February 20, 2014

Could magnetic monopoles exist?

Note from Terry, (t.sloan@lancaster.ac.uk).

Abstract

The behaviour of Dirac monopoles (of magnetic charge $g_D = 137e/2$) in the the Galaxy is studied using the latest estimates of the Galactic Magnetic Field (GMF). It is shown that for masses less than 10^{18} GeV the monopoles are rapidly expelled by the GMF in times 1.7 10^{-7} $M^{1/2}$ Myr where M is the magnetic monopole (MM) mass in GeV. For greater masses the gravitational force overcomes the magnetic force and the monopoles are retained within the Galaxy with speeds in the range $0.3\ 10^{-3} < v/c < 10^{-3}$ in the vicinity of the orbit of the solar system. It is shown that there is a distinct possibility that such particles exist but will have escaped detection due to the effects of either the GMF or their large kinetic energy. The possibility that MM could make a contribution to the dark matter in the Galaxy is discussed.

1 Introduction

Dirac showed that the quantisation of charge and angular momentum would be natural if a magnetic monopole (MM) existed [1]. The strength of such a MM to account for this naturalness would be expected to be $g_D = 137e/2$ in natural units (3.29 10^{-8} emu in cgs units or 3.29 10^{-9} Amp-m in SI units) where e is the electronic charge. Subsequently, 't Hooft and Polyakov [2] showed that such MMs are obligatory in certain low energy theories. Despite such strong theoretical motivation MMs have never been observed although many searches for them have been undertaken [3].

MMs, if they exist, must have masses greater than TeV otherwise they would have been detected in accelerator searches. In the theories of [2] the masses are expected to be at the GUT scale i.e. from 10^{16} to 10^{19} GeV. In this paper we study the behaviour of such large mass monopoles in the Galaxy.

Upper limits on the flux of Galactic monopoles are available via the Parker bound [4]. Fluxes above this value would short circuit the Galactic magnetic field (GMF).

In this paper we profit from the improved knowledge of the GMF [6, 7] and the Galactic mass distribution [8] to study the behaviour of magnetic monopoles in the Galaxy. The aim is to see if magnetic monopoles (MMs) could exist in the Galaxy. Throughout it is assumed that the MMs interact with matter predominamtly through their gravitational and electromagnetic fields and that strong interaction effects are not significant.

2 Simulation of the behaviour of a magnetic monopole in the Galactic magnetic and gravitational fields

A simulation of the motion of a MM in the Galaxy was made.[1] The starting positions and starting velocity could be chosen arbitrarily and the monopole tracked. The monopole was tracked in small increments of time (2 10^{10} seconds). (It was checked that the results are insensitive to the value of this time increment). In each increment the acceleration was computed from the vector sum of the gravitational and the magnetic forces on the monopole. Each force was calculated from the parameterisation of the GMF in [6] and mass distribution in [8].

The acceleration of the monopole was used to compute the change in velocity. The new velocity was then used to compute the change in position in the small time increment. In this way the monopole position and velocity could be tracked as a function of time. A second simulation was also made which used a Runge-Kutta integration of the equations of motion from the MM start point. The two programs gave identical results when compared.

The gravitational force at radius R in the Galaxy was computed from the gravitational acceleration given by $g(R) = GM_G/R^2$ where G is the Newtonian gravitational constant and M_G is the total Galactic mass contained within the radius R computed in [8]. The escape velocity from the Galaxy is then given by

$$v_{escape} = \sqrt{2\int_{\infty}^{R} g(R)dR} \tag{1}$$

This is shown in figure 1 as a function of radius R. The force due to the GMF, $g_D\mathbf{B}$ where $\mathbf{B}$ is the local magnetic field, was computed from the parameterisation given in [6]. The random component given in [7] was not used since this would be expected to average to zero over time [9].

As an illustration fig 2 (upper plot) shows the behaviour of a monopole of mass 10^{16} GeV in an orbit identical to that of the solar system in the gravitational field of the Galaxy but with a very weak GMF (the true GMF multiplied by 10^{-3}). It can be seen that the magnetic field perturbs the orbit so that the monopole “walks” out of the Galaxy. This is similar to a particle beam in a badly constructed particle accelerator. The lower plot shows the same monopole in a full strength GMF. It can be seen that, in this case, the monopole is quickly expelled from the Galaxy. Here the orbit of the Solar System was assumed to be at a radius of 8.5 kpc from the Galactic centre where the gravitation acceleration is 1.9 10^{-10} $\mathrm{m\ sec}^{-2}$.

An electric field in the Galaxy could affect the motion of a monopole in the Galaxy. The Lorenz force on a monopole of pole strength g moving with velocity $\mathbf{v}$ in an electric field $\mathbf{E}$ is $g\mathbf{v}\mathrm{x}\mathbf{E}/\mathrm{c}^2$. For a Dirac monopole moving with speed $\beta \sim 10^{-3}$ a Galactic electric field of order 100 V/m would be required to produce a force on the monopole of similar magnitude to the magnetic force. The Galactic electric field strength is unknown. However, an order of magnitude estimate can be obtained by assuming that the energy density in the Galactic electric

[1] A right handed Cartesian coordinate system (x, y, z) is used, where the Galactic centre is at the origin, Galactic north is in the positive z direction and the Sun is located at x=-8.5 kpc.

field is of a similar magntitude to that for the magnetic field. The energy density of the Galactic magnetic field $0.5B^2/\mu_0 \sim 3\ 10^{-14}$ J/m^3. The electric field strength to give a similar energy density ($0.5\epsilon_0 E^2$) is $E \sim 0.1$ V/m. This would give a force 3 orders of magnitude less than the magnetic force. Hence it seems unlikely that the Galactic electric field will have a significant effect on the trajectory of a magnetic monopole.

3 Life time of a Dirac Monopole in the Galaxy

Three classes of monopoles are considered.

