,1,n)) ,
Q(z)~ = sum(Q_k(z), k,1,n)~ +~ sum(sum(Q_kl_pair(z),l,1,m),k,1,n)

];

$$[\,\mathrm{Q}(z)=Q_s(z)+Q_b(z)\,,\mathrm{Q}(z)=mnP_{\mathrm{kl}}(z)\,,\mathrm{Q}(z)=\frac{M_{\mathrm{kl}}mn\overline{\mathrm{a\_}_k(z)a_l(z)}}{4}\,,\mathrm{Q}(z)=\frac{M_{\mathrm{kl}}mn\,\left(\mathrm{realpart}\left(a_k(z)\right)\mathrm{realpar}\right.}{} \quad (\%\text{o}241)$$

(%i242) cmt_ch04_44103:[
N_kl = conjugate(N_lk), M_kl = conjugate(M_lk)
];

$$[N_{\mathrm{kl}} = N_{\mathrm{lk}}\,, M_{\mathrm{kl}} = M_{\mathrm{lk}}] \qquad (\%\ o242)$$

(%i243) cmt_ch04_44104:[
N_kl= N_kl_EM +~ N_kl_PM
];

$$[N_{\mathrm{kl}} = \mathrm{N\_kl\_PM} + \mathrm{N\_kl\_EM}] \qquad (\%\ o243)$$

(%i244) cmt_ch04_44105:[
N_kl_EM = integrate(
(
conjugate(E_bar_k)~ ~ H_bar_l_P)
+~
E_bar_l ~ conjugate(H_bar_ck_p)~
,S,1,N)~
+~
(~
conjugate(ϕ_hat_k(%i*ω*D_hat_l_p))
+~
conjugate(ϕ_hat_l(%i*ω*D_hat_k_p))
)
+
(
conjugate(ψ_hat_k(%i*ω*B_hat_l))
+~
conjugate(ψ_hat_l(%i*ω*B_hat_k))
. e_z, S)
];

$$[\mathrm{N\_kl\_EM} = \overline{\phi\mathrm{\_hat}_k\left(\%i\mathrm{D\_hat\_l}_p\omega\right)} + \overline{\phi\mathrm{\_hat}_l\left(\%i\mathrm{D\_hat\_k}_p\omega\right)} + S + \left(\mathrm{E\_bar}_l \sim \mathrm{H\_bar\_ck}_p + \mathrm{E\_bar}_k \sim \mathrm{H\_} \qquad (\%\ o244)$$

(%i245)

```
cmt_ch04_44106:[
N_kl_EM = 'integrate(
(
(conjugate(E_ck_hat)~ ~ Hn_l_p_hat) +~ (E_l_hat ~ conjugate(H_k_p_hat)))
+
(conjugate(φ_k_hat(%I*ω*D_l_p_hat))~ +~ conjugate(φ_l_hat(%i*ω*D_k_p_hat)))~
+
(conjugate(ψ_k_hat(%i*ω*B_l_hat))~ +~ conjugate(ψ_l_hat(%i*ω*B_k_hat))
)
. e_z, S )
];
```

$$[\text{N\_kl\_EM} = S\,(\,\left(\overline{\psi\text{\_k\_hat}\,(\%i\text{B\_l\_hat}\omega)} + \overline{\psi\text{\_l\_hat}\,(\%i\text{B\_k\_hat}\omega)}\right).e_z + \overline{\phi\text{\_k\_hat}\,(\%\text{I}\,\text{D\_l\_p\_hat}\omega)} + \overline{\phi\text{\_}$$

(% o245)

(%i246)

```
cmt_ch04_44107:[
N_kl_PM = 'integrate( ( ((conjugate(V_k_a_bar_hat) . J_l_a_hat) + (V_l_a_bar_hat . conjugate(J_k_a_hat)))
+
(conjugate( V_m_k_bar_hat) . J_l_m_hat)~ + (V_l_m_bar_hat~ . conjugate(J_k_m_hat))
+
(conjugate(V_k_e_bar_hat)~ . J_l_e_hat)~ +~ (V_l_e_bar_hat . conjugate(J_k_e_hat)))~ . e_z, S )
];
```

$$[\text{N\_kl\_PM} = S\,(\,(\,\text{V\_m\_k\_bar\_hat.J\_l\_m\_hat}+\text{V\_l\_m\_bar\_hat.J\_k\_m\_hat}+\text{V\_l\_e\_bar\_hat.J\_k\_e\_}$$

(% o246)

(%i247)

```
cmt_ch05_44108:[
M_kl = M_kl_b +~ M_kl_s
];
```

$$[M_{\text{kl}} = \text{M\_kl}_s + \text{M\_kl}_b] \qquad (\%\ o247)$$

Here he uses colon (:) for the term-by-term tensor productMaxima will not accept that syntax, will create function tbyt or prdt to do thisFor now using dot product or multiplication, but this has to be changed later.INCOMPLETE cmt_ch04_44109~

(%i248) cmt_ch04_44109:[
M_kl_b = 2 *~ 'integrate((ω^2 .~ (~ conjugate(S_k_bar_hat) . η_bar . S_l_bar_hat)) ,S)
];

$$\left[\mathrm{M\_kl}_b = 2S \left(\omega^2 . \mathrm{S\_k\_bar\_hat} . \begin{pmatrix} \eta_{11} & \eta_{12} & \eta_{13} \\ \eta_{21} & \eta_{22} & \eta_{23} \\ \eta_{31} & \eta_{32} & \eta_{33} \end{pmatrix} . \mathrm{S\_l\_bar\_hat} \right) \right] \quad (\%\ o248)$$

(%i249) cmt_ch04_44110:[
M_kl_s = - 'integrate((((n . conjugate(T_hat_k_Σ)) . U_hat_t +~ (n . T_hat_l_Σ)~ . conjugate(U_hat_k))
+
λ_ex *~ (~ (n . grad(conjugate(M_hat_k)))~ . (%i*ω*μ_0 *~ M_hat_t)~ +~ (n . grad(M_hat_t)) . conjugate(%i*ω*μ_0*M_hat_k))
+
(m/e)~ *~ (v_r ^2 / ρ_0)~ *~ (~ (n . conjugate(p_hat_k))~ . ~ (%i*ω*ρ_hat_l)~ +~ (n . ρ_hat_l)~ . conjugate(%i*ω*ρ_bar_k))
) ,S)~
];

$$[\mathrm{M\_kl}_s = -S \left(\frac{m\ (\%i\,((n.\mathrm{p\_hat}_k)\,.\rho\mathrm{\_hat}_l\omega) - \%i\,((n.\rho\mathrm{\_hat}_l)\,.\rho\mathrm{\_bar}_k\omega))\,{v_r}^2}{e\rho_0} + (\%i\,((n.\,\mathrm{grad}\,(\mathrm{M\_hat}_k))\,.\mathrm{M\_h} \right. \quad (\%\ o249)$$

–>

(%i250) cmt_ch04_44111:[P_k(z) = P_kk(z), P_kk(z) = 1/4 *~ N_kk . conjugate(a_k(z)) *~ a_k(z)];

$$\left[P_k(z) = P_{\mathrm{kk}}(z)\,, P_{\mathrm{kk}}(z) = \frac{\left(N_{\mathrm{kk}} . \overline{a_k(z)} \right) a_k(z)}{4} \right] \quad (\%\ o250)$$

(%i251) cmt_ch04_44112:[Q_k(z)~ = Q_kk(z), Q_kk(z)~ = 1/4 * M_kk . conjugate(a_k(z)) *~ a_k(z)];

$$\left[Q_k(z) = Q_{\mathrm{kk}}(z)\,, Q_{\mathrm{kk}}(z) = \frac{\left(M_{\mathrm{kk}} . \overline{a_k(z)} \right) a_k(z)}{4} \right] \quad (\%\ o251)$$

(%i252) cmt_ch04_44113:[P_kl(z) = conjugate(P_lk(z))~ , P_kl(z) = (1/4)~ *~ N_kl *~ conjugate(a_k(z)~) *~ a_l(z)]
;

$$\left[P_{\mathrm{kl}}(z)=\overline{P_{\mathrm{lk}}(z)}\,,P_{\mathrm{kl}}(z)=\frac{N_{\mathrm{kl}}a_l(z)\overline{a_k(z)}}{4}\right] \quad (\%\ o252)$$

(%i253) cmt_ch04_44114:[Q_kl(z) = conjugate(Q_lk(z)) ,Q_kl(z) = (1/4) * M_kl * conjugate(a_k(z)) * a_l(z)];

$$\left[Q_{\mathrm{kl}}(z)=\overline{Q_{\mathrm{lk}}(z)}\,,Q_{\mathrm{kl}}(z)=\frac{M_{\mathrm{kl}}a_l(z)\overline{a_k(z)}}{4}\right] \quad (\%\ o253)$$

Example Using tex - Output Latex From Derived ExpressionsLatex strings can be created using maxima and the wxmaxima notebook gets saved as pdflatex.Then the exported notebook with latex can be processed by the pdflatex command to generate documents.You can also call maxima from SageMath or Maxima-Jupyter and export those notebooks in pdf or latex format.SageTex also works well and can be integrated into TexMaker and TexStudio for a full publishing environment

(%i256) tex([Q_kl(z) = conjugate(Q_lk(z))] ,false);
tex([Q_kl(z) = conjugate(Q_lk(z))],false) ;
tex([Q_kl(z) = (1/4) * M_kl * conjugate(a_k(z)) * a_l(z)],false) ;

(% o254)

"$ $

\left[{\itQ_kl}\left(z\right) = {\itQ_lk}\left(z\right)^\star\right]$$"

(% o255)

"$ $ \left[{\itQ_kl}\left(z\right) = {\itQ_lk}\left(z\right)^\star\right]$$"

(% o256)

"$ $ \left[{\itQ_kl}\left(z\right) = {{{\itM_kl}\,{\ita_l}\left(z\right)\,{\ita_k}\left(z\right)^\

(%i257) cmt_ch04_44115:[P_kl_pair(z) = P_kl(z)~ +~ P_lk(z) , P_kl_pair(z) = (1/2)~ *~ Re(N_kl *~ conjugate(a_k(z)) *~ a_l(z))]
;

$$[\mathrm{P\_kl\_pair}(z)=P_{\mathrm{lk}}(z)+P_{\mathrm{kl}}(z)\,,\mathrm{P\_kl\_pair}(z)=\frac{N_{\mathrm{kl}}\left(\mathrm{realpart}\left(a_k(z)\right)\mathrm{realpart}\left(a_l(z)\right)+\mathrm{imagpart}\left(a_k(z)\right)\mathrm{imag}}{2}$$

(% o257)

(% i258) cmt_ch04_4416:[Q_kl_pair(z) = Q_kl(z) +~ Q_lk(z) , Q_kl_pair(z) = 1/2 * Re(M_kk . conjugate(a_k(z)) * a_k(z)~)] ;

$$[\,\mathrm{Q\_kl\_pair}(z) = Q_{\mathrm{lk}}(z)+Q_{\mathrm{kl}}(z)\,, \mathrm{Q\_kl\_pair}(z) = \frac{\mathrm{realpart}\left(M_{\mathrm{kk}}.\overline{a_k(z)}\right)\mathrm{realpart}\left(a_k(z)\right) - \mathrm{imagpart}\left(M_{\mathrm{kk}}.\overline{a_k(}\right.}{2} \quad (\%\ o258)$$

