Project 2. STUDYING THE PENDULUM OSCILLATIONS

Main goal: Measuring the period of oscillation of a pendulum.

The basic experimental setup is presented in Figure:

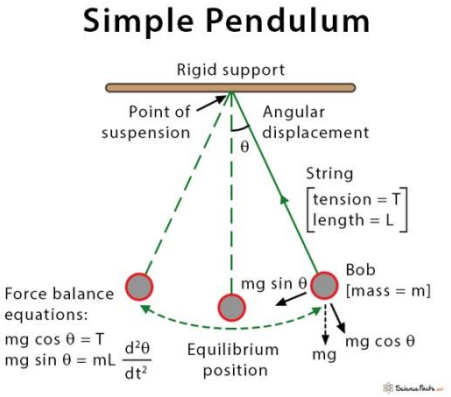

Figure 1. *Principle scheme of the experimental setup.*

Consider the string to be massless, such that all mass of the pendulum is represented with the bob mass m.

Remark: An ***oscillation*** is a motion that characterized by repeatability over time of physical values quantities (the angle θ of deviation from the vertical line in our case) that determine movement or state. Oscillations are called ***periodic*** if the value θ versus time t changes such that the oscillation process is repeated through an equal period T of time: $\theta(t+T) = \theta(t)$.

Mathematical model: Let's consider the simplified model where the force returning the pendulum to the equilibrium position is linear with respect to the deviation angle θ:

$$F_{\text{return}} = -k\theta \ , \tag{1}$$

with some positive constant k. Then the equation of motion for the pendulum is given in the form of a second derivative term and the function θ itself:

$$\frac{d^2\theta(t)}{dt^2} + \omega^2\theta(t) = 0 \ . \tag{2}$$

Here the frequency $\omega = 1/T$.

MATH153 Task:

1. Check that the function:

$$\theta(t) = B\cos(\omega t) \ , \tag{3}$$

with a constant B is a solution to Eq. [=equation] (2).

2. Suppose that the maximum deviation angle is θ_{max}, and the pendulum length is L. Find the function of the path $s(t)$ (along the arc) and the sector area function $A(t)$ (based on the path arc) for a single period range $[0,T]$.

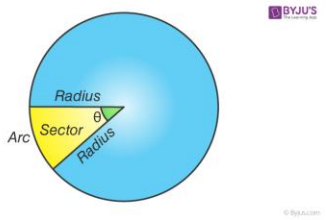

Figure 2. *Geometrical definition of an arc and a sector.*

PHYS105 Theoretical Task:

1. Derive the equation for the kinetic energy $K(t)$, potential gravity energy $U(t)$ and the mechanical energy $E(t) = K(t) + U(t)$.

2. What can you say about the mechanical energy E conservation for the model (2)?

3. Analyzing units, find how the pendulum frequency ω depends on the following parameters: m, L, θ_{max} and g (the gravity acceleration).

4. What is the physical meaning of B in Eq.(3)?

5. How the period T of the pendulum oscillation depends on m and L?

PHYS105 Experimental Task:

Install the experimental setup and track the oscillations of the bob as function $\theta(t)$.

1. Make a conclusion if the real $\theta(t)$ follows Eq.(3) or not.

2. Evaluate the mechanical energy conservation for your experimental setup. If the mechanical energy is not conserved then evaluate the average % of its losing per a single period.

For the tasks 1-2, one period could be too short for practical measurement, you are recommended to observe the system a longer time (like $10T$) and repeat your evaluation few times.

3. Explain the difference in the behavior of a real pendulum and the theoretical solution (3). What could be the reason for that?

MATH103 Task:

1. Construct a matrix whose form is given below. (You are free to choose various t variables.)

$$\begin{bmatrix} \theta(t_1) & K(t_1) & U(t_1) \\ \theta(t_2) & K(t_2) & U(t_2) \\ \theta(t_3) & K(t_3) & U(t_3) \end{bmatrix}$$

a. Find the inverse of this matrix.
b. Transform this matrix to LDU form.

2. Find the complete solution for the system given below. (You are free to choose various t variables. The variables must be different than the ones of 1st question.)

$$\begin{bmatrix} \theta(t_1) & K(t_1) & U(t_1) & s(t_1) & A(t_1) \\ \theta(t_2) & K(t_2) & U(t_2) & s(t_2) & A(t_2) \\ \theta(t_3) & K(t_3) & U(t_3) & s(t_3) & A(t_3) \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 0 \end{bmatrix}$$

EE 101 Task:

1. Your task is to read the file_list.csv file. Then, you need to create a CSV file containing a list of students, their respective groups, and the questions answered by each student. For example:

   a_1_2122011022.cpp
   a_2_2111011011.txt.cpp
   B_2_2111011243.cpp.txt
   A_3_2111011011.txt

The output file should be structured as follows:

Student list	GroupName	QuestionsAnswered
2122011022	a	1
2111011011	a	2, 3
2111011243	b	2

Link to file_list.csv:
https://drive.google.com/file/d/1Jee2ez13q9_nX205IHj2IqnF5bAeKmT0/view?usp=sharing