

Italian Healthcare S.p.A.
Executive Stock Option evaluation

Marco Contadini 919908

Guglielmo Del Sarto 919842

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Marco Contadini
G.D.S.

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Preliminary hypothesis and remarks.

Throughout our report we work under the following hypothesis:

1. There is no taxation on investments. Therefore, the investors' IRR will not be influenced by the taxation.

We place this hypothesis because we do not know which jurisdiction our investors are subject to (across countries the tax rate is highly different). After doing the evaluation in a tax-free world, we compute in section 3 the price of the ESO option in case the investors are subject to Italian fiscal regime.

2. We do not take into consideration the exit year $T = 1$. Indeed, as it has probability zero, ignoring it will not alter our valuation.

3. The rates, unless otherwise stated, are continuous rates. Time periods (often referred as t or T) are all expressed in year (i.e. $t = 1.5$ means one year and a half).

4. The company will not pay any dividend. From the given details, we are not able to discover Italian Healthcare's financials (and so we would have to really guess a dividend rate¹).

This assumption seems more than reasonable if we look to investors's intention to sell their positions: they are not really interested in the occurring dividends but in maximizing the equity value being prone to pushing investments (therefore high retention rate).

Moreover, using the usual convention, if a dividend is payed, we would observe a drop in the stock value of the same amount. But, being the investors the holders of the stock, the dividend will go anyway into their pockets leaving their final IRR almost unvaried².

5. All the computations are developed in Python using version 3.8.2 (latest available). Please find attached the code and its explanation in this [Google Drive folder](#). In order to run the code smoothly, it is necessary to have installed on the machine the following Python packages:

- numpy
- scipy
- pandas
- math

If you prefer, you can also run the code online using Jupyter Notebook. For further details, please find information in the pdf file "*Code_analysis_MC_GDS.pdf*" available in the linked folder.

¹The sector dividend rate may not be accurate if we don't know the liquidity status of the object company.

²Only the time difference of receiving dividends before the exit year. The impact would be nevertheless small.

1 Acquisition profile and macroeconomics variables.

Italian Healthcare S.p.A., leading firm in Italian biotechnology sector, will be acquired by a consortium of investors.

The acquisition will be delivered through the usage of a Special Purpose Vehicle, the newly born *HoldCo*. The consortium of investors willing to acquire the company is composed by three actual shareholders (namely, shareholder A, C and D) and a private investment fund.

1.1 Law profile of acquisition.

The investors have planned to acquire the target company through a *leverage buyout*. Indeed, there will be three main steps of the acquisition:

1. HoldCo will be endowed with the monetary resources needed in order to successfully make the acquisition transaction. There will be a mix of debt and equity. Precisely:
 - 8 500 000 € will be given as equity injection;
 - 13 600 000 € will be arisen as HoldCo debt.
2. HoldCo will buy 100% of Italian Healthcare S.p.A. equity (through the monetary resources it was given by its investors and financiers).
3. After twelve to fifteen months from the acquisition, HoldCo will merge with Italian Healthcare into one single entity.

Though the nominal value of the stock is 1 €, for our evaluation we use its market value. From the information available, we can state that the equity market value of HoldCo is 8.5 Mlns €.

The company has 1 Mln outstanding shares.

It should be stressed the fact that the investor are isolating the potential risk of a *leverage buyout* into the SPV. Indeed, the debt taken will be re-paid only using the value creation potential of IH S.p.A. (or, in case of default, by its assets).

The investor are going to be exposed only for their equity conferment.

1.2 Economic profile of the acquisition.

In the present section, we show key factors of the biomedical and healthcare sectors.

At the end, we present our thoughts and our company parameters estimation.

1.2.1 Sector outlook.

According to the consultancy Deloitte³ and Fortune business unit⁴ the health care sector is expected to grow at a solid 5% CAGR over the period 2019 - 2023, peaking at 612.7 billions dollars in 2020 (up from 425.5 today).

Deloitte's report shows that one of the major cause of death across the world is heart related (strokes and Ischemic heart disease on top). The demand for heart failure caring system is then expected to be vast.

