Nanoscience and Nanotechnology

Quantum Cellular Automata

The "GPU" of quantum computing

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Abstract

Contrary to sequential quantum computing models (such as quantum circuits), quantum cellular automata (QCA) perform parallel qudit operations on local subsystems according to a global rule G. As with other contemporary paradigms, noise reduction remains a major hurdle, but recent studies propose theoretical error corrected models to mitigate such disruptive effects [1]. The initial QCA lattice continuously updates over discrete timesteps via particle interactions, where the global rule is enacted depending on adjacent cell states. Additionally, the system's simultaneous evolution must be "causal (it propagates information at a bounded speed) and translation-invariant (it acts everywhere the same)" [2]. This ensures coherent transfer of logical information, where the global rule dictates quantum error correction effects. Currently, neutral atom arrays, specifically Rydberg-atom platforms, and trapped ions are the two most promising architectures. The advent of functional QCA not only concretizes a complementary template of quantum computing, but also hints to eventual advanced physical simulations. Nonetheless, exploiting its "massive parallelism and locally constrained instructions" [3] remains limited in both scale and efficiency.

1 Introduction

The emergence of complex behaviour/structure in nature bears an inherent beauty, with arguably philosophical implications. Reductionism aside, the existence of any phenomenon undeniably depends on the rules that constrain its environment. In physics, these rules are fundamental theorems that allow us to innovate. Following these simple design principles, one such innovation aims to revolutionize quantum computing: Quantum Cellular Automata. This distributed quantum computing paradigm excels at propagating simple rules over a lattice simultaneously, so in parallel. The array of finite-dimensional quantum subsystems, called qudits, continuously evolves through local interactions with neighbouring cells. Classical cellular automata were first suggested as a discretized space for physical models, since they effectively divide systems into a discrete number of cells. However, they lacked the necessary mechanics to describe fundamentally quantum interactions. Thus, quantum simulations became an ambitious, but logical, application of QCAs. Although this approach is an appealing discretization of both space and time, showing that all physics can be simulated this way is a non-trivial endeavor [5]. Nevertheless, QCAs could, in theory, reproduce quantum walks, lattice gasses, Floquet dynamics, quantum field theory [4] and many other setups. Aside from physical use, QCA can also benefit practically any quantum algorithm. For example, quantum machine learning, cryptography breaches, secure computation and quantum interferometry all gain from parallel commutation operators [6]. Thus, this secures QCAs as a complementary distributed architecture to quantum circuits.

2 Science Background

QCA are a powerful alternative QC paradigm that is proven to be Quantum Turing complete [3] [2]. Quantum Turing completeness defines the axioms of all quantum computers, so QCAs are considered universally applicable to quantum computing. Pr. Pablo Arrighi shows this by constructing a partial QCA (PQCA) from which he builds the universal quantum logic gates [2]. In this minimal example, incoming information gets scattered by a fixed 'scattering unitary operator' U acting on a specific cell for each moment t. Essentially, this defines the quantum processing unit in the QCA architecture as a cell of 2x2 qubits. The local propagation unitaries are shown in figure 1. This is analogous to the classical CA rules, extensively featured in Conway's Game of Life.

