Project 2

Obtaining a uniform magnetic field in a certain volume is a task that is often encountered both in setting up a physical experiment and in ensuring the operation of a number of electrical and electronic devices. Depending on the required value of magnetic induction $\mathbf{B}$, and other system parameters, this problem is solved in different ways.

A system for producing a small region of space with nearly uniform magnetic field, called **Helmholtz Coils**, consists of two identical coils of radius R on the same z-axis as shown in Figure 1.

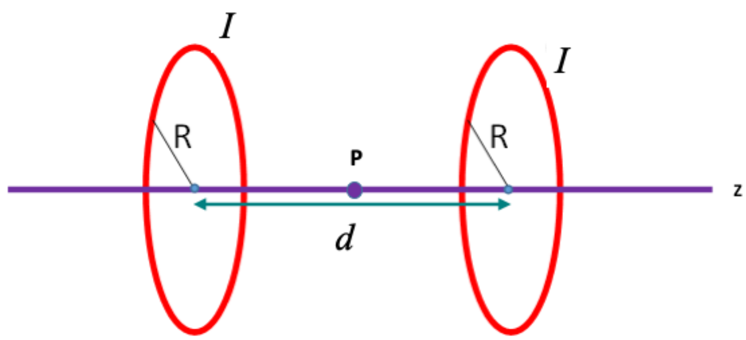

Figure 1. Basic setup for the Helmholtz Coils

In general case, the magnetic field produced in such setup is not homogeneous, see Figure 2.

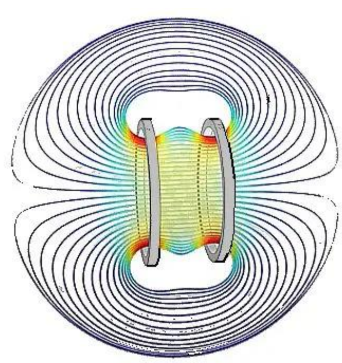

Figure 2. The magnetic field lines for the basic Helmholtz Coil setup.

Source: https://www.testandmeasurementtips.com/whats-all-this-helmholtz-coil-stuff-faq/

Preliminary task. Determine the magnitude of the magnetic field induction $\mathbf{B}$ on the axis of a single circular coil of radius R with current I depending on the distance z orthogonal to its plane.

Basic model for the Helmholtz coil. Consider now two identical circular coils of radius R, the current in each of them is equal to I. The coils are arranged so that their planes are parallel and their centers lie on the same z-axis at a distance d from each other, see **Figure 1**. Let's choose a coordinate system so that its z-axis coincides with the axis of the coils.

- Find the general equation for the magnetic field $B(z)$.
- Let's assuming that the currents in the coils flow in one direction. Determine at which ratio d/R the magnetic field in the center of the system on the axis of the turns will be as uniform as possible.

<u>*Remark:*</u> To do that, consider the magnetic field $B(x)$ to be expanded into the ***Taylor series*** in a neighborhood of the axis z = 0. The field in the neighborhood of $z = 0$ is as more homogeneous, as more derivatives of B in the Taylor expansion are equal to zero.

- Based on $B'(0)$ and $B''(0)$ determine d/R.
- Make a reasonable guess about the homogeneity of the magnetic field in the direction perpendicular to the z-axis.
- Find the magnetic field on the axis of coils in the neighborhood of the center of the system as the function of μ_0, I and R for the determined d/R.
- Now consider two identical thin coils of N turns each, placed coaxially to each other at a distance equal to their average radius. Find the magnetic field on the axis of the coils in the neighborhood of the center of the system as the function of μ_0, I and R for the determined d/R.
- Derive the vector function $\mathbf{B}(r,\theta,z)$ in the ***cylindrical coordinate system*** for the field shifted from the axis z = 0 by a small distance Δ. Evaluate the radial component B_r of this magnetic field for small shifts Δ as a function of r.
- Evaluate the ***partial derivatives*** $\partial B/\partial r$, $\partial B/\partial z$ and $\partial B/\partial \theta$ for the whole vector $\mathbf{B}$.
- Calculate the inductance of one of the coils theoretically.
- Propose a measurement system employing oscilloscope, signal generator, and basic circuit elements to measure the inductance.

Analysis of Helmholtz coil in a circuit. Connect a 10Ω resistor in series with the designed Helmholtz coil. Apply a 0-5V symmetric square wave and investigate the voltage responses across both the coil and the resistor while varying the frequency of the square wave. Model the Helmholtz coil in SPICE and implement each step in simulation too.

- Specify the frequency of the complete charging and discharging of coil.
- Change the period of the square-wave to $T=10L/R$, $T=L/R$, $T=L/10R$ and draw the waveforms for coil and resistor.
- Reduce the amplitude of the square wave to half, repeat the measurements, and explain the observed changes.