Problem Chosen	2022 MCM/ICM Summary Sheet	Team Control Number
C		2200524

The Midas Touch: A Trading Strategy in Gold and Bitcoin Market

Summary

In recent years, gold and bitcoin have become more and more popular in the investment market. How to use historical information to allocate investment assets so as to maximize returns is a problem worth pondering.

This paper aims to propose reasonable and effective trading strategies for traders by establishing a mathematical model, making use of the market price data from 2016 to 2021. Facing the intricate market environment, our strategy successfully balances benefits and risks and offers the safest investment strategies to the trader.

Our work includes the following steps: indicator building, Analytic Hierarchy Process, scoring, investment strategy proposition.

First, to obtain the indicators needed for further analysis, a bull and bear market judgment model, a 5-day/15-day moving average trading method, a risk assessment model and a time series forecast model are introduced. Then, indicators including moving average tendency, investment risk and Expected profit rate are abstracted from the results of the models above.

Next, to quantify the importance of each indicator, calculate the relevant weights of all indicators using the AHP model. We arranged a judgement matrix according to the relative importance between each factor after passing the Consistency Judgment, and used the arithmetic mean method to find the weights.

Then, after proper positivization, calculate the weighted average of all indicators, which is regarded as the daily score of investing in gold and investing in Bitcoin.

Finally, according to the scores of the investments, we deduced the daily investment strategies, earning 1770.87% by the end of our 5-year plan.

In addition, we evaluated the advantages and disadvantages of the model and proposed some suggestions, and carried out a sensitivity analysis of the model to the commission rate, thereby proved the reliability and stability of the model.

Keywords: trading strategies; AHP; ARIMA; 5/15 MA; sensitivity analysis; SPSS; MATLAB

Contents

1 Introduction

1.1 Problem Background

To get the most bang for the buck, market traders frequently buy and sell volatile assets, including the extremely popular investment commodity **gold** and **Bitcoin**, which has become popular in recent years.

Of all precious metals, gold is the most popular investment. Investors typically buy gold as a way to diversify risk, especially through the use of futures contracts and derivatives. The gold market is as susceptible to speculation and volatility as any other.

Bitcoin is a decentralized digital currency with no central bank or single administrator that can be sent from one user to another on the peer-to-peer Bitcoin network without intermediaries. Transactions are verified by network nodes and recorded in a public distributed ledger called the blockchain. Cryptocurrency was invented in 2008 by an unknown person or group of people using the name Satoshi Nakamoto. The currency came into use in 2009 when its implementation was released as open source software. [1]

It is crucial for traders to determine the asset allocation plan for the day based on the previous intraday price of the asset, and in fact, there is a commission on every purchase and sale. Therefore, establishing a mathematical model to infer the optimal investment portfolio based on the existing data will become a very effective way for the trader.

Figure 1: Gold

Figure 2: Bitcoin

1.2 Restatement of the Problem

This problem is to develop a mathematical model for a trader to choose a daily investment scenario, **based solely on the past flow of intraday prices** to date to determine whether the trader should buy, hold or sell the underlying asset.

At the start time, September 11, 2016, you have $1000 in cash. During a 5-year trading period, the trader will have a trading portfolio [C,G,B] consisting of cash, gold, and Bitcoin in USD, troy ounces, and Bitcoin. A corresponding commission is charged for each

[1] Wikipedia:Bitcoin

transaction. The commission is $\alpha\%$ of the transaction amount, 1% for gold, 2% for Bitcoin, and there is no cost to hold assets. Bitcoin can be traded every day, while gold can only be traded on specific open days. On this basis, the problems are as follows:

- Develop a model that gives the best daily trading strategy based only on the previous data. Using your model and strategy, how much the initial $1,000 investment will be worth on September 10, 2021?
- Give evidence that your model provides the best strategy.
- Determine the sensitivity of the strategy to transaction costs, how do transaction costs affect the strategy and outcomes?
- Communicate your strategies, models and results to traders in a memo of no more than two pages.

2 General Assumptions and Model Overview

2.1 Assumptions

To simplify the problem, we make the following basic assumptions, each of which is properly justified.

1. The daily price of Bitcoin and gold is regarded as a constant and the daily price can be known at the time of transaction.
2. Traders are rational people with risk aversion.
3. There is no inflation.
4. There are no major changes in the market within these five years.

2.2 Model Overview

This article mainly assigns points to the decision-making of daily trading strategies on gold and bitcoin, then deduced daily investment decisions based on score values. Before this, AHP was used to obtain various indicators and their weights to measure the pros and cons of trading decisions. In order to quantify these indicators, a *bull and bear market judgment model*, a *5-day/15-day moving average trading method*, a *risk assessment model* and a *time series forecast model* are introduced. Finally, by searching the relevant information, according to the comparison between the score and the threshold, we can judge how the two assets should be traded, and the daily asset trading strategy is obtained.

In summary, the whole modeling process can be shown as follows:

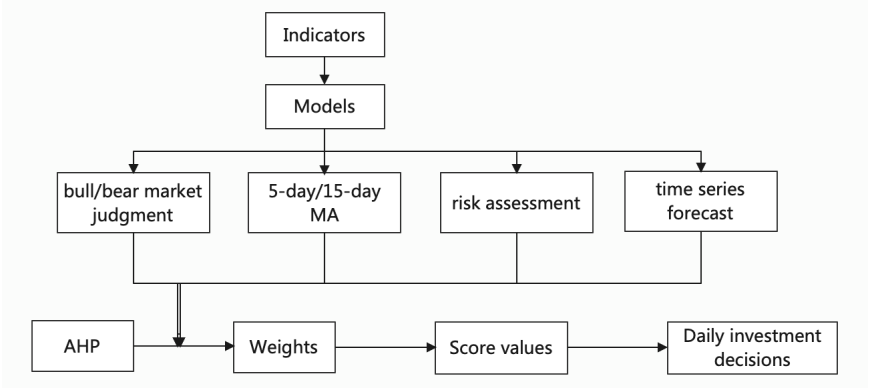

Figure 3: Model Overview

3 Model Preparation

3.1 Notations

Important notations used in this paper are listed in Table 1,

Symbols	Definitions	Unit
P_g	the intraday price of gold	USD
P_b	the intraday price of bitcoin	USD
w_j	The weight of indicator j	
r_g	The investment risks of Investing in Gold	
r_b	The investment risks of Investing in Bitcoin	
$R_g^{'}$	The 15-day profit rate of investing in Gold	USD
$R_b^{'}$	The 5-day profit rate of investing in Bitcoin	USD
P_{g15}	The price of gold on 15-day moving average line	USD
P_{b5}	The price of Bitcoin on 5-day moving average line	USD
T_g	Investment tactic of gold(0,1,-1)	
T_b	Investment tactic of Bitcoin(0,1,-1)	
$C(i)$	The amount of cash on day i	USD
$B(i)$	The amount of Bitcoin on day i	$\triangle B$(Satashi)
$G(i)$	The amount of Gold on day i	troy ounce

