Manual: Network Contributor Rewards

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1 Overview

This manual explains the DoubleZero rewards model for network contributors, which is a mechanism that uses **Shapley values** to distribute rewards among network contributors (e.g. private link operators) in proportion to the *marginal value* each contributor adds to the network's efficiency. This is operationalized in `network_shapley.py` at https://github.com/doublezerofoundation/network-shapley.

Users who only wish to use the code should proceed directly to the Github link that contains the core code, a short readme file that explains how to run the code, and some example inputs. This longer manual is for readers who want to understand the complexities and theory behind the approach.

- Section 2 motivates the choice of Shapley values over the carried traffic model to reward network contributors.
- Section 3 walks through a minimal model with three nodes and three links.
- Section 4 layers on five complications that the model also accounts for: operator withdrawals, non-contiguous networks, latency measurements, prioritized and future demand, and multicast.
- Section 5 introduces the full model and describes some simulation results.

2 Motivating Shapley Values

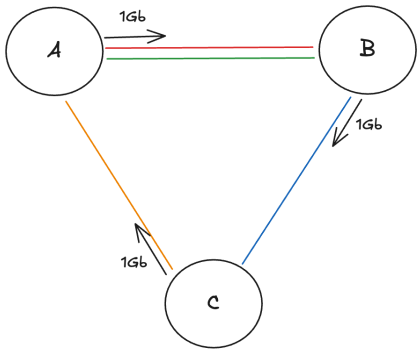
The rewards model is the key to making the DoubleZero network useful in a decentralized manner. At a high level, the rewards model accumulates revenue from users and distributes that to network contributors (net of rewards removed to prevent over-concentration or inorganic traffic attacks). But the critical question is how rewards are distributed *amongst* contributors.

The rewards model uses the concept of **Shapley values** from the field of cooperative game theory to do so. This framework gives rewards to a network contributor *fairly* and in proportion to its *marginal* contribution to the global value function.

Shapley values may appear to be needlessly complex in favor of a simpler approach, which is to pay out rewards on the basis of carried traffic. However, we believe the carried traffic model – while easier to understand – is insufficiently discriminating and does not incentivize long-term value.

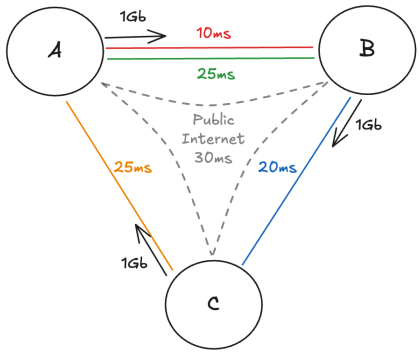
To illustrate the point, consider a scenario with four links connecting cities A, B, and C, depicted in Figure 1. The red and green links connect A and B; an orange link connects A and C; and a blue link connects B and C. Suppose the red link is more performant than the green link. Finally, each city wants to send one gigabit of traffic to its clockwise neighbor. Under this topology and using the carried traffic model, the division of rewards is straightforward: the green link receives no rewards (as it carries no traffic) while each of the other links receives one-third of the rewards (as they each carry one gigabit).

Figure 1: Three-City Example



But these rewards do not map to true value created. For instance, consider the world without each of the links sequentially. Should the red link drop out, the green link becomes much more important. Should the orange or blue links drop out, users will care if the public internet pathway connecting those cities is meaningfully worse; but they will not care if the performance is comparable. Indeed, to augment the scenario, suppose the public internet pathway connecting each city has a one-way latency of 30ms. Suppose the red line is substantially more performant than the green line (with a latency of 10ms rather than 25ms); and that the blue line is more performant (20ms) than the orange line (25ms). This is depicted in Figure 2.

