

# Numerical study on 2D Riemann problems using state of the art solvers

Ferre Van Assche, Vergauwen Bob

KU Leuven

January , 2015

# Outline

Introduction

Numerical schemes

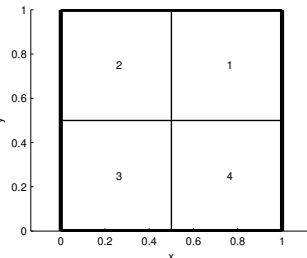
# Introduction

## Riemann problem in 2D

A Riemann problem consists of a system of conservation equation together with a piecewise constant initial condition.

$$U_t + F(U)_x + G(U)_y = 0$$

$$U_0(x, y) = \begin{cases} u_1 & (x, y) \in [0.5, 1]^2 \\ u_2 & (x, y) \in [0, 0.5] \times [0.5, 1] \\ u_3 & (x, y) \in [0, 0.5]^2 \\ u_4 & (x, y) \in [0.5, 1] \times [0, 0.5] \end{cases}$$



# The equations

## Euler equation for compressible fluids

$$U_t + F(U)_x + G(U)_y = 0$$

with

$$U = \begin{pmatrix} \rho \\ \rho u \\ \rho v \\ e \end{pmatrix} \quad F = \begin{pmatrix} \rho u \\ \rho u^2 + p \\ \rho uv \\ u(e + p) \end{pmatrix} \quad G = \begin{pmatrix} \rho v \\ \rho uv \\ \rho v^2 + p \\ v(e + p) \end{pmatrix}$$

Equation describing the dynamics of a compressible fluid.

Conserved quantities are:  $\rho$  density,  $\rho u$  momentum in  $x$ ,  $\rho v$  momentum in  $y$  and the total energy  $e$ .

# Numerical methods

5 different numerical schemes tested:

- ▶ TVDLF
- ▶ HLL
- ▶ HLLC
- ▶ TVD-MUSCL
- ▶ FD

# TVDLF - Total Variation Diminishing (TVD) concept

Total variation of numerical approximation of  $u$ :

$$TV(u^n) = \sum_{i=0}^N |u_{i+1}^n - u_i^n|$$

Scheme is total variation diminishing in time if

$$TV(u^{n+1}) \leq TV(u^n) \quad \forall n$$

# TVDLF

First order Lax-Friedrichs scheme:

$$U_j^{n+1} = U_j^n - \frac{\Delta t}{\Delta x} (F_{j+1/2} - F_{j-1/2}) + \frac{1}{2} (\Phi_{j+1/2} - \Phi_{j-1/2})$$

where

$$F_{j+1/2} = \frac{F_j + F_{j+1}}{2}$$

$$\Phi_{j+1/2} = U_{j+1} - U_j$$

Scheme is TVD  $\Leftrightarrow$  CFL condition satisfied

Scheme is first order accurate

Reduce numerical diffusion:

$$\Phi_{j+1/2} = \frac{\Delta t}{\Delta x} c_{j+1/2}^{\max} (U_{j+1} - U_j)$$

# TVDLF - second order spatial accuracy

d



# TVD-MUSCL

content...

# HHL and HLLC

content...

# FD

content...

# MPI-AMRVAC and HPC

Maar heel even aanhalen... Iedereen kent dit toch al.

## Personal working method

Talk about the scrip we wrote to set up the project and run everything.

Tell them we ran the code succsefully for small problem on the cw network on 5 computers.  $\Rightarrow$  network was not stable or something else went wrong.