Table 1: Notations

3.2 Data Preprocessing

The data we use includes the data files given as:

1. LBMA-GOLD.csv
2. BCHAIN-MKPRU.csv.

These two files give the dollar price of gold and Bitcoin respectively on the corresponding date. But before using it, the data needs to be preprocessed.

Firstly, convert the time into date-type data in a unified time format; then merge the two tables, keep the Bitcoin price unchanged, and complete the gold price according to the price of the last trading day. Then fill the missing value with the last non-missing value; at the same time, add a column of indicators: transaction, if it is **not** a gold trading day, fill in 0, and if it **is** a gold trading day, fill in 1. This indicator is related to subsequent transactions, and it should be judged whether the current gold is tradable in each transaction.

For the convenience of data processing, the date format in the data needs to be readjusted, so the date is numbered. Taking September 11, 2016 as the first day, then numbering them in sequence,so the last day, September 10, 2021, is the 1826th day. In the subsequent processing process, expressions similar to "day i" are involved, which are obtained by numbering the date according to this idea.

In order to quantify the investment strategy, for any asset,we set **+1 for buying, 0 for holding, and -1 for selling**, so that the investment strategy can be quantitatively expressed, and then used as a useful indicator.

4 Step I:Bull and Bear Market Judgment Model

4.1 Introduction to the concept of bull and bear market

The so-called bull market and bear market, both represent **market trends**.

A **market trend is** a perceived tendency of finance markets to move in a particular direction over time. These trends are classified as *secular* for long time frames, *primary* for medium time frames, and *secondary* for short time frames. Traders attempt to identify market trends using technique analysis, a framework which characterizes market trends as predictable price tendencies within the market when price reaches support and resistance levels, varying over time.

The terms "bull market" and "bear market" describe upward and downward market trends, respectively, and can be used to describe either the market as a whole or specific sectors and securities. A **bull market** is a period of generally rising prices, generally begins when stocks rise 20% from their low, and ends when stocks drawdown 20%. A **bear market** is a general decline in the stock market over a period of time.

4.2 Introduction to MA(Moving Average)

Moving Average, referred to as MA, is a method of statistical analysis, which averages the security prices (indexes) in a certain period and connects the averages of different times to form a MA, which is a technical indicator used to observe the trend of a security's price change. The **moving average theory** is one of the most commonly used technical indicators nowadays. It helps traders confirm the existing trend, forecast the trend that will appear and find the trend that is overextended and about-to-reverse .

Commonly used moving average lines are 5-day, 10-day, 30-day, 60-day, 120-day and 240-day indicators. Among them, the 5-day and 10-day short-term moving averages are

the reference indicators for short-term operations and are called daily moving averages; 30-day and 60-day moving averages are medium-term moving averages, called quarterly moving averages; 120-day and 240-day moving averages are The long-term moving average is called the annual moving average.

The most commonly used method of moving average is to compare the relationship between the moving average of the security price and the price of the security itself. A buy signal occurs when a security's price rises above its moving average. A sell signal is generated when the price of a security falls below its moving average.

4.3 Judgement of Bull and bear market

Because gold and bitcoin trading is similar to stock market trading, we can borrow related concepts from the stock market, such as bull and bear markets and moving averages.

We can borrow the *moving average* concept to determine whether the market is bullish or bearish, based on the 60-day line (that is, the curve obtained by averaging the prices of the current day and the previous 60 days, and connecting the averages over time). If the price curve of the day is above the 60-day line, it is considered a bull market, and the market is optimistic , which means there are more opportunities. Otherwise, it is a bear market, and the market is negative which contains fewer opportunities. [1]

The calculation method is listed below:

$$\overline{P}_{60} = \frac{P_{i-59} + P_{i-58} + \cdots + P_i}{60} \tag{1}$$

We made the 60MA of gold and Bitcoin,then made the price-time curve of these two assets. By comparison, we can clearly see the bull and bear market trend in these 5 years, as is shown in the figures below:

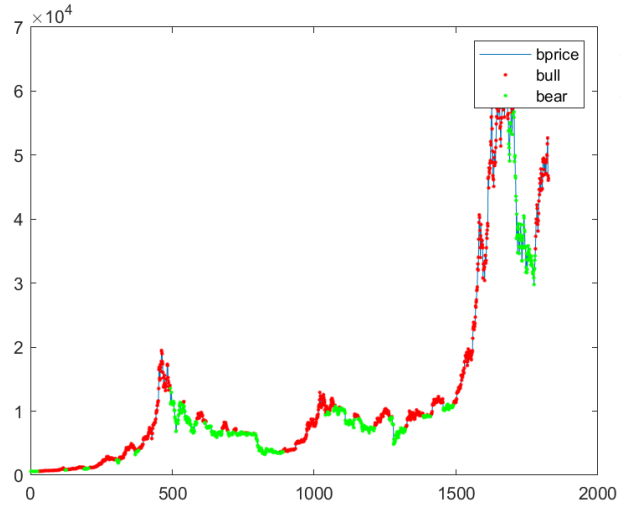

Figure 4: Bull Bear of Bitcoin

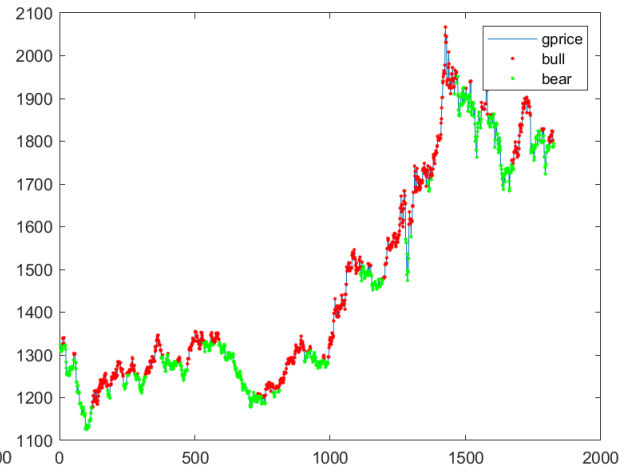

Figure 5: Bull Bear of Gold

5 Step II: 5MA (15MA) trading method

There is such concept as 5-day moving averages in stock trading, which can also be used as evaluative indicators for investment decisions in this problem. Because Bitcoin has a large fluctuation, it is best to use short-term trading, while gold has less volatility, so it is better to use intermediate-short term trading. Therefore, the 5-day moving average is better used as an important trading evaluation indicator for **Bitcoin**, and the 15-day moving average is better used as an important trading evaluation indicator for **gold**.

Similar to stocks, the 5-day (or 15-day) moving average can also play a judging role in gold (or Bitcoin) investment decisions.

For Bitcoin:

- When the intraday price is higher than the 5-day moving average but lower than 1.15 times of it, it means that the overall market is profitable and suitable for buying.
- When the intraday price is lower than the 5-day moving average :
    1. then if it is a bull market at this time, hold the asset.
    2. If it is a bear market, and the intraday price is lower than the moving average but higher than 0.75 times the moving average, hold the asset;
    3. if the intraday price is lower than 0.75 times the moving average , it's suitable for buying.
- When the intraday price is higher than 1.15 times the 5-day moving average, it's suitable for selling.

As for gold:

- When the intraday price is higher than the 15-day moving average but lower than 1.05 times of it, it means that the overall market is profitable and suitable for buying.
- When the intraday price is lower than the 15-day moving average
    1. then if it is a bull market at this time, hold the asset.
    2. If it is a bear market, and the intraday price is lower than the moving average but higher than 0.85 times the moving average, hold the asset;
    3. if the intraday price is lower than 0.85 times the moving average , it's suitable for buying.
- When the intraday price is higher than 1.05 times the 15-day moving average, it's suitable for selling.

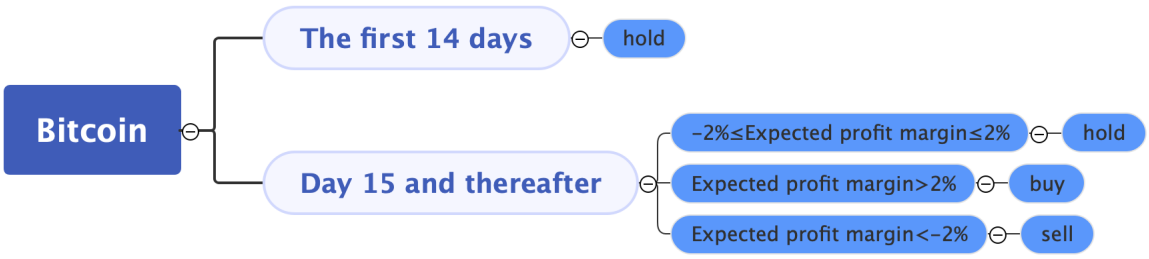

Figure 6: Method Illustration of Bitcoin

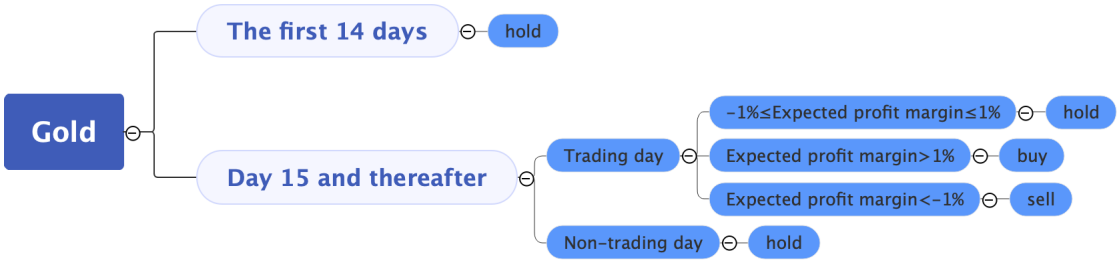

Figure 7: Method Illustration of Gold

According to the above method, the buying and selling decisions of gold and Bitcoin can be obtained based on the price of the day and the 5-day and 15-day moving averages. As is shown below:

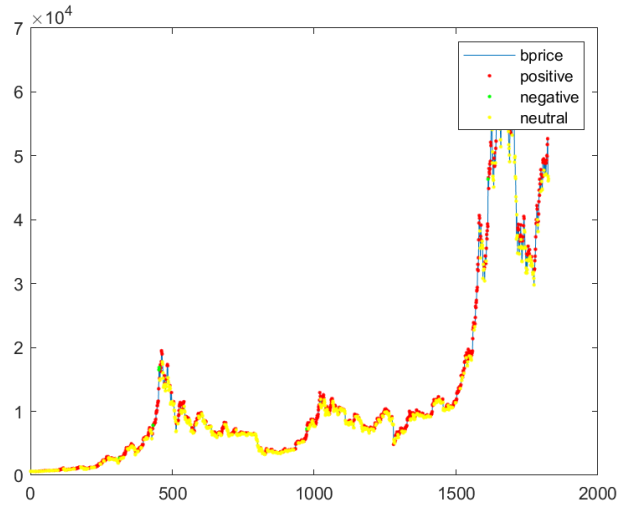

Figure 8: Bitcoin Trading Method

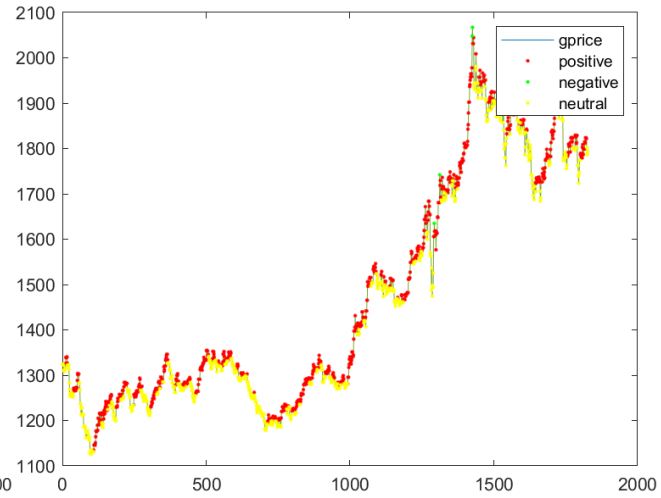

Figure 9: Gold Trading Method

6 Step III:Time-Series Forecasting Model

6.1 The concept of Time Series

A **time series** is a sequence of numbers listed in time order. Most commonly, a time series is a sequence taken at successive equally spaced points in time. Thus it is a sequence of discrete-time data. **Time series analysis** includes methods for analysing time series data to extract meaningful statistics and other characteristics of the data. **Time series forecasting** is the use of a model to predict future values based on previously observed values.

Time series analysis is one of the quantitative forecasting methods. It includes general statistical analysis (such as autocorrelation analysis, spectral analysis, etc.), the establishment and inference of statistical models, and the optimal prediction, control and filtering of time series. Classical statistical analysis assumes the independence of data series, while time series analysis focuses on the interdependence of data series. The latter is actually a statistical analysis of the stochastic process of discrete indicators, so it can be regarded as a component of stochastic process statistics. For example, the rainfall of the first month, the second month, ..., the Nth month of a certain area is recorded, and the rainfall of each month in the future can be predicted by using the time series analysis method.

6.2 Stationarity of time series

If the time series x_t meets the following three conditions:

$$E(x_t) = E(x_{t-s}) = u \tag{2}$$

$$Var(x_t) = Var(x_{t-s}) = \delta^2 \tag{3}$$

$$Cov(x_t, x_{t-s}) = \gamma_s \tag{4}$$

then it's called *covariance stationary*, or weak stationary for short.

In *Time Series Analysis*, a time series that satisfies the above conditions is called "stationary", and the stationary mentioned in the *time series field* is generally weakly stationary by default.

6.3 Model Building

The price data given in this topic is arranged in time order, which *is* a typical time series. Therefore, with the help of the time series analysis idea, we can use the known past price data to predict future price data, which is very helpful for trading decisions.

Since the price in this topic is not a stationary time series, the data needs to be differentiated and converted into a stationary time series before modeling. Therefore, we adopt the ARIMA (Autoregressive Integrated Moving Average Model) model for prediction. The model has the following form:ARIMA(p,d,q) [2]

$$y_t^{'} = \alpha_0 + \sum_{i=1}^{p} \alpha_i y_{t-i}^{'} + \epsilon_t + \sum_{i=1}^{q} \beta_i \epsilon_{t-i} \tag{5}$$

$$y_t^{'} = \triangle^d y_t = (1-L)^d y_t \tag{6}$$

By integrating these two equations, we can obtain the general expression of the ARIMA model.

$$(1 - \sum_{i=1}^{p} \alpha_i L^i)(1-L)^d y_t = \alpha_0 + (1 + \sum_{i=1}^{q} \beta_i L^i)\epsilon_t \tag{7}$$

Using the SPSS Expert Modeler function, the *p, d, and q* parameters of the ARIMA model can be determined. It is calculated that gold satisfies ARIMA(0,1,16), while Bitcoin satisfies ARIMA(0,1,10). By substituting the parameter values into the above formula, the time series forecast values of gold and Bitcoin prices can be obtained respectively:

$$(1-L)y_t = \alpha_0 + (1 + \sum_{i=1}^{16} \beta_1 L)\epsilon_t \ (forGold) \tag{8}$$

$$(1-L)y_t = \alpha_0 + (1 + \sum_{i=1}^{10} \beta_i L^i)\epsilon_t \ (forBitcoin) \tag{9}$$

6.4 Forecasting Results

According to the 5 years price data of gold and Bitcoin given in the problem. Take the numbered dates as the horizontal axis and the price as the vertical axis to make a price-time line chart. Meanwhile, on the same time axis, with the predicted price data as the vertical axis, a price-time line chart of the predicted data is made. As is shown by the two figures below. At the same time, using SPSS to calculate the goodness of fit (R^2) :

gold: 0.998, Bitcoin: 0.997

According to the image and R^2, it is obvious that the model fits very well.

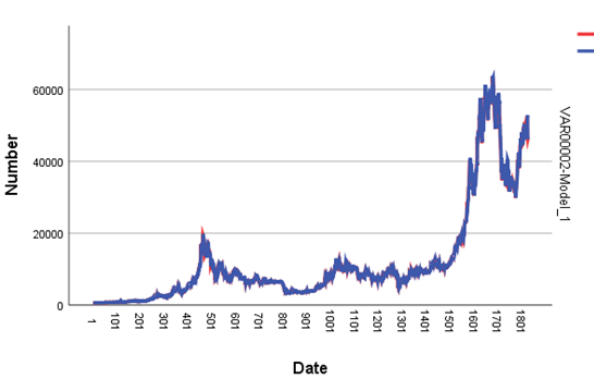

Figure 10: Bitcoin Forecast

Figure 11: Gold Forecast

According to the results of the time series prediction, we can calculate the price P_g of gold in the next 15 days and the price P_b of Bitcoin in the next 5 days.

7 Step IV:Risk Measurement Model

When traders invest, the investment risk of an asset is a very important factor, and it is also an important indicator for us to measure related issues.

We take the **Coefficient of variation** of relevant assets as a quantitative representation of investment risk. After experiments,we decide to take 30 days as a time unit. Let the prices' standard deviation be S_p and the average value be $\overline{p}$, then the coefficient of variation can be expressed as the ratio of the standard deviation and the mean, the formula is as follows [3]

$$CV = \frac{S_p}{\overline{p}}$$

It should be noted that when the known data volume is less than 15, the volatility reliability is not good because the data volume is too small, so the indicator is calculated from the 15th day. For the first 15 days, traders are advised to sit still and not to trade.

