Integrated Project #1

Kinematics

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Contents

Chapter 1

Introduction

In this experiment, we are trying to calculate movement of a basic projectile shot in an ideal area with no friction or other factors and try to find out what is the difference between this ideal area and the real life and derive a function/matrix from those experiments.

Chapter 2

Preparing the Experiment

2.1 Objective

To study projectile motion in real life, considering the effects of air resistance.

2.2 Theory

2.2.1 Projectile Motion without Air Resistance

The motion of an object thrown into the air is called projectile motion. Neglecting air resistance, the projectile follows a parabolic path due to the constant acceleration of gravity. The equations of motion for projectile motion are:

$$\begin{aligned} x(t) &= v_0 \cdot \cos(\theta) \cdot t \\ y(t) &= v_0 \cdot \sin(\theta) \cdot t - \frac{1}{2} g t^2 \end{aligned} \tag{2.1}$$

where:

- $x(t)$ and $y(t)$ are the horizontal and vertical positions of the projectile at time t.
- v_0 is the initial velocity of the projectile.
- θ is the launch angle.
- g is the acceleration due to gravity (approximately $9.81\,m/s^2$).
- t is the time of flight.

2.2.2 Projectile Motion with Air Resistance

In the presence of air resistance, the projectile experiences an additional force that opposes its motion. This force is proportional to the speed of the projectile and acts in the opposite direction of the projectile's velocity. The equations of motion for projectile motion with air resistance are more complex and cannot be solved analytically.

Experimental Procedure:

- Set up the spring gun at an angle θ with the horizontal.
- Measure the distance R from the gun to the point where the ball lands.
- Measure the height H of the highest point reached by the ball.
- Repeat steps 1–3 for different initial angles θ.
- Calculate the average initial velocity v_0 of the ball by firing the gun horizontally and measuring the distance it travels.
- Measure the mass m of the ball.
- Compare the theoretical and experimental results for R, H, and θ. Discuss the effects of air resistance on the trajectory of the projectile.

Chapter 3

Theoretical Part

3.1 Projectile Shot In An Area Without Air Resistance

Without the air resistance, the maximum distance that projectile can go, R, is calculated as follows:

$$x_f = x_{\max} = R = v_0 \cdot \cos(\theta) \cdot t \tag{3.1}$$

With this expression in mind, we can find the total flight time as follows:

$$t = \frac{R}{v_0 \cdot \cos(\theta)} \tag{3.2}$$

And if we use this time to calculate the maximum height, H, we can find it as following:

$$\begin{aligned} H = y_f &= y_0 + v_0 \cdot \sin(\theta) \cdot t - \frac{g}{2} t^2 \\ &= y_0 + v_0 \cdot \sin(\theta) \cdot \frac{R}{v_0 \cdot \cos(\theta)} - \frac{g}{2} \left(\frac{R}{v_0 \cdot \cos(\theta)} \right)^2 \\ &= \boxed{y_0 + R \cdot \tan(\theta) - \frac{g}{2} \cdot \frac{R^2}{v_0^2} \cdot \frac{1}{\cos^2(\theta)}} \end{aligned} \tag{3.3}$$

3.2 Finding the General $\tan(\theta)$ Function

If we leave $\tan(\theta)$ alone in the expression above, we can get a general function for $\tan(\theta)$ as follows:

$$\begin{aligned} \tan(\theta) R &= H - y_0 + \frac{g}{2} \cdot \frac{R^2}{v_0^2} \cdot \frac{1}{\cos^2(\theta)} \\ \tan(\theta) &= \boxed{\frac{H - y_0}{R} + \frac{g}{2} \cdot \frac{R}{v_0^2} \cdot \frac{1}{\cos^2(\theta)}} \end{aligned} \tag{3.4}$$

3.3 Developing the $y = f(x)$ Function

We already have the required function. If we change it a bit, we can develop a general function y as it depends on x:

$$y(x) = y_0 + x \cdot \tan(\theta) - \frac{g}{2} \cdot \frac{x^2}{v_0^2} \cdot \frac{1}{\cos^2(\theta)} \tag{3.5}$$

3.4 Developing the Envelope Curve

In order to develop a function, we can manipulate our $y = f(x)$ function a little bit:

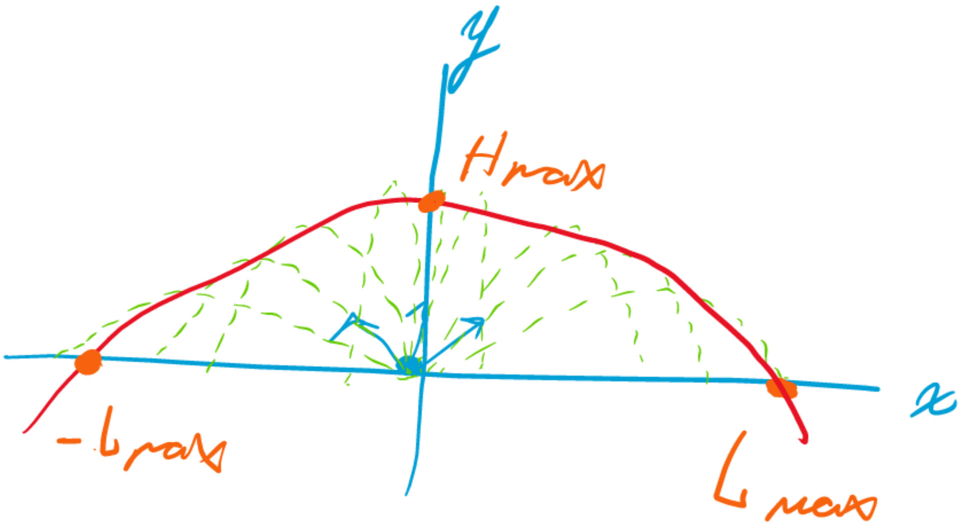

Figure 3.1: Envelope curve of the projectile motion

$$\begin{aligned}
y(x) &= \tan(\theta)x - \frac{g}{2} \cdot \frac{x^2}{v_0^2} \cdot \frac{1}{\cos^2(\theta)} \\
\cos^2(\theta) &= \frac{1}{1 + \tan^2(\theta)} \\
y(x) &= \tan\theta x - \frac{g}{2} \cdot \frac{x^2}{v_0^2} \cdot (1 + \tan^2(\theta)) \\
y(x) &= \frac{1}{2} \cdot \frac{v_0^2}{g} - \frac{g}{2} \cdot \frac{x^2}{v_0^2}
\end{aligned}$$

3.4.1 Finding the Derivative

$$\begin{aligned}
\frac{d}{dx} y(x) &= \frac{-g}{v_0^2} \cdot x \\
\frac{d^2}{dx^2} y(x) &= \frac{-g}{v_0^2}
\end{aligned}$$

Chapter 4

Experimental Part

4.1 Projectile Shot In An Area With Air Resistance

With air resistance in mind, we can't easily make calculations of the movement. For this purpose, we have made 4 different projectile shots and tried to make calculations according to their movement.

	15°	30°	45°	50°
L	100.21 cm	97.41 cm	58.25 cm	58.13 cm
H	3.56 cm	7.96 cm	9.43 cm	11.75 cm
v_0	$367\frac{cm}{s}$	$312\frac{cm}{s}$	$322\frac{cm}{s}$	$366.2\frac{cm}{s}$

Table 4.1: Projectile Shot With Air Resistance

- Mass of the projectile: $2.7g$
- Average acceleration: $424\frac{cm}{s^2}$

Due to air resistance, rolling motion of the projectile and other factors, real results are different from the theoretical results. Both L and H is much smaller than is should be because of these factors.

You can access experiment video via this url:
`https://youtu.be/2aXTh4kZMRA`

Chapter 5

MATH103: Matrix Operations and Linear Systems

$$A = \begin{pmatrix} 100 & 3.5 & 15 \\ 97 & 7.9 & 30 \\ 58 & 7.6 & 45 \end{pmatrix} \tag{5.1}$$

These values are from the experimental part of this project.

5.1 Inverse of Matrix A

5.1.1 Elimination Matrices

$$E_1 = \begin{pmatrix} 1 & 0 & 0 \\ -0.97 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{5.2}$$

$$E_2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -0.96202 & 1 \end{pmatrix} \tag{5.3}$$

5.1.2 Taking the Inverse of Matrix A

$$I = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{5.4}$$

$$(A|I) = \left(\begin{array}{ccc|ccc} 100 & 3.5 & 15 & 1 & 0 & 0 \\ 97 & 7.9 & 30 & 0 & 1 & 0 \\ 58 & 7.6 & 45 & 0 & 0 & 1 \end{array}\right) \tag{5.5}$$

$$E_2 \cdot E_1 \cdot (A|I) = \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 0.01645 & -0.00561 & -0.00174 \\ 0 & 1 & 0 & -0.33881 & 0.46853 & -0.19941 \\ 0 & 0 & 1 & -0.03601 & -0.07189 & 0.05814 \end{array}\right) = \left(I|A^{-1}\right) \tag{5.6}$$

$$A^{-1} = \begin{pmatrix} 0.01645 & -0.00561 & -0.00174 \\ -0.33881 & 0.46853 & -0.19941 \\ -0.03601 & -0.07189 & 0.05814 \end{pmatrix} \tag{5.7}$$