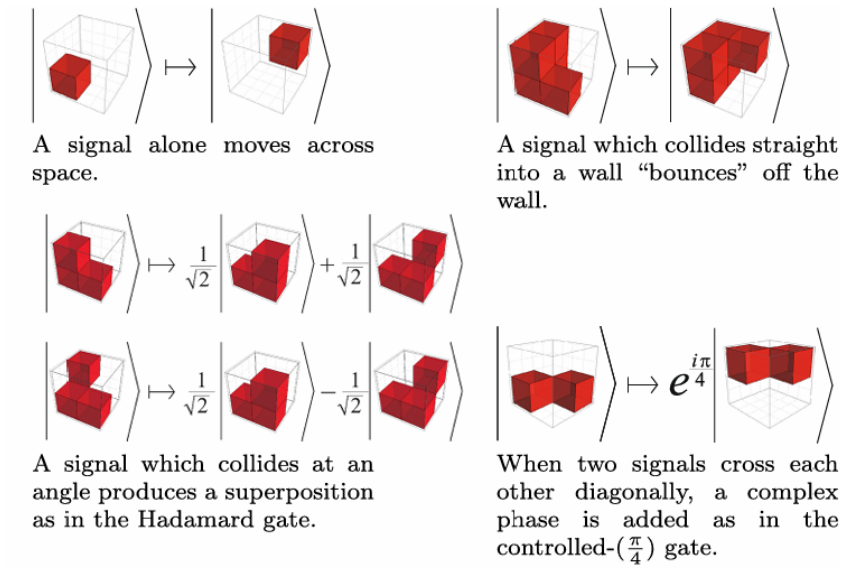

Figure 1: "The Quantum Game of Life": Scattering unitaries of the universal 3D PQCA. Each cell contains one qubit, red if in the $|1\rangle$ state, and white if $|0\rangle$. The application of lattice operator U is staggered in time. When t is even, the QPCA cells $\{0,1\}^3\ \forall(2\mathbb{Z})^3$ translations are updated. Conversely, when odd t, the QPCA cells $\{1,2\}^3\ \forall(2\mathbb{Z})^3$ translations are updated.

QCA therefore constitute a valid framework for processing any quantum algorithm in a distributed system. Additionally, since the PQCA is minimal, we can reduce any concurrent PQCA to another. As universal quantum gates can be constructed (Fig. 2), there also exists an equivalent reduction to quantum circuits and vice versa [4].

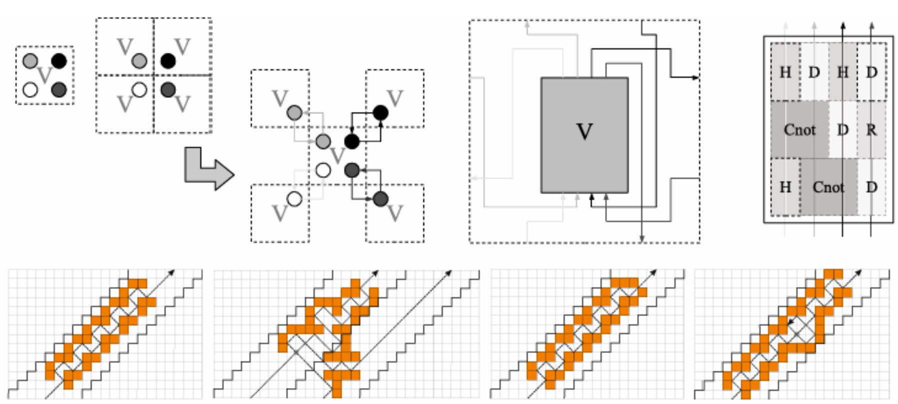

Figure 2: "Intrinsic simulation of a 2D PQCA by another. Top left: The partitions at odd and even steps of the PQCA with scattering unitary V overlap, but in the simulating PQCA they will be laid out side-by side. Top right: The simulating PQCA emulates each V in parallel, as well as the wirings between the V. Bottom: It does so by combining tiles (of fixed shapes and taking a fixed number of steps to be traversed, implementing universal quantum gates) into a layout that implements V" [2]

Conditions for universal QCAs are much more restrictive and will be expanded upon in section 3. Notably, unit cells contain 2 qubits in quantum circuits and 8 qubits in QCAs. Yet, the main advantage of QCAs over quantum circuits is their lower demand on hardware [3]. Indeed, such distributed QC architectures rely on the combination of parallel single-device quantum computers which communicate between eachother. In addition, single site addressability is much more feasible without the need to directly manipulate qubits [4]. There is therefore signifcant value in the development of various QCA structures.

3 Putting the Quantum in the Computing

A global unitary operator which iteratively acts upon the QCA must have precise properties supported by quantum mechanics. For instance, discrete-time properties arise in QCAs due to causality. Since the global operator G is causal, there exists an upper bound on the speed at which information is transmitted in a QCA [4] [2]. This is because information cannot travel faster than a fixed number of cells per-time step, dictated by G. This upper limit can also be chosen depending on the setting. For example, light speed will bind a relativistic system or the speed of sound can limit non-relativistic approximations [4]. Causality ensures that locality is preserved when updating states based on adjacent cells. Another generally accepted condition is translation-invariance, but some studies deem it optional [4]. Translationnally invariant dynamics act on all qudits of the lattice in the same way, such that G remains globally consistent. This definition includes not only euclidean space translations, but also requires consistency over all discrete time steps [2] [1]. This type of homogeneity is crucial for physical simulations where the rules of physics are the same everywhere simultaneously. The unitary axiom is often paired with causality to prove localizability [2]. This property enforces local mechanisms by mapping G to a single cell while only considering adjacent information within the same time step. This "unit" of space-time automatically evolves in parallel to every other mapping in the qudit array. Qudit coupling mechanisms therefore distribute information over the lattice in a controlled way. Otherwise, recent developments in quantum error correction (QEC) establish a measurement-free correction method. Independence from measurements reduces noise, which is a major hurdle in the production of QCA (Sections 4,5).

