Coding Boot Camp Chapter 6

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Q 6.1.

solution:

1) Arrange the 20 jars in a linear way numbering it from 1 to 20.
2) Now pick #no of pills from each jar according to the order they are arranged.
for ex pick 1 pill from jar 1, 2 from jar 2, 3 from jar 3 and so on.
3) Suppose if all the jars have equal weight of pills then the total weight is supposed to be:

Applying rule N (N + 1) / 2

20(20 + 1) / 2 = 210 grams

4) Now weight the pills that has been collected from these 20 jars, Let the total weight be X grams.
5) Deduct 210 grams from X grams. Let the result be Y.
6) The difference value Y will indicate which bottle is heavier.

Q 6.2

Solution:

Suppose the possibility for you to get one shot and make the hoop is p, $p \in [0, 1]$
Game 1 the possibility to win is p ,
Game 2 have several situations:

(1) Three shots and make three shots, the possibility is p*p*p;
(2) Three shots and make two shots, it maybe misses any shot, so the possibility is 3*p*p*(1-p);

The total possibility for Game 2 is $\mathrm{p} * \mathrm{p} * \mathrm{p} + 3 * \mathrm{p} * \mathrm{p} * (1 - \mathrm{p}) = 3p^2 - p^3$

So if
$$p > 3p^2 - p^3$$
$$(2p - 1)(p - 1) > 0$$
$$p < 0.5 \; or \; p > 1 (remove)$$

so when $0 < p < 0.5$, we should play Game 1;
when $0.5 < p < 1$, we should play Game 2;
otherwise, if $p = 0, 0.5, 1$, we can choose any game.

Q 6.3

Solution:

According to the problem we have 8*8=64-2=62 squares, it seems we can use 31 dominos to cover all the squares, but we should consider the limit that each dominos should cover exactly two squares, so these two squares, so these two square must near each other, but we cut the opposite corner square, so considering the following diagram, we remove two yellow square due to the requirement of the problem(opposite corner), so we left total 62 squares, but there is 30 yellow one and

32 red one, we need to use 31 dominos to cover, each dominos should cover one yellow and one red, so we cannot exactly cover two squares.

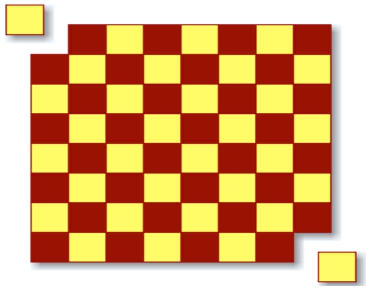

Fig.1 8*8 board

Q 6.4

Solution:

(1) Three vertex:

Because all the ants have the same speed, each of them has two directions, so only when two or more ants choose the opposite direction, which means there are only one situation for them to avoid collision, all the ants choose the same direction, if we assume the two direction is 0 and 1, so the possibility for each ant choose 0 or 1 is 0.5, so the possibility for they all choose 0 is 0.5*0.5*0.5;

And all choose 1 is 0.5*0.5*0.5,

The possibility of collision is 1-0.5*0.5*0.5-0.5*0.5*0.5 = 1-0.5^2 = 0.75;

(2) n vertex:

So the same with n-vertex polygon, the possibility of avoiding collision is $0.5^n \times 2 = 0.5^{n-1}$,

The possibility of collision is $1 - 0.5^{n-1}$.

Q 6.5

Solution:

(1) First, we can use the five-quart jug and fill with water,

(2) Second we use the five-quart water to fill the three-quart jug, then the water left in the five quart should be two- quart (5-3 = 2),

(3) Then we empty the three-quart jug, and fill this two-quart water into the three-quart jug, now the three-quart jug has two-quart water

(4) Finally we fill with five-quart water into the five-quart jug, and fill with the three-quart jug, which means we pour one-quart (3-2=1) water into three-quart jug, so we the water left in five-quart jug is exactly four quarts of water.

Q 6.6

Case1: only one person and maybe have 0 or 1 blue person

If we consider the case of just one blue-eyed person on the island, and we suppose all the person and smart enough to realize himself maybe the blue-eye person. We can show that he obviously leaves the first night, because he knows he's the only one maybe the blue-eye person. He looks around and sees no one else, and knows he should leave. So: (**Case 1)** If there is one blue-eyed person, he leaves the first night.

Case2: two persons and maybe have equal or less two blue-eye persons
If there are two blue-eyed people, they will each look at the other. They will each realize that "if I don't have blue eyes (**Suppose 1**), then that guy is the only blue-eyed person. And if he's the only person, by (**Case 1)** he will leave tonight." They each wait and see, and when neither of them leave the first night, each realizes "My **Suppose 1** was incorrect. I must have blue eyes." And each leaves the second night.
So: **(Case 2)**: If there are two blue-eyed people on the island, they will each leave the 2nd night.

Case 3: general case, have n people and maybe have equal or less n blue-eye persons
If there are n blue-eyed people, each one will look at the other n-1 and go through a process similar to the one above. Each considers the two possibilities -- "I have blue eyes" or "I don't have blue eyes." He will know that if he doesn't have blue eyes, there are only n-1 blue-eyed people on the island -- the n-1 he sees. So he can wait n-1 nights, and if no one leaves, he knows he must have blue eyes -- **(Case 2)** says that if he didn't, the other guys would have left. When he sees that they didn't, he knows his eyes are blue. All n of them are doing this same process, so they all figure it out on day n and leave.

