A global search framework for practical three-dimensional packing with variable carton orientations

A Global Search Framework for Practical Three-Dimensional Packing with Variable Carton Orientations

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Abstract

This article aims to tackle a practical three-dimensional packing problem, where a number of cartons of diverse sizes are to be packed into a bin with fixed length and width but open height. Each carton is allowed to be packed in any one of the six orientations, and the carton edges are parallel to the bin edges. The allowance of variable carton orientations exponentially increases the solution space and makes the problem very challenging to solve. This study first elaborately devises the packing procedure which converts an arbitrary sequence of cartons into a compact packing solution and subsequently develops an improved genetic algorithm (IGA) to evolve a set of solutions. Moreover, a novel global search framework (GSF), utilizing the concept of evolutionary gradient, is proposed to further improve the solution quality. Numerical experiments indicate that IGA provides faster and better results and GSF demonstrates its superior performance, especially in solving relative large-size and heterogeneous instances. Applying the proposed algorithms to some benchmarking cases of the three-dimensional strip packing problem also indicates that the algorithms are robust and effective compared to existing methods in the literature.

Keywords: packing; loading; three-dimensional; genetic algorithm; global search; evolutionary gradient

1. Introduction

Bin packing, or container/pallet loading, is a very common but important task in the manufacturing and logistics industries. However, even after the problem has been researched for several decades, many of packing tasks are still accomplished manually, i.e., the workers stow bins or containers according to their experience and judgement. It is evident from operational observations and the literature that manual packing may not optimally utilize the valuable bin/container space, and the volume utilization can be significantly improved with the support of computer-generated solutions. Exact algorithms, due to the complexity and combinatorial explosive nature, can only solve small-size problems to optimality with the current computing technology. For most practical applications, heuristics and meta-heuristics are still the best choices to achieve optimal or near-optimal solutions within reasonable time.

This study solves a practical three-dimensional packing problem by using heuristics and meta-heuristics with the purpose of obtaining high quality solutions as fast as possible. In the investigated problem, a number

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of cartons of diverse sizes are to be stowed into a paperboard bin with fixed length and width but open height (subject to physical limit), i.e., the bin height can be trimmed to fit the content of the bin to save transport cost (e.g., when air transportation is used). Each carton is allowed to be packed in any one of the six orientations, and the carton edges are parallel to the bin edges (we will call this as variable orientations hereafter). The cartons need not be stowed in layers or walls, and no so-called guillotine constraint is imposed to the packing process.

According to the typology of cutting and packing problems introduced by Wäscher et al. (2007), our investigated problem is a three-dimensional packing problem with one variable dimension, different from the traditional three-dimensional bin packing problem (3D-BPP) where a selected subset of a number of cartons are packed into a bin with fixed length, width and height to maximize the volume utilization, or where a large number of cartons are packed into identical bins with the objective to minimize the number of bins used. For this problem, the variable bin height poses a greater challenge in providing high-quality solutions since usually no selection of cartons is allowed, i.e., all cartons need to be packed, and when the bin is not fully packed the extra height will be trimmed. The flexibility of carton orientations further expands the solution space significantly and thus increases the difficulty in finding the optimal solutions. Up to now, such a problem with both variable bin height and variable carton orientations has only been studied by a few researchers.

It should be noted that the problem under investigation possesses some characteristics of the three-dimensional strip packing problem (3D-SPP). However, compare with the traditional 3D-SPP in the literature, it presents two important features: a) the physical bin height (which corresponds to strip length in 3D-SPP) limit is relatively short due to the fact that paperboard bins are used, which limits the maximum loading height/weight; whereas in 3D-SPP the strip length is usually much longer than both the strip width and the strip height, thus the term 'strip' is used; b) the cartons that are to be packed are 'generated' on the fly in various dimensions, which presents a very heterogeneous packing problem; in contrast in traditional 3D-SPP it is common to see boxes (in small bunches) have the same dimensions. The demand to pack highly heterogeneous cartons in a relatively small paperboard bin makes layer-building heuristics unsuitable (e.g., in extreme case, every carton to be packed may belong to a different carton type, this may make layer-building almost impossible). Although the current problem represents a special case of 3D-SPP, the concept of 'bin' will still be used for ease of description in this paper for two reasons: a) paperboard bins are used in practical packing; and b) there is no significant difference between the maximum (variable) bin height and the fixed bin length and fixed bin width.

The solution techniques for packing problems can be classified into three types:

Exact methods based on mathematical programming belong to the first type. Chen et al. (1995) provided a mixed integer linear programming (MILP) model to solve the 3D-BPP with variable orientations. Due to the strong NP-hardness of the problem, this MILP model only solved a small instance with 6 cartons to optimality. Martello et al. (2000) and den Boef et al. (2005) presented an exact algorithm for loading a single bin and discussed the lower bounds for the 3D-BPP. Martello et al. (2007) presented an algorithm to

solve moderate instances to optimality for the general 3D-BPP and its robot-packable variant. Fekete et al. (2007) developed a two-level tree search algorithm for solving higher-dimensional packing problems to optimality, but their computational results reported were on the optimal solutions for two-dimensional test problems from the literature. Ceselli and Righini (2008) presented a branch-and-price algorithm for the exact optimization of the ordered open-end bin-packing problem defined by Yang and Leung (2003), where the items to be packed are sorted in a given order and the capacity of each bin can be exceeded by the last item packed into the bin.

Heuristics and approximate algorithms comprise the second type. Lim et al. (2005) conducted a comprehensive review on the heuristics studied by many researchers, from which it could be seen that most heuristics were on the wall- or layer-building of the traditional 3D-BPP to pack a selected subset of a number of cartons into a bin/container pursuing high volume utilization. For example, George and Robinson (1980) presented a wall-building approach heuristic without restrictions on the carton orientations for identical containers. Bischoff and Marroitt (1990) evaluated 14 different heuristics based on the George-Robinson framework. Bischoff et al. (1995) proposed a pallet loading heuristic for non-identical cartons where the stability of the pallet was considered. Pisinger (2002) offered a heuristic based on the wall-building approach, where strips and layers were created so that the 3D-BPP could be decomposed into smaller sub-problems. Baltacioglu et al. (2006) developed a new heuristic algorithm using rules to mimic human intelligence to solve the 3D-BPP. Faroe et al. (2003) provided a heuristic for packing cartons into a minimum number of identical containers based on guided local search, however, no variable orientation was allowed. Huang and He (2009) developed a heuristic algorithm for the traditional 3D-BPP without orientation constraints based on the concept of caving degree inspired by an old adage “Gold corner, silver side and grass belly” from Chinese Weiqi (a board game), which achieved very high volume utilization for the benchmark instances provided by Bischoff and Ratcliff (1995) and later solved by so many researchers, such as Lim et al. (2005), Bortfeldt and Gehring (1998, 2001) and Zhang et al. (2007).

The third choice is ***meta-heuristics***, such as genetic algorithms, tabu search and simulated annealing, are widely used. Gehring and his colleagues have published a number of papers on meta-heuristic methods for 3D-BPP. Gehring and Bortfeldt (1997) first presented a genetic algorithm and then (2002) provided a parallel genetic algorithm for single container loading with stronger heterogeneous cartons. Bortfeldt and Gehring (1998) first applied tabu search and then (2001) presented a hybrid genetic algorithm for single container loading, finding that the former was suitable to weakly heterogeneous cases but the latter suitable to strongly heterogeneous cases. Lodi et al. (2002) provided a tabu search framework by exploiting a constructive heuristic to evaluate the neighborhood where the carton orientations were fixed. Zhang et al. (2007) proposed a simulated annealing algorithm combined with a bin-loading heuristic to solve the classic 3D-BPP. Crainic et al. (2008a) introduced a two-level tabu search where the first level aimed to reduce the number of bins and the second optimized the bin packing of cartons with fixed orientations. Egeblad and Pisinger (2009) examined 2D/3D knapsack packing problems using simulated annealing. Goncalvez and Resende (2012) proposed a parallel random-key-based genetic algorithm using layer-building heuristics.

Another classification for 3D loading problems by Fanslau and Bortfeldt (2010) divides packing algorithms into exact and heuristic methods. The latter can be further divided into three groups: conventional heuristics, meta-heuristics and ***tree search methods***. Bortfeldt and Mack (2007) presented a layer-building heuristic method for the 3D-SPP, where tree search algorithms were adopted to determine the best layer depths and to obtain the best strip directions and strip dimensions for a given layer. Their method did not involve any supplementary loading restrictions, particularly no supporting constraints are imposed. Regarding the heuristic packing approaches (Pisinger, 2002), wall building, stack building, horizontal layer building, block building and guillotine cutting were often used.

Allen et al. (2011) proposed a hybrid placement strategy for the 3D-SPP, which is actually a decomposition method: one bigger sub-set of items are packed via three-dimensional best fit heuristic, the other smaller sub-set of items are packed through a tabu search algorithm, thus saving the search time.

Through analyzing the existing solution methods, the following observations can be made: (1) Most of heuristics are based on the wall-building with some modifications, which are in nature greedy techniques that might stuck at local optimal solutions and away from the global optimality; (2) Exact methods are based on mathematical programming and graph theory by reducing the original problem to a theoretically solvable model, which at the moment would be virtually impossible to guarantee the global optimality for practical 3D problems, especially for those with variable orientations; (3) Meta-heuristics are the present best choice, which can search much larger feasible space than the simple heuristics and converge much faster than the exact methods, hence make it realistic to obtain optimal or near-optimal solutions within reasonable search time.

However, in practical applications of the meta-heuristics, the users usually have to tune the algorithm parameters according to the problem conditions and carry out a number of tests to select the best solution as the real stowage plan. In general, the users hope to have lower variance of solutions in the tests, which may imply the algorithm has higher search performance and convergence. But the search ability of an algorithm depends on the prevailing problem conditions. As long as the solution variance exists, all the better solutions via the algorithm are still possible. So how to fully utilize the search ability until the average deviation of solutions turns to be null will be considered in this paper.

This paper further studies the problem proposed in Wu et al. (2010) in which an exact mathematical model based on Chen et al. (1995) was first developed and a genetic algorithm was subsequently designed and implemented with a heuristic packing procedure (or decoding routine). In this paper, we first improve the decoding procedure aiming at tighter packing and higher computational efficiency; and then redesign and implement a genetic algorithm with well tailored components based on the work of He (2007 and 2009). Further, we propose a global search framework based on the concept of evolutionary gradient (He and Hui, 2009 and 2010) and devise several scenarios of the framework for simulation experiments. A suitable scenario is selected for the practical 3D packing problem through numerical experiments.

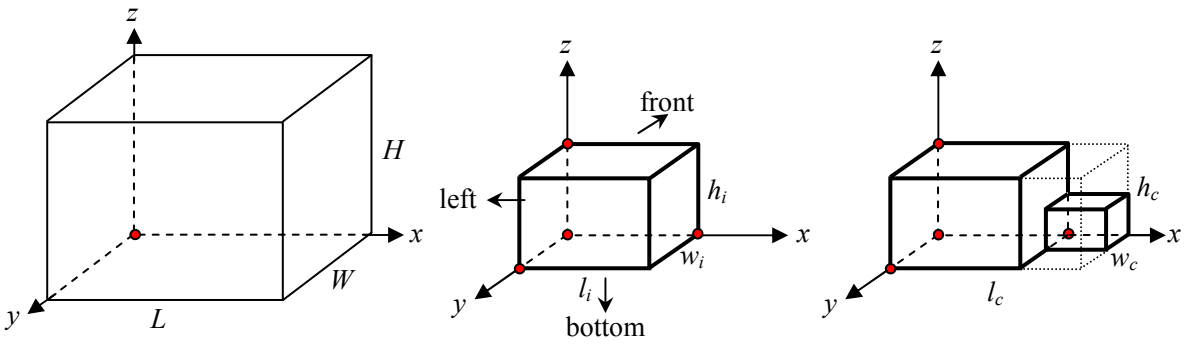
2. Problem Description

The investigated problem originates from a real-world application, for which Wu et al. (2010) provided an MILP model and a basic GA. The packing objective is to minimize the height H of the paperboard bin used when packing N cartons so that logistics costs can be saved since the packed bins are transported via air. As mentioned earlier, the height of bins is subject to physical limit.

2.1 Bins and Cartons

In the practical problem, a rectangular ***bin*** with fixed length and width ($L\times W$) is available to pack a number (N) of rectangular ***cartons*** for shipping (see Fig. 1a). The height (H) of the bin can be trimmed to fit the cartons packed, but cannot exceed the physical limit height, H_{max}.

Each ***carton*** i has a fixed size of $l_i\times w_i\times h_i$ (centimeter), where l_i, w_i and h_i are the length, width and height of carton i. Without loss of generality, we assume $l_i\geq w_i\geq h_i$ (see Fig. 1b). Each carton may be packed into the bin in any one of the six orientations that keep the carton edges parallel to the bin edges. The cartons need not be stowed in layers or walls, and no so-called guillotine constraint is imposed to the packing process.



(a) A bin with $L\times W\times H$ (b) A carton i with $l_i\times w_i\times h_i$ (c) A cage bin with $l_c\times w_c\times h_c$

Fig. 1 A bin, a carton and a cage bin

2.2 Reference Points, Cage Bin and Orientations

The Cartesian coordinate system is used to locate a carton in the bin (see Fig. 1a). Martello et al. (2000) proposed ***corner points*** as candidate locations where a carton could be placed, which needs be calculated by using a 2D algorithm and bears the shortcoming that some space of the bin is not covered and thus cannot be exploited. Zhang et al. (2007) presented possible ***placement points*** for a carton to be placed, a selected point of which is just a possible location for a carton, but may not be the ***final position*** because of the later ***parallel moves*** for compact packing. Therefore placement points belong to a kind of ***reference points***. Crainic et al. (2008b) introduced ***extreme points*** and corresponding heuristics for 3D-BPP without any further parallel moving, even though extreme points are generated in the same way as placement points. We adopt the concept of ***reference points*** in this work.

When each carton i is packed into the bin, the carton edges are parallel to the bin edges, as shown in Fig. 1(b). A ***reference point*** in the bin is denoted by its coordinates (x, y, z). An empty bin has only one reference point (0, 0, 0), and every time when the *left-front-bottom* corner of carton i with $l_i\times w_i\times h_i$ is assigned at (x_i, y_i, z_i), three new reference points will be generated depending on the carton orientation, such as (x_i+l_i, y_i, z_i), (x_i,

y_i+w_i, z_i) and (x_i, y_i, z_i+h_i).

