Mid-term exam, Electronic Circuit, Spring semester, 2019

April 15, 2019

We have 40 questions. The exam ends at AM 10:30. Write down your answers on the answer sheets. Explicitly show the unit of your answer. For each question, use the designated answer slot.

In the mid-term exam, no partial credit will be given.

When the numbers are provided, just answer a numerical value and a correct unit.

If not stated otherwise, use the long-channel IV characteristics of MOSFETs.

Some useful facts are summarized below:

For a resistor, $V = IR$. For a capacitor, $I = C\, dV/dt$. For an inductor, $V = L\, dI/dt$.

A good value for the thermal voltage, V_T, at room temperature is 0.02585 V. You may use an approximate value.

Approximately, $\ln 10 \approx 2.3$. Moreover, $e^{-0.7} \approx 0.5$.

The vacuum permittivity is approximately $8.85 \times 10^{-12}\ \mathrm{F/m}$.

The (absolute) elementary charge is approximately $1.6 \times 10^{-19}\ \mathrm{C}$.

At a given time, the power dissipation is defined by $P(t) = I(t)V(t)$.

At room temperature, the intrinsic carrier density of silicon, n_i, is 10^{10} /cm^3.

The built-in potential in a pn junction is given by $V_0 = V_T \ln \frac{N_A N_D}{n_i^2}$.

1. Consider the crystalline silicon. The position vector of any silicon atom can be written as either $\left(\frac{j+k}{2}, \frac{i+k}{2}, \frac{i+j}{2}\right) a$ or $\left(\frac{j+k}{2}+\frac{1}{4}, \frac{i+k}{2}+\frac{1}{4}, \frac{i+j}{2}+\frac{1}{4}\right) a$ in the Cartesian coordinate system. In this problem, i, j, and k are arbitrary integers and a is 0.543 nm. Calculate the shortest distance between silicon atoms.

2. Consider a silicon cube, whose volume is given by 10 nm X 10 nm X 10 nm. When the sample is n-type doped with a doping density of 10^{20} cm^{-3}, estimate the number of electrons in the sample. Of course, in this problem, the electrons represent the ones found in the conduction band.

3. Calculate the node voltage of the "out" node.

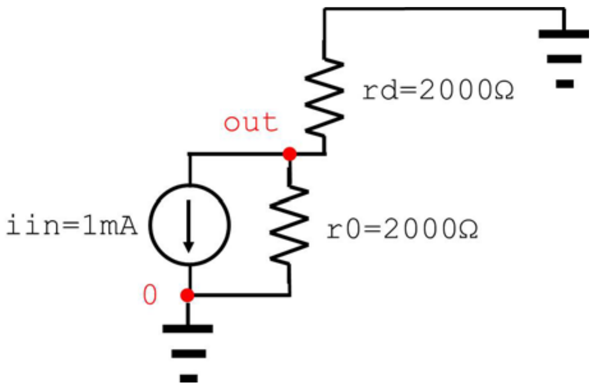

Fig. P3.

