

# מבוא לראייה ממוחשבת – 22928

## 2016א

מנחה: אמיר אגוזי

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מפגש מס' 5

# בפעם שעברה:

- זיהוי פנים – Viola & Jones
- Adaboost
- Detection by classification
- Sliding window
- Global vs. part-based

# היום

- Texture synthesis
- Image quilting
- Image transformations
- Estimation
- RANSAC
- Camera model

# Overview

- Texture synthesis [Efros & Leung, ICCV'99]
- Quilting [Efros & Freeman 2001]
- Image Analogies [Hertzmann et al. 2001]
- Super-resolution [Freeman et al. 2002]
- Scene completion [Hays & Efros 2007]

Slides from: Alyosha Efros, Bill Freeman, James  
Hayes

[http://www.cs.nyu.edu/~fergus/teaching/comp\\_photo/index.html](http://www.cs.nyu.edu/~fergus/teaching/comp_photo/index.html)

# Texture

- Texture depicts spatially repeating patterns
- Many natural phenomena are textures



radishes



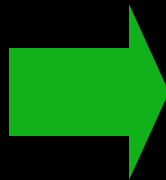
rocks



yogurt

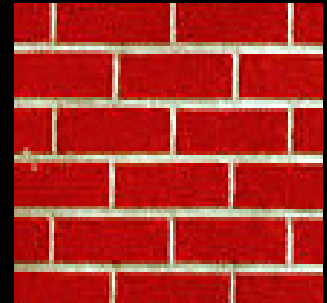
# Texture Synthesis

- Goal of Texture Synthesis: create new samples of a given texture
- Many applications: virtual environments, hole-filling, texturing surfaces



# The Challenge

- Need to model the whole spectrum: from repeated to stochastic texture



**repeated**

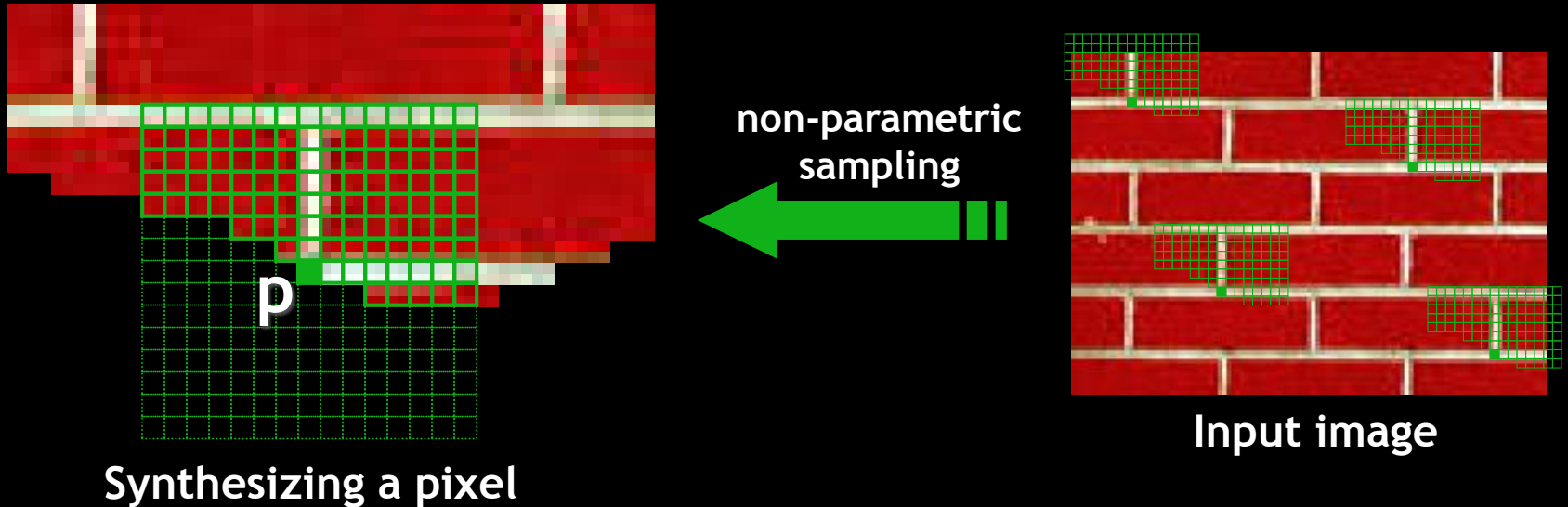


**stochastic**



**Both?**

# Efros & Leung Algorithm



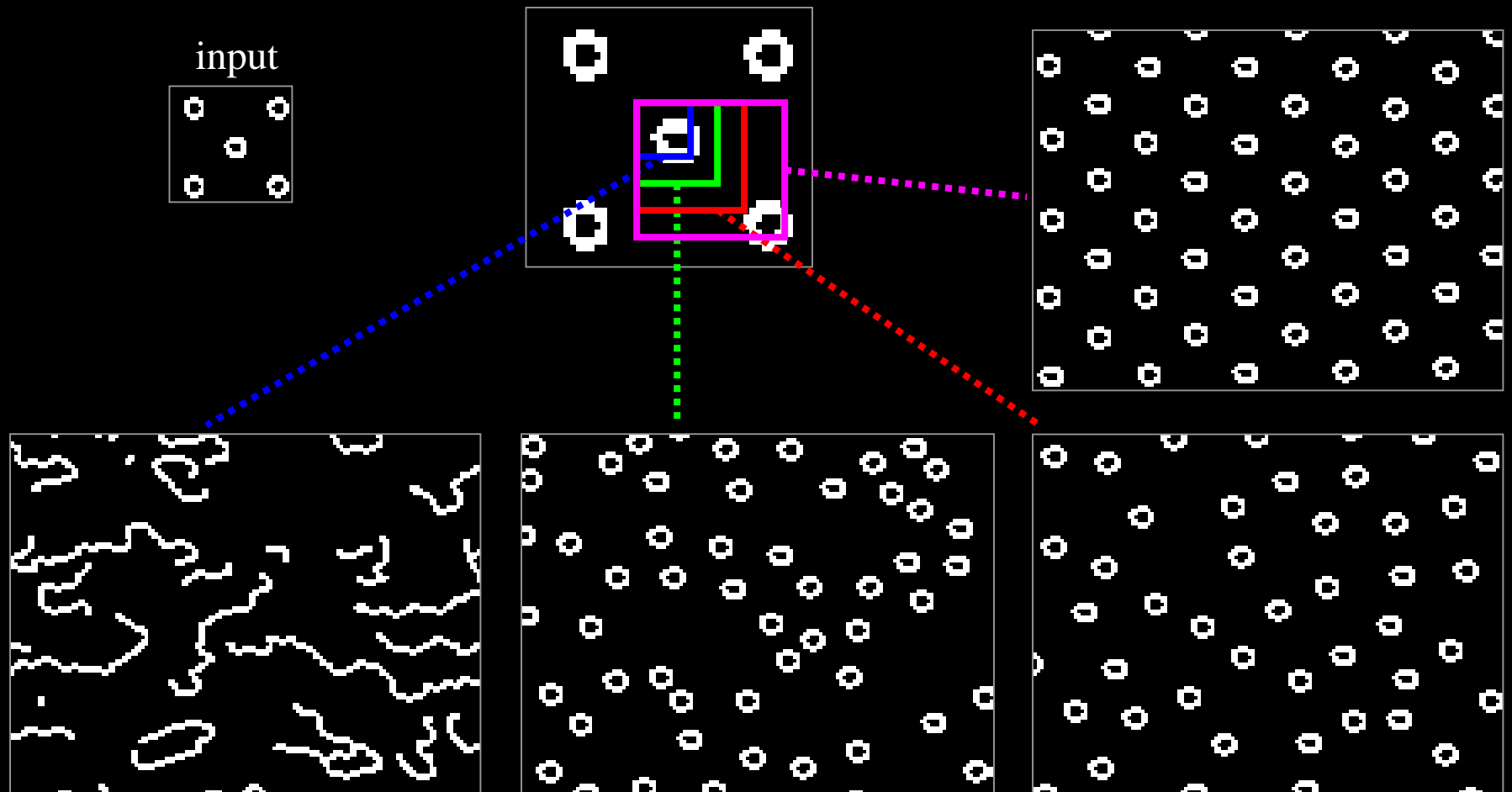
- Assuming Markov property, compute  $P(\mathbf{p}|\mathbf{N}(\mathbf{p}))$ 
  - Building explicit probability tables infeasible
  - Instead, we *search the input image* for all similar neighborhoods — that's our pdf for **p**
  - To sample from this pdf, just pick one match at random



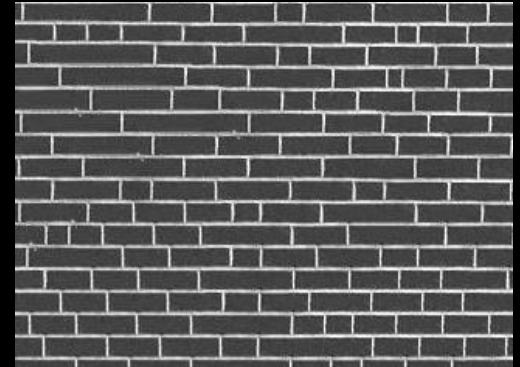
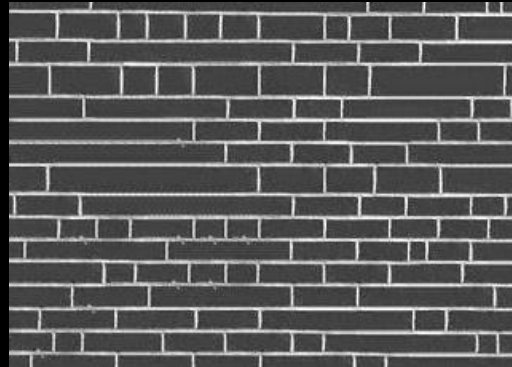
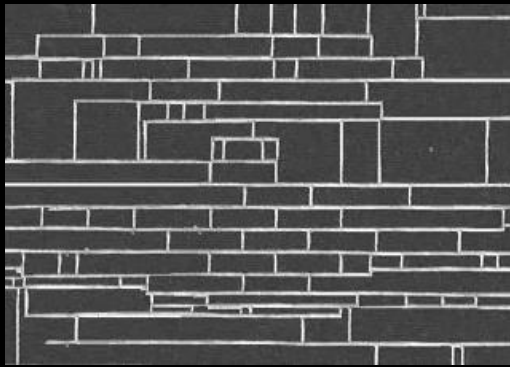
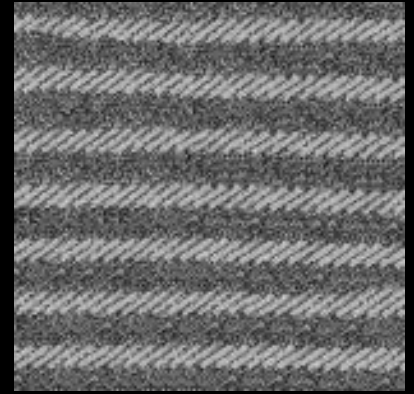
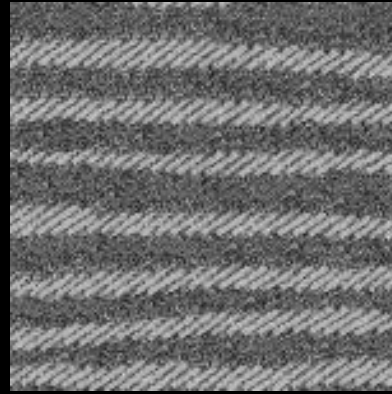
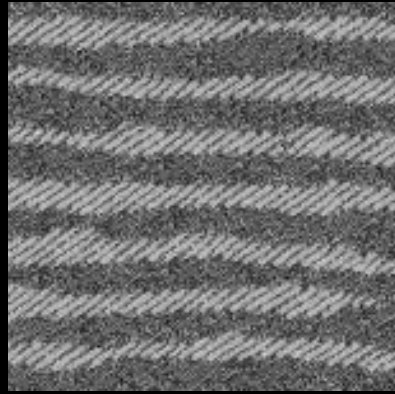
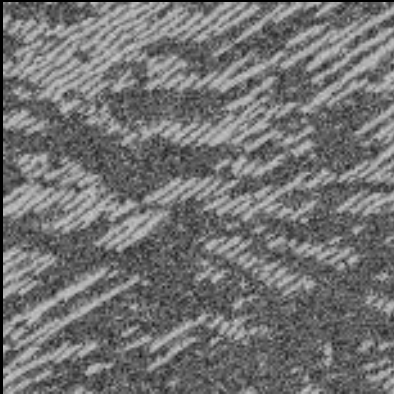
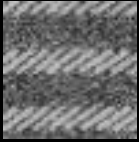
# Some Details

- Growing is in “onion skin” order
  - Within each “layer”, pixels with most neighbors are synthesized first
  - If no close match can be found, the pixel is not synthesized until the end
- Using *Gaussian-weighted* SSD is very important
  - to make sure the new pixel agrees with its closest neighbors
  - Approximates reduction to a smaller neighborhood window if data is too sparse

# Neighborhood Window



# Varying Window Size

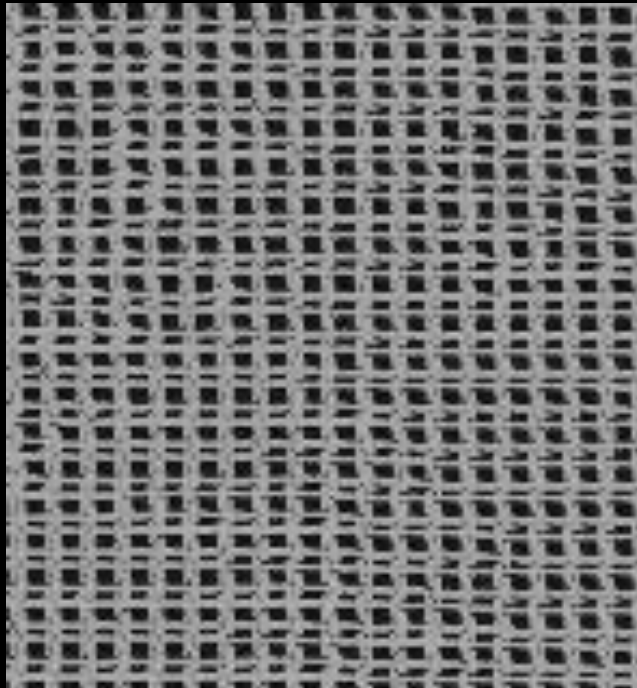
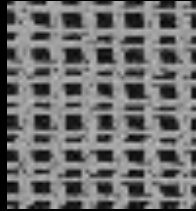


Increasing window size

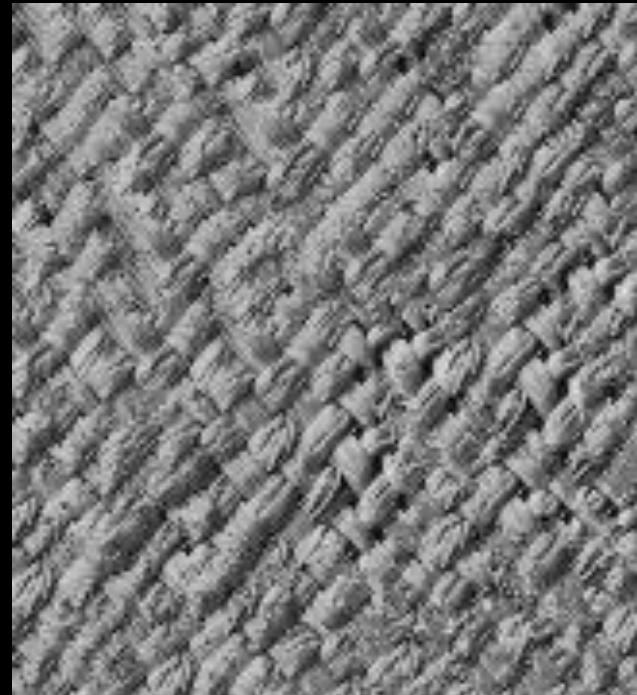
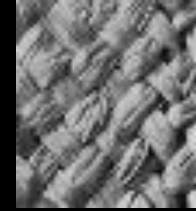


