Discrete Shells

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Content

- Overview
- Previous Work
- Discrete Shell Model
  - Membrane Energy
  - Flexure Energy
  - Dynamics
- Implementation
- Results
- Conclusion

1.Overview

- What is the problem?
  - Simulate the thin flexible structures
- Basic Concepts
  - Thin Shell v.s. Thin Plate
- Why is it difficult?
  - The "*thinness*" -- cannot apply 3-D models
  - The structural rigidity -- cannot apply Thin plate Model
- Solution?
  - Small change in the existing cloth animation algorithms

1.1 What is the problem?

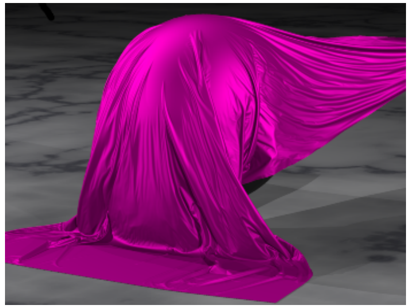
- Simulate the thin flexible structures which are characterized by a curved undeformed configuration.

1.2
Thin flexible objects

Thin Shell

- Thin flexible structure
- High ratio of width to thickness (>100)
- Curved, non-flat rest configuration
- Naturally curved

Thin plate



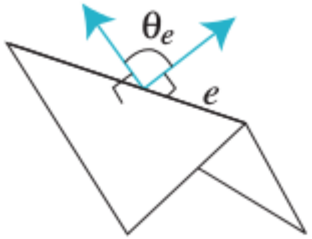
- Similar to shell
- Main difference: flat rest configuration
- Successfully modeled by mass-spring network
- Naturally flat

1.3 Why is it hard?

"thinness"

- Degeneracy in one dimension
- Not admit to straightforward tessellation
- Cannot model as 3-D solid

Structural Rigidity



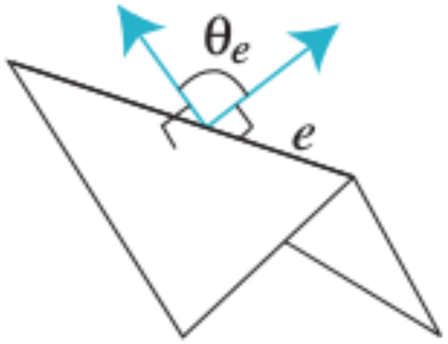
- Thin plate modeled by mass-spring network
- Resistance to bending is effected by diagonal springs
- diagonal springs are insensitive to dihedral angles between faces

2. Previous Work

- Continuum-based Approaches
  - Kirchoff-Love consitutive equations
  - F. Cirak et al. 2000, Subdivision surfaces: A new paradigm for thin-shell finite-element analysis
  - Seth Green et al. 2002, Subdivision-based multilevel methods for large scale engineering simulation of thin shells
  - Grinspun et al. 2002, CHARMS
- Complex, computational expensive
- Challenging & costly to simulate

3. Discrete Shell Model

- Arbitrary 2-manifold triangle mesh
- Governed by non-linear two energies:
  - Membrane
    - Intrinsic deformations
    - Resist shearing and stretching
  - Flexural
- Deformation defined by piecewise-affine deformation map
  - every vertex of the undeformed mesh -> corresponding vertex of the deformed mesh



3.1 Membrane

Elastic surfaces resist change in area and shearing

Energy stored by edges

- One-Dimensional Spring:

$$W = \frac{(||e|| - ||\bar{e}||)^2}{||\bar{e}||}$$

$$||e|| = ||\varphi(\bar{e})||$$

- Two-Dimensional Energy:
  - energy stored by mesh edges

$$W_L(x) = \frac{1}{2}\sum_{\bar{e}} \frac{(||e|| - ||\bar{e}||)^2}{||\bar{e}||}$$

  - ||e|| is the deformed edge length
  - ||e|| bar is the undeformed edge length

Energy stored by faces

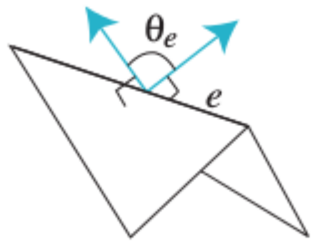
- Two-Dimensional Energy:
  - energy stored by mesh faces

$$W_A(x) = \frac{1}{2}\sum_{\bar{f}} \frac{(||f|| - ||\bar{f}||)^2}{||\bar{f}||}$$