4. Consider a netlist shown below. Calculate the DC cathode voltage.

```
v1 batt 0 1.5
rchar batt anode 1e6
cchar anode 0 1e-9
rpcss anode cathode 21
rload cathode 0 50
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5. Consider a RC circuit. When the voltage source is given by $V(t) = V_{DC} + v_{amp} \sin \omega t$, calculate the current, $I(t)$.

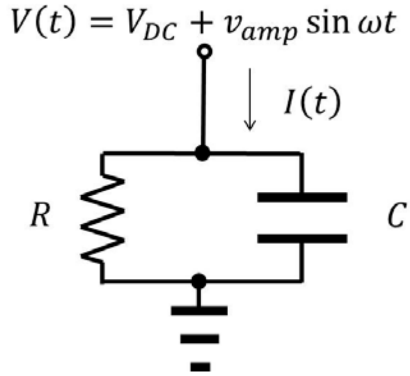

Fig. P5.

6. The reverse saturation current of the diode is 0.5 fA. Calculate the output voltage. Write your answer in the mV scale.

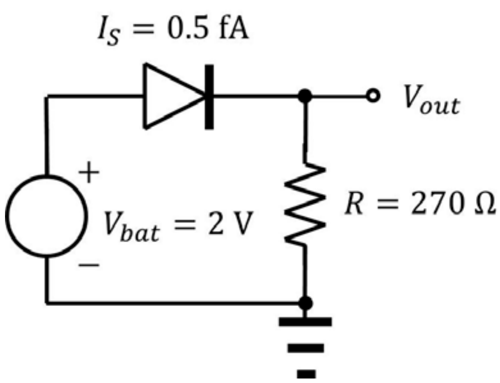

Fig. P6.

7. Instead of the diode used in Fig. P6, a new two-terminal element is introduced. (It is a fictitious one for the exam.) Its current-voltage relation is found to be $I = I_0 \frac{\exp(V/V_T)}{\exp(V/V_T)+1}$. Of course, V_T is the thermal voltage. In this problem, I_0 is 1 mA. Calculate the output node voltage.

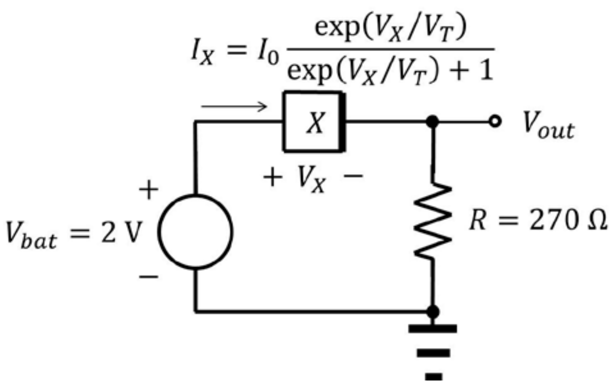

Fig. P7.

8. Three curves represent the electrostatic potential profiles for a pn diode. (Neglect the offset value of the electrostatic potential.) One of them is obtained at equilibrium. Another curve is for the forward mode (+0.1 V) and the other for the reverse mode (-0.1 V). Among the three curves, which one is the curve for the forward mode? Specify the correct curve index.

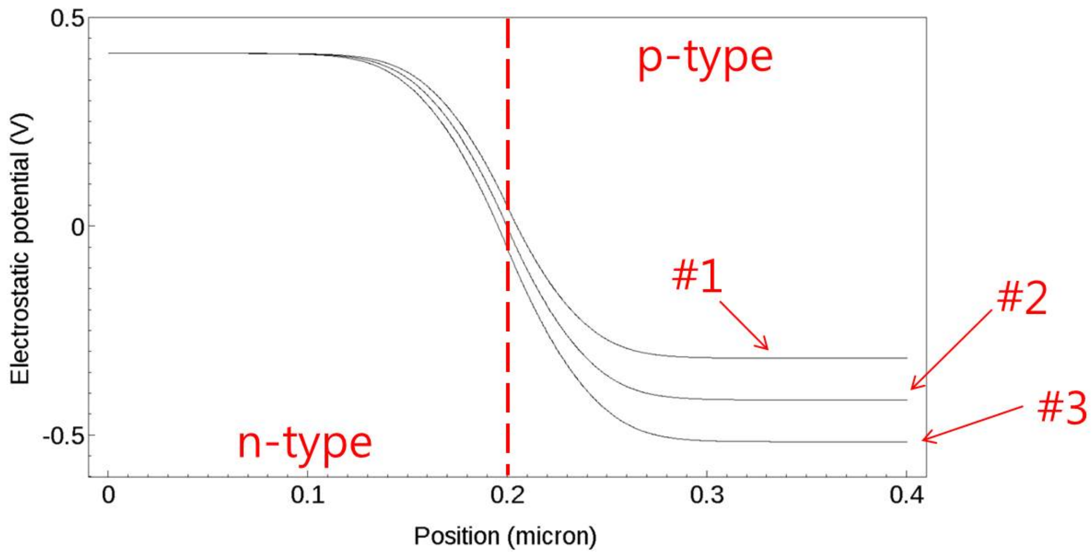

Fig. P8.