# Synthesis Results

french canvas



rafia weave

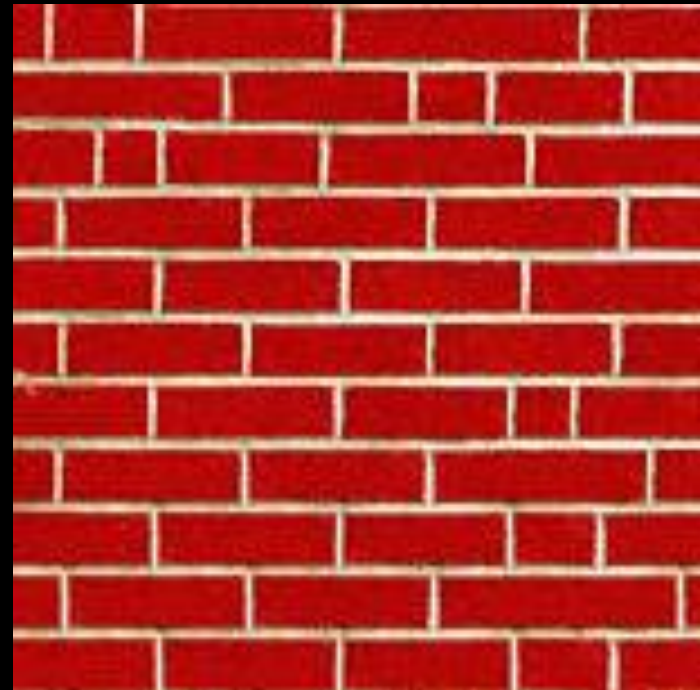
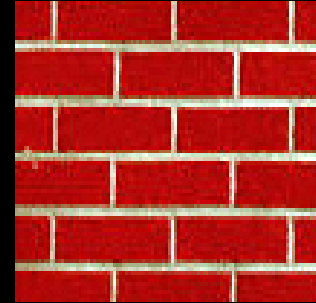


# More Results

white bread



brick wall



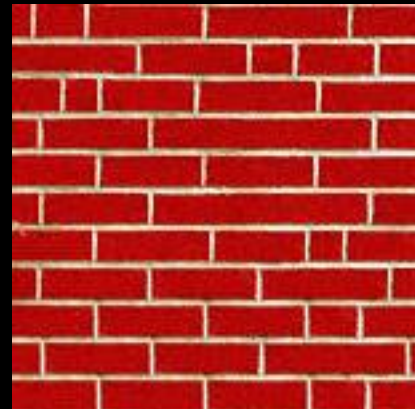


# Homage to Shannon

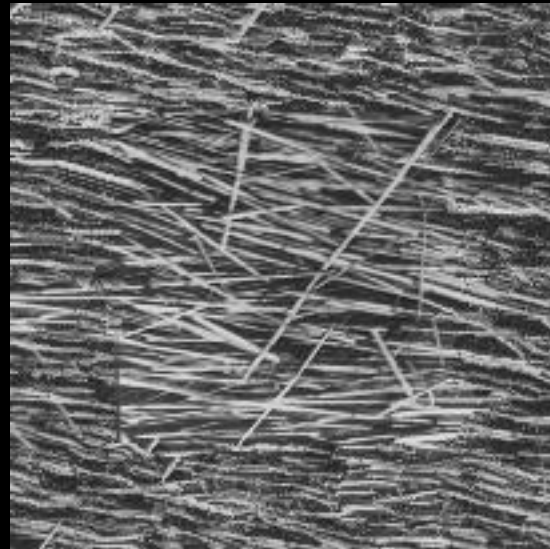
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# Hole Filling



# Extrapolation





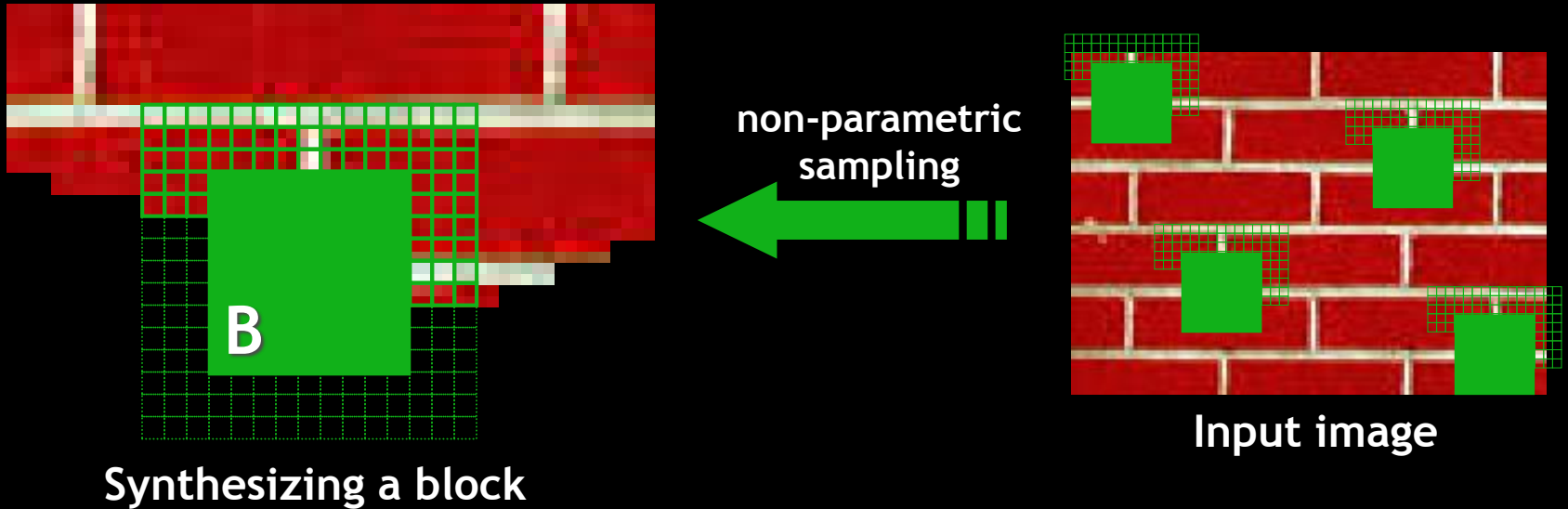
# Summary

- The Efros & Leung algorithm
  - Very simple
  - Surprisingly good results
  - Synthesis is easier than analysis!
  - ...but very slow

# Overview

- Texture synthesis
- Quilting
- Image Analogies
- Super-resolution
- Scene completion

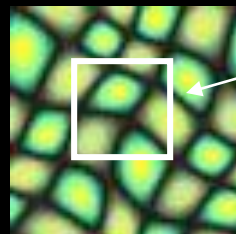
# Image Quilting [Efros & Freeman]



- Observation: neighbor pixels are highly correlated

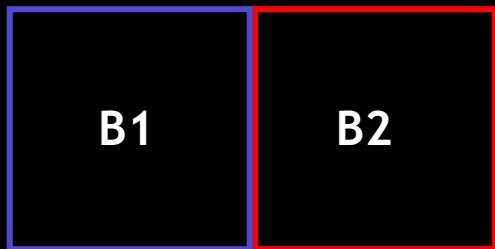
Idea: unit of synthesis = block

- Exactly the same but now we want  $P(B|N(B))$
- Much faster: synthesize all pixels in a block at once
- Not the same as multi-scale!

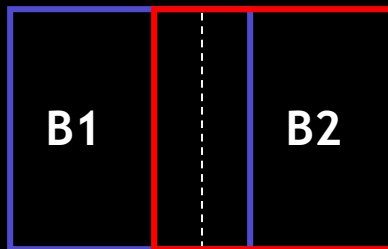


block

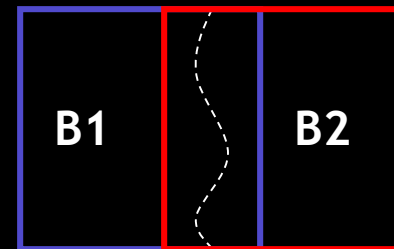
Input texture



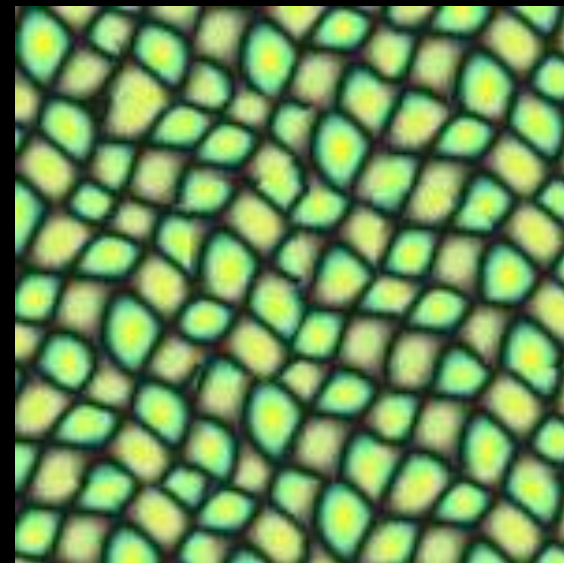
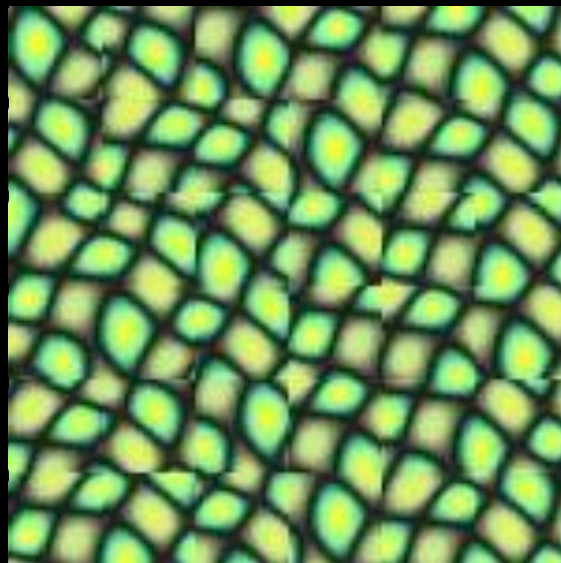
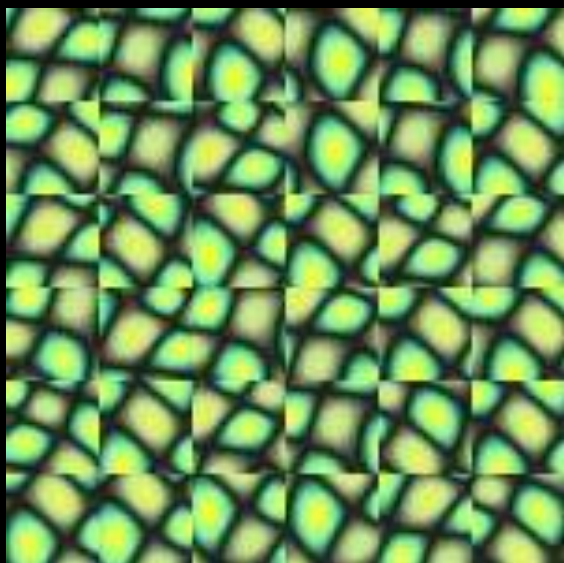
Random placement  
of blocks



Neighboring blocks  
constrained by overlap

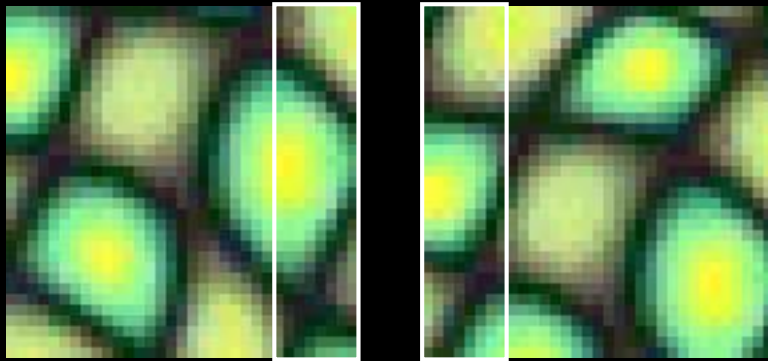


Minimal error  
boundary cut

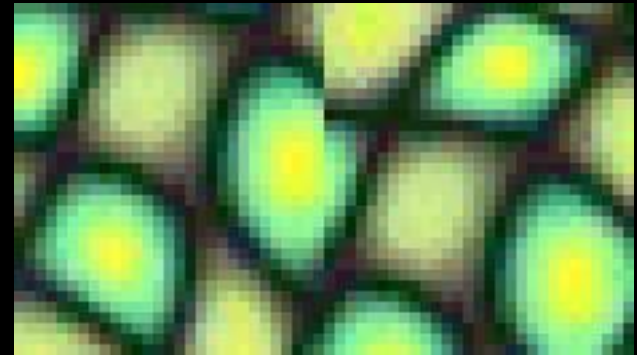


# Minimal error boundary

overlapping blocks

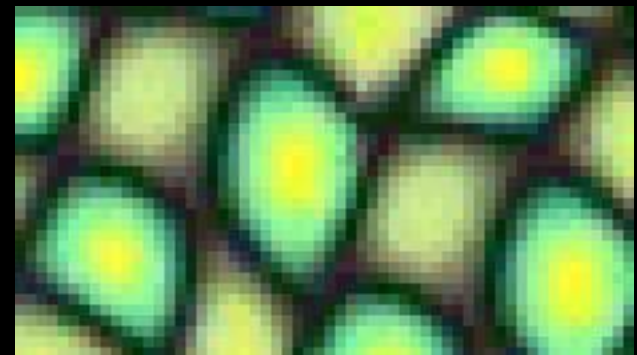


vertical boundary



A diagram showing the calculation of overlap error. Two vertical blocks are shown inside large square brackets, with a minus sign between them. Two yellow arrows point from the overlapping blocks in the top image to these two blocks. To the right of the brackets is a superscript '2', followed by an equals sign and a small vertical strip of the image showing a jagged red boundary line. This represents the squared difference between the two blocks in the overlap region.

