

# Cosmographiko

*A three dimensional, spatial audio, algorithmic,  
generative and musical synthesis and composition.*

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## Control Levels

### Control0:

A simple Euler Identity circle x,y (real, imaginary), on the 3D plane with a spin and synchronous rotation to control1, having as rotation axis (quaternion) the sum of the three rotation axis vectors of control1.

### Control1:

*Silver star* (Astrum Argentum) a 6D moving and modulated array of Spherical coordinate in the 3D form of  $y=1/x$  graph across the 3D plane with independent axis of rotation (quaternion) for each x, y, z axis.

### Control2:

Control array, an array of Spherical coordinated in golden angle distribution modulated by spherical harmonics, according to their order, and mapped to control1 coordinates via *neighboring point logic* and its proximity range.

Neighboring point logic:

```
////////localization areas on the sphere array
(
{~points = ~az.collect({ |az, i|Spherical(1, az, ~el[i]);}).value;};

(
{var n=1.neg,neighbors;
~neighbors=[];
(360).do{n=n+1;
~center = Spherical(1, ~az.[n], ~el.[n] );

~threshold = 0.5;//////////proximity range

neighbors = ~points.select { |pt|
Quaternion(0,pt.x,pt.y,pt.z).distance(Quaternion(0,~center.x,~center.y,~center.z)) < ~threshold};

~neighbors=~neighbors.add(neighbors);
}.value;};

~thetaArray = ~neighbors.collect(_.theta);
~phiArray = ~neighbors.collect(_.phi);
```

### Control3:

Material arrays, again mapped as before, to the control array via neighboring point logic, they are arrays of Spherical point coordinates with magnitude iterations in order to have depth, and various wrappings in their forms, in order to represent cosmographically astral objects like atomic clouds, stars (first suns) and supermassive black holes, galaxies, suns and black holes, planets. Each material array forms a different viewpoint on the cosmos during the same timeline. This means that the values these arrays create, which are the final values passed on several arguments of the synths, are passed via an order of cosmographic viewership which happens according to the following principle, that of location.

Through the layers of mappings from the cycle, to the silver star, through the control array to the material arrays, the location chosen corresponds to a specific point on the cosmic material cloud initially, as this cloud is morphed to create first stars, a location can be a star, which in turn has its own spherical locations, as such stars collapse and form galaxies, a location can correspond to a galaxy which in turn has its own spherical locations being suns or simply cloud matter. Following the suns have their own spherical locations corresponding to their structure and possibly the sun locations can be planets, with in turn their own structure, as each suns potentially corresponds to a solar system.

In order to structure this viewership and state of auditory information, Routines come in play. Control0 via and Control1 set an initial location proximity which is then used by specified Routine for each astral object. A master Routine and Task specify the astral object used and viewed throughout the timeline of the cosmographic and locational level of the composition. The musical synthesis of the composition regarding concepts like rhythm, melody, chords timing and tempo are partly independent from locational data, something that will further described and explained at a later part of this document.

Returning to the master Routine and Task, these create a composition framework in the choice of astral object data used, according to the state of the universe cosmography temporally.

This means that for example at an initial state, the only possible material array corresponding to the location data from previous control levels is that of the cloud matter. Once first suns appear, there is a probability that a chosen location is a star. At that moment a star Routine is initiated where location data are mapped to the stars coordinate structure and consequent synth argument values.

This part of the timeline can be compositional and have a specific period, after which, and according to the universe timeline, new possible routines and their respective material arrays emerge, like the one of a galaxy, or subroutines to these corresponding to further material arrays like those of suns and planets.

All these data, routines and task follow a cosmographic timeline cycle which defines the lifecycle of the universe with corresponding modulation values of every type of array argument.

A simplified version of the above described cosmographic timeline composition would be to place the viewer at only the direct perception of one level of macrocosm/microcosm the different material arrays point. This especially helpful for diffusion and panning, as the VBAP location could follow only eg. galaxy location data, regardless of the oscillator arguments, thus placing the viewer at a constant space and not making things too complex to perceive, by zooming in and out to different astral objects.

To make thing clearer on the above layering and time based triggering of different routines and subroutines in a task, one can visualize each material array being a different tonal argument on the oscillator structure or a morphing of its tonal argumentation. For example when a location is chosen through the control layers, the tonal arguments this coordinate values create corresponding to each material array, are different. For example, if the location, throughout the layers of the material array routines, corresponds to a galaxy, and a sun on this galaxy, then the tonal arguments on the oscillator include both those created by the galaxy as a location, the sun as a location on the galaxy and a point on the sun structure as a location on the sun, so three levels and types of argument control. These levels are therefore present according to the presence of material arrays at a specified location through the timeline. Furthermore, each material level array can correspond to a different range of tonal musical temperament (confluence) in order 12,56,992... and fundamental multiplier.

As stated before, for the sake of simplicity this time based inter-wiring of arguments concerns only the tonal arguments of the synth oscillator and not the panning of the synth. The pan locations (VBAP) remain in that sense one dimensional and at one level of material array viewership., for example these spatial sound locations can correspond only to chosen galaxies/stars, or have a more abstract nature and be simply locations of the control array, thus not concerning cosmographic materiality, and being purely abstract and mathematical in its geometric aesthetic.