9. Three curves represent the electron and hole densities for a one-dimensional pn diode. The doping density in both n-type and p-type regions is 10^{17} cm^{-3}. One of them is obtained at equilibrium. Another curve is for the forward mode (+0.1 V) and the other for the reverse mode (-0.1 V). Estimate the small-signal capacitance per area.

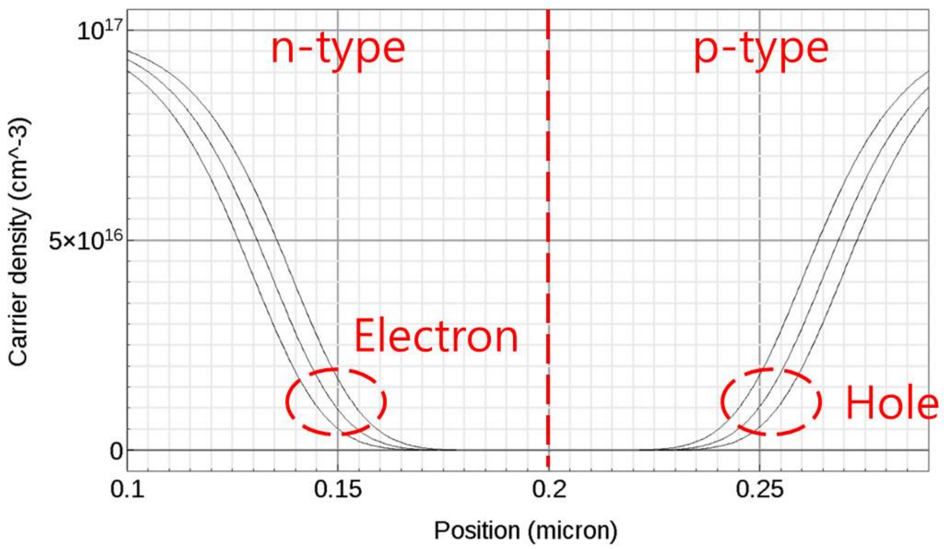

Fig. P9.

10. Three curves represent the diode currents as functions of the diode forward voltage. Although the diode is the same, the temperature is different for each curve. (250 K, 300 K, and 350 K) Among the three curves, which one is for 250 K? Specify the correct curve index.

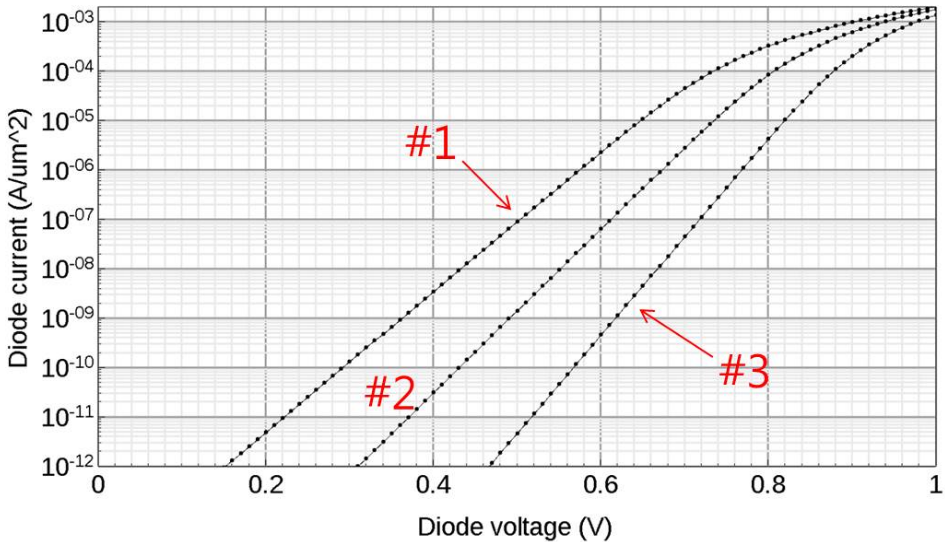

Fig. P10.

11. For a diode, the terminal current and the terminal voltage are related by $I = I_S\left(\exp(\frac{V}{V_T}) - 1\right)$. Assume that the DC diode voltage is zero. Express the small-signal resistance.

12. In the lecture, it has been stressed that the diode current-voltage graph shows a slope of 60 mV/dec. (It is drawn in the semi-log scale.) However, as we already know, “60 mV” is a rough value valid only at room temperature. In this problem, write the EXACT expression for “60 mV”, in terms of the thermal voltage.

13. The left figure shows the circuit schematic. Four cases for the capacitance are considered. 1 pF, 10 pF, 100 pF, and 1 nF. The output voltages are shown in the right figure as black lines. They are labeled with #1, #2, #3, and #4, from the bottom to the top. (The red line in the right figure shows the sinusoidal input voltage.) Among the four black curves, which one is for 1 nF? Specify the correct curve index.