Part of the calculation results are as follows:

	E	F	G	H	I	J	K	L	M	N	O
1	Date	9/25/16	9/26/16	9/27/16	9/28/16	9/29/16	9/30/16	10/1/16	10/2/16	10/3/16	10/4/16
2	risk_b	0.010864	0.010996	0.01067	0.010374	0.010142	0.009947	0.010142	0.010129	0.010032	0.009849
3	risk_g	0.008627	0.008959	0.008709	0.008468	0.008302	0.008094	0.007901	0.007721	0.007714	0.009656

Figure 12: Calculation Results of risks

8 Step V: Calculating Prospective Profits

The long-term volatility of gold is quite small, while the volatility of Bitcoin is relatively large. Therefore, Bitcoin is suitable for short-term investment, and gold is suitable for intermediate-short term investment. Choose 5 days as the investment cycle for Bitcoin and 15 days as the investment cycle for gold.

According to the future price data obtained by the ARIMA model, the *5-day profit* of Bitcoin is defined as the Bitcoin price after 5 days minus the Bitcoin price of that day, and the *15-day profit* of gold is the gold price after 15 days minus the gold price of that day. The formula is as follows:

$$R_b = p_b^{'} - p_b \tag{10}$$

$$R_g = p_g^{'} - p_g \tag{11}$$

The profit rate is defined as profit divided by the asset price of the day, thus, the Bitcoin 5-day profit rate and the gold 15-day profit rate formula are as follows:

$$R_b^{'} = \frac{R_b}{p_b} \tag{12}$$

$$R_g^{'} = \frac{R_g}{p_g} \tag{13}$$

By this way,use the intraday price of gold and Bitcoin,we can calculate the profit of any given date.Part of the calculation results are shown below:

	E	F	G	H	I	J	K	L	M	N	O
1	Date	9/25/16	9/26/16	9/27/16	9/28/16	9/29/16	9/30/16	10/1/16	10/2/16	10/3/16	10/4/16
2	profit_b	0.007079	0.020067	0.018219	0.015404	0.016809	0.002642	-0.00947	0.001664	0.011686	0.012106
3	profit_g	-0.0595	-0.06005	-0.05501	-0.05028	-0.04332	-0.05406	-0.05331	-0.05331	-0.04439	-0.01971

Figure 13: Calculation Results of Profits

9 Step VI:Analytic Hierarchy Process

The problem to be solved by the AHP is the relative weight of the lowest layer to the highest layer. The weight can sort the various schemes and measures in the lowest level, so as to make choices in different schemes or form the principle of selecting schemes. Based on the model established above, we can summarize **six indicators related to this problem**:

Measure Bitcoin: **5-day moving average tendency, Bitcoin investment risk, expected profit rate after 5 days**.

Measure gold: **15-day moving average tendency, gold investment risk, expected profit rate after 15 days**.

Using these indicators, the weight of each indicator can be obtained by using Analytic Hierarchy Process (AHP). [4] Among these six indicators, the three indicators that measure Bitcoin and the three indicators that measure gold have a one-to-one correspondence, so the weights obtained by the AHP should be the same. Below we only use three indicators for AHP, and the other three indicators' weights can be obtained in the same way.

9.1 Build a Hierarchical Table

To build an indicator hierarchy . According to the actual goals, criteria and alternatives to achieve the goals, the direct relationship between each factor is arranged at different levels from top to bottom.

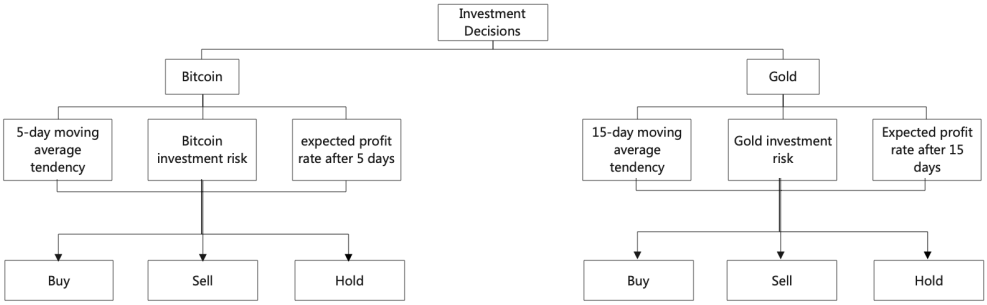

Figure 14: Hierarchy Table

9.2 Construct Judgment Matrix

When determining the weights between factors at various levels, if it is only a qualitative result, it is often not easy to be accepted by others. Therefore, Saaty proposed a consistent matrix method, namely:

1. Do not compare all factors together, but compare them in pairs.
2. Use relative scales to minimize the difficulty of comparing factors of different nature with each other to improve accuracy.

The judgment matrix is a comparison of the relative importance of all factors in this layer to a certain factor in the previous layer. The relative importance level table was first proposed by Saaty.T.L, who proposed the AHP, and later studies were also used based on this table. It compares the importance of all factors in the same dimension to obtain a judgment matrix. The basic scales include five items, namely Equal Strong, Weak Strong, Strong, Very Strong, and Absolution.

The relative importance increases from 1 to 9, as shown in the following table:

Scale	Definition
1	Indicates that two elements are equally strong
3	Indicates that one element is weakly stronger than the other
5	Indicates that one element is stronger than the other
7	Indicates that one element is significantly stronger than the other
9	Indicates that one element is absolutely stronger than the other

Table 2: Data Source Collection

After the AHP model is established and perfected, it is necessary to make corresponding comparisons in the elements of each layer, that is, to perform certain assignment processing according to the degree of importance of the two elements, and in this way to create the corresponding Decision matrix, the assignment of each element needs to be provided by decision makers and experts in the industry. The relative importance of each element of the investment provided by industry experts is as follows:

$$\begin{bmatrix} 1 & 2 & 3 \\ 1/2 & 1 & 3/2 \\ 1/3 & 2/3 & 1 \end{bmatrix} \tag{14}$$

9.3 Consistency Judgement

Consistency judgment is a test of the logical consistency of the matrix to prevent logical errors such as A>B, B>C, and C>A . A logical error in a matrix will affect the weight of

the entire decision-making model, and even lead to decision-making failure, so making consistency judgments is a must.