Figure 2: Three-City Example (Detailed)



With this added information and under the lens of counterfactual scenarios, the division of rewards on the basis of traffic carried no longer seems optimal. The green link should get more than zero for its insurance

value; the blue link should earn more rewards than the orange link when benchmarked against the public internet alternative; and the red link should be rewarded for its very strong performance over both the public internet and the green link counterfactual scenarios. But this is all incompatible with the carried traffic model. The carried traffic model could be modified with ad-hoc adjustments to distribute rewards for redundancy value, improved latency over the public internet and peers, etc (e.g. insurance lines get paid out at 10% of non-insurance lines). But this general approach would require substantial tuning of its many parameters. Moreover, this approach quickly erodes the simplicity of the carried traffic model anyways.

By contrast, the Shapley value methodology handles these complexities without needing additional components. Shapley values distribute gains of a value function only amongst players, based on each player's marginal contribution to that value function across different coalitions of other players.[1] This avoids the need for managing a wide range of scenarios manually. Contributors only get paid for improvements to the global value function that they specifically drive.

3 Simple Model

There are two key steps to identifying an entity's marginal contribution: defining the aggregate value function and computing its value under different coalitions of network contributors. This section walks through both steps for the model in Figure 2.

The value function starts simple: traffic times the negative latency of the traffic, summed up over all traffic flows. This is a literal manifestation of "Increase Bandwidth Reduce Latency" and represented in Equation (1).

$$V = -\sum_i t_i l_i \tag{1}$$

To compute the value of this function under different coalitions of network contributors, we take the traffic flows as given. Traffic can always reach its destination through the public internet, which has effectively unlimited bandwidth but poor latency. But different coalitions of contributors can improve on the latency on some or all of those flows through their private links. Implicitly, then, DoubleZero only distributes rewards for improvements over the public internet; and not for routes that perform equivalently or worse than the public internet.

In order to compute how an arbitrary coalition of contributors can improve on the value function, we set up and solve a standard linear program that routes traffic and thus delivers a value that the coalition achieves. Critically, this linear program determines only the *hypothetical* routing of traffic, rather than the actual routing of traffic. This is for two reasons. First, all simulated coalitions except for one have only a strict subset of network contributors operating, and so there is no corresponding real-world outcome to measure. Second, for the sole coalition in which all network contributors do operate, the hypothetical and actual routing of traffic may diverge for engineering considerations from time to time.[2] However, the goal

[1] Formally, Shapley values do so by satisfying key desirable properties like symmetry (players with equal contributions get equal rewards), dummy player (those who create no value get no reward), and additivity across both settings and value functions. Indeed, they were a major reason that their creator, Lloyd Shapley, won the Nobel Prize in Economics in 2012.

[2] There are three particular salient cases where the true and simulated routing may diverge. First, traffic may be routed over slightly slower but higher throughput lines in practice, to better accommodate surges; but the simulated routing would optimize latency instead. Second, hybrid routes that mix private and public routes are permitted in the simulated routing,

of the hypothetical routing logic in this particular scenario is to generally correspond closely to the actual routing logic.

The linear program has the standard structure, noted in Equation (2), where the vector of decision variables x refers to the amount of traffic to route over each link for each traffic type. In this formulation, c refers to the cost across each link, A_{eq} and b_{eq} ensures that the traffic demands are met, and A_{ub} and b_{ub} ensures that the bandwidth constraints of each link are respected.

$$\min_{x, x \geq 0} cx \quad \text{s.t. } A_{eq}x = b_{eq} \text{ and } A_{ub}x \leq b_{\text{ub}} \tag{2}$$

To illustrate, consider the example in Figure 2, which can be laid out in the following table. For simplicity, the name of the operator is the same color as the link. Suppose we wish to solve the linear program assuming all four operators and their associated four links are made available, i.e. the largest possible coalition of contributors.