The concept of ***cage bin*** is utilized in our work. A cage bin is a virtual bin and is defined as the smallest rectangular box (with dimensions of $l_c \times w_c \times h_c$) that is able to encase all the cartons already packed in the bin; therefore, the cage bin changes as the packing of cartons proceeds. For example, Fig. 1c shows the cage bin encasing two cartons packed so far in the bin.

Each carton can be placed into the bin in any one of its six orientations. That is to say, although the original size ($l_i \times w_i \times h_i$) of carton i is fixed, its orientation in the bin is not pre-fixed. To make the total packing compact, each carton i can take any one of its six orientations as depicted in Fig. 2, where γ_i denote the orientation number, $\gamma_i \in \{1, 2, 3, \ldots, 6\}$, $i = 1, 2, 3, \ldots, N$.

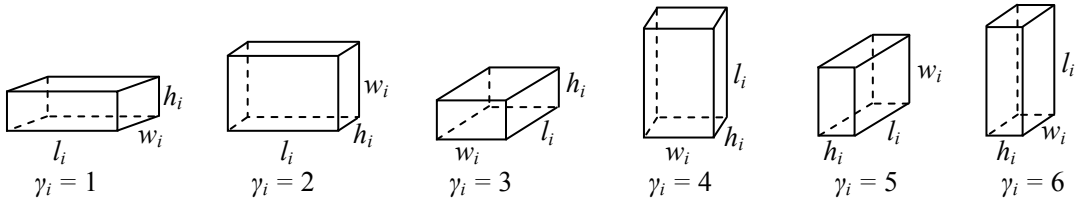


Fig. 2 Six orientations of carton i

2.3 An MILP Solvable Example

Mathematically proven global optimal solutions, no matter how difficult to derive, are always desirable for first of all, providing a benchmark for other methods; and secondly, applying to the actual problem when the solutions are relatively easy to achieve. Wu et al. (2010) adopted the MILP model presented in Chen et al. (1995) and coded in GAMS. Though computing resources have advanced significantly in the past two decades, the mathematical approach still seems to be less practical, as shown by the following example.

Table 1 presents the original sizes of a set of cartons to be packed into a small bin with fixed length and width $L \times W = 80 \times 58$ cm, the $H_{max} = 95$ cm. This example (called SM00) will be used to illustrate the proposed solution techniques in this paper.

Table 1. The sizes of the cartons in SM00

Carton i	$l_i \times w_i \times h_i$	Carton i	$l_i \times w_i \times h_i$
1	36×36×36	6	40×31×21
2	59×39×20	7	31×31×17
3	54×40×21	8	31×17×16
4	58×37×21	9	26×23×14
5	52×33×20	10	33×21×4

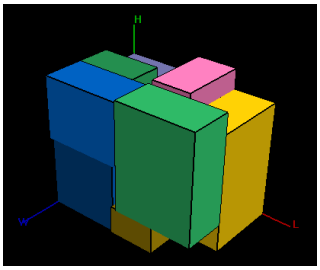


Fig. 3 Optimal packing for SM00

The above example was solved by the commercial solver CPLEX 11.2 based on the GAMS model in Wu et al. (2010). CPLEX 11.2 finds the solution with the height 68 cm after 1006 seconds, and proves the optimality at 1144 second. Fig. 3 shows the packing configuration of the optimal solution.

It can be observed from the example that mathematical approach still takes a long time to solve a relatively small practical instance. An optimistic estimation of the maximum number of cartons the mathematical model can handle under current computational resources would be below 20, or even 15. In

Wu et al. (2010), such a maximum number was 12 for CPLEX 11.0 with 2 hours' running time and in a number of cases where less than 12 cartons were presented, CPLEX 11.0 failed to find the proven optimal solutions within the specified running time. Therefore, to provide solutions for practical problems within reasonable time, alternative approaches must be used.

3. The GA Coding and Decoding Procedures

To solve a problem by GA, the first task is to represent a solution of the problem as a chromosome, P. This representation is called *coding*. For combinatorial optimization, there are several representation methods, among which permutation-based representation (Gen and Cheng, 1997) and random-key (Bean, 1994) are often used. The procedure to change a chromosome P into a phenotype solution with an objective value $f(P)$ is called *decoding*.

3.1 Coding of an Arbitrary Solution

A mixed chromosome, which includes both the carton packing order and the carton orientations, is adopted to represent a packing solution in this research. The mixed chromosome $P = (P_1, P_2)$ consists of two parts: $P_1=(\pi_1, \pi_2, \pi_3, ..., \pi_N)$ is a carton sequence, $\pi_i \in \{1, 2, 3, \ldots, N\}$, $i = 1, 2, 3, \ldots, N$; $P_2 = (\gamma_1, \gamma_2, \gamma_3, ..., \gamma_N)$ is an orientation sequence, $\gamma_i \in \{1, 2, 3, \ldots, 6\}$, $i = 1, 2, 3, \ldots, N$. Fig. 4 shows a sample chromosome for a problem with 10 cartons, where each carton π_i in P_1 is given a corresponding orientation γ_i in P_2.

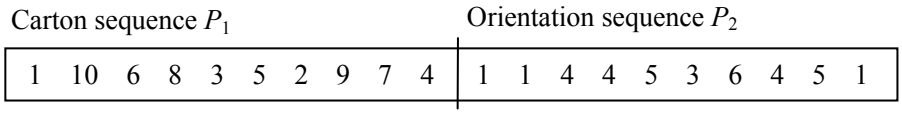


Fig. 4 A sample mixed chromosome

3.2 Decoding Procedure

The decoding routine converts a chromosome into a feasible packing solution. Cartons in the given chromosome are processed sequentially, and their coordinates and relative positions are determined at the same time.

3.2.1 Carton List and Reference Point List

To decode a chromosome, two data lists, a ***carton list*** (c-list) and a ***reference point list*** (ref-point-list), are used to store the relevant data. A c-list corresponds to a chromosome. Each element i in the c-list contains the following information: carton no. π_i, orientation no. γ_i, original size ($l_i \times w_i \times h_i$), orientational dimensions ($ln_i \times wn_i \times hn_i$) (derived from orientation no. γ_i and original size), coordinates (x_i, y_i, z_i) (to be determined), and relative positions (left-carton-no, front-carton-no, bottom-carton-no) (to be determined). Table 2 shows a sample c-list for the sample chromosome in Fig. 4. The coordinates and relative positions of the cartons are variables to be determined so that all cartons do ***not intersect*** with each other while maintaining a packing solution as compact as possible. When the coordinates and relative positions of the cartons in the c-list are determined through the decoding procedure (i.e. the packing procedure), the c-list is changed into a packing solution.

Table 2. The sample carton list for the sample chromosome in Fig. 4

			Original size			New dimensions			Coordinates			Relative positions		
i	π_i	γ_i	l_i	w_i	h_i	ln_i	wn_i	hn_i	x_i	y_i	z_i	left	front	bottom
1	**1**	**1**	36	36	36	**36**	**36**	**36**						
2	**10**	**1**	33	21	4	**33**	**21**	**4**						
3	**6**	**4**	40	31	21	**31**	**21**	**40**						
4	**8**	**4**	31	17	16	**17**	**16**	**31**						
5	**3**	**5**	54	40	21	**21**	**54**	**40**						
6	**5**	**3**	52	33	20	**33**	**52**	**20**						
7	**2**	**6**	59	39	20	**20**	**39**	**59**						
8	**9**	**4**	26	23	14	**23**	**14**	**26**						
9	**7**	**5**	31	31	17	**17**	**31**	**31**						
10	**4**	**1**	58	37	21	**58**	**37**	**21**						

Based on the concept of reference points in §2.2, a ref-point-list is adopted to store the information of the ref-points generated in the process of packing: coordinates, the type of the point, the temporary indices, occupied status. As stated earlier, an empty bin has one ref-point (0, 0, 0), and the first carton is assigned to this point hence marked as "occupied". And then every time when carton π_i with $ln_i \times wn_i \times hn_i$ is assigned at (x_i, y_i, z_i), three new reference points, (x_i+ln_i, y_i, z_i), (x_i, y_i+wn_i, z_i) and (x_i, y_i, z_i+hn_i), are generated and added to the ref-point-list as "not occupied yet". A ***feasible ref-point*** for the current carton to be packed is the one which ensures that the carton will not intersect with any cartons packed so far or the bin itself. If a ref-point is selected to assign a carton, this point may not be the ***final position*** of the carton because of the later ***parallel moves*** (introduced in §3.2.2), which try to move the carton along the x-, y- and z-axis to make the packing more compact.

To efficiently conduct the parallel moving, we classify the ref-points into three types: type 1, (x_i+ln_i, y_i, z_i); type 2, (x_i, y_i+wn_i, z_i); and type 3, (x_i, y_i, z_i+hn_i). Furthermore, the just packed carton π_i is recorded as the adjacent carton of these three points, (x_i+ln_i, y_i, z_i), (x_i, y_i+wn_i, z_i) and (x_i, y_i, z_i+hn_i), which is convenient to determine the relative positions of the packed cartons. Therefore, each ref-point k in the ref-point list should include the following information: coordinates (x_k, y_k, z_k), the adjacent carton to this point (adj_carton), type no. (type 1, 2 or 3), two indices index1 and index2 (for reference point selection) and the assignment flag (as_flag).

3.2.2 Parallel Moves, Relative Positions and Reference Point Selection

Every time a carton is assigned to a ref-point, parallel moves can be performed for two purposes: one is to make the packing more compact by moving the carton as close as possible to the left, front and bottom of the bin; the other is to determine the relative positions of the carton packed.

Parallel moving was also conducted in Wu et al. (2010) where a carton was tried to move along x-, y- and z-axis in turn (x→y→z) after it was packed on a ref-point. However, fixed-sequence moves along x-, y- and z-axis might not be the optimal configuration for all cartons. The example in Fig. 5 illustrates the shortcomings of the simple sequential moves x→y→z. In Fig. 5, four cartons have been packed in the bin,

assume ref-points 1, 2 and 3 are selected to assign a carton and then conduct parallel moves. Fig. 5(a) and (c) shows the situation of the moves x→y→z to the cartons assigned at ref-points 1, 2 and 3; Fig. 5(b) presents the result of the moves y→x→z to the cartons assigned at ref-points 1 and 2; and Fig. 5(d) displays the case of the moves x→z→y to the carton assigned at point 3.

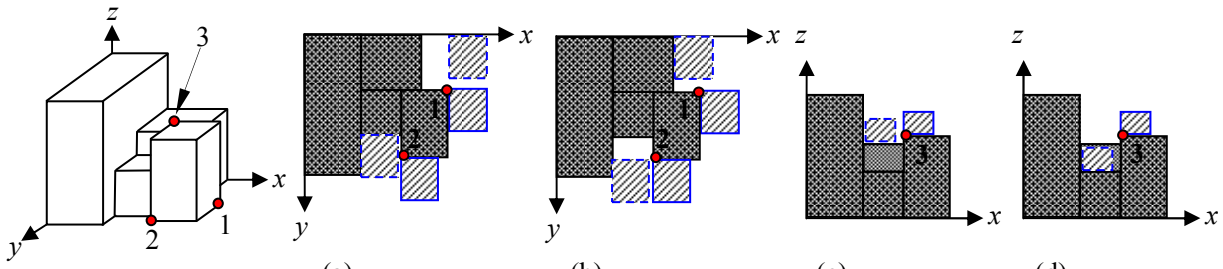


Fig. 5 Different moves

From these cases we note that: (1) different sequence of parallel moves leads to different outcome; (2) one or two moves among the three moves in turn make no difference; (3) a further move may fill an additional gap; (4) one move may fail to fill any gap, but it can surely tell which carton is touched during the process.

Table 3. Different moves according to ref-point types

Ref-point type	Moves in turn	Interpretation
Type 1	y→z→x(→z)	With a left carton, no need for x move at first.
Type 2	x→z→y(→z)	With a front carton, no need for y move at first.
Type 3	x→y→z (→x)	With a bottom carton, no need for z move at first.
(0, 0, H_c)	z move only	On the top of the cage bin, no need for other moves.

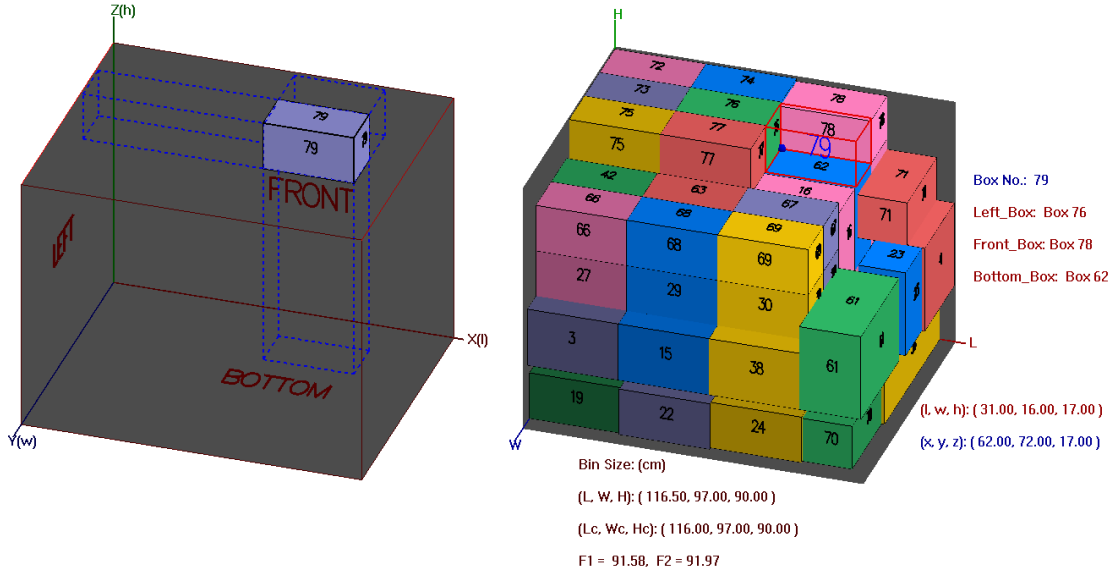


Fig. 6 Illustration of relative positions in packing

Based on the observation, a new parallel moving strategy is proposed as follows. First, we classify the

ref-points generated by the packed cartons into three types as stated in §3.2.1. We then conduct different moves to a carton in line with the type of the ref-point selected, as shown in Table 3.

Relative positions are intended to tell which cartons are touched on the ***left***, in ***front*** and at the ***bottom*** of a carton (see Fig. 6), which are meaningful to guide the (especially manual) packing process. We denote the bin itself as carton "0" so that every carton packed in the bin is associated with three cartons to identify its relative positions: one on the left, one in front and one at the bottom.