## Array structures

- Silver star (as in `astrum argentum.scd` file has 6D modulation whereas each of the axis of the 3D exponential graph star has an independent rotation axis (rotation via quaternion), These rotation axis are wrapped in the range of azimuth: 0-90 angle degrees and elevation 0-90 angle degrees.
- Control array as in `choice array.scd` file is a sphere with coordinate point distribution driven by golden angle azimuthal and elevation intervals, has depth through magnitude iteration of some interval  $>1$ , eg. square root of two or any interval of choice times a sawtooth sequence  $L$ , that is  $L$  is an iteration sequence  $L=L+1$ , so  $magnitude=interval*L$ . The control array is modulated via spherical harmonics at certain LFO speeds, spread of harmonic (elliptic or spherical bubble) and at certain ambisonic order according to the composition cosmographic timeline (big bang to heat death period).
- Material array as in `material arrays` files, has as its structure basis the same coordinate point creation as the control array, only this time, via the silver ratio and the total spherical object is to represent a certain astral object or situation, or collection of astral objects eg. a galaxy. This means the spherical object is shaped accordingly to form the respective object and its modulation is spin (VBAP panning of azimuth and elevation) and a Bessel function as a form of rippling

Bessel:

```
~magxx = ~magx.collect { |val, i|  
  var waveSpeed = 1;  
  var waveFreq = 6;  
  var amp=0.4;  
  var theta = ~az[i].degrad;  
  var phi = ~el[i].degrad;  
  // var wave = sin(waveFreq * theta + waveFreq * phi - waveSpeed * t);  
  var wave = amp*sin(waveFreq * theta + waveFreq * phi - waveSpeed * t);  
  val * wave.postln;  
};
```

## **Geometric interpolation of arrays**

A gradual mapping (according to cosmographic timeline) via neighbor point logic and its proximity limit as argument. Each of the arrays, *controls*, gradually, shift from or to, their spherical point distributions towards a specified platonic solid or polytope, with maybe the addition of in between symmetrical distributions like geodesies and Goldberg polyhedra.

The *control0* cycle shifts towards specified polygons like a single line, a triangle, a square etc., if we consider its initial state before a shift being a total cyclic and tight distribution like via the golden angle, or simply many gradient cyclic points via a +1 degree angle azimuthal shift or even smaller degree. The control array shifts towards a tetrahedron, with its respective rotation, spin and spherical harmonic modulations.

Regarding the material arrays, each astral object type follows a different such geometric interpolation, and symbolized as such via the respective polygon. The monadic general spatial could matter shifts towards a cube or from an octahedron. If we consider compound polytope geometries a cube is a hull and a octahedron a core, thus the cube can be a final state of universal heat-death, following a spatiotemporal collapse and end of one universal cycle, following a new big bang, geometrically interpolated and symbolized via the core nature of the octahedron (z axis is the birth line of big bang and the azimuthal square x,y of the octahedron being its sudden burst of expansion). In a similar fashion of symbolism, at the state of heat-death and hull cube interpolation state.

A first sun (star) and consequent galaxies (supermassive black hole) are interpolated to a rhombic dodecahedron (hull) and a cuboctahedron (core) respectively. Suns, black holes and planets (a state between a sun and a black hole) are interpolated as polytope compounds via the rhombic triacontahedron (hull) and the icosidodecahedron (core), respectively.

## Time

**Silver star** remains in the synthesis a constant modulation source and seed array for music related data like rhythm, chords and melody sequences.

The **control array's** modulations and arguments change throughout the timeline of the cosmographic composition period. Arguments like the spherical harmonic oscillation speed, the spherical harmonic order as well as the overall geometric interpolation towards a tetrahedral distribution are such realms of change.

The **material arrays** change through the timeline also through their respective geometric interpolations, but also in their lifecycle as astral object through initiation from respective routines to their different states, for example a sun that collapses to a black hole. Other forms of change include the ripple Bessel effect oscillations they process, as well as in detail their change of form while in a particular routing state, like in the case of a star growing in size, a galaxy growing, or a planet being born. All these latter forms of change happen on the same material array type and are not termed as changes of one material array to the other. To be concrete a star is a different array than a galaxy and also a different array from a sun or a cosmic matter cloud. These different arrays are interchanged via routines and sub routines that set a different array as data source, not via array specific modulators.

## Tonality and space

*Silver Star* is an abstract object and particle system array which lies at the bottom levels of the control layers, before the control array object, and is responsible, apart from setting the initial azimuth and elevation values for the spatial argument chain of the composition, mainly responsibly for the factors that render the algorithmic synthesis to a musical composition.

Silver star is essentially a  $y=1/x$  starry form of the 3D axial system, where each axis can rotate independently on the the 3D plane via quaternion rotation and a rotation axis, whereas named a 6D system six axis come into play (in complex terms three real and three imaginary, or more philosophically three spatial and three temporal), three coordinate axis and three rotation axis. This objects forms starry constellation and symbolizes free will as an underlying factor of the universe.

As a choice of coordinate on the silver star is initiated the coordinate is mapped to the control array which in turns generates the musical values. This is achieved through three types of arrays whose values are derived from the silver stars particle coordinate choice. Each point has a 3D coordinate of  $x,y,z$  as a Spherical object.

When the magnitude of this Spherical objects, through iteration of the whole control array, is set to be a power of 2, then a sequence of  $[x_0,y_0,z_0,x_1,y_1,z_1\dots]$  derived from the silver star coordinate choice sets a rhythmic pattern of three beats per bar. The amount of bars per rhythmic cycle is determined by the control array and its geometric interpolation. The magnitude when for example 1 then the rhythmic time signature is  $\frac{3}{4}$ , when four then the time signature is  $\frac{3}{1}$ . For the sake of simplicity and quick adaptation to different rhythmic musical patterns one can set the creation of the rhythmic pattern directly from a pure tetrahedron array, or any other solid, without the need of interpolating the control array to such geometries, and keeping things separate.

Form of modulation would be in this case, as similarly in the case of the silver star, rotation and spin of the seed polyhedron for the rhythmic pattern array. For example one cycle could be one position of the tetrahedron with four bars, then for the next bar a different position of the tetrahedron on the 3D plane can be used, creating thus varying rhythmic according to specified patterns derived from the polyhedron rotation. Of course, the seed polyhedron and its array can remain static throughout the composition, entailing thus a stable rhythm where only form of modulation would be the overall tempo via the SC TempoClock. Essentially the silver star works as a frame of choice, which then maps to the control array, from which the musical data arrays are derived, modulated further by the control arrays modulators.