Start	End	Latency	Operator
A	B	10	Red
A	B	25	Green
B	C	20	Blue
A	C	25	Orange
A	B	30	Public
B	C	30	Public
A	C	30	Public

Under this formulation, these seven links can be turned into a matrix representing the flow of traffic to and from the various nodes. The matrix has three rows (each representing a node) and fourteen columns (each representing a link, and where the original seven bidirectional links are turned into fourteen directed links). In turn, each entry is a zero if the link is unrelated to the node, and a positive or negative one if it respectively carries traffic away or towards the city. For the reader's convenience, the columns are tagged by the first letter in the link name, and apostrophes represent links facilitating the reverse flow of traffic.

$$\tilde{A}_{eq} = \begin{array}{c} \\ A \\ B \\ C \end{array} \begin{array}{c} \begin{array}{cccccccccccccc} r & g & b & o & r' & g' & b' & o' & p_1 & p_2 & p_3 & p_1' & p_2' & p_3' \end{array} \\ \begin{bmatrix} 1 & 1 & 0 & -1 & -1 & -1 & 0 & 1 & 1 & 0 & -1 & -1 & 0 & 1 \\ -1 & -1 & 1 & 0 & 1 & 1 & -1 & 0 & -1 & 1 & 0 & 1 & -1 & 0 \\ 0 & 0 & -1 & 1 & 0 & 0 & 1 & -1 & 0 & -1 & 1 & 0 & 1 & -1 \end{bmatrix} \end{array}$$

The demand requirements are set against the matrix that embeds the flow logic. This scenario has the following demand matrix, where the row is the origin and the column is the destination.

but in practice, the routing logic only looks for end-to-end routes on the network in practice. Third, traffic in the Solana validator case often follows complex multi-hop routes (i.e. Turbine); but to keep the simulated routing tractable, we utilize the reduced-form demand of senders and recipients.

	A	B	C
A	0	1	0
B	0	0	1
C	1	0	0

However, these three types of traffic represent distinct traffic types and cannot be comingled, as otherwise the outbound traffic from A to B could be offset against the inbound traffic from C to A. Thus, we must represent the demand requirements of each traffic type uniquely. We first focus on the first traffic type, wherein one gigabit of traffic moves from A to B, and generate the following right-hand side expression. In this, the first node A pushes one unit of traffic onto the network, the second node B takes that unit off, and the third node C neither adds nor removes to this particular traffic flow.

$$\tilde{b}^1_{eq} = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$$

This successfully defines a flow constraint matrix for a single traffic type. From this, we can expand to multiple traffic types by letting x represent not just a single type of traffic being sent over a set of edges, but instead *multiple* types of traffic each being sent over a set of edges independently. This allows us to define a full constraint matrix for all traffic flows in Equation (3).

$$A_{eq} = \begin{bmatrix} \tilde{A}_{eq} & 0 & 0 \\ 0 & \tilde{A}_{eq} & 0 \\ 0 & 0 & \tilde{A}_{eq} \end{bmatrix} \qquad b_{eq} = \begin{bmatrix} \tilde{b}^1_{eq} \\ \tilde{b}^2_{eq} \\ \tilde{b}^3_{eq} \end{bmatrix} \tag{3}$$

The next step in this illustration is to embed bandwidth constraints. These are straightforward. For any particular traffic type, the eight links have their own bandwidth constraints. Public links of course have no constraints.

$$\tilde{A}_{ub} = \begin{array}{c} \begin{matrix} r & g & b & o & r' & g' & b' & o' & p_1 & p_2 & p_3 & p'_1 & p'_2 & p'_3 \end{matrix} \\ \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

This scenario has not yet specified the bandwidth limits. For the sake of argument, suppose all links can sustain five gigabits of traffic in either direction, except for the green link, which can only sustain two.

$$b_{ub} = \begin{bmatrix} 5 & 2 & 5 & 5 & 5 & 2 & 5 & 5 \end{bmatrix}^T$$

Together, these define the bandwidth constraint matrix for a single traffic type. As before, we can expand to multiple traffic types by stacking the bandwidth constraints horizontally, as all types of traffic on a single line share that bandwidth. There are no modifications needed for the righthand side constraint.