As Italian Healthcare has an expertise on such devices, we can be optimistic about future revenue growth. Despite such growth potentials, the sector may present some difficulties.

First of all it must be pointed out that, as PwC⁵ report shows, there has been (and it will surely continue to be) an intense M&A activity. This leads to a substantial capital injection inside the sector boosting competition. As patent-discovery is vital to the success of an investment in biomedical devices, competition leads to uncertainty of investments project (less probability that IH will obtain the patent faster than its competitors).

³[Here](#) the report.

⁴[Here](#) the article.

⁵[Here](#) the report.

1.2.2 Company parameters.

We do not have at our disposal the balance-sheet of Italian Healthcare S.p.A. Making prediction about its possible growth rate and its equity volatility is not an easy exercise.

We have decided to benchmark those parameters with the biomedical sector average. We have then made some corrections using Italian listed biomedical companies data.

Sector average parameters have been downloaded from [Damodaran](#) database. Financials and stock prices of public listed companies have been downloaded from [Yahoo Finance](#).

We list the sector key figures:

- **Adjusted ROE:** 7.12%
- **Fundamental growth:** 4.15%
- **Standard deviation in firm value**⁶: 47.46%
- **Standard deviation in equity**⁶: 52.33%

We found four Italian listed biotechnology firms. Namely: Recordati, DiaSorin, Amplifon and Saes group. We downloaded historical stock prices of the past five years. We then calculated the daily stock volatility (as standard deviation of daily returns) for each company. We annualized them using usual convention:

$$\sigma_a = \sigma_d \cdot \sqrt{252}$$

where σ_a is the annual volatility and σ_d the daily one.

We averaged those value (simple mean). We got:

- **Comparable stock standard deviation:** 29.32%

The data seems to confirm our intuition that the biotechnology sector is characterized by a great volatility (especially if compared with historically stable sectors such as food & beverage or commercials).

As we know from theory, the volatility is a key driven of an option price. Greater the value of the option.

As volatility benchmark for the evolution of stock value of Italian Healthcare, we are going to use:

$$\text{Benchmark Vol} = 0.7 \cdot 47.46\% + 0.3 \cdot 29.32\% = 42.02\% = 0.4202$$

This value is obtained through a weighted average between:

1. Standard deviation in firm value \rightarrow weight 0.7 (this is indeed the best fitting driven factor of equity growth dispersion for IH's equity.)
2. Comparable stock volatility \rightarrow weight 0.3 (we add this correction factor in order to adapt European data to an Italian company.)

1.3 Discounting factor: risk-free.

We price the executive stock option with the usual risk-neutral valuation approach. We need to define the risk free rates. We use such rates both to discount financial sums across periods and to shape the drift of our stock (see sec. 2.1.2 for detail).

Following usual practice, being Italian Healthcare revenue mostly made in the Italian market, we define an Italian Government zero rate as risk free rate.

We downloaded the data from [World Government bonds](#)⁷. Precisely:

Maturity (year)	Zero rate
2	0.587%
3	0.717%
4	1.010%
5	1.201%

⁶Explicitly calculated by Damodaran for option pricing models. European data.

⁷Values in the table are an average of bootstrapped zero rate over the period 28th April 2020 and 8th May 2020.

2 Executive Stock Option.

As it is known from general theory⁸, ESO are call options whose underlying are usually newly issued shares. In our case, instead, the underlying are already-existing shares. Precisely, it is the investors who grant the management the right to buy their shares under the following conditions:

1. the “investors” will realize a minimum IRR of at least 20% when selling their shares (on “exit”);
2. the “bad leaver” clause has not been triggered by the management;
3. the underlying value will be greater than the strike price at expiry date (convenience set).

If all these conditions are satisfied, the option is activated by the holder.⁹

2.1 Building risk-neutral pricing model.

As stated, we value the ESO using the risk neutral approach. By definition we have:

$$\pi_0(ESO) = \mathbb{E}_q[\pi_T(ESO) \cdot DF_{0,T}]$$

where we define $\pi_t(ESO)$ as the value of the option at time t and $DF_{0,t}$ the risk free discounting factor for the period $[0, t]$. q is the risk neutral measure.