1. Monopoles which start in a Galactic orbit.
2. Monopoles falling into the Galaxy from extra-Galactic space.
3. Monopoles ejected from Supernovae.

3.1 Monopoles starting in a Galactic orbit

Monoples in a Galactic orbit are simulated by starting the monopole at an orbit position (with a velocity to be chosen) and then tracking it as it is influenced by the GMF and Galactic gravitational field. The time taken for it to reach a radius of 25 kpc is used as a measure of the lifetime of a monopole in the Galaxy. At radii greater than 20 kpc the GMF is close to zero [6], so little further magnetic acceleration takes place beyond this radius. The simulation was limited to lifetimes of the monopole of less than 1000 Myr.

Figure 3 (upper plot) shows this time plotted as a function of monopole mass for 3 different starting velocities from the Solar System radius (8.5 kpc). It can be seen that the lifetime is roughly independent of starting velocity and varies approximately as $\sqrt{M}$ for masses up to $\sim 10^{17.5}$ GeV. The velocity on reaching a radius of 25 kpc (lower plot) is also approximately independent of the starting velocity up to this mass. At this mass the velocity of the monopole at radius 25 kpc becomes of similar magnitude to the escape velocity at this radius ($v/c = 0.0015$, see fig1).

For masses greater than 10^{18} GeV the simple smooth relation between lifetime or exit velocity (velocity at 25 kpc radius) and mass breaks down. The reason for this is that for lower masses the GMF accelerates the monopoles to high speed and they are hurled out of the Galaxy since their speeds become greater the the escape velocity. However, the higher mass MM are retained within the Galaxy since the gravitational force is strong enough to overcome the disturbing effects of the GMF. The trajectories are not simple orbital trajectories but rather the monopoles diffuse in random paths within the Galaxy. (The approach to confinment for higher mass MMs is illustrated in the next section and figure 5.)

Monopoles with initial velocity perpendicular to the Galactic plane starting at the Solar radius and with Solar velocity were confined into orbit at somewhat lower masses ($M \sim 10^{18}$) and at $M = 10^{19.5}$ GeV the trajectory was a very recognisable orbit with mean radius close to the starting radius and a rather small scatter of the trajectory about the orbit (RMS ~ 0.3 kpc). The reason for this is that with these trajectories the monopoles experience the strong magnetic field in the Galactic plane for shorter times that monopoles moving in this plane. Hence their orbits are perturbed less.

The approach to confinement in an orbit is illustrated in figure 4. Monopoles were followed out to a radius of 1000 kpc where the gravitational force is small. Monopoles in orbit never reach this radius. Figure 4 shows the maximum radius reached (upper plot) and the mean (centre plot) and RMS (lower plot) radii as a function of the monopole masses for monopoles with initial motion in the Galactic plane starting at the solar radius (8.5 kpc). It can be seen that the onset of confinement in the Galaxy begins at masses of order $10^{18.5}$ GeV. For masses less than this value the monopoles are thrown to beyond radii of 1000 kpc and with velocity such that they never return to the Galaxy i.e. they are unconfined. For monopoles of mass greater than $10^{18.5}$ GeV they remain confined in the Galaxy but with ill defined orbits as illustrated by the mean and RMS radii. However, as the mass increases to $10^{19.5}$ and beyond the orbit becomes more and more recognisable and with paths closer to the Galactic centre.

3.2 Monopoles falling into the Galaxy

Monopoles falling into the Galaxy from extra-Galactic space only reach significant magnetic fields at radii of less than 20 kpc. At smaller radii they experience both gravitational and magnetic forces. A series of tests was made with the monopoles falling into the Galaxy. The monopoles were started outside the region of significant magnetic field at radius 21 kpc and at the escape velocity for this radius (2.2 times the orbit speed).

The precise value of the mass before a monopole can reach the solar orbit radius depends on the direction of approach of the monopole to the Galaxy. However, for masses less than $\sim 10^{17}$ GeV the falling monopoles never reach the radius of the solar orbit. They are forced back and rapidly expelled from the Galaxy by the GMF.

The situation is different for higher monopole masses. Figure 5 illustrates this. It shows the velocity $\beta = v/c$ and radius of a monopole of mass 10^{18} GeV as a function of time for monopoles falling towards the Galactic centre. Higher mass monopoles can penetrate to radii less than that of the solar orbit due to their increased kinetic energies. However, the combined effect of the gavitational and magnetic fields means that the monopoles tend to have diffuse paths rather than orbits. These paths can be long so that the monopole dwells for a long time within the Galaxy.

As typical examples the upper plots in figure 5 show the situation for monopoles falling towards the Galactic centre from different directions. It can be seen that the MM often rebounds several times off the Galactic centre until eventually it finds a path where it can escape from the Galaxy. The time taken to reach extra-Galactic space is of order Gyr so such heavy monopoles are effectively confined within the Galaxy albeit spending most of their time at rather large radii. The lower plots in figure 5 show the same plots for MMs falling perpendicular to the Galactic plane. The right hand lower plot shows a monopole falling in the direction of the magnetic field (South-North, +z direction) while the left hand plot shows a monopole falling opposite to the direction of the magnetic field. In the first case the GMF accelerates the monopole so it exits the magnetic field without any rebound. In the second case a rebound occurs since the GMF decelerates the monopole until it finds an exit path after of order 1 Gyr.

In all these cases the monopoles are held in a cloud at rather large radii.

3.3 Monopoles ejected from Supernovae

The velocity of ejecta from supernovae explosions is in the vicinity $\beta \sim 10^{-2}$ [10]. It is assumed that monopoles trapped in a star which undergoes such an explosion will be ejected with such a velocity.