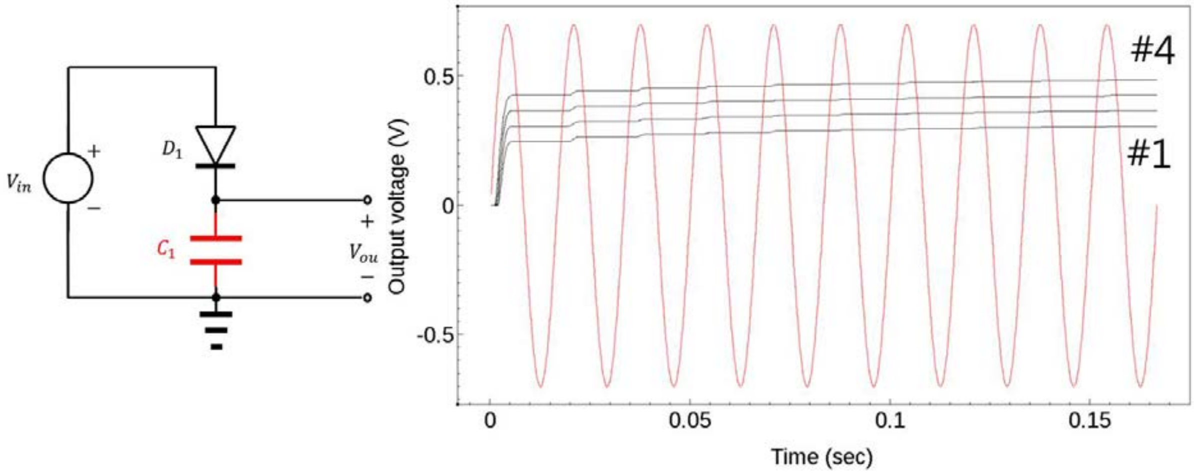

Fig. P13.

14. Consider a diode whose reverse saturation current is 0.5 fA. A researcher performs the small-signal analysis on the diode biased as 0.4 V. The small-signal resistance is extracted. After extracting it, the researcher calculates the diode current at 0.7 V by using the small-signal resistance. Of course, this is wrong and the result is not correct. Let us call this wrong current I_{wrong}. Calculate the following ratio:

$$\text{Ratio} = \frac{I_D(0.7\ \text{V})}{I_{wrong}}$$

15. Consider a capacitor made of two parallel metal plates. Between the metal plates, a 2-nm-thick silicon dioxide layer is found. Area of each metal plate is 400 nm^2. Calculate the capacitance. Remember that the vacuum permittivity is approximately $8.85 \times 10^{-12}\ \text{F/m}$. The relative permittivity of the silicon dioxide is 3.9.

16. Consider a MOS structure. The threshold voltage is 0.23 V and the p-type substrate is doped with a density of 10^{18} cm^{-3}. The gate voltage is 0.0 V. In this case, +4X10^{-7} C/cm^2 is found in the gate. Estimate the depth of the depleted region in the substrate.

17. Consider a MOS structure. The inversion charge per area is plotted. Estimate the threshold voltage.

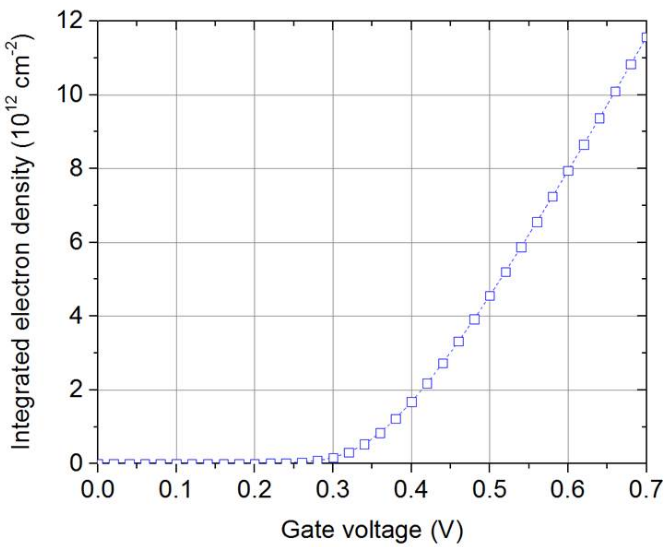

Fig. P17.

18. Consider the same graph in P17. Calculate the thickness of the dielectric layer. The relative permittivity of the dielectric material is 25.