  - ||f|| is the deformed area
  - ||f|| bar is the undeformed area

3.2 Flexure Bending Energy

- Ciarlet, 2000
  - *The stored energy function of a nonlinearly elastic flexural shell is thus remarkably simple: it is a quadratic and positive definite expression in terms of the exact difference between the curvature tensor of the deformed middle surface and that of the undeformed one*
- Bending Energy
  - Measure of the difference in curvature



- Curvature
  - Differential of the Gauss map -- Shape operator
- Gauss Map
  - Maps a surface Omega in Euclidean space R^3 to the unit sphere S^2.

3.2.2 Bending Energy

- Principle curvature:
  - eigenvectors of the differential of the gauss map
  - the diagonal elements of the shape operator
- Mean curvature:
  - half of the principle curvature
  - $H(p) = \frac{1}{2} Tr(S(p))$ --$Tr(S)$ is the matrix trace (sum of diagonal elements)
- Bending Energy: the squared difference of mean curvature

3.2.2 Bending Energy

- The squared difference of mean curvature

$$[Tr(\varphi^* S) - Tr(\bar{S})]^2 = (2H - 2\bar{H})^2$$

- S and S bar are the shape operators evaluated over the undeformed and deformed configurations.
- H and H bar are the mean curvatures
- phi is a diffeomorphism which is a map between topological space that is differentiable and has a differentiable inverse. Used here for the consistent basis of coordinates.
- phi*S is the pull-back of S onto Omega bar.
- Tr is the matrix trace

3.2.3 Flexural Energy

- To use the above formula, we should....?
- Discrete the area integral over the reference

domain

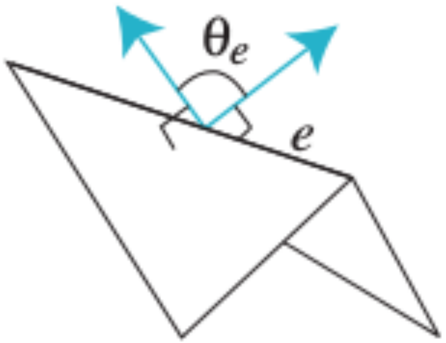
$$\int_{\bar{\Omega}} (2H - 2\bar{H})^2 \mathrm{d}\bar{A}$$

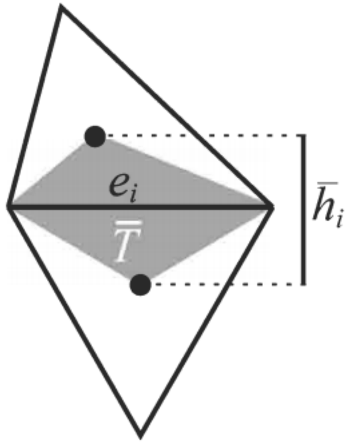
- Consider two adjacent triangles
  - theta_e is the dihedral angle between the faces.
  - we have pointwise:

$$(2H - 2\bar{H})^2 = (\theta_e - \bar{\theta}_e)^2$$

  - summation over edge -> flexural energy

$$W_B(x) = \sum_{\bar{e}} (\theta_e - \bar{\theta}_e)^2 ||\bar{e}||$$





3.3 Dynamics

- Shell model is sum of membrane and flexural energies.