The simulation was run for monopoles of velocity $\beta = 10^{-2}$ from different starting points in the Galaxy. Their behaviour for masses less than $10^{15.5}$ GeV was similar to that observed for monopoles starting from Galactic orbit. For such masses the acceleration due the GMF dominates and the monopoles are flung out of the Galaxy at high velocity (eg $\beta \sim 0.1$ for mass 10^{14} GeV) in times of order a few megayears.

For masses greater than 10^{17} GeV the kinetic energy of the monopoles with this velocity is so high that the gravitational and magnetic accelerations only slightly perturb the trajectories. Hence these monopoles deviate little from linear trajectories and experience little change in velocity.

Hence monopoles trapped in large stars and ejected in subsequent supernovae explosions will have short dwell times in the Galaxy and will travel with high velocities ($\beta \sim 10^{-2}$).

4 Monopole stopping power and range in material

The stopping power, dT/dx, of MMs in material has been discussed by a number of authors [11–18]. In this paper the calculations of [18] for silicon and iron are used. The range, R, was computed from the integral of these stopping powers

$$R = \int_{T_{min}}^{T} \frac{dT}{dT/dx} \tag{2}$$

where T is the kinetic energy of the monopole. This is shown as a function of the monopole's initial velocity in figure 6. In principle the value of the minimum kinetic energy T_{min} should be zero. However, the stopping power has not been calculated for velocities $\beta < 10^{-5}$. Here, T_{min} was taken to be the kinetic energy corresponding to $\beta = 10^{-6}$ and the stopping power was extrapolated to this value from the curves in [18]. The loss of range due to this approximation will be quite small.

Assuming that the ranges shown in figure 6 are representative of all materials the following may be deduced. A monopole passing diametrically through the centre of the Sun (Earth) sees $2\ 10^{11}$ ($7\ 10^{9}$) g/cm^2 of material. This means that a MM is required to have mass less than 10^{16} ($2\ 10^{14}$) GeV to stop in the Sun (Earth) for MMs travelling in the Galaxy with the velocities described above ($\beta \sim 10^{-3}$). Hence their ranges are much greater than the sizes of planetary objects and small stars and they will only stop in larger stars. This implies that the induction techniques used to search for monopoles trapped in the Earth [21] or in meteorites [22] are less likely to find them than searches in a continuously sensitive apparatus such as MACRO [23] or SLIM [24] which can detect a passing monopole.

5 Discussion of the Results

MMs must have masses greater than of order 10^3 GeV otherwise they would have been detected in accelerator experiments. MMs in the mass range $10^3 < M < 10^{17}$ GeV are quickly accelerated out of the Galaxy by the GMF. A linear fit to the data in figure 3 gives their lifetime in the Galaxy as 1.7 $M^{1/2}$ Myr with the mass in GeV, where the lifetime is defined as the time to reach a radius of 25 kpc. Hence MMs in this mass range which were present in the Galaxy at its formation will have been rapidly expelled from it. The same is true for MMs in this mass range falling into the Galaxy. They never reach the solar orbital radius and are quickly expelled from the Galaxy. MMs ejected in supernova are similarly rapidly expelled from the Galaxy. Hence it is plausible that MMs in this mass range could exist in the Universe but would be undetectable since Earthbound detectors would be shielded from them by the GMF.

For more massive MMs (mass $> 10^{18}$ GeV) the stronger gravitational force overcomes the magnetic force from the GMF. In this case the MMs remain longer within the Galaxy. However, they tend to spend a major fraction of their dwell time in the Galaxy at radii which is much larger than the solar orbit radius. Hence their flux in the vicinity of the Earth will be limited.

Furthermore, since they have large kinetic energies their ranges in matter before they stop are very long so that they will be unable to stop in planets or small stars. Hence induction methods of detection of MMs are unlikely to see any even if they exist. However, experiments which are sensitive in real time such as MACRO or SLIM [23, 24] could see a passing MM. Indeed one candidate event has been reported [28].

MM could stop in larger stars and be emitted during supernova explosions. However, such MMs are shown to have very short lifetimes in the Galaxy making their detection difficult.

Could MMs contribute to the dark matter (DM) in the Galaxy? This would only be the case for MMs with mass more than 10^{18} GeV since smaller masses would be expelled from the Galaxy too quickly. The total mass density at the solar orbit radius is thought to be of the order of 0.1 solar masses per pc^3 [26] and 10% of this is thought to be dark DM [27]. A MM of mass $10^{18.3}$ GeV gives a monopole density which fits well the Galactic mass distribution [8] at radii of greater than 30 kpc. This mass also gives a monopole density roughly compatible with that of the dark matter at the solar orbit radius within a factor 2. However, the MM flux with velocity $v/c \sim 10^{-3}$ at the solar orbit radius would then be 5 10^{-13} (cm^2 s sr)$^{-1}$. This is larger by a factor of 30 than the Parker bound [4]. Furthermore, the flux is greater than the upper limits from continuously sensitive experiments [23, 24]. Hence it is unlikely that MMs contribute significantly to the DM in the Galaxy.

We conclude therefore that magnetic monopoles may exist but are difficult to detect since detectors on Earth are shielded from them by the effects of the GMF. Hence, given the strong theoretical motivation for them, they may still exist and be hidden from Earth bound experiments.

6 Acknowledgements

Thank Glennys Farrer and Ronnie Jannson for help with their parameterisation of the GMF. Who else?

References

[1] P.A.M. Dirac, Proc. Phys. Soc. London A133 (1931) 60: Phys. Rev. 74 (1948) 817.

[2] G. ’t Hooft, Nucl. Phys. B79 (1974) 276; B105 (1976) 538; A. Polyakov JETP Lett. 20 (1974) 194.

[3] “Review of Particle Properties”, J. Beringer et al. (Particle Data Group) Phys. Rev. D**86** (2012) 0100001.

[4] E.N. Parker Astrophys. J. 160 (1970) 383; 163 (1971) 225; 166 (1971) 295.