19. The area of a source region is 1600 nm^2. An abrupt doping profile is introduced in the source region. The dopant density is 2 X 10^{20} cm^{-3}. The depth of the source region is 50 nm. Then, how many n-type dopants are found in the source region?

20. The threshold voltage of a MOS structure is dependent on the substrate doping density. Consider a certain MOS structure. The p-type doping density in the substrate region is intentionally doubled. What happens to the threshold voltage? Increased? Decreased? Or doesn't change?

21. The threshold voltage of a NMOSFET is 0.8 V. $\mu_n C_{ox} \frac{W}{L}$ is $4\ \mathrm{mA/V^2}$. Assuming the long-channel MOSFET, calculate the drain current at V_{GS} = 1.8 V and V_{DS} = 0.1 V.

22. For the same MOSFET in P21, calculate the drain current at V_{GS} = 1.8 V and V_{DS} = 1.8 V.

23. Assuming the long-channel MOSFET, write down the small-signal output resistance at V_{DS} = 0 V. Of course, the gate voltage is larger than the threshold voltage.

24. The I_D–V_{DS} curve of a MOSFET is shown. For three gate bias voltages (1.09 V, 1.1 V, and 1.11 V), the drain voltages are swept from 0 V to 1.1 V. The right figure is the magnified version of the original graph. Calculate the transconductance at V_{GS} = 1.1 V and V_{DS} = 1.1 V.

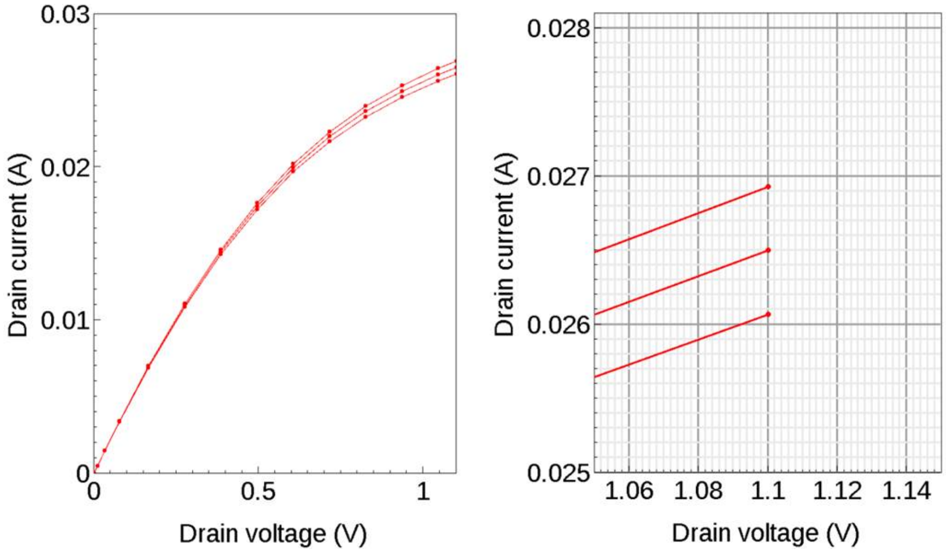

Fig. P24.

