

# Neural Subdivision

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[https://www.dgp.toronto.edu/~hsuehli/pdf/neuralSubdiv\\_95mb.pdf](https://www.dgp.toronto.edu/~hsuehli/pdf/neuralSubdiv_95mb.pdf)

## Intro

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- **what is neural subdivision?** Neural Subdivision, a novel framework for data-driven coarse-to-fine geometry modeling
  - Neural subdivision 避免了经典subdivision的缺陷：
    - volume shrinkage
    - over-smoothing
    - amplification of tessellation artifacts
- 该论文的创新性：
  - non-linear subdivision beyond simple linear averaging used in classical techniques
  - only requires a set of high-resolution meshes for learning network weights
  - output is a surface mesh with deterministic connectivity based on the input, enabling direct use in the standard graphics pipelines such as texture mapping
  - ensure rotation and translation invariance(encode vertex position data in a local frame rather than the entire shape)

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## Background

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(经典)

- subdivision surface: recursive up-sampling of a discrete surface mesh

- classic methods:
  - each input mesh element is divided into many vertices
    - split edges
    - add vertices
  - smooth positions of the mesh vertices by taking a **linear weighted average** of their neighbors's positions(weight scheme based purely on the local mesh connectivity)
- 应用:
  - 交互式建模 A modeler may start with a very coarse cage, adjust vertex positions, then subdivide once, adjust the finer mesh vertices, and repeat this process until satisfied
- 缺陷:
  - 容易overly smooth the entire shape
  - do not identify details to maintain细节丢失
  - 全局使用 fixed on-size-fits-all weighting rules 丢失了大量信息的使用

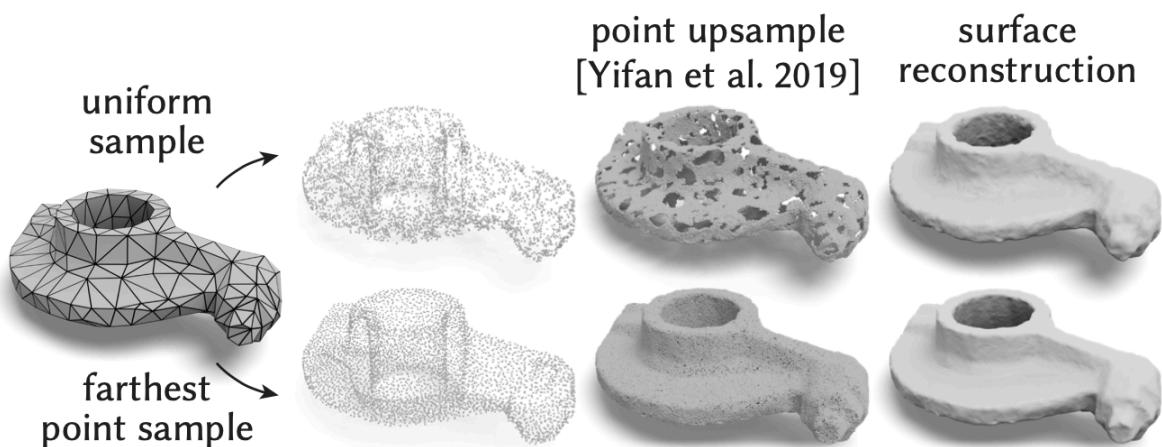
(神经)

- **basic process:**
  - input: coarse triangle mesh
  - Loop subdivision: recursively subdivide by applying the fixed topological, but predicting vertex positions using a neural network conditioned on the local geometry of a patch
- 问题: 如何collect training data pairs? 虽然现在3D模型很多, 但是都不包含需要的信息
  - 解决: 在训练时, 仅对 origin / 连接性高的曲面网格进行自我监督; 随机生成训练样本的低分辨率版本, 同时保持其表面间的双射