4.5 Development of SDAM Excitation Theory For SDAM Waveguides4.5.1 Generalized Reciprocity Theorem In Complex Conjugate FormIn the region of exciting sources in the bulk material Maxwell's equations take this form:

(% i259) cmt_ch04_451:[express(curl(E_1)) = -%i*ω*B_1 - J_b1_m];

$$[\left[\frac{d}{dy}\mathrm{E1}_z - \frac{d}{dz}\mathrm{E1}_y\,, \frac{d}{dz}\mathrm{E1}_x - \frac{d}{dx}\mathrm{E1}_z\,, \frac{d}{dx}\mathrm{E1}_y - \frac{d}{dy}\mathrm{E1}_x\right] = \left[-\%i\mathrm{B1}_x\omega - \mathrm{J\_b1m}_x\,, -\%i\mathrm{B1}_y\omega - \mathrm{J\_b1m}_y\,, -\%i\mathrm{B}\right. \quad (\%\ o259)$$

(% i260) cmt_ch04_452:[express(curl(~ H_1_p))~ = %i*ω*D_1_p +~ J_b1_e];

$$[\left[\frac{d}{dy}\mathrm{H1\_p}_z - \frac{d}{dz}\mathrm{H1\_p}_y\,, \frac{d}{dz}\mathrm{H1\_p}_x - \frac{d}{dx}\mathrm{H1\_p}_z\,, \frac{d}{dx}\mathrm{H1\_p}_y - \frac{d}{dy}\mathrm{H1\_p}_x\right] = \left[\%i\mathrm{D1\_p}_x\omega + \mathrm{J\_b1e}_x\,, \%i\mathrm{D1\_p}\right. \quad (\%\ o260)$$

(% i261) cmt_ch04_453:[express(div(D_1_p))~ = ρ_b1_e];

$$\left[\frac{d}{dz}\mathrm{D1\_p}_z + \frac{d}{dy}\mathrm{D1\_p}_y + \frac{d}{dx}\mathrm{D1\_p}_x = \rho\mathrm{\_b1}_e\right] \quad (\%\ o261)$$

(% i262) cmt_ch04_454:[express(div(B_1))~ = ρ_b1_m];

$$\left[\frac{d}{dz}\mathrm{B1}_z + \frac{d}{dy}\mathrm{B1}_y + \frac{d}{dx}\mathrm{B1}_x = \rho\mathrm{\_b1}_m\right] \quad (\%\ o262)$$

The charge and current densities (electric and magnetic) are related to each other by:

(%i263) cmt_ch04_455:[
%i*ω*ρ_b1_e +~ express(div(J_b1_e)) = 0,
%i*ω*ρ_b1_m +~ express(div(J_b1_m))~ = 0
];

$$[\%i\rho\_b1_e\omega+\frac{d}{dz}\mathrm{J\_b1e}_z+\frac{d}{dy}\mathrm{J\_b1e}_y+\frac{d}{dx}\mathrm{J\_b1e}_x=0\,,\%i\rho\_b1_m\omega+\frac{d}{dz}\mathrm{J\_b1m}_z+\frac{d}{dy}\mathrm{J\_b1m}_y+\frac{d}{dx}\mathrm{J\_b1m}_x=0\,]$$

(% o263)

The field-intensity vectors E_1, H_1_p = H_1 - p_1 x v_0 are linked with the flux-density vectors D_1_p and B_1 by:

(%i264) cmt_ch05_456:[D_1_p = ϵ_0 *~ E_1 +~ P_net, D_1_p = ϵ_0 *~ E_1 +~ (P_1 +~ p_1)];

$$[\,[\mathrm{D1\_p}_x\,,\mathrm{D1\_p}_y\,,\mathrm{D1\_p}_z]=[\mathrm{E1}_x\epsilon_0+\mathrm{Pnet}_x\,,\mathrm{E1}_y\epsilon_0+\mathrm{Pnet}_y\,,\mathrm{E1}_z\epsilon_0+\mathrm{Pnet}_z]\,,[\mathrm{D1\_p}_x\,,\mathrm{D1\_p}_y\,,\mathrm{D1\_p}_z]=[\mathrm{E}$$

(% o264)

–>

(%i265) cmt_ch05_457:[B+1 = μ_0 *~ (H_1_p +~ M_net)~ , B_1 = μ_0~ *~ (H_1 +~ M_1)];

$$[\,[B_x+1\,,B_y+1\,,B_z+1]=\left[(\mathrm{Mnet}_x+\mathrm{H1\_p}_x)\,\mu_0\,,\left(\mathrm{Mnet}_y+\mathrm{H1\_p}_y\right)\mu_0\,,(\mathrm{Mnet}_z+\mathrm{H1\_p}_z)\,\mu_0\right],[\mathrm{B1}_x\,,\mathrm{B1}_y\,,\mathrm{B}$$

(% o265)

where P_1, M_1 and p_1 describe physical properties of the medium being analyzed.To obtain the conjugate reciprocity theorem it is is necessary to consider another (small-signal)~system denoted with subscript 2The dynamic equations of this second system all take the complex conjugate form initially used for the BAM analysis.The conventional procedure applied to combine the two systems of equations gives equation (3.5.27).Equation (3.5.27)~is re-written here using equations (4.5.6) and (4.5.7) as follows:

(%i266) cmt_ch04_458:[
express(div(express(E_1 ~ conjugate(H_2_p)) + express(conjugate(E_2) ~ H_1_p))) =
-%i*ω* (P_1 . conjugate(E_2))~ - (conjugate(P_2)~ . E_1)
- %i*ω* (~ (M_1 . conjugate(H_2))~ - (conjugate(M_2) . H_1))
-%i *ω*~ ((~ p_1 . conjugate(E_2_primed)) - (conjugate(p_2) . E_1_primed))
- (~ (J_b1_e . conjugate(E_2))~ +~ (conjugate(J___b2_e) . E_1))
- ~ (J_b1_m . conjugate(H_2_p))~ +~ (conjugate(J_b2_m)~ . H_1_p)
];

$$[\,\frac{d}{dx}\left(\mathrm{E1}_y\mathrm{H2p}_z-\mathrm{E1}_z\mathrm{H2p}_y+\mathrm{E2}_y\mathrm{H1\_p}_z-\mathrm{E2}_z\mathrm{H1\_p}_y\right)+\frac{d}{dy}\left(-\mathrm{E1}_x\mathrm{H2p}_z+\mathrm{E1}_z\mathrm{H2p}_x-\mathrm{E2}_x\mathrm{H1\_p}_z+\mathrm{E2}_z\mathrm{H1\_p}_x\right.$$

(% o266)

where the electric fields E_1_primed = E_1 +~ v_0 x B_1 and E_2_primed = E_2 +~v_0 x B_2 acmt_re measured relative to anobserver moving with the non-relativistic velocity v_0

(% i268) S_12:[S_12_x, S_12_y, S_12_z];
H_2_p:[H_2p_x,H_2p_y, H_2p_z];

$$[\mathrm{S\_12}_x\,, \mathrm{S\_12}_y\,, \mathrm{S\_12}_z] \tag{\% o267}$$

$$[\mathrm{H\_2p}_x\,, \mathrm{H\_2p}_y\,, \mathrm{H\_2p}_z] \tag{\% o268}$$

(% i269) cmt_ch04_459:[express(div(S_12))~ +~ q_12_b = r_12_b];

$$\left[\mathrm{q\_12}_b + \frac{d}{dz}\mathrm{S\_12}_z + \frac{d}{dy}\mathrm{S\_12}_y + \frac{d}{dx}\mathrm{S\_12}_x = \mathrm{r\_12}_b\right] \tag{\% o269}$$

(% i270) cmt_ch04_4510:[S_12 = S_12_EM +~ S_12_PM];

$$[[\mathrm{S\_12}_x\,, \mathrm{S\_12}_y\,, \mathrm{S\_12}_z] = \mathrm{S\_12\_PM} + \mathrm{S\_12\_EM}] \tag{\% o270}$$

(% i271) cmt_ch04_4511:[
S_12_EM~ = express(E_1 ~ conjugate(H_2_p))~ +~ express(conjugate(E_2)~ ~ H_1_p)
];

$$[\mathrm{S\_12\_EM} = [\mathrm{E1}_y\mathrm{H\_2p}_z - \mathrm{E1}_z\mathrm{H\_2p}_y + \mathrm{E2}_y\mathrm{H1\_p}_z - \mathrm{E2}_z\mathrm{H1\_p}_y\,, -\mathrm{E1}_x\mathrm{H\_2p}_z + \mathrm{E1}_z\mathrm{H\_2p}_x - \mathrm{E2}_x\mathrm{H1\_p}_z + \mathrm{E2}_z\mathrm{H} \tag{\% o271}$$

(% i272) cmt_ch04_4512:[
S_12_PM~ = (V_1_a . conjugate(J_2_a))~ +~ (conjugate(V_2_a)~ . J_1_a)
];

$$[\mathrm{S\_12\_PM} = \mathrm{V\_2}_a.\mathrm{J\_1}_a + \mathrm{V\_1}_a.\mathrm{J\_2}_a] \tag{\% o272}$$

(%i273) cmt_ch04_4512:[
S_12_PM = ((V_bar_1_a . conjugate(J_2_a)) + (conjugate(V_bar_2_a) . J_1_a))
+
((V_bar_1_m . conjugate(J_2_m)) +~ (conjugate(V_bar_2_m)~ . J_1_m))
+
((v_bar_1_e . conjugate(J_2_e))~ +~ (conjugate(V_bar_2_e)~ . J_1_e))

];

$$[\mathrm{S\_12\_PM} = \mathrm{v\_bar\_1}_e.\mathrm{J\_2}_e + \mathrm{V\_bar\_2}_m.\mathrm{J\_1}_m + \mathrm{V\_bar\_2}_e.\mathrm{J\_1}_e + \mathrm{V\_bar\_2}_a.\mathrm{J\_1}_a + \mathrm{V\_bar\_1}_m.\mathrm{J\_2}_m + \mathrm{V\_}$$

(%o273)

(%i274) cmt_ch04_4513:[
q_12_b = 2 *~ (ω^2 *~ S_bar_1 .η_bar . conjugate(S_2))~ +~ (ρ_m0 *~ U_1 . (1/τ)~ . conjugate(U_2))
+
2*ν_M~ *~ μ_0 *~ (ω/ω_M)^2 *~ (M_1 . conjugate(M_2))
+
(1/μ_0)~ *~ (~ (~ v_1 - G_1*v_0)~ . conjugate(J_2_e) +~ (conjugate(v_2)~ - conjugate(G_2)*v_0)~ . J_1_e)
];

$$[\mathrm{q\_12}_b = \frac{2\left(\left[\mathrm{M1}_x, \mathrm{M1}_y, \mathrm{M1}_z\right].M_2\right)\mu_0\nu_M\omega^2}{{\omega_M}^2} + 2\left(\mathrm{S\_bar}_1.\begin{pmatrix}\eta_{11} & \eta_{12} & \eta_{13}\\ \eta_{21} & \eta_{22} & \eta_{23}\\ \eta_{31} & \eta_{32} & \eta_{33}\end{pmatrix}.S_2\right)\omega^2 + \left(U_1.\frac{1}{\tau}.U_2\right)\rho\mathrm{\_m0} + \frac{[v_2 -}{}$$

(%o274)

