INTERNATIONAL STANDARD **ISO**

Working Draft

Space Environment (Natural and Artificial)

Model of Earth's Magnetosphere Magnetic Field

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MOSCOW 2000

SPACE ENVIRONMENT
(NATURAL AND ARTIFICIAL)
MODEL OF THE EARTH'S MAGNETOSPHERE
MAGNETIC FIELD

1 Applicability Scope

The present standard is intended to calculate induction of the magnetic field in the Earth's magnetosphere. The standard sets the parameters of magnetospheric large-scale current systems in accord with conditions in the space environment and can be used to investigate physical processes in the Earth's magnetosphere as well as in calculations, developing, testing and estimating the results of exploitation of spacecrafts and other equipment operating in the space environment.

Calculations in terms of the magnetic field model proposed in the standard can be used to forecast radiation situation in the space, including the periods of intense magnetic disturbances (magnetic storms), when developing systems of spacecraft magnetic orientation, when forecasting the influence of magnetic disturbances on transcontinental pipings and powertransmission lines.

The goals of standardization of the Earth's magnetospheric magnetic field are:

- providing the unambiguous presentation of the geomagnetic field of currents flowing inside the Earth and the magnetic field of magnetospheric currents;

- providing comparability of results of interpretation and analysis of space experiments;

- providing less labour-consuming character of calculations of the magnetic field of magnetospheric currents in the space at geocentric distances of 1-6.6

Earth's radii (R_E);

- providing the most reliable calculations of all elements of the geomagnetic field in the space environment.

2 Definitions, Notations and Abbreviations

2.1. Vector of magnetic field induction $\vec{B}_M$ in the Earth's magnetosphere is calculated by the formula

$$\vec{B}_M = \vec{B}_1 + \vec{B}_2, \quad nT \tag{1}$$

where $\vec{B}_1$ is the vector of induction of the geomagnetic field of currents flowing inside the Earth,

$\vec{B}_2$ is the vector of induction of the magnetic field of magnetospheric currents.

2.2. The magnetic field of currents flowing inside the Earth, $\vec{B}_1$, is presented in the form of a series of spherical harmonic functions. The expansion coefficients (IGRF model) undergo very slight changes in time and their values are approved by the International Association of Geomagnetism and Aeronomy (IAGA) every 5 years.

2.3. The magnetic field of the magnetospheric currents is calculated in terms of the paraboloid model of the magnetosphere.

3 Main Statements

3.1. The model presents the vector of induction of magnetospheric currents as a function of the solar-magnetospheric coordinates.

3.2. The model of the magnetic field of magnetospheric currents (referred to below as "model") describes a regular part of the magnetic field, its dependence on the parameters of the interplanetary medium and reflects compression of the Earth's magnetosphere in the dayside due to interaction with the solar wind, asymmetry day-night (field on the nightside is weakened), day and season variations in the region from 1 to 6.6 R_E.

3.3. The model takes into account the angle of inclination of the geomagnetic dipole to the plane orthogonal to the Earth-Sun line (geomagnetic dipole tilt angle), varying in the range from -35° to +35°.

3.4. Vector of induction of the magnetic field of magnetospheric currents is calculated by the formula

$$\vec{B}_2 = \vec{B}_{sd}(\psi, R_1) + \vec{B}_t(\psi, R_1, R_2, \Phi_\infty) + \vec{B}_r(\psi, b_r) + \vec{B}_{sr}(\psi, R_1, b_r)\,. \quad (2)$$

Here $\vec{B}_{sd}$ is the field of currents on the magnetopause shielding the dipole field; $\vec{B}_t$ is the field of a current system of the magnetospheric tail (currents across the tail and their closure currents on the magnetopause); $\vec{B}_r$ is the field of the rimg current; $\vec{B}_{sr}$ is the field of currents on the magnetopause, shielding the ring current field.

3.5. The components of the magnetic field of magnetospheric currents, $\vec{B}_{sd}$, $\vec{B}_t$, $\vec{B}_r$, $\vec{B}_{sr}$, are calculated separately in terms of the paraboloid model of the magnetosphere in the form of series in the Bessel functions. Each large-scale magnetospheric current system has its own system of current screening on the magnetopause.