## 2.2 Neural Geometry Learning

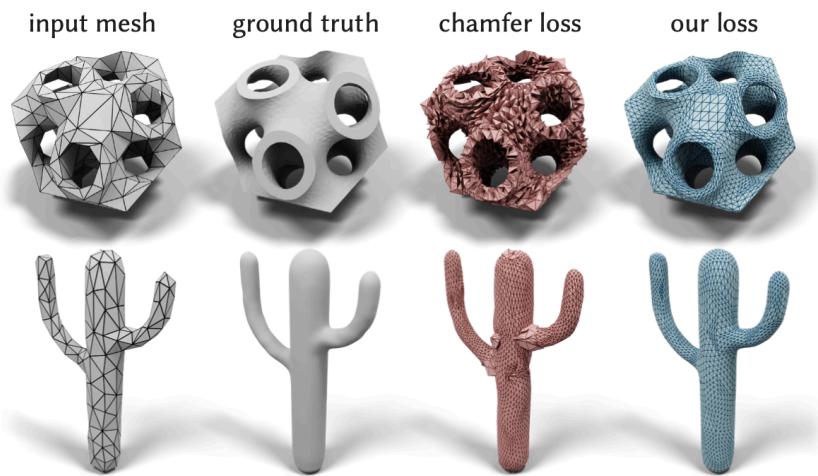
(寻找神经网络的输入，还是上面这个问题)

- deep point cloud upsampling 缺点
  - lack connectivity information
  - require the neural network to estimate the structure of the underlying manifold
  - post process is often required to convert the output of point-based methods to meshes (not end-to-end trainable system)



- deform global template
  - predict vertex coordinates
  - warps the entire 3D domain conditioned on a latent vector that encode the deformation target
  - 缺点
    - output is limited to deformations of a single shape
- others
  - using a local or global parameterization to unfold a mesh into 2D grid
  - apply graph-based techniques adapted for meshe
- 论文的优点
  - refine the mesh locally, input could be arbitrary
  - don't require co-aligned training data with a well-defined object space

- output is translation and rotation invariant(describe in local coordinate system)
- 论文采用MeshCNN的变形
  - **MeshCNN** learn filter over the local mesh structure via undirected edges -> deterministic tasks
  - 论文正好相反 generative tasks, develop features over the half-flaps(an edge along with its two adjacent triangles)[each half-flap has a canonical orientation, 对运动不变性很有帮助]
- loss距离
  - surface-to-surface distances(chamfer distance): define via closest-point queries
    - 问题: match many points to the same point, while leaving other points unmatched, resulting in self-overlaps and unrepresented areas



## 3. Neural Subdivision Overview

- **training and loss**
  - train with  $l^2$  loss: distance between each predicted vertex position at every level of subdivision and its corresponding point on the original shape
  - dataset: (no existing dataset) develop a training data comprising of coarse and fine meshes tied by bijective mappings between

them

- **data-generation**

- each subdivided mesh at any level can be mapped back to the initial coarse mesh
  - mid-point-to-vertex
  - vertex-to-vertex
- 坐标转换
  - $v$ : subdivision新生成的点
  - $g$ : subdivided-to-coarse bijective mapping
  - $f$ : coarse-to-origin bijective mapping

通过随机选边和点的方式将origin进行coarse，得到一序列 coarse model

computing a conformal map between the 1-ring edge neighborhood (before the collapse) and the 1-ring vertex neighborhood (after the collapse)

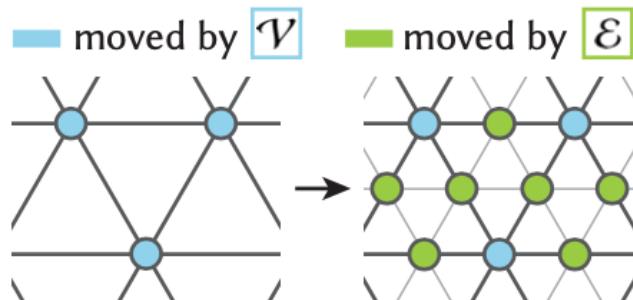
- $f(g(v))$ : unique point on the original mesh corresponding to  $v$

- **network architecture**

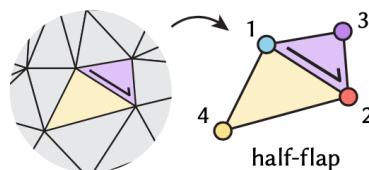
- operate over atomic local mesh neighborhoods and predict differential features(represent geometry in the local coordinates not in world coordinates)
- **Initialization step**
  - compute differential pre-vertex quantities based on local coordinate
  - model  $I$  is applied to the 1-ring neighborhood of every vertex to map to a *high-dimensional feature vector*

encode local geometry of the patch, and differential quantities which directly represent local geometry to reconstruct the vertex coordinates

- split edges at midpoint, subdivide each triangle into 4



- **Vertex step**
  - use  $V$  to predict vertex features based on 1-ring neighborhood
- **Edge step**
  - use the module  $\varepsilon$
- half-flap
  - directed edge: define the local coordinate frame
  - concatenate 4 vertices' features and feed them into shallow MLP(multi-layer perceptrons)