From the exercise we have the information that the ESO can be exercised only at the moment of exit of the investors. This occurs only at predetermined stages, namely: $T \in \{2, 3, 4, 5\}$.

It must be noted that we are not able to work directly on previous expectation because we do not know which will be the exact exit time.

Using the tower property of the expectation, we can write:

$$\pi_0(ESO) = \mathbb{E}_q[\pi_T(ESO) \cdot DF_{0,T}] = \mathbb{E}_p[\mathbb{E}_q[\pi_T(ESO) \cdot DF_{0,T} \mid T]] \quad (1)$$

where we define p the measure given in the exercise when presenting exit probabilities.

We can now come up with a value for:

$$\mathbb{E}_q[\pi_T(ESO) \cdot DF_{0,T} \mid T]$$

Indeed, given a particular time T we can use Monte Carlo simulation to recover a possible value of S_T and then of $\pi_T(ESO)$.

Since exit times set is discrete, we can then write:

$$\pi_0(ESO) = \sum_{k=2}^5 \mathbb{E}_q[\pi_k(ESO) \cdot DF_{0,k} \mid k] \cdot \mathbb{P}(T = k) \quad (2)$$

From now on, we concentrate in finding $\pi_T(ESO)$ and $\mathbb{E}_q[\pi_T(ESO) \mid T]$.

2.1.1 Pay-off.

Executive stock options are call options. This means that the pay-off of the option at maturity is given by:

$$\pi_T(ESO) = (U_T - K)^+ = (S_T \cdot N_s^{IRR} - K^u \cdot N_s^{IRR})^+ = [(S_T - K^u) \cdot N_s^{IRR}]^+$$

where N_s^{IRR} is the number of share in a particular state of the world (given by the value of IRR) and K^u the unit strike price.

As we can see, in our case the strike price is scenario-dependent.

Indeed, from the report we know that the management is entitled to buy a number of shares which depends on the Internal Rate of Return (IRR) achieved by the investors when selling their position. Therefore, according to the number of shares underlying the option, the strike price becomes:

$$K = K^u \cdot N_s^{IRR}$$

We have now to define IRR , N_s^{IRR} , K^u and S_T .

⁸See for example: John C. Hull, *Option futures and other derivatives* (Pearson Education, 2017), chapter 12.

⁹Note that condition 1. and 3. overlap for an at the money option. Precisely, $1. \Rightarrow 3.$

- We calculate the annual IRR achieved by investors starting from the following consideration:

$$S_T = S_0 \cdot (1 + IRR)^T$$

so that:

$$IRR = \left(\frac{S_T}{S_0} \right)^{\frac{1}{T}} - 1$$

This because the investors had a negative cash flow of S_0 at the moment of the company creation and they are left with S_T . Their annual rate of return is indeed the one presented in the above formulas.

- N_s^{IRR} is recovered using the linearity assumption. In particular we have that, for $IRR \in (a, b)$:

$$N_s^{IRR} = N_s^a + \left[(N_s^b - N_s^a) \cdot \frac{IRR - a}{b - a} \right] \quad (3)$$

Note that $N_s^{<20\%} = 0$ and $N_s^{\geq 40\%} = 125000$.

- The ESO is now at the money (ATM). This information is vital in order to correctly derivate option's strike price.

Indeed, from ATM definition, we have:

$$K = U_0$$

where K is the strike price and U_0 is the current underlying value.

We start from the consideration that:

$$K = N_s^{IRR} \cdot K^u \quad (4)$$

and

$$U_0 = N_s^{IRR} \cdot S_0 \quad (5)$$

In this way, setting the equality $K = U_0$ allows us to simplify for the random variable N_s^{IRR} .

Knowing that $S_0 = 8.5 \text{ €}$ we obtain:

$$K^u = 8.5 \text{ €}$$

For the sake of completeness, we have calculated the share price as:

$$S_0 = \frac{EM_0}{N_{shares}}$$

where EM_0 is the current equity market value and N_{shares} is the number of outgoing shares.

We leave the derivation of S_T in the following section.

