Parameters

$$\begin{align}
a &> 0 \tag{1}\\
b &> 0 \tag{2}\\
K &\geq 0 \tag{3}\\
D_0 &\geq 0 \tag{4}\\
\tau &\geq 0 \tag{5}
\end{align}$$

Potential

$$V(x) = \frac{a}{4}x^4 - \frac{b}{2}x^2 = \frac{a}{4}x^2\left(x - \sqrt{\frac{2b}{a}}\right)\left(x + \sqrt{\frac{2b}{a}}\right) \tag{6}$$

Force

$$F(x) = -\frac{\partial V}{\partial x} = -ax^3 + bx \tag{7}$$

We have 2 Hypervolumes

$$\begin{align}
\mathcal{V}_t &\subset \mathbb{R}^{n_t} \tag{8}\\
\mathcal{V}_\tau &\subset \mathbb{R}^{n_\tau} \tag{9}\\
\mathcal{V} &= \mathcal{V}_t \times \mathcal{V}_\tau \tag{10}\\
n_t &\geq n_\tau \tag{11}
\end{align}$$

$$\begin{align}
\dot{X}_t &= F(X_t, X_\tau)\,\mathrm{d}t + \sqrt{2D_0}\,\mathrm{d}B_t \tag{12}\\
&= -aX_t^3\,\mathrm{d}t + bX_t\,\mathrm{d}t + K(X_t - X_\tau)\,\mathrm{d}t + \sqrt{2D_0}\,\mathrm{d}B_t \tag{13}
\end{align}$$

$$F(x, x_\tau) = F_1(x) + F_2(x_\tau) = -ax^3 + (b+K)x - Kx_\tau \tag{14}$$

$$\begin{align}
\frac{\partial}{\partial t}\rho_1 &= -\frac{\partial}{\partial x}\int\limits_{\mathcal{V}_\tau}\mathrm{d}x_\tau\, F(x, x_\tau)\rho_2(x, t; x_\tau, t-\tau) + D_0\frac{\partial^2}{\partial x^2}\rho_1(x,t) \tag{15}\\
&= -\frac{\partial}{\partial x}F_1(x)\int\limits_{\mathcal{V}_\tau}\mathrm{d}x_\tau\,\rho_2(x,t;x_\tau,t-\tau) - \frac{\partial}{\partial x}\int\limits_{\mathcal{V}_\tau}\mathrm{d}x_\tau\,F_2(x_\tau)\rho_2(x,t;x_\tau,t-\tau) + D_0\frac{\partial^2}{\partial x^2}\rho_1(x,t) \tag{16}\\
&= -\frac{\partial}{\partial x}F_1(x)\rho_1(x,t) - \frac{\partial}{\partial x}\int\limits_{\mathcal{V}_\tau}\mathrm{d}x_\tau\,F_2(x_\tau)\rho_2(x,t;x_\tau,t-\tau) + D_0\frac{\partial^2}{\partial x^2}\rho_1(x,t) \tag{17}
\end{align}$$

$$\frac{\partial}{\partial t}\rho_1 = -\frac{\partial F_1}{\partial x}\rho_1(x,t) - F_1(x)\frac{\partial}{\partial x}\rho_1(x,t) - \frac{\partial}{\partial x}\int\limits_{\mathcal{V}_\tau}\mathrm{d}x_\tau\,F_2(x_\tau)\rho_2(x,t;x_\tau,t-\tau) + D_0\frac{\partial^2}{\partial x^2}\rho_1(x,t) \tag{18}$$

$$\begin{align}
\frac{\partial}{\partial t}\rho_2 &= -\frac{\partial}{\partial x}F(x,x_\tau)\rho_2(x,t;x_\tau,t-\tau) + D_0\frac{\partial^2}{\partial x^2}\rho_2(x,t;x_\tau,t-\tau) + \frac{\partial}{\partial x_\tau}\rho_2(x,t;x_\tau,t-\tau)\int\limits_{\mathcal{V}_t}\mathrm{d}\dot{x}_\tau\,\rho_1(\dot{x}_\tau,t-\tau)\dot{x}_\tau \tag{19}\\
&= -\frac{\partial}{\partial x}F(x,x_\tau)\rho_2(x,t;x_\tau,t-\tau) + D_0\frac{\partial^2}{\partial x^2}\rho_2(x,t;x_\tau,t-\tau) + \frac{\partial}{\partial x_\tau}\rho_2(x,t;x_\tau,t-\tau)\left(\int\limits_{\mathcal{V}_t}\mathrm{d}\dot{x}_\tau\,\rho_1(\dot{x}_\tau,t-\tau)\dot{x}_\tau\right) \tag{20}\\
&= -\frac{\partial}{\partial x}F_1(x)\rho_2(x,t;x_\tau,t-\tau) - F_2(x_\tau)\frac{\partial}{\partial x}\rho_2(x,t;x_\tau,t-\tau) + D_0\frac{\partial^2}{\partial x^2}\rho_2(x,t;x_\tau,t-\tau) + \frac{\partial}{\partial x_\tau}\rho_2(x,t;x_\tau,t-\tau)\left(\int\limits_{\mathcal{V}_t}\mathrm{d}\dot{x}_\tau\,\rho_1(\dot{x}_\tau,t-\tau)\dot{x}_\tau\right) \tag{21}
\end{align}$$

$$\begin{align}
\frac{\partial f}{\partial x} &\sim \frac{f(x_{i+1}) - f(x_{i-1})}{2\Delta x} \tag{22}\\
\frac{\partial^2 f}{\partial x^2} &\sim \frac{2f(x_i) - f(x_{i-1}) - f(x_{i+1})}{\Delta x^2} \tag{23}
\end{align}$$

$$\int\limits_{\mathcal{V}}\mathrm{d}x\,f(x) = \Delta x\sum_{i=1}^{N_{\mathcal{V}}} f(x_i) \tag{24}$$