25. The input voltage is given as $V_{in}(t) = A\sin\omega t$. Of course, $\omega = 2\pi f$ is the angular frequency. The period, T, is given as an inverse of f. After an ideal rectifier, the output voltage is given as $V_{out}(t) = A\sin\omega t\,\Theta(\sin\omega t)$. Here, Θ is the step function, which is unity for a non-negative argument and zero otherwise. Calculate the time-averaged value of $V_{out}(t)$:

$$\text{Time} - \text{averaged value} = \frac{1}{T}\int_0^T V_{out}(t)dt$$

26. Consider the same situation in P25. Instead of the time-averaged value, calculate the second harmonic component:

$$\text{Second harmonic} = \frac{1}{2T}\int_0^T V_{out}(t)\cos 2\omega t\, dt$$

27. Calculate the node voltage.

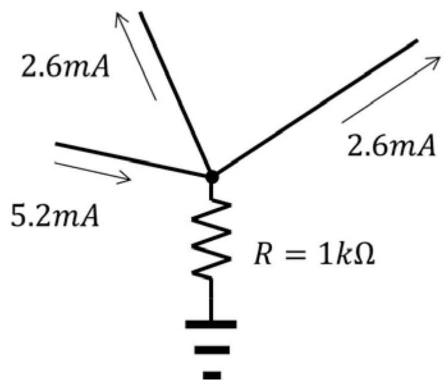

Fig. P27.

28. Consider a pn junction. Its n-type region is uniformly doped with a donor density of $2X10^{19}$ cm^{-3}. Its p-type region is also uniformly doped. However, the acceptor density is 10^{18} cm-3. As a certain reverse bias, the depletion layer inside the p-type region is 40-nm-long. Then, what is the length of the depletion layer inside the n-type region?

29. In this problem, the maximum limit of the drain current is studied. It is well known that the maximum speed of an electron in the Si crystal is about $1 \text{ X } 10^7$ cm/sec. In this problem, let us ASSUME that all the inversion electrons in the MOSFET move at a speed of $1 \text{ X } 10^7$ cm/sec. The oxide capacitance (per area) is $3.2 \text{ X } 10^{-6}$ F/cm^2. The threshold voltage is 0.3 V. Also V_{DD} is 0.7 V. Then, estimate the maximum drain current of the MOSFET with L = 20 nm and W = 20 nm.