For reference point selection, Wu et al. (2010) defined an index which we call *index*1 in this research. For ref-point k, let l_{ck} , w_{ck}, h_{ck} denote the length, width and height of the cage bin ***after*** the carton π_i is placed at this point, V_i denote the total volume of cartons packed so far including carton π_i, *index*1 is defined as:

$$index1 = (l_{ck} \times w_{ck} \times h_{ck}^2)/V_i \,, \qquad (1)$$

which is a measure of the cage bin height over the fill rate of the current cage bin. In their work, the ref-point with the highest packing quality among the feasible ref-points is selected as the packing point for carton π_i. In Fig. 7, the third carton with $ln_3 \times wn_3 \times hn_3$ is being packed, and ref-points 3, 4, 5, 6 and 7 are the feasible points, of which ref-points 3, 4 and 5 are obviously not so suitable as 6 and 7. Compared with ref-point 7, ref-point 6 has a higher value of ***index***1 thus is selected to pack the third carton.

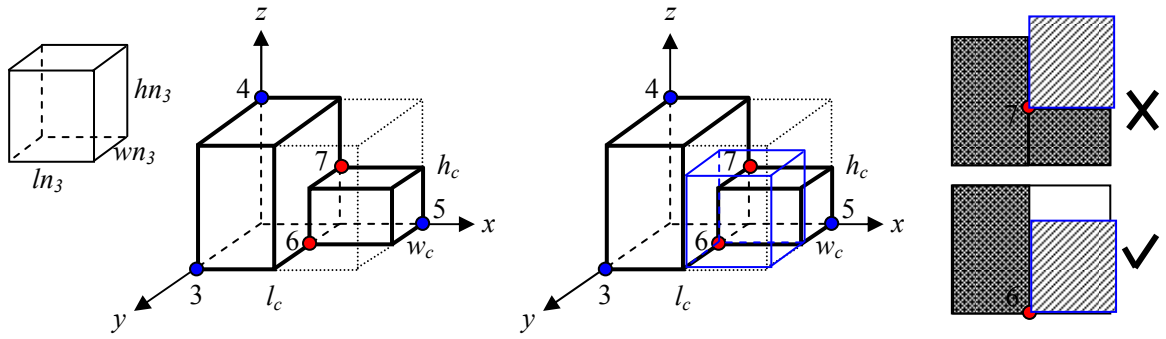


Fig. 7 Pack a carton by using *index*1

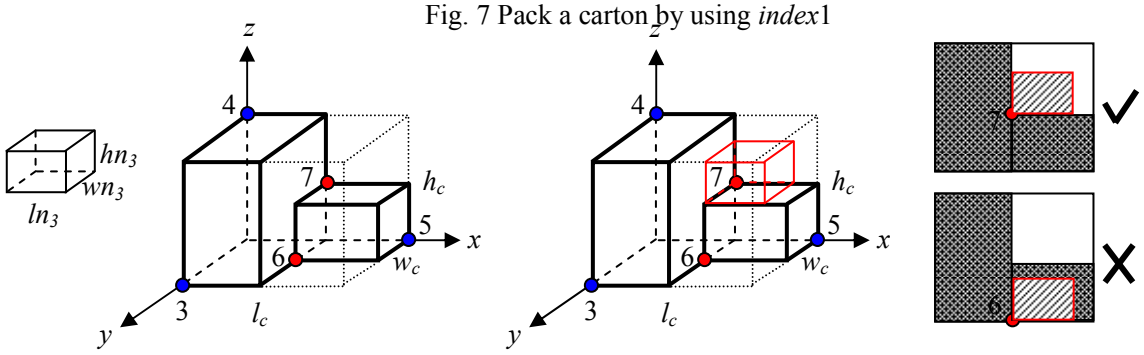


Fig. 8 Pack a carton by using *index*2

However, only using *index*1 for ref-point selection is not comprehensive enough. In the case that there are several ref-points for packing carton π_i, whose cage bins are the same (i.e. when carton π_i is assign to anyone of these points, it will not exceed the current cage bin), these points have the same value of *index*1. In this case, *index*1 is not able to offer differentiating information for these points, and we need a new index to effectively select one of these points.

Assume that the coordinates of ref-point k is (x_k, y_k, z_k), carton π_i has dimensions of $ln_i \times wn_i \times hn_i$, the

current cage bin ***before*** packing carton π_i has dimensions of $l_c \times w_c \times h_c$. Given that $(x_k + ln_i) \le l_c$, $(y_k + wn_i) \le w_c$, $(z_k + hn_i) \le h_c$. A ***spare volume*** is defined as dynamic residual space with dimensions of $l_s \times w_s \times h_s$, where $l_s = (l_c - x_k)$, $w_s = (w_c - y_k)$ and $h_s = (h_c - z_k)$. The *index*2 is defined as:

$$index2 = (l_s \times w_s \times h_s^2)/(ln_i \times wn_i \times hn_i) , \quad (3)$$

which is a measure of the spare-height over the filled rate of the spare volume. In our work, the ref-point with the lowest *index*2 among the feasible ref-points is selected as the packing point for carton π_i. Fig. 8 shows an example to select a ref-points using *index*2.

When packing a carton, we first choose a spare volume with lowest *index*2 to assign it; if no such a spare volume, then we select a ref-point with the highest packing quality *index*1; if no ref-point is feasible to put the current carton, we then put it at (0, 0, H_c), where H_c is the height of the current cage bin.

3.2.3 Detailed Decoding Procedure

Fig. 9 presents the detailed flowchart of the decoding procedure. To measure the quality of a packing solution, ***volume utilization*** (Bischoff and Ratcliff, 1995) is quite often used by the researchers, which is defines as:

$$F_1 = [V_{total} / (L \times W \times H)] \times 100\% \quad (4)$$

where V_{total} is the total volume of the cartons, ($L \times W \times H$) is the volume of the bin used. We also note that Loh and Nee (1992) used "***packing density***", which is defined as:

$$F_2 = [V_{total} / (L_c \times W_c \times H)] \times 100\% \quad (5)$$

where ($L_c \times W_c \times H$) is the volume of the final cage bin, the smallest rectangular box enveloping all the cartons packed in the bin.

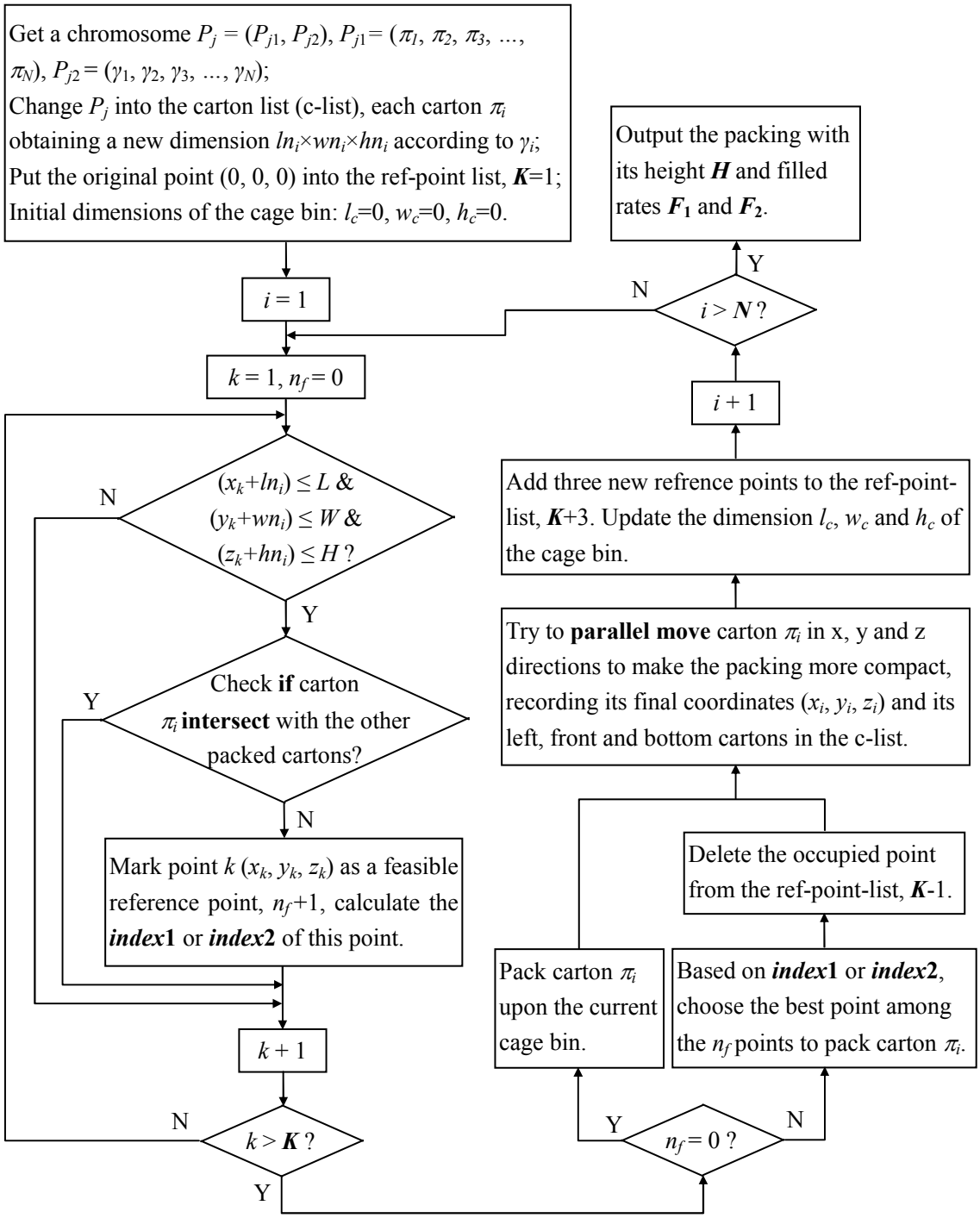


Fig. 9 The decoding procedure

In this work, both F_1 and F_2 are used to measure the quality of a packing. F_1 is equivalent to H, but F_1 is more distinctly to reveal the utilization rate of net space of the bin. Evidently, F_2 reflects the real tightness of the packing, hence should be used as one of the criteria to evaluate the performance the solution methods, regardless of the argument that F_2 overestimates the usual volume utilization (Bischoff and Ratcliff, 1995).

Table 4 presents the packing solution for the sample chromosome in Fig. 4, with the coordinates and relative positions available.

Table 4. The packing solution for the sample chromosome in Fig. 4

			Original size			New dimensions			Coordinates			Relative positions		
i	π_i	γ_i	l_i	w_i	h_i	ln_i	wn_i	hn_i	x_i	y_i	z_i	left	front	bottom
1	**1**	**1**	36	36	36	36	36	36	**0**	**0**	**0**	**0**	**0**	**0**
2	**10**	**1**	33	21	4	33	21	4	**0**	**0**	**36**	**0**	**0**	**1**
3	**6**	**4**	40	31	21	31	21	40	**0**	**36**	**0**	**0**	**1**	**0**
4	**8**	**4**	31	17	16	17	16	31	**31**	**36**	**0**	**6**	**1**	**0**
5	**3**	**5**	54	40	21	21	54	40	**0**	**0**	**40**	**0**	**0**	**10**
6	**5**	**3**	52	33	20	33	52	20	**21**	**0**	**40**	**3**	**0**	**10**
7	**2**	**6**	59	39	20	20	39	59	**54**	**0**	**0**	**5**	**0**	**0**
8	**9**	**4**	26	23	14	23	14	26	**48**	**39**	**0**	**8**	**2**	**0**
9	**7**	**5**	31	31	17	17	31	31	**36**	**0**	**0**	**1**	**0**	**0**
10	**4**	**1**	58	37	21	58	37	21	**21**	**0**	**60**	**3**	**0**	**5**
$L = 80$, $W = 58$, $H = 81$, $F_1 = 74.33\%$; $L_c = 79$, $W_c = 57$, $F_2 = 76.60\%$														

3.3 Sorting Heuristics and Random Search

Sorting the cartons according to certain rules has demonstrated some advantages in providing better starting solutions for meta-heuristics. In fact, with the decoding procedure, we can conduct packing simulation experiments to test the sorting heuristics of ordering the cartons. For instance, we can compare the packing of the carton sequence in decreasing volume order and with various orientations. We also can randomly search a large number of random solutions (chromosomes) within very short computational time and select the best one for real packing. From the subsequent numerical experiments, we can see that the random search combined with a sorting heuristic demonstrates better performance in solving large-size packing problems than MILP model.

4. The Improved GA (IGA)

4.1 Tailored Components of IGA

4.1.1 Generation

At the beginning of the GA, an initial generation of chromosomes are often randomly generated. In IGA, a new mechanism for producing the initial generation is designed considering both "first fit decreasing" and diversity: (1) Six chromosomes with the same carton sequence in decreasing volume order, and each with an orientation sequence of the same orientation γ_j, $\gamma_j = 1, 2, 3, \ldots, 6$. That means, for each chromosome, the cartons are packed in decreasing volume order and in the same orientation. Six orientations are exactly covered by the six chromosomes. (2) The remainder chromosomes are produced randomly as usual.

Let *popsize* denote the number of individuals in the initial generation, which is a critical parameter to control the solution quality, and is given according to problem size. At every iteration of GA, a new generation is re-produced through crossover, mutation and selection, and *popsize* is usually kept constant.

4.1.2 Selection

Selection is the process to select most of the better individuals in each generation for crossover, part of the generation (usually worse individuals) for mutation. Common selection methods include the

roulette-wheel selection (Goldberg, 1989) and the *tournament* method (Goldberg and Deb, 1991). The *tournament* method is adopted in IGA due to the fact that in the *tournament* method the objective value of a chromosome can be directly used as the selection criterion and thus time-saving.

In selection, the crossover and mutation rates are two crucial performance parameters that influence the convergence speed of the GA. Assume the number of the chromosomes that are selected to crossover is *xsize*; the number of the chromosomes that are selected to mutate is *msize*. The ratio C_r = *xsize/popsize* is called crossover rate, often $C_r \in [0.5, 0.9]$; and the ratio M_r= *msize/popsize* called mutation rate, often $M_r \in [0.1, 0.5]$. In general, if M_r increases, GA will converge slowly, thus has more chances to approach better solutions; but if M_r is too large, GA tends to behave like a random search.