2.1.2 Stock price.

Being consistent with the risk neutral approach¹⁰ we define the dynamic of the stock price as:

$$dS_t = r_f \cdot S_t \cdot dt + \sigma \cdot S_t \cdot dW_t \quad (6)$$

where :

- r_f is the risk free rate during the period $[0, t]$;
- σ is the volatility of the stock (expressed, empirically, in terms of annualized returns standard deviation);
- W_t is a Wiener process defined under the measure q and dW_t is its dynamic.

From Ito's stochastic calculus we know that the unique solution of eqn. 6 is represented by:

$$S_t = S_0 \cdot \exp\left\{ \sigma \cdot W_t + \left(r_f - \frac{1}{2} \sigma^2 \right) \cdot t \right\} \quad (7)$$

thanks to eqn. 7, we are able to recover the stock price at every instant of time t .

As a matter of fact, we have defined the risk free rate in sec. 1.3 and the stock volatility in sec. 1.2.2. The only variable left to be found is W_t .

Since W_t is a random variable distributed as $N(0, t)$, we need to find a way to simulate it. This can be done building:

$$\epsilon \cdot \sqrt{t} \stackrel{d}{=} N(0, t) \text{ where } \epsilon \sim N(0, 1)$$

Keeping this in mind, we apply Monte Carlo method as described in sec. 2.2.

¹⁰See for example: M. Musiela and M. Rutkowski, *Martingale Methods In Financial Modelling* (Springer, 2008), chapter 3.

2.2 Computational approach using Monte Carlo simulation.

In Python, inside the library [Numpy](#), we have at our disposal the package *random* which contains the function *randn*. *Randn* is a function designed to randomly pick one element from the distribution $N(0, 1)$. This function can allow us to define the ϵ needed to simulate the random walk.

Recall that we are now searching for a way to simulate W_T since we are interested in discovering S_T .

We can proceed as follows:

1. We build a vector of normally distributed $N(0, 1)$ random variables (our ϵ). The length of the vector is fixed at 3,000,000 corresponding to the number of simulations we are going to do for each exit year. We refer to this vector as *EpsVect*.
2. We then apply eqn. 7 setting $t = T \in \{2, 3, 4, 5\}$ as our exit times.
In this way, we obtain four vectors (each one of same dimension of *EpsVect*). We refer to this new vectors as *FinalStockVect_T*. For clarity, we have:

- *FinalStockVect_2* containing 3,000,000 different possible stock values in $T = 2$.
- *FinalStockVect_3* containing 3,000,000 different possible stock values in $T = 3$:

and so on so forth. For a graphic representation, each vector is of the form:

$$\begin{matrix} \text{path 1} & \text{path 2} & \dots & \text{path 3,000,000} \\ \left(\begin{matrix} S_T^1 & S_T^2 & \dots & S_T^{3,000,000} \end{matrix} \right) \end{matrix}$$

where S_T^i represents the stock value in the state of the world i .

3. For each value of each vector described in the previous point, we calculate the pay-off following the rule defined in sec. 2.1.1. We end up with four different vectors (which we call *FinalPayoffVect_T*) each one containing 3,000,000 different pay-offs for a given exit year T . Again, for clarity:
 - *FinalPayoffVect_2* containing 3,000,000 different possible ESO pay-offs, given investors exit at $T = 2$.
 - *FinalPayoffVect_3* containing 3,000,000 different possible ESO pay-offs, given investors exit at $T = 3$.

and so on so forth. For a graphic representation, each vector is of the form:

$$\begin{matrix} \text{path 1} & \text{path 2} & \dots & \text{path 3,000,000} \\ \left(\begin{matrix} \pi_T(ESO)^1 & \pi_T(ESO)^2 & \dots & \pi_T(ESO)^{3,000,000} \end{matrix} \right) \end{matrix}$$

where $\pi_T(ESO)^i$ represents the ESO pay-off in the state of the world i .

4. We average the values inside each of the four vectors (this is done using *np.mean*). We come up with four different values:

$$\mathbb{E}_q[\pi_T(ESO) \mid T] \text{ with } T \in \{2, 3, 4, 5\}$$

each one representing the expectation of the option value (at maturity) given the exit year T .

