Assignment on Calibration & Hedging

Calibration of local volatility

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class

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1 Main result

Dupire (1994) proposed a model to evaluate the local volatility:

$$\frac{\partial V}{\partial T} + rK\frac{\partial V}{\partial K} - \frac{1}{2}\sigma^2_{local}(K,T)K^2\frac{\partial^2 V}{\partial K^2} = 0 \tag{1}$$

In this report, we focus on the parametric calibration method, in which volatility is assumed to follow the function of strike price (K) and maturity (T):

$$\sigma_{local}(K,T) = \frac{\beta_1}{K^{\beta_2}}$$

Then, the idea is to compare the market price with the price obtained by equation (1) and solve the minimisation problem to "best" estimate β_1 and β_2.
Use Crank-Nikolson method, we discretise T to N and K to I divisions; thereby we have (N+1) and (I+1) points equi-distanced for T and K respectively and find the following equations:

$$\begin{aligned}\frac{V_i^{n+1} - V_i^n}{\Delta T} + rK_i\frac{1}{2}\Big(\frac{V_{i+1}^n - V_{i-1}^n}{2\Delta K} + \frac{V_{i+1}^{n+1} - V_{i-1}^{n+1}}{2\Delta K}\Big) \\ - \frac{1}{2}\sigma^2_{local}(K_i,T_n)K_i^2\frac{1}{2}\Big(\frac{V_{i+1}^n - V_{i-1}^n}{2\Delta K} + \frac{V_{i+1}^{n+1} - V_{i-1}^{n+1}}{2\Delta K}\Big) = 0\end{aligned} \tag{2}$$

We impose the boundary conditions on equation (2).

$$\begin{cases} V(K, T=0) = V_i^0 = max(S_0 - K_i, 0) \\ V(K \to +\infty, T) = V_{I+1}^n = 0 \\ V(K \to 0, T) = V_0^n = S_0 \end{cases}$$

Equation (2) results in I equations for each node n

$$A_i V_{n+1}^{i+1} + B_i V_{n+1}^i + D_i V_{n+1}^{i-1} = C_n^i \tag{3}$$

where

$$\begin{cases} A_i = \frac{\Delta T}{4}\Big(r\frac{K_i}{\Delta K} - \sigma_i^2\frac{K_i^2}{\Delta K^2}\Big) \\ B_i = -\frac{\Delta T}{4}\Big(r\frac{K_i}{\Delta K} + \sigma_i^2\frac{K_i^2}{\Delta K^2}\Big) \\ D_i = 1 + \frac{\Delta T}{2}\sigma_i^2\frac{K_i^2}{\Delta K^2} \\ C_i = V_i^n - \frac{\Delta T}{4}\big(rK_i(V_n^{i+1} - V_n^{i-1}) - \sigma_i^2\frac{K_i^2}{\Delta K^2}(V_n^{i+1} - V_n^{i-1})\big) \end{cases}$$

By solving the system of equation (3), we will find the price for all strike and maturity. Call price is now dependent on β, we can solve the minimization problem by Levenberg-Marquardt algorithm

$$\begin{cases} J(1,i) = \frac{\partial V_i}{\partial \beta_1} = \frac{\partial V}{\partial \sigma_i}\frac{\partial \sigma_i}{\partial \beta_1} = Vega_i\frac{1}{K^{\beta_2}} \\ J(2,i) = \frac{\partial V_i}{\partial \beta_2} = \frac{\partial V}{\partial \sigma_i}\frac{\partial \sigma_i}{\partial \beta_2} = Vega_i\frac{\beta_1 \ln K_i}{K^{\beta_2}} \end{cases}$$

where Vega is calculated as

$$Vega = \frac{V(K,T,\sigma) - V(\sigma - \Delta h)}{\Delta h}, \qquad \Delta h = 0.01$$

Table 1: Call price for different K at T=0.5

K_i	7	7.5	8	8.5	9	9.5	10
V^i_{market}	3.3634	2.9092	2.4703	2.0536	1.6666	1.3167	1.0100
10.5	11	11.5	12	12.5	13	13.5	14
0.7504	0.5389	0.3733	0.2491	0.1599	0.0986	0.0584	0.0332