overlap error

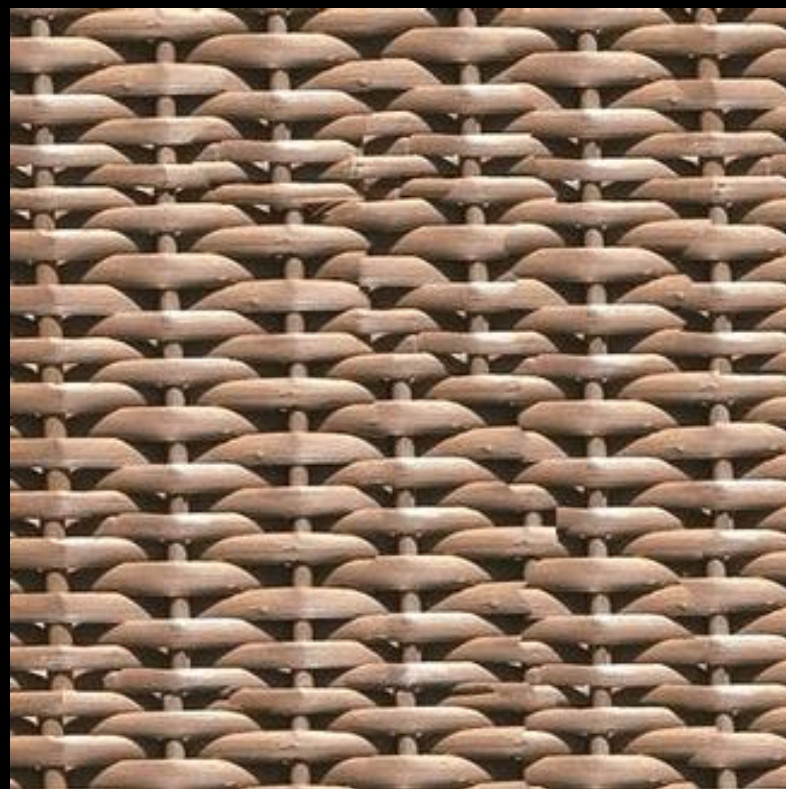
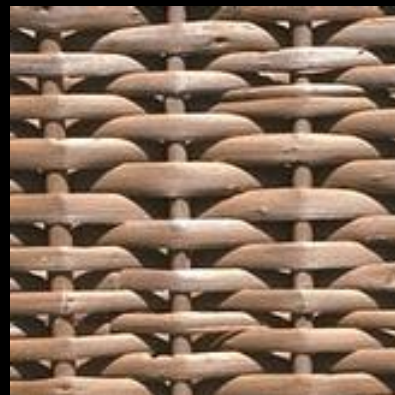


min. error boundary

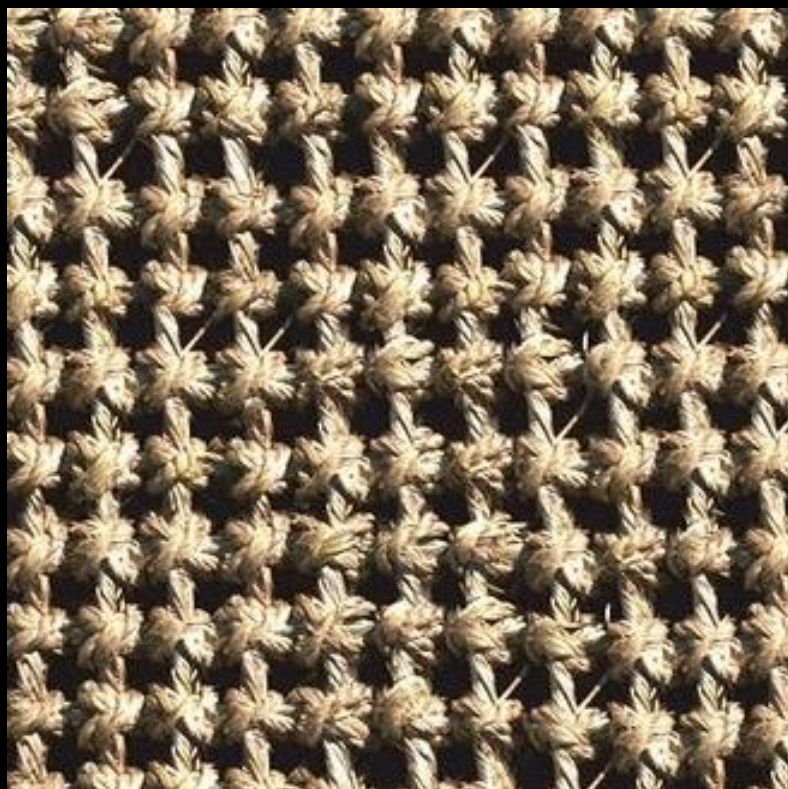
# Our Philosophy

- The “Corrupt Professor’s Algorithm”:
  - Plagiarize as much of the source image as you can
  - Then try to cover up the evidence
- Rationale:
  - Texture blocks are by definition correct samples of texture so problem only connecting them together











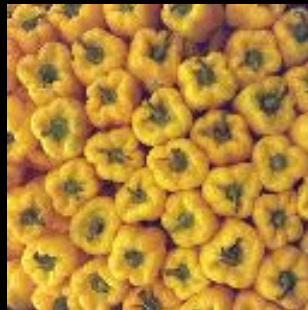
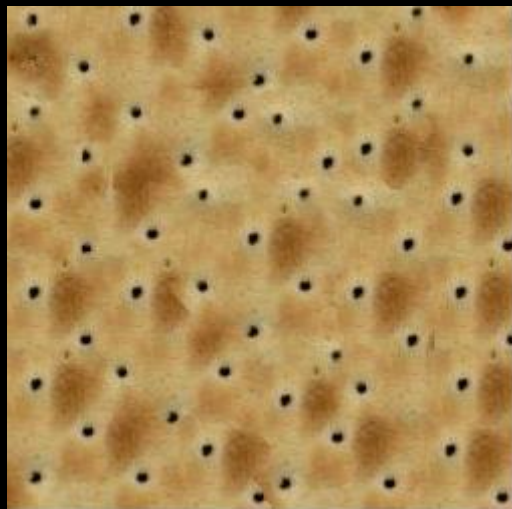
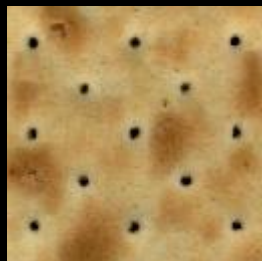












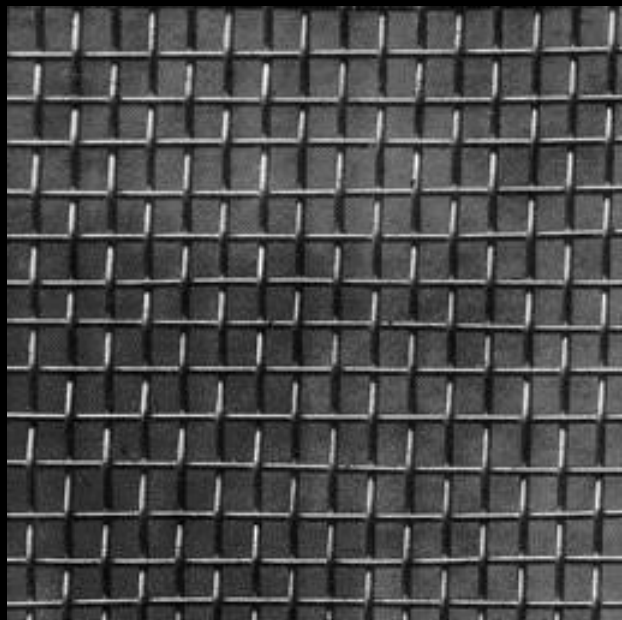




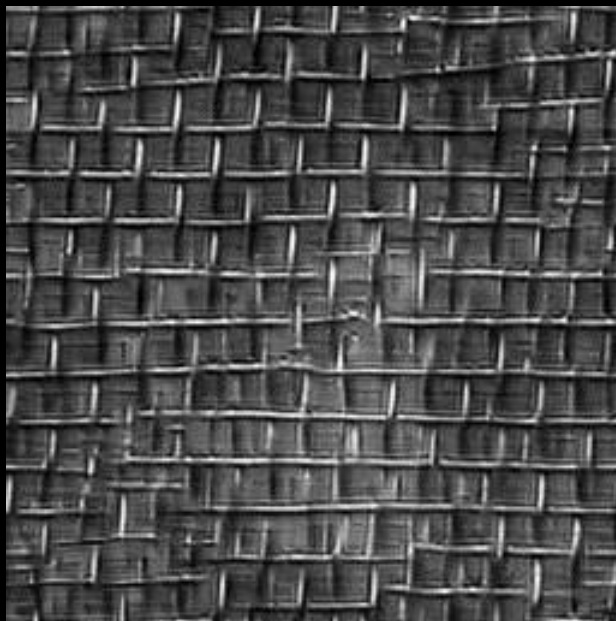
# Failures

(Chernobyl Harvest)