4 Manufacturing QCA

While QCA are feasible paradigms, they are overshadowed by the more popular, yet comparable, sequential paradigms such as quantum circuits. Nonetheless, the prospect of building practical QCA relies on first the composition of the lattice and second the global unitary operator.

Currently, the most promising setups involve reconfigurable 2D neutral-atom arrays. Qubit assemblies of this type are characteristically identical, which is great for scalability. Moreover, they remain coherent for long times, even when subjected to dense multidimensional conditions [9]. For instance, experiments with Rydberg atoms demonstrate "highly parallel coherent qubit transport and entanglement enabling a powerful quantum information architecture" [8]. Global Rydberg laser pulses generate entanglement interactions, wherein information is exchanged. This allows for full control over the system at every discrete time step (past, present and future).

Alternatively, trapped ion arrangements create uniformity by laser-cooling the lattice to its motional ground state. In fact, ion trap qubits exhibit good quantum memory due to their long decoherence time. From this initial ensnarement, two quantum operators are achieved by applying two counter-propagating optical Raman beams: qubit rotations and entanglement interactions [10]. To draw a comparison to our definition of QCA, one beam is global while the other is split into independent and individual addressing beams per qubit. The primary propagation formalism here is the discrete quantum walk. This framework inscribes possible superimposed paths over multi-qubit states and is the basis for a wide variety of quantum algorithms [10] [3]. This dynamic evolution corresponds to a discretized Dirac equation with Hamiltonian $H = P\sigma_z + m\sigma_x$, where P and mass (m) are controllable parameters [10] [4].

More QCA systems exist, but the aforementioned constructions are of particular interest because they can integrate QEC methods advanced by T. L. M. Guedes *et al* [1]. Not only does this paper prove that QEC is compatible under the QCA architecture, but also presents a viable candidate. The quantum two-line voting (QTLV) rule displayed measurement-free characteristics under phenomenological bit-flip noise and depolarizing circuit noise. This makes it the first fully quantum QCA approach, leaving classical decoding interfaces behind.

5 Discussion

Although QCA tackle the quantum computing problem from a fresh angle, they unfortunately suffer from many of the same plagues. Quantum computers are still far from being practical and require at least 1000 **high-quality** qubits to perform any meaningful tasks. Case in point, modern cutting-edge QC are often referred to as noisy intermediate scale quantum (NISQ) devices; they are noisy and experience loss of information [6]. Distributed QC paradigms attempt to cut down on these restrictions by pairing together to increase overall computing resources. However, the effectiveness of this method is bottlenecked by quantum communication capabilities. Since most of the noise originates from communication links, it will be crucial to minimize information transfers between subsystems [6]. On a positive note, aforementioned QEC procedures are a step in the right direction for the future of QCAs.

Considering the feasibility of physical simulations, it is important to question if the entirety of physics can truly be discretized. For instance, Dirac fermions and non-abelian gauge theories are both quantum field theories that can be represented by self-updating quantum interactions. However, fermion doubling has yet to be simulated in any way [4], and general relativity will be difficult to reconcile with QCA [2].

Another challenge QCAs face is their necessary robustness. Relaxing these stringent conditions will be an interesting task for researchers willing to push their limits. Approximate locality preserving dynamics, symmetries, defining irreversible QCAs, etc. are all edge-cases worth investigating [4]. Another interesting research avenue is the pursuit of multidimensional and/or topological QCA, which are presently limited to 2D planes. We are far from hearing the end of emergent properties in QCAs.

Nonetheless, this is still a field in its infancy and it wouldn't be surprising to see noisy implementations of this paradigm within 10 years [2]. Assuming quantum advantage will eventually be achieved, I can imagine QCA being suitable for algorithms that deal with many blocks of commuting operations. This complements quantum circuits, which are more geared towards algorithms containing many gates between different pairs of qubits [6]. We can draw an analogy between this synergy and the collaboration between the CPU and GPU. Perhaps future quantum computing designs may incorporate similar task divisions as their classical ancestors.

References

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Note that Farrelly poses 8 open ended questions at the end of his review. Worth the read!