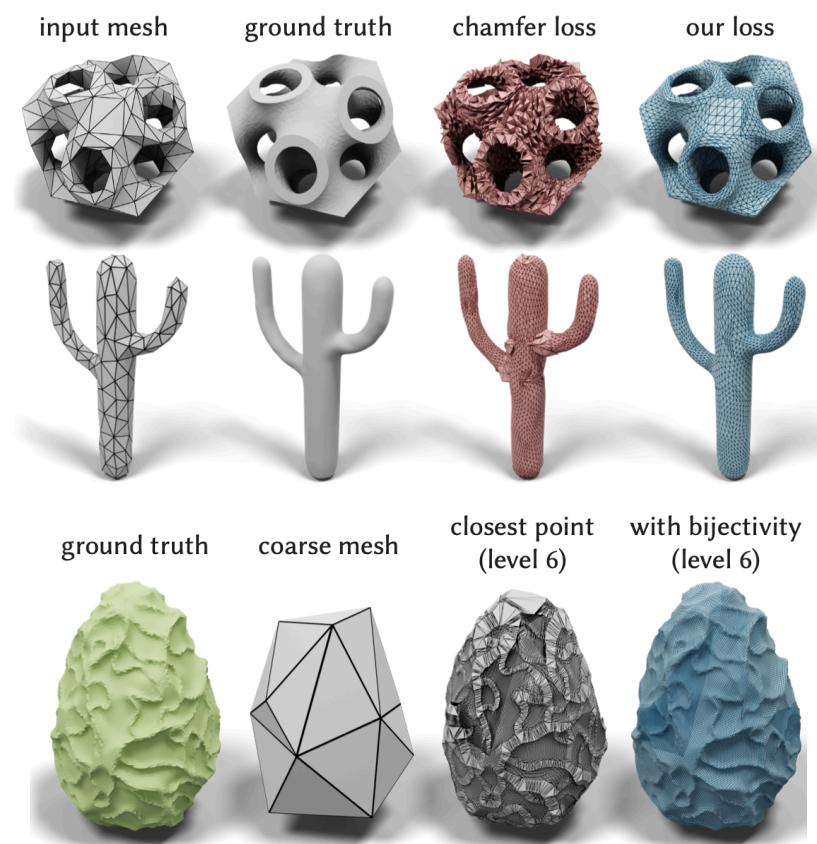


- $I, V$  process all half-flaps, use average pooling to combine the half-flap features into per-vertex features;  $\varepsilon$  combine feature from two half-flaps(边的两个方向) via average pooling

## 4. Data Generation and Training

- **naive subdivision training**

- 随机取样 pairs of coarse/fine meshes
- measure the distance between the network's predicted subdivision and the ground truth
- iterate over coarse/fine pairs while optimizing the loss
- 缺陷
  - chamfer-like loss or point-to-mesh distance will lead to incorrect and self-overlapping matches



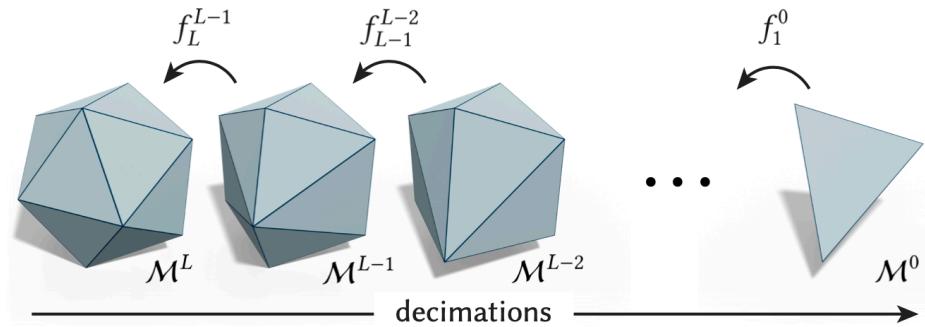
- **Successive Self-Parameterization**

- construct a map between different discretizations of the same shape
- combine MAPS and successive mapping
- **two-step module**
  - input: a triangle mesh and an edge collapse algorithm
  - output: decimated mesh with a corresponding bijective map between the input and the decimated model