Another musical realm the silver star, through the control array, is responsible for is the creation of melodies. Again in the same manner as the above described rhythmic pattern, a tone array is created, from the  $x,y,z$  sequences that each particle coordinate create, only this time the applied Spherical object that exports these  $x,y,z$  data based on the silver star point coordinates, must have a tonality related magnitude, this would be an octave of range of musical temperament to the formula  $2.\text{pow}(x / \text{order of musical temperament})$ , with possible numbers of 12, for classical 12 tonic system or larger numbers for bigger amounts of microtonality, like 56 from number 28, 992 from the number 496 etc according to perfect number sequence. The magnitude exports  $x,y,z$ , according to the musical temperament range and then the final value for the tonal array sequence is derived through the tonal musical temperament formula  $2.\text{pow}(n/12)$  for  $x,y$  and  $z$  respectively. As described in a previous chapter, the choice of musical temperament can depend on the material array present, or simply be a constant throughout the timeline and any involved material array objects. Again, as before in the case of rhythmic patterns, periodic modulation of the silver star through rotation and spin, can create periodic change of the melody pattern and geometric interpolation to a tetrahedral distribution, helps create clear 4 bar  $\frac{3}{4}$  melodic pattern periods.

Final musical parameter is that of chords. The above described sequences of coordinated on each bar and period of modulation can also form a chord of three tones and not a sequence of three chords. This parameter can works through the logic of assigning the tonal values to three parallel oscillators.

One can make different setups with this concept, either playing at one bar duration a three note sequence, and on another bar a chord or both a chord and a three note sequence via two separate synths, or having the chord of each bar being one tonal argument and the tree note

sequence of the bar a second, temporal argument of tone on the same three oscillators of the chord.

For the sake of CPU usage economy and being able to have faster tempos and longer decays without the use of more than one server to run the code of the composition, something cumbersome both for production and recording, in essence any other application other than real time playback, where again separating the composition in various serves is a tedious task on its own, the way of using one server is chosen. For this reason, having three oscillators for every note played is too heavy for the CPU, especially when the later described addition of additive synthesis via the spherical points that form the depth of the spherical object at an azimuth and elevation coordinate, are added to the synth oscillator bank.

Therefore for the sake of this composition it will follow to use two separate synths. The first one with one basic oscillator forming the three note tonal sequences and a second synth with three oscillators, forming only chords from the specified coordinates, not at every but at some periodic or semi-periodic interval or some kind of boolean sequence of presence or non presence like an L-System (Figure1). Note that the duration of the chord as a note played follows a sum of the correspondent coordinate rhythmic x,y,z sequence, lasting thus a whole bar, at this duration argument a variable can be added which randomly increases or decrease a bit this duration, thus creating extra variance and lack of or more reverberance on the chord progressions.

About the additive synthesis. As described before the control array gains depth as spherical object by iteration of its fundamental magnitude in a saw-tooth manner towards a maximum via an  $n=L*\text{interval}$ ;  $L=L+1$ ; logic whereas interval is an interval of choice with value higher than 1, eg.  $3/2$ , or the square root of two, each can give different timbre.

Furthermore, all these above mentioned tonal and musical properties derived from coordinates are described on the control level of the silver star and the control array, for the sake of simplicity in description. In fact the coordinates are mapped through all layers of control, from the silver star, through the control array, to each respective material arrays where, through this mapping via neighbor point proximity logic the final coordinate data are derived. These data are the final values used in the synth for the fore-mentioned algorithmic music formulas and sequences. In this manner also respective to the material array type and form in hand according to location and universe timeline, ranges to the additive synthesis iterations are set, musical temperament ranges and magnitude of coordinates which is to be another yet tonal argument derived from a coordinate.

To sum up, coordinates chosen on the silver star level and mapped up to the material array, derive a rhythmic pattern array, a tone/melody pattern array, a respective chord for each coordinate, a magnitude of coordinate tonal scalar and fundamental for the depth related additive sawtooth synthesis (overtone), and finally a VBAP coordinate for panning the sound source of the tone played, which as described before, for the sake of simplicity is kept on an abstract level on one level of control, the silver star. Spectatorship on the auditory level is easier that way as it is not really possible to derive a specific material array from its VBAP information, therefore the journey of mappings through the control levels concerns all arguments other than VBAP coordinates.

Concluding, final note on the oscillator arguments other than frequency (all these tonal variables happen on a specified fundamental frequency) are the amplitude and the phase. The amplitude is defined by the product of reciprocal tonal arguments (in musical temperament value format) and phase is defined from the coordinate data by adding x,y and z raw values when mapped linearly from 0..1 to 0..2pi. Other modulators include those on the panned VBAP location which essentially are no extra LFOs but derive from the modulation of the silver star. Note here that although reading a seed coordinate on the silver star is a musical contract and follows a systematic period, the modulation of the silver star is independent, therefore, while each oscillator related value change happens at specific intervals, the VBAP location is a continuous panning modulation with no intervals and bars.



Lindenmayer's original L-system for modelling the growth of algae.