(%i275) cmt_ch04_4514:[
r_12_b = - ((J_b1_e . conjugate(E_2)) +~ (conjugate(J_b2_e)~ . E_1))
-
((J_b1_m . conjugate(H_2_p)) +~ (conjugate(J_b2_m)~ . H_1_p))

];

$$[\mathrm{r\_12}_b = -\mathrm{J\_b2}_m.\left[\mathrm{H1\_p}_x, \mathrm{H1\_p}_y, \mathrm{H1\_p}_z\right] - \mathrm{J\_b2}_e.\left[\mathrm{E1}_x, \mathrm{E1}_y, \mathrm{E1}_z\right] - \mathrm{H\_2p}_z\mathrm{J\_b1m}_z - \mathrm{H\_2p}_y\mathrm{J\_b1m}_y - \mathrm{H\_2}$$

(%o275)

(% i276)

```
cmt_ch04_4515:[
express(div( express(E_1 ~ ~ conjugate(H_2_p)~ +~
express(conjugate(E_2)~ ~ ~ H_1_p))))
=
((ϕ_1*~ conjugate(%i*ω*D_2_p)) +~ (conjugate(ϕ_2)~ *~
(%i*ω*D_1_p))) + (( ψ_1* conjugate(%i*ω*B_2)) +~ (conjugate(ψ_2)~
*~ (%i*ω*B_1)))
+~
((conjugate(J_b1_e)~ . express(grad(conjugate(ϕ_2)))) +~
(conjugate(J_b2_e)~ . express(grad(ϕ_1))) ) + ((J_b1_m .
express(grad(conjugate(ψ_2)))) +~ (conjugate(J_b2_m)~ .
express(grad(ψ_1))) )
-
((~ (%i*ω*ρ_b1_e) *~ conjugate(ϕ_2) ) ~ +~ (conjugate(%i*ω*ρ_b2_e)
*~ ϕ__1)) - ( (%i*ω*ρ_b1_m)~ *~ conjugate(ψ_2)) +~
(conjugate(%i*ω*ρ_b2_m) *~ ψ_1)
];
```

$$[\frac{d}{dx}\left(\mathrm{E1}_y\mathrm{H\_2p}_z-\mathrm{E1}_z\mathrm{H\_2p}_y+\mathrm{E2}_y\mathrm{H1\_p}_z-\mathrm{E2}_z\mathrm{H1\_p}_y\right)+\frac{d}{dy}\left(-\mathrm{E1}_x\mathrm{H\_2p}_z+\mathrm{E1}_z\mathrm{H\_2p}_x-\mathrm{E2}_x\mathrm{H1\_p}_z+\mathrm{E2}_z\right.$$

(% o276)

(% i280)

```
E_c1:[E_c1_x, E_c1_y, E_c1_z];
E_c2:[E_c2_x, E_c2_y, E_c2_z];
H_c2_p:[H_c2p_x, H_c2p_y, H_c2p_z];
H_c1_p:[H_c1p_x, H_c1p_y, H_c1p_z];
```

$$\left[\mathrm{E\_c1}_x, \mathrm{E\_c1}_y, \mathrm{E\_c1}_z\right] \tag{\% o277}$$

$$\left[\mathrm{E\_c2}_x, \mathrm{E\_c2}_y, \mathrm{E\_c2}_z\right] \tag{\% o278}$$

$$\left[\mathrm{H\_c2p}_x, \mathrm{H\_c2p}_y, \mathrm{H\_c2p}_z\right] \tag{\% o279}$$

$$\left[\mathrm{H\_c1p}_x, \mathrm{H\_c1p}_y, \mathrm{H\_c1p}_z\right] \tag{\% o280}$$

(%i281) cmt_ch04_4516:[
S_12_EM = (express(E_c1 ~ conjugate(H_c2_p))) +~
express(conjugate(E_c2)~ ~ ~ H_c1_p)
+
((ϕ_1*conjugate(%i*ω*D_2_p)) +~ conjugate(ϕ_2) *~ (%i*ω*D_1_p))
+
(ψ_1*conjugate(%i*ω*B_2)) +~ (conjugate(ψ_2)~ *~ (%i*ω*B_1))
];

$$[\mathrm{S\_12\_EM} = [\,\%i\mathrm{B1}_x\psi_2\omega - \%iB_2\psi_1\omega + \%i\mathrm{D1\_p}_x\phi_2\omega - \%i\mathrm{D\_2}_p\phi_1\omega + \mathrm{E\_c1}_y\mathrm{H\_c2p}_z - \mathrm{E\_c1}_z\mathrm{H\_c2p}_y + \mathrm{E\_c2}_y\mathrm{H} \qquad (\%\ o281)$$

(%i282) cmt_ch04_4517:[
r_12_b = - (J_b1_e . conjugate(E_c2)) +~ (conjugate(J_b2_e)~ . E_c1)
- (J_b1_m . conjugate(H_c2_p)) +~ (conjugate(J_b2_m)~ . H_c1_p)
+
((%i*ω*ρ_b1_e)~ *~ conjugate(ϕ_2)) +~ (conjugate(%i*ω*ρ_b2_e)~ *
ϕ_1)~
+
((%i*ω*ρ_b1_m) * conjugate(ψ_2)) + (conjugate(%i*ω*ρ_b2_m) * ψ_1)

];

$$[\mathrm{r\_12}_b = \%i\rho\_\mathrm{b1}_m\psi_2\omega - \%i\rho\_\mathrm{b2}_m\psi_1\omega + \%i\rho\_\mathrm{b1}_e\phi_2\omega - \%i\rho\_\mathrm{b2}_e\phi_1\omega + \mathrm{J\_b2}_m.\left[\mathrm{H\_c1p}_x, \mathrm{H\_c1p}_y, \mathrm{H\_c1p}_z\right] + \mathrm{J} \qquad (\%\ o282)$$

4.5.2 Quasi-Orthogonality and Orthogonality of Modes In SDAM Waveguides
4.5.2.1 Mode quasi-orthogonality relations for lossy waveguides

(%i283) cmt_ch04_4518:[
'diff(P_kl(z),z)~ +~ Q_kl(z)~ = 0
];

$$\left[\frac{d}{dz}P_{\mathrm{kl}}(z) + Q_{\mathrm{kl}}(z) = 0\right] \qquad (\%\ o283)$$

(%i284) cmt_ch04_4519:[
P_kl(z)~ = (1/4)~ *~ 'integrate(conjugate(S_kl_EM (r_t,z)) . e_z,S, 1,N)
+
(1/4) * 'integrate(conjugate(S_kl_PM (r_t,z)) . e_z, S, 1, N),
P_kl(z)~ = P_kl_EM +~ P_kl_PM,
P_kl(z) = (1/4) * (N_kl_EM + N_kl_PM) * conjugate(A_k) * A_l * exp(-(conjugate(γ_k) + γ_l) * z),
P_kl(z)~ = (1/4)~ *~ N_kl*~ conjugate(a_k(z))~ *a_l(z)~

];

$$[P_{\mathrm{kl}}(z)=\frac{(N-1)\left(\overline{\mathrm{S\_kl\_PM}\,(r_t\,,z)}.e_z\right)}{4}+\frac{(N-1)\left(\overline{\mathrm{S\_kl\_EM}\,(r_t\,,z)}.e_z\right)}{4}\,,P_{\mathrm{kl}}(z)=\mathrm{P\_kl\_PM}+\mathrm{P\_kl\_EM}\,,$$

(%o284)

(%i285) cmt_ch04_520:[
Q_kl(z) = (1/4) * 'integrate(conjugate(q_kl_b(r_t,z)) ,S, 1,N)
+
(1/4) * 'integrate(conjugate(q_kl_s(r_t,z)) , S, 1, N),
Q_kl(z)~ = Q_kl_b +~ Q_kl_s,
Q_kl(z)~ = (1/4) * (M_kl_b + M_kl_s) * conjugate(A_k) * A_l * exp(-(conjugate(γ_k) + γ_l) * z),
Q_kl(z)~ = (1/4) * M_kl* conjugate(a_k(z)) *a_l(z)
];

$$[Q_{\mathrm{kl}}(z)=\frac{(N-1)\,\overline{\mathrm{q\_kl}_s\,(r_t\,,z)}}{4}+\frac{(N-1)\,\overline{\mathrm{q\_kl}_b\,(r_t\,,z)}}{4}\,,Q_{\mathrm{kl}}(z)=\mathrm{Q\_kl}_s+\mathrm{Q\_kl}_b\,,Q_{\mathrm{kl}}(z)=\frac{A_k A_l\,(\mathrm{M\_kl}_s+\mathrm{M\_}}{4}$$

(%o285)

(%i286) cmt_ch04_520_A:[
'diff(N_kl * exp(-(conjugate(γ_k) * γ_l) * z),z) + M_kl * exp(-(conjugate(γ_k) + γ_l) * z) = 0
];

$$\left[\frac{d}{dz}\left(N_{\mathrm{kl}}\%e^{-z\gamma_k\gamma_l}\right)+M_{\mathrm{kl}}\%e^{z\,(-\gamma_l-\gamma_k)}=0\right] \quad (\%o286)$$

(%i287) cmt_ch04_521:[
(conjugate(γ_k)~ +~ γ_l)~ *~ N_kl = M_kl, (conjugate(γ_k) +~ γ_l)~ *~ P_kl = Q_kl
];

$$[N_{\mathrm{kl}}\,(\gamma_l+\gamma_k)=M_{\mathrm{kl}}\,,P_{\mathrm{kl}}\,(\gamma_l+\gamma_k)=Q_{\mathrm{kl}}] \quad (\%o287)$$

(%i288) cmt_ch04_4522:[
(conjugate(γ_k) + γ_l) *~ (N_kl_EM +~ N_kl_PM)~ = M_kl_b +~ M_kl_s
];

$$[(\mathrm{N\_kl\_PM}+\mathrm{N\_kl\_EM})\,(\gamma_l+\gamma_k)=\mathrm{M\_kl}_s+\mathrm{M\_kl}_b] \qquad (\%o288)$$

4.5.2.2 Mode Orthogonality Relations For Lossless Waveguides

(%i289) cmt_ch04_4523:[
(conjugate(γ_k) + γ_l) * N_kl = (conjugate(γ_k) + γ_l) * (N_kl_EM + N_kl_PM),
(conjugate(γ_k) + γ_l) * N_kl = 0
];

$$[N_{\mathrm{kl}}\,(\gamma_l+\gamma_k)=(\mathrm{N\_kl\_PM}+\mathrm{N\_kl\_EM})\,(\gamma_l+\gamma_k)\,,N_{\mathrm{kl}}\,(\gamma_l+\gamma_k)=0] \qquad (\%o289)$$

(%i290) cmt_ch04_4524:[
(β_k - β_l)~ *~ N_kl = (β_k - β_l)~ *~ (N_kl_EM + N_kl_PM),
(β_k - β_l) * N_kl = 0
];

$$[N_{\mathrm{kl}}\,(\beta_k-\beta_l)=(\mathrm{N\_kl\_PM}+\mathrm{N\_kl\_EM})\,(\beta_k-\beta_l)\,,N_{\mathrm{kl}}\,(\beta_k-\beta_l)=0] \qquad (\%o290)$$

(%i291) cmt_ch04_4525:[
N_kl = n_KK *~ δ_kl,
if l != k then N_kl = 0,
if l = k then N_kk = N_k
];

$$[N_{\mathrm{kl}}=n_{\mathrm{KK}}\delta\mathrm{\_kl}\,,\mathrm{false}\,,\mathrm{false}] \qquad (\%o291)$$