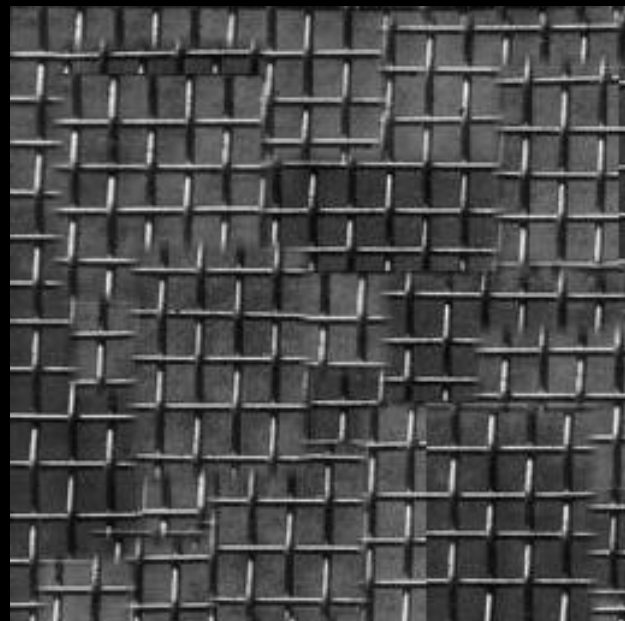




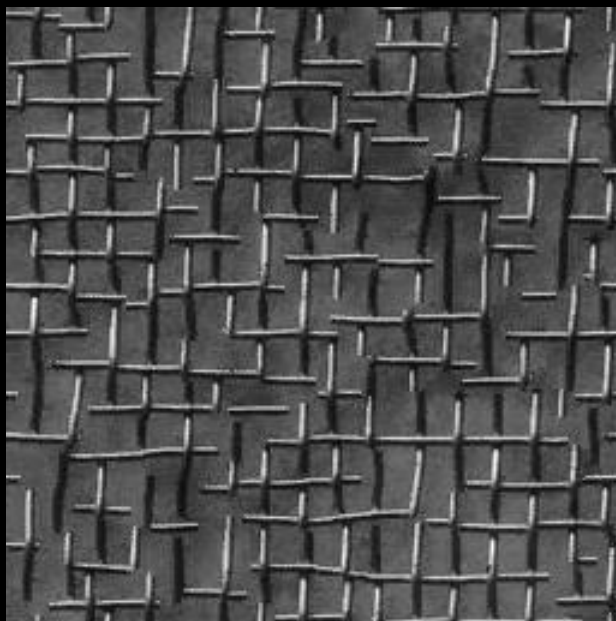
**input image**



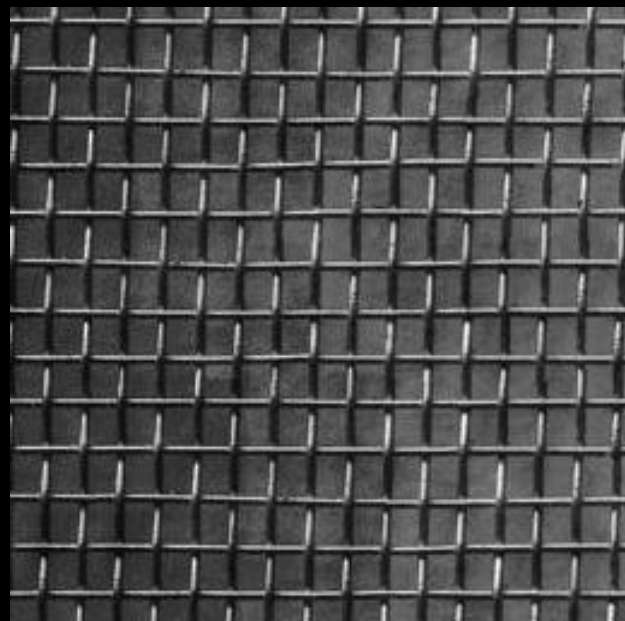
**Portilla & Simoncelli**



**Xu, Guo & Shum**

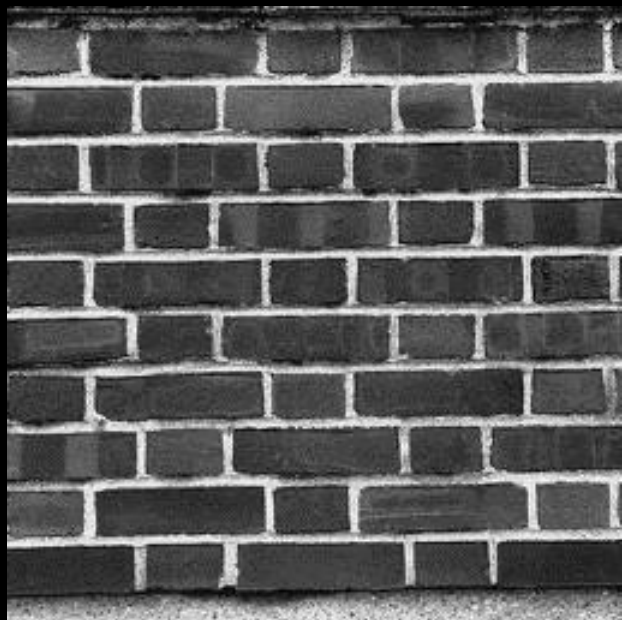


**Wei & Levoy**



**Our algorithm**

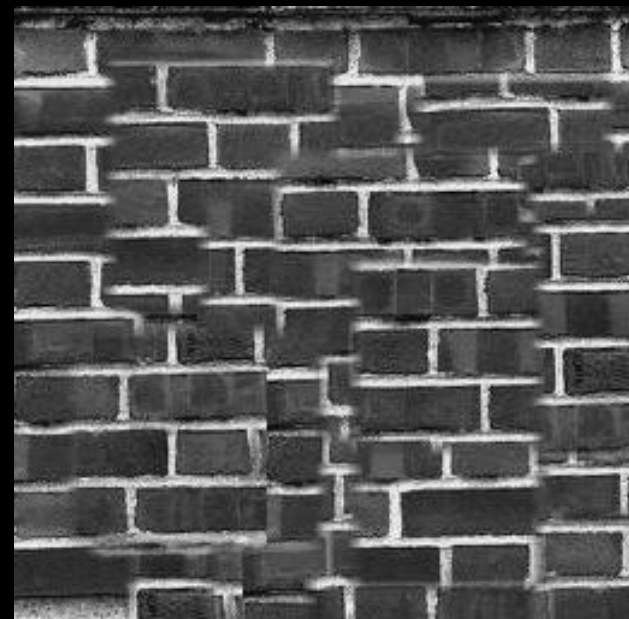




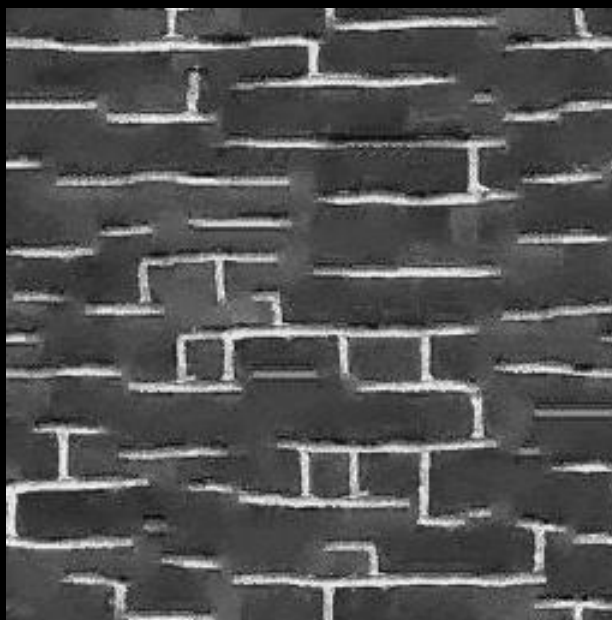
**input image**



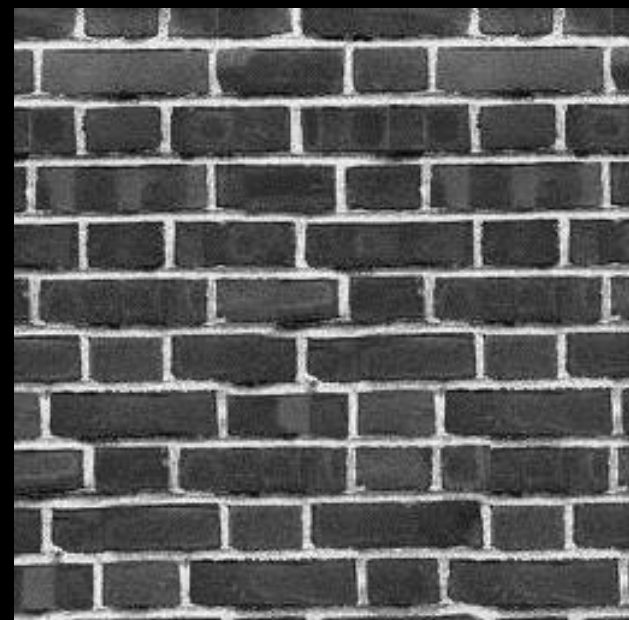
**Portilla & Simoncelli**



**Xu, Guo & Shum**



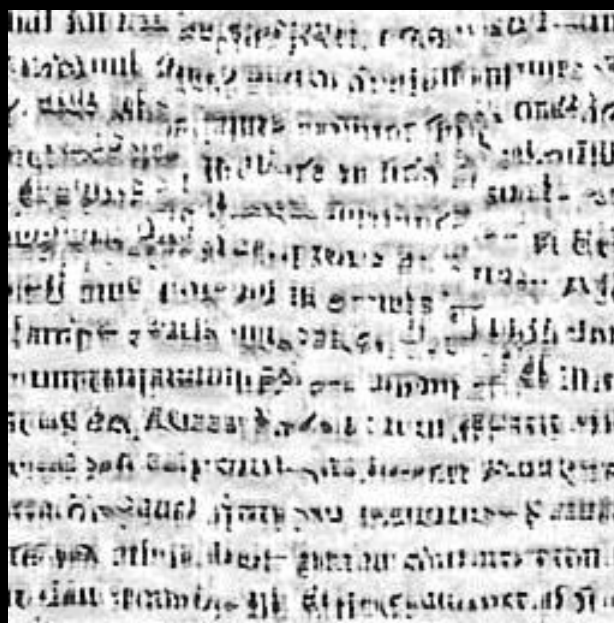
**Wei & Levoy**



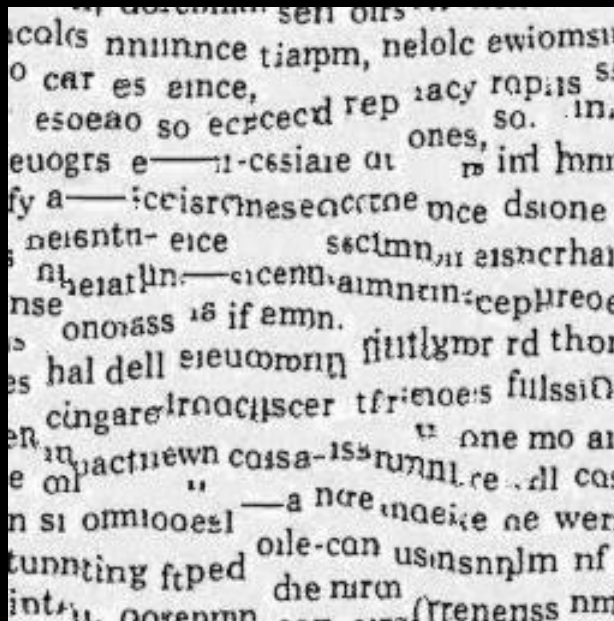
**Our algorithm**

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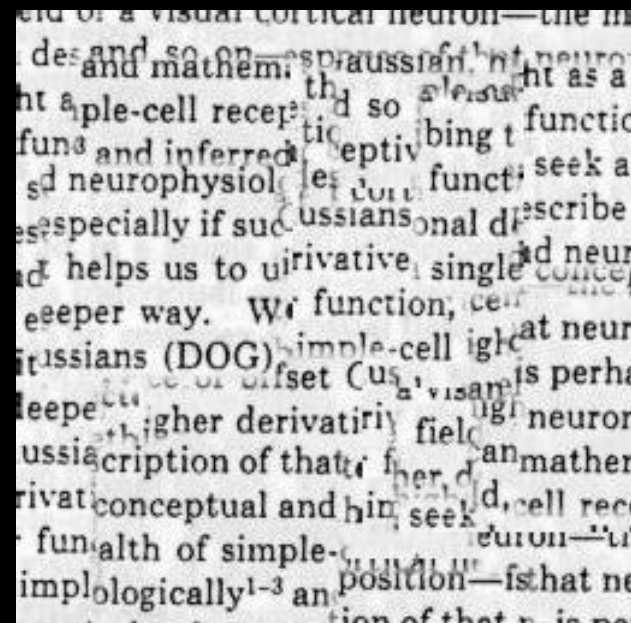
input image



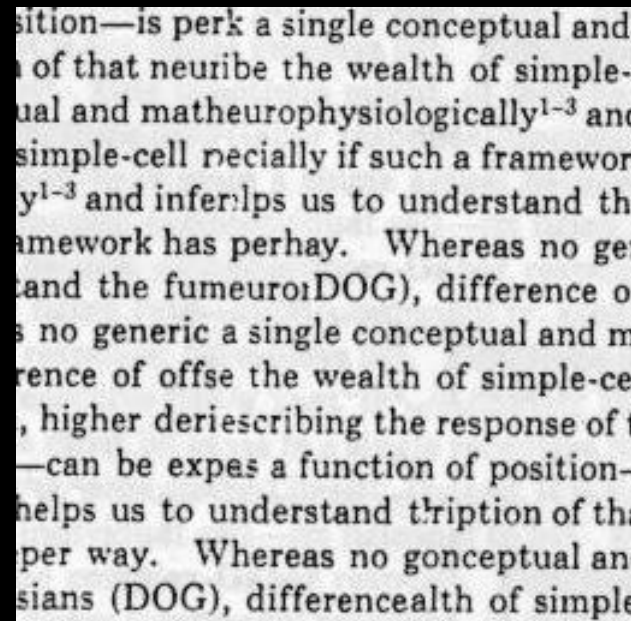
Portilla & Simoncelli



Wei & Levoy



Xu, Guo & Shum



Our algorithm



# Political Texture Synthesis!

## Bush campaign digitally altered TV ad

President Bush's campaign acknowledged Thursday that it had digitally altered a photo that appeared in a national cable television commercial. In the photo, a handful of soldiers were multiplied many times.

This section shows a sampling of the duplication of soldiers.



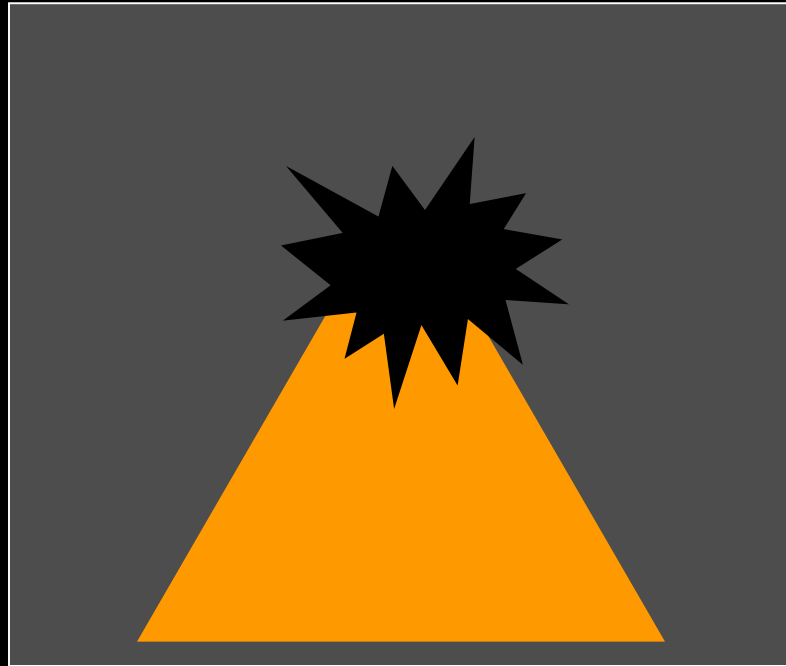
Original photograph

# Fill Order



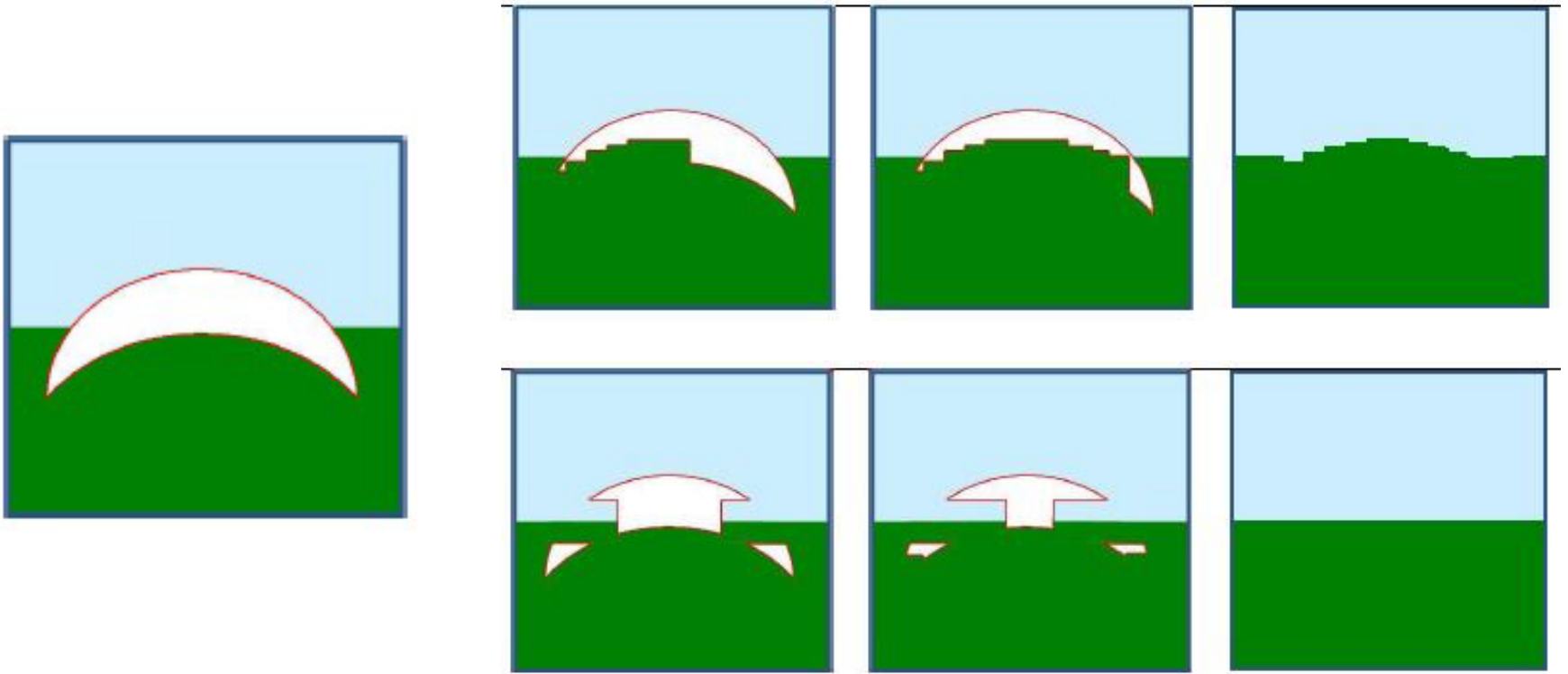
- In what order should we fill the pixels?

# Fill Order



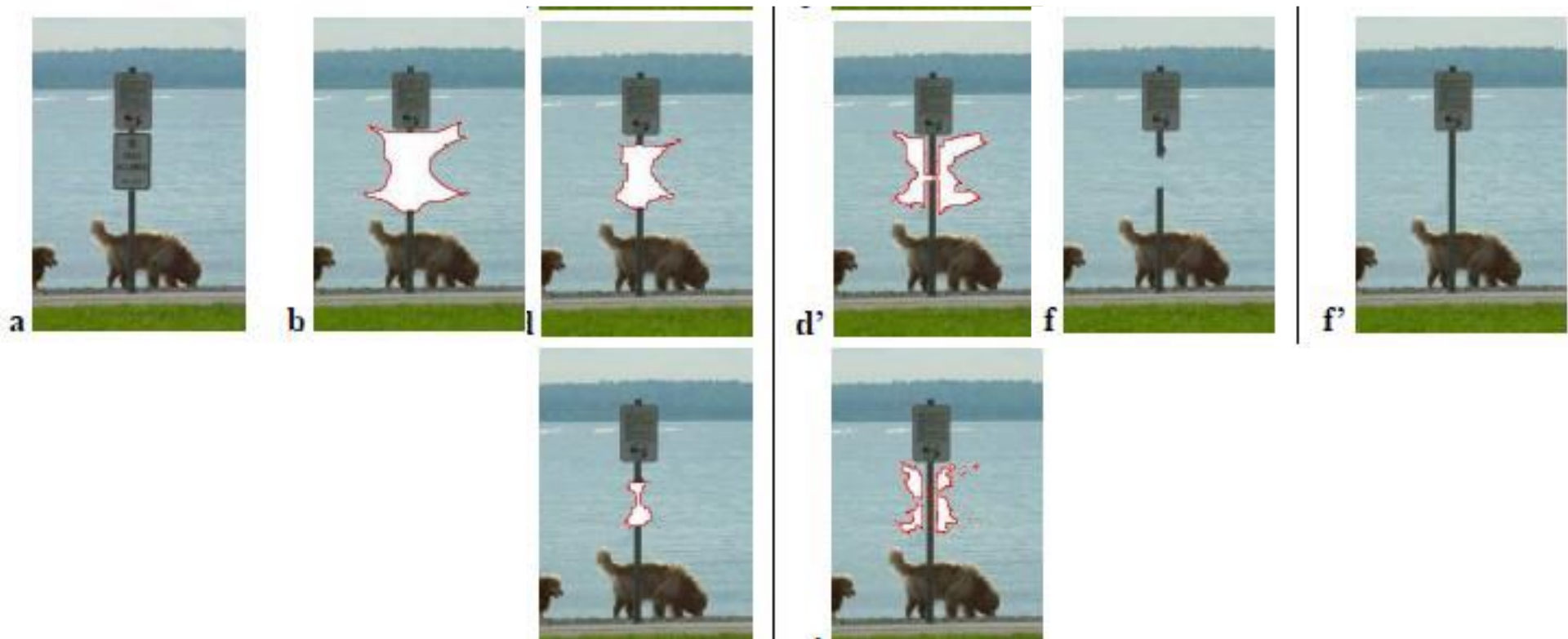
- In what order should we fill the pixels?
  - choose pixels that have more neighbors filled
  - choose pixels that are continuations of lines/curves/edges

# Image quilting – order problem



Region Filling and Object Removal by  
Exemplar-Based Image Inpainting, Criminisi et al. '04

# Image quilting – order problem



Region Filling and Object Removal by  
Exemplar-Based Image Inpainting, Criminisi et al. '04