**variables** : A B

**constants** : none

**axiom** : A

**rules** :  $(A \rightarrow AB), (B \rightarrow A)$

which produces:

$n = 0$  : A

$n = 1$  : AB

$n = 2$  : ABA

$n = 3$  : ABAAB

$n = 4$  : ABAABABA

$n = 5$  : ABAABABAABAAB

$n = 6$  : ABAABABAABAABABAABABA

$n = 7$  : ABAABABAABAABABAABAABABAABAABAAB

*Figure 1*

## Choice

As fore-mentioned an index of the silver star array is chosen in sequence corresponding to the specified coordinate which in turns moves all the argument gears of the composition. A different level of, more or less, modulator gears 'state of being' gears and other arguments, not so much related to music, but to the cosmographic timeline are defined by the timeline itself and its corresponding overall envelope task and routines. Such arguments, as mentioned before, include the state of the material arrays form, their modulation characteristics, spherical harmonic oscillations orders and spreads, Bessel ripples and other automata, that are more or less automata and do not depend on some choice or interface and are defined purely through the chapters of progression in the universal timeline and randomness, eg. on each different cosmic period these automata may give different values at different phases. Their nature stands before the eyes of the beholder but they are not beholden in any other way from the viewer or composer other than algorithmic oscillation and noise.

Back to the silver star, reading of the index on the array can be linear from bottom to top and some preference in this does not affect the concept of choice in a composing manner in any way, as the actual variance is created by the silver star being an automaton and modulated object, not by how its array of values are read. The same path of index reading can return different values for the chosen coordinates at different phases of modulation, therefore musical variance melodies and rhythms that have difference and repetition are achieved without the need of further specifying the array reading index order or scrambling it. Possible only inversion of the array order is a possible reading alternative, if a Mobius strip loop is desirable for the composition musical sequences.

Therefore to escape the clutches of this automaton, where essentially the only means of synthesis control is the dialing of the oscillation knobs of the silver star's modulators, a further interface to the silver star is needed, and thus a final and bottom line level of control.

Before the introduction of this level a view words and abstractions are needed on the silver star nature, what is behavior is and what it means to choose a point from its cluster. As all other arrays in higher control levels have a geometric interpolation towards or away from a polyhedron and polytope, the silver star does not have a geometric interpolation towards or away from anything other than itself. This means that the interpolation is essentially how zoomed in or zoomed out from our perception its object form lies. Like breaking the fourth wall in a sense. The silver star object and consequent coordinates array are therefore either very zoomed out, being formally very close to having no coordinates like the 3D axis themselves, near the axis, and a very thin halo of points on the cycle of rotation of each axis. On the other hand, very zoomed in means that we see the center of the star limit up close, the 3D axis are there but the underlying sphere is visible and the axis rotation halos are kinda lost from visibility through the noise especially if the interdependent 3D axes are modulated as oscillating really fast or chaotically in relation to each other.

In any case, no matter the realm of perception of the silver star, and consequent array values in relation to the origin, the choice interface is always there. This is to be described as simply a cycle, a one dimensional ring of sorts that floats in 3D space with the same center of origin as the silver star, and has spin around any possible rotation axis vector on the 3D plane coming from the origin outwards, unlike the silver star where all three rotation axis for x axis, y axis and z axis respectively, are wrapped in the quarter area of the hemisphere of azimuth 0..90 degrees and elevation 0..90 degrees.

This interface cycle therefore can rotate around any axis and spin around any axis around the origin. For example in visual terms if this cycle was spinning really fast and its rotation axis was spinning also really fast, we would not see a cycle floating any more, but a sphere, because our eyes would not be able to distinguish the frames.

Choosing a point on the silver star can be described via a 2D analog as choosing a point on the  $y=1/x$  graph, or if lamdoma (Figure 4) intervals are used, the graph  $y=(x+a)/x$  (Figures 2 and 3). In that choice the further the coordinate is from an equilibrium  $y=x$ , the more  $x$  increases its overtone value and  $y$  its subharmonic value, and in analogy, according to trigonometry, complex number theory and the Euler Identity  $\exp(\text{Complex}(0,1)*\text{angle})$  (Figure 5), the more the  $y$  overtone value is increase, the more the  $x$  subharmonic value is increased. Similarly this happens on the

silver star, according also to its specific point distribution which is increments of one degree for elevation and increments of the silver ratio for its azimuthal coordinates, these happening for each axis and +/- orientation, creating thus a silver star.