4.1.3 Crossover

Crossover is the process during which two parents generate two offspring, such that the children inherit a set of building blocks from each parent. Genetic operators are related to representation schemes. Poon and Carter (1995) presented a survey of crossover operators for ordering applications. Regarding the permutation-based representation, the following crossover operators have been widely used: partially matched crossover (PMX) intending to keep the absolute positions of elements and linear order crossover (LOX) intending to respect relative positions. In IGA, PMX is basically adopted for the carton sequences; but for the orientation sequences, *concomitant* PMX is performed, so that the orientation of a carton is kept unchanged, as shown in Fig. 10. The orientation change for a carton is implemented through the following mutation operator.

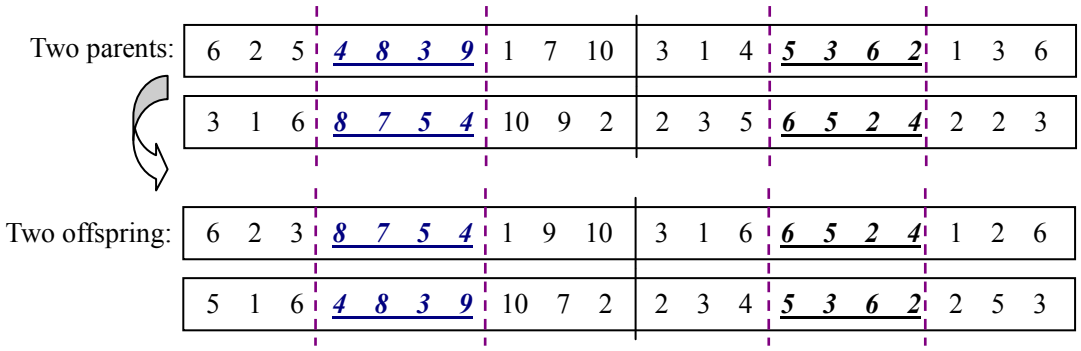


Fig. 10 Illustration of the process of concomitant PMX

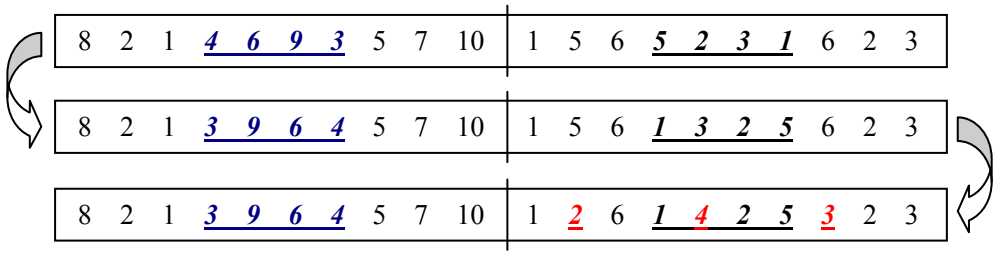
4.1.4 Mutation

Mutation is the process to change some genes in a chromosome and produce a new chromosome. There are also a number of methods to mutate (Gen and Cheng, 1997), such as reversion, swap and insertion.

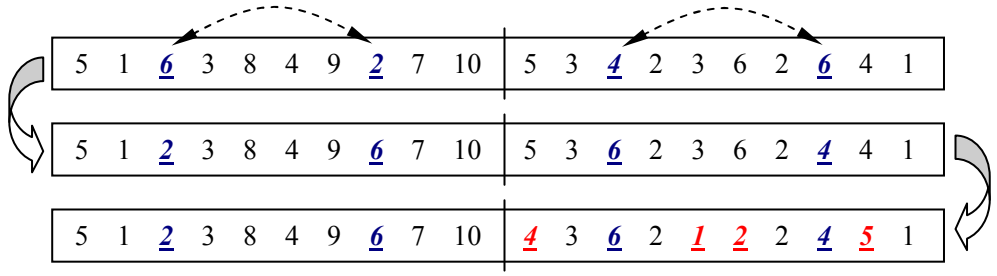
He (2007 and 2009) has conducted a comparative study on reversion and swap mutations in solving symmetrical and asymmetrical combinatorial problems, such as the traveling salesman problem (TSP) and the zero-wait scheduling problems (ZWSP). According to our experience, it is evident that the swap mutation is favorable to the problems with asymmetrical feature, especially in solving the large-size instances. The 3D-BPP is of highly asymmetrical feature, hence the swap mutation is found to be advantageous.

The mutation in this work has two functions: one is to change the order of the cartons to be packed; the

other is to assign new orientations for a carton. Following reversion or swap, random change of orientations is conducted, making it possible for each carton to be packed in different orientations, and allowing the algorithm to select the right orientation for a carton. Fig. 11 illustrates the mutation procedures. Considering the following random change of orientations, the reversion or swap can be only carried out to the carton sequence.



(a) Reversion and random change of orientations



(b) Swap and random change of orientations

Fig. 11 Illustration of the process of mutation

4.1.5 Termination Condition

Termination condition for IGA is that the algorithm stops when the objective value difference between the worst chromosome and the best one in the current generation is equal to or less than a small value, like 0.1 or 0.01, which is dependent on the precision of the objective values.

4.2 Procedure of IGA

With GA components customized, the GA procedure is laid out as follows:

Step 1. Initial generation and evaluation: Produce an initial generation comprising *popsize* chromosomes, and evaluate all the chromosomes using the decoding procedure.

Step 2. Selection: Select *xsize* better chromosomes in each generation to crossover, and *msize* worse chromosomes to mutate.

Step 3. Crossover and Mutation: Conduct *concomitant* PMX to the *xsize* better chromosomes generating *xsize* new chromosomes. Conduct mutation to the *msize* worse chromosomes generating *msize* new chromosomes.

Step 4. Evaluation and new generation: The offspring (*xsize*+*msize* new chromosomes) from crossover and mutation are decoded. Sort the parents and the offspring according to their objective values, and select the former *popsize* best chromosomes as the new generation.

Step 5. Test the termination condition: If the termination condition is met, then go to Step 6; else go

to Step 2 for a new iteration.

Step 6. Output the results: Output the best chromosome and its corresponding packing solution.

4.3 Solution of SM00 by IGA

IGA, inclusive of the decoding procedure is implemented in C language. Table 5 presents the results of 10 runs of IGA for SM00, where the parameter settings are *popsize*=200, *xsize*=160, *msize*=40, with swap mutation used, and *t* denote the iterations in each test *l*.

Table 5. The results of 10 tests of IGA for SM00

Test *l*	*t*	CPU time (s)	H_l	RD_l
1	29	0.187	72	1.408
2	23	0.109	76	7.042
3	28	0.110	74	4.225
4	28	0.109	75	5.634
5	18	0.078	77	8.451
6	23	0.094	74	4.225
7	25	0.094	76	7.042
8	35	0.140	**71**	0.000
9	30	0.125	76	7.042
10	26	0.094	76	7.042
Average	26.5	0.114		5.211

To evaluate the performance of meta-heuristics, a number of computational tests are carried out. Each test is one run of the algorithm with one final solution. The relative deviation (denoted by RD_l) from the best solution of the tests is used to evaluate each test, which is defined as:

$$RD_l = [(H_l - H_{best})/H_{best}] \times 100\%, \qquad (6)$$

where H_l is the objective value of the solution in test *l*, H_{best} is the best of the tests. A lower average relative deviation is generally expected.

5. The Global Search Framework for IGA

Besides the improvements on the genetic algorithm, the global search framework (GSF, He and Hui, 2010) is adopted to achieve better solutions for the large-size and highly heterogeneous packing problems. In this framework, both evolved solutions and dynamic statistic information (the so-called evolutionary gradient) are harnessed to guide the global search.

As well known, single meta-heuristic is problem dependent, its robustness is limited, and premature convergence is another main drawback, especially for cases of large-scale problems. Hybridization, parallelization and other search frameworks are the main effective measures to enhance the robustness and to overcome the drawback of *premature convergence*. The proposed framework aims to address two issues: the first goal is to solve larger problems within reasonable computational time; the second is to make the algorithm more robust and less calibration effort. In fact, this framework has realized some parallel ideas in the sequential environment.

It is well known that every run of the exact model for the same problem obtains the same solution; but different runs of a meta-heuristic algorithm may obtain different solutions. That is why we often run the meta-heuristic algorithm several times for the same problem and select the best solution for use. However, in practical application, the user is more willing to run the packing program for only one time and then obtain the expected packing solution.

The distinction between the solutions from a number of computational tests is the impetus for us to pursue much better solutions. In other words, the existing distinction implies that much better solutions are possible if more computational tests are conducted. At the moment of seeing the distinction between the solutions, what we should consider is *how* to effectively conduct the subsequent computational tests and *when* to stop computing. Regarding these two questions, we proposed the concept of *evolutionary gradient* and then devised a *global search framework* (He and Hui, 2010). In this framework, when the evolutionary gradient vanishes, the algorithm stops. In this work, five scenarios of GSF are devised and tested, and the best scenario is selected for the practical 3D packing problem.

5.1 Evolutionary Gradient

As well known, the concept of gradient is adopted in numerical methods, where the gradient is regarded as the criterion to determine the termination of the algorithm – if the gradient turns to be zero, the optimum or local optimum is deemed to have been found. Similarly, in evolutionary algorithms, the distinction between the solutions obtained so far from computational tests can be treated as the criterion to determine the termination of the algorithms, thus the concept of ***evolutionary gradient*** is proposed.

When a number of computational tests are carried out to evaluate the given algorithm, the relative deviation (*RD*) of each test can be calculated with respect to the ***current best*** solution:

$$RD(\%) = [(objective\ \ value - current\ \ best)/current\ \ best] \times 100\%. \quad (7)$$

Let N_L denote the number of computational tests of the single IGA. The mean relative deviation is defined as ***evolutionary gradient***, denoted by E_{dev}:

$$E_{dev} = \frac{1}{N_L}\sum_{l=1}^{N_L} RD_l\ , \quad (8)$$

where RD_l is the relative deviation of test l calculated by Eq. 7. In fact, E_{dev} can be regarded as an important sign which indicates whether it is necessary to conduct more computational tests for better solutions. If the N_L tests obtain solutions with the same objective value, then $E_{dev} = 0$, which implies that it is not necessary to carry out more tests; otherwise, $E_{dev} > 0$, which indicates the probability of better solutions through additional computational tests. The initial N_L depends on the problem size. For large-size problems, we usually witness $E_{dev} > 0$, and additional runs are expected to provide better solutions. The additional N_L can be determined by the subsequent Eq. 9.

5.2 The Global Search Framework

Based on the concept of evolutionary gradient, three phases of computational tests are organized. In ***phase* 1**, which is called rough search, N_L tests of IGA are carried out to see whether the evolutionary gradient exists ($E_{dev} > 0$). If yes, go on with ***phase* 2** which consists of one or more refining searches, each refining search involves $N_L = f(E_{dev})$ tests of IGA, until the evolutionary gradient turns to be zero ($E_{dev} = 0$). Despite the zero evolutionary gradient approached at the end of *phase* 2, ***phase* 3** can follow with further ambition to find more refined solutions. In *phase* 3, if no new solution is found, only one refining search is to

be executed; otherwise, with new solution(s) obtained, several refining searches are to be conducted until the new evolutionary gradient becomes zero. The rough search and refining searches can be considered as local searches, while the whole organization of the three phases is called the global search framework. Each rough search or refining search is seen as a *global iteration*, denoted by *g*; and a single IGA test as a *local iteration*, denoted by *l*.

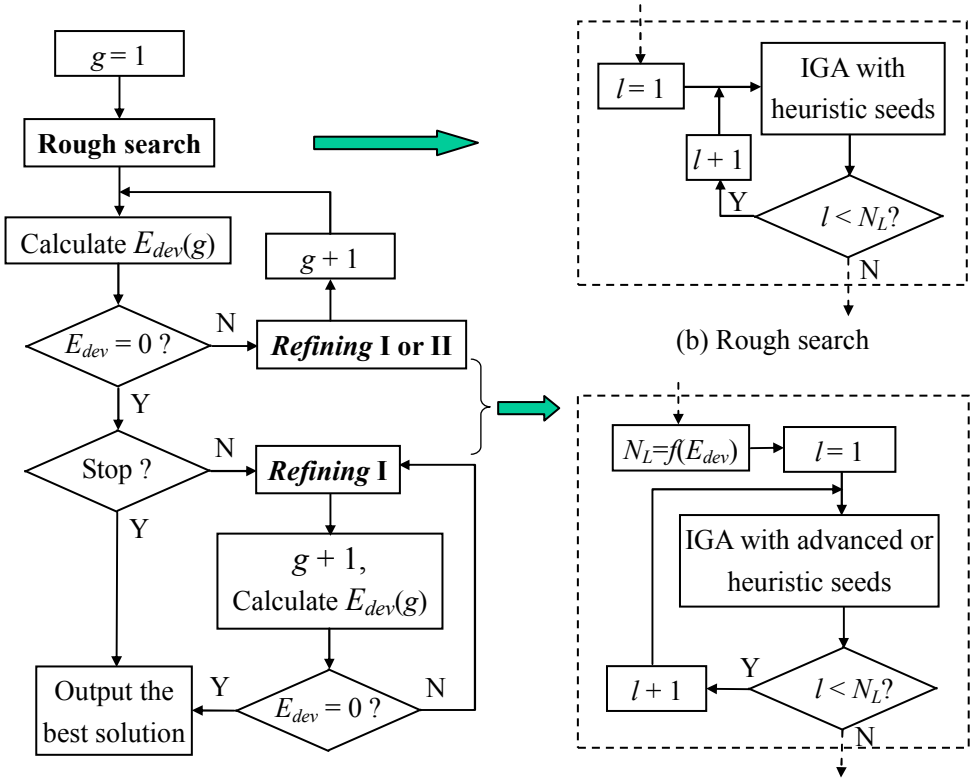


Fig. 12 Flow chart of the global search framework

The procedure of the GSF is shown in Fig. 12(a). The rough search, as shown in Fig. 12(b), comprises N_L tests of IGA, each of which starts with an initial generation including the ***six heuristic chromosomes*** and (*popsize*-6) random chromosomes, where the six heuristic chromosomes can be treated as excellent "breed" or "seeds" to expedite the evolution. Each refining search, as shown in Fig. 12(c), comprises N_L=$f(E_{dev})$ tests of IGA, each test starts with an initial generation including ***advanced chromosomes*** from last global iteration and random chromosomes. $N_L = f(E_{dev})$ can be determined by Eq. 9,

$$f(E_{dev}) = \begin{cases} N_{LL}\left(1+\dfrac{1}{E_{dev}}\right), & if \quad E_{dev} \geq \dfrac{N_{LL}}{N_{LU}-N_{LL}}; \\ N_{LU}, & else \end{cases} \tag{9}$$

where N_{LL} and N_{LU} are respectively the lower and upper bounds of the number of IGA tests. N_L increases with the decreasing of E_{dev}, but has an upper bound. N_L may also be set as a constant parameter in the search process. In this work, N_L is set as a constant number.