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A Applications

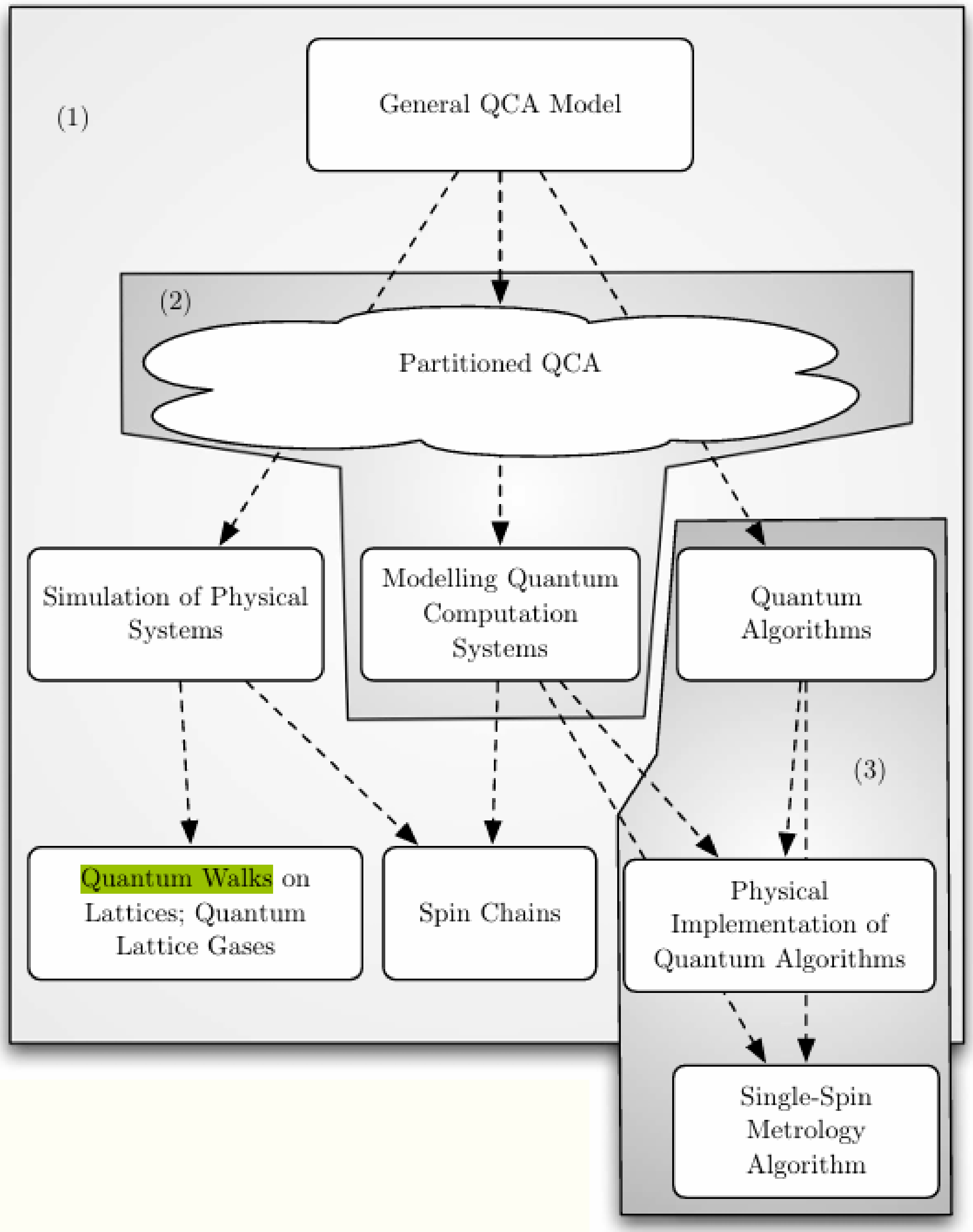

Figure 3: Visualization of potential quantum cellular automata use cases.

B Revisions

- Several general style changes.
- Contextualized words such as "parallelism" and "discretization" to appeal to a broader audience.

  Ex: **Old:** "This distributed quantum computing paradigm excels at immense parallelism through the propagation of simple rules over a lattice." **New:** This distributed quantum computing paradigm excels at propagating simple rules over a lattice simultaneously, so in parallel.
- Clarified that the 4 example systems in the introduction were examples and not the only applications possible.
- Specified universal logic quantum gates.
- added the precision that the fixed scattering unitary is an operator
- I decided to leave information in the caption without mentioning it in the text as I deem it redundant. Also, I believe it is good practice to write very detailed captions, since they considerably enrich a figure.
- I didn't agree with some of Kind Burgundy Raspberry's comments, and considering the two other reviewers thought my methodology was strong, I won't be changing its structure.
- Hopefully these changes made my paper more approachable to readers less familiar with quantum computing. It is challenging to write on niche concepts within an already niche field while still making accessible to all!