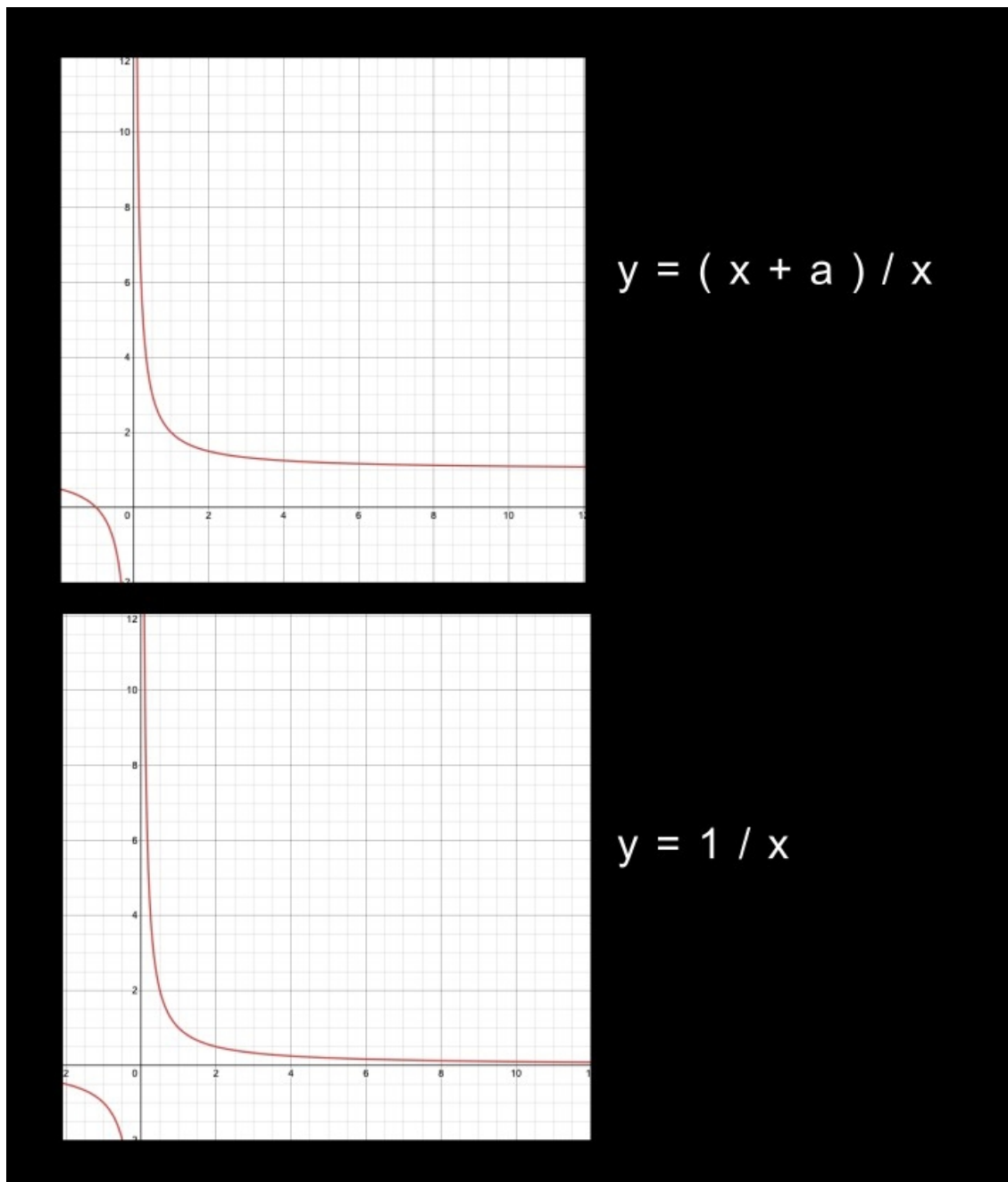
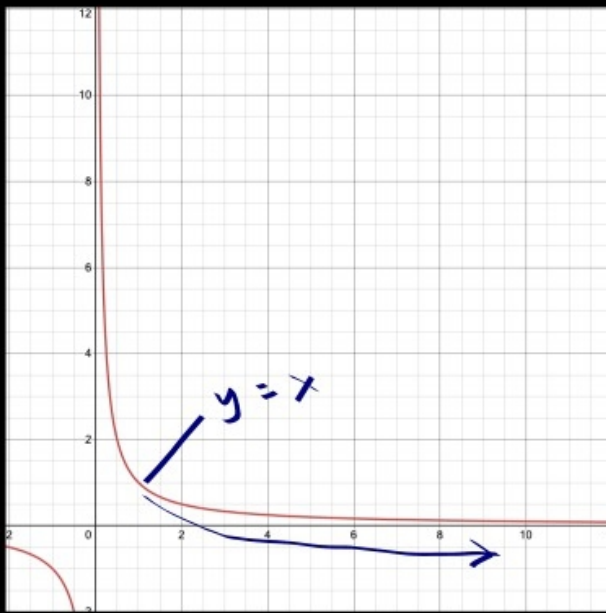
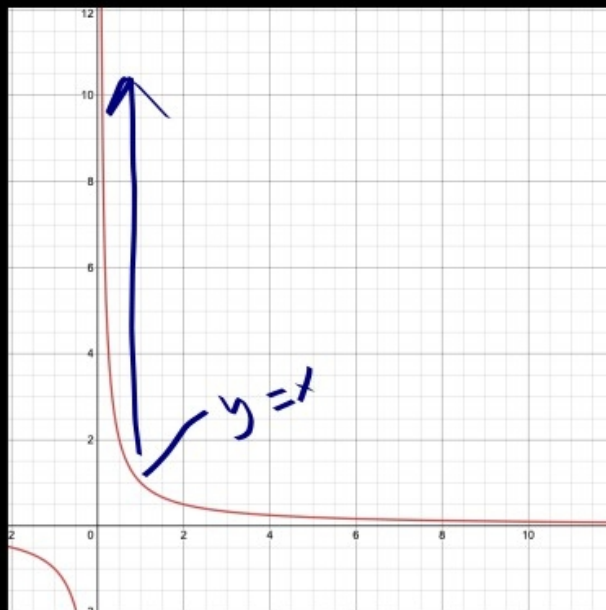