It should be emphasized that, in addition to GAs, other hybrid solution methods can also be incorporated into the GSF. Using such a framework, much better solutions can be obtained in succession,

until the evolutionary gradient vanishes.

5.3 Scenarios of GSF Considering Evolution Speed

The "*refining I*" and "*refining II*" in Fig. 12 are two kinds of refining searches with different breed of selection strategies. For *refining I*, only the best solution found in iteration g is put into the initial generation of each test in iteration g+1; but for *refining II*, all the N_L evolved solutions obtained in iteration g are put into the initial generation of each test in iteration g+1. For *refining II*, considering N_L advanced seeds introduced into a new evolutionary environment, it seems that the six heuristic seeds are not necessary to the coming global iterations, and that N_L can be reduced to a relative smaller number, both of which can expedite evolution or solution speed. With these considerations, the scenarios presented in Table 6 are laid out for simulation experiments to determine which one is the best. Subsequent numerical experiments demonstrate that, in *refining II*, indeed N_L can be reduced to a relative smaller number, but the six heuristic seeds are still vital to the coming global iterations.

Table 6. Scenarios of GSF for simulation experiments

Scenario	Option of *phases* 1, 2 or 3, *refining I* or *II*	N_L	Heuristic seeds or not	Performance
GGA1	*Phase* 1+2 (*refining I*): slow and nice solution	30	Yes	***
GGA2	*Phase* 1+2 (*refining II*): fast	10	No	**
GGA3	*Phase* 1+2 (*refining II*): fast	20	No	***
GGA4	*Phase* 1+2 (*refining II*): fast and nice solution	20	Yes	****
GGA5	*Phase* 1+2 (*refining II*)+3 (*refining I*)	20	Yes	*****

Furthermore, one may doubt the necessary existence of *phase* 3 in GGA5. For easy instances *phases* 1 and 2 are sufficient to obtain satisfactory solutions, indeed *phase* 3 additionally leads to another refining search wasting some CPU time. However, for hard instances *phase* 3 gives rise to more chances to find much better solutions. As known from the computational experiences in exact methods, even if the MILP model has found the best solution to an instance, nevertheless, a long time is still needed to prove the optimality. Compared with such a long time, the time for an additional refining search in *phase* 3 is very short.

5.4 Solutions of SM00 by GSF under Diverse Scenarios

Table 7 presents solutions of SM00 obtained under the various scenarios. GGA1 finds the best over a longer time because of the bigger N_L and only one advanced seed for further evolution. GGA2 has a short time due to the smaller N_L and N_L advanced seeds for further evolution, but with a dissatisfactory solution. The difference of GGA3 from GGA2 is only that N_L is doubled, thus the solution is improved. In GGA2 and GGA3, with N_L advanced seeds for the coming global iterations, the evolution speed has indeed been expedited. The difference of GGA4 from GGA3 is that, six heuristic seeds together with N_L advanced seeds are used for the coming global iterations, consequently further enhancing the evolution speed. GGA5 with the merits of both GGA4 (fast speed) and GGA1 (ability for better solution) has found the best solution in shorter time.

Table 7. Solutions of SM00 by GSF under diverse scenarios

Scenario	N_L	G	CPU time CT (s)	H
GGA1	30	1+2=3	9.70	70
GGA2	10	1+2=3	2.91	72
GGA3	20	1+2=3	5.31	71
GGA4	20	1+2=3	4.95	71
GGA5	20	1+2+1=4	7.20	70

6. Computational Experiments

In this section, we apply the proposed methods to the cases solved in Wu et al. (2010), and expand the experiments to some larger and more heterogeneous new cases. Moreover, GGA5 is tested with the benchmark instances BR01~10 (Bischoff and Ratcliff, 1995).

6.1 Problem Data

In Wu et al. (2010), 10 packing cases of small bins ($L\times W\times H_{max}$=80×58×95 for SM04 and SM05, and $L\times W\times H_{max}$=78×57×95 for other small cases) and 27 cases of big bins ($L\times W\times H_{max}$=116.5×97×95) were collected from a company where the workers ***manually pack*** cartons in the bin based on a rule of thumb and their own packing experience. In these cases, there exist 13 types of cartons, each case involves no more than 9 types of cartons, and the number of cartons to be packed into a bin is no more than 40 ($N \le 40$). We denote the original GA in Wu et al. (2010) as OGA.

The new cases in this work include 6 packing cases of small bins ($L\times W\times H_{max}$=80×58×95) and 12 cases of big bins ($L\times W\times H_{max}$=116.5×97×95). In contrast to the old small-bin cases which contain 3 ~ 5 carton types, the new small-bin packing involves 8~11 carton types; and each new big-bin case involves 7~13 carton types instead of 2~9 carton types for the old big-bin cases. For the new cases, up to 40 cartons are loaded into a small bin, and up to 80 cartons are packed into a big bin.

To evaluate the performance of the methods for more heterogeneous instances, two particular big-bin packing cases, BM40 and BM41, are tested. In BM40 and BM41, 20 cartons to be packed involve 20 carton types, i.e., each carton belongs to a different carton type.

6.2 Algorithm Parameter Setting

The IGA and the global search framework are coded in C language and compiled with Microsoft Visual VC++ 8.0. All the numerical experiments are conducted on a computer with an Intel T7500 2.20GHz Duo CPU and 1.96GB RAM.

Parameter setting is an important issue which impacts the performance. In the GA, the setting of the population size (denoted by *popsize*) in a generation depends on the problem size. The crossover rate (C_r) and mutation rate (M_r) are two crucial factors that influence solution quality and convergence speed. In the GSF, the number (N_L) of IGA tests within a global iteration is also an important parameter to determine the solution time and quality.

In OGA, M_r is given differently according to the mutation methods, reversion (Rvs) or swap (Swp), and

a fixed number of iterations (denoted by *iter.*) is set for each OGA test. Specifically, in OGA with Rvs mutation, (*popsize*, C_r, M_r, *iter.*) = (120, 0.9, 0.5, 200) for small-bin cases (SM), and (200, 0.9, 0.5, 500) for big-bin cases (BM); in OGA with swap mutation, (*popsize*, C_r, M_r, *iter.*) = (120, 0.9, 0.9, 200) for SM, and (200, 0.9, 0.9, 500) for BM. A notable setting of M_r is that it was set up to 0.9! The best solution by OGA is selected from 20 computational tests.

In IGA, M_r has been tuned from 0.1 to 0.5, with decreasing convergence speed, but not improving the solution quality so much, thus M_r is given as 0.2. Due to the termination condition of *until-no-improvement* adopted in IGA, the computation of the algorithm stops at the iteration of "no improvement". So in IGA, (*popsize*, C_r, M_r) = (200, 0.8, 0.2) for SM, and (400, 0.8, 0.2) for BM. The best solution by IGA is also selected from 20 computational tests.

In the different scenarios of GSF, (*popsize*, C_r, M_r) = (200, 0.7, 0.3) for SM, and (400, 0.7, 0.3) for BM, with a little change of C_r and M_r to allow the algorithms to search more combinations of carton orientations. N_L is set as 30 for GGA1, 10 for GGA2, 20 for GGA3, GGA4 and GGA5.

We also list the search results by the MILP, the manual packing (when possible) and a controlled random search. The MILP model is limited by a running time of 7200 seconds (two hours). In the random search (Rdm) combined with sorting heuristics based on the decoding procedure, 500 feasible solutions are searched for SM, 1000 for BM, including the six heuristic solutions.

6.3 Results and Analysis

With the parameter setting, numerical experiments have been performed. The comparative study is carried out from three aspects: the packing height (H), filled rates (F_1 and F_2) and computational time (CT). Tables 8 and 9 show the best solutions (H) of the cases obtained via a range of methods or scenarios. The best solution for an instance by OGA or IGA is selected from 20 computational tests, so the computational time for OGA or IGA is the total running time of the 20 tests. The other methods, including the random search, MILP and the scenarios of GSF, obtain the best or optimal solution for an instance at the end of their execution. Tables A-1 to A-5 in Appendix A provide the corresponding filling rates and computational times of these methods.

6.3.1 Comparison of Packing Height

From Tables 8 and 9, the following observations can be noted:

(1) Random search vs. manual packing. For the old cases, the manual packing heights are presented in Table 8. Compared with the manual packing, the random search has found better solutions for 22 cases among the 37 cases. Moreover, from filled-rate comparison (see Table A-1), it is noted that the random search gets a higher average F_1 than the manual packing. The random search is completed within one second for the old cases, or within 3 seconds for new cases (see Tables A-4 and A-5). For the moderate size instances (BM13~27), the random search approaches almost the same average F_1 as the manual packing. Hence, for the new cases where the manual packing data are not available, the results by the random search are treated as the ones by the manual packing.

(2) Reversion mutation vs. swap mutation: Both in OGA and IGA, the two mutation methods have compared, and corresponding results are presented in Tables 8 and 9 from which it is verified again that swap mutation is more favorable to the solution quality of the algorithms.

(3) IGA(swp) vs. OGA(swp). IGA(swp) performs better than OGA(swp) both in solution quality and speed. For 31 cases among the 55 cases, IGA(swp) finds better solutions than OGA(swp). Only for BM24, IGA(swp) obtains a worse solution than OGA(swp).

(4) Different scenarios of GSF: Five scenarios of GSF as shown in Table 6 are also tested with all the cases to reveal their performance in solving diverse instances. The experiment results show that GGA1 and GGA4 have complementary merits: GGA1, further refining searches (local search actually) with one evolved solution consecutively, can find good solutions but with longer search time; GGA4, further refining searches with a number of evolved solutions consecutively, can obtain good solutions with higher speed. GGA5, first conducting refining searches with a number of evolved solutions and then with one evolved solution, has the merits of both GGA4 and GGA1, thus has found new solutions for most of the cases. For SM04, SM06 and SM10, GGA5 finds the same proven optimal solutions as MILP model, but the other GA-based methods have not found these solutions.

(7) GGA5 vs. GAs. For most of the instances, the GGA5 has achieved better solutions than OGA (see Tables 8 and 9, better solutions by GGA5 vs. OGA are marked with $^{\#}$), but the computational times are much shorter (see Tables A-4 and A-5). Only the solution ($H = 97$) for SM15 by GGA5 exceeds H_{max}, leaving this instance as a challenge for coming work. Furthermore, compared to IGA, GGA5 has achieved better solutions for almost all of the new cases (see Table 8, better solutions by GGA5 vs. IGA are marked with $^{\$}$), which demonstrates that the GSF actually helps improve the solution quality, especially in solving the large-size cases.

Table 8. The best solutions (heights) of the old cases by different methods MILP

					OGA		IGA		GSF				
	N	**Manual**	**Rdm**	**MILP[&]**	**Rvs**	**Swp**	**Rvs**	**Swp**	**GGA1**	**GGA2**	**GGA3**	**GGA4**	**GGA5**
SM01	8	91	92	80*	80	80	80	80	80	80	80	80	80
SM02	8	56	40	37*	37	37	37	37	37	37	37	37	37
SM03	9	69	53	47*	47	47	47	47	47	47	47	47	47
SM04	9	91	103	82	83	83	83	83	82	83	83	83	82[#]
SM05	10	56	61	52*	52	52	52	52	52	52	52	52	52
SM06	12	53	48	38*	40	40	40	40	40	40	40	40	38[#]
SM07	17	70	62	62	62	62	62	62	62	62	62	62	62
SM08	17	91	77	71	68	69	64	62	62	64	62	62	62[#]
SM09	19	71	62	62	52	52	51	51	51	51	51	51	50[#]
SM10	21	85	78	62	64	63	64	63	62	63	62	62	62[#]
BM01	5	91	84.1	84.1*	84.1	84.1	84.1	84.1	84.1	84.1	84.1	84.1	84.1
BM02	11	91	99	83.1	84.1	84.1	84.1	84.1	84.1	84.1	84.1	84.1	84.1
BM03	12	45	47	32*	32	32	32	32	32	32	32	32	32
BM04	13	91	98	77.4	83.4	83.4	83.4	77.4	77.4	79	77.4	77.4	77.4[#]
BM05	15	91	90	79	80	79	80	79	78	79	79	78	78[#$]
BM06	16	67	47	40	40	40	40	40	40	40	40	40	40
BM07	21	48	40	40	38	40	38	38	37	37	37	37	37[#$]
BM08	22	59	54	53	53	51	53	51	48	51	50	48	48[#$]
BM09	23	94	80	82	71	72	72	71	71	71	71	71	71[#]
BM10	23	91	62	63.1	61	61	61	61	61	61	61	61	61
BM11	25	91	80.1	78	73.4	77.4	73.4	73.4	73.4	73.4	73.4	73.4	73.4[#]
BM12	26	94	66	68	61	61	61	61	61	61	61	61	61
BM13	27	83	85	78	73	71	69	69	68	70	69	68	68[#$]
BM14	28	95	94	88.4	79	78.4	78.4	78.4	78.4	80	78.4	78.4	78.4
BM15	29	91	94	87	77	75	75	74	74	74	73	73	73[#$]
BM16	29	63	54	52	48	48	47	47	47	47	47	47	47[#]
BM17	29	91	101	88.4	82	80	79	78.4	78.4	79	80	80	78.4[#]
BM18	30	70	70	73.4	67	63.1	63	63	63	63	63	63	63[#]
BM19	30	94	78.4	79	63.4	63.1	63.4	63.1	63	63	63	63	63[#$]
BM20	30	94	94	87	80	79	79	79	78	80	78	78	78[#]
BM21	30	95	94	87	82	78.4	78.4	78.4	78.4	78.4	78.4	78.4	78.4
BM22	31	91	85	87	78	78	78	78	77	78	78	77	77[#$]
BM23	33	95	94	93.4	78	78.4	78	77.4	77.4	77.4	77.4	77	77[#$]
BM24	35	91	94	89	84.1	80	81	82	79	85	79	85	79[#$]
BM25	36	94	87	79	76	76	75	73	75	77	71	71	71[#$]
BM26	38	91	101	97	85	82	80	80	80	80	80	80	80[#]
BM27	40	91	100	109	85	85	84	85	84	84	85	84	84[#$]

[&] *Solved by CPLEX 11.2*;

***** *Proven optimal solution*;

[#] *New solution compared to OGA*;

[$] *New solution compared to IGA*.