$$A_{ub} = \begin{bmatrix} \tilde{A}_{ub} & \tilde{A}_{ub} & \tilde{A}_{ub} \end{bmatrix}$$

Finally, the cost matrix in the objective function is simply the vector representation of the latencies for each link.

$$\tilde{c} = \begin{matrix} r & g & b & o & r' & g' & b' & o' & p_1 & p_2 & p_3 & p_1' & p_2' & p_3' \\ \begin{bmatrix} 10 & 25 & 20 & 25 & 10 & 25 & 20 & 25 & 30 & 30 & 30 & 30 & 30 & 30 \end{bmatrix} \end{matrix}$$

As before, they need to be stacked horizontally to represent the three traffic types in this scenario.

$$c = \begin{bmatrix} \tilde{c} & \tilde{c} & \tilde{c} \end{bmatrix}$$

These five terms of c, A_{eq}, b_{eq}, A_{ub}, and b_{ub} jointly define the linear program. Any standard solver can be used to find the solution. This is a problem where we minimize positive cost, and so we can take the negative value at the optimal point to instead have a term we wish to maximize.

We thus iterate through each of the sixteen possible coalitions – representing the power set of the four operators – and compute out the value function assuming their respective links are present or absent. Figure 3 yields the value function for each of the sixteen combinations. It also indicates the latency that each particular traffic flow must endure, as a helpful intermediate step.

Figure 3: Shapley Value by Coalition

Coalition	ℓ_{AB}	ℓ_{BC}	ℓ_{CA}	Value
No Operators	30	30	30	−90
Red	10	30	30	−70
Green	25	30	30	−85
Blue	30	20	30	−80
Orange	30	30	25	−85
Red, Blue	10	20	30	−60
Red, Orange	10	30	25	−65
Green, Blue	25	20	30	−75
Green, Orange	25	30	25	−80
Blue, Orange	30	20	25	−75
Red, Green	10	30	30	−70
Red, Blue, Orange	10	20	25	−55
Green, Blue, Orange	25	20	25	−70
Red, Green, Blue	10	20	30	−60
Red, Green, Orange	10	30	25	−65
All Operators	10	20	25	−55

The final step is to take this table of coalition-level values and compute operator-level Shapley values. To illustrate, consider a single operator, e.g. the Green operator. In Figure 4, we take all coalitions that

exclude the green operator, and measure the increase in the value function when the coalition *includes* it. For instance, the Blue and Orange coalition has a value of -75; and this improves to -70 when the Green operator joins the coalition. This improvement of 5 is weighted by the relative number of permutations that correspond to this coalition, assuming every possible ordering of operators.[3]

Figure 4 compares each coalition that includes and excludes the Green operator, along with the marginal improvement and weight. This can be summed to a total Shapley value of 2.5 for the Green operator.

Figure 4: Marginal Value of Green Link

With G	Without G	ΔV	Weight
$V(G,R,B,O)$	$V(R,B,O)$	$(-55)-(-55)=0$	0.250
$V(G,R,B)$	$V(R,B)$	$(-60)-(-60)=0$	0.083
$V(G,R,O)$	$V(R,O)$	$(-65)-(-65)=0$	0.083
$V(G,B,O)$	$V(B,O)$	$(-70)-(-75)=5$	0.083
$V(G,R)$	$V(R)$	$(-70)-(-70)=0$	0.083
$V(G,O)$	$V(O)$	$(-80)-(-85)=5$	0.083
$V(G,B)$	$V(B)$	$(-75)-(-80)=5$	0.083
$V(G)$	$V(\text{none})$	$(-85)-(-90)=5$	0.250
		Sum	**2.50**

The procedure can be repeated for each other network contributor: the high-performance Red operator has a value of 17.5; the semi-performant Blue operator has a value of 10; and the Orange operator, which is only slightly better than the public internet, has a value of 5. In fractional terms, that means that Red operator earns 50% of the available rewards, the Blue operator earns 29%, the Orange operator earns 14%, and the Green operator earns 7%.

