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Reflection of Fast Sausage Modes from the Solar Transition Region

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Abstract Using the two-layer model for representation of the solar transition region, an expression for the reflection coefficient of fast sausage modes is derived. This derived expression is compared with observation. It is found that fast sausage modes with typical periods in the tens of seconds are not significantly reflected from the transition region, but its effect is not negligible. It is further concluded that more observations need to be made of fast sausage modes reflecting off of the transition region before the derived expression can be properly validated.

1. Introduction

In recent decades, the solar atmosphere has been observed transmitting a number of low-frequency wave phenomena. This has allowed for the use of coronal seismology in determining various plasma parameters. In particular, density-enhanced magnetic flux tubes, coronal loops, act as natural waveguides for these oscillations.

Under the linear magnetohydrodynamic (MHD) wave theory, oscillations in coronal loops generally fall under four main groups, torsional Alfvén modes, slow modes, fast kink modes, and fast sausage modes. Of these waves, the present discussion focuses on fast sausage modes which are characterized by axisymmetric, periodic expansion and compression of the coronal loop's minor radius. Additionally, the magnetic field, density, and pressure of the coronal loop are also perturbed and oscillate in antiphase with the minor radius perturbations.

Another important property of sausage modes is that they possess a cutoff wavenumber. This causes axial wavenumbers less than the overtone-specific cutoff to become leaky and radiate fast waves into the surrounding plasma. In this paper, only non-leaky modes, or trapped modes, are consider. This dispersion property makes sausage modes of great seismological interest as a means to derive plasma parameters of coronal loops.

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Given these motions, it follows that sausage modes are highly compressible. This leads to emissions in the microwave (e.g., Melnikov et al., 2005), EUV (e.g., Yuan et al., 2013 and Nisticò, Pascoe, and Nakariakov, 2014), and soft X-ray bands (e.g., Hayes et al., 2016) that have been observed within all layers of the solar atmosphere. For a recent review on fast sausage modes, see Li et al., 2020.

The transition region (TR), a thin interface layer between the upper chromosphere and corona, is an important factor when considering wave propagation in the solar atmosphere. Because it separates the relatively cool, dense chromosphere from the hot, thin corona, it serves as a boundary from which incident waves may be reflected. Therefore, to properly understand how waves propagate and transmit energy from the chromosphere to the corona, a quantitative understanding of this reflection must be attained. As of now, however, interaction between sausage modes and the TR has not be properly analyzed. This analysis has been conducted for torsional Alfvén waves by Hollweg, 1978, Hollweg, 1984, and more recently Tsap and Kopylova, 2021, but not yet for the slow modes and fast kink and sausage modes.

The need for this information comes from the lack of closure on the various plausible, but untested in the statistical sense, excitation scenarios whereby the excited sausage mode must propagate through the TR. For example, in Berghmans, de Bruyne, and Goossens, 1996, an analytical model for sausage modes generated by axisymmetric coronal loop footpoint motions is presented. It is concluded that sausage modes generated within the lower layers of the atmosphere may spend a considerable amount of time reflecting back and forth between them due to their significantly decreased Alfvén speed. In Shestov et al., 2017, torsional Alfvén waves, which are present in the upper chromosphere (see Liu et al., 2019), are shown to generate fast sausage modes through the ponderomotive force and nonlinear phase-mixing. These two possible excitation methods present in the lower atmosphere may be capable generating waves that transmit through the TR and into the corona. Therefore, the results of this study may shed light on whether this is possible.

Within this study, an analytical expression is obtained for the reflection coefficient of a fast sausage mode reflecting off of a sharp density discontinuity. The expression's validity is tested against an observed reflection event and yields matching results. It must be noted that only a few of these events are recorded in the literature, severely limiting the extent to which the validity and/or accuracy of the expression may be tested.

2. Methods

The coronal loop is modeled as a straight, density-enhanced cylinder. The assumption of a straight rather than curved coronal loop (hereafter tube) is taken as the effects due to curvature are out of the scope of this study (see Díaz, Zaqarashvili, and Roberts, 2006, Díaz, 2006, and Thackray and Jain, 2017). A cylindrical coordinate system (r, φ, z) is used with the axis of the tube taken to be the axis of the coordinate system. The tube is taken to have a minor radius of a and thus a boundary separating the interior of the tube from the exterior at $r = a$.

The plasma perturbations making up the sausage wave are governed by the ideal MHD equations in the cold plasma limit ($\beta = 0$). By using the cold plasma limit, the plasma pressure in the system vanishes and the magnetic pressure is assumed to be the only pressure force. This assumption can be made given that a finite β has been shown to have only negligble effects on the propagation of sausage modes (see Chen et al., 2016 and Inglis et al., 2009). The magnetic field of the coronal loop is taken to of the form $B\hat{\boldsymbol{e}}_z$.