30. Consider the circuit shown in Fig. P30. All three MOSFETs have the same parameters, except for the channel width, W. Calculate a sum of all drain currents, when $V_{G1} = V_{G2} = V_{G3} = 1$ V.

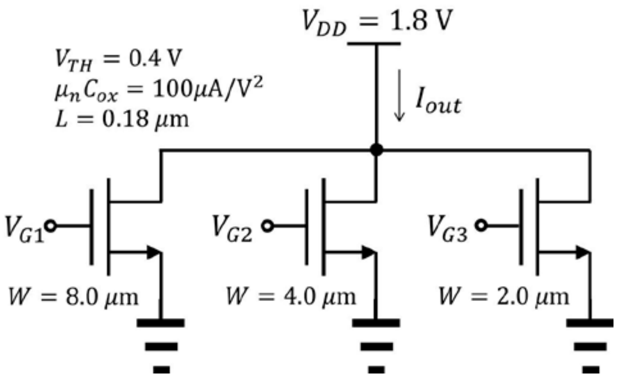

Fig. P30.

31. Consider the same circuit in Fig. P30. In this problem, we may apply either 0 V or 1 V to each gate terminal, individually. Then, a triplet (V_{G1} , V_{G2} , V_{G3}) is converted into the output current. There can be eight cases, for example, (0 V, 0 V, 0 V), (0 V, 0 V, 1 V), (0 V, 1 V, 0 V), and so on. Then, what is the smallest difference between two closest output currents?

32. Initially, the gate voltage is 0 V. Also it is assumed that 50 fC is initially stored in the 25 fF capacitor. At t = 0^+, the high gate voltage (1.8 V) is applied. After ten seconds later, what would be the voltage across the capacitor?

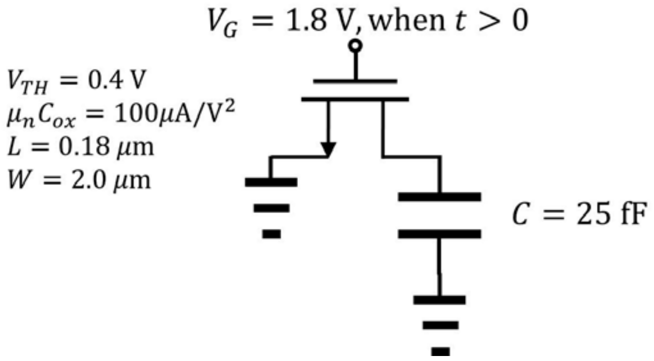

Fig. P32.

33. Consider a chain of MOSFETs. Three identical MOSFETs are serially connected. The drain terminal of the third MOSFET is connected to a DC voltage source, whose value is small as v_D. Calculate the small-signal resistance by evaluating $\frac{v_D}{i_D}$.

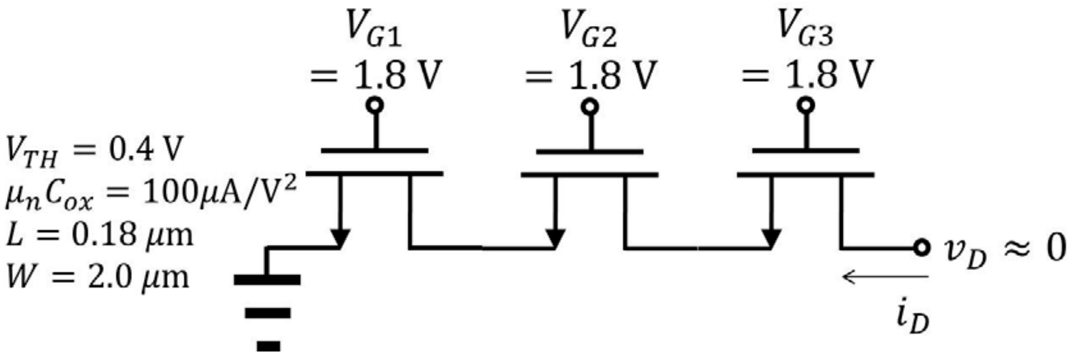

Fig. P33.

34. By a certain means, we increase the threshold voltage of the second MOSFET. It is now 1.6 V. Then, we expect the small-signal resistance (given by $\frac{v_D}{i_D}$) is increased. Calculate the following ratio:

$$\text{Ratio} = \frac{\text{Small} - \text{signal resistance in P34}}{\text{Small} - \text{signal resistance in P33}}$$

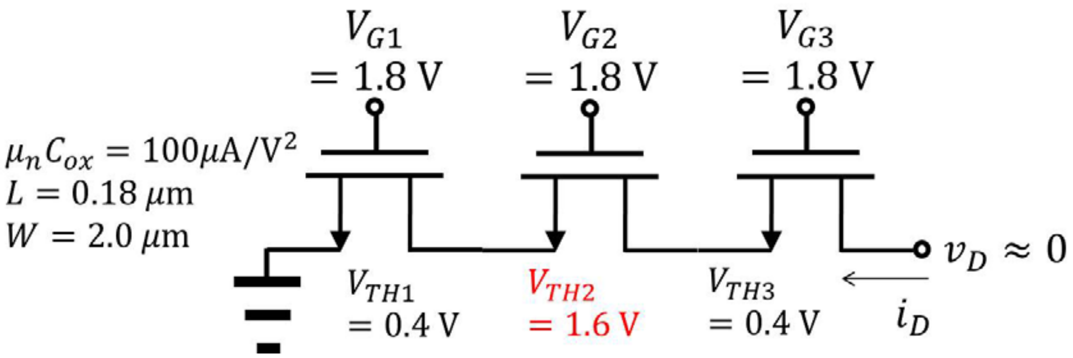

Fig. P34.

35. Consider a MOS structure whose dielectric thickness is 2 nm. Assume that the permittivity of the dielectric material is 3.2 X 10^{-13} F/cm. The threshold voltage is 0.4 V. When the inversion electron density per area is 2 X 10^{12} cm^{-2}, what is the gate voltage?

36. The transconductance of a MOSFET (whose width is 1 micron) is shown as a function of the gate voltage. The drain bias voltage is fixed. Find the optimal gate voltage to maximize the transconductance.

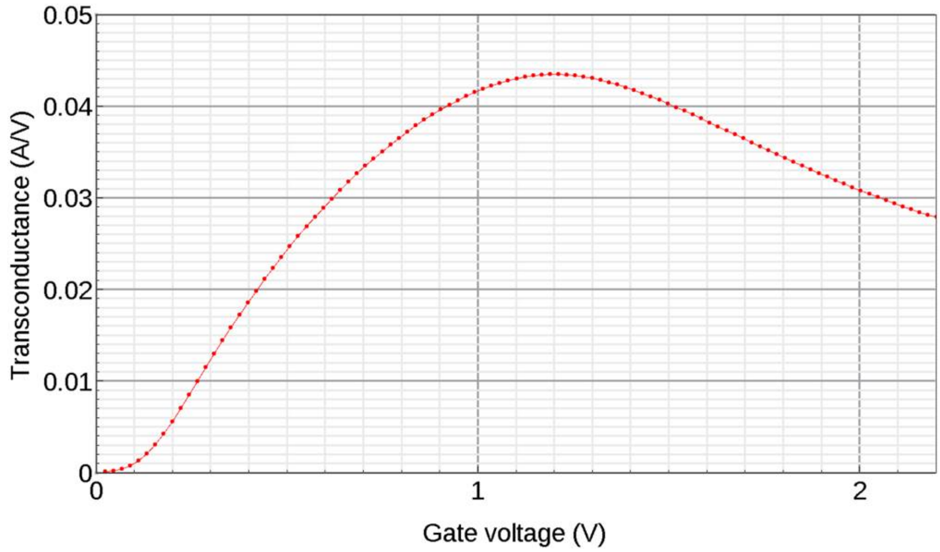

Fig. P36.

37. Based on the answer of the previous P36, the gate bias voltage is fixed to the optimal value. The drain bias voltage is also fixed to that of P36. However, the width of the MOSFET is not 1 micron but 40 nm. When a small-signal voltage excitation of sin(377t) mV is applied to the gate terminal, write the small-signal current response of the drain current.

38. Calculate the input voltage, V_{in}.

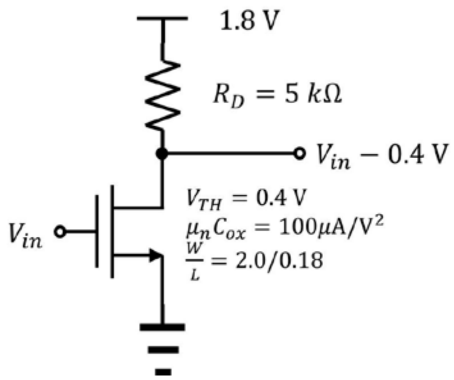

Fig. P38.

39. Calculate the input voltage, V_{in}.

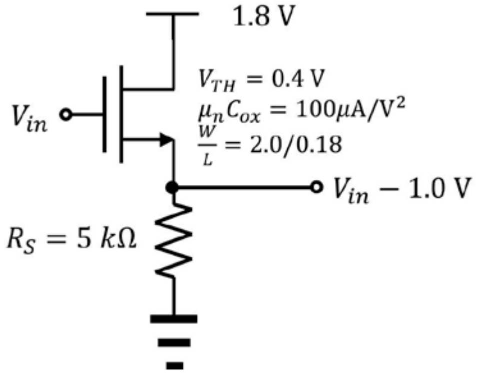

Fig. P39.

40. The drain current is normalized with the width. Consider a MOSFET whose width is 20 nm. It is operated in the saturation mode. Then, estimate its small-signal output resistance at the gate voltage of 0.7 V.

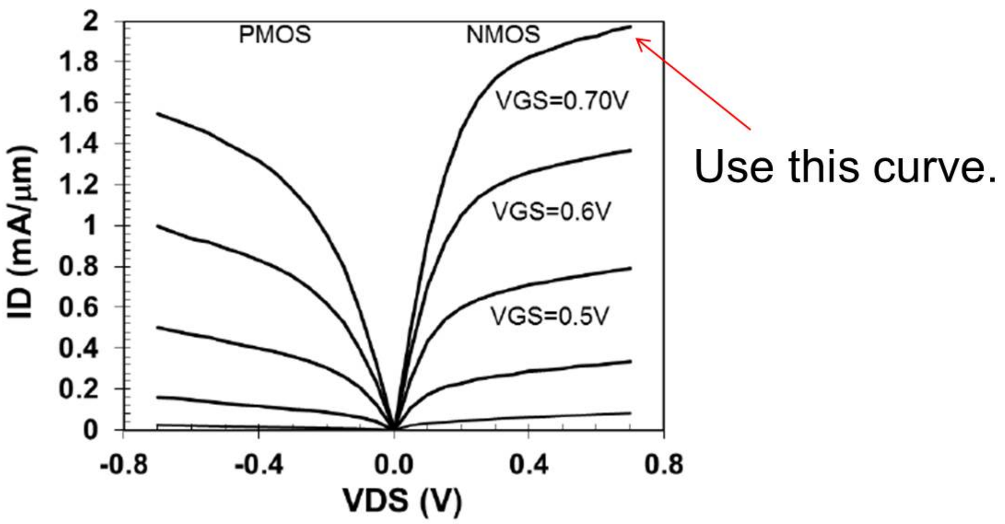

Fig. P40.