(%i293) E_ck_hat:[Eckh_x, Eckh_y, Eckh_z];
H_ck_p_hat:[H_ckph_x, H_ckph_y, H_ckph_z];

$$[\mathrm{Eckh}_x\,,\mathrm{Eckh}_y\,,\mathrm{Eckh}_z] \qquad (\%o292)$$

$$[\mathrm{H\_ckph}_x\,,\mathrm{H\_ckph}_y\,,\mathrm{H\_ckph}_z] \qquad (\%o293)$$

(%i294) cmt_ch04_4526:[
N_k = N_k_EM +~ N_k_PM,
N_k = 2 *~ Re(
integrate(
(express(conjugate(E_ck_hat)~ ~ H_ck_p_hat)
+~ conjugate(ϕ_k_hat)~ *~ (%i*ω*D_k_p_hat)
+~ ψ_k_hat *~ (%i*ω*B_k_hat))~ . e_z, s,1,N))
];

$$[N_k = \mathrm{N\_k\_PM{+}N\_k\_EM}, N_k = 2(N-1)\,\mathrm{realpart}\,([\%i\mathrm{B\_k\_hat}\psi\mathrm{\_k\_hat}\omega+\%i\mathrm{D\_k\_p\_hat}\phi\mathrm{\_k\_hat}\omega+$$ (%o294)

(%i295) cmt_ch04_4527:[
V_k_a_bh = -T_k_Σ_bh,
J_k_a_hat = %i*ω*u_k_hat,
J_k_a_hat = U_k_hat
];

$$[\mathrm{V\_k\_a\_bh} = -\mathrm{T\_k\_\_bh}, \mathrm{J\_k\_a\_hat} = \%i\mathrm{u\_k\_hat}\omega, \mathrm{J\_k\_a\_hat} = \mathrm{U\_k\_hat}]$$ (%o295)

(%i296) cmt_ch04_4528:[
V_k_m_hat = -λ_cx *~ M_k_hat,
J_k_m_hat = %i*ω*μ_0 *~ M_k_hat
];

$$[\mathrm{V\_k\_m\_hat} = -\lambda\mathrm{\_cx}\ \mathrm{M\_k\_hat}, \mathrm{J\_k\_m\_hat} = \%i\mathrm{M\_k\_hat}\mu_0\omega]$$ (%o296)

(%i297) cmt_ch04_4529:[
V_k_e_bh = (m/e)~ *~ (v_0*v_k_p_hat +~ (v_T^2 / o_0)~ *~ p_k_hat
* I_bar),
J_k_e_hat = %i*ω*p_k_hat
];

$$[\mathrm{V\_k\_e\_bh} = [\frac{m\left(\mathrm{v0}_x\mathrm{v\_k\_p\_hat} + \frac{I_{\mathrm{bar}}\mathrm{p\_k\_hat}{v_T}^2}{o_0}\right)}{e}, \frac{m\left(\mathrm{v0}_y\mathrm{v\_k\_p\_hat} + \frac{I_{\mathrm{bar}}\mathrm{p\_k\_hat}{v_T}^2}{o_0}\right)}{e}, \frac{m\left(\mathrm{v0}_z\mathrm{v\_k\_}\right.}{}$$ (%o297)

(%i298) cmt_ch04_4530:[
P_k = (1/4)~ *~ N_k *~ abs(a_k)^2];

$$\left[P_k = \frac{N_k {a_k}^2}{4}\right]$$ (%o298)

(%i299) cmt_ch04_4531:[
P_kk = conjugate(P_kk), P_kk = (1/4)~ *~ N_kk *~ conjugate(a_k)~ *
a_k
];

$$\left[P_{\mathrm{kk}} = P_{\mathrm{kk}}\,, P_{\mathrm{kk}} = \frac{N_{\mathrm{kk}} {a_k}^2}{4}\right] \tag{\% o299}$$

(%i300) cmt_ch04_4532:[
N_kl = N_kk_tilde *~ δ_k_tilde_l,
if l ≠ k then N_kl = 0,
if l = k then N_kk_tilde = N_k
];

$$[N_{\mathrm{kl}} = \mathrm{N\_kk\_tilde}\delta\mathrm{\_k\_tilde}_l\,, N_{\mathrm{kl}} = 0\,, \mathrm{false}] \tag{\% o300}$$

(%i301) H_ck_tilde_p_hat:[H_cktph_x, H_cktph_y, H_cktph_z];

$$[\mathrm{H\_cktph}_x\,, \mathrm{H\_cktph}_y\,, \mathrm{H\_cktph}_z] \tag{\% o301}$$

(%i302) cmt_ch04_4533:[
N_k = N_kk_EM +~ N_kk_PM,
N_k = 'integrate(
(
express(conjugate(E_ck_hat) ~ H_ck_tilde_p_hat) +~ express(E_ck_hat ~ conjugate(H_ck_p_hat))
+
(conjugate(ϕ_hat) *(%i*ω*D_k_tilde_p_hat)~ +~ ϕ_k_tilde_hat~ *~ conjugate(%i*ω*D_k_p_hat))
+
(conjugate(ψ_k_hat)~ *~ (%i*ω*B_k_tilde_hat) +~ ψ_k *~ conjugate(%i*ω*B_k_hat))). e_z,

S, 1, N)
+
'integrate(
(
(conjugate(V_k_a_bh) . J_k_tilde_a_hat~ +~ V_k_a_bh . conjugate(J_k_a_hat))
+
(conjugate(V_k_m_bh) . J_k_tilde_m_hat +~ V_k_tilde_m_bh . conjugate(J_k_m_hat))
+
(conjugate(V_k_e_bh)~ . J_k_tilde_e_hat +~ V_k_tilde_e_bh . conjugate(J_k_e_hat))~
) . e_z, S, 1,N
)
]
;

$$[\, N_k = \text{N\_kk\_PM}+\text{N\_kk\_EM}\,, N_k = (N-1)\,(\,[\,\%i\text{B\_k\_tilde\_hat}\psi\text{\_k\_hat}\omega-\%i\text{B\_k\_hat}\psi_k\omega-\%i\text{D\_k\_p\_} \tag{\% o302}$$

(%i303) cmt_ch04_4534:[
P_kk_pair = P_k_k_tilde +~ P_k_tilde_k~ ,
P_kk_pair = (1/2)~ *~ Re(~ N_k_k_tilde *~ conjugate(a_k)~ *~ a_k_tilde)~ ,
P_kk_pair = (1/2)~ *~ Re(N_k *~ conjugate(a_k)~ *~ a_k_tilde)
];

$$[\, \text{P\_kk\_pair} = \text{P\_k\_tilde}_k+\text{P\_k\_k\_tilde}\,, \text{P\_kk\_pair} = \frac{\text{N\_k\_k\_tilde}a_k\text{a\_k\_tilde}}{2}\,, \text{P\_kk\_pair} = \frac{N_k a_k \text{a\_}}{2} \tag{\% o303}$$

(%i304) cmt_ch04_4535:[
Φ_k(r_t,z)~ = Φ_k_bar(r_t) *~ exp(-γ_k*z)
];

$$\left[\Phi_k\left(r_t\,,z\right)= \ \_\mathrm{k\_bar}\left(r_t\right)\%e^{-z\gamma_k}\right] \quad (\%o304)$$

(%i305) cmt_ch04_4536:[
Φ_1(r_t,z)~ = Φ_a(r_t,z)~ +~ Φ_b(r_t,z)~ ,
Φ_1(r_t,z)~ = 'sum(A_l(z) *~ Φ_l (r_t,z)~ + Φ_b(r_t,z),l,1,N),
Φ_1(r_t,z) = A_l(z)~ *~ Φ_l_hat(r_t)~ *~ exp(-γ_k*z) +~ Φ_b(r_t,z),
Φ_1(r_t,z) = 'sum(a_l(z) *~ Φ_l_hat(r_t)~ +~ Φ_b(r_t,z), l,1,N)
];

$$[\,\Phi_1\left(r_t\,,z\right)=\Phi_b\left(r_t\,,z\right)+\Phi_a\left(r_t\,,z\right),\Phi_1\left(r_t\,,z\right)=N\,\left(\Phi_l\left(r_t\,,z\right)A_l(z)+\Phi_b\left(r_t\,,z\right)\right),\Phi_1\left(r_t\,,z\right)= \ \_\mathrm{l\_hat}\left(r_t\right)A_l(z) \quad (\%o305)$$

(%i319) n_s_plus:~ [n_sp_x, n_sp_y, n_sp_z];
E_c1_plus: [E_c1p_x, E_c1p_y, E_c1p_z];
n_s_minus: [n_sm_x, n_sm_y, n_sm_z];
E_c1_minus: [E_c1m_x, E_c1m_y, E_c1m_z];
H_c1_p_plus:[H_c1pp_x, H_c1pp_y, H_c1pp_z];
H_c1_p_minus:[H_c1pm_x, H_c1pm_y, H_c1pm_z];
D_1_p_plus:[D_1pp_x, D_1pp_y, D_1pp_z];
D_1_p_minus:[D_1pm_x, D_1pm_y, D_1pm_z];
B_1_plus:[B_1p_x, B_1p_y, B_1p_z];
B_1_minus:[B_1m_x, B_1m_y, B_1m_z];
ϕ_1_plus:[ϕ_1p_x, ϕ_1p_y, ϕ_1p_z];
ϕ_1_minus:[ϕ_1m_x, ϕ_1m_y, ϕ_1m_z];
ψ_1_plus:[ψ_1p_x, ψ_1p_y, ψ_1p_z];
ψ_1_minus:[ψ_1m_x, ψ_1m_y, ψ_1m_z];

$$\left[\mathrm{n\_sp}_x\,,\mathrm{n\_sp}_y\,,\mathrm{n\_sp}_z\right] \quad (\%o306)$$

$$\left[\mathrm{E\_c1p}_x\,,\mathrm{E\_c1p}_y\,,\mathrm{E\_c1p}_z\right] \quad (\%o307)$$

$$\left[\mathrm{n\_sm}_x\,,\mathrm{n\_sm}_y\,,\mathrm{n\_sm}_z\right] \quad (\%o308)$$

$$\left[\mathrm{E\_c1m}_x\,,\mathrm{E\_c1m}_y\,,\mathrm{E\_c1m}_z\right] \quad (\%o309)$$

$$\left[\mathrm{H\_c1pp}_x , \mathrm{H\_c1pp}_y , \mathrm{H\_c1pp}_z \right] \tag{\% o310}$$

$$\left[\mathrm{H\_c1pm}_x , \mathrm{H\_c1pm}_y , \mathrm{H\_c1pm}_z \right] \tag{\% o311}$$

$$\left[\mathrm{D\_1pp}_x , \mathrm{D\_1pp}_y , \mathrm{D\_1pp}_z \right] \tag{\% o312}$$

$$\left[\mathrm{D\_1pm}_x , \mathrm{D\_1pm}_y , \mathrm{D\_1pm}_z \right] \tag{\% o313}$$

$$\left[\mathrm{B\_1p}_x , \mathrm{B\_1p}_y , \mathrm{B\_1p}_z \right] \tag{\% o314}$$

$$\left[\mathrm{B\_1m}_x , \mathrm{B\_1m}_y , \mathrm{B\_1m}_z \right] \tag{\% o315}$$

$$\left[\phi\mathrm{\_1p}_x , \phi\mathrm{\_1p}_y , \phi\mathrm{\_1p}_z \right] \tag{\% o316}$$

$$\left[\phi\mathrm{\_1m}_x , \phi\mathrm{\_1m}_y , \phi\mathrm{\_1m}_z \right] \tag{\% o317}$$

$$\left[\psi\mathrm{\_1p}_x , \psi\mathrm{\_1p}_y , \psi\mathrm{\_1p}_z \right] \tag{\% o318}$$

$$\left[\psi\mathrm{\_1m}_x , \psi\mathrm{\_1m}_y , \psi\mathrm{\_1m}_z \right] \tag{\% o319}$$