[5] M.S. Turner, E.N. Parker and T.J. Bogdan Phys. Rev. D26 (1982) 1296.

[6] G. Farrar and R. Jansson Astrophysical J. arXiv:1204.3662

[7] G. Farrar and R. Jansson Astrophysical J. Letters **761** (2012) L11.

[8] Malcolm Fairbairn, Tom Douce and Jace Swift arXiv:1206.2693.

[9] G. Farrer private communication.

[10] A. Hayato et al., arxiv 1009.6031.

[11] S.P. Ahlen Phys. Rev. D14 (1975) 2935.

[12] S.P. Ahlen Phys. Rev. D17 (1978) 229.

[13] S.P. Ahlen and K. Kinoshita Phys. Rev. D26 (1982) 2347.

[14] S.P. Ahlen and G. Tarlé Phys. Rev. D27 (1983) 688.

[15] D.J. Ficenec, S.P. Ahlen A.A. Marin, J.A. Musser and G. Tarlé, “Observation of electroniic excitation by extremely slow protons with applications to the detection of supermassive charged partilces”, Phys. Rev. D36 (1987) 311.

[16] S.D. Drell, N.M. Kroll, M.T. Mueller, S.J. Parke and M.J. Ruderman “Energy loss of slowly moving magnetic monopoles in matter”, Phys. Rev. Letts. **50** (1983) 644.

[17] L. Bracci, G. Fiorentini and R. Tripiccione, “On the energy loss of very slowly moving magnetic monopoles”, Nucl. Phys. **B238** (1984) 167.

[18] J. Derkaoui, G. Giacomelli, T. Lari, A. Margiotta, M. Ouchrif, L. Patrizii, V. Popa and V. Togo, “Energy losses of magnetic monopoles and dyons in the Earth”, Astroparticle Physics **9** (1998) 173.

[19] R.M. Sternheimer and R.F. Peierls, Phys. Rev. **B3** (1971) 3681.

[20] J. Lindhard, Mat. Fys. Medd. Dan. Vid. Selak, bf 28 no. 8 and J. Lindahrd and M. Scharff, Phys. Rev. **124** (1961) 128.

[21] P. Mermod et al. Phys. Rev. Letts.

[22] Meteorite searches

[23] MACRO

[24] SLIM

[25] S.P. Ahlen and G. Tarlé, “Can grand unification monopoles be detected with plastic scintillators?”, Phys. Rev. **D27** (1983) 688.

[26] Holmberg and Flynn MNRAS 313 (2000) 209 and 35 (2004) 440.

[27] Dark matter flux of 0.1 Msolar per pc^3.

[28] B. Cabrera.

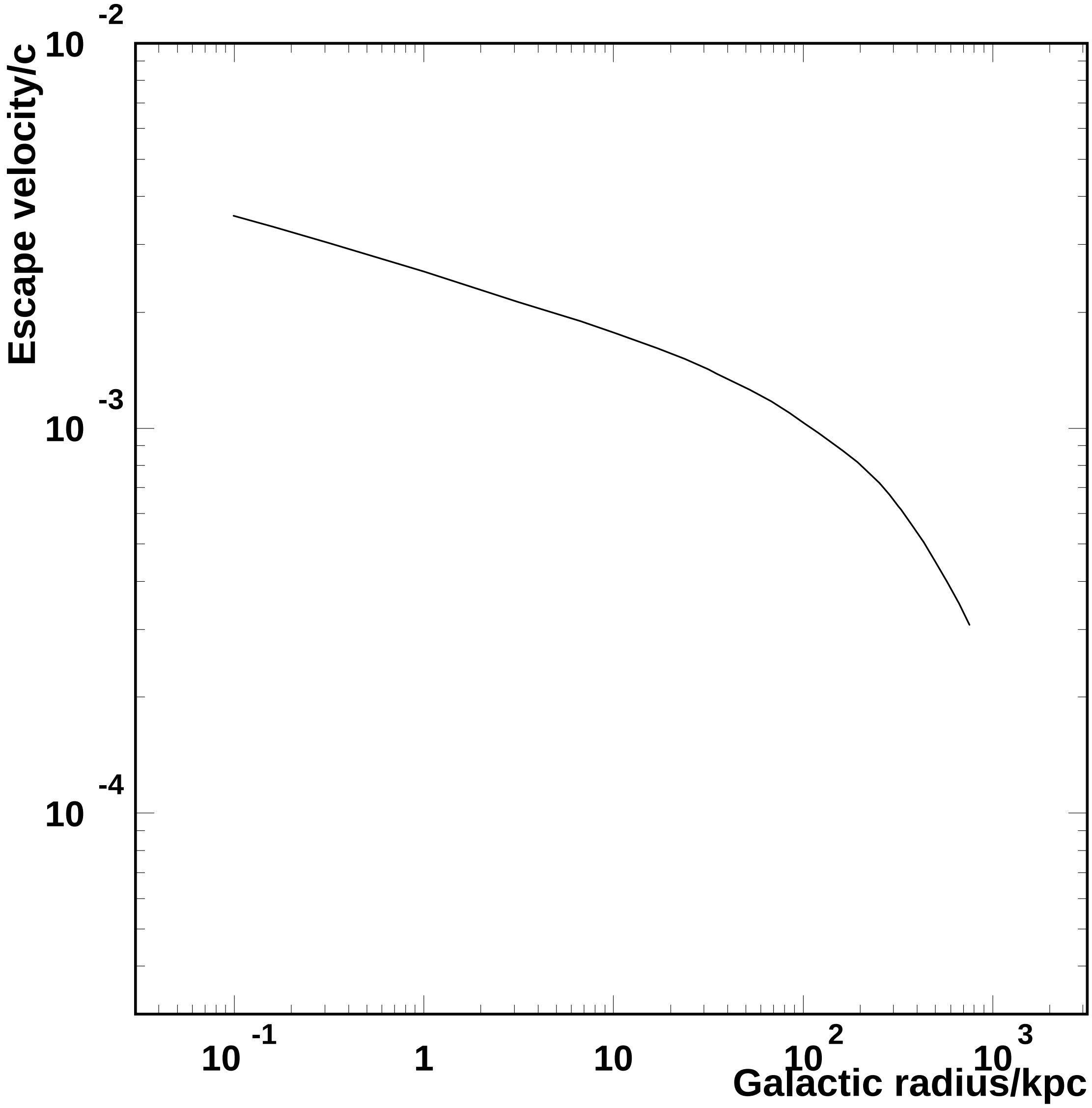
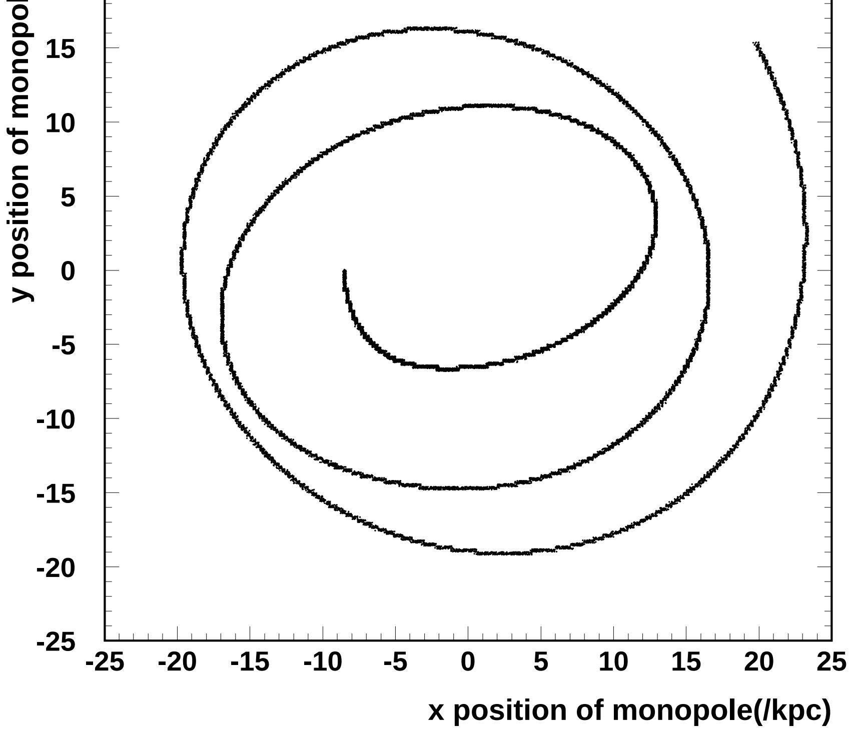