# Patch-based image analysis

Super resolution from single image

<http://www.wisdom.weizmann.ac.il/~vision/SingleImageSR.html>

The patch transform, Cho et al. '08

<http://people.csail.mit.edu/taegsang/patchTransform.html>

Space-time video completion, Wexler et al. '04

<http://www.wisdom.weizmann.ac.il/~vision/VideoCompletion.html>

Seam carving, Avidan & Shamir '07

<http://www.faculty.idc.ac.il/arik/site/seam-carve.asp>

# Image transformations

# Image transformations

image filtering: change **range** of image

$$g(x) = T(f(x))$$

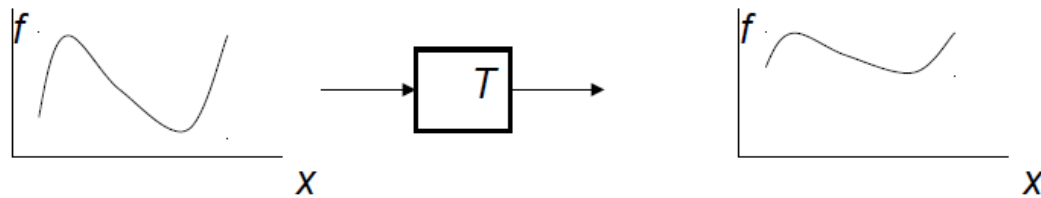
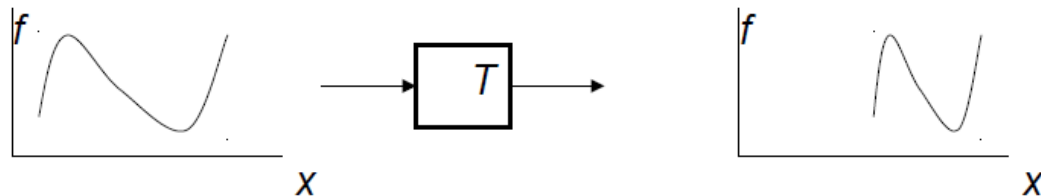


image warping: change **domain** of image

$$g(x) = f(T(x))$$





# Image transformations

image filtering: change **range** of image

$$g(x) = T(f(x))$$

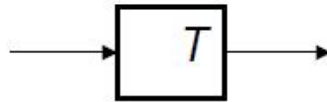
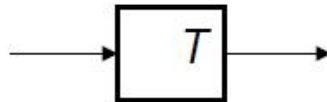
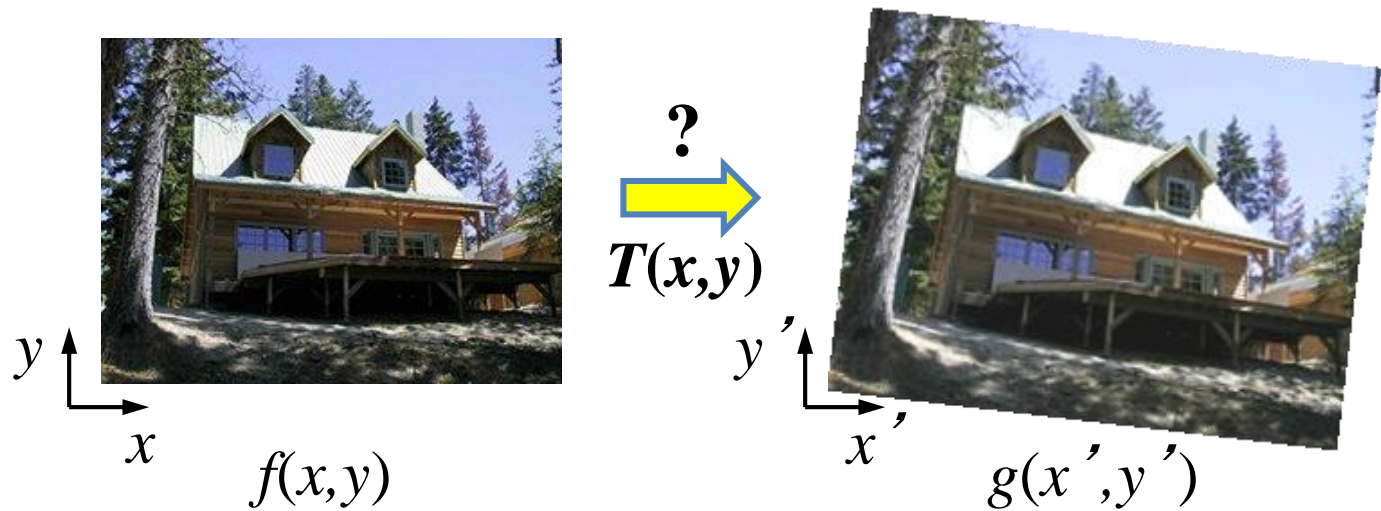


image warping: change **domain** of image

$$g(x) = f(T(x))$$

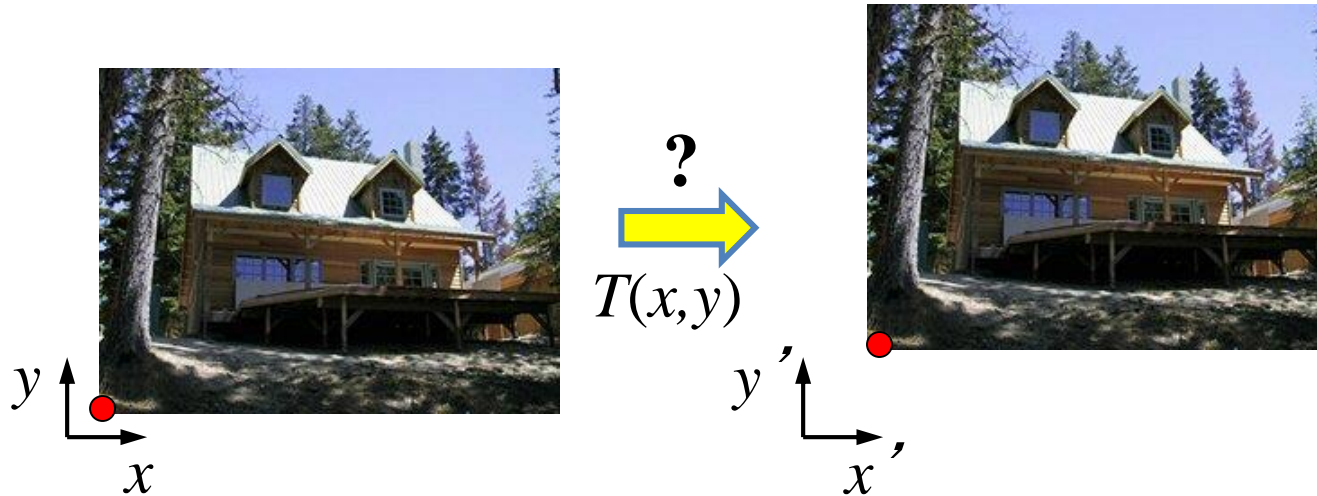


# Recovering Transformations



- What if we know  $f$  and  $g$  and want to recover the transform  $T$ ?
  - e.g. better align photographs you've taken
  - willing to let user provide correspondences
    - How many do we need?

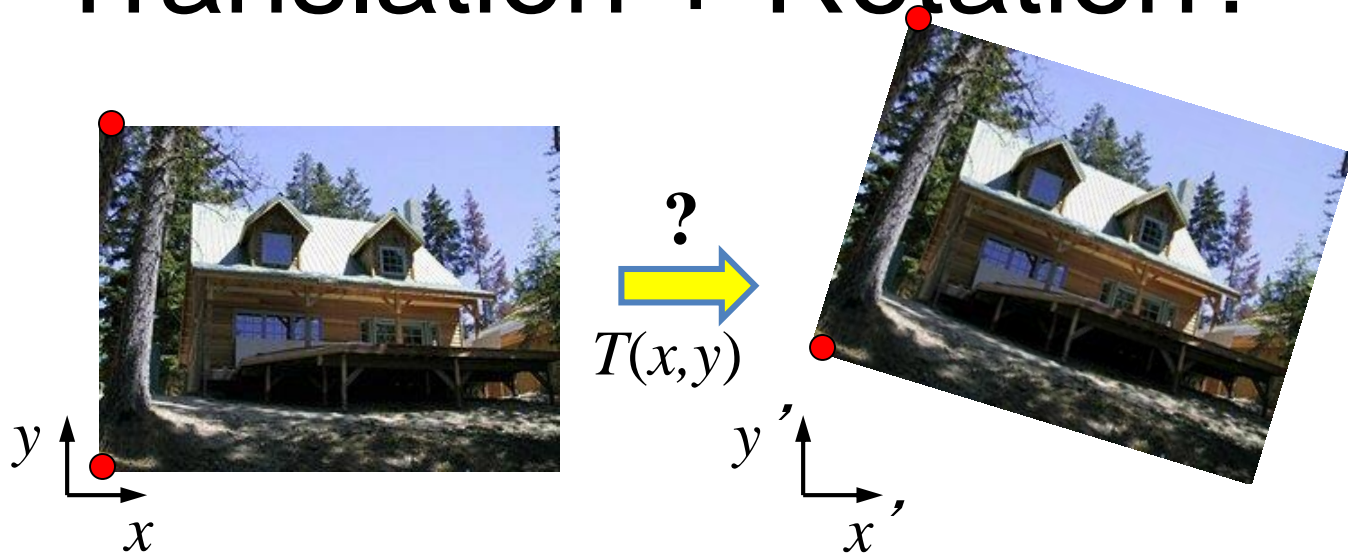
# Translation: # correspondences?



- How many correspondences needed for translation?
- How many Degrees of Freedom?
- What is the transformation matrix?

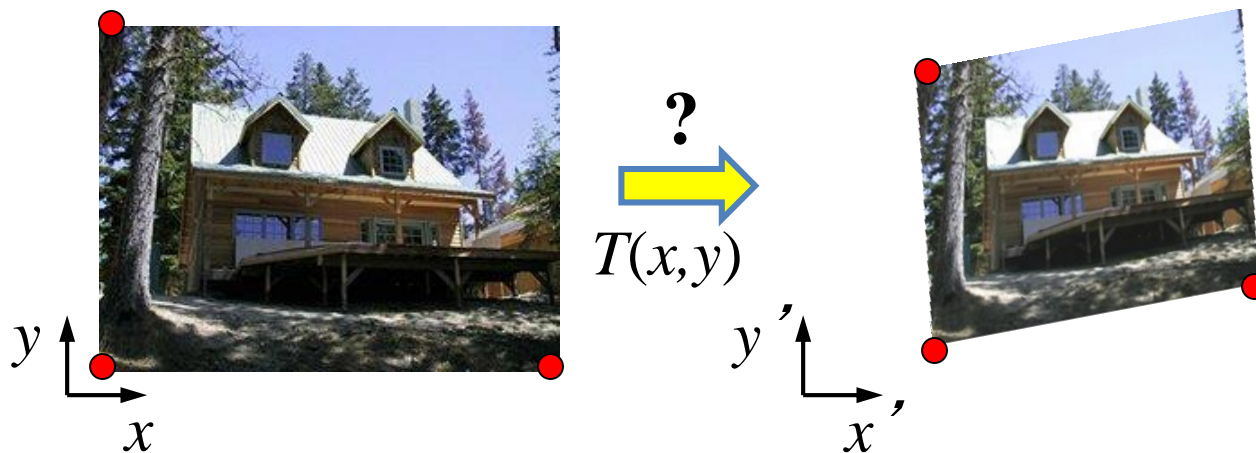
$$\mathbf{M} = \begin{bmatrix} 1 & 0 & p'_x - p_x \\ 0 & 1 & p'_y - p_y \\ 0 & 0 & 1 \end{bmatrix}$$

# Translation + Rotation?



- How many correspondences needed for translation+rotation?
- How many DOF?

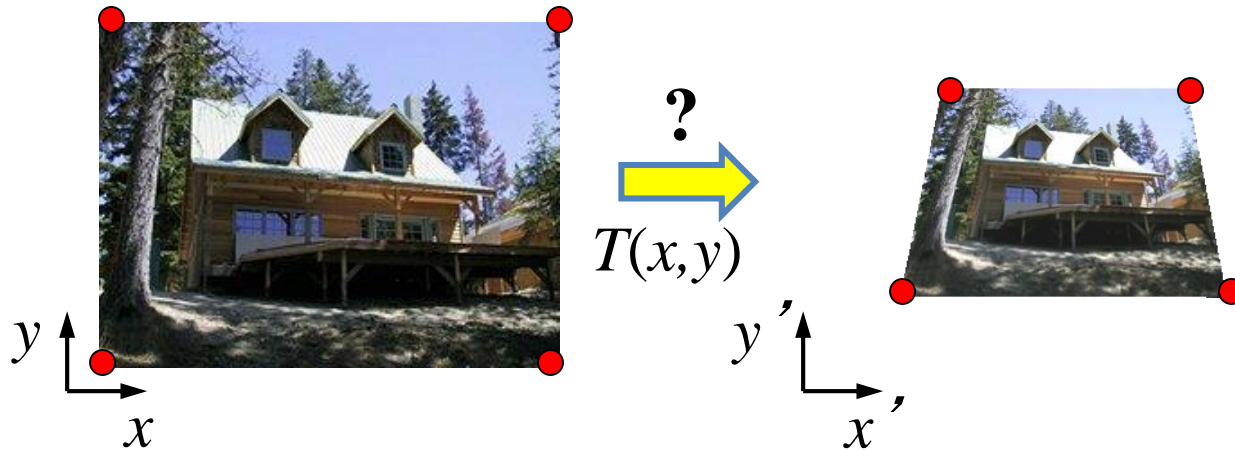
# Affine: # correspondences?



- How many correspondences needed for affine transform?
- How many DOF?

$$T(x, y) = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

# Projective / Homography



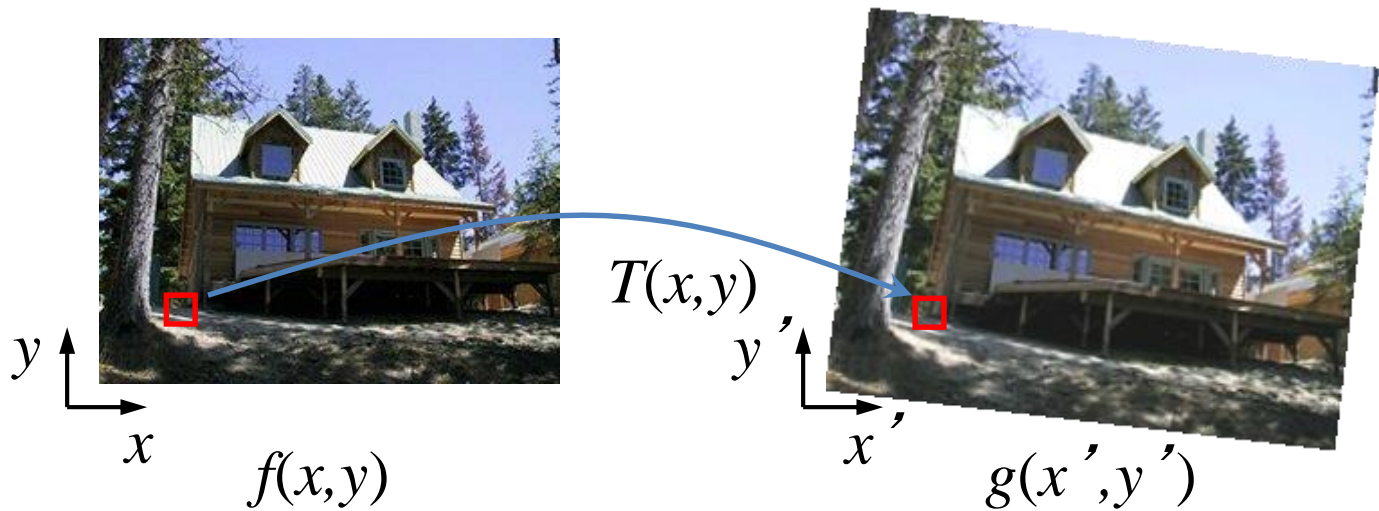
- How many correspondences needed for projective? How many DOF?

$$T(x, y) = h \left( \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \right)$$

$$h(x, y, z) = (x / z, y / z)$$

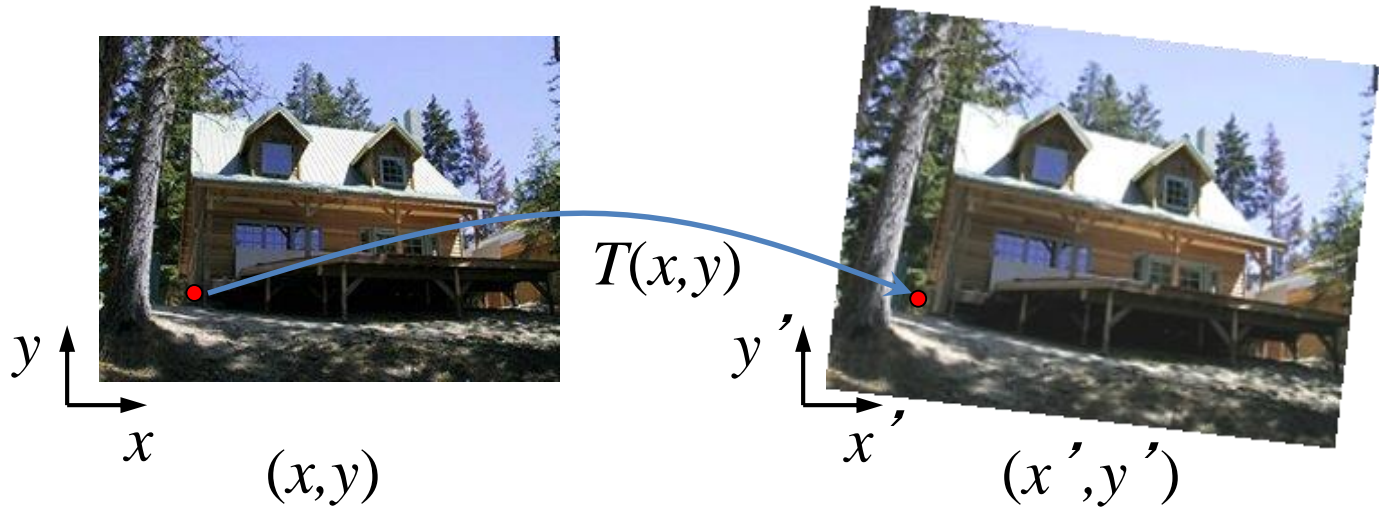


# Image Warping



- Given a coordinate transform  $(x',y') = T(x,y)$  and a source image  $f(x,y)$ , how do we compute a transformed image  $g(x',y') = f(T(x,y))$ ?