Table 9. The best solutions (heights) of the new cases by different methods

				OGA		IGA		GSF				
	N	**Rdm**	**MILP[&]**	**Rvs**	**Swp**	**Rvs**	**Swp**	**GGA1**	**GGA2**	**GGA3**	**GGA4**	**GGA5**
SM00	10	80	68*	72	72	72	71	70	72	71	71	70[#$]
SM11	11	96	80	84	88	83	83	80	83	80	83	80[#$]
SM12	20	110	90	89	89	89	89	89	90	89	89	87[#$]
SM13	20	113	104	98	98	98	95	95	98	97	96	94[#$]
SM14	31	109	105	91	91	90	90	90	90	89	89	89[#$]
SM15	40	126	118	106	103	100	100	98	100	100	100	**97**[#$]
BM28	13	77.4	61	64	64	63.1	63.1	61	63.1	61	61	61[#$]
BM29	30	106	100	90	89	91	89	89.4	90	89.4	87.4	87.4[#$]
BM30	35	96	99	89	85	86	85	85	84	83	83	83[#$]
BM31	40	87	98	78	78	78	76	76	78	78	76	75[#$]
BM32	45	99	102	97	89	90	88	88	89	89	89	88[#]
BM33	50	88	95	76	73	72	71	71	71	71	71	70[#$]
BM34	55	108	116	99	96	94	94	93	94	91	91	90[#$]
BM35	60	86	95	81	78	74	74	73	74	74	74	73[#$]
BM36	65	91	104	87	83	83	82	80	82	81	81	79[#$]
BM37	70	105	112	91	89	88	88	86	88	87	88	86[#$]
BM38	75	96	107	91	90	88	88	86	88	87	86	86[#$]
BM39	80	107	121	102	95	93	93	90	92	90	90	90[#$]
BM40	20	107.4	96	90	92	91	91	90	90	90	90	90[#$]
BM41	20	103.1	87	86	85.4	85	85	84.1	85.4	85	85.4	84.1[#$]

[&] *Solved by CPLEX 11.2*;

* *Proven optimal solution*;

[#] *New solution compared to OGA*;

[$] *New solution compared to IGA*.

6.3.2 Comparison of Filled Rates F_1 and F_2

Tables A-1~3 show the filled rates comparison of the methods. As defined in §3.2.3, F_1 is calculated with respect to the net volume of the bin used, while F_2 is calculated with respect to the volume of the final cage bin, which reflects the actual degree of the packing tightness.

With the packing height H available and given $L\times W$ of the bin, and the total volume of the packed cartons, F_1 can be calculated using Eq. 4. But to calculate F_2 with Eq. 5, $L_c\times W_c$ of the cage bin has to be given. For manual packing with only the height H, rather than the detailed solution with coordinates (x, y, z), $L_c\times W_c$ is impossible, hence F_2 is not given for manual packing. For the solutions by MILP and OGA, a conversion program is designed to change the solutions into new format with F_1, F_2 and relative position. For the MILP model, it is found that most of them have the equal F_1 and F_2. This phenomenon is plausible from the constraints and the Big-M used in the MILP model. The results by the random search are to be compared with those by manual packing and MILP, thus without F_2 presented.

The cases studied can be categorized into five groups as listed in Table 10, where the old cases are divided into two groups, and the new cases into three groups.

Table 10. Groups of the cases studied

Group	Cartons *N*	Bin Type	Results	Old or New Cases
1	$N \le 26$	small or big-bin	Tables 8, A-1, A-4	old
2	$27 \le N \le 40$	big-bin	Tables 8, A-1, A-4	old
3	$10 \le N \le 40$	small-bin and BM28, BM40, BM41	Tables 9, A-2, A-5	new
4	$30 \le N \le 80$	big-bin	Tables 9, A-2, A-5	new
5	$10 \le N \le 20$ Heterogeneous	small or big-bin	Tables 9, A-3, A-5	new

The old cases in Table A-1 can be classified into two groups, ***group* 1** ($N \le 26$) with a small number of cartons to be packed, ***group* 2** ($27 \le N \le 40$) with a moderate number of cartons.

In ***group* 1** ($N \le 26$), a lower average F_1 (65.68% for small-bin packing and 65.71% for big-bin packing) by the manual operation for both small-bin packing and big-bin packing; as a contrast, the random search achieves a higher average F_1 (72.05% and 74.25%). In ***group* 1**, MILP model can obtain much higher average F_1 (81.99% and 81.06%) than the random search. The average F_1 by OGA (82.88% and 82.61%) and IGA (83.99% and 83.95%) increases a bit. The average F_1 by GGA5 (84.84% and 84.64%) increases about 2% against OGA. Fig. 13 shows the situation of the average F_1 for ***group* 1**, where it can be seen that MILP, OGA, IGA and GGA5 increase F_1 by 15%~ 19% with respect to the manual packing.

In ***group* 2** ($27 \le N \le 40$), the packing by the manual operation, the random search and MILP model has similar lower average F_1 (74.31%, 74.48% and 77.73% respectively); but OGA, IGA and GGA5 increase by 14%~16% with respect to the manual packing. For these moderate-size instances, our random search obtains almost the same average F_1 as the manual packing. That is why the packing solutions by the random search for the new cases can be regarded as the real manual packing solutions. It should be noticed is that GGA5 achieves up to an average F_1 of 90%. Fig. 14 shows the situation of the average F_1 for ***group* 2**. It is obvious that GA-based methods take great advantage over the others.

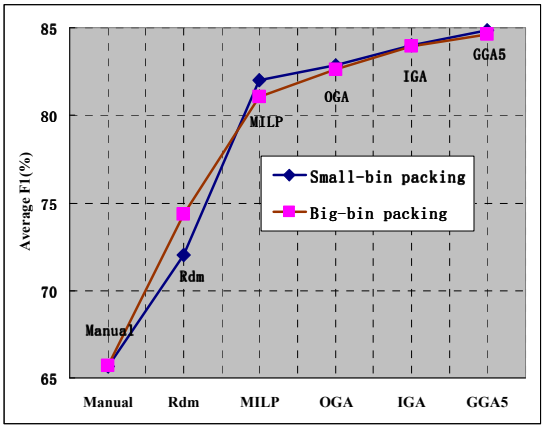


Fig. 13 Average F_1 for small-size old cases (***group* 1**)

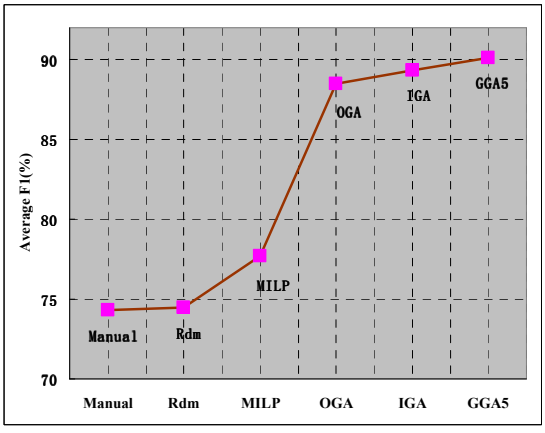


Fig. 14 Average F_1 for moderate-size old cases (***group* 2**)

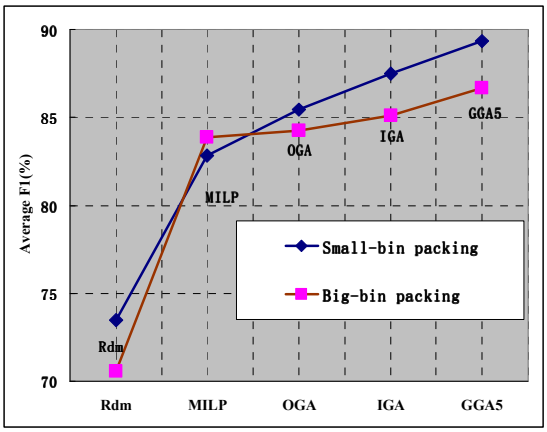


Fig. 15 Average F_1 for small-size new cases (***group* 3**)

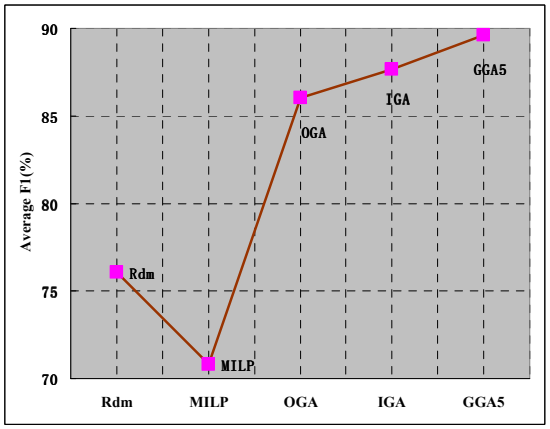


Fig. 16 Average F_1 for large-size new cases (***group* 4**)

The new cases in Table A-2 can also be classified into two groups: the small-bin packing cases ($10 \le N \le 40$) and the big-bin cases (BM28, BM40 and BM41) with more heterogeneous cartons ($13 \le N \le 20$) are put into ***group* 3** (small-size instances in Table A-2), the big-bin packing cases (BM29~39) with a large number of cartons are put into ***group* 4** ($30 \le N \le 80$ in Table A-2, big-bin packing).

In ***group* 3**, the situation of the average F_1 is similar to ***group* 1**, it is also can be seen that MILP, OGA, IGA and GGA5 achieve higher average F_1 (83%~90%, see Fig. 15).

The situation of ***group* 4** varies: the MILP model's performance declines severely. For this group of large-size instances, OGA, IGA and GGA5 demonstrate distinct advantage over the other techniques, as shown in Fig. 16.

Table A-3 provides statistic results of the heterogeneous cases in which each carton belongs to a different carton type. These cases are put into ***group* 5** (with $10 \le N \le 20$ different cartons). For this group, MILP shows its dominance with higher average F_1 over OGA and IGA. However, GGA5 is still the best! Fig. 17 shows the situation of the average F_1 for ***group* 5.**

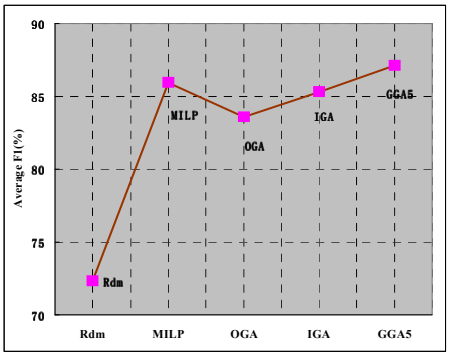


Fig. 17 Average F_1 for heterogeneous ***group* 5**

From ***group* 1** to ***group* 5**, IGA obtains higher average F_1 than OGA, and GGA5 higher than IGA logically and rationally. Another observation worth noticing is that, when OGA and IGA obtain the same packing height (H) for a packing case, thus the same value of F_1, but IGA achieves higher value of F_2 than OGA (see the cases SM01, 07 and 10, BM02, 06, 08, 10, 12, 14, 19~22, 27, 29 and 30, totally 16 cases). That means IGA pack cartons much tighter than OGA owing to the improved decoding procedure, especially because of *index2* and new parallel moving strategy adopted.

6.3.3 Comparison of Computational Time

Computational time is an important factor to evaluate an algorithm. Tables A-4 and A-5 present the computational times (*CT*) of five methods in solving the old and new instances, and the time is in seconds. From these two tables, we can see the following facts:

(1) The random search based on the decoding procedure can be finished within very short time (CT_R < 3s). So the decoding procedure can be used for packing simulation experiments conveniently.

(2) For most cases, MILP model reach the limited running time, 2 hours (7200s). Taking the volume utilization into account, MILP model can only find satisfactory solutions for the small-size instances in ***groups* 1, 3** and **5** within the limited time.

(3) The solution speed of IGA is about **14 times faster** than that of OGA. From the comparison of filled rates, IGA achieves higher filled rates (F_1 and F_2) than OGA.

(4) The solution speed of **GGA5** is about **4 times faster** than that of OGA. What is more, GGA5 improves the average F_1 by approximate 2~4% with respect to OGA. When OGA is applied to the large-size instances BM37, 38 and 39, the solution times are 8437 seconds (**2 hours** and 20 minutes), 11189 seconds (about **3 hours**), 14601 seconds (about **4 hours**) respectively, all over 2 hours. But for these three cases, **GGA5** has completed the search within **2016** seconds (about **33 minutes**).

(5) The average number of global iterations *g* in GGA5 is about 4 (3.84 for the old cases, 4.45 for the new cases). For the cases BM08, BM24 and SM15, the global iterations *g* of GGA5 are 6, 9 and 7 respectively. These data reveal the fact that the global iterations of GGA5 are indeed essentially necessary to achieve the high quality solutions.

6.3.4 Comprehensive Comparison

From the comparison and analysis of the packing height (*H*), filled rates (F_1 and F_2) and computational time (*CT*), the following comprehensive recognitions can be drawn out:

(1) The random search has an equal or even higher average F_1 than the manual packing, with very short search time (less than 3 seconds). This means that the decoding procedure is effective in simulating a packing solution.

(2) MILP model can only obtain satisfactory solutions for the small-size instances ($N \leq 26$) within acceptable time (e.g. 2 hours); but for moderate-size or large-size instances, the exact model, which is limited by the current computing resources, such as computational time, memory and CPU speed, shows its apparent inferior position in contrast to the GA-based methods.

(3) OGA is indeed effective and efficient in solving the original small- and moderate-size instances, but has still left a larger space for further improvement. For the new cases including more heterogeneous cases, OGA shows its drawbacks in terms of solution speed and quality. As a contrast, IGA witnesses improvement not only on solution speed but also on solution quality.

(4) The novel GSF (GGA5) has obtained the best solutions for **most of the old and new cases**

among the methods tested, within acceptable time (less than 33 minutes), hence is the best method tested in our work.

It should also be mentioned that in Wu et al. (2010), the achieved bin height for each test case was fed into the container loading algorithm provided by Martello et al. (2007) for runs of two hours. In most cases, the Martello et al. (2007) code was not able to provide a solution to load all the cartons within the given height limit. Since the results provided in this paper are better than those of Wu et al. (2010), it can be reasonably concluded that it would be even more difficult for the algorithm described in the Martello et al. (2007) to provide feasible results under the same settings in Wu et al. (2010).