Figure 1: The escape velocity from Galaxy computed from the mass distribution of [8].



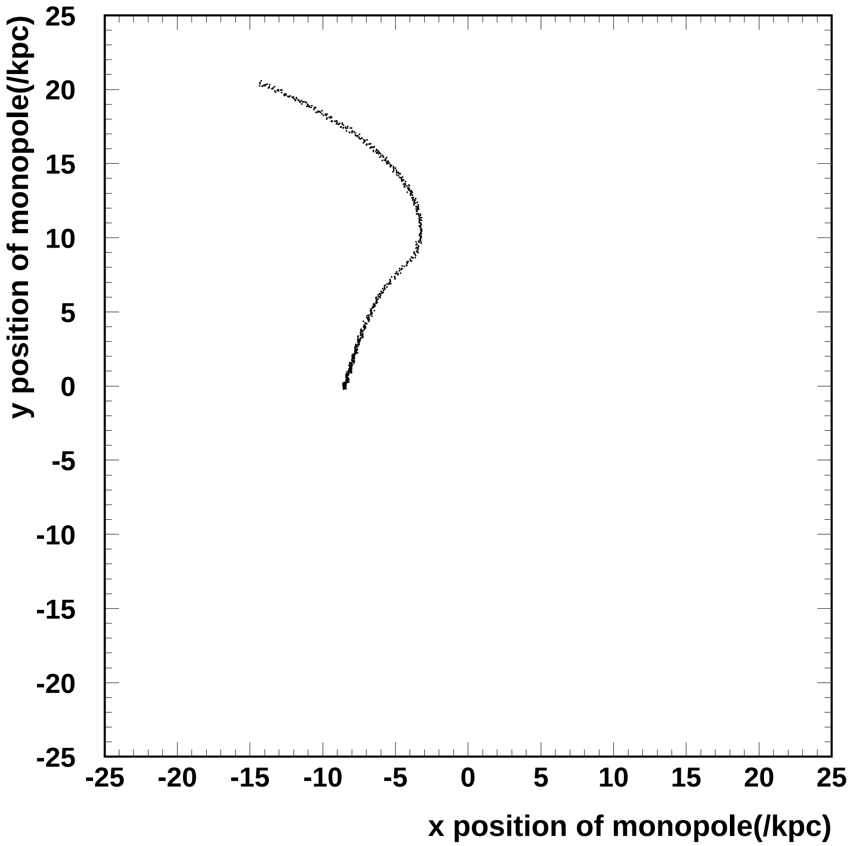


Figure 2: The orbit perturbations are illustrated (upper plot) by the trajectories of a magnetic monopole in a very weak magnetic field (10^{-3} times the normal GMF [6]). The lower plot shows the trajectory in the full strength GMF [6]. In each case the monopole starts in orbit at the solar system coordinates and the vertical component of the GMF is set to zero so that the monopole is constrained to move in the Galactic plane.