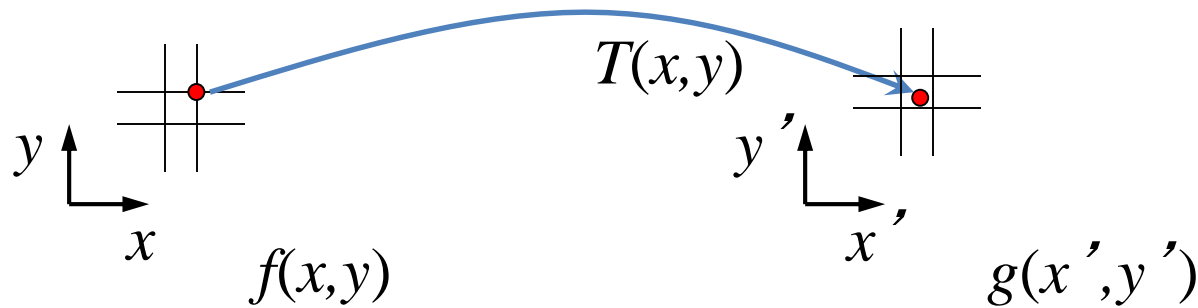
# Forward warping



- Send each pixel  $(x, y)$  to its corresponding location

$(x', y') = T(x, y)$  in the second image

# Forward warping



Q: what if pixel lands “between” two pixels?

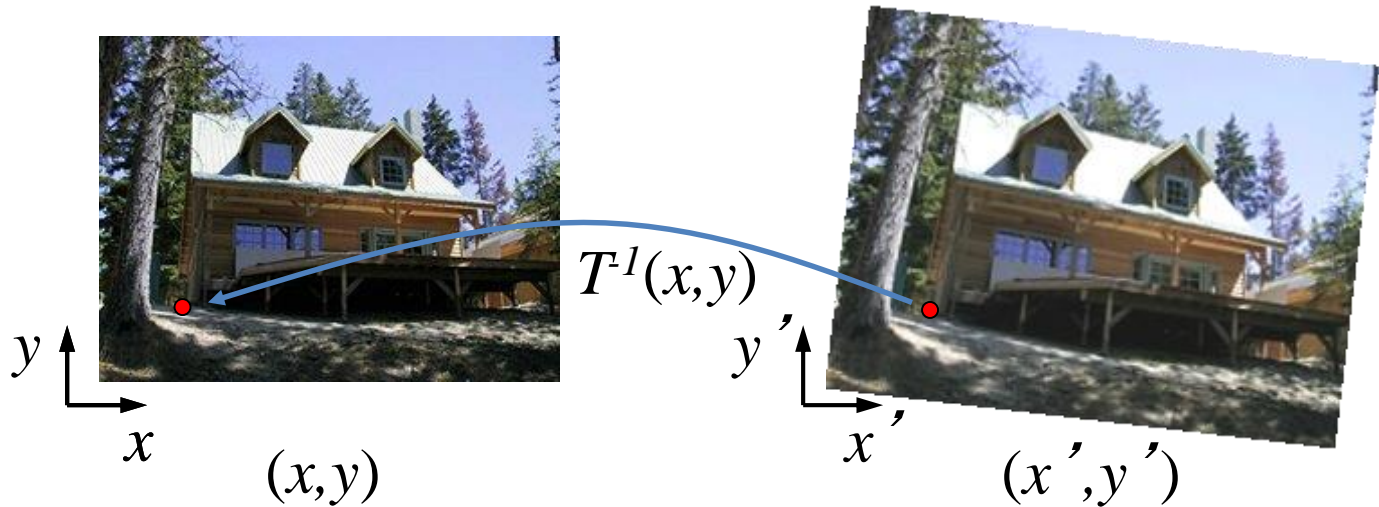
A: distribute color among neighboring pixels  $(x',y')$

- Known as “splatting”

- Can also interpolate points in target image:

[griddata](#) (Matlab), [scipy.interpolate.griddata](#) (Python)

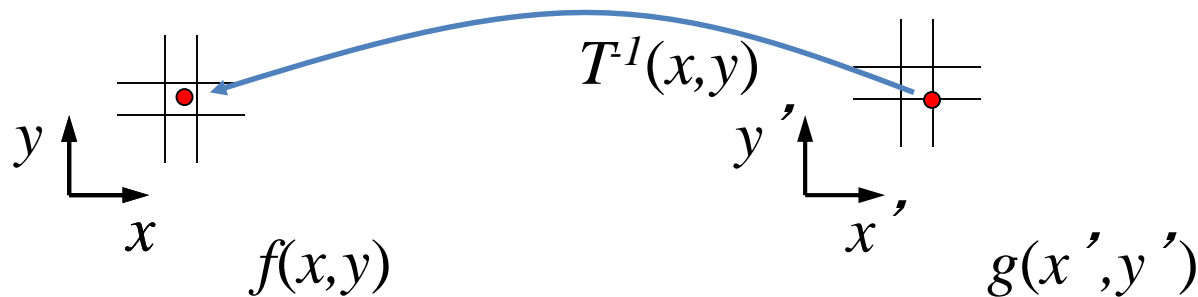
# Inverse warping



- Get each pixel color  $g(x', y')$  from its corresponding location

$(x, y) = T^{-1}(x', y')$  in the first image

# Inverse warping



Q: what if pixel comes from “between” two pixels?

A: *Interpolate* color value from neighbors

- nearest neighbor, bilinear, Gaussian, bicubic
- See [interp2](#) (Matlab),  
[scipy.interpolate.interp2d](#) (Python)

# Forward vs. inverse warping

- Q: Which is better?

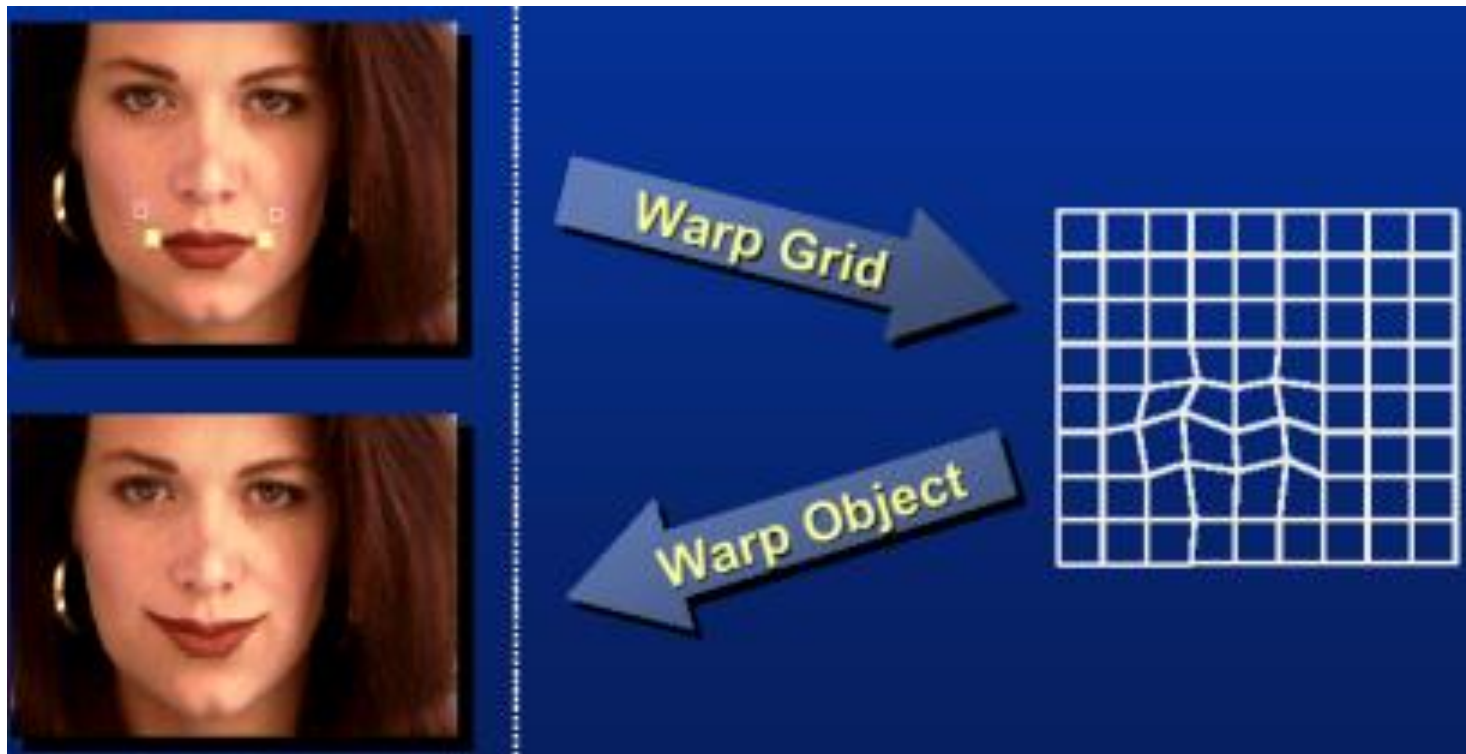


# Forward vs. inverse warping

- Q: Which is better?
- A: Usually inverse – eliminates holes
  - However, it requires an invertible warp function
  - Not always possible

# How to Obtain Warp Field?

- Move control points to specify a spline warp
- Spline produces a smooth vector field  $T(x, y)$



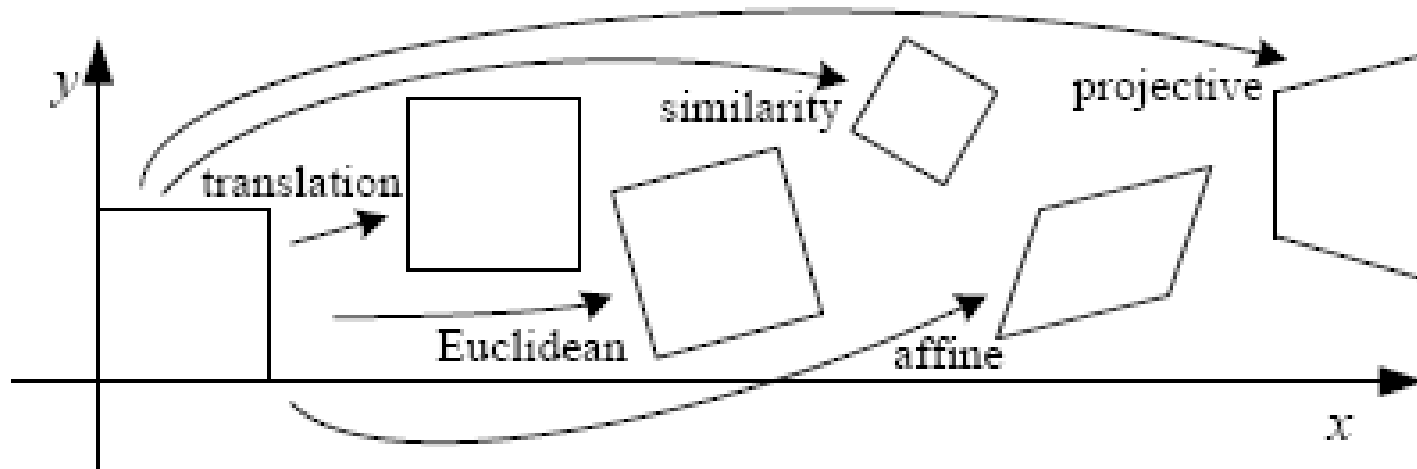
# Warp as Interpolation

- **We are looking for a warping field**
  - A function that given a 2D point, returns a warped 2D point
- **We have a sparse number of correspondences**
  - These specify values of the warping field
- **This is an interpolation problem**
  - Given sparse data, find smooth function

# Geometry transformations

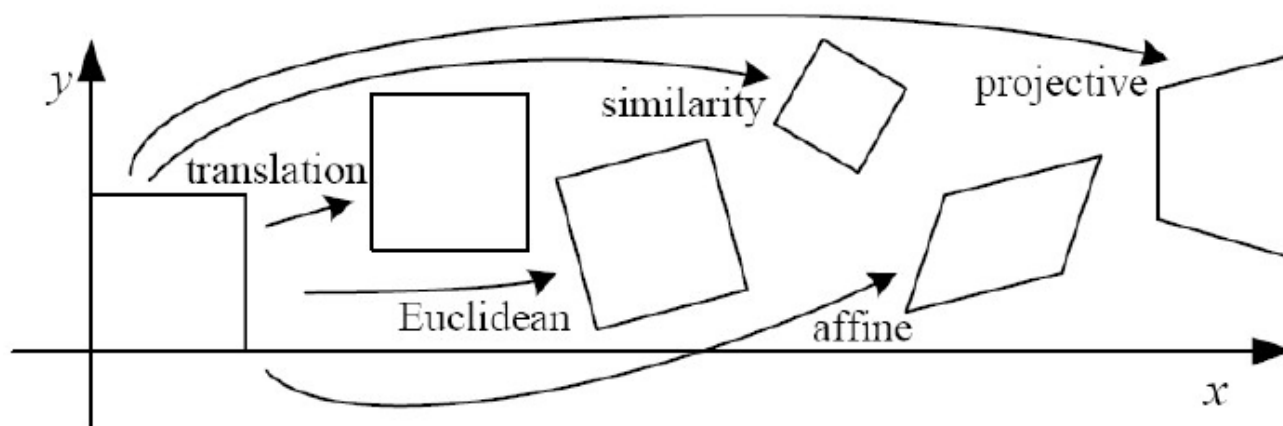
Hartley, R., and Zisserman, A. Multiple View Geometry in Computer Vision, Cambridge University Press, 2004, Chapters 1–3

# 2D Geometry transformations



Translation  $\subset$  Euclidean  $\subset$  Similarity  $\subset$  Affine  $\subset$  Projective

# 2D Geometry transformations



Name	Matrix	# D.O.F.	Preserves:	Icon
translation	$\begin{bmatrix} I & t \end{bmatrix}_{2 \times 3}$	2	orientation + ...	
rigid (Euclidean)	$\begin{bmatrix} R & t \end{bmatrix}_{2 \times 3}$	3	lengths + ...	
similarity	$\begin{bmatrix} sR & t \end{bmatrix}_{2 \times 3}$	4	angles + ...	
affine	$\begin{bmatrix} A \end{bmatrix}_{2 \times 3}$	6	parallelism + ...	
projective	$\begin{bmatrix} \tilde{H} \end{bmatrix}_{3 \times 3}$	8	straight lines	

Translation  $\subset$  Euclidean  $\subset$  Similarity  $\subset$  Affine  $\subset$  Projective



# Basic 2D transformations as 3x3 matrices

Translate

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Scale

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Rotate

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos(\theta) & -\sin(\theta) & 0 \\ \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Transformations can be combined by matrix multiplication

# Affine Transformations

- Affine transformations are combinations of ...

- Linear transformations, and
- Translations

$$\begin{bmatrix} x' \\ y' \\ w' \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ w \end{bmatrix}$$

- Properties of affine transformations:

- Origin does not necessarily map to origin

- Lines map to lines
- Parallel lines remain parallel
- Ratios are preserved
- Closed under composition
- Models change of basis

- Will the last coordinate  $w$  always be 1?