The equilibrium density of the system ρ_0 is assumed to be of the form

$$\rho_0 = \begin{cases} \rho_h, & z < 0, \\ \rho_c, & z > 0, \end{cases} \qquad \rho_h > \rho_c, \tag{1}$$

where subscripts of h and c denote chromospheric and coronal values, respectively, and $z = 0$ marks the modeled TR's density discontinuity. This approximation of the system's density as a simple step function was used considering the results found by Schwartz, Cally, and Bel, 1984 for Alfvén waves. Additionally, the temperature jump across the TR is not accounted for due to the assumption of a cold plasma.

Given the assumptions made, an expression for the reflection coefficient was found using analytical techniques. The validity of these analytical expressions was then tested against an observed case of fast sausage modes generated in a flare event interacting with the TR.

3. Reflection Coefficient

3.1. General Equations

To find an analytical expression for the reflection coefficient, we focus on the following small perturbations about the equilibrium: the Lagrangian plasma displacement $\boldsymbol{\xi}$, the magnetic field perturbation $\boldsymbol{b}$, and the total pressure perturbation P. In the cold plasma limit, plasma pressure vanishes and the total pressure perturbation takes the form of the magnetic pressure perturbation $P = Bb_z/\mu_0$. The linearized, ideal MHD equations for a cold plasma are

$$\frac{\partial^2 \boldsymbol{\xi}}{\partial t^2} = -\frac{1}{\rho_0}\nabla_\perp P + v_A^2 \frac{\partial^2 \boldsymbol{\xi}}{\partial z^2}, \tag{2}$$

$$\boldsymbol{b}_\perp = B\frac{\partial \boldsymbol{\xi}}{\partial z}, \tag{3}$$

$$P = -\rho_0 v_A^2 \nabla \cdot \boldsymbol{\xi} \tag{4}$$

where

$$v_A^2 = \frac{B^2}{\mu_0 \rho_0} \tag{5}$$

is the squared Alfvén speed, μ_0 is the vacuum permeability, $\nabla_\perp = \nabla - \hat{\boldsymbol{e}}_z \partial/\partial z$, and $\boldsymbol{b}_\perp = \boldsymbol{b} - b_z \hat{\mathbf{e}}_z$ (e.g., Ruderman and Erdélyi, 2009 and Oliver, Ruderman, and Terradas, 2015).

All perturbations are assumed to be of the form

$$P(t, r, \varphi, z) = \hat{P}(r) e^{i(-\omega t \pm kz + m\varphi)} \tag{6}$$

where the sign preceeding k is positive (negative) for a wave propagating in the positive (negative) z direction. For fast sausage modes, $m = 0$ resulting in all perturbations lacking azimuthal dependence and the φ components of $\boldsymbol{\xi}$ and $\boldsymbol{b}$ vanishing (Equations 2 and 3).

With P and ξ in the form of Equation 6, Equations 2 and 4 can be used to obtain

$$r^2 \frac{\mathrm{d}^2 \hat{P}}{\mathrm{d}r^2} + r \frac{\mathrm{d}\hat{P}}{\mathrm{d}r} + (l^2 r^2 - 1)\hat{P} = 0. \tag{7}$$

where

$$l^2 = \pm \frac{\omega^2 - k^2 v_A^2}{v_A^2} \tag{8}$$

is the square of the radial wavenumber with the preceeding sign positive or negative for internal ($r \leq a$) or external ($r > a$) values, respectively. In this paper, only internal values are considered and the sign preceeding Equation 8 is taken to always be positive.

Equation 7 is of the form of Bessel's differential equation. Requiring the solution to be both regular at $r = 0$ and non-divergent as $r \to \infty$, Equation 7 is satisfied by

$$\hat{P} = \begin{cases} A_i J_0(l_i r), & r \leq a, \\ A_e K_0(l_e r), & r \geq a, \end{cases} \tag{9}$$

where J_α is the Bessel function of the first kind and order α, K_α is the modified Bessel function of the second kind and order α, l_i is the internal radial wavenumber, l_e is the external radial wavenumber, and A_i and A_e are arbitrary constants. As previously mentioned, only internal values are considered and the subscript of i will be omitted. From Equation 9, we can find both $\hat{\xi}_r$ and $\hat{b}_r$ with

$$\hat{\xi}_r = \frac{1}{\rho_0 v_A^2 l^2} \frac{\mathrm{d}\hat{P}}{\mathrm{d}r}, \qquad \hat{b}_r = ikB\hat{\xi}_r \tag{10}$$

given by Equations 2–4.