Figure 1: Calibration of local volatility - Dupire formula

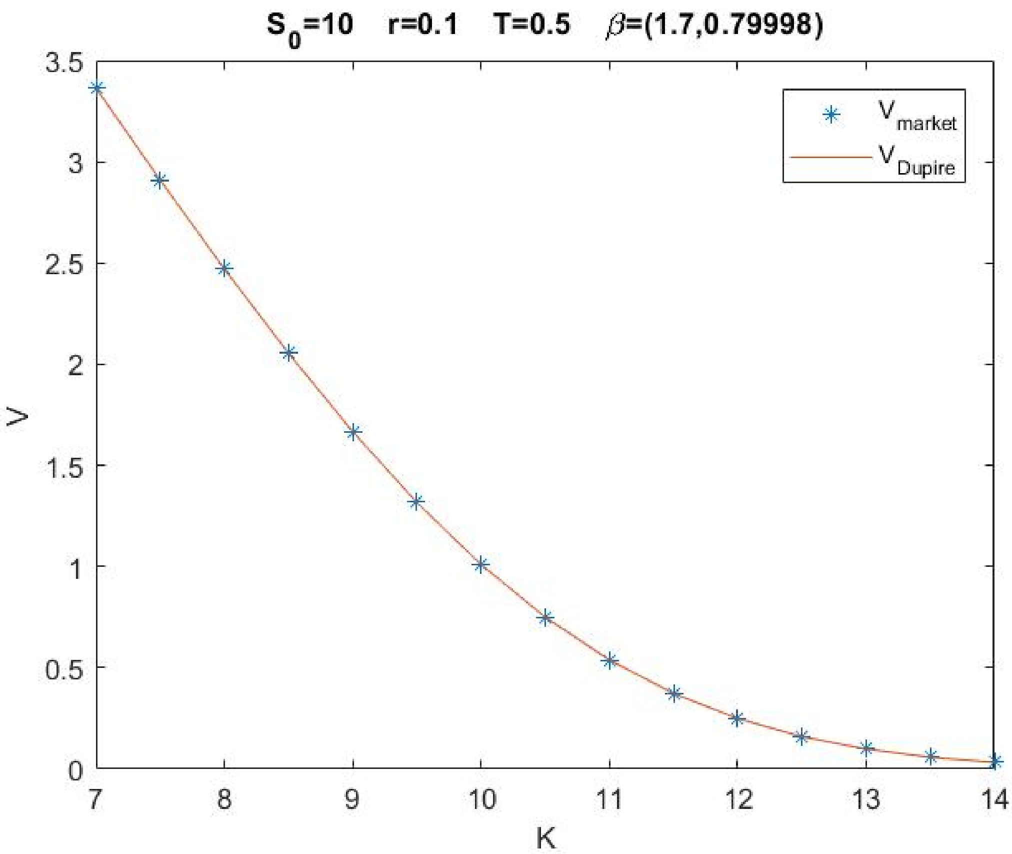

Figure 2: Call price by Dupire formula