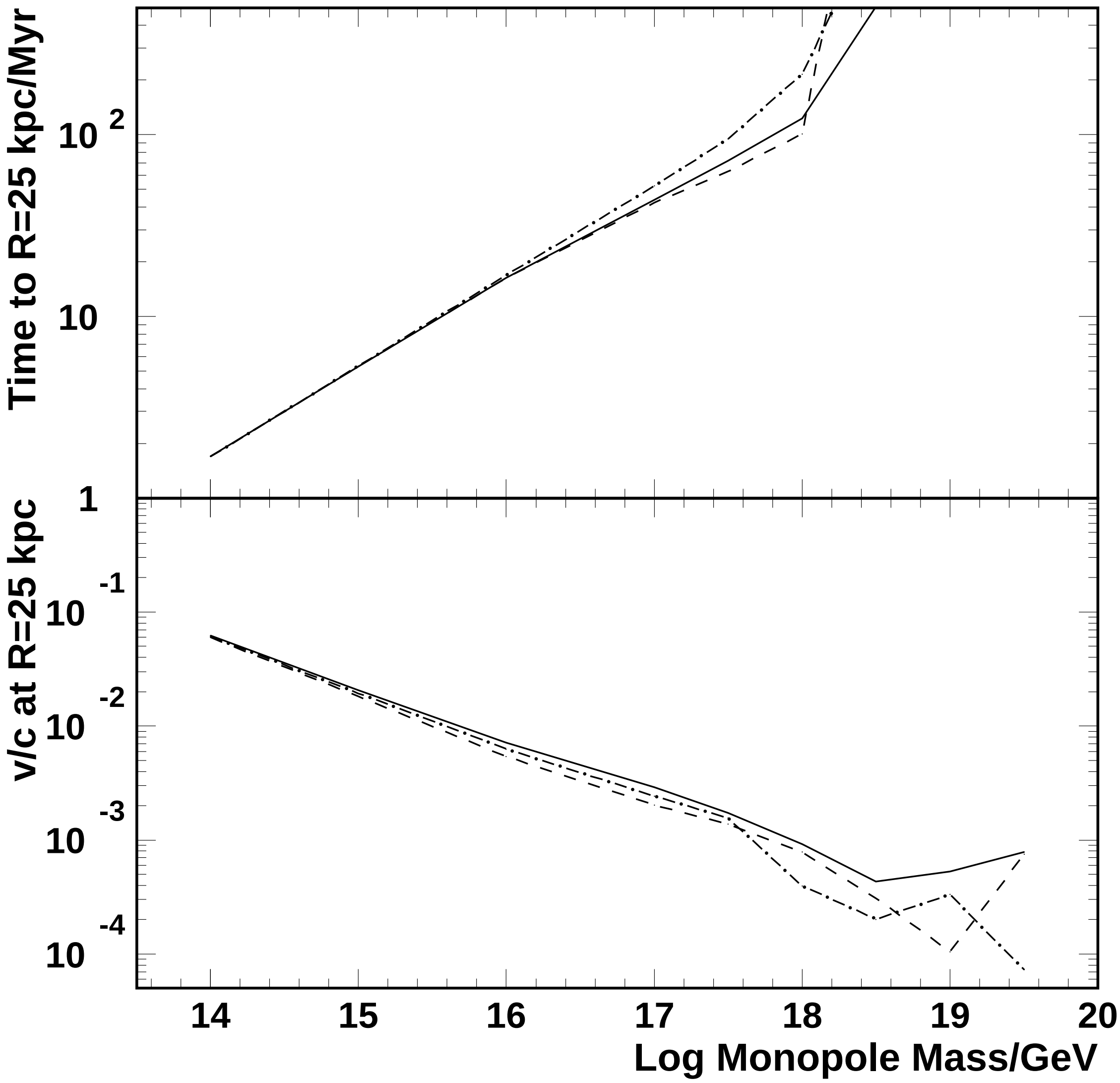


Figure 3: (Upper plot) Time for a monopole in the Galaxy to reach radius 25 kpc as a function of monopole mass. Each curve is for a monopole starting at the same Galactic coordinates as the Solar System (radius 8.5 kpc). The solid (dashed) curve is for a monopole starting with the same (opposite) speed and direction as the Solar System. The dash dotted curve is for a monopole starting at rest at the position of the Solar System. (Lower plot) shows the velocities at radius 25 kpc in the same 3 designations as a function of monopole mass.