For the current sheet

(% i320) cmt_ch04_4537:[express(n_s_plus ~ E_c1_plus) +~ express(n_s_minus ~ E_c1_minus) = -J_s_m];

$[[-\mathrm{E\_c1p}_y\mathrm{n\_sp}_z+\mathrm{E\_c1p}_z\mathrm{n\_sp}_y-\mathrm{E\_c1m}_y\mathrm{n\_sm}_z+\mathrm{E\_c1m}_z\mathrm{n\_sm}_y , \mathrm{E\_c1p}_x\mathrm{n\_sp}_z-\mathrm{E\_c1p}_z\mathrm{n\_sp}_x+\mathrm{E\_c1m}_x$

(%o320)

(%i321) cmt_ch04_4538:[express(n_s_plus ~ H_c1_p_plus) + express(n_s_minus ~ H_c1_p_minus) = -J_s_E];

$[[-\mathrm{H\_c1pp}_y\mathrm{n\_sp}_z+\mathrm{H\_c1pp}_z\mathrm{n\_sp}_y-\mathrm{H\_c1pm}_y\mathrm{n\_sm}_z+\mathrm{H\_c1pm}_z\mathrm{n\_sm}_y, \mathrm{H\_c1pp}_x\mathrm{n\_sp}_z-\mathrm{H\_c1pp}_z\mathrm{n\_sp}_x+$

(%o321)

For the charge sheet

(%i322) cmt_ch04_4539:[express(n_s_plus ~ D_1_p_plus) + express(n_s_minus ~ D_1_p_minus) = ρ_s_e];

$[[-\mathrm{D\_1pp}_y\mathrm{n\_sp}_z+\mathrm{D\_1pp}_z\mathrm{n\_sp}_y-\mathrm{D\_1pm}_y\mathrm{n\_sm}_z+\mathrm{D\_1pm}_z\mathrm{n\_sm}_y, \mathrm{D\_1pp}_x\mathrm{n\_sp}_z-\mathrm{D\_1pp}_z\mathrm{n\_sp}_x+\mathrm{D\_1p}$

(%o322)

(%i323) cmt_ch04_4540:[express(n_s_plus ~ B_1_plus) + express(n_s_minus ~ B_1_minus) = ρ_s_m];

$[[-\mathrm{B\_1p}_y\mathrm{n\_sp}_z+\mathrm{B\_1p}_z\mathrm{n\_sp}_y-\mathrm{B\_1m}_y\mathrm{n\_sm}_z+\mathrm{B\_1m}_z\mathrm{n\_sm}_y, \mathrm{B\_1p}_x\mathrm{n\_sp}_z-\mathrm{B\_1p}_z\mathrm{n\_sp}_x+\mathrm{B\_1m}_x\mathrm{n\_sm}_z$

(%o323)

For the dipole sheet

(%i324) cmt_ch04_4541:[express(n_s_plus ~ ϕ_1_plus) + express(n_s_minus ~ ϕ_1_minus) = (1/ϵ_0)~ *~ η_s_e];

$[[\mathrm{n\_sp}_y\phi\mathrm{\_1p}_z-\mathrm{n\_sp}_z\phi\mathrm{\_1p}_y+\mathrm{n\_sm}_y\phi\mathrm{\_1m}_z-\mathrm{n\_sm}_z\phi\mathrm{\_1m}_y, -\mathrm{n\_sp}_x\phi\mathrm{\_1p}_z+\mathrm{n\_sp}_z\phi\mathrm{\_1p}_x-\mathrm{n\_sm}_x\phi\mathrm{\_1m}_z+$

(%o324)

(%i325) cmt_ch04_4542:[express(n_s_plus ~ ψ_1_plus) + express(n_s_minus ~ ψ_1_minus) = (1/ϵ_0) * η_s_m];

$[[\mathrm{n\_sp}_y\psi\mathrm{\_1p}_z-\mathrm{n\_sp}_z\psi\mathrm{\_1p}_y+\mathrm{n\_sm}_y\psi\mathrm{\_1m}_z-\mathrm{n\_sm}_z\psi\mathrm{\_1m}_y, -\mathrm{n\_sp}_x\psi\mathrm{\_1p}_z+\mathrm{n\_sp}_z\psi\mathrm{\_1p}_x-\mathrm{n\_sm}_x\psi\mathrm{\_1m}_z$

(%o325)

(%i327) S_lk_EM:[S_lkem_x, S_lkem_y, S_lkem_z];
S_lk_PM:[S_lkpm_x, S_lkpm_y, S_lkpm_z];

$$[\mathrm{S\_lkem}_x, \mathrm{S\_lkem}_y, \mathrm{S\_lkem}_z] \quad (\%o326)$$

$$[\text{S\_lkpm}_x, \text{S\_lkpm}_y, \text{S\_lkpm}_z] \tag{\% o327}$$

(% i328) cmt_ch04_4543:[
'integrate(express(div(S_lk_EM~ +~ S_lk_PM)) , S, 1, N) +~
'integrate(q_lk_b, S ,1, N)~ +~ 'integrate(r_lk_b, S, 1, N)
];

$$[(N-1)\,\text{r\_lk}_b+(N-1)\,\text{q\_lk}_b+(N-1)\left(\frac{d}{dz}(\text{S\_lkpm}_z+\text{S\_lkem}_z)+\frac{d}{dy}\left(\text{S\_lkpm}_y+\text{S\_lkem}_y\right)+\frac{d}{dx}(\text{S\_lkp} \tag{\% o328}$$

(% i329) cmt_ch04_4544:[S_lk_EM];

$$[[\text{S\_lkem}_x, \text{S\_lkem}_y, \text{S\_lkem}_z]] \tag{\% o329}$$

(% i330) cmt_ch04_4545:[
S_lk_PM = ((V_bar_l_a . conjugate(J_k_a))~ +~
(conjugate(V_bar_k_a) . J_l_a))
+
((V_bar_l_m~ . conjugate(J_bar_k_m)~) +~ (conjugate(V_bar_k_m)~ .
J_l_m))
+
((V_bar_l_e . conjugate(J_k_e)) +~ (conjugate(V_bar_k_e)~ . J_l_e))
];

$$[[\text{S\_lkpm}_x, \text{S\_lkpm}_y, \text{S\_lkpm}_z] = \text{V\_bar\_l}_m.\text{J\_bar\_k}_m+\text{V\_bar\_l}_e.\text{J\_k}_e+\text{V\_bar\_l}_a.\text{J\_k}_a+\text{V\_bar\_k}_m.\text{J\_} \tag{\% o330}$$

The dot product below between S_bar_l and η_bar and S_bar_kshould be the term by term tensor product :~(colon)

(%i331) cmt_ch04_4546:[
q_lk_b = 2 *~ (ω^2 *S_bar_l . η_bar . conjugate(S_bar_k)~ +~ ρ_m0 *~ U_1 . (1/τ_bar) . conjugate(U_k))
+~
2 *~ ν_M~ *~ μ_0 *~ (ω/ω_M)^2 *~ (M_l . conjugate(M_k))
+
(1/μ_e)~ *~ (~ (v_l - j_l *~ v_0)~ . conjugate(J_k_e)~ +~ (conjugate(v_k)~ - conjugate(j_k))~ *~ v_0) . J_l_e

];

$$\left[\, \mathrm{q\_lk}_b = \frac{2\,(M_l.M_k)\,\mu_0 \nu_M \omega^2}{{\omega_M}^2} + 2\left(\left(\mathrm{S\_bar}_l.\begin{pmatrix}\eta_{11} & \eta_{12} & \eta_{13}\\ \eta_{21} & \eta_{22} & \eta_{23}\\ \eta_{31} & \eta_{32} & \eta_{33}\end{pmatrix}.\mathrm{S\_bar}_k\right)\omega^2 + \left(U_1.\frac{1}{\tau\_\mathrm{bar}}.U_k\right)\rho\_\mathrm{m0}\right) + \frac{([\mathrm{v0}_x}{}\right. \tag{\% o331}$$

(%i332) cmt_ch04_4547:[
r_lk_b = -(J_bl_e . conjugate(E_ck)~ +~ J_bl_m . conjugate(H_ck_p)~)
+
((%i*ω*ρ_bl_e)*conjugate(ϕ_k) +~ (%i*ω*ρ_bl_m)~ *~ conjugate(ψ_k))
];

$$\left[\mathrm{r\_lk}_b = \%i\rho\_\mathrm{bl}_m\psi_k\omega + \%i\rho\_\mathrm{bl}_e\phi_k\omega - \mathrm{J\_bl}_m.\mathrm{H\_ck}_p - \mathrm{J\_bl}_e.E_{\mathrm{ck}}\right] \tag{\% o332}$$

Apply divergence theorem (3.2.25)~to the expression for S_lk_EM in Eq (4.5.44)

(%i336) E_cl:[E_cl_x, E_cl_y, E_cl_z];
E_ck:[E_ck_x, E_ck_y, E_ck_z];
H_ck_p:[H_ckp_x, H_ckp_y, H_ckp_z];
H_cl_p:[H_clp_x, H_clp_y, H_clp_z];

$$\left[\mathrm{E\_cl}_x\,,\mathrm{E\_cl}_y\,,\mathrm{E\_cl}_z\right] \tag{\% o333}$$

$$\left[\mathrm{E\_ck}_x\,,\mathrm{E\_ck}_y\,,\mathrm{E\_ck}_z\right] \tag{\% o334}$$

$$\left[\mathrm{H\_ckp}_x\,,\mathrm{H\_ckp}_y\,,\mathrm{H\_ckp}_z\right] \tag{\% o335}$$

$$\left[\mathrm{H\_clp}_x\,,\mathrm{H\_clp}_y\,,\mathrm{H\_clp}_z\right] \tag{\% o336}$$

The three line integrals in the following expression have a minus sign~(-) at the end of the expression inside the integral.This denotes a boundary condition where we are 'looking' toward one side of a boundary.One side is denoted as the +~direction or side, the other is the - direction or side.The first section of each line integral is 'looking' toward the +~side of the boundaryThe second section of each line integral is 'looking' toward the -~side of the boundaryNOTE: At this point I'm not sure how to represent this in maxima,but may end up using unit vectors perpendicular to the boundary.Then multiplying the expressions inside the line integrals by the appropriate unit vectorThe other approach is to use sub-expressions named with the +~and - or p and m to hold the +~and - parts of the integral expressions.