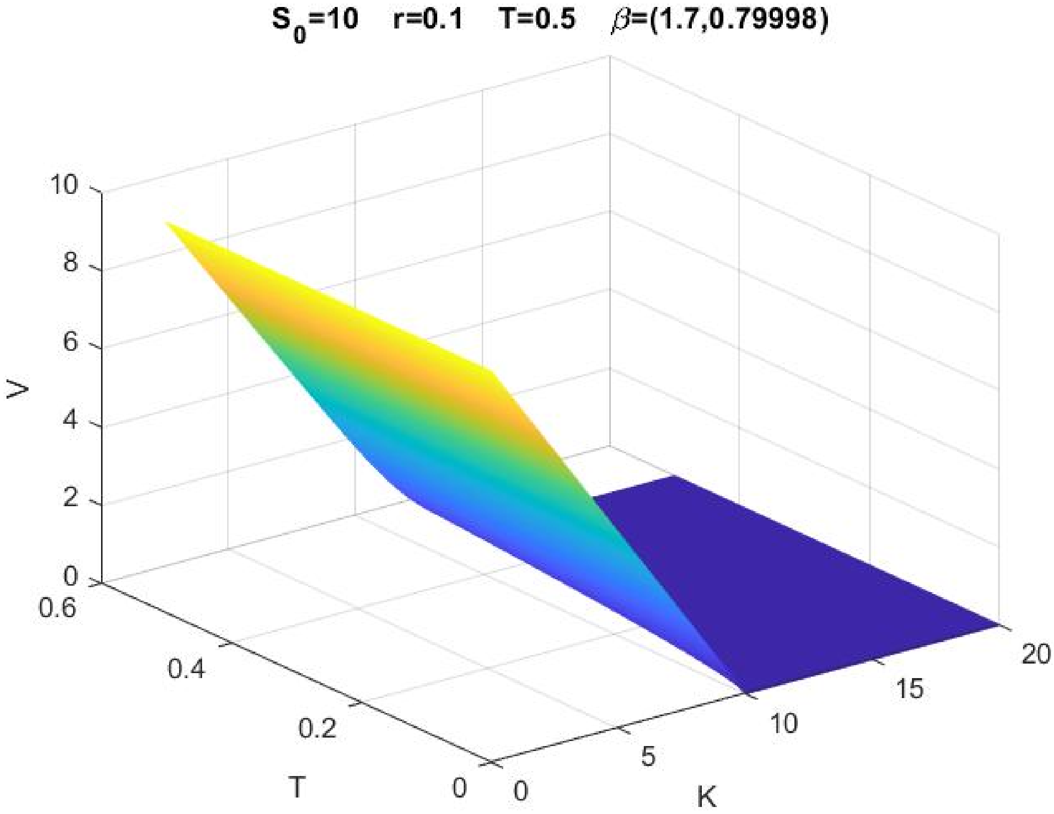

Figure 3: Vega

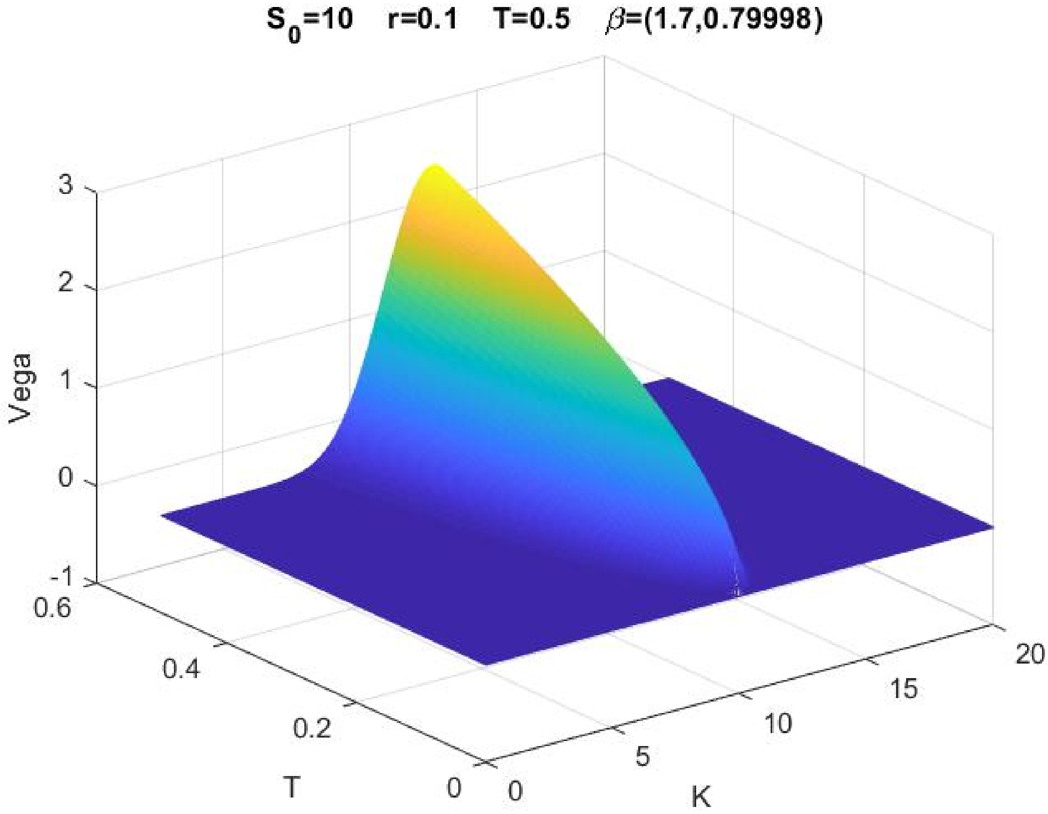

Figure 4: Volatility as a function of K and T