Figure 2



$x = 1, \dots$



$x = [0] \dots 1$

Figure 3

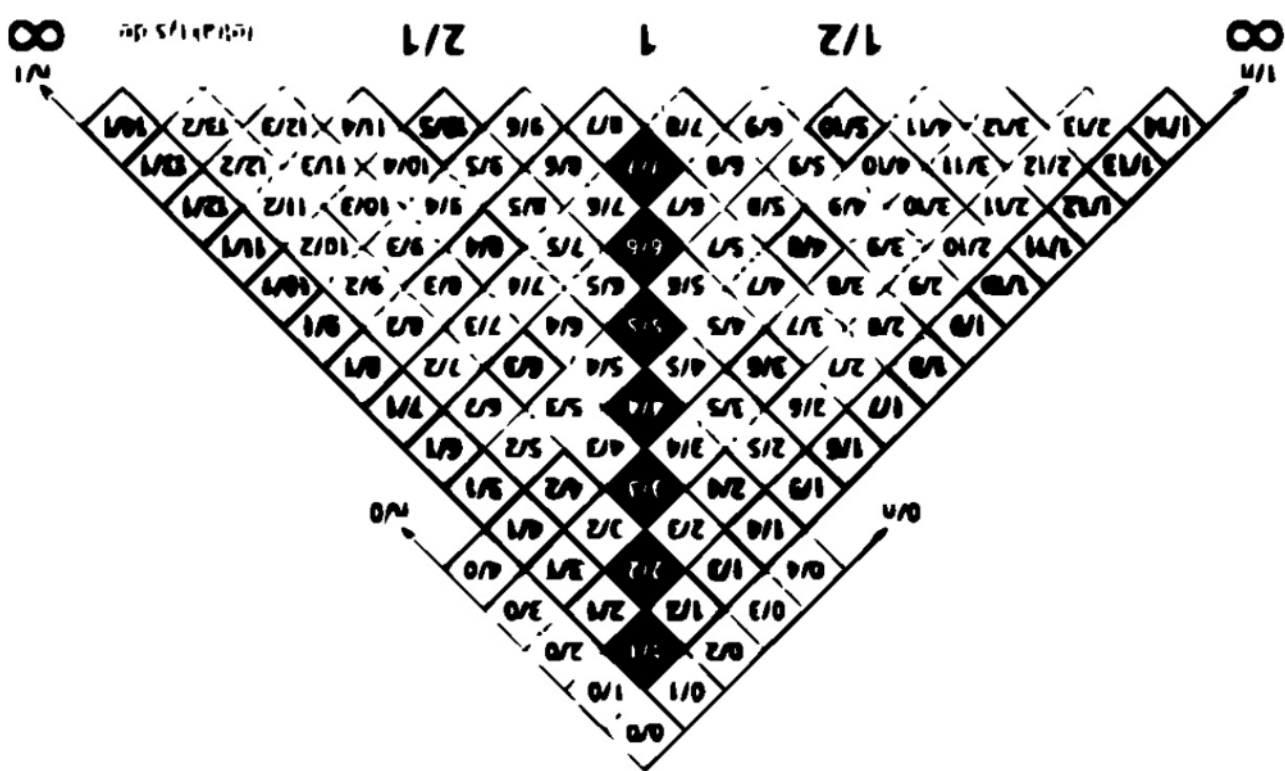


Figure 4

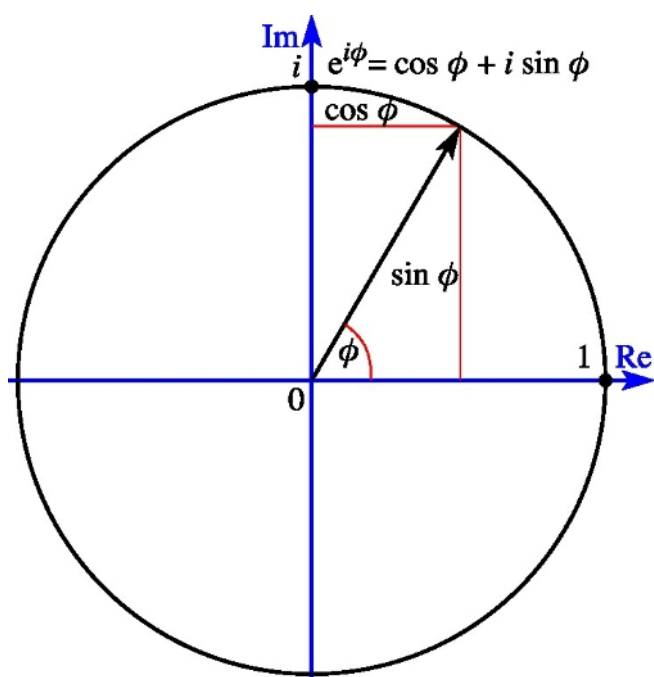


Figure 5

Moving back to the free floating cycle on the 3D plain, interface to the choice of point on the silver star. Having described the nature of the cycle as an object abstractly, in detail it is an azimuthal iteration of angular increments with constant elevation eg. elevation=0. Via quaternion product on any rotation axis this azimuthal ring can be placed on any orientation and spin around any axis, spin would be a modulation of the object in that case as in any other similar case of spherical object modulation of its array. The angular increments for creating the cycle can be a simple 360 times iteration with 1 degree increments, or infinite limit points via golden angle increments (the one ring to rule them all), or simply a polygon, like a line, a triangle, a square etc., Or every further, including temporality and the cosmographic timeline in mind the cycle structure can also take part in this generative composition and be subject to change in its form, like from some distribution to another such as geometric interpolation of a golden angle ring, to a polygon.

The logic of choice, of this cycle as an interface to the silver star is simple. Similarly as in previous control arrays, the cycle object array index is read at a specific rate, some tempo, and the modulation automaton responsible for the spin and rotation of the cycle does the rest by choosing a location on the silver star, again via neighbor point and proximity logic.

Essentially this means that the whole composition, algorithm, synthesis and generative artwork, contains randomness only where needed and aesthetically reasonable and by choice, and any controls are reduced to one interface of a 2D cycle on the 3D plane, a ring.

Its controls are the tempo of reading through its array index values, which feeds the values for all other control levels and finally the synth arguments, and the speed of spin around a chosen axis, a modulation which created variance in the array values read in ordinance to their mapping to the silver star. So, tempo and two modulators, speed of spin, and oscillation (panning) of rotation axis. For the sake of simplicity this rotation axis of the cycle can be sync to the modulation of the silver star, in particular the oscillation of its three rotation axis, by being the sum of these rotation axis as vectors. This sync would also wrap the probable locations of the cycle rotation axis to the same quarter hemisphere area as the rotation axis of the silver star, but there is no reason why not to further develop its range keeping the sync intact. Meaning, the rotation axis of the cycle interface control level can be the sum of the silver star rotation vectors, but itself as a vector itself, can mirror its location to any other quarter hemisphere, becoming thus ever present on the 3D plane as initially defined while keeping a sync to the silver star as modulated objects.

Concluding, master controls of the whole composition through the cycle interface (Figure6) are its array index reading routines, its spin speed and where on the 3D plane its rotation axis is mirrored, with initial state the sum of rotation vectors of the silver star.