We denote the input triangle mesh as  $\mathcal{M}^L = (\mathbf{V}^L, \mathbf{F}^L)$ , where  $\mathbf{V}^L, \mathbf{F}^L$  are vertex positions and face information respectively at the original level  $L$ . The input mesh  $\mathcal{M}^L$  is successively simplified into a series of meshes  $\mathcal{M}^l = (\mathbf{V}^l, \mathbf{F}^l)$  with  $0 \leq l \leq L$ , where  $\mathcal{M}^0 = (\mathbf{V}^0, \mathbf{F}^0)$  is the coarsest mesh. For each edge collapse  $\mathcal{M}^l \rightarrow \mathcal{M}^{l-1}$ , we compute the bijective map  $f_l^{l-1} : \mathcal{M}^{l-1} \rightarrow \mathcal{M}^l$  (see Fig. 12) on the fly. The final map  $f_L^0 : \mathcal{M}^0 \rightarrow \mathcal{M}^L$  is computed via composition,

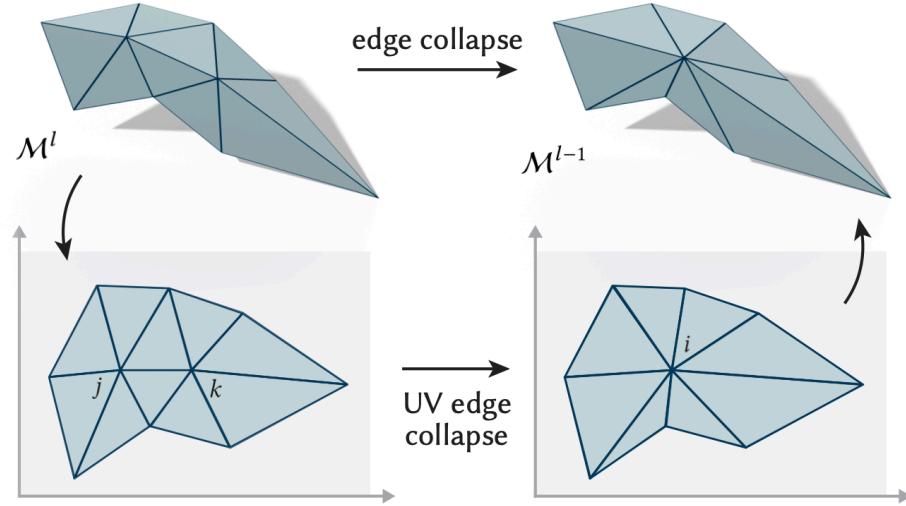
$$f_L^0 = f_{L-1}^{L-1} \circ \cdots \circ f_1^0. \quad (1)$$

We now focus our discussion on the computation of a bijective map for a single edge collapse.



- **Single Edge Collapse**

- terms
  - $N(i)$ : neighboring vertices of a vertex  $i$
  - $N(j, k) = N(j) \cup N(k)$ : neighboring vertices of an edge  $(j, k)$
- compute the bijective map for the edge's 1-ring  $N(j, k)$ 
  1. parameterize the neighborhood  $N(j, k)$  into 2D
  2. perform the edge collapse both on the 3D mesh and in UV space



- boundary vertices of  $N(j, k)$  before the collapse become the boundary vertices of  $N(i)$  after the collapse

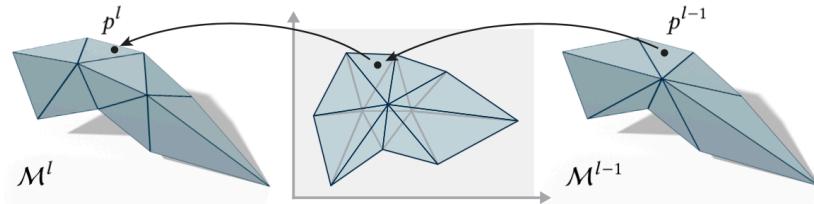
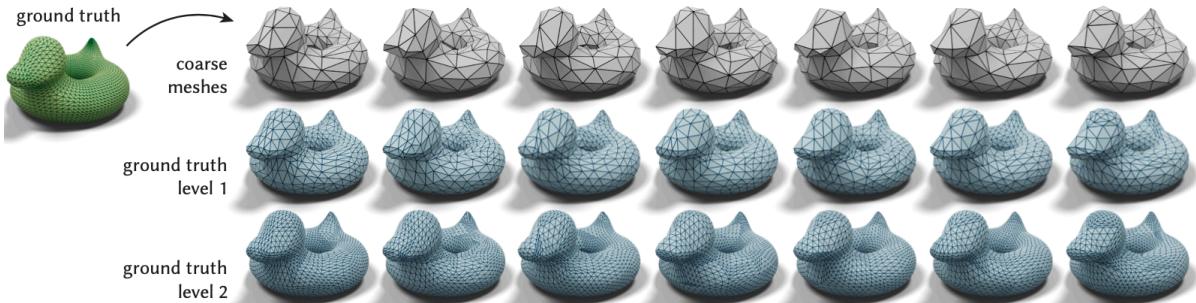