4 Adjustments and Complications

In practice, the simplified model in Section 3 is extended to address five complications.

4.1 Operator Withdrawals

Private links can disappear without notice, for commercial, operational, or regulatory reasons. To guard against this, the methodology preemptively rewards redundancy across operators. Formally, it does this by modifying the value function from Equation (1) to Equation (4).

$$V = -\mathbb{E}\left(\sum_i t_i l_i\right) \tag{4}$$

Operationally, this is done by treating the output from the analysis in Figure 3 as the scenario-level values. In other words, this is the value of that coalition *assuming the coalition survives*. By contrast, the **expected** value of the coalition accounts for the fact that certain operators may drop out. That is, suppose

[3]There are 4! = 24 orderings, and two – Blue, Orange, Green, Red; and Orange, Blue, Green, Red – correspond to this particular scenario; and so the weighting is 2/24.

a given coalition C is scheduled, then we multiply all subcoalitions within that one times the probability of that particular subcoalition appearing.

$$\mathbb{E}\left(V(C)\right) = \sum_{S \subseteq C} p^{|S|}(1-p)^{|C|-|S|}V(S)$$

In the current model, there is a common operator probability p. In future iterations, this can be made into an operator-specific probability if there is substantial variation between operators.

Computationally, this step can be the primary bottleneck in the methodology if these calculations are done sequentially. Thus, the code recasts the problem to be able to compute the equation for all coalitions in a single pass. Suppose v is the scenario-level values written in cardinal order and B is a diagonal matrix where each element is $p^{|S_k|}$, i.e. the "all-survive" baseline. We then define C to be the Möbius inversion on the Boolean lattice, with the following definition. This can be precomputed to save runtime cycles.

$$C_{ij} = \begin{cases} (-1)^{|S_j|-|S_i|} & \text{if } S_i \subseteq S_j \\ 0 & \text{otherwise} \end{cases}$$

This allows us to compute the expected value for all coalitions in a single operation.

$$\mathbb{E}(V) = CBv$$

4.2 DZXs, Edges, and Contiguity

The DoubleZero network does not just care about high-performance inter-city links. Transit *within* a metro area (also known as "DZXs") is important, as is transit at the edges of the network. Without this, new bottlenecks in communication would emerge at these chokepoints.

However, in the base implementation of the methodology, these sorts of links are either poorly incentivized or not incentivized. Edge connections, e.g. high-quality dedicated internet access, are entirely out-of-scope. Intra-city DZX links are nominally included, but given the short distances, the ability to improve over public latency is highly limited.

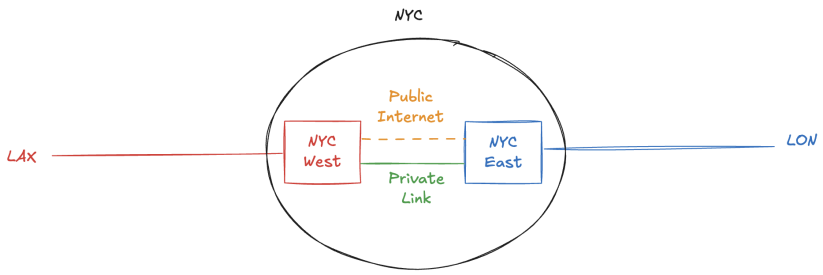
To address this, the methodology adds extra rewards for *contiguity*. Contributors are encouraged to support traffic reaching its destination entirely on DoubleZero, i.e. entirely over a contiguous network. This is operationalized by assessing an explicit latency penalty for routes that mix network links with public links to reach their endpoints. The justification for this penalty is strongly grounded in the practical operation of hybrid routing. In the short run, the public internet does not give the protocol visibility into bottlenecks, congestion, potential peering disputes, or other problems with traffic flows that could otherwise be proactively solved. In the long run, the public internet does not support the same MTU size that DoubleZero aspires to support, which will introduce traffic limitations or engineering complexities. Both problems are solved when traffic is routed entirely on DoubleZero.

