Numerical study on 2D Riemann problems using state of the art solvers

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Outline

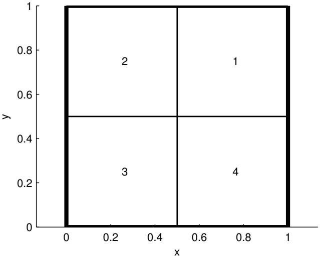
Introduction

Riemann problem in 2D

A Riemann problem consists of a system of conservation equation together with a piecewise constant initial condition.

$$U_t + F(U)_x + G(U)_y = 0$$

$$U_0(x,y) = \begin{cases} u_1 & (x,y) \in [0.5,1]^2 \\ u_2 & (x,y) \in [0,0.5] \times [0.5,1] \\ u_3 & (x,y) \in [0,0.5]^2 \\ u_4 & (x,y) \in [0.5,1] \times [0,0.5] \end{cases}$$



The equations

Euler equation for compressible fluids

$$U_t + F(U)_x + G(U)_y = 0$$

with

$$U = \begin{pmatrix} \rho \\ \rho u \\ \rho v \\ e \end{pmatrix} \quad F = \begin{pmatrix} \rho u \\ \rho u^2 + p \\ \rho u v \\ u(e+p) \end{pmatrix} \quad G = \begin{pmatrix} \rho v \\ \rho u v \\ \rho u^2 + p \\ v(e+p) \end{pmatrix}$$

Equation describing the dynamics of a compressible fluid. Conserved quantities are: ρ density, ρu momentum in x , ρv momentum in y and the total energy e.

Numerical methods

5 different numerical schemes tested:

- TVDLF
- HLL
- HLLC
- TVD-MUSCL
- FD

TVDLF - Total Variation Diminishing (TVD) concept

Total variation of numerical approximation of u:

$$TV(u^n) = \sum_{i=0}^{N} |u_{i+1}^n - u_i^n|$$

Scheme is total variation diminishing in time if

$$TV(u^{n+1}) \leq TV(u^n) \quad \forall n$$