The consistency ratio (CR) is mainly used to measure the influence of random factors. Generally speaking, when CR<0.1, the consistency test is passed, and the weight is valid.

The eigenvector corresponding to the maximum eigenroot λmax of the judgment matrix is normalized (so that the sum of the elements in the vector is equal to 1) as λ. Since λ continuously depends on a_{ij}, the more λ is larger than n, the more serious the inconsistency of A is. The eigenvector corresponding to the largest eigenvalue is used as the weight vector of the influence degree of the compared factor on a certain factor in the upper layer. The greater the inconsistency, the greater the judgment error caused. Therefore, the inconsistency of A can be measured by the value of $\lambda - n$. This defines the consistency index CI:

$$CI = \frac{\lambda - n}{n - 1} \tag{15}$$

To measure the size of CI, the random consistency indicator RI is introduced. The method is to construct 500 pairwise comparison matrices, $A_1, A_2, ...A_{500}$, then the consistency indicators $CI_1, CI_2, ..., CI_{500}$ can be obtained, and RI is defined as the average of them:

$$RI = \frac{CI_1 + CI_2 + \cdots + CI_{500}}{500} = \frac{\frac{\lambda_1 + \lambda_2 + \cdots + \lambda_{500}}{500} - n}{n - 1} \tag{16}$$

Saaty's results are as follows:

n	1	2	3	4	5	6	7	8	9	10	11
RI	0	0	0.58	0.90	1.12	1.24	1.32	1.41	1.45	1.49	1.51

Table 3: RI's Example

The consistency ratio CR can be calculated using CI and RI:

$$CR = \frac{CI}{RI} \tag{17}$$

For the judgment matrix in this topic: After calculation, the consistency ratio(CR) is $-2.1350e^{-15}$<0.1, which is a strictly consistent matrix, so the consistency test is passed.

9.4 Get Indicator Weights

According to the judgment matrix, use the arithmetic mean method to find the weights

Step 1: Normalize the judgment matrix by column, that is, divide each element by the sum of its column.

Step 2: Add the normalized columns and sum them by row.

Step 3: Divide each element in the vector by n to get the weight vector.

The weight vector obtained according to the arithmetic mean method is:

$$\begin{bmatrix} 0.5455 \\ 0.2727 \\ 0.1818 \end{bmatrix} \tag{18}$$

So let the weights of *moving average tendency*, *investment risk* and *expected profit* be $\omega_1, \omega_2, \omega 3$, respectively,then: $\omega_1 = 0.5455, \omega_2 = 0.2727, \omega_3 = 0.1818$

10 Step VII:Scoring by Simple Weighted Method

According to the analytic hierarchy process (AHP), we obtained the weight relationship among the three indicators of moving average tendency, investment risk and expected profit rate, and also worked out the specific values of these three indicators for each day. Then we can use Simple Weighted Method to calculate the daily score of gold and bitcoin. After that, every day, we can judge whether to buy or sell an asset by comparing the score with the threshold value, and then get the daily investment decision.

Now we choose Gold as an example , and bitcoin works the same way.

10.1 Indicator Positivization

Different indicators have different mathematical characteristics: For some of them, the larger, the better, such indicators are called positive indicators. For the other, the smaller, the better, this is called negative indicators. For the convenience of the study, we converted all the indicators into positive indicators. This process is called **indicator positivization** .

Among the three indicators in this problem, the investment risk is a negative indicator, –>the smaller the better, which needs to be transformed into positive.

We can take the inverse number of the risk indicator to convert it into a positive indicator, that is, to complete the forward processing of the indicator.

10.2 Score calculation

The simple weighting method, as the name implies, is to multiply the indicator data by their weight, then add them up, so the obtained value is the score of the investment decision.

Assuming that the weight of each indicator is w_j, the score of trading gold is $score_g$, and the score of trading bitcoin is $score_b$, the expression of the score can be written as:

$$SCORE_g = T_g\omega_1 + r_g\omega_2 + R_g\omega_3 \tag{19}$$

$$SCORE_b = T_b\omega_1 + r_b\omega_2 + R_b\omega_3 \tag{20}$$

Substituting the daily relevant indicator values into the equation, we can get the daily score of investing in gold and investing in Bitcoin, which will be used as an important reference for making investment decisions.

11 Results:Investment Strategy

11.1 Initial Investment

In the previous step, we obtained the daily scores of investing in gold and investing in Bitcoin. According to the scores of the investments, we can deduce the daily investment strategies.

On the first day, we held $1000 in cash, Bitcoin was $621.65 on that day, and Gold was $1324.6. At the beginning of the investment, due to the lack of valid data, a tentative investment strategy was adopted. There are two strategies at the beginning, hold the cash or buy evenly. which is:

1. In the initial time, do not perform any blind operation until the necessary conditions for the prediction of the time series forecast are met.

2. Invest according to a certain ratio, but there will be commissions for buying, so you need to find a suitable investment ratio.

After experiment, we chose to invest $200 in bitcoin and gold, respectively, leaving $600 in cashand don't make any investment for the first 15 days. After that, when the gold market is not open, there is also no gold transaction.

Due to the commission of buying and selling, try to conduct buying and selling operations as few times as possible.

11.2 Buy or Sell

Take gold as an example:

Compare the expected profit margin obtained earlier with the commission rate. When the expected profit rate is higher than the commission rate, buy, $T_g = 1$; when the expected profit rate is lower than the negative commission rate, then there must be a loss, timely selling can minimize the loss, $T_g = -1$; if The profit rate is the same as the commission, then hold the asset, $T_g = 0$.Meanwhile when the gold market is not open, there is also no gold trade.

Bitcoin is judged in the same way as gold.

11.3 Determine the Trade Amount

First, add an appropriate constant to the score item, at here the constant is taken as 1, so that the scores of Bitcoin and Gold are both positive numbers. **Let the amount of buying and selling be proportional to the score**, then T_g and T_b are multiplied by the score to obtain a decision-making auxiliary variable that includes both transaction direction information and transaction amount information.