This contiguity bonus further brings edges and DZXs into scope for the methodology. For edges, contributors who invest in high-quality DIA access facilitate direct ingress and egress to users and avoid the penalty. Contributors who do not invest instead require traffic to ingress or egress short of its origin or destination point, and thus that traffic suffers the penalty. For DZXs, contributors who furnish them allow traffic to

move through metro areas over a contiguous network, and thus that traffic avoids the penalty. Without those DZXs, that traffic must hit the public internet to switch between inter-city links and so it suffers the penalty.

To illustrate the methodology, consider a simple example in Figure 5 wherein two contributors provide links: one from LA to New York, terminating in a device on the west side of New York; and one from London to New York, terminating in a device on the east side of New York. The devices are of course connected by the public internet, but a contributor is determining whether to install the green link, that directly connects them privately.

Figure 5: Contiguity Example

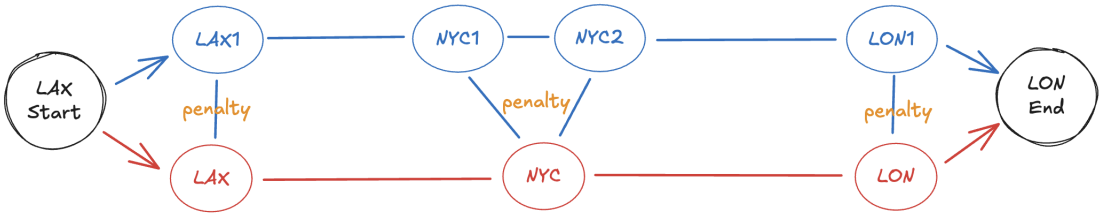


While the rewards methodology formally handles this case, the contributor may be insufficiently motivated to install the green link. The improvements over the public internet latency – which govern the contributor's rewards – may be small over short distances, despite the large positive externalities it unlocks for the network as a whole.

The rewards model motivates this by applying a latency penalty to move from the devices to the public internet. This delivers a strong incentive for contributors to jointly build contiguous networks. The penalty effectively limits rewards for non-contiguous paths (even if they have low latency) and only contiguous paths will earn the maximum potential rewards. Thus, the contributor who installs the green link internalizes some of that surplus directly.

This can be visualized in Figure 6, and this logic is operationalized in the code. DoubleZero devices are connected to each other of course, and connected to traffic senders and receivers via local DIA connections. Those senders and receivers are also connected to the public internet. However, to move from the devices to the public internet costs a fixed latency penalty (and is again constrained by the device's DIA connection), which can only be avoided by using purely private or purely public paths. As such, the joint private path still needs to beat the public path in order to earn rewards; but the division of those rewards is slightly equalized amongst the different link contributors to encourage contiguity.

Figure 6: Contiguity Example (Programmed Representation)



This approach has some broader benefits. First, by essentially awarding a baseline value to all links regardless of length, it better matches the cost profile of network contributors, who have fixed costs like devices that are similarly incurred regardless of route length. Second, this is a market-based solution. This approach, rather than issuing a handful of coarse rules, allows for more flexibility in achieving optimal outcomes. For instance, in certain markets, contiguous connectivity might be outrageously expensive and so a public connection is the better option. The resulting non-contiguous network will not earn rewards at the maximal rate, but it still will be admitted to the network and used ultimately to route traffic.

4.3 Link Jitter

The model in Section 3 assumes that link latencies can be measured as a single number. In practice, there is wide variation around any particular latency measurement, particularly on the public internet. During periods with low congestion, latency might be low; and during periods with high congestion, latency might be high. Moreover, this becomes more complex as the size of the traffic varies. A single packet may face very few queues in its delivery; a stream of packets, though, will see at least some of its members delayed in delivery.