Fig. 14. Since both the pre-collapse and post-collapse parameterizations of the 1-ring map it into the same 2D domain, we can easily use the shared UV space to map a point back and forth between  $\mathcal{M}^l$  and  $\mathcal{M}^{l-1}$ .

- use conformal flattening to compute the UV parameterization
- **4.3 Implementation**

- edge collapse algo.  $\rightarrow O(N \log N)$
- flatten  $\rightarrow$  constant cost
- total successive self-parameterization  $\rightarrow O(N \log N)$

- **4.4 Training Data & Loss Computation**

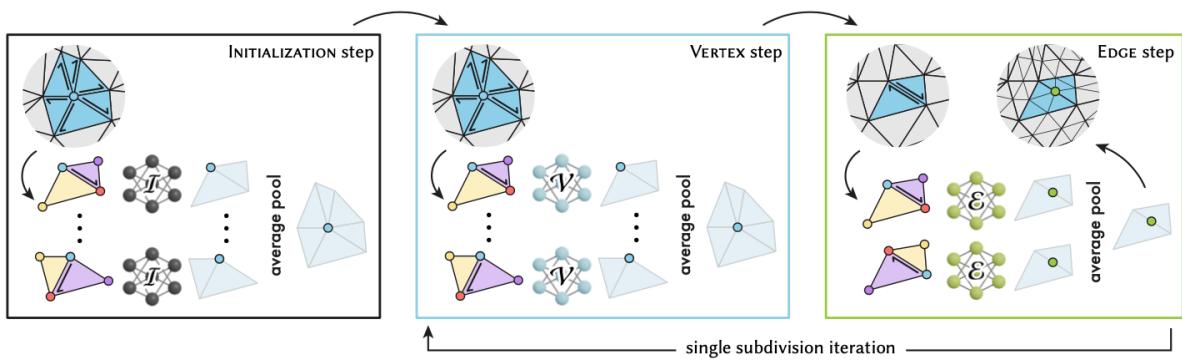


1. use QSLIM with a random sequence of edge collapses to construct several different decimated models

2. plug in self-parameterization to obtain a high-quality bijective map for each coarse and fine pair
3. use Loop topology to retrieve the correspondences
4. use barycentric coordinates  $b$  on the coarse mesh to obtain  $f(b)$  on the fine mesh
5. use  $l^2$  distance  $\|f(b) - \varepsilon(b)\|_2$  to measure per-vertex loss

## 5. Network Architecture

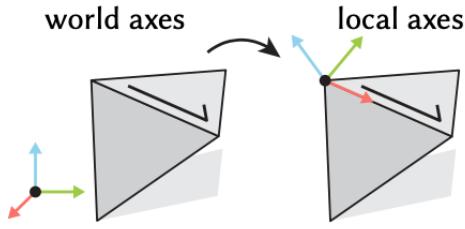
- **INITIALIZATION step**
  - input: mesh at a previous level of subdivision + topological update rule(Loop use mid-point subdivision)
  - $I$ : map input per-vertex features to high-dimensional feature vector at each vertex
- **VERTEX step**
  - $V$ : in each subdivision iteration, update features at corners of triangles of the input mesh
- **EDGE step**
  - $\varepsilon$ : compute features of vertices generated at mid-points of edges of the input mesh



- classical subdivision algorithm(在此基础上改进)
  - even vertices from previous iterations
  - newly inserted odd vertices
  - different: paper apply  $V$  and  $\varepsilon$  in sequence, instead of parallel.