5.1.3 ALU Factorization of Matrix A

$$A = \begin{pmatrix} 100 & 3.5 & 15 \\ 97 & 7.9 & 30 \\ 58 & 7.6 & 45 \end{pmatrix} = L \cdot U \tag{5.8}$$

$$L = \begin{pmatrix} 1 & 0 & 0 \\ 0.97 & 1 & 0 \\ 0.58 & 1.2364 & 1 \end{pmatrix} \tag{5.9}$$

$$U = \begin{pmatrix} 100 & 3.5 & 15 \\ 0 & 4.505 & 15.45 \\ 0 & 0 & 17.1976 \end{pmatrix} \tag{5.10}$$

5.2 Linear System Using Data From Experiment

let

$$\begin{aligned} t &= 1s \\ v_0 &= 20\frac{m}{s} \\ g &= 10\frac{m}{s^2} \end{aligned}$$

let $\theta = 45^\circ$ for $R_{\max}$

$$x(2) = 20 \cdot \cos(45^\circ) = R_{\max} = 10\sqrt{2}m \tag{5.11}$$

let $\theta = 90^\circ$ for $H_{\max}$

$$y(2) = 20 \cdot \sin(90^\circ) - \frac{1}{2} \cdot 10 = H_{\max} = 15m \tag{5.12}$$

$$Ax = b \tag{5.13}$$

$$\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 10\sqrt{2} \\ 15 \end{pmatrix} \tag{5.14}$$

$$(A|b) = \left(\begin{array}{cc|c} 1 & 2 & 10\sqrt{2} \\ 3 & 4 & 15 \end{array}\right) \tag{5.15}$$

$$P_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \tag{5.16}$$

$$P_1 \cdot (A|b) = \left(\begin{array}{cc|c} 3 & 4 & 15 \\ 1 & 2 & 10\sqrt{2} \end{array}\right) \tag{5.17}$$

$$E_{12} = \begin{pmatrix} 1 & -4 \\ 0 & 1 \end{pmatrix} \tag{5.18}$$

$$E_{21} = \begin{pmatrix} 1 & 0 \\ -\frac{1}{3} & 1 \end{pmatrix} \tag{5.19}$$

$$E_{12} \cdot E_{21} \cdot P_1 \cdot (A|b) = \left(\begin{array}{cc|c} 1 & 0 & 15 - 20\sqrt{2} \\ 0 & 1 & 15\sqrt{2} - \frac{15}{2} \end{array}\right) \tag{5.20}$$

$$\tag{5.21}$$

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 15 - 20\sqrt{2} \\ 15\sqrt{2} - \frac{15}{2} \end{pmatrix} \tag{5.22}$$

$$x_1 = 5 \cdot (3 - 4\sqrt{2}) \tag{5.23}$$

$$x_2 = 15 \cdot (\sqrt{2} - \frac{1}{2}) \tag{5.24}$$

Chapter 6

EE101: Taking the Inverse of a Matrix Using Gauss-Jordan Method

Code for this part can be found at the following link:

`https://gist.github.com/barbarbar338/b66e2a404abb3a7f7c4d542204a3626a`

Chapter 7

Conclusion

In conclusion, this experiment aimed to investigate projectile motion in real-life scenarios, taking into account the effects of air resistance. The theoretical tasks involved deriving mathematical equations to describe the initial angle (θ) and the envelope curve ($y = f(x)$) of projectile motion. Calculus I task focused on determining the maximum range (R) and height (H) for all possible initial angles θ, along with analyzing the envelope curve function and its derivatives.

The experimental tasks included measuring the average initial velocity (v_0) of the spring gun, obtaining real measurements of distance (L) and height (H) for different initial angles θ, and evaluating the mass (m) of the ball to determine the average acceleration component due to air resistance.

Air resistance has a significant impact on the motion of projectiles. By understanding the effects of air resistance, we can more accurately predict the trajectory of objects moving through the air.

MATH103 tasks involved matrix operations, including finding the inverse of matrix A using the Gauss-Jordan method, determining elimination matrices, and performing LU factorization. Additionally, a linear system was created to model the experimental data.

In the EE101 tasks, a C++ program was developed to facilitate matrix calculations, ensuring user-friendly input for matrix size and elements, and providing the inverse matrix using the Gauss-Jordan method.

Through these combined efforts, the experiment provided a comprehensive understanding of projectile motion, integrating theoretical concepts with practical measurements. The results obtained from the mathematical models and experimental data were compared, shedding light on the accuracy of theoretical predictions in real-world scenarios. The designed C++ program demonstrated the practical applicability of computational tools in solving complex mathematical problems related to physics and engineering. Overall, this experiment not only enhanced our understanding of projectile motion but also underscored the importance of considering factors such as air resistance in real world physics applications.