2.3 Option value.

We now have everything needed to recover the option price.

Using the *Discounting Function*¹¹ we calculate $\pi_0^T(ESO)$ where the notation stands for "the pay-off present value given the exit time at T ".

At the end we have a value for each of the following:

$$\begin{aligned} \pi_0^2(ESO) &= \mathbb{E}_q[\pi_2(ESO) \mid T = 2] \cdot DF_{0,2} \\ \pi_0^3(ESO) &= \mathbb{E}_q[\pi_3(ESO) \mid T = 3] \cdot DF_{0,3} \\ \pi_0^4(ESO) &= \mathbb{E}_q[\pi_4(ESO) \mid T = 4] \cdot DF_{0,4} \\ \pi_0^5(ESO) &= \mathbb{E}_q[\pi_5(ESO) \mid T = 5] \cdot DF_{0,5} \end{aligned}$$

¹¹Continuous compounding convention. Please refer to code explanation attached [here](#) for further details.

It has to be noted that we were able to bring the discounting factors outside the expectations since they are, throughout our report, assumed to be deterministic.
From eqn. 2 we get (using the given exit probabilities):

$$\pi_0(ESO) = \pi_0^2(ESO)\mathbb{P}(T = 2) + \pi_0^3(ESO)\mathbb{P}(T = 3) + \pi_0^4(ESO)\mathbb{P}(T = 4) + \pi_0^5(ESO)\mathbb{P}(T = 5)$$

Our simulation produced:

$$\pi_0(ESO) = 203592.29 \text{ €}$$

2.3.1 Confidence interval.

We also build a confidence interval at 95% for each $\pi_0^i(ESO)$ in this way:

1. we calculate (through the function `np.sdev`) the standard deviation of the values in `FinalPayoffVect_T`
2. we recover the boundary of the confidence set as:

$$Bound_T = \pi_0^T(ESO) \pm 1.96 \cdot SD(\text{FinalPayoffVect_T})$$

where 1.96 comes from a t-student distribution with infinite degrees of freedom.

3. we discount the boundary and we get our confidence interval for $\pi_0^i(ESO)$

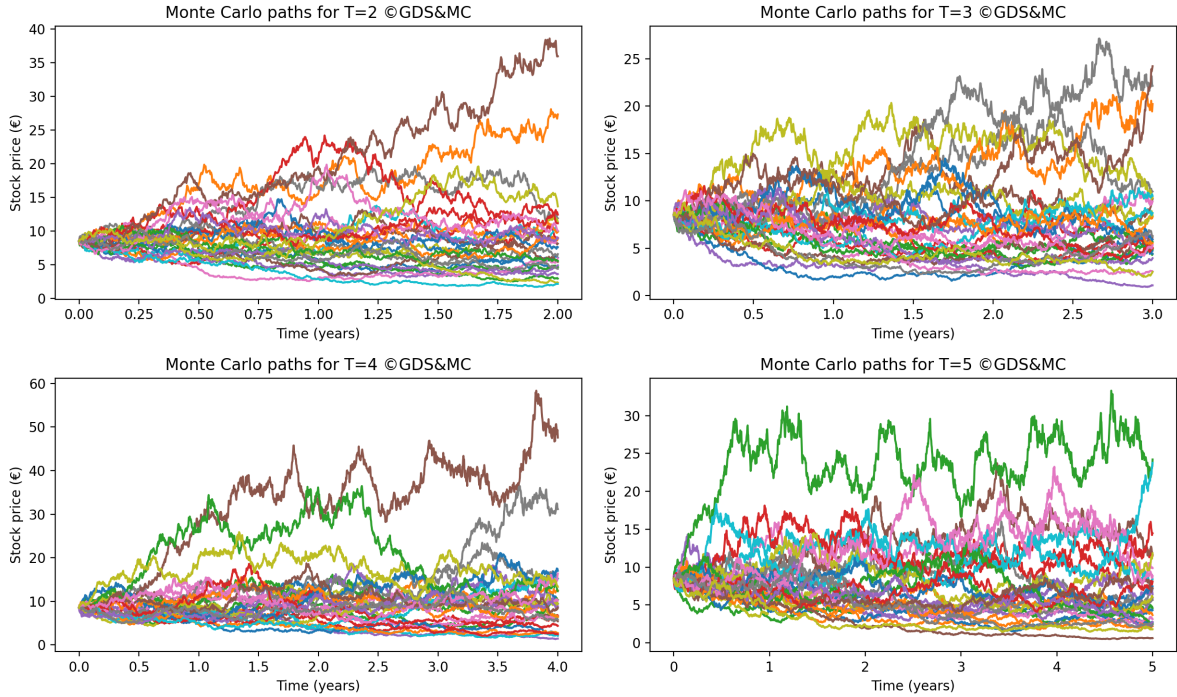
We average the boundary and get:

$$CI_{95\%} = [203272.34, 203912.24]$$

2.4 Script output and key pictures.

2.4.1 Stock evolution.

Stock dynamics for different maturities¹² (only 30 paths plotted for each maturity).



¹²Obtained using different code with respect to the one provided.

2.4.2 Script outcome.

A possible (simulation produces always different results) script outcome:

```
Hello. Nice to meet you. I am ESO_pricer_bot.
I am here to help you discover the price of the ESO option described in the assignment.
Let us first check that everything works fine. Would you like make a test? (logic 1/0): 1
Let me price a plain vanilla option.
I will compare my result with BS: let's see whether I am good or not.
Let's start!
The option you want to price is call (c or C) or put (p or P)? c
What is the current value (€) of the underlying? 307.07
What is the strike price? 330.00
What is the volatility of the stock? Please, use decimal values (24% -> 0.24) 0.25
What is the maturity of the contract? (please, value in years) 2
What is the risk free rate between today and maturity? 0.02
Perfect. I will be back in few seconds.
The price according to BS is: 38.9701 €.
The price according to my own method is: 38.9726 €
...
Let me build a t-test. I will show my precision testing
H_0: my price = BS price
The result of the test is:
t-statistic: 0.1822
pvalue of: 0.85542.
Seems that I was quite accurate.
...
Now, refresh my memory and give me details about ESO option!
What is HoldCo today's stock value (€)? 8.5
What is HoldCo equity volatility? 0.4202
How many outstanding shares does HoldCo have? 1000000
Which are the possible exit years? (value separated by space: 1 3...) 2 3 4 5
Now, FOLLOWING THE ORDER in which you have insert THE YEARS, please:
indicate the exit probability (decimal value separated by space: 0.4 0.6...): 0.05 0.15 0.3 0.5
indicate the risk free rate for [0,T] (decimal values separated by space: 0.4 0.6...): 0.00587 0.00717 0.0101 0.01201
Allow me few seconds please.

The ESO option price is in the interval (values in €) ( 203272.34251763078 ; 203912.23682168784 )
with a confidence level of 95%
The centered value is: 203592.28966965934 €
The ESO option price in the case in which our investors are subject to Italian fiscal regime
is in the interval (values in €) ( 159127.57080490844 ; 159728.86409199866 )
with a confidence level of 95%
The centered value is: 159428.21744845356 €
>>>
```

Statistical precision of the method

They refer to the same year T=2

ESO price and its confidence interval

3 Final remarks.

3.1 Bad leaver clause.

Up to now we have ignored this clause of the contract. We did so on purpose.

As it is known, bad leaver clause is a clause strictly related to personal inclination of the manager. Assign a probability of "bad leaver" activation would be surely a biased guess. We would surely assign a probability reflecting our own expectations and moralities.

We prefer to build a simple mathematical derivation in order to give, to each manager, a way to establish its own value of the option.

It must bear in mind, however, that the fair value of the option is the one calculated in previous section. This is because the ESO option represents a prize for the management and its valuation should not take into consideration the triggering of the "bad leaver" clause. The next calculations are intended just for self-valuation from the manager point of view.