3.6. The expansion coefficients of the components of the magnetic field of magnetospheric currents $\vec{B}_{sd}$, $\vec{B}_t$, $\vec{B}_r$, $\vec{B}_{sr}$ are determined by the values of parameters of the magnetospheric current systems: ψ is the geomagnetic dipole tilt angle; R_1 is the distance to the subsolar point at the magnetopause; R_2 is the distance to the earthward edge of the magnetospheric tail current sheet; Φ_∞ is the magnetic flux in the tail lobes, defining the current intensity in the magnetotail; b_r being for the intensity of the ring current magnetic field at the Erth's center.

3.7. Instant values of the parameters of the magnetospheric curent systems, $\psi, R_1, R_2, \Phi_\infty, b_r$, are determined using a limited set of empirical data in terms of so-called submodels (see Appendix 1).

3.8. The magnetospheric dynamics is determined to be a sequence of its instant states.

4 Calculation of Induction of the Magnetic Field of the Magnetospheric Currents

4.1. The magnetic field of the magnetopause shielding currents, $\vec{B}_{sd}$, is calculated as

$$\vec{B}_{sd} = -\nabla U_{sd},$$

where the scalar potential U_{sd} of the magnetic field of magnetopause currents is presented in spherical variables R, θ, φ (θ being the polar angle plotted from X_{GSM} axis, φ is the azimuthal angle plotted counterclockwise from Z_{GSM}) reads:

$$U_{sd} = -\frac{B_0 R_E^3}{R_1^2} \sum_{n=1}^{\infty} \left(\frac{R}{R_1}\right)^n \left[d_n^{\|} \sin\psi P_n(\cos\theta) + d_n^{\perp} \cos\psi_n^1(\cos\theta)\right], \quad (3)$$

$P_n(x) = (2^n n!)^{-1} \cdot \left(d^n \left(x^2-1\right)^n / dx^n\right)$, $P_n^1(x) = \sqrt{1-x^2} \cdot (dP_n/dx)$.

B_0 is the magnetic field at the geomagnetic equator of the Earth. The first six dimensionless coefficients $d_n^{\|}$ and $d_n^{\perp}$ are listed in the Table 1.

Table 1:

n	$d_n^{\perp}$	$d_n^{\parallel}$
1	0.6497	0.9403
2	0.2165	-0.4650
3	0.0434	0.1293
4	-0.0008	-0.0148
5	-0.0049	-0.0160
6	-0.0022	-0.0225

Formulae for transition from spherical to the solar-magnetospheric coordinates are presented in Appendix 2.

4.2. The magnetic field of the tail current system $\vec{B}_t$ is calculated from the equation:

$$\vec{B}_t = -\nabla U_t + \vec{B}_{t_{in}}$$

Where U_t is determined by the series:

$$U_t = b_t R_1 \begin{cases} \sum\limits_{k,n=1}^{\infty} (b_{nk} + c_{nk} K_n'(\lambda_{nk}\alpha_0)\lambda_{nk}) \cos\ n\varphi\ J_n(\lambda_{nk}\beta) I_n(\lambda_{nk}\alpha), & \alpha < \alpha_0 \\ \beta_t\ \alpha_0\ ln\ \alpha\ sign(\frac{\pi}{2} - |\varphi|) + \\ \qquad \sum\limits_{k,n=1}^{\infty} c_{nk} \cos n\varphi I_n'(\lambda_{nk}\alpha_0)\lambda_{nk} J_n(\lambda_{nk}\beta) K_n(\lambda_{nk}\alpha), & \alpha \geq \alpha_0 \end{cases}$$

here:$c_{nk} = b_{nk}\lambda_{nk} I_n(\lambda_{nk}\alpha_0)$, $\quad b_{nk} = \dfrac{2\lambda_{nk} \int\limits_0^1 \int\limits_{-\pi}^{\pi} J_n(\lambda_{nk}\beta) f(\beta,\varphi) cos\, n\varphi\ d\varphi d\beta}{\pi(\lambda_{nk}^2 - n^2) J_n^2(\lambda_{nk}) I_n'(\lambda_{nk}\alpha_0)}$

