

Assignment 2 (Due: May 31, 2020)

1. **(Math)** In the augmented Euclidean plane, there is a line $x - 3y + 4 = 0$, what is the homogeneous coordinate of the infinity point of this line?
2. **(Math)** A, B, C and D are four points in 3D Euclidean space, their coordinates are $(x_i, y_i, z_i), i = 1, 2, 3, 4$, respectively. Please prove that:

$$\text{These four points are coplanar} \Leftrightarrow \begin{vmatrix} x_1 & y_1 & z_1 & 1 \\ x_2 & y_2 & z_2 & 1 \\ x_3 & y_3 & z_3 & 1 \\ x_4 & y_4 & z_4 & 1 \end{vmatrix} = 0$$

3. **(Math)** On the normalized retinal plane, suppose that \mathbf{p}_n is an ideal point of projection without considering distortion. If distortion is considered, $\mathbf{p}_n = (x, y)^T$ is mapped to $\mathbf{p}_d = (x_d, y_d)^T$ which is also on the normalized retinal plane. Their relationship is,

$$\begin{cases} x_d = x(1 + k_1 r^2 + k_2 r^4) + 2\rho_1 xy + \rho_2 (r^2 + 2x^2) + xk_3 r^6 \\ y_d = y(1 + k_1 r^2 + k_2 r^4) + 2\rho_2 xy + \rho_1 (r^2 + 2y^2) + yk_3 r^6 \end{cases}$$

where $r^2 = x^2 + y^2$

For performing nonlinear optimization in the pipeline of camera calibration, we need to compute the Jacobian matrix of \mathbf{p}_d w.r.t \mathbf{p}_n , i.e.,

$$\frac{d\mathbf{p}_d}{d\mathbf{p}_n^T}$$

It should be noted that in this question \mathbf{p}_d is the function of \mathbf{p}_n and all the other parameters can be regarded as constants.

4. **(Math)** In our lecture, we mentioned that for performing nonlinear optimization in the pipeline of camera calibration, we need to compute the Jacobian of the rotation matrix (represented in a vector) w.r.t its axis-angle representation. In this question, your task is to derive the concrete formula of this Jacobian matrix. Suppose that

$$\mathbf{r} = \theta \mathbf{n} \in \mathbb{R}^{3 \times 1}, \text{ where } \mathbf{n} = \begin{bmatrix} n_1 \\ n_2 \\ n_3 \end{bmatrix} \text{ is a 3D unit vector and } \theta \text{ is a real number denoting the rotation angle.}$$

With Rodrigues formula, \mathbf{r} can be converted to its rotation matrix form,

$$\mathbf{R} = \cos \theta \mathbf{I} + (1 - \cos \theta) \mathbf{n} \mathbf{n}^T + \sin \theta \hat{\mathbf{n}}$$

$$\text{and obviously } \mathbf{R} \triangleq \begin{bmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{bmatrix} \text{ is a } 3 \times 3 \text{ matrix.}$$

Denote \mathbf{u} by the vectorized form of \mathbf{R} , i.e.,

$$\mathbf{u} \triangleq (R_{11}, R_{12}, R_{13}, R_{21}, R_{22}, R_{23}, R_{31}, R_{32}, R_{33})^T$$

Please give the concrete form of Jacobian matrix of \mathbf{u} w.r.t \mathbf{r} , i.e., $\frac{d\mathbf{u}}{d\mathbf{r}^T} \in \mathbb{R}^{9 \times 3}$.

In order to make it easy to check your result, please follow the following notation requirements,

$$\alpha \triangleq \sin \theta, \beta \triangleq \cos \theta, \gamma \triangleq 1 - \cos \theta$$

In other words, the ingredients appearing in your formula are restricted to $\alpha, \beta, \gamma, \theta, n_1, n_2, n_3$.

5. **(Programming)** RANSAC is widely used in fitting models from sample points with outliers. Please implement a program to fit a straight 2D line using RANSAC from the following sample points:
(-2, 0), (0, 0.9), (2, 2.0), (3, 6.5), (4, 2.9), (5, 8.8), (6, 3.95), (8, 5.03), (10, 5.97), (12, 7.1), (13, 1.2), (14, 8.2), (16, 8.5), (18, 10.1). Please show your result graphically.