$$W = K_L W_L + K_A W_A + K_B W_B$$

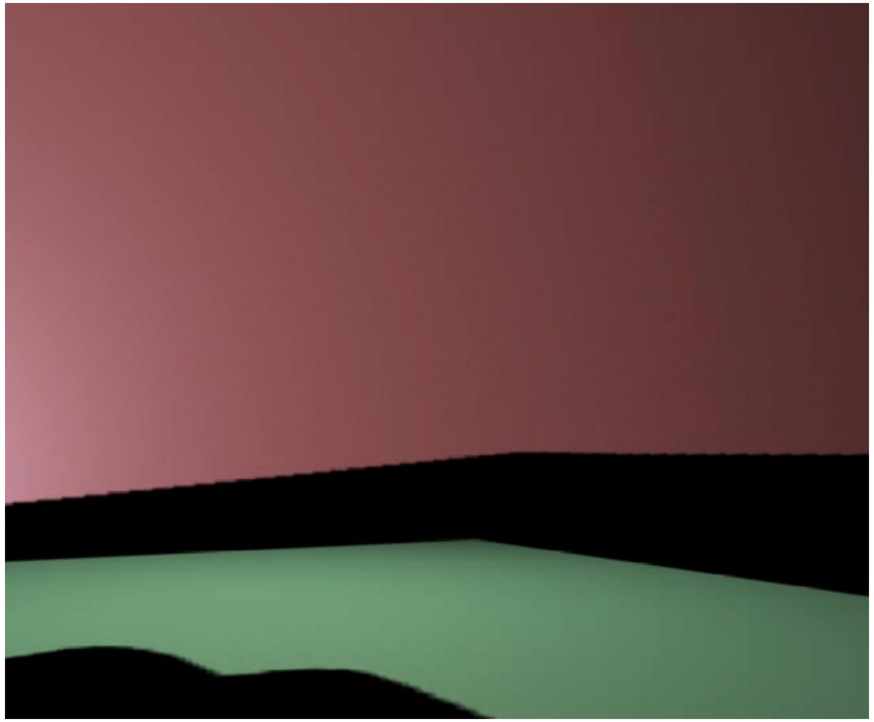
  - K_L: stretching stiffness
  - K_A: shearing stiffness
  - K_B: fexural stiffness
- Go from elastic sheets to aluminum-like sheets by tuning these parameters.

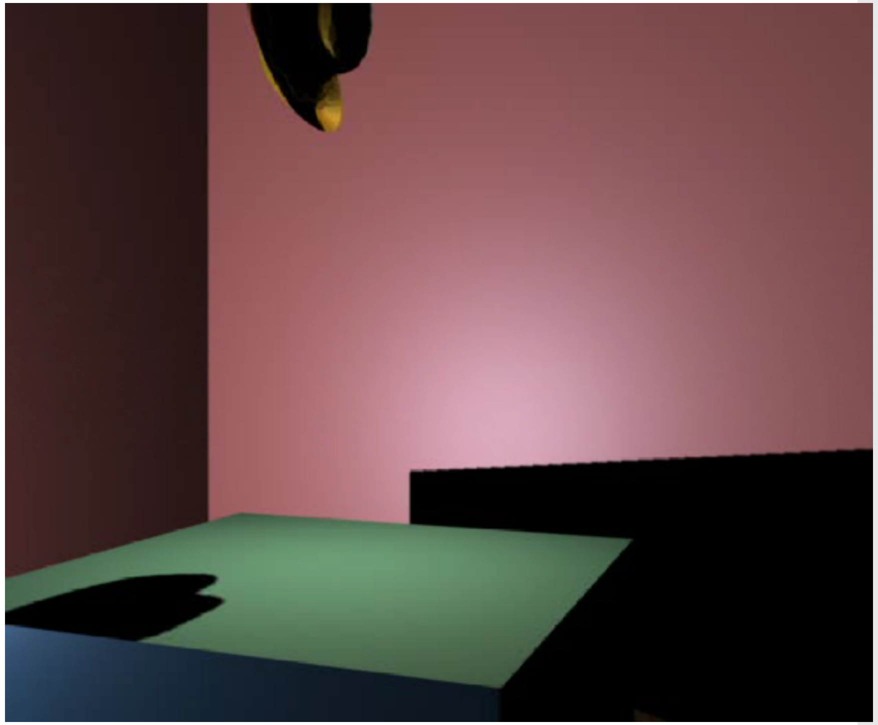
4. Implementation

- Simply take the existed code for cloth simulation
- Replace the bending energy:

$$W_B(x) = \sum_{\bar{e}} (\bar{\theta}_e)^2 ||\bar{e}|| \qquad W_B(x) = \sum_{\bar{e}} (\theta_e - \bar{\theta}_e)^2 ||\bar{e}||$$

5. Results





6. Conclusion

- Simple discrete bending energy
- fast computation
- simplicity of implementation

Questions?

- Thank you!