(%i337)

```
cmt_ch04_4548_A:[
'integrate(div(S_lk_EM),S)~ =
'diff(
'integrate(
(
(express(E_cl ~ conjugate(H_ck_p)) +~ express(conjugate(E_ck~ ~
H_cl_p)))
+~
(φ_l*~ conjugate(%i*ω*D_k_p) +~ conjugate(φ_k) *~ (%i*ω*D_l_p))
+~ (ψ_l~ *~ conjugate(%i*ω*B_k)~ +~ conjugate(ψ_k)~ *~ (%i*ω*B_l))
)
*~ e_z,S)
,z)
-
'integrate(
n_s_p *~ ~ (express(E_cl ~ conjugate(H_ck_p)) +~
express(conjugate(E_ck) ~ H_cl_p))
+~
n_s_m * (express(E_cl ~ conjugate(H_ck_p)) +~ conjugate(E_ck)
~ ~ H_cl_p),L,1,N)

-
'integrate(
n_s_p * (φ_l* conjugate(%i*ω*D_k_p) + conjugate(φ_k) * (%i*ω*D_l_p))
+
n_s_m * (φ_l * conjugate(%i*ω*D_k_p) + conjugate(φ_k) * (%i*ω*D_l_p))
,L,1,N)
-
'integrate(
n_s_p * (ψ_l* conjugate(%i*ω*B_k) + conjugate(ψ_k) * (%i*ω*B_l))
+
n_s_m * (ψ_l * conjugate(%i*ω*B_k) + conjugate(ψ_k) * (%i*ω*B_l))
,L,1,N)
];
```

$[\, S\, \mathrm{div}\left([\mathrm{S\_lkem}_x\, , \mathrm{S\_lkem}_y\, , \mathrm{S\_lkem}_z]\right) = \mathrm{diff}\backslash(\, [\, S e_z\, (\, -\%i B_k \psi_l \omega + \%i B_l \psi_k \omega - \%i\mathrm{D\_k}_p \phi_l \omega + \%i \mathrm{D\_l}_p \phi_k \omega + \mathrm{E\_c}$

(%o337)

Using the boundary conditions (4.5.37)~to (4.5.42)~allows for this re-arrangement of the line integrals in cmt_ch04_4548_A

(%i338) cmt_ch04_4548_B:[
'integrate(express(div(S_lk_EM))~ , S,1,N)~
=
'diff(
'integrate(S_lk_EM . e_z, S,1,N) - 'integrate(r_kl_s, L,1,N)
)
];

$$[(N-1)\left(\frac{d}{dz}\mathrm{S\_lkem}_z+\frac{d}{dy}\mathrm{S\_lkem}_y+\frac{d}{dx}\mathrm{S\_lkem}_x\right)=\mathrm{del}\left((N-1)\left(\left[\mathrm{S\_lkem}_x,\mathrm{S\_lkem}_y,\mathrm{S\_lkem}_z\right].e_z\right)-($$
(%o338)

(%i339) cmt_ch04_4549:[
τ_lk_s = -(express(J_s_e . conjugate(E_ck)) +~ express(J_s_m . conjugate(H_ck_p)))
+
(%i*ω*p_s_e *~ conjuate(ϕ_k) +~ %i*ω*p_s_m)
+~
((η_s_e / ϵ_0)~ . conjugate(%i*ω*D_k_p)) ~ +~ ((η_s_m/μ_0) . conjugate(%i*ω*B_k))
];

$$[\tau\_\mathrm{lk}_s=\%i\mathrm{p\_s}_e\,\mathrm{conjuate}\left(\phi_k\right)\omega+\%i\mathrm{p\_s}_m\omega-\%i\left(\frac{\eta\_\mathrm{s}_m}{\mu_0}.B_k\omega\right)-\%i\left(\frac{\eta\_\mathrm{s}_e}{\epsilon_0}.\mathrm{D\_k}_p\omega\right)-\mathrm{J\_s}_m.\left[\mathrm{H\_ckp}_x,\mathrm{H\_ck}\right.$$
(%o339)

(%i340) S_lk_PM:[S_lkpm_x, S_lkpm_y, S_lkpm_z];

$$\left[\mathrm{S\_lkpm}_x,\mathrm{S\_lkpm}_y,\mathrm{S\_lkpm}_z\right] \quad (\%o340)$$

(%i341) cmt_ch04_4550:[
'integrate(express(div(S_lk_PM)),S),
'diff(~ 'integrate(S_lk_PM . e_z,S)~ ,z)
+~
'integrate(q_lk_s,L)
];

$$[S\left(\frac{d}{dz}\mathrm{S\_lkpm}_z+\frac{d}{dy}\mathrm{S\_lkpm}_y+\frac{d}{dx}\mathrm{S\_lkpm}_x\right),L\mathrm{q\_lk}_s+\frac{d}{dz}\left(S\left(\left[\mathrm{S\_lkpm}_x,\mathrm{S\_lkpm}_y,\mathrm{S\_lkpm}_z\right].e_z\right)\right)]$$

(% o341)

(% i342) cmt_ch04_4551:[
q_lk_s = n . S_lk_PM,
q_lk_s = - ((n . T_bar_l_Σ)~ . conjugate(U_k) +~ conjugate(n . T_bar_lk_Σ) . U_l)
+~
λ_ex *~ ((n . express(grad(M_l))) . conjugate(%i*ω*μ_0~ *~ M_l) +~ (n . conjugate(express(grad(M_k))) . (%i*ω*μ_0*M_l)))
-
((n . p_l)~ *~ conjugate(%i*ω*p_k)~ +~ (n-conjugate(p_k)) *~ (%i*ω*p_l)
)
];

$$[\,\mathrm{q\_lk}_s = n.\left[\mathrm{S\_lkpm}_x\,,\mathrm{S\_lkpm}_y\,,\mathrm{S\_lkpm}_z\right],\mathrm{q\_lk}_s = -\%i\,(n-p_k)\,p_l\omega+\%i\,(n.p_l)\,p_k\omega+\Bigg(\%i\,\Bigg(n.\left[\frac{d}{dx}M_k\,,\frac{d}{dy}M\right.$$

(% o342)

(% i343) cmt_ch04_4552:[
'diff(P_kl(xz)~ ,z)~ +~ Q_lk(z)~ = R_lk(z)
];

$$\left[Q_{\mathrm{lk}}(z)+\frac{d}{dz}P_{\mathrm{kl}}\,(\mathrm{xz})=R_{\mathrm{lk}}(z)\right] \qquad (\%\ \mathrm{o343})$$

(% i344) cmt_ch04_4553:[
P_lk= P_lk_EM +~ P_lk_PM,
P_lk = 'integrate((S_lK_EM~ +~ S_lk_PM)~ . e_z, S)
];

$$[\,P_{\mathrm{lk}}=\mathrm{P\_lk\_PM}+\mathrm{P\_lk\_EM}\,,P_{\mathrm{lk}}=S\,\left(\left[\mathrm{S\_lkpm}_x+\mathrm{S\_lK\_EM}\,,\mathrm{S\_lkpm}_y+\mathrm{S\_lK\_EM}\,,\mathrm{S\_lkpm}_z+\mathrm{S\_lK\_E}\right.\right.$$

(% o344)

(% i345) cmt_ch04_4554:[
Q_lk = Q_lk_b +~ Q_lk___s,
Q_lk = 'integrate(q_lk_b, S)~ +~ 'integrate(q_lk_s, S)
];

$$[Q_{\mathrm{lk}}=\mathrm{Q\_lk}_b+\mathrm{Q\_lk\_}_s\,,Q_{\mathrm{lk}}=S\mathrm{q\_lk}_s+S\mathrm{q\_lk}_b] \qquad (\%\ \mathrm{o345})$$

(% i346) cmt_ch04_4555:[
R_lk = R_lk_EM +~ R_lk+PM,
R_lk = 'integrate(r_lk_b, S_b)~ +~ 'integrate(r_lk_s, L_s)
];

$$[R_{\mathrm{lk}}=\mathrm{R\_lk\_EM}+R_{\mathrm{lk}}+\mathrm{PM}\,,R_{\mathrm{lk}}=L_s\mathrm{r\_lk}_s+S_b\mathrm{r\_lk}_b] \qquad (\%\ \mathrm{o346})$$

(%i347) cmt_ch04_4556:[
P_lk = P_ak +~ P_bk,
P_lk = P_ak_EM +~ P_bl_PM,
P_lk = 'integrate((S_ak_EM +~ S_bk_PM)~ . e_z, S)
];

$$[P_{\mathrm{lk}} = P_{\mathrm{bk}} + P_{\mathrm{ak}}\,, P_{\mathrm{lk}} = \mathrm{P\_bl\_PM} + \mathrm{P\_ak\_EM}\,, P_{\mathrm{lk}} = S\,\left(\left(\mathrm{S\_bk\_PM} + \mathrm{S\_ak\_EM}\right).e_z\right)] \quad (\%\ o347)$$

(%i348) cmt_ch04_4557:[
Q_lk = Q_ak +~ Q_bk,
Q_lk = Q_ak_EM +~ Q_bk_PM,
Q_lk = 'integrate(q_ak_b,S)~ +~ 'integrate(q_ak_s, L)
];

$$[Q_{\mathrm{lk}} = Q_{\mathrm{bk}} + Q_{\mathrm{ak}}\,, Q_{\mathrm{lk}} = \mathrm{Q\_bk\_PM} + \mathrm{Q\_ak\_EM}\,, Q_{\mathrm{lk}} = L\mathrm{q\_ak}_s + S\mathrm{q\_ak}_b] \quad (\%\ o348)$$

(%i349) cmt_ch04_4558:[
'diff(P_bk(z),z)~ +~ Q_bk(z)~ = 0
]

;

$$\left[\frac{d}{dz} P_{\mathrm{bk}}(z) + Q_{\mathrm{bk}}(z) = 0\right] \quad (\%\ o349)$$

(%i350) cmt_ch04_4559:[
'diff(P_ak(z),z)~ +~ Q_ak(z)~ =~ R_lk(z)
];

$$\left[\frac{d}{dz} P_{\mathrm{ak}}(z) + Q_{\mathrm{ak}}(z) = R_{\mathrm{lk}}(z)\right] \quad (\%\ o350)$$

(%i351) cmt_ch04_4560:[
P_ak(z)~ = P_ak_EM +~ P_ak_PM(z),
P_ak(z)~ = 'integrate((S_ak_EM(z)~ +~ S_ak_PM(z)) . e_z,s),
P_ak(z)~ = 'sum(A_l(z),l, -N,N)~ *~ integrate((S_lk_EM +~ S_lk_PM)~ . e_z,S) ,
P_ak(z) = 'sum(A_l(z),l, -N,N) * integrate((conjugate(S_lk_EM) + conjugate(S_lk_PM)) . e_z,S),
P_ak(z)~ = 'sum((N_kl_EM + N_kl_PM) * A_l(z) * exp(-(conjugate(γ_k) + γ_l) * e_z), l,-N,N),
P_ak(z)~ = ('sum(N_kl *~ A_l(z)~ *~ exp(-γ_kl *~ z) , l, -N,N)) *~ exp(-conjugate(γ_k)~ *~ z)
];

$$[P_{\mathrm{ak}}(z) = \mathrm{P\_ak\_PM}(z)+\mathrm{P\_ak\_EM}\,, P_{\mathrm{ak}}(z) = s\;((\mathrm{S\_ak\_PM}(z) + \mathrm{S\_ak\_EM}(z))\,.e_z)\,, P_{\mathrm{ak}}(z) = (2N+1)\,S\,(\tag{\% o351}$$