in order to harness neighborhood information from previous steps

- Design choices(及时训练集很少也能使效果不错)
  - operate over local mesh patches and share weights rather than global => even a single training pair provides many local mesh patches to train our neural modules
  - mesh discretization: don't require re-parameterizing or re-sampling the surface
  - represent vertices using differential quantities to a local coordinate frame instead of using global coordinates => invariant
- component of neural model: half-flap 优点
  - provide a unique canonical orientation for the 4 vertices
  - provide well-defined local coordinate fram
  - each is a shallow MLP -> define over features of four ordered points
  - train operators per model ( $I, V, \varepsilon$ ) across all levels of subdivision and training examples
- **average pooling:** aggregate features from different half-flaps to per-vertex features
  - 1,2 step apply the half-flap operator to every outgoing edge in a 1-ring neighborhood of a vertex, and average pooling aggregates per-half-flap outputs into a per-vertex feature
  - 3th step apply half-flap operator for each directions of the edge and again uses average pooling to get the vertex feature
- **representation**
  - local differential quantities in local frame of each half-flap => ensure invariance to rightd transformation
    - x-axis: half-edge direction
    - z-axis: average two adjacent face normal 得到的边缘法线
    - y-axis: cross product



- input
  - $I$ : three edge vectors and differential coordinates of each vertex(discrete curvature information)
  - $V$  and  $\varepsilon$ : edge vectors and per-vertex high-dimensional learned features
- output
  - $V$  and  $\varepsilon$ : high-dimensional learned features and differential quantities that can be used to reconstruct the vertex position

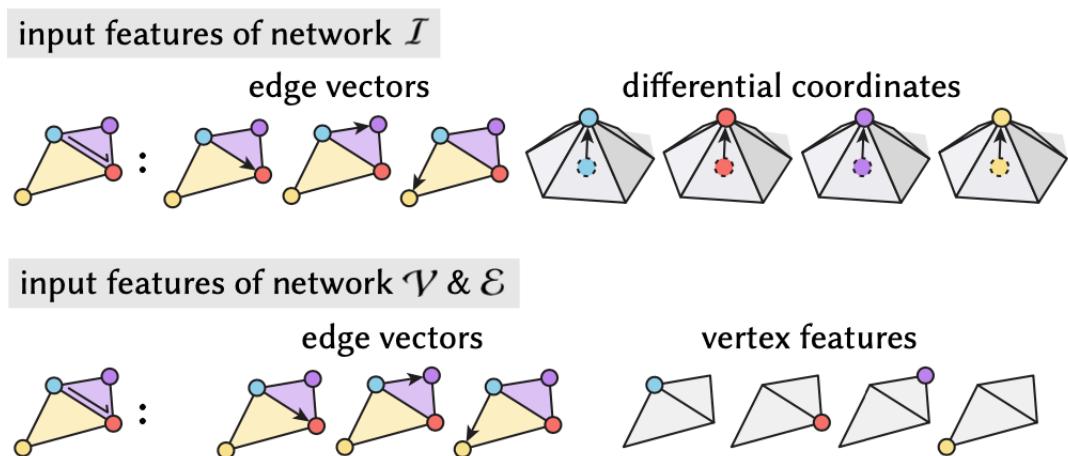


Fig. 19. The input feature to module  $\mathcal{I}$  consists of three edge vectors from the source vertex (blue) and vectors of the differential coordinates for the four vertices. The input features to module  $\mathcal{V}$  and  $\mathcal{E}$  are three edge vectors with per-vertex high-dimensional features from the previous steps.

## 6. Evaluations

- **isometric deformations**: train on a single pose => generalize to unseen poses under isometric deformations
- **non-isometric deformations**

- shapes from different classes
  - shapes from different types of distretizations
  - multiple shapes and categories
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## 7. Limitations & Future Work

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- extension
  - quadrilateral meshes
  - surface with boundaries
- limitation
  - 整合global information
  - harness information from a wider neighborhood and to dive to a deeper subdivision level

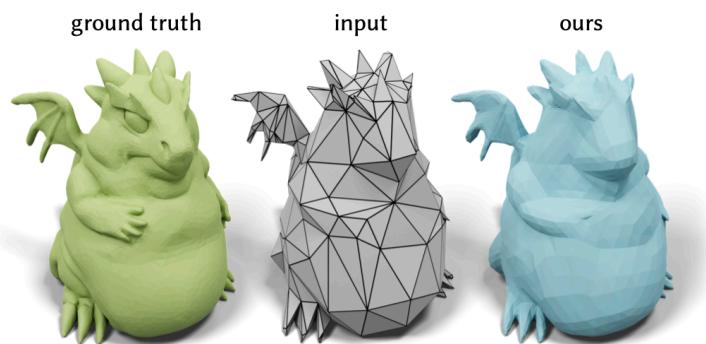


Fig. 30. Our approach is based on local geometry, and thus fails to hallucinate semantic features. ©Bratty Dragon by Splotchy Ink under CC BY.