$$f(\beta,\varphi) = \begin{cases} \dfrac{\alpha_0}{\beta_t}\beta cos\,\varphi, & \alpha_0\beta cos\,\varphi < \beta_t \\ sign(\frac{\pi}{2} - |\varphi|), & \alpha_0\beta cos\,\varphi \geq \beta_t, \end{cases},$$

where λ_{nk} are zeros of the of the $J_n{}' = 0$ equation, $\alpha_0 = \sqrt{1 - 2R_2/R_1}$ is the parabolic coordinate of the inner edge of the tail current sheet, $\beta_t = \dfrac{d}{R_1}$, d is the half thickness of the current sheet, $b_t = \frac{2\Phi_\infty}{\pi R_1^2}\sqrt{R_1/(2R_2 + R_1)}$, is a magnetic field in the tail lobe at the inner edge of the tail current sheet.

The magnetic field inside the current sheet, B_2, is calculated from the relations:

$$B_{t_{in}\alpha} = b_t \frac{\alpha_0}{\alpha} \frac{\beta}{\beta_t} \frac{cos\,\varphi}{\sqrt{\alpha^2 + \beta^2}},\ B_{t_{in}\beta} = 0,\ B_{t_{in}\varphi} = 0.$$

Description of the paraboloid coordinates and formulae for transition to the solar-magnetospheric coordinates are presented in Appendix 2.

4.3. Vector of the ring current magnetic field $\vec{B}_r$ is determined by the expressions:

$$\vec{B}_r = \frac{M_R}{M_E} \cdot \begin{cases} \left(\dfrac{R}{R_{rc}}\right)^5 \cdot \vec{B}_d + 2B_0 \frac{R_E^3}{R_2^3} \left(\dfrac{R_2^5}{R_{rc}^5} - 1\right) \vec{e}_z & 0 \leq R \leq R_2 \\ \\ \vec{B}_d & R \geq R_2 \end{cases} \tag{4}$$

where $R_{rc} = \sqrt{0.5(R^2 + R_2^2)}$, $M_R = 0.5 b_r \cdot R_2^3/(4\sqrt{2} - 1)$ is the magnetic moment of the ring current, $M_E = B_0 \cdot R_E^3$ is the magnetic moment of the geomagnetic dipole, $\vec{B}_d$ is the magnetic field of the geomagnetic dipole, $\vec{e}_z$ being a unite vector directed opposite to the geomagnetic dipole.

Expressions for $\vec{B}_d$ and $\vec{e}_z$ in the solar-magnetospheric coordinates are presented in Appendix 3.

4.4. The magnetic field of the magnetopause currents shielding ring current $\vec{B}_{sr}$ is calculated from the equation

$$\vec{B}_{sr} = -\nabla U_{sr},$$

where scalar potential U_{sr} of the magnetic field of magnetopause currents is presented in spherical variables $R, \theta,\ \varphi$ (see p.4.1.) reads:

$$U_{sr} = -\frac{M_R}{R_1^2} \sum_{n=1}^{\infty} \left(\frac{R}{R_1} \right)^n \left[d_n^{\|} \sin\psi P_n(\cos\theta) + d_n^{\perp} \cos\psi P_n^1(\cos\theta) \right]. \qquad (5)$$

Coefficients $d_n^{\|}$ and $d_n^{\perp}$ are listed in the Table 1.

Appendix 1

Submodels: Calculation of the Main Parameters of Magnetospheric Current Systems

In the paraboloid model of the magnetosphere the values of parameters of the magnetospheric current systems are calculated using submodels. The submodels represent empirical relations or auxiliary models to relate parameters of the magnetospheric current systems to the measured data.