(%i352) cmt_ch04_4561:[
Q_ak(z)~ = Q_ak_b(z) +~ Q_ak_s(z) ,
Q_ak(z) = 'integrate(q_ak_b(z), S,1,N) +~ 'integrate(q_ak_s(z), L,1,N),
Q_ak(z)~ = 'sum(A_l(z)~ * ('integrate(q_ak_b(z), S,1,N) + 'integrate(q_ak_s(z), L,1,N)) ,L,1,N),
Q_ak(z)='sum((M_kl_b +~ M_kl_s)*A_l(z)*exp(-(conjugate(γ_k)+γ_l)*z),L,1,N),
Q_ak(z)~ = 'sum((M_kl *~ A_l(z)*exp(-γ_l *z)) ,L,1,N) *~ exp(-conjugate(γ_k)*z)
];

$$[Q_{\mathrm{ak}}(z) = \mathrm{Q\_ak}_s(z)+\mathrm{Q\_ak}_b(z)\,, Q_{\mathrm{ak}}(z) = (N-1)\,\mathrm{q\_ak}_s(z)+(N-1)\,\mathrm{q\_ak}_b(z)\,, Q_{\mathrm{ak}}(z) = N A_l(z)\,((N-1)\,\mathrm{q\_} \tag{\% o352}$$

(%i353) cmt_ch04_4562:[
R_lk(z)~ = R_lk_b(z)~ +~ R_lk_s(z),
R_lk(z) = (R_lk_b(z) + R_lk_s(z)) *~ exp(-conjugate(γ_k)*z),
R_lk(z)~ = R_lk(z)~ *~ exp(-conjugate(γ_k)*z)
];

$$\left[R_{\mathrm{lk}}(z) = \mathrm{R\_lk}_s(z) + \mathrm{R\_lk}_b(z)\,, R_{\mathrm{lk}}(z) = (\mathrm{R\_lk}_s(z) + \mathrm{R\_lk}_b(z))\,\%e^{-z\gamma_k}\,, R_{\mathrm{lk}}(z) = R_{\mathrm{lk}}(z)\%e^{-z\gamma_k}\right] \tag{\% o353}$$

(%i354) cmt_ch04_4563:[
R_k_b(z)~ = 'integrate(
-((J_b_e . conjugate(Eck_hat)) +~ (J_b_m . conjugate(H_ck_p_hat)))~
+
(%i*ω*ρ_b_e~ *~ conjugate(ϕ_k_hat)~ +~ %i*ω*ρ_s_m *~
conjugate(ψ_k_hat))
,S)
]
;

$$[\mathrm{R\_k}_b(z) = S\,\left(\%i\rho\_\mathrm{s}_m\psi\_\mathrm{k\_hat}\omega + \%i\rho\_\mathrm{b}_e\phi\_\mathrm{k\_hat}\omega - \mathrm{J\_b}_m.\left[\mathrm{H\_ckph}_x\,,\mathrm{H\_ckph}_y\,,\mathrm{H\_ckph}_z\right] - \mathrm{J\_b}_e.\mathrm{Ec}\right. \tag{\% o354}$$

(%i355) cmt_ch04_4564:[
R_k_s(z) = 'integrate(
-(J_b_e . conjugate(E_hat_ck) +~ J_b_m . conjugate(H_hat_ck_p))
+~
(%i*ω*ρ_b_e *~ conjugate(ϕ_k) +~ %i*ω*ρ_b_m *~ conjugate(ψ_k))
+
((η_s_e / ϵ_0) . conjugate(%i*ω*D_hat_k_p) + (η_s_m/μ_0) .
conjugate(%i*ω*B_hat_k))
,L
)
];

$$[\mathrm{R\_k}_s(z) = L\,(\,\%i\rho\_\mathrm{b}_m\psi_k\omega + \%i\rho\_\mathrm{b}_e\phi_k\omega - \%i\left(\frac{\eta\_\mathrm{s}_m}{\mu_0}.\mathrm{B\_hat}_k\omega\right) - \%i\left(\frac{\eta\_\mathrm{s}_e}{\epsilon_0}.\mathrm{D\_hat\_k}_p\omega\right) - \mathrm{J\_b}_m.\mathrm{H\_hat\_}\right. \tag{\% o355}$$

(%i356) cmt_ch04_4565:[
'sum(N_kl *~ 'diff(A_l.z)~ - (((conjugate(γ_k) +γ_l)~ *~ N_kl - M_kl)~
*~ A_l),L,1,N)~ * exp(-γ_l*z)~
];

$$\left[N\%e^{-z\gamma_l}\left(N_{\mathrm{kl}}\,\mathrm{del}\left(A_l.z\right) - A_l\left(N_{\mathrm{kl}}\left(\gamma_l + \gamma_k\right) - M_{\mathrm{kl}}\right)\right)\right] \tag{\% o356}$$

(%i357) cmt_ch04_4566:[
'sum('sum(N_kl *~ 'diff(A_l(z),z) *~ exp(-γ_l*z)~ ,l,1,N) , k,1,N) = R_z(z)
];

$$\left[N^2 N_{\mathrm{kl}}\left(\frac{d}{dz}A_l(z)\right)\%e^{-z\gamma_l} = R_z(z)\right] \tag{\% o357}$$

(%i358) cmt_ch04_4567:[
'sum('sum(N_kl * ('diff(a_l(z),z) +~ γ_l*a_l(z)) ,K,1,N)~ ,L,1,N)
];

$$\left[N^2 N_{\mathrm{kl}} \left(a_l(z)\gamma_l + \frac{d}{dz} a_l(z)\right)\right] \tag{\% o358}$$

(%i362) E_hat_ck:[E_hck_x, E_hck_y, E_hck_z];
E_hat_cl:[E_hcl_x, E_hcl_y, E_hcl_z];
H_hat_cl_p:[H_hclp_x, H_hclp_y, H_hclp_z];
H_hat_ck_p:[H_hckp_x, H_hckp_y, H_hckp_z];

$$\left[\mathrm{E\_hck}_x , \mathrm{E\_hck}_y , \mathrm{E\_hck}_z\right] \tag{\% o359}$$

$$\left[\mathrm{E\_hcl}_x , \mathrm{E\_hcl}_y , \mathrm{E\_hcl}_z\right] \tag{\% o360}$$

$$\left[\mathrm{H\_hclp}_x , \mathrm{H\_hclp}_y , \mathrm{H\_hclp}_z\right] \tag{\% o361}$$

$$\left[\mathrm{H\_hckp}_x , \mathrm{H\_hckp}_y , \mathrm{H\_hckp}_z\right] \tag{\% o362}$$

(%i363)

```
cmt_ch04_4568:[
N_kl = 'integrate(
(
express(conjugate(E_hat_ck) ~ H_hat_cl_p)
+~
express(E_hat_cl~ ~ conjugate(H_hat_ck_p))
)
+
(
conjugate(φ_hat_k)~ *~ (%i*ω*D_hat_l_p)~ +~ φ_hat_l *~
conjugate(%i*ω*D_hat_k_p)
)
+
(
conjugate(ψ_hat_k) *~ (%i*ω*B_hat_l)~ +~ ψ_hat_l *~
conjugate(%i*ω*B_hat_k)
) . e_z
,S,1,N)
+
'integrate(
(
(
(conjugate(V_bar_hat_k_a) . J_hat_ll_a)
+~
(V_bar_hat_l_a . conjugate(J_hat_k_a))
)
+
(
(conjugate(V_bar_hat_k_m) . J_hat_l_m)
+
(V_bar_hat_l_m . conjugate(J_hat_k_m))
)
+
(
(conjugate(V_bar_hat_k_e)~ . J_hat_l_e)~
+
(V_bar_hat_l_e . conjugate(J_hat_k_e))
)
) . e_z,
S,1,N)
];
```

$$[\,N_{\mathrm{kl}} = [\,(N-1)\,(\,(\%i\mathrm{B\_hat}_l\psi\mathrm{\_hat}_k\omega - \%i\mathrm{B\_hat}_k\psi\mathrm{\_hat}_l\omega)\,.e_z - \%i\mathrm{D\_hat\_k}_p\phi\mathrm{\_hat}_l\omega + \%i\mathrm{D\_hat\_l}_p\phi\mathrm{\_hat}_k \qquad (\%\,o363)$$

(% i364)
```
cmt_ch04_4569:[
R_k(z)~ = 'integrate(
-(J_b_e . conjugate(E_hat_ck)  +~ J_b_m . conjugate(H_hat_ck_p))
+
(%i*ω*ρ_b_e *~ conjugate(ϕ_k) +~ ~ %i*ω*ρ_b_m *~ conjugate(ψ_k))
,S_b,1,N)
+
'integrate(
-(J_b_e . conjugate(E_hat_ck)  + J_b_m . conjugate(H_hat_ck_p))
+
(%i*ω*ρ_b_e * conjugate(ϕ_k) +  %i*ω*ρ_b_m * conjugate(ψ_k))
+~
(   (η_s_e/ϵ_0)   .   conjugate(%i*ω*D_hat_k_p)   +~   (η_p_m/μ_0)   *~
conjugate(%i*ω*B_hat_k) )
,
L,1,N)
];
```

$$[\,R_k(z) = (N-1)\,(\,\%i\rho\_\mathrm{b}_m\psi_k\omega+\%i\rho\_\mathrm{b}_e\phi_k\omega-\frac{\%i\mathrm{B\_hat}_k\eta\_\mathrm{p}_m\omega}{\mu_0}-\%i\left(\frac{\eta\_\mathrm{s}_e}{\epsilon_0}.\mathrm{D\_hat\_k}_p\omega\right)-\mathrm{J\_b}_m.\,[\mathrm{H\_hckp} \tag{\% o364}$$

(% i368)
```
E_hat_k:[E_hk_x, E_hk_y, E_hk_z];
H_hat_l_p:[H_hlp_x, H_hlp_y, H_hlp_z];
H_hat_k_p:[H_hkp_x, H_hkp_y, H_hkp_z];
H_hat_l:[H_hl_x, H_hl_y, H_hl_z];
```

$$\left[\mathrm{E\_hk}_x\,,\mathrm{E\_hk}_y\,,\mathrm{E\_hk}_z\right] \tag{\% o365}$$

$$\left[\mathrm{H\_hlp}_x\,,\mathrm{H\_hlp}_y\,,\mathrm{H\_hlp}_z\right] \tag{\% o366}$$

$$\left[\mathrm{H\_hkp}_x\,,\mathrm{H\_hkp}_y\,,\mathrm{H\_hkp}_z\right] \tag{\% o367}$$

$$\left[\mathrm{H\_hl}_x\,,\mathrm{H\_hl}_y\,,\mathrm{H\_hl}_z\right] \tag{\% o368}$$

(%i369) cmt_ch04_4570:[

N_kl = 'integrate((express(conjugate(E_hat_k) ~ H_hat_l_p) +~ express(H_hat_l ~ conjugate(H_hat_k_p)))~ . e_z, S, 1,N)
+
'integrate(
(
((conjugate(V_hat_bar_k_a) . J_hat_l_a)~ +~ (V_bar_hat_l_a . conjugate(J_hat_k_a)))
+~
((conjugate(V_bar_hat_k_m)~ . J_jat_l_m)~ +~ (V_bar_hat_l_m . conjugate(J_hat_k_m)))
+
((conjugate(V_bar_hat_k_e) . J_hat_l_e)~ +~ (V_bar_hat_l_e . conjugate(J_hat_k_e)))
)
~ . e_z, S,1,N)
];