The smallest unit of Bitcoin is Satoshi, 1 Satoshi = 0.00000001 Bitcoin. Since theoretically gold can be cut infinitely, the smallest unit of its transaction is negligible.

Let C(i) be the amount of cash on the day i, B(i) be the amount of bitcoin (in Satoshi), G(i) be the amount of gold (in ounces), x_b be the addition ratio of Bitcoin to the score, and x_g be the addition ratio of Gold to the score. Assume 1 point of the score corresponds to investing in Bitcoin $x_b \cdot B(i-1)$ ($\triangle B$, unit: Bitcoin), and 1 point of the score corresponds to investing in gold $x_g \cdot G(i-1)$ ($\triangle G$, unit: troy ounce) .

That is to say, the **ratio** of the amount of *additional bitcoin or gold* to the *amount of bitcoin or gold in the previous period* can be determined by the score.

There are the following three constraints: B(i)>0; G(i)>0; C(i)>0. Take the initial value of $x_b = x_g = 0.0001$, and increase it with a step size of 0.0001 respectively, then find the values of x_b and x_g that maximize the Total Earning. Use the enumeration method to confirm the optimal x_b and x_g values that meet the conditions,

that is:

$$X_b = 0.0005; x_g = 0.3188$$

.

Calculate our daily equity

$$E(i) = C(i) + B(i)P_b(i) + G(i)P_g(i) \tag{21}$$

The net value of all the investor's assets on the last day is 1.8709×10^4, and the net rate of return is 1770.87%, which shows that the investment plan is very effective.
Chart of changes in net assets value:

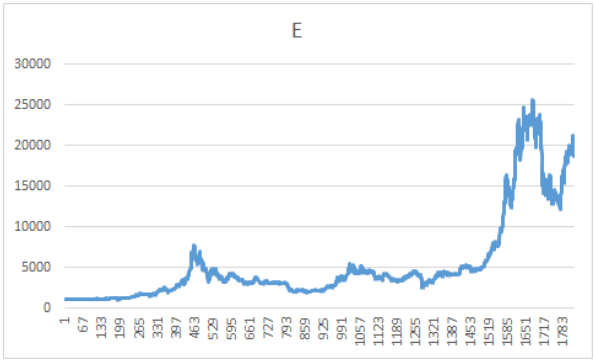

Figure 15: Chart of changes in net assets value

12 Testing the Results

In this investment, various financial plans can be enumerated as follows:

- The amount of products held remains unchanged throughout the whole process
  1. Invest all in bitcoin, and the calculated rate of profit is 7,358.97%, but this "all eggs in one basket" approach is extremely risky.
  2. Invest all in gold, and the calculated profit rate is 35.48%, with lower risk but lower profit compared with plan 1.
  3. Invest part in bitcoin and part in gold, it can be seen that the profit and risk are both between the above schemes.
  4. Invest part in bitcoin, part in gold, and hold a certain proportion of cash: Since the price of bitcoin and gold increases, while the value of cash remains unchanged, the risk and yield of this strategy are lower than plan 3.

- The amount of bitcoin and gold held changes, while the proportion of cash remains unchanged throughout the process.Under the same risk exposure, the optimal solution is between scheme 1 and Scheme 3. Compared with the method proposed in this paper, the return rate is lower because the asset allocation is not flexible enough.

  Cash holdings are always 600, and the remaining assets are allocated according to the scoring method given in this paper. On day 1, C_0=600,B_0=200/621.65 (day 1 Bitcoin price), G_0=200/1324.6 (day 1 gold price). For the first 15 days, due to the lack of data, we temporarily stayed put. The net asset value of the last day is 1.6946×10^4, and the return rate is 1594.59%. The profit rate is lower than that proposed in this paper.

- The proportion of bitcoin, gold and cash can be flexibly changed throughout the process.This includes but is not limited to the methods proposed in this paper. This method is the best strategy which combines the risk aversion of rational people and the maximization of profit.

13 Sensitivity Analysis

In the previous modeling process, we assumed a 2% commission for Bitcoin transactions and 1% for gold transactions. But, does the modeling process still apply if the commission rate changes? Below we will perform a sensitivity analysis of the model to the commission rate.

Now we will change the commission rate. First, keep the gold commission unchanged, and change the Bitcoin commission to 1.5% and 2.5%, respectively, and get new situations A1 and A2. Similarly, keep the Bitcoin commission unchanged, and change the gold commission rate to 0.5% and 1.5%, respectively, and get New situation A3, A4.

Using the same modeling method, we can obtain the total net assets on the last day, September 10, 2021, and calculate the deviation rate of the net asset value before the commission change. The results are shown in the following table:

A1	A2	A3	A4
0.072182	0.040253	0.033605	0.070521

Table 4: Deviation Rate of Total Asset Value

It can be seen from the sensitivity analysis results that the net income on the last day is not highly sensitive to transaction costs. Therefore, changes in the commission rate will not significantly change the model results, so the model is still applicable. At the same time, we also found that the sensitivity to Bitcoin transaction costs is higher than that of gold transaction costs, which is speculated to be due to the large proportion of Bitcoin purchased in this strategy.

14 Strengths and Weaknesses

14.1 Strengths

1. The investment plan is comprehensive enough and the transaction value is calculated based on the score obtained by integrating multiple indicators.

2. Considering that traders are more probably to be rational people with risk aversion, the relatively conservative investment plan proposed in this paper is more appropriate, which does not consider loans and shorting so it effectively reduces risks.

3. Using the recursive model, the amount of additional investment in each phase is based on the amount of total investment in the previous phase, and the amount of total investment directly affects the decision of each phase.

14.2 Weaknesses

1. This paper assumes that asset prices do not fluctuate within a day and the price of the day is known, while the actual trading price, opening price and closing price may be different in fact.

2. Gold is not traded on non-trading days, but if it is appropriate to trade on those days, the volume should be included in the first trading day thereafter. Such operations result in loss of earnings.

3. The determination of trading volume on the first day has strong subjectivity.

15 A Memorandum to the Trader

Invest Wisely, Earn Steadily

This is a professional asset allocation team from MCM. Our goal is to make investment decisions that maximize your expected returns and minimize risk. Therefore, many factors should be considered when investing, including the coefficient of variation of the product price (i.e. risk), market sentiment, a reasonable forecast of the future price, etc..