Q 6.7
Solution:
The gender ratio of the new generation should be 1, because when the families number is very huge, the families contribute would be exactly one girl and on average one boy.
To consider this problem, because each family will stop when they have one girl, so we can think the children of these families would be (G means girl, and B means boy) G, BG, BBG, BBBG, BBBBG…., one way to think about this is to imagine that we put all the gender sequence of each family into one giant string. So if family 1 has BG, family 2 has BBG, and family 3 has G, we would write BGBBGG.
Actually, we don't really care about the grouping families because we're concerned about the population as a whole, as soon as a child is born, we can just append its gender (B or G) into the string.
So, if the odds of having a boy and girl is the same, then the odds of the next character being a G is 50%. Therefore, roughly half of the string should be Gs and half should be Bs, the gender ratio would be 1.

(The simulation code show in soulution07.java.)

Q 6.8
Solution:
The following is a description of the instance of this famous puzzle involving n=2 eggs and a building with N=100 floors.

Suppose that we wish to know which stories in a 100-floor building are safe to drop eggs from, and which will cause the eggs to break on landing. We make a few assumptions:

(1) An egg that survives a fall can be used again.
(2) A broken egg must be discarded.
(3) The effect of a fall is the same for all eggs.
(4) If an egg breaks when dropped, then it would break if dropped from a higher floor.
(5) If an egg survives a fall, then it would survive a shorter fall.
(6) It is not ruled out that the first-floor windows break eggs, nor is it ruled out that the 100th-floor do not cause an egg to break.

If only one egg is available and we wish to be sure of obtaining the right result, the experiment can be carried out in only one way. Drop the egg from the first-floor window; if it survives, drop it from the second floor window. Continue upward until it breaks. In the worst case, this method may require 100 droppings. Suppose 2 eggs are available. What is the least number of egg-droppings that is guaranteed to work in all cases?

The problem is not actually to find the critical floor, but merely to decide floors from which eggs should be dropped so that total number of trials are minimized.

In this post, we will discuss solution to a general problem with n eggs and k floors. The solution is to try dropping an egg from every floor (from 1 to N) and recursively calculate the minimum number of droppings needed in worst case. The floor which gives the minimum value in worst case is going to be part of the solution.

In the following solutions, we return the minimum number of trails in worst case; these solutions can be easily modified to print floor numbers of every trials also.

So, we can consider this problem as:

When we drop an egg from a floor 100, there can be two cases: (1) The egg breaks, (2) The egg doesn’t break.

1) If the egg breaks after dropping from Nth floor, then we only need to check for floors lower than N with eggs; so the problem reduces to N-1 floors and 2 eggs

2) If the egg doesn’t break after dropping from the Nth floor, then we only need to check for floors higher than N; so the problem reduces to 100-N floors and 2 eggs.

Since we need to minimize the number of trials in *worst* case, we take the maximum of two cases. We consider the max of above two cases for every floor and choose the floor which yields minimum number of trials.

(Analysis code shows in solution08.java.)

Q 6.9

Solution:

Let’s just start by choosing a random locker, and let’s determine whether it will end up open or closed. Let’s choose locker # 6.

Let’s go through each pass:

Pass # 1: all lockers are opened, including locker # 6

Pass # 2: all even numbered lockers are closed, including locker # 6

Pass # 3: every 3rd locker is toggled…so 3, 6, 9, ….96, 99. includes locker # 6.
Pass # 4: 4, 8, 12, etc. are all toggled. Excludes #6.
Pass # 5: 5, 10, 15 are all toggled. Excludes # 6 again.
Pass # 6: 6, 12, 18, etc. are all toggled. Includes # 6.
Passes greater than 6: Locker #6 will not be toggled again, since those will all start farther down the hall.
So, one thing you may notice after running through this example is that locker #6 is only toggled when the number of the pass (also called "x") that you are on is a factor of the # 6 – you can see that 1,2, 3, and 6 are all factors of 6. And those are all the passes on which the locker 6 is toggled – the sequence is open, close, open and then close. So, locker # 6 ends up closed.
Now that sounds like promising information. Since we are dealing with factors here, why not try a prime number – since a prime number only has 2 factors – itself and '1'. Let's try the number 13 as our prime number. The factors are 1 and 13 – which means that the operations are open and then close for any pass greater or equal to 13. So, it ends up being closed.
So, we know one thing for sure: that the lockers are only toggled on passes that are factors of the individual locker number.

Q 6.10
Solution:
(1) Simple approach(28 days)
There are some approaches for this problem. The simple approach is:
Dividing the bottles across the 10 test strips, first in groups of 100. Then, we wait seven days, when the result come back we look for a positive result across the test strips. We select the bottles associated with the positive test strip, "toss" all the other bottles, and repeat the process. We perform this operation until there is only one bottle left in the test set.

1. Divide bottles across available test stripes, one drop per test strip.
2. After seven days, check the test strips for results.
3. On the positive test strip: select the bottles associated with it into a new set of bottles. If this set size is 1, we have located the poisoned bottled, if this greater than one, go back to step 1.

(2) Optimal approach(7 days)
We can take each bottle number and look at its binary representation. If there's a 1 in the ith digit, then we will add a drop of this bottle's contents to test strip i. Observe that 2^10 is 1024, so 10 test strips will enough to handle up to 1024 bottles.
We wait seven days, and then read the results. If test strip I is positive, then set bit I of the result value, reading all the test strips will give us the ID of the poisoned bottle.

Follow up:
(Code shows in solution 10.java.)