$[N_{\mathrm{kl}} = (N-1)\,(\,(\,\mathrm{V\_hat\_bar\_k}_a . \mathrm{J\_hat\_l}_a + \mathrm{V\_bar\_hat\_l}_m . \mathrm{J\_hat\_k}_m + \mathrm{V\_bar\_hat\_l}_e . \mathrm{J\_hat\_k}_e + \mathrm{V\_bar}$ (%o369)

(%i370) cmt_ch04_4571:[

R_kl(z) = 'integrate((J_b_e . conjugate(E_hat_k)) +~ (J_b_m . conjugate(H_hat_k_p)), S_b, 1,N)
-
'integrate((J_s_e . conjugate(E_hat_k)) +~ (J_s_m . conjugate(H_hat_k_p)) ,L_s,1,N)
-
'integrate((J_s_eff_e . conjugate(E_hat_k)) +~ (J_s_eff_m . conjugate(H_hat_k)) , L_b_s,1,N)

];

$[R_{\mathrm{kl}}(z) = -\left(\mathrm{J\_s}_m . \left[\mathrm{H\_hkp}_x, \mathrm{H\_hkp}_y, \mathrm{H\_hkp}_z\right] + \mathrm{J\_s}_e . \left[\mathrm{E\_hk}_x, \mathrm{E\_hk}_y, \mathrm{E\_hk}_z\right]\right)(N-1) - \left(\mathrm{J\_s\_eff}_m . \mathrm{H\_}\right.$ (%o370)

(%i371) cmt_ch04_4572:[
(η_s_e/ϵ_0) . conjugate(%i*ω*D_hat_k_p) + (η_s_m/μ_0) * conjugate(%i*ω*B_hat_k)
=
ϕ_s*n_o~ *conjugate(%i*ω*D_hat_k_p) +~ ψ_s *~ n_o . conjugate(%i*ω*B_hat_k)
];

$$\left[-\frac{\%i\mathrm{B\_hat}_k \eta\_\mathrm{s}_m \omega}{\mu_0} - \%i\left(\frac{\eta\_\mathrm{s}_e}{\epsilon_0}.\mathrm{D\_hat\_k}_p\omega\right) = -\%i\mathrm{D\_hat\_k}_p n_o \phi_s \omega - \%i\left(n_o.\mathrm{B\_hat}_k\omega\right)\psi_s\right] \quad (\%\ o371)$$

4.5.3.2 Equations of mode excitation for lossless SDAM waveguides (Space Dispersive Active Media)

--> cmt_ch04_4573:[
'diff(A_k(z)~ ,z) =
(1/N_k)~ *~ 'integrate(
-(J_b_e . conjugate(E_ck)~ +~ J_b_m . conjugate(H_ck_p))
+
(%i*ω*ρ_b_e *~ conjguate(ϕ_k) +~ %i*ω*ρ_s_m~ *~ conjugate(ψ_k))
,S_b, 1,N)
+
(1/N_k) * 'integrate(
-(J_b_e . conjugate(E_ck) + J_b_m . conjugate(H_ck_p))
+
(%i*ω*ρ_b_e * conjguate(ϕ_k) + %i*ω*ρ_s_m * conjugate(ψ_k))
((~ η_s_e/ϵ_0)~ . conjugate(%i*ω*D_k_p) +~ (η_s_m/μ_0) . conjugate(%i*ω*B_k))

,L_s, 1,N)

];

$$[\frac{d}{dz}A_k(z) = \left((N-1)\left(\%i\rho\_\mathrm{s}_m\psi_k\omega + \%i\rho\_\mathrm{b}_e\,\mathrm{conjguate}\left(\phi_k\right)\omega - \mathrm{J\_b}_m.\left[\mathrm{H\_ckp}_x, \mathrm{H\_ckp}_y, \mathrm{H\_ckp}_z\right] - \mathrm{J\_b}_e. \quad (\%\ o376)$$

```
-->     cmt_ch04_4574:[

        'diff(a_k(z) ,z)  +~ %i*βk*a_k(z)~ =
        (1/N_k) * 'integrate(
        -(J_b_e . conjugate(E_ck) + J_b_m . conjugate(H_ck_p))
        +
        (%i*ω*ρ_b_e * conjguate(φ_k) + %i*ω*ρ_s_m * conjugate(ψ_k))
        ,S_b, 1,N)
        +
        (1/N_k) * 'integrate(
        -(J_b_e . conjugate(E_ck) + J_b_m . conjugate(H_ck_p))
        +
        (%i*ω*ρ_b_e * conjguate(φ_k) + %i*ω*ρ_s_m * conjugate(ψ_k))
        +
        (( η_s_e/ϵ_0) . conjugate(%i*ω*D_hat_k_p) + (η_s_m/μ_0) .
        conjugate(%i*ω*B_hat_k))

        ,L_s, 1,N)
        ];
```

$$[\%i a_k(z)\beta\mathrm{k}+\frac{d}{dz}a_k(z)=((N-1))/N_k+\big((N-1)\ \big(\%i\rho\_\mathrm{s}_m\psi_k\omega+\%i\rho\_\mathrm{b}_e\,\mathrm{conjguate}\,(\phi_k)\,\omega-\mathrm{J\_b}_m.\,\big[\mathrm{H\_ckp}_x \qquad (\%\ \mathrm{o}396)$$

(%i394) cmt_ch04_4575:[
N_k = 2 * re(
'integrate(
(
express(conjugate(E_hat_ck) ~ conjugate(H_hat_ck_p))
+
conjugate(ϕ_hat_k) * (%i*ω*D_hat_k_p)
+
conjugate(ψ_hat_k) * (%i*ω*B_hat_k)
) . e_z
,S,1,N)
)
+
2 * re(
'integrate(
(
(conjugate(V_bar_hat_k_a)~ . J_hat_k_a)
+~
(conjugate(V_bar_hat_k_m)~ . J_hat_k_m)~
+
(conjugate(V_bar_hat_k_e)~ . J_hat_l_e)

) . e_z
,S,1,N)
)

];

$[\,N_k = 2\,\mathrm{re}\,(\,(N-1)\,(\,[\,\%i\mathrm{B\_hat}_k\psi\mathrm{\_hat}_k\omega+\%i\mathrm{D\_hat\_k}_p\phi\mathrm{\_hat}_k\omega+\mathrm{E\_hck}_y\mathrm{H\_hckp}_z-\mathrm{E\_hck}_z\mathrm{H\_hckp}_y\,,\%i\mathrm{B}$ (%o394)

−>

```
cmt_ch04_4576:[
'diff(A_k(z) ,z) =
(1/N_k) * 'integrate(
-(J_b_e . conjugate(E_ck) + J_b_m . conjugate(H_ck_p))
+
(%i*ω*ρ_b_e * conjguate(φ_k) + %i*ω*ρ_s_m * conjugate(ψ_k))
,S_b, 1,N)
+
(1/N_k) * 'integrate(
-(J_b_e . conjugate(E_ck) + J_b_m . conjugate(H_ck_p))
+
(%i*ω*ρ_b_e * conjguate(φ_k) + %i*ω*ρ_s_m * conjugate(ψ_k))
+
(( η_s_e/ϵ_0) . conjugate(%i*ω*D_k_p) + (η_s_m/μ_0) .
conjugate(%i*ω*B_k))

,L_s, 1,N)




];
```

$$[\frac{d}{dz}A_k(z) = ((N-1))/N_k + ((N-1)\ (\%i\rho\_\mathrm{s}_m\psi_k\omega + \%i\rho\_\mathrm{b}_e\,\mathrm{conjguate}\,(\phi_k)\,\omega - \mathrm{J\_b}_m.\,[\mathrm{H\_ckp}_x\,,\mathrm{H\_ckp}_y\,,\mathrm{H} \tag{\% o395}$$

(%i398) cmt_ch04_4577:[
a_k = A_k(z)~ *~ exp(-γ_k8z),
'diff(a_k(z) ,z) + γ_k *a_k(z) =
(1/N_k) * 'integrate(
-(J_b_e . conjugate(E_hat_ck) + J_b_m . conjugate(H_hat_ck_p))
+
(%i*ω*ρ_b_e * conjguate(ϕ_hat_k) + %i*ω*ρ_s_m * conjugate(ψ_hat_k))
,S_b, 1,N)
+
(1/N_k) * 'integrate(
-(J_s_e . conjugate(E_hat_ck) + J_s_m . conjugate(H_hat_ck_p))
+
(%i*ω*ρ_b_e * conjguate(ϕ_hat_k) + %i*ω*ρ_s_m * conjugate(ψ_hat_k))
+
((η_s_e/ϵ_0) . conjugate(%i*ω*D_hat_k_p) + (η_s_m/μ_0) . conjugate(%i*ω*B_hat_k))

,L_s, 1,N)

];

$$[\, a_k = A_k(z)\%e^{-\gamma\_k8z} \,, a_k(z)\gamma_k + \frac{d}{dz}a_k(z) = ((N-1))\,/N_k + ((N-1))\,/N_k \,] \qquad (\%\ o398)$$

(%i399) cmt_ch04_461:[
N_hat . Z(z)~ =R(z)
]
;

$$[N_{\mathrm{hat}}.\,\mathrm{Z}(z) = \mathrm{R}(z)] \qquad (\%\ o399)$$

--> cmt_ch04_462:[
Z(k)= 'diff(a_k(z)~ ,z)~ +~ γ_k * a_k(z)~ ,
Z(k) = 'diff(A_k(z),z) *~ exp(-γ_k*z)
];

$$\left[\mathrm{Z}(k) = a_k(z)\gamma_k + \frac{d}{dz}a_k(z) \,, \mathrm{Z}(k) = \left(\frac{d}{dz}A_k(z)\right)\%e^{-z\gamma_k}\right] \qquad (\%\ o400)$$

(% i409) cmt_ch04_463:[
if is_active(N_kl)~ then N_kl = N_k *~ δ_kl,
if is_reactive(N_kl) then N_kl = R_k_bar * δ_k_bar_l,
if is_reactive(N_k) then N_k = N_k_k_bar,
if is_reactive(Z_k) then Z_k = R_k_bar/N_k_bar,
N_k_k_bar = conjugate(N_k_bar_k),
conjugate(N_kbar_k) = conjugate(N_k_bar)
];

$$[\text{if is\_active}\,(N_{\mathrm{kl}})\ \text{then}\ N_{\mathrm{kl}} = N_k \delta\text{\_kl}\ , \text{if is\_reactive}\,(N_{\mathrm{kl}})\ \text{then}\ N_{\mathrm{kl}} = \text{R\_k\_bar}\delta\text{\_k\_bar}_l\ , \text{if is\_reactive}\,(N_k$$

(% o409)

(% i410) cmt_ch04_464:[
conjugate(γ_k)~ +~ γ_k_bar = 0,
if is_active(Z_k)~ then Z_k = Rk/N_k,
if is_reactive(Z_k) then Z_k = Rk/N_k_bar
];

$$\left[\gamma\text{\_k\_bar} + \gamma_k = 0\,, \text{if is\_active}\,(Z_k)\ \text{then}\ Z_k = \frac{\text{Rk}}{N_k}\ , \text{if is\_reactive}\,(Z_k)\ \text{then}\ Z_k = \frac{\text{Rk}}{\text{N\_k\_bar}}\right] \quad (\%\ o410)$$