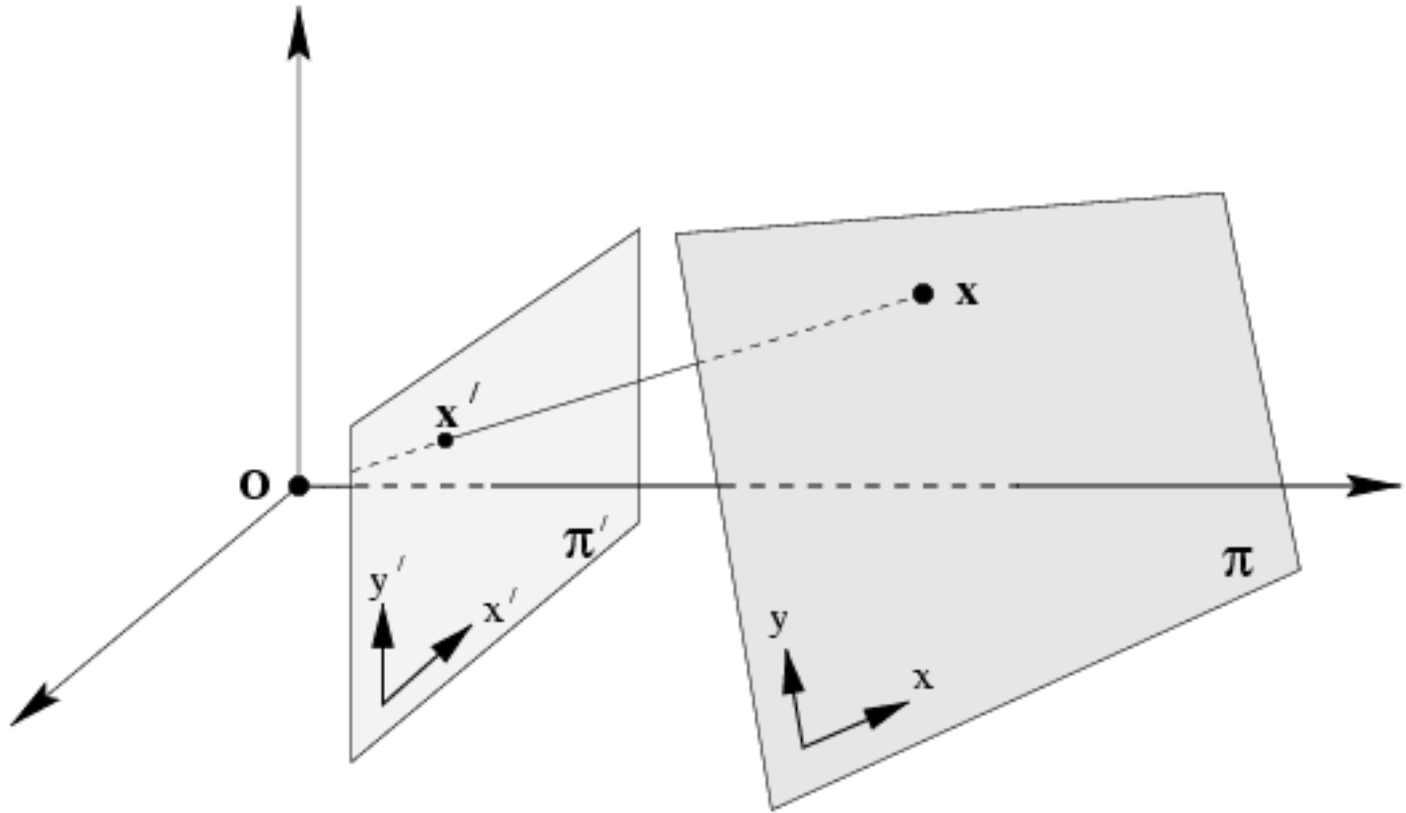
# Projective Transformations

- Projective transformations ...
  - Affine transformations, and
  - Projective warps

$$\begin{bmatrix} x' \\ y' \\ w' \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} x \\ y \\ w \end{bmatrix}$$

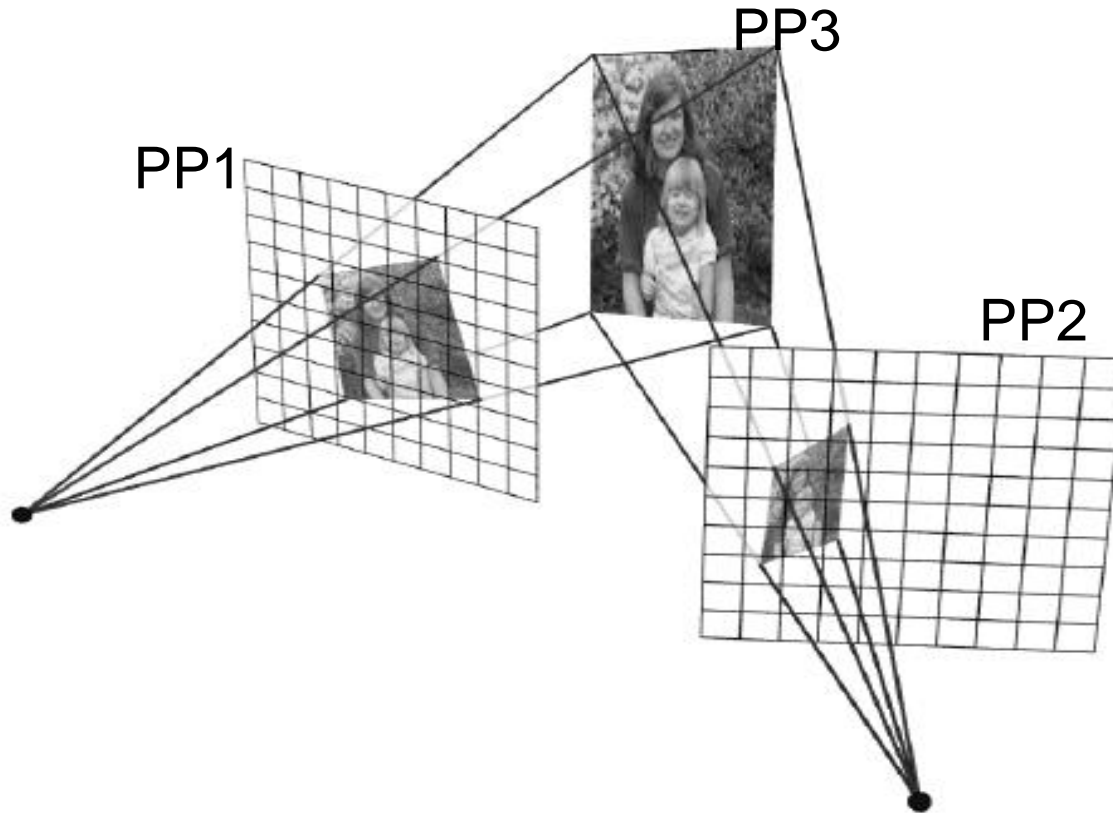
- Properties of projective transformations:
  - Origin does not necessarily map to origin
  - Lines map to lines
  - Parallel lines do not necessarily remain parallel
  - Ratios are not preserved
  - Closed under composition
  - Models change of basis

# Mapping between planes



*central projection* may be expressed by  $x' = Hx$   
(application of theorem)

# Planar scene (or far away)



- PP3 is a projection plane of both centers of projection, so we are OK!
- This is how big aerial photographs are made



# Image warping with homographies

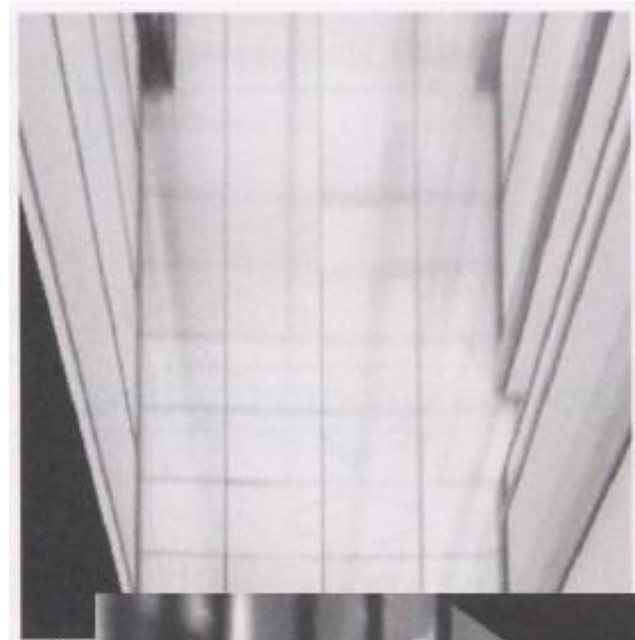


image plane in front



black area  
where no pixel  
maps to



Source: Steve Seitz

# Removing projective distortion



select four points in a plane with know coordinates

$$x' = \frac{x'_1}{x'_3} = \frac{h_{11}x + h_{12}y + h_{13}}{h_{31}x + h_{32}y + h_{33}} \quad y' = \frac{x'_2}{x'_3} = \frac{h_{21}x + h_{22}y + h_{23}}{h_{31}x + h_{32}y + h_{33}}$$

$$\begin{aligned} x'(h_{31}x + h_{32}y + h_{33}) &= h_{11}x + h_{12}y + h_{13} \\ y'(h_{31}x + h_{32}y + h_{33}) &= h_{21}x + h_{22}y + h_{23} \end{aligned} \quad (\text{linear in } h_{ij})$$

(2 constraints/point, 8DOF  $\Rightarrow$  4 points needed)

Remark: no calibration at all necessary,  
better ways to compute (see later)

# Solving for homographies

$$\mathbf{p}' = \mathbf{H}\mathbf{p}$$
$$\begin{bmatrix} wx' \\ wy' \\ w \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

- Can set scale factor  $i=1$ . So, there are 8 unknowns.
- Set up a system of linear equations:

$$\mathbf{A}\mathbf{h} = \mathbf{b}$$

- where vector of unknowns  $\mathbf{h} = [a, b, c, d, e, f, g, h]^T$
- Need at least 8 eqs, but the more the better...
- Solve for  $\mathbf{h}$ . If overconstrained, solve using least-squares
- Use RANSAC to throw outliers

# Parameter estimation

- 2D homography  
Given a set of  $(x_i, x_i')$ , compute  $H$   
(optimize  $x_i' = Hx_i$ )
- 3D to 2D camera projection  
Given a set of  $(X_i, x_i)$ , compute  $P$  ( $x_i = PX_i$ )
- Fundamental matrix  
Given a set of  $(x_i, x_i')$ , compute  $F$   
( $x_i'^T F x_i = 0$ )
- Trifocal tensor  
Given a set of  $(x_i, x_i', x_i'')$ , compute  $T$

# Projective geometry



# Homogeneous representation of lines and points

- Equation of line in the plane  $ax + by + c = 0$
- As an inner product of vectors

$$(x, y, 1)^T (a, b, c) = 0$$

- This is true for any  $(kx, ky, k)^T, k \neq 0$
- Therefore, the set of vectors  $(kx, ky, k)^T, k \neq 0$  can represent the point  $(x, y) \in \mathbb{R}^2$
- The set of equivalent vectors are called *homogeneous vectors*.

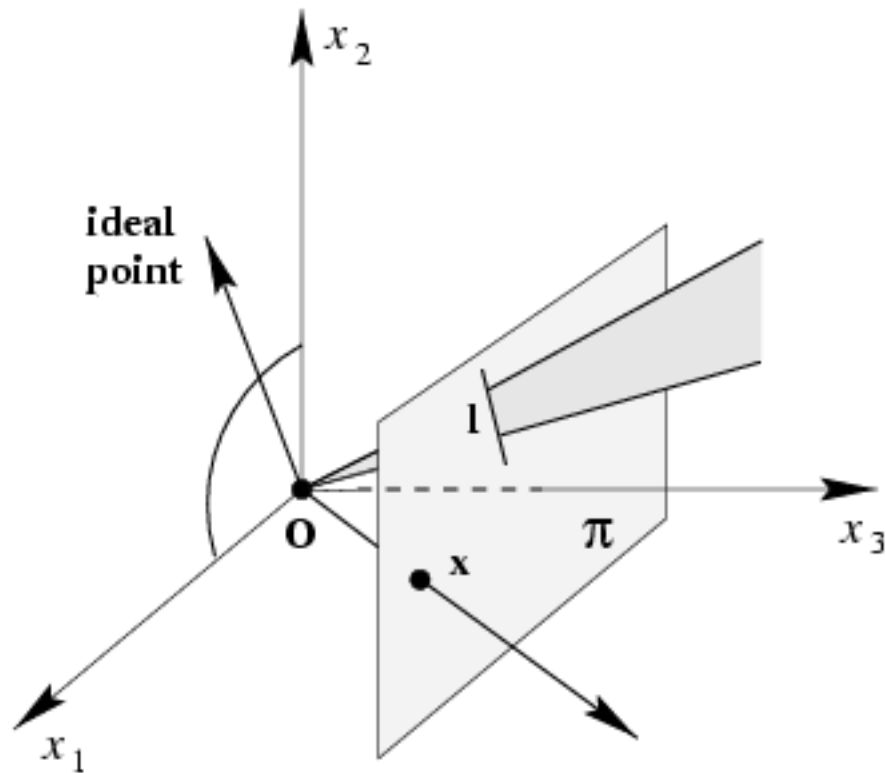
# The projective plane

The set of equivalence classes of vectors in

$$\mathbb{R}^3 - (0, 0, 0)^T$$

forms the projective space  $\mathbb{P}^2$

# A model for the projective plane



exactly one line through two points  
exactly one point at intersection of two lines

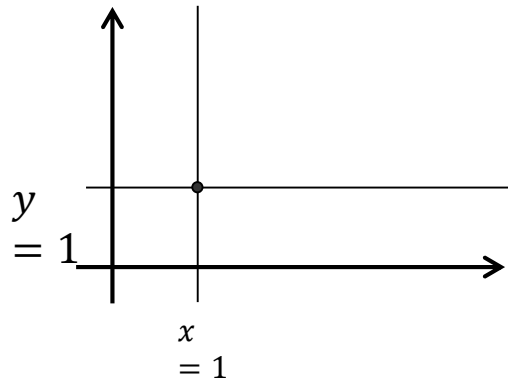
# Points and lines

- The point  $\mathbf{x}$  lies on the line  $\mathbf{l}$  if and only if

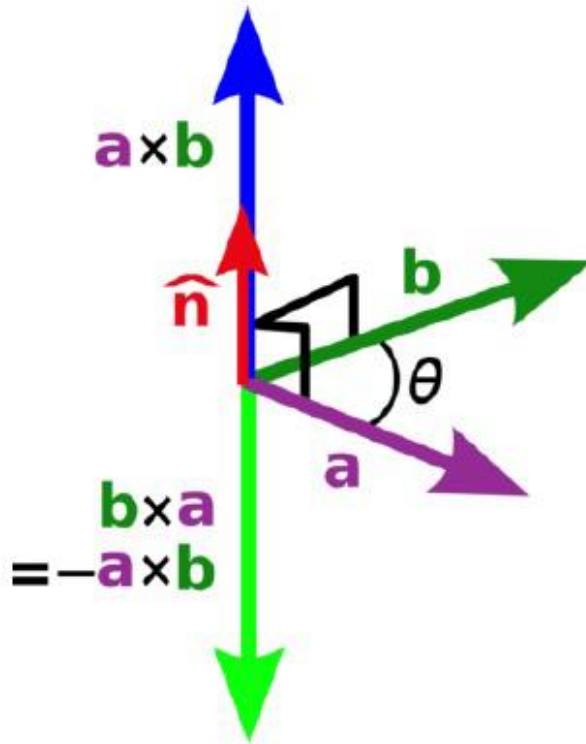
$$\mathbf{x}^T \mathbf{l} = \mathbf{l}^T \mathbf{x} = 0$$

- The intersection of two lines  $\mathbf{x} = \mathbf{l} \times \mathbf{l}'$
- The line through two points  $\mathbf{l} = \mathbf{x} \times \mathbf{x}'$

Example



– מכפלה ווקטורית  
cross product



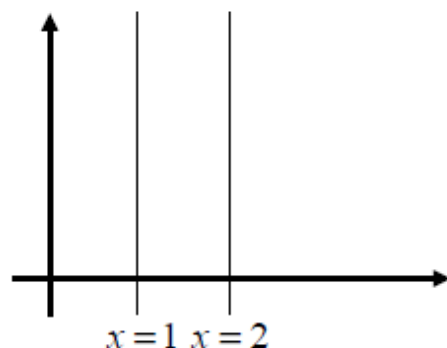


# Ideal points and the line at infinity

Intersections of parallel lines

$$l = (a, b, c)^T \text{ and } l' = (a, b, c')^T \quad l \times l' = (b, -a, 0)^T$$