6.4 Tests on 3D-SPP Benchmark Instances

6.4.1 Result Comparison

As explained earlier, the problem proposed in this paper is similar to the 3D-SPP but has some distinct features. For heuristics algorithms, it is common to exploit the problem features to ensure effectiveness and efficiency. Although the proposed algorithms did not take 3D-SPP into consideration at the design phase, one of the algorithms, GGA5, is applied to testify its effectiveness on some of the strip packing instances from Bischoff and Ratcliff (1995) as an additional way for benchmarking. We have to mention that the use of genetic algorithms actually has some challenges when the number of cartons to be packed increases and therefore the tests on the 3D-SPP benchmarking instances can be viewed as a 'stretch' for the current algorithms. We first apply the GGA5 to the selected 3D-SPP cases without imposing the orientation constraints (marked as 'w/o orient. con.') and subsequently consider the orientation constraints (marked as 'with orient. con.') in both generating chromosomes and mutating according to the specified orientation constraints, where the GA parameters remain the same, ($popsize$, C_r, M_r) = (400, 0.7, 0.3). We then compare the results with those presented in Bortfeldt and Mack (2007) and Allen et al. (2011) where some of the state of the art results are available. We also notice that a working paper by Bortfeldt and Jungmann (2010) has seemingly improved the results significantly. Table 11 shows the volume utilization (F_1) for all these methods.

For GGA5, most of the cases where the orientation constraints are not imposed generate slightly better results than those with orientation constraints imposed, except for BR03 and BR10. Look across GGA5 with orientation constraints, Bortfoldt and Mark (2007) and Allen et al. (2011), the volume utilizations achieved by GGA5 for BR01~03 and BR05 are higher than those obtained by Bortfeldt and Mack (2007) and Allen et al. (2011). We also get close results for the rest cases. In terms of overall average volume utilization, ours is also higher than that of Bortfeldt and Mack (2007) and Allen et al. (2011). Computational time wise, ours is longer than that of Bortfeldt and Mack (2007) and Allen et al. (2011) due to operations involved in the genetic algorithms. The results from Bortfeldt and Jungmann (2010), where the algorithm tries to solve a series of container loading problems iteratively (in reduced length), seem to be the best among all methods presented in Table 11.

Table 11. Results for the test cases BR01~10 from Bischoff and Ratcliff (1995)

	Types	*N*	GGA5 F_1 (%)		B&M (2007)	Allen et al. (2011)	B&J (2010)
			w/o orient. con.	with orient. con.	F_1 (%)	F_1 (%)	F_1 (%)
BR01	3	139.4	***91.70***	**91.52**	87.3	90.0	*94.2*
BR02	5	140.1	***91.85***	**90.84**	88.6	89.6	*94.8*
BR03	8	135.4	***90.53***	**90.53**	89.4	89.9	*94.7*
BR04	10	132.2	***90.31***	89.49	**90.1**	88.8	*94.2*
BR05	12	127.8	***89.91***	**89.57**	89.3	88.5	*93.9*
BR06	15	133.8	***89.75***	89.47	**89.7**	88.6	*93.5*
BR07	20	129.2	*88.59*	88.23	**89.2**	88.7	*92.6*
BR08	30	138.0	*87.44*	87.32	87.9	**88.3**	*91.8*
BR09	40	127.5	*86.54*	86.19	87.3	**87.9**	*91.5*
BR10	50	129.2	*86.30*	86.41	87.6	**87.9**	*91.3*
Average	19.3	133.3	***89.29***	**88.96**	88.64	88.82	*93.2*

6.4.2 Difference Analysis

In terms of measuring difficulty of packing problems, Smith-Miles and Lopes (2012) review some common combinatorial optimization problems. One particular factor proposed for bin packing in that paper is the average carton volume against the bin volume as one of the measurements for difficulty. Fig. 18 presents the carton volume against the bin volume for each carton in cases of SM14, BM31, BM39, BR06-008 and BR10-009. It can be observed that in terms of number of cartons for packing, SM13 and BM34 (and most other cases investigated in this paper) represent a category of problem with relatively small number of cartons but significantly differ in terms of carton sizes and number of cartons available per carton type. On the contrary, BR06-008 and BR10-009 represent typical strip packing problems, where more cartons are to be packed and each carton type usually includes quite a number of cartons; moreover, the size difference between types is relatively lower. These features were effectively utilized by the algorithms proposed in Bortfeldt and Mack (2007) and Allen et al. (2011), and other algorithms with effective layer-packing and strip building.

Fig. 19 further indicates the difference between our investigated cases and those of traditional 3D-SPP, where the maximum, minimum and average carton volumes against the 'bin' (container) volume are presented. It can be observed that the maximum cartons in some selected cases from this paper have a volume that is more than 20% of the 'bin' itself; most of the maximum cartons are more than 10% of the 'bin' volume, while none of the cases selected from BR01 to BR10 has a carton volume more than 3% of the overall container volume.

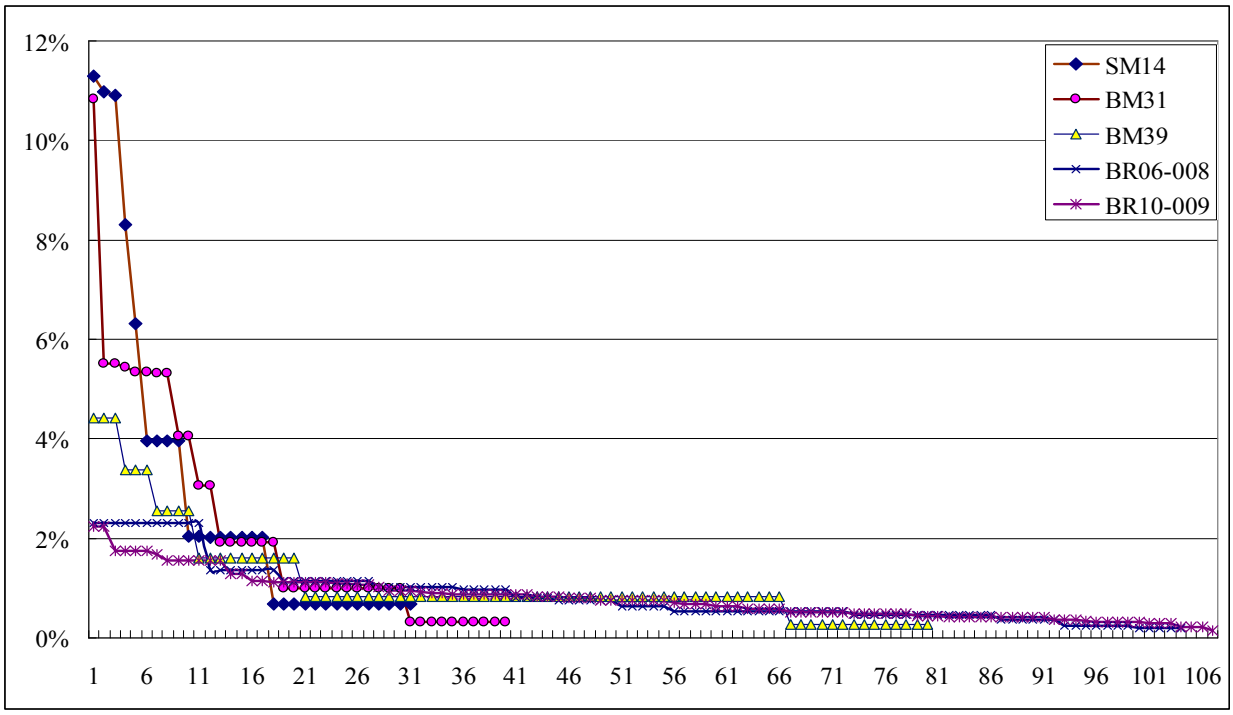
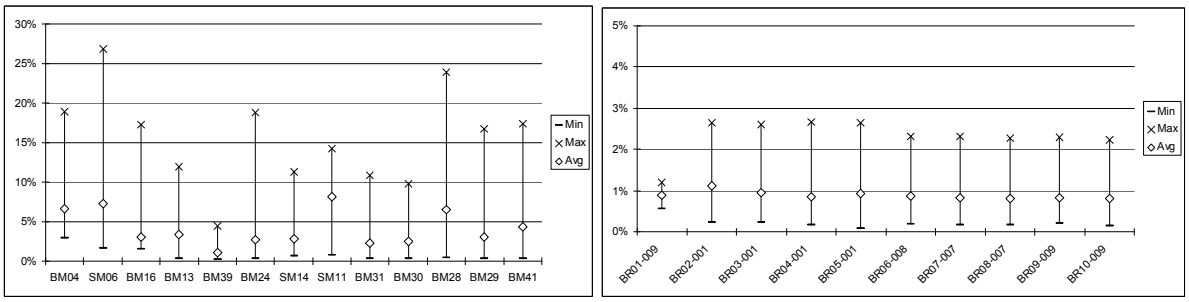


Fig. 18 Carton vs. bin volume comparison for SM14, BM31, BM39, BR06-008 and BR10-009



(a) Cases from this paper (b) Cases from BR01 to BR10

Fig. 19 Maximum, minimum and average volume against 'bin' volume

7. Conclusion

This study, with the aim to solve the challenging real-world 3D packing problem with variable carton orientations, has made meticulous improvement to the packing (decoding) procedure, devised a particular genetic algorithm with customized selection, crossover and mutation operators, and adopted a novel global search framework based on the concept of evolutionary gradient. The numerical experiments indicate that the selected scenario of the global search framework is able to solve the investigated packing problem effectively and efficiently. The novelty and contribution of this work can be summarized as follows:

(1) A novel index (*index*2) is defined for the selection of reference points in locating a carton in the bin, which is the crucial reason that makes the packing more compact.

(2) A new parallel moving strategy is proposed to reduce possible gaps and to avoid apparently

invalid moves thus save computational time and provide faster and more compact packing.

(3) The relative positions which indicate the cartons that are touched on the left, in front and at the bottom of a particular carton can be reported via the improved decoding procedure. This, to our knowledge, is the first attempt to presenting relative positions, which provides to be very convenient and handy to guide manual packing in practice.

(4) Appropriate genetic components have been selected and well combined together with simultaneous consideration of solution speed and precision.

(5) The concept of evolutionary gradient has been proposed considering the usual existing diversity of the solutions obtained by meta-heuristics, e.g. the genetic algorithm. The global search framework has subsequently been devised in order to make full use of the solution ability of the selected genetic algorithm.

(6) Several scenarios of the GSF have been constructed and examined with the purpose to improve the solution speed. A suitable scenario (GGA5) has then been selected and tested with a wide range of packing cases, including larger-size and more heterogeneous instances. For most of the instances, the GGA5 has achieved better solutions than OGA, and the computational times are much shorter.

With the above novelty, the **GSF** is able to achieve packing solutions with average volume utilization up to **85~90% within half an hour**; for some cases, the volume utilization reaches **92~95%**. In contrast, the **manual packing** only has average volume utilization less than **75%**, with very few exceptional cases reaching 80~82%. The average increment of volume utilization by GGA5 can be **more than 15%** against the current practical operations.

To sum up, the GSF can provide high-fill-rate packing solutions with information of the relative positions, orientations and coordinates of the cartons. Furthermore, with the ability to handle arbitrary orientations and heterogeneous cartons, the proposed method can be easily tailored and applied to other types of bin packing problems.

The GSF can be easily transformed into a synchronous parallel version, because the synchronization and communication, which are the difficult issues in parallel computing, are already available. The further interesting work is to design an asynchronous parallel version of this framework.

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Appendix A: Solution Results by Different Methods

Table A-1. Comparison of filled rates of the old cases by different methods

		Manual		Rdm		MILP		OGA(Swp)			IGA(Swp)			GGA5		
	N	*H*	F_1(%)	*H*	F_1(%)	*H*	F_1(%)	*H*	F_1(%)	F_2(%)	*H*	F_1(%)	F_2(%)	*H*	F_1(%)	F_2(%)
SM01	8	91	69.13	92	68.38	80	78.64	80	78.64	83.00	80	78.64	87.49	80	78.64	87.49
SM02	8	56	54.66	40	76.52	37	82.73	37	82.73	82.73	37	82.73	82.73	37	82.73	82.73
SM03	9	69	57.25	53	74.54	47	84.05	47	84.05	84.05	47	84.05	84.05	47	84.05	84.05
SM04	9	91	78.33	103	69.20	82	86.93	83	85.88	85.88	83	85.88	85.88	82	86.93	86.93
SM05	10	56	80.91	61	74.28	52	87.13	52	87.13	90.53	52	87.13	90.53	52	87.13	90.53
SM06	12	53	62.42	48	68.93	38	87.06	40	82.71	84.19	40	82.71	84.19	38	87.06	87.06
SM07	17	70	64.24	62	72.53	62	72.53	62	72.53	80.32	62	72.53	88.84	62	72.53	88.84
SM08	17	91	63.74	77	75.33	71	81.70	69	84.06	85.56	62	93.55	93.55	62	93.55	93.55
SM09	19	71	60.73	62	69.55	62	69.55	52	82.92	82.92	51	84.55	84.55	50	86.24	86.24
SM10	21	85	65.34	78	71.20	62	89.57	63	88.15	89.30	63	88.15	91.36	62	89.57	89.57
Avg01~10			**65.68**		**72.05**		**81.99**		**82.88**	**84.85**		**83.99**	**87.32**		**84.84**	**87.70**
Increase					**6.37**		**16.31**		**17.21**			**18.32**			**19.17**	
BM01	5	91	68.61	84.1	74.24	84.1	74.24	84.1	74.24	80.15	84.1	74.24	80.15	84.1	74.24	80.15
BM02	11	91	77.92	99	71.63	83.1	85.33	84.1	84.32	85.65	84.1	84.32	88.78	84.1	84.32	89.56
BM03	12	45	63.47	47	60.77	32	89.25	32	89.25	92.50	32	89.25	92.50	32	89.25	92.50
BM04	13	91	72.92	98	67.71	77.4	85.73	83.4	79.57	83.71	77.4	85.73	87.92	77.4	85.73	87.92
BM05	15	91	72.23	90	73.03	79	83.20	79	83.20	89.53	79	83.20	89.53	78	84.27	89.74
BM06	16	67	48.62	47	69.31	40	81.44	40	81.44	81.79	40	81.44	82.65	40	81.44	83.52
BM07	21	48	63.56	40	76.27	40	76.27	40	76.27	78.76	38	80.29	86.95	37	82.46	84.55
BM08	22	59	70.44	54	76.97	53	78.42	51	81.49	83.28	51	81.49	85.03	48	86.59	86.96
BM09	23	94	66.64	80	78.30	82	76.39	72	87.00	88.29	71	88.23	90.31	71	88.23	90.31
BM10	23	91	58.11	62	85.29	63.1	83.80	61	86.68	86.76	61	86.68	87.89	61	86.68	88.73
BM11	25	91	72.08	80.1	81.89	78	84.10	77.4	84.75	85.78	73.4	89.37	92.54	73.4	89.37	92.54
BM12	26	94	53.93	66	76.80	68	74.54	61	83.10	84.33	61	83.10	85.21	61	83.10	85.96
Avg01~12			**65.71**		**74.35**		**81.06**		**82.61**	**85.04**		**83.95**	**87.46**		**84.64**	**87.70**
Increase					**8.64**		**15.35**		**16.90**			**18.23**			**18.93**	
BM13	27	83	73.88	85	72.14	78	78.62	71	86.37	87.50	69	88.87	89.26	68	90.18	91.51
BM14	28	95	74.42	94	75.22	88.4	79.98	78.4	90.18	90.49	78.4	90.18	91.06	78.4	90.18	91.06
BM15	29	91	71.86	94	65.97	87	75.17	75	87.20	88.33	74	88.37	88.76	73	89.58	90.91
BM16	29	63	65.29	54	76.17	52	79.10	48	85.69	86.81	47	87.51	89.43	47	87.51	89.43
BM17	29	91	79.13	101	71.30	88.4	81.46	80	90.01	91.03	78.4	91.85	93.12	78.4	91.85	93.16
BM18	30	70	80.17	70	80.17	73.4	76.45	63.1	88.93	89.01	63	89.07	89.94	63	89.07	89.94
BM19	30	94	60.11	78.4	72.07	79	71.52	63.1	89.54	90.18	63.1	89.54	90.49	63	89.69	90.07
BM20	30	94	74.56	94	74.56	87	80.56	79	88.71	89.10	79	88.71	90.81	78	89.85	91.18
BM21	30	95	73.95	94	74.74	87	80.75	78.4	89.61	90.48	78.4	89.61	90.93	78.4	89.61	92.20
BM22	31	91	73.93	85	79.15	87	77.33	78	86.26	86.63	78	86.26	88.15	77	87.38	90.38
BM23	33	95	74.14	94	74.93	93.4	75.41	78.4	89.84	90.15	77.4	91.00	91.39	77	91.47	92.66
BM24	35	91	81.83	94	79.22	89	83.67	80	93.08	93.40	82	90.81	91.69	79	94.26	95.18
BM25	36	94	68.72	87	74.25	79	81.76	76	84.99	85.36	73	88.48	88.87	71	90.98	92.32
BM26	38	91	80.81	101	72.81	97	75.81	82	89.68	90.07	80	91.92	93.12	80	91.92	93.12
BM27	40	91	81.81	100	74.44	109	68.30	85	87.58	87.96	85	87.58	88.87	84	88.62	89.93
Avg13~27			**74.31**		**74.48**		**77.73**		**88.51**	**89.10**		**89.32**	**90.39**		**90.14**	**91.54**
Increase					**0.17**		**3.42**		**14.20**			**15.01**			**15.84**	