To account for this, we intend on measuring the distribution of link latencies, especially over the public internet, across several sampled time intervals and for a given traffic flow that has meaningful size. We then intend on using some upper percentile value, e.g. the p95, as the "measured" latency for the purposes of the methodology.

4.4 Demand Prioritization and Scale

The model in Section 3 can be extended in two further ways, with respect to demand.

First, not all traffic may be equal. Certain traffic might be more important, e.g. if there is meaningful stake weight associated with the receiver or if the sender is willing to pay for a higher tier of service. This can be handled by extended the value function to embed a notion of relative priority p in the traffic flows, taking the value function from Equation (4) to Equation (5).

$$V = -\mathbb{E}\left(\sum_i p_i t_i l_i\right) \tag{5}$$

Second, the rewards methodology distributes rewards for contributors who meet the demand profile as it exists today. However, the procurement for network links can be long, given the substantial capital expenditure needed, and this squares poorly with a fast-growing network. To help grow the capacity of the network in tandem with demand, we propose to scale up current traffic demand by a constant factor γ for routing purposes. Scaling demand helps identify chokepoints or bottlenecks, and distributes rewards in advance to contributors who alleviate those issues. This scaling will evolve as time goes on, and eventually – as the network becomes sufficiently mature – it will be effectively phased out (i.e. set to $\gamma = 1$).

4.5 Multicast

One of DoubleZero's features is multicast: the ability for the network devices, rather than the sender, to replicate packets, using hardware embedded in the network. As brief context, the devices that link fiber cables to one another are highly-specialized and performant ASICs. They are able to replicate packets at the line rate of information flow, which is far faster than the slower CPUs that sit inside most users' nodes.

As part of that, the devices perform the replication intelligently. Rather than naively replicating packets at the source, it does so as late as possible in the topology and close to the receiver. For instance, if a Singaporean sender needs to send a packet to two receivers in London, a unicast paradigm would require it to send a packet to each one sequentially. In multicast, the sender sends a single packet to London; and only in the local London data center is that packet replicated as they traverse the last few meters to their respective destinations.

This does not change the rewards methodology, but it does modify how bandwidth constraints are embedded in the linear program. In terms of outcomes, it allows links with little capacity but low latency to be much more effective, because their lack of capacity no longer becomes a bottleneck in many cases.

Note that not all demand flows will utilize multicast. However, all devices on the DoubleZero network can support multicast for multicast-enabled flows. The only flows on DoubleZero that cannot use multicast are flows off the network, i.e. from a DoubleZero device onto the public internet or towards end users.

5 Full Model

The full model, with adjustments as described in Section 4, can be found at:
https://github.com/doublezerofoundation/network-shapley/.

The model can be simulated on a simple network per the script below. These files – corresponding to the list of the private links, the list of devices, the list of public links, and two different demand profiles – are included for user convenience. It is important to note that they do not necessarily represent the current or future map of DoubleZero or anticipated latencies. They are simply cities that are chosen for illustrative purposes, and latencies are computed by finding the distances between their latitude and longitudes and applying some scaling factor with noise for private links and public links alike.