3.2. Reflection Coefficient

Consider a propagating fast sausage mode generated by an unspecified source in the chromosphere, $z < 0$. Above this source and below the TR, the sausage mode can be defined by the superposition of an incident wave moving in the positive z direction and a wave reflected off of the TR moving in the negative

z direction. Given Equation 6 and restricting our focus to the internal solution, we can write the total pressure perturbation in the chromosphere as

$$P_h(t, r, z) = A\hat{P}_h(r)e^{i(-\omega t + k_h z)} + B\hat{P}_h(r)e^{i(-\omega t - k_h z)} \tag{11}$$

where subscripts of h denote values taken in the chromosphere and A and B are constants that define the contributions of the incident wave and the reflected wave, respectively. The chromospheric magnetic field perturbation $b_{rh}(t, r, z)$ takes the same form as Equation 11 and can be obtained by applying Equation 10.

In the corona, $z > 0$, the transmitted wave originating from the chromosphere only propagates away from the TR in the positive z direction. Therefore, the total pressure perturbation in the corona is

$$P_c(t, r, z) = C\hat{P}_c(r)e^{i(-\omega t + k_c z)} \tag{12}$$

where subscripts of c denote values in the corona and C is a constant that defines the amplitude of the transmitted wave. As was the case with the chromosphere, the coronal magnetic field perturbation $b_{rc}(t, r, z)$ can be obtained by applying Equation 10 to Equation 12.

At the TR, the perturbed total pressure and magnetic field are set to have matching conditions such that $P_h(t, r, 0) = P_c(t, r, 0)$ and $b_{rh}(t, r, 0) = b_{rc}(t, r, 0)$. Solving for C in both expressions, setting the results equal to each other, and further solving for B/A leads to

$$\frac{B}{A} = \frac{Q_h - Q_c}{Q_h + Q_c} \tag{13}$$

where

$$Q_h = k_h l_c \rho_c v_{Ac}^2 J_0(l_c r) J_1(l_h r) \tag{14}$$

and

$$Q_c = k_c l_h \rho_h v_{Ah}^2 J_0(l_h r) J_1(l_c r). \tag{15}$$

Using Equation 13, the reflection coefficient is taken to be $R = |B|^2/|A|^2$, or

$$R = \frac{|Q_h - Q_c|^2}{|Q_h + Q_c|^2}. \tag{16}$$

This expression is equivalent to the ratio of reflected energy flux to incident energy flux, ignoring any absorption by the TR. Similarly, the transmission coefficient T, or the ratio of transmitted energy flux to incident energy flux, can be expressed as $1 - R$.

4. Results

The reflection coefficient given in Equation 16 provides a way to analyze and predict observed reflection events. Generally, Equation 16 suggests that fast sausage modes, with typical periods in the tens of seconds, can effectively penetrate from the chromosphere to the corona. Further, as the period tends to 0 (or $\omega \to \infty$), the reflection coefficient vanishes regardless of the set density difference. This is very different from torsional Alfvén waves, which will have the reflection coefficient tend towards a certain value defined by the density difference between the chromosphere and corona (Tsap and Kopylova, 2021).

In the flare event analyzed in Kumar and Innes, 2015, a propagating fast sausage mode is seen to travel upwards along a coronal loop from a flare site and be partially reflected off of the TR. Using matching parameters, the reflection coefficient was calculated to be $R \approx 0.2$ implying a small but not negligble portion of the wave was reflected off of the TR. This result matches the observation of Kumar and Innes, 2015.

However, the very low number of observed fast sausage modes severely restricts the extent to which Equation 16 can be validated. The present analysis demonstrates a general agreement with a single observation, but may differ for other scenarios. For example, the coronal loop present within a flare event such as the one analyzed in Kumar and Innes, 2015 has significantly different plasma conditions than one in the quiet Sun (Aschwanden, Nakariakov, and Melnikov, 2004). Therefore, it is concluded that further comparison needs to be done with observation to adequately prove the extent to which the predictive power of Equation 16 is valid.

5. Conclusions

In this article we have derived an expression for the reflection coefficient for a propagating sausage mode interacting with the TR. The modeled TR accounts for the sharp density decrease between the chromosphere and corona. It has been shown that the results obtained from the derived expression match that with the flare-associated event analyzed in Kumar and Innes, 2015. However, it is concluded that more comparison with observed reflection events needs to be done in order to properly consider the extent to which its resulting values are valid. Until further observations of reflecting fast sausage modes are made, however, this cannot be done.

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