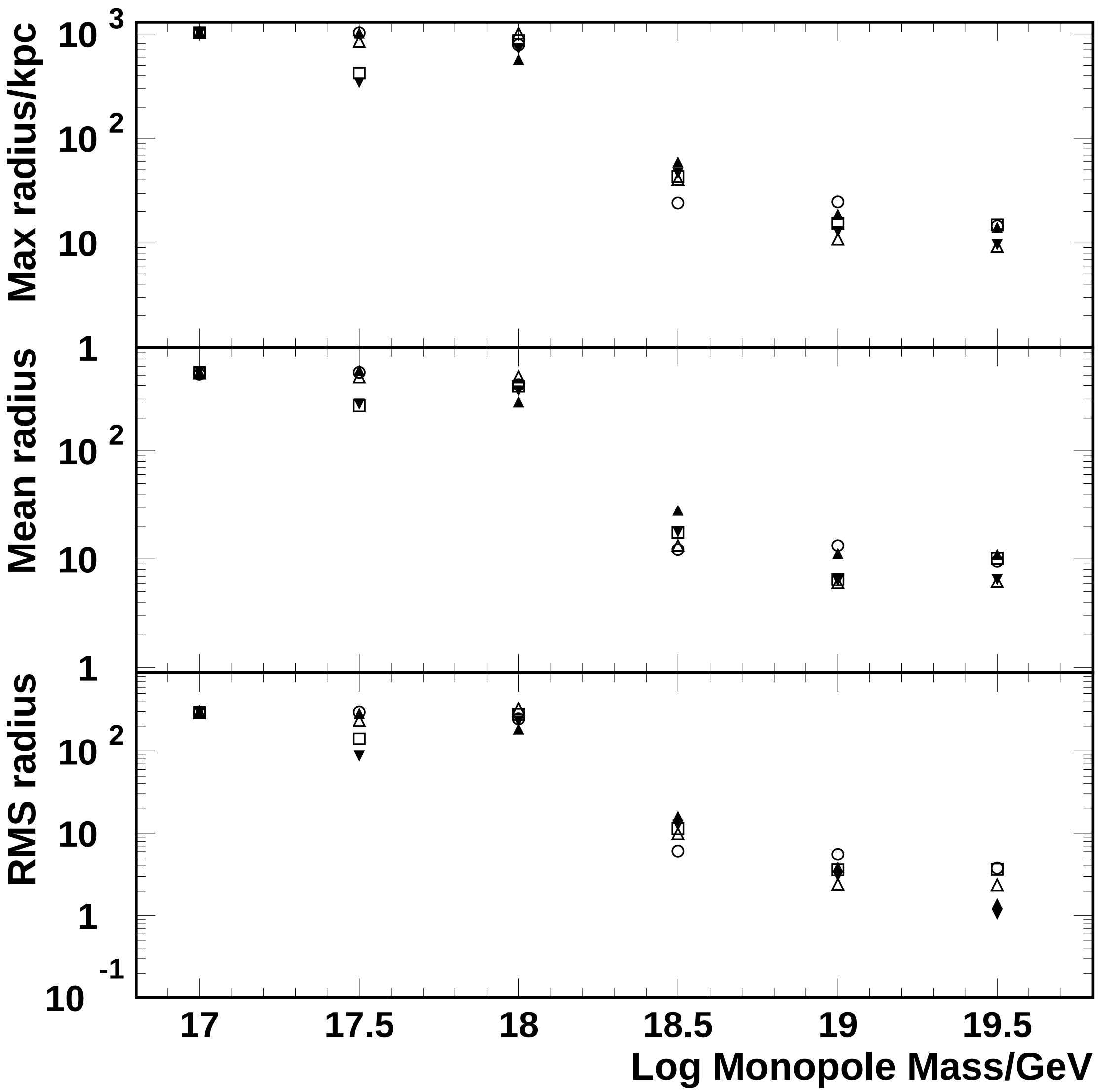


Figure 4: (Upper plot) Maximum radius in kpc, (centre plot) mean radius and (lower plot) RMS value of the radius as a function of $\log_{10}$ Mass. The closed triangles show the values for monopoles starting in a solar orbit, closed inverted triangles are for those starting in a solar orbit in the opposite direction to the Sun, open circles are for those directed towards the Galactic centre with the Sun's velocity, open squares are for those directed away from the centre with the Sun's velocity and open triangles are for those starting from rest.