Recall that we have built the model starting from eqn. 1. We rearrange it as follows:

$$\pi_0^{BL}(ESO) = \mathbb{E}_u[\mathbb{E}_q[\pi_T(ESO) \mid \text{bad leaver} = 0]] \cdot DF_{0,T}$$

where u is the measure representing the morality and personal preferences of the management. We shaped the random variable "bad leaver" as a logical (1 if the clause is activated).

In this way:

$$\pi_0^{BL}(ESO) = \mathbb{E}_q[\pi_T(ESO)] \cdot DF_{0,T} \cdot \mathbb{U}(\text{bad leaver} = 0) = \pi_0(ESO) \cdot \mathbb{U}(\text{bad leaver} = 0)$$

assuming independence of bad leaver clause and time to maturity (this may not be true, but it is a fair approximation in order to let computation be as smooth as possible).

Now, every manager has the same $\pi_0(ESO)$ (calculated in 2.3). It just have to, based on his/her morality and personal preferences, decide its probability to not activate the bad leaver and he/she will be able to recover its very own ESO value (which, again, is not the fair value of the contingent claim).

3.2 Value under Italian fiscal regime.

For the sake of completeness, we derive the value of the contingent claim in the situation in which the investors are subject to Italian taxation.

As it is known, Italian tax rate for financial investments is $t_r = 26.00\%$ ¹³.

The taxation does not impact the stock dynamics and so the final stock prices remain unvaried. In our model the introduction of taxation has an impact only on the IRR obtained by the investors (and so on N_s^{IRR}).

In particular, since capital gain is taxed, we have to differentiate from *IRR* and the *net internal rate of return* which we define as IRR_N .

We analyze cash flows made from investors:

Time	Today	T
Gross CF	$-S_0$	S_T
Net CF	$-S_0$	$S_T - \text{Tax}$

Where Tax to be payed is calculated on the capital gain as:

$$\text{Tax} = (S_T - S_0) \cdot t_r$$

In this way we can recover

$$S_T - \text{Tax} = S_0 \cdot (1 + IRR_N)^T$$

so that:

$$IRR_N = \left[\frac{S_T \cdot (1 - t_r)}{S_0} + t_r \right]^{\frac{1}{T}} - 1$$

¹³See for example [Borsa Italiana](#).

We have then plugged IRR_N into eqn. 3 obtaining the number of shares underlying the option in the case investors are under Italian fiscal regime.

The ESO value would surely be lower (since $IRR_N < IRR$ for $t_r > 0$ the number of stock underlying the option will always be smaller). In particular, we recover (using pay-off rule in sec. 2.1.1) that:

$$\pi_0(ESO) = 159428.22 \text{ €}$$

$$CI_{95\%} = [159127.57, 159728.86]$$

3.3 Statistical precision of Monte Carlo.

Before performing any calculation regarding the ESO valuation, we performed an analysis to test the precision of our algorithm.

We have set the following parameters (an ideal call option on Apple inc. stock):

- $S_0 = 307.07 \text{ €}$
- $K = 330.00 \text{ €}$
- $\sigma = 0.25$
- $T = 2$
- $r_f = 0.02$

and we checked for the following test:

$$H_0 : \text{Price}^{MC} = \text{Price}^{BS}$$

$$H_1 : \text{Price}^{MC} \neq \text{Price}^{BS}$$

where Price^{BS} is the price of a Call option derived using Black-Scholes closed formula and Price^{MC} is the price for the same option derived using our Monte Carlo algorithm.

As we know, we can perform this test using a classical t-statistic of the form:

$$t^{stat} = \frac{\text{Price}^{MC} - \text{Price}^{BS}}{\sigma_{MC}}$$

where σ_{MC} is the standard deviation associated with the different outcomes of the simulation.

Given a certain $\alpha \in (0, 1)$, if we find a p-value smaller than α , we then have evidence against H_0 with confidence level of $1 - \alpha$. If the p-value is instead greater than α , we do not have evidence against the goodness of our model.

It turned out, after performing several trials with different values, that we always had a p-value greater than 0.334 with a mean p-value of 0.567. We are highly confident that our model is truly representing the stochastic model object of our analysis.