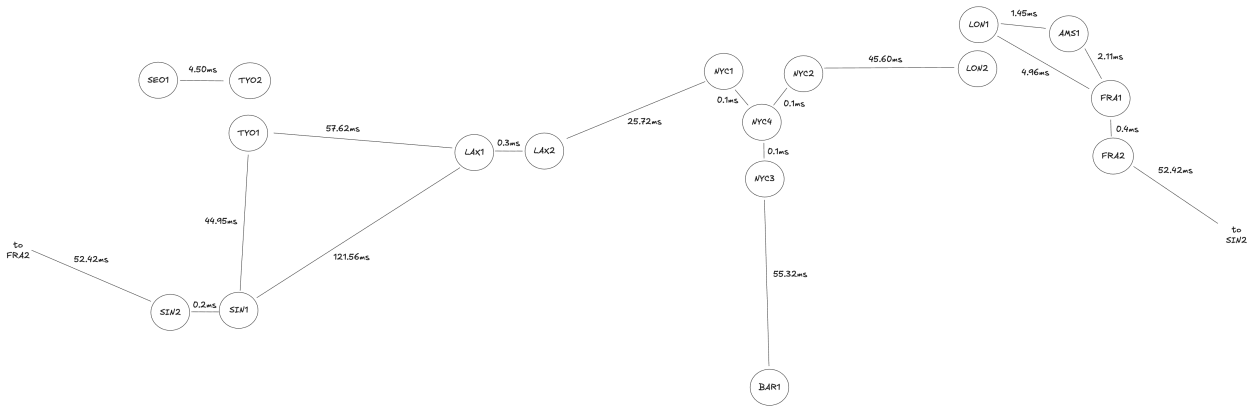
```
import pandas as pd
from network_shapley import network_shapley

private_links = pd.read_csv("private_links.csv")
devices       = pd.read_csv("devices.csv")
public_links  = pd.read_csv("public_links.csv")
demand1       = pd.read_csv("demand1.csv")
demand2       = pd.read_csv("demand2.csv")

result1 = network_shapley(
    private_links     = private_links,
    devices           = devices,
    demand            = demand1,
    public_links      = public_links,
    operator_uptime   = 0.98, # optional
    contiguity_bonus  = 5.0,  # optional
    demand_multiplier = 1.2,  # optional
)
print(result1)

result2 = network_shapley(
    private_links     = private_links,
    devices           = devices,
    demand            = demand2,
    public_links      = public_links,
    operator_uptime   = 0.98, # optional
    contiguity_bonus  = 5.0,  # optional
    demand_multiplier = 1.2,  # optional
)
print(result2)
```

As added context, the map of private links can be visualized in Figure 7. The public internet links are not visualized for simplicity.

Finally, the function `network_shapley` takes in three additional optional parameters: the probability of any given operator remaining in the network for a given epoch, the latency penalty associated with public links in a hybrid route, and the scaling multiplier for a demand. These correspond to the various complications in Section 4.

The results of these simulated scenarios are in Figure 8. In the first simulation, which approximates the Solana map with a leader based in Singapore, Gamma and Theta earn most of the rewards. Gamma provides the valuable Singapore to Frankfurt route that connects the leader to central Europe (where most of the Solana stake lives). Theta provides the connections around the Pacific, which are also critically important given the stake in Tokyo and in the United States.

Figure 7: Simulated DoubleZero Network



In the second simulation, there are two concurrent usage patterns: a leader in New York sending blocks to the Solana validator set, and a leader in Buenos Aires broadcasting blocks to several other locations in an equal-weighted manner. Now, other providers become critical. Zeta, which connects New York to Buenos Aires, becomes the most important provider now. In addition, healthy rewards accrue to Beta and Theta, who provide the cross-Atlantic and cross-Pacific connectivity. Finally, it is worth noting that Epsilon, which is effectively a DZX provider in New York, does remarkably well because of its ability to make the network highly contiguous in delivering traffic.

Figure 8: Network Contributor Simulated Rewards

Operator Name	**Value #1**	**Percent #1**	**Value #2**	**Percent #2**
Alpha	21.5370	2.80%	2.0154	0.16%
Beta	10.6595	1.03%	187.1199	15.01%
Delta	13.5257	1.31%	111.6727	8.96%
Epsilon	0.0407	0.00%	88.5022	7.10%
Gamma	487.1094	47.01%	23.0343	1.85%
Kappa	0.0603	0.01%	10.6422	0.85%
Theta	503.1153	48.55%	333.5523	26.76%
Zeta	0.1393	0.01%	490.0349	39.13%

Contributors and users of DoubleZero are encouraged to build simulations that are more realistic for their purposes.