Table A-2. Comparison of filled rates of the new cases by different methods

		Rdm		MILP		OGA(Swp)			IGA(Swp)			GGA5		
	N	*H*	F_1(%)	*H*	F_1(%)	*H*	F_1(%)	F_2(%)	*H*	F_1(%)	F_2(%)	*H*	F_1(%)	F_2(%)
SM00	10	80	75.26	68	88.54	72	83.63	85.09	71	84.8	85.88	70	86.01	86.01
SM11	11	96	74.6	80	89.52	88	81.38	82.41	83	86.28	86.28	80	89.52	89.52
SM12	20	110	70.35	90	85.98	89	86.95	86.95	89	86.95	86.95	87	88.94	88.94
SM13	20	113	74.5	104	80.94	98	85.9	87.41	95	88.61	88.61	94	89.55	89.55
SM14	31	109	72.9	105	75.68	91	87.32	87.32	90	88.29	88.29	89	89.28	89.28
SM15	40	126	73.19	118	76.23	103	87.33	87.33	100	89.95	89.95	97	92.73	92.73
Average			**73.47**		**82.82**		**85.42**	**86.09**		**87.48**	**87.66**		**89.34**	**89.34**
Increase					**9.35**		**11.95**			**14.01**			**15.87**	
BM29	30	106	74.47	100	78.94	89	88.70	89.08	89	88.70	89.91	87.4	90.32	91.56
BM30	35	96	76.92	99	74.59	85	86.87	87.25	85	86.87	88.01	83	88.97	89.35
BM31	40	87	77.20	98	68.53	78	86.11	86.48	76	88.37	89.68	75	89.55	89.94
BM32	45	99	77.29	102	75.02	89	85.98	86.35	88	86.96	87.33	88	86.96	87.33
BM33	50	88	70.71	95	65.50	73	85.24	85.60	71	87.64	88.02	70	88.89	89.27
BM34	55	108	75.90	116	70.66	96	85.38	85.75	94	87.20	87.58	90	91.07	91.47
BM35	60	86	76.45	95	69.21	78	84.29	84.65	74	88.95	90.16	73	90.06	91.39
BM36	65	91	78.27	104	68.49	83	85.82	86.19	82	86.87	88.00	79	90.16	90.55
BM37	70	105	73.00	112	68.43	89	86.12	86.49	88	87.10	87.47	86	89.12	89.51
BM38	75	96	79.62	107	71.44	90	84.93	85.30	88	86.86	88.15	86	88.88	89.27
BM39	80	107	77.03	121	68.12	95	86.76	87.13	93	88.63	89.93	90	91.58	91.97
Avg29~39			**76.08**		**70.81**		**86.02**	**86.39**		**87.65**	**88.57**		**89.60**	**90.15**
Increase					**-5.27**		**9.94**			**11.57**			**13.52**	
BM28	13	77.4	67.36	61	85.47	64	81.46	81.74	63.1	82.63	84.28	61	85.47	88.31
BM40	20	107.4	73.84	96	82.61	92	86.20	86.50	91	87.15	88.28	90	88.12	88.42
BM41	20	103.1	70.50	87	83.54	85.4	85.11	85.18	85	85.51	86.41	84.1	86.42	88.01
Average			**70.57**		**83.87**		**84.26**	**84.47**		**85.10**	**86.32**		**86.67**	**88.25**
Increase					**13.31**		**13.69**			**14.53**			**16.10**	

Table A-3. Comparison of filled rates of the heterogeneous cases by different methods

		Rdm		MILP		OGA(Swp)			IGA(Swp)			GGA5		
	N	*H*	F_1(%)	*H*	F_1(%)	*H*	F_1(%)	F_2(%)	*H*	F_1(%)	F_2(%)	*H*	F_1(%)	F_2(%)
SM00	10	80	75.26	68	88.54	72	83.63	85.09	71	84.8	85.88	70	86.01	86.01
SM11	11	96	74.60	80	89.52	88	81.38	82.41	83	86.28	86.28	80	89.52	89.52
BM28	13	77.4	67.36	61	85.47	64	81.46	81.74	63.1	82.63	84.28	61	85.47	88.31
BM40	20	107.4	73.84	96	82.61	92	86.20	86.50	91	87.15	88.28	90	88.12	88.42
BM41	20	103.1	70.50	87	83.50	85.4	85.11	85.18	85	85.51	86.41	84.1	86.42	88.01
Average			**72.31**		**85.93**		**83.56**	**84.18**		**85.27**	**86.23**		**87.11**	**88.05**
Increase					**13.62**		**11.24**			**12.96**			**14.80**	

Table A-4. Comparison of computational times of the old cases by different methods

		Rdm		MILP		OGA(Swp)		IGA(Swp)		GGA5			Comparison	
	N	H	CT_R	H	CT_M	H	CT_O	H	CT_I	H	CT_G	g	CT_O/CT_I	CT_O/CT_G
SM01	8	92	0.031	80*	153	80	11.12	80	1.25	80	5.16	4	8.90	2.16
SM02	8	40	0.031	37*	547	37	14.80	37	1.56	37	3.81	3	9.49	3.88
SM03	9	53	0.031	47*	1	47	15.84	47	1.39	47	3.44	3	11.40	4.60
SM04	9	103	0.032	82	7200	83	31.85	83	1.84	82	6.06	3	17.31	5.26
SM05	10	61	0.046	52*	734	52	16.36	52	2.74	52	6.23	3	5.97	2.63
SM06	12	48	0.047	38*	285	40	29.56	40	3.02	38	17.27	4	9.79	1.71
SM07	17	62	0.063	62	7200	62	56.48	62	3.86	62	9.11	3	14.63	6.20
SM08	17	77	0.063	71	7200	69	49.32	62	7.27	62	20.27	3	6.78	2.43
SM09	19	62	0.125	62	7200	52	79.12	51	4.95	50	38.83	4	15.98	2.04
SM10	21	78	0.078	62	7200	63	77.88	63	14.41	62	51.03	5	5.40	1.53
BM01	5	84.1	0.031	84.1*	3	84.1	24.04	84.1	1.02	84.1	2.73	3	23.57	8.81
BM02	11	99	0.062	83.1	7200	84.1	96.88	84.1	5.42	84.1	14.44	3	17.87	6.71
BM03	12	47	0.109	32*	368	32	141.76	32	5.09	32	17.63	3	27.85	8.04
BM04	13	98	0.078	77.4	7200	83.4	108.00	77.4	10.39	77.4	37.27	3	10.39	2.90
BM05	15	90	0.094	79	7200	79	130.20	79	18.81	78	60.75	4	6.92	2.14
BM06	16	47	0.109	40	7200	40	253.40	40	9.36	40	45.38	4	27.07	5.58
BM07	21	40	0.328	40	7200	40	472.36	38	18.13	37	72.16	4	26.05	6.55
BM08	22	54	0.734	53	7200	51	493.72	51	16.39	48	115.55	6	30.12	4.27
BM09	23	80	0.188	82	7200	72	408.52	71	17.83	71	101.69	3	22.91	4.02
BM10	23	62	0.188	63.1	7200	61	475.36	61	26.20	61	85.69	3	18.14	5.55
BM11	25	80.1	0.218	78	7200	77.4	488.52	73.4	28.33	73.4	89.28	3	17.24	5.47
BM12	26	66	0.234	68	7200	61	648.08	61	33.25	61	89.73	3	19.49	7.22
BM13	27	85	0.250	78	7200	71	567.60	69	39.00	68	118.69	3	14.55	4.78
BM14	28	94	0.265	88.4	7200	78.4	544.92	78.4	56.30	78.4	200.02	5	9.68	2.72
BM15	29	94	0.359	87	7200	75	660.53	74	44.72	73	193.80	5	14.77	3.41
BM16	29	54	0.360	52	7200	48	992.68	47	60.52	47	174.53	3	16.40	5.69
BM17	29	101	0.265	88.4	7200	80	610.40	78.4	59.19	78.4	241.58	5	10.31	2.53
BM18	30	70	0.297	73.4	7200	63.1	816.08	63	54.17	63	130.94	3	15.07	6.23
BM19	30	78.4	0.297	79	7200	63.1	800.32	63.1	66.28	63	195.64	4	12.07	4.09
BM20	30	94	0.297	87	7200	79	657.93	79	50.89	78	206.34	5	12.93	3.19
BM21	30	94	0.328	87	7200	78.4	661.87	78.4	50.13	78.4	141.19	3	13.20	4.69
BM22	31	85	0.313	87	7200	78	684.30	78	62.66	77	182.74	4	10.92	3.74
BM23	33	94	0.359	93.4	7200	78.4	769.54	77.4	90.92	77	259.89	4	8.46	2.96
BM24	35	94	0.375	89	7200	80	895.13	82	57.88	79	492.23	9	15.47	1.82
BM25	36	87	0.422	79	7200	76	1071.30	73	89.08	71	551.45	5	12.03	1.94
BM26	38	101	0.453	97	7200	82	1015.65	80	106.28	80	245.70	3	9.56	4.13
BM27	40	100	0.500	109	7200	85	1185.35	85	97.69	84	334.41	4	12.13	3.54
Average												**3.84**	**14.62**	**4.19**

Table A-5. Comparison of computational times of the new cases by different methods

		Rdm		MILP		OGA(Swp)		IGA(Swp)		GGA5			Comparison	
	N	H	CT_R	H	CT_M	H	CT_O	H	CT_I	H	CT_G	g	CT_O/CT_I	CT_O/CT_G
SM00	10	80	0.031	68*	1144	72	22.19	71	2.47	70	7.20	4	8.98	3.08
SM11	11	96	0.031	80	7200	88	18.50	83	2.69	80	9.31	5	6.88	1.99
SM12	20	110	0.062	90	7200	89	96.79	89	11.79	87	30.38	4	8.21	3.19
SM13	20	113	0.063	104	7200	98	141.32	95	8.99	94	25.55	4	15.72	5.53
SM14	31	109	0.188	105	7200	91	264.47	90	22.06	89	111.38	4	11.99	2.37
SM15	40	126	0.235	118	7200	103	742.48	100	49.42	97	225.91	7	15.02	3.29
BM28	13	77.4	0.093	61	7200	64	126.49	63.1	9.45	61	43.13	5	13.39	2.93
BM29	30	106	0.328	100	7200	89	1199.79	89	57.66	87.4	237.34	5	20.81	5.06
BM30	35	96	0.391	99	7200	85	856.47	85	72.56	83	285.92	5	11.80	3.00
BM31	40	87	0.500	98	7200	78	1278.47	76	101.77	75	424.41	5	12.56	3.01
BM32	45	99	0.625	102	7200	89	2602.36	88	151.75	88	600.91	4	17.15	4.33
BM33	50	88	0.797	95	7200	73	2607.48	71	215.23	70	1016.83	5	12.11	2.56
BM34	55	108	0.984	116	7200	96	4717.27	94	235.84	90	802.78	4	20.00	5.88
BM35	60	86	1.297	95	7200	78	4326.64	74	318.31	73	1057.34	4	13.59	4.09
BM36	65	91	1.500	104	7200	83	4689.35	82	409.11	79	1380.56	3	11.46	3.40
BM37	70	105	1.797	112	7200	89	**8436.69**	88	605.63	86	**1868.84**	5	13.93	4.51
BM38	75	96	2.140	107	7200	90	**11188.73**	88	600.88	86	**2014.80**	4	18.62	5.55
BM39	80	107	2.500	121	7200	95	**14601.00**	93	741.09	90	**2015.97**	4	19.70	7.24
BM40	20	107.4	0.141	96	7200	92	488.04	91	23.08	90	62.31	4	21.15	7.83
BM41	20	103.1	0.140	87	7200	85.4	248.97	85	22.52	84.1	97.41	4	11.06	2.56
Average												**4.45**	**14.21**	**4.07**