I believe you also understand that investment in gold and bitcoin markets contains both infinite opportunities and countless risks. To get the best investment results, you need to be both brave and cautious. In any case, detailed modeling and analysis based on known data can often lead to the most scientific investment strategy. Therefore, we hope to obtain an investment strategy that balance both benefits and risks through the analysis of past market prices.

The products considered for this investment are gold and bitcoin. Gold as a precious metalwhose price is relatively stable, suitable for intermediate-short term investment diversification risk; Bitcoin's price volatility is larger, so it's suitable for short-term investment to expand earnings. They complement each other in terms of risk and profit.

On the first day of investment, all assets should be allocated in a reasonable way. In the first 15 days of investment, due to the lack of known data, it is suggested to stay still and preserve assets in order to avoid blind losses.

After 15 days, there is a certain amount of data, then buying and selling actions can be taken. First, according to the comparison between expected profit rate and commission,

determine whether the trade of bitcoin and gold takes place or not that day. If there is a profit margin, which is, the absolute value of expected profit rate is greater than the commission, then it is suggested to take trading actions, otherwise sit still. Next, determine the trading amount. Indicators including 5-day moving average tendency (P_{b5}), Bitcoin investment risk (R_b), Expected profit rate after 5 days ($R_b^{'}$), 15-day moving average tendency (P_{g15}) , Gold investment risk (R_g) and Expected profit rate after 15 days ($R_g^{'}$) need to be calculated. Among all the indicators mentioned above, indicators P_{b5} and P_{g15} are obtained from the comparison between the bitcoin/gold price over the day and the average price of the past 5/15 days; indicators R_b and R_g are obtained from the variation coefficient of the price data of the past 30 days; indicators $R_b^{'}$ and $R_g^{'}$ are calculated from the expected price after 5/15 days predicted by time series and the bitcoin/gold price on the day. Then according to the weight given, the three indicators of gold and bitcoin are weighted and summed up to calculate the score of gold and bitcoin trading on the day targeted. Finally, confirm the proportion of additional investment in the amount of investment according to the scores, formulate the optimal investment strategy.

We strongly recommend that you invest according to the strategy give. It is our honor to support you. Hope that you can make a lot of money and have a good experience during the process!

References

[1] S. Yunsheng, "Market judgment and operation based on moving averages," *China Business*, no. 9, p. 44, 2009.

[2] T. Jakaša, I. Andročec, and P. Sprčić, "Electricity price forecasting — arima model approach," in *2011 8th International Conference on the European Energy Market (EEM)*, pp. 222–225, 2011.

[3] H. Y. Hu Lirong, "Comparison and application of financial investment risk assessment technology," *Economic Forum*, no. 9, pp. 66–69, 2017.

[4] D. Shengye, "Assembly component supplier evaluation system based on ahp and entropy weight method," 2020.

Appendices

1.Judge the Bull & bear market

```
clc;clear
load data;   % date;b_price;g_price;transaction
mkt_b=zeros(1826,1);
mkt_g=zeros(1826,1);
sdate=42624;   % 2016-9-11
edate=44449;   % 2021-9-10
lag=1;
for date=sdate:lag:edate
    [i,j]=find(time_price(:,1)==date);
    b_price=time_price(i,2);
    g_price=time_price(i,3);
    if i<60
        b_b=sum(time_price(1:i,2))/i;
        g_b=sum(time_price(1:i,3))/i;
    else
        b_b=sum(time_price(i-59:i,2))/60;
        g_b=sum(time_price(i-59:i,3))/60;
    end
    if b_b>b_price
        mkt_b(i,1)=-1;   % bear
    elseif b_b<b_price
        mkt_b(i,1)=1;    % bull
    else
        mkt_b(i,1)=0;
    end
    if g_b>g_price
        mkt_g(i,1)=-1;   % bear
    elseif g_b<g_price
        mkt_g(i,1)=1;   % bull
    else
        mkt_g(i,1)=0;
    end
end
figure(1)
n1=find(mkt_b==1);
n2=find(mkt_b==-1);
plot(time_price(:,2))
hold on
plot(n1,time_price(n1,2),'.r')
plot(n2,time_price(n2,2),'.g')
legend('bprice','bull','bear')

figure(2)
n1=find(mkt_g==1);
n2=find(mkt_g==-1);
plot(time_price(:,3))
hold on
plot(n1,time_price(n1,3),'.r')
```

```
plot(n2,time_price(n2,3),'.g')
legend('gprice','bull','bear')
```

2.Determining the moving averages tendency

```
clc;clear
load data;    % date;b_price;g_price;transaction
load mkt;    % mkt_b;mkt_g
MA_5=zeros(1826,1);
sdate=42624;  % 2016-9-11
edate=44449;  % 2021-9-10
lag=1;
for date=sdate:lag:edate
    [i,j]=find(time_price(:,1)==date);
    b_price=time_price(i,2);
    if i>4
        MA5_b=sum(time_price(i-4:i,2))/5;
    else
        MA5_b=sum(time_price(1:i,2))/i;
    end
    if b_price > MA5_b && b_price<1.15*MA5_b
        MA_5(i,1)=1;
    elseif b_price >= MA5_b*1.15
        MA_5(i,1)=-1;
    elseif mkt_b(i,1)==-1 && b_price<0.75*MA5_b
        MA_5(i,1)=1;
    else  MA_5(i,1)=0;
    end
end

n1=find(MA_5==1);
n2=find(MA_5==-1);
n3=find(MA_5==0);
plot(time_price(:,2))
hold on
plot(n1,time_price(n1,2),'.r')
plot(n2,time_price(n2,2),'.g')
plot(n3,time_price(n3,2),'.y')
legend('bprice','positive','negative','neutral')
```

3.AHP analysis

```
clc;clear
n=3;
A=[1  2  3;
 1/2  1  3/2;
 1/3  2/3  1];
Sum_A = sum(A);
[n,n] = size(A);
SUM_A = repmat(Sum_A,n,1);
Stand_A = A ./ SUM_A;
sum(Stand_A,2);
disp(sum(Stand_A,2) / n)
```