1 Vector: 3 perspectives - Physics, Math, CS

$$\begin{bmatrix} 7 & 8 & 6 \\ 6 & 8 & 4 \\ 7 & 5 & 4 \end{bmatrix}$$

1.1 physics

- vectors are arrows in space with length and direction
- so vectors can be moved around in space without any issues

1.2 cs

- vectors are ordered lists of numbers

1.3 math

- seeks to generalize both of these views and defines vector operations such as addition and multiplication

1.4 In linear algebra

- graham suggests to view vector as arrow with tail always fixed at origin
- maybe i can imagine a vector as a operation of shifting the origin
- thus when we do vector addition we will start the second vector from the start of first vector, since the origin has been shifted by the first vector
- Note: "but" it could really be shifting of all the points in the coordinate system (as told by graham)

1.5 Scalars

- The numbers that we multiply the vector with to scale the vector in its original direction
- since it's used frequently it just interchangeable with number

2 Linear Combinations, Spans, Basis Vectors

2.1 Vector Coordinates

- vector coordinates are the number numbers present in a vector
- Each of the vector coordinates is also a scalar that scales the basis vectors of the coordinate system $\hat{i}, \hat{j}$

2.2 Span

Span of $\vec{v}$ and $\vec{w}$ are the set of all of their linear combinations

$$a\vec{v} + b\vec{w}$$

Note: Its common to think a collection of vectors as points, due to clustering/noise

2.3 Linearly dependent Vectors

- If the third vector can be formed by linear combination of the other vectors then the vectors are said to be linearly dependent
- If we cannot get a vector by linear combination of other vectors then then those vectors are called as linearly independent

2.4 Basis

- basis of a vector space is a set of linearly independent vectors that span the full space

3 Linear Transformation and matrices

Transformations are just "functions" just suggests to visualize every function as a movement of coordinates(changing of basis vectors, thus changing of all of coordinate system)

3.1 Linear transformation in graph

- every lines will remain parallel
- and origin will remain at center

3.2 Matrix Transformation

- A 2 dimensional linear transformation of a coordinates system can be entirely described just 4 numbers (a 2d matrix)
- In a matrix each of the column is the final landing basis vector
- thus, matrix $\implies$ transformation data
- $$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \underbrace{x \begin{bmatrix} a \\ c \end{bmatrix} + y \begin{bmatrix} b \\ d \end{bmatrix}}_{\text{intuition}} = \begin{bmatrix} ax + by \\ cx + dy \end{bmatrix}$$

  Note:
- by summary: Matrix $\implies$ set of transformed basis vectors
- thus giving transformation data of the space(coordinate system)

4 Matrix Multiplication as Composition

4.1 Composition matrix

- when a series of transformation occurs the "overall" transformation matrix is called as composition matrix

- this composition matrix is formed by matrix multiplication of each transformation matrix –from right to left

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$$\overset{\substack{f(g(x)) \\ \text{Read right to left}}}{\overleftarrow{\underbrace{\begin{bmatrix}1 & 1\\0 & 1\end{bmatrix}}_{\text{2nd transformation}} \quad \underbrace{\begin{bmatrix}0 & -1\\1 & 0\end{bmatrix}}_{\text{1st transformation}}}} = \begin{bmatrix}1 & -1\\1 & 0\end{bmatrix}$$

- thus we can prove the associative nature of matrix multiplication by just intuition alone

$$(AB)C = A(BC)$$

5 Three dimensional linear transformations

Consider a linear transformation with 3d vector as input and output

$$\underset{\text{Input}}{\begin{bmatrix} 2 \\ 6 \\ -1 \end{bmatrix}} \xrightarrow{L(\vec{v})} \underset{\text{Output}}{\begin{bmatrix} 3 \\ 2 \\ 0 \end{bmatrix}}$$

- This means with the given transformation function L. The vector point (2, 6, -1) is transformed to (3, 2, 0)
- In matrix form L must be a 3 by 3 matrix, since there are 3 basis vectors in 3 dimensions
- Ex:

$$\begin{bmatrix} 0 & 1 & 2 \\ 3 & 4 & 5 \\ 6 & 7 & 8 \end{bmatrix}$$

6 Determinant