1. The tilt angle of geomagnetic dipole, ψ, is calculated by the formula

$$\sin\psi = -\sin\beta\cos\alpha_1 + \cos\beta\sin\alpha_1\cos\varphi_m \tag{6}$$

where α_1= 11,43° is the angle between the Earth's axis and the geodipole moment,

β is the Sun's deflection ($\sin\beta = \sin\alpha_2\cos\varphi_{se}$),

α_2 = 23,5° is the angle between the Earth's axis and the normal to the ecliptic plane,

φ_{se} = 0,9856263(172 - I_{day}) is the angle between the Sun-Earth line and the Earth's axis projection on the ecliptic plane,

I_{day} is the number of day in year,

$\varphi_m = UT\cdot 15^\circ - 69,76^\circ$ is the angle between the midday-midnight and northern magnetic polar meridians,

UT is the universal time in hours.

2. The distance from the Earth to the subsolar point on the magnetopause, R_1, is calculated from the balance between the solar wind dynamic pressure and the magnetic pressure in the magnetosphere

$$2kP = B_{0m}^2/2\mu_0.$$

Here k is the coefficient describing "a degree of elasticity" of solar wind particle interaction with the magnetopause ($k = 1$ when elastic reflection is assumed and $k = 0.5$ for a case when inelastic reflection is assumed), P is the solar wind dynamic presuure, B_{0m} being the value of the magnetospheric magnetic field on the magnetopause. Using data on the solar wind velocity v and proton concentration n one can obtain

$$R_1 = 100/(nv^2)^{1/6} \tag{7}$$

(where R_1 is determined in R_E; n, in cm^{-3}; and v, in km/s).

3. The distance to the earthward edge of the geomagnetic tail current sheet, R_2, is calculated by the formula

$$R_2 = 1/\cos^2\varphi_k\,, \tag{8}$$

where R_2 is expressed in R_E, and φ_k is the latitude of the equatorward boundary of the auroral oval in midnight.

4. Magnetic flux through the magnetotail lobes, Φ_∞, is calculated by the formula

$$\Phi_\infty = \Phi_0 + \Phi_s\,, \tag{9}$$

where Φ_0 is the magnetic flux in the magnetotail during quiet periods, Φ_s being the time-dependent magnetic flux in the lobes associated with intensification of the magnetotail current system during disturbances:

$$\Phi_0 = 3,7\cdot 10^8 Wb$$

$$\Phi_s = -AL\frac{\pi R_1^2}{14}\sqrt{\frac{2R_2}{R_1}+1}\,, \tag{10}$$

where AL is the auroral index of geomagnetic activity.

5. The ring current intensity is characterized by the value of ring current magnetic field at the Earth's center, which is calculated by the Dessler-Parker-Scopke relation

$$b_r = -\frac{2}{3} B_0 \frac{\varepsilon_r}{\varepsilon_d} \tag{11}$$

where ε_r is the total energy of ring current particles, $\varepsilon_d = \frac{1}{3} B_0 M_E$ being the geomagnetic dipole energy.

Appendix 2.

Coordinate Systems Used Here

Spherical coordinates R, θ, φ with the polar axis plotted along the Sun-Earth axis are determined by the expressions

$$\begin{aligned} x/R_1 &= R\cos\theta \\ y/R_1 &= R\sin\theta\sin\varphi \\ z/R_1 &= R\sin\theta\cos\varphi, \end{aligned} \tag{12}$$

where x, y, z are the solar-magnetospheric (GSM) coordinates.

Paraboloid coordinates α, β, φ with the polar axis plotted along the Sun-Earth axis are determined by the expressions.

$$\begin{aligned} 2x/R_1 &= \beta^2 - \alpha^2 + 1 \\ y/R_1 &= \alpha\beta\sin\varphi \\ z/R_1 &= \alpha\beta\cos\varphi, \end{aligned} \tag{13}$$

where x, y, z are GSM coordinates. In the magnetospheric paraboloid model magnetopause is the surface $\beta = 1$.

Appendix 3.

Magnetic Field of Geomagnetic Dipole

$\vec{B}_d$ and $\vec{e}_{z_{sm}}$ in the solar-magnetospheric coordinates are described by the expressions

$$
\begin{aligned}
&\vec{B}_d = -\nabla V_d \\
&V_d = (\tfrac{R_E}{R})^3 B_0 \cdot (z\cos\psi - x\sin\psi) \\
&\vec{e}_z = (-\sin\psi; \quad 0; \quad \cos\psi).
\end{aligned}
\tag{14}
$$