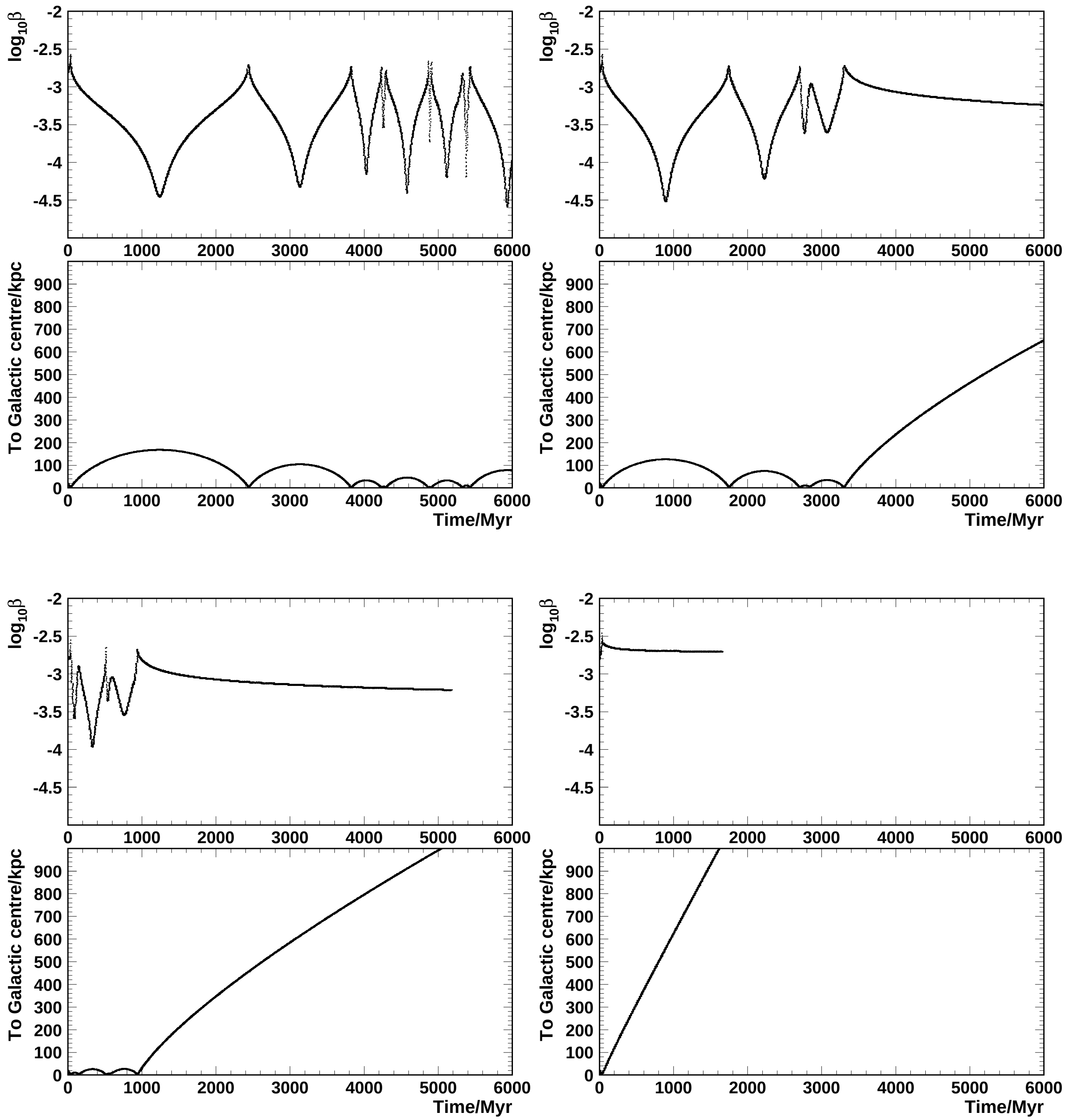


Figure 5: Four pairs of plots showing distance from the Galactic centre (lower of the pair) and total velocity ($\beta = v/c$) (upper of the pair) as a function of time for a monopole of mass 10^{18} Gev falling into the Galaxy. The monopoles were each started at a radius of 21 kpc with velocity towards the Galactic centre of magnitude 2.24 times the solar orbit velocity (the escape velocity at this radius)- (Upper left) Monopole falling along the +x axis. - (Upper right) Monopole falling along the -x axis. - (Bottom left) Monopole falling along the +z axis. - (Bottom right) Monopole falling along the -z axis. The x axis is defined as the line between the solar system and the Galactic centre. The z axis is perpendicular to the Galactic plane.

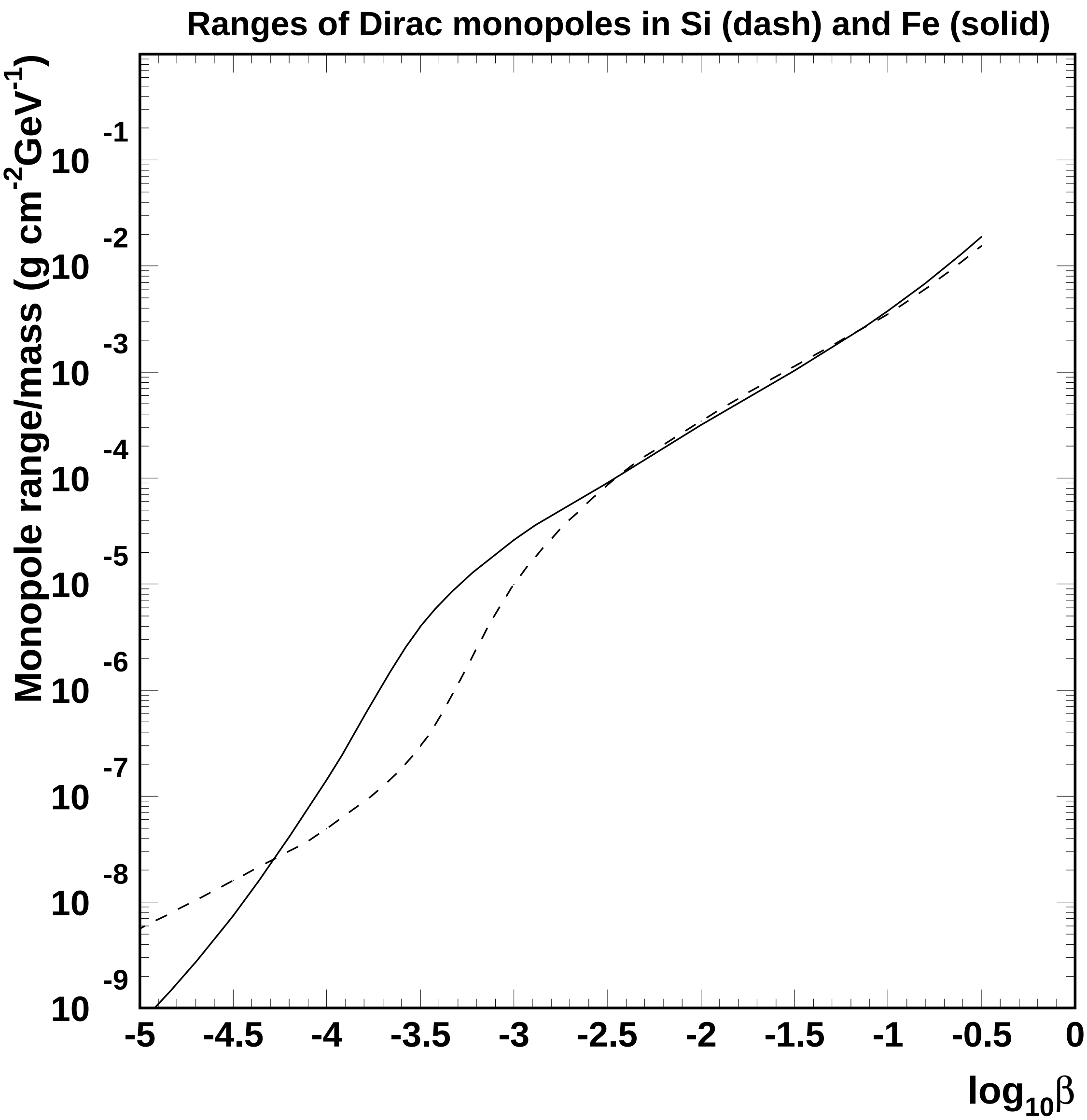


Figure 6: Range of Dirac monopoles in iron and silicon as a function of monopole velocity. The range is computed from the stopping powers given in [18] and it is defined as the thickness of material to slow the monopole to a speed speed $\beta < 10^{-6}$.