Finally to harmonize the cycle as an interface, according to the lamdoma matrix, a mathematical state and most basic symbol and matrix of intervals and harmony, across arts and sciences, the following calibrations must be made:

the cycle object having a linear azimuthal increment in its creation defined as  $az$  and its magnitude defined as  $mag$ , then when magnitude  $a = 0$  then only azimuth and elevation values can be passed to the silver star and therefore the magnitude information of the cycle which would complete the definition of spherical coordinate mapping to the silver star must be defined through a random number, as 0 is no point at all, or because of the latter reason, as a lack of mapping, a gap in the sequence, silence and pause in musical terms. If  $mag$  is any other number, a lamdoma interval number following a formula of  $mag = (n+a)/n$ , then the lamdoma matrix and interval logic is incorporated to the cycle interface as a variable. Thus the control cycle has a speed of speed, an interval of magnitude and an orientation on the 3D plane, or three controls defining a point on the silver star under a tempo, and setting the whole thing described so far in a generative motion, for as many cosmic cycles as possible.

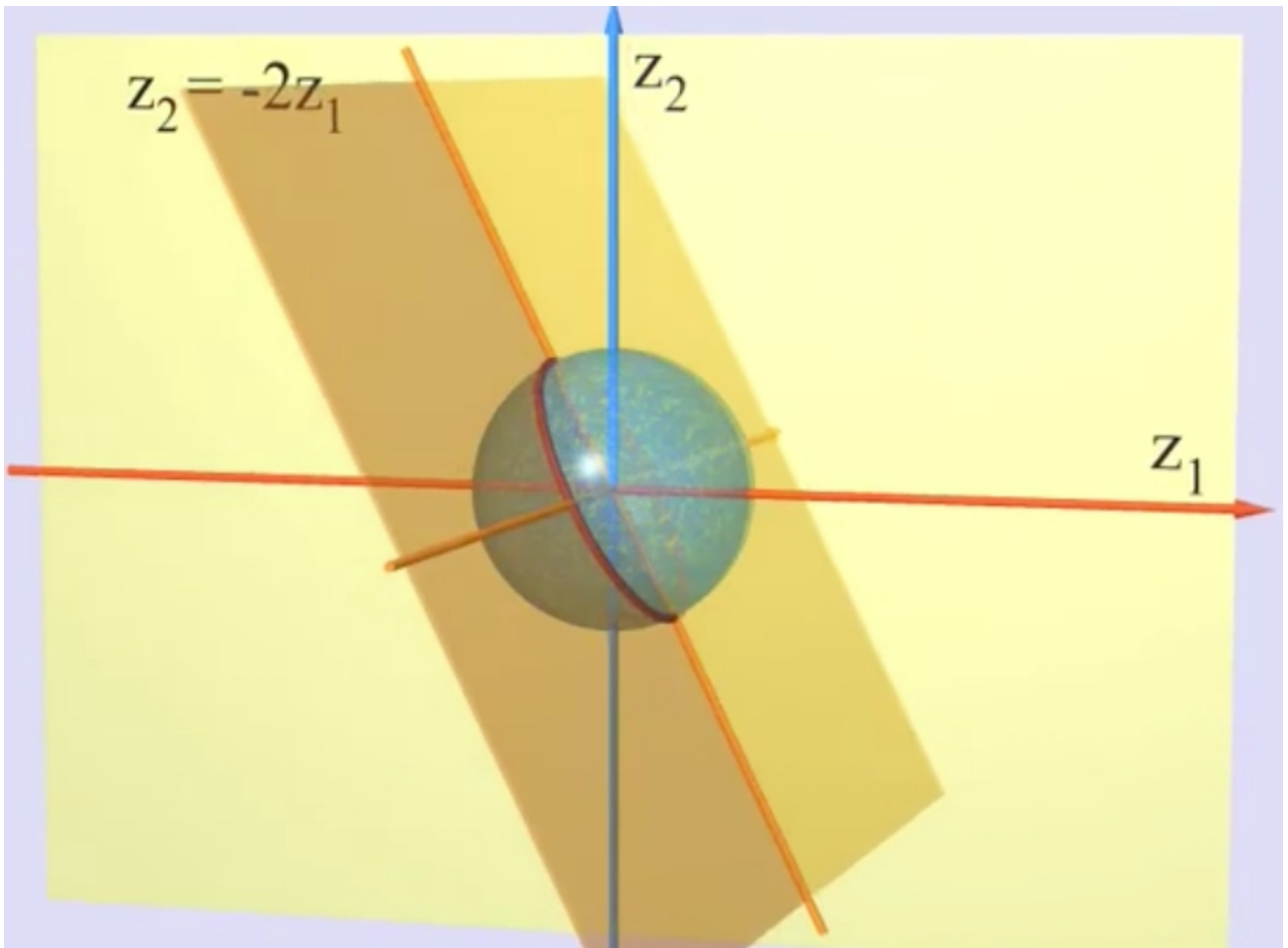
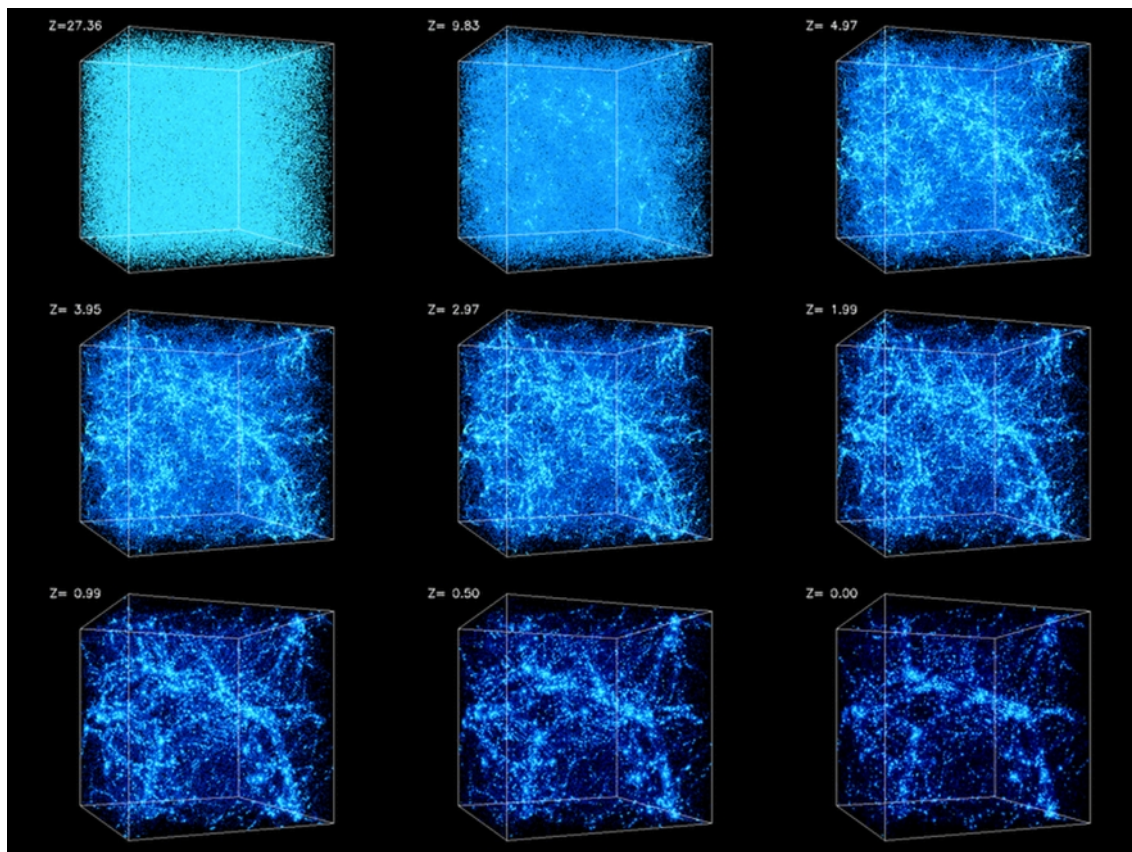
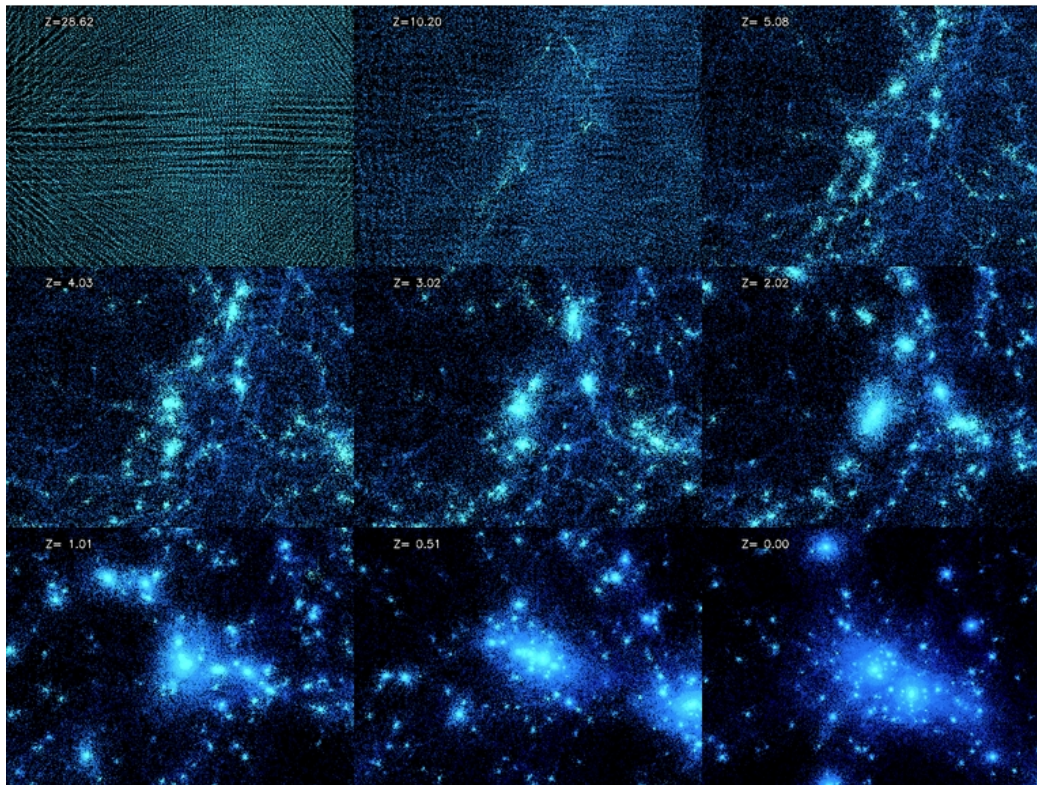


Figure 6

*For reference here are some scientific visuals regarding cosmography and astral structure. I am sure you can find plenty and more accurate ones or closer to your interest, at public libraries, museums, universities, labs, exhibitions and astronomy forums.*





The movie stills pictured above illustrate the formation of clusters and large-scale filaments in the Cold Dark Matter model with dark energy. The frames show the evolution of structures in a 43 million parsecs (or 140 million light years) box from redshift of 30 to the present epoch (upper left  $z=30$  to lower right  $z=0$ ). The simulation was performed at the National Center for Supercomputer Applications by Andrey Kravtsov (the University of Chicago) and Anatoly Klypin (New Mexico State University).