Example



$(b, -a)$  tangent vector  
 $(a, b)$  normal direction

Ideal points  $(x_1, x_2, 0)^T$

Line at infinity  $l_\infty = (0, 0, 1)^T$

$$\mathbf{P}^2 = \mathbf{R}^2 \cup l_\infty$$

Note that in  $\mathbf{P}^2$  there is no distinction  
between ideal points and others

## נקודות אידיאליות

- הישרים:  $\mathbf{l}_1 = [a, b, c_1]^T$  ו-  $\mathbf{l}_2 = [a, b, c_2]^T$  הם ישרים מקבילים. איפה הם נחתכים?
- $\mathbf{l}_1 \times \mathbf{l}_2 = (c_2 - c_1)[b, -a, 0]^T \approx [b, -a, 0]^T$
- נקודות עם קואורדינטה שלישית 0 נקראות
  - נקודות אידיאליות
  - כיוונים
  - נקודות באינסוף
- אין להן ייצוג בגיאומטריה האוקלידית

# הישר באינסוף

- כל הנקודות באינסוף נמצאות על אותו הישר,

$$l_\infty = [0, 0, 1]^T: \text{"הישר באינסוף"}$$

- קל לודא על ידי  $l_\infty^T p = [0, 0, 1][x, y, 0]^T = 0$

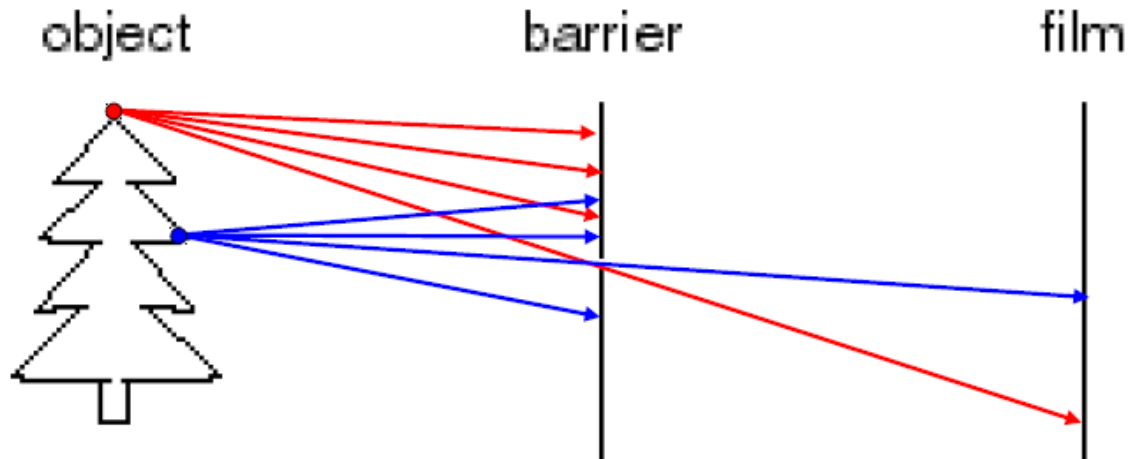
- נקרא גם: קו האופק

- לא בהכרח נמצא בתוך התמונה



Camera model

# Pinhole Camera

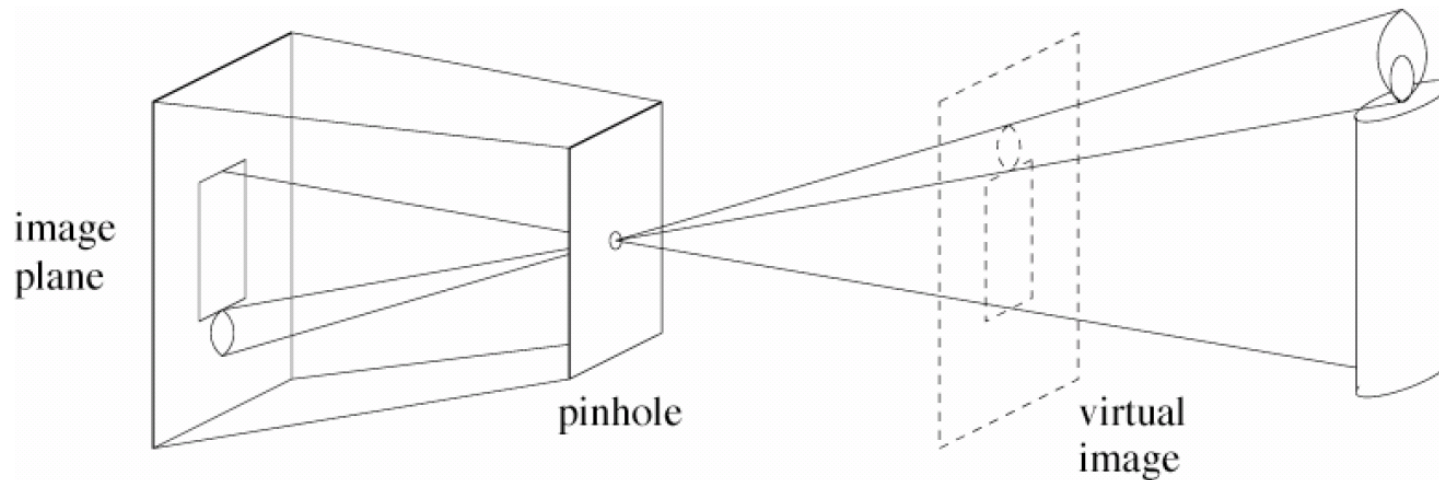


- Add a barrier to block off most of the rays
  - This reduces blurring
  - The opening is known as the **aperture**
  - How does this transform the image?



# Pinhole Camera

- Pinhole camera is a simple model to approximate imaging process, perspective **projection**.



- If we treat pinhole as a point, only one ray from any given point can enter the camera.

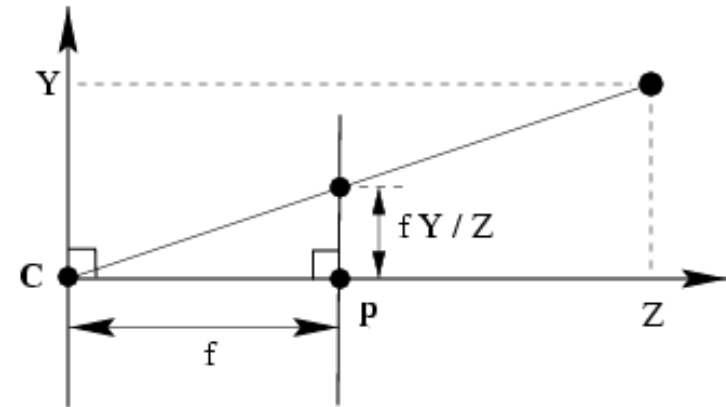
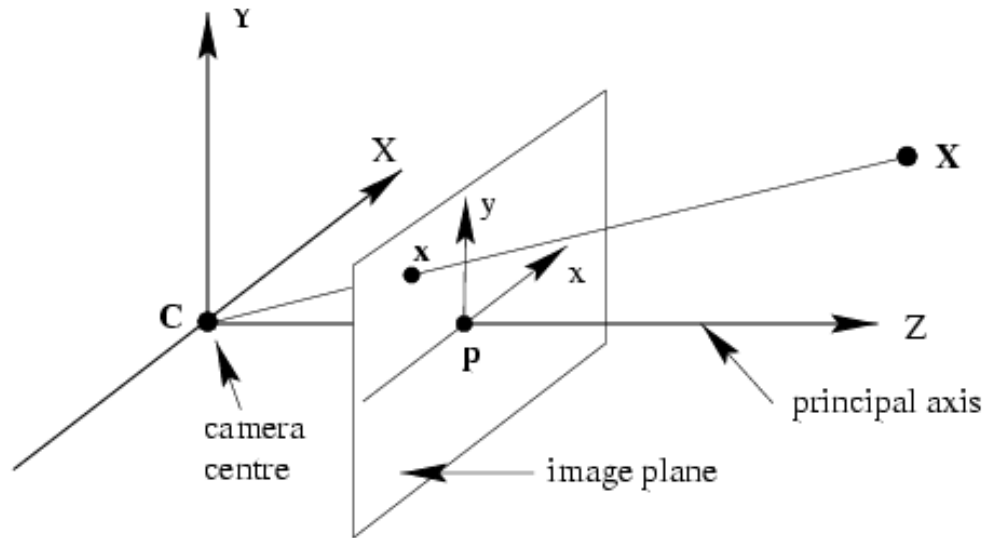
# Perspective effects



# Perspective effects



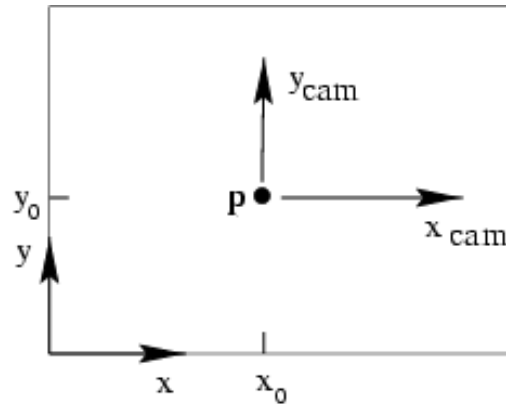
# Pinhole camera model



$$\begin{bmatrix} X & Y & Z \end{bmatrix}^T \mapsto \begin{bmatrix} fX/Z & fY/Z \end{bmatrix}^T$$

$$\begin{bmatrix} fX \\ fY \\ Z \end{bmatrix} = \begin{bmatrix} f & 0 & 0 \\ 0 & f & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

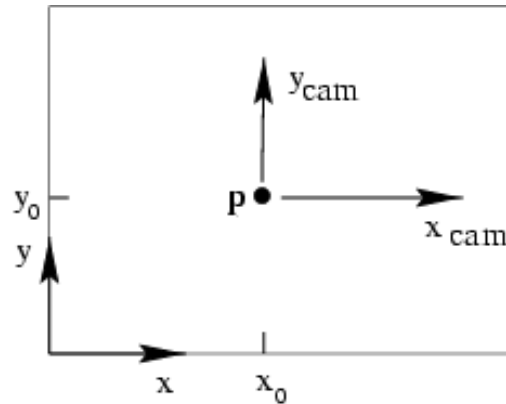
# Principle point offset



$$\begin{bmatrix} X & Y & Z \end{bmatrix}^T \mapsto \begin{bmatrix} fX/Z + p_x & fY/Z + p_y \end{bmatrix}^T$$

$$\begin{bmatrix} fX + Zp_x \\ fY + Zp_y \\ Z \end{bmatrix} = \begin{bmatrix} f & & p_x & 0 \\ & f & p_y & 0 \\ & & 1 & 0 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

# Principle point offset



$$\mathbf{x} = K [I \mid \mathbf{0}] \mathbf{X}_{cam}$$

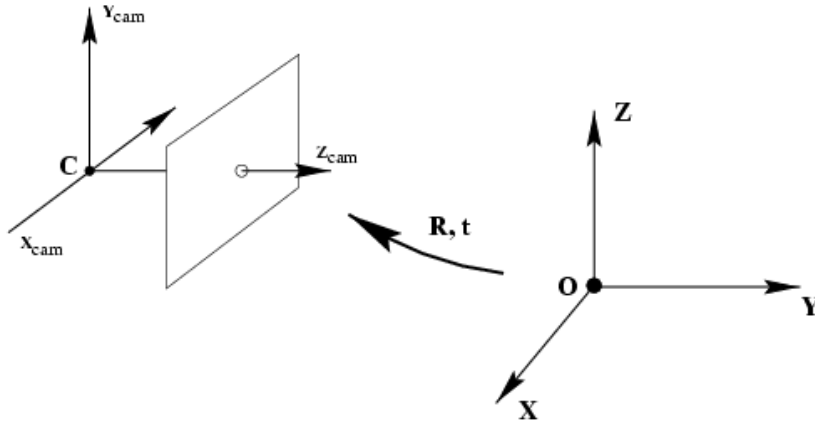
$$\begin{bmatrix} fX + Zp_x \\ fY + Zp_y \\ Z \end{bmatrix} = \begin{bmatrix} f & 0 & p_x \\ 0 & f & p_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

$$K = \begin{bmatrix} f & 0 & p_x \\ 0 & f & p_y \\ 0 & 0 & 1 \end{bmatrix}$$

Calibration  
matrix



# Camera rotation and translation



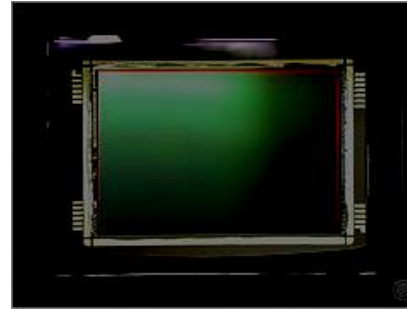
$$\mathbf{X}_{cam} = R(\mathbf{X} - C)$$

$$\mathbf{X}_{cam} = \begin{bmatrix} R & -RC \\ 0 & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} = \begin{bmatrix} R & -RC \\ 0 & 1 \end{bmatrix} \mathbf{X}$$

$$\mathbf{x} = K[I \mid \mathbf{0}]\mathbf{X}_{cam} = K[R \mid -RC]\mathbf{X} = P\mathbf{X}$$

$$P = K[R \mid \mathbf{t}], \quad \mathbf{t} = -RC$$

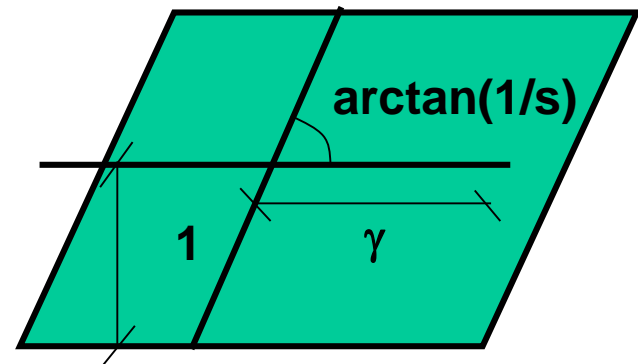
# CCD camera



$$K = \begin{bmatrix} m_x & & \\ & m_y & \\ & & 1 \end{bmatrix} \begin{bmatrix} f \\ f \\ 1 \end{bmatrix} \begin{bmatrix} p_x \\ p_y \\ 1 \end{bmatrix} = \begin{bmatrix} \alpha_x & & x_0 \\ & \alpha_y & y_0 \\ & & 1 \end{bmatrix}$$

When skew angle is not zero:

$$K = \begin{bmatrix} \alpha_x & s & x_0 \\ & \alpha_y & y_0 \\ & & 1 \end{bmatrix}$$



# Finite projective camera

$$K = \begin{bmatrix} \alpha_x & s & x_0 \\ & \alpha_y & y_0 \\ & & 1 \end{bmatrix}$$

$$P = K[R \mid \mathbf{t}], \quad \mathbf{t} = -RC$$

11 DOF (5 + 3 + 3)

## תרגיל

- מצלמת חריר אידיאלית (ideal pinhole camera) בעלת מרחק מוקד של 7mm.
- גודל כל פיקסל הוא 0.02mm X 0.03mm.
- ומרכז הצילום נמצא ב  $550 \times 650$  - כאשר הקורדינטות מתחילות מפינה שמאלית עליונה ב  $(0,0)$ .
- מהי מטריצת הקליברציה של המצלמה?

## תרגיל -תשובה

- מצלמת חריר אידיאלית (ideal pinhole camera) בעלת מרחק מוקד של 7mm.
- גודל כל פיקסל הוא 0.02mm X 0.03mm.
- ומרכז הצילום נמצא ב  $550 \times 650$  -כאשר הקורדינטות מתחילות מפינה שמאלית עליונה ב-(0,0).
- מהי מטריצת הקליברציה של המצלמה?

**תשובה:**

$$K = \begin{bmatrix} f \cdot k_u & 0 & x_0 \\ 0 & f \cdot k_v & y_0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 \cdot \frac{1}{0.02} & 0 & 550 \\ 0 & 7 \cdot \frac{1}{0.03} & 650 \\ 0 & 0 & 1 \end{bmatrix}$$

# סיכום

- טקסטורה
- Image transformations
- Geometry
- Projective geometry
- Camera model



## בפעם הבאה (והאחרונה):

- Camera model (cont.)
- Stereo vision
- מצגות פרויקט