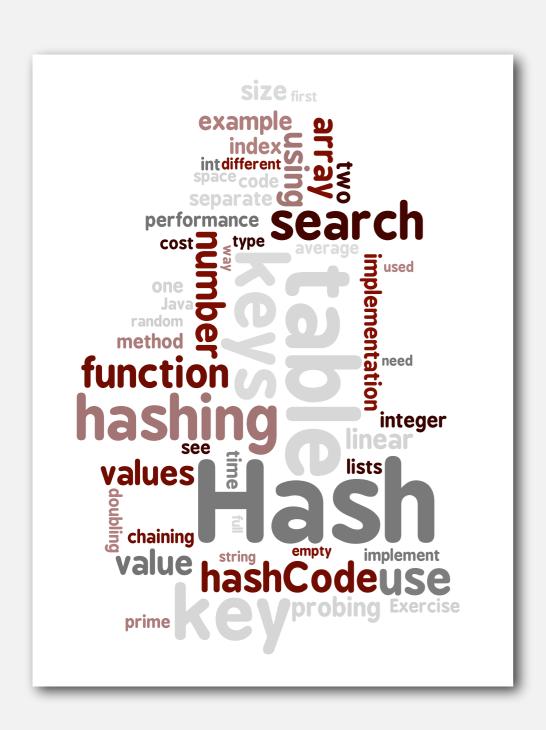
3.4 Hash Tables



- hash functions
- separate chaining
- linear probing
- applications

Optimize judiciously

"More computing sins are committed in the name of efficiency (without necessarily achieving it) than for any other single reason—including blind stupidity." — William A. Wulf

"We should forget about small efficiencies, say about 97% of the time: premature optimization is the root of all evil." — Donald E. Knuth

"We follow two rules in the matter of optimization:

Rule 1: Don't do it.

Rule 2 (for experts only). Don't do it yet - that is, not until you have a perfectly clear and unoptimized solution. " -M. A. Jackson

Reference: Effective Java by Joshua Bloch



ST implementations: summary

implementation		guarantee			average case	ordered	operations	
	search	insert	delete	search hit	insert	delete	iteration?	on keys
sequential search (linked list)	N	N	N	N/2	N	N/2	no	equals()
binary search (ordered array)	lg N	N	N	lg N	N/2	N/2	yes	compareTo()
BST	N	N	N	1.38 lg N	1.38 lg N	?	yes	compareTo()
red-black tree	2 lg N	2 lg N	2 lg N	1.00 lg N	1.00 lg N	1.00 lg N	yes	compareTo()

- Q. Can we do better?
- A. Yes, but with different access to the data.

Hashing: basic plan

Save items in a key-indexed table (index is a function of the key).

Hash function. Method for computing array index from key.

hash("it") = 3

"it"

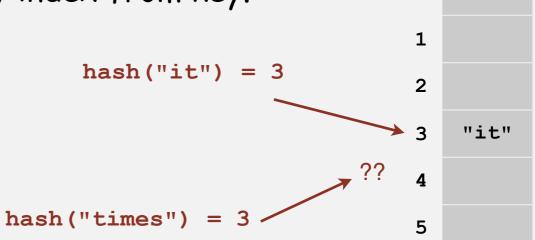
Issues.

- Computing the hash function.
- Equality test: Method for checking whether two keys are equal.

Hashing: basic plan

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Hash function. Method for computing array index from key.



0

Issues.

- Computing the hash function.
- Equality test: Method for checking whether two keys are equal.
- Collision resolution: Algorithm and data structure to handle two keys that hash to the same array index.

Classic space-time tradeoff.

- No space limitation: trivial hash function with key as index.
- No time limitation: trivial collision resolution with sequential search.
- Space and time limitations: hashing (the real world).

hash functions

- separate chaining
- linear probing
- applications

Computing the hash function

Idealistic goal. Scramble the keys uniformly to produce a table index.

- Efficiently computable.
- Each table index equally likely for each key.

thoroughly researched problem, still problematic in practical applications

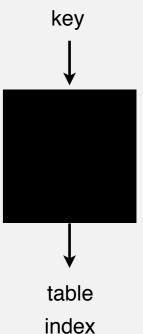


- Bad: first three digits.
- Better: last three digits.

573 = California, 574 = Alaska(assigned in chronological order within geographic region)

- Ex 2. Social Security numbers. ←—
- Bad: first three digits.
- Better: last three digits.

Practical challenge. Need different approach for each key type.

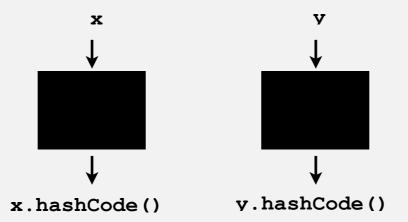


Java's hash code conventions

All Java classes inherit a method hashcode (), which returns a 32-bit int.

Requirement. If x.equals(y), then (x.hashCode() == y.hashCode()).

Highly desirable. If !x.equals(y), then (x.hashCode() != y.hashCode()).



Default implementation. Memory address of x.

Trivial (but poor) implementation. Always return 17.

Customized implementations. Integer, Double, String, File, URL, Date, ...

User-defined types. Users are on their own.

Implementing hash code: integers, booleans, and doubles

```
public final class Integer
{
   private final int value;
   ...

public int hashCode()
   { return value; }
}
```

```
public final class Boolean
{
    private final boolean value;
    ...

public int hashCode()
    {
        if (value) return 1231;
        else return 1237;
    }
}
```

```
public final class Double
{
   private final double value;
   ...

public int hashCode()
   {
     long bits = doubleToLongBits(value);
     return (int) (bits ^ (bits >>> 32));
   }
}
```

convert to IEEE 64-bit representation; xor most significant 32-bits with least significant 32-bits

Implementing hash code: strings

```
public final class String
{
    private final char[] s;
    ...

public int hashCode()
    {
        int hash = 0;
        for (int i = 0; i < length(); i++)
            hash = s[i] + (31 * hash);
        return hash;
    }
}</pre>
```

char	Unicode
'a'	97
'b'	98
'C'	99

- Horner's method to hash string of length L: L multiplies/adds.
- Equivalent to $h = 31^{L-1} \cdot s^0 + ... + 31^2 \cdot s^{L-3} + 31^1 \cdot s^{L-2} + 31^0 \cdot s^{L-1}$.

War story: String hashing in Java

String hashCode() in Java 1.1.

- For long strings: only examine 8-9 evenly spaced characters.
- Benefit: saves time in performing arithmetic.

```
public int hashCode()
{
   int hash = 0;
   int skip = Math.max(1, length() / 8);
   for (int i = 0; i < length(); i += skip)
      hash = s[i] + (37 * hash);
   return hash;
}</pre>
```

• Downside: great potential for bad collision patterns.

War story: String hashing in Java

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  return hash;
}</pre>
```

Downside: great potential for bad collision patterns.

Implementing hash code: user-defined types

```
public final class Transaction
   private final long who;
   private final Date when;
   private final String where;
   public Transaction(long who, Date when, String where)
   { /* as before */ }
   public boolean equals(Object y)
   {    /* as before */ }
   public int hashCode()
                                  nonzero constant
                                                                    for primitive types, use
      int hash = 17;
                                                                    hashCode()
      hash = 31*hash + ((Long) who).hashCode(); 

                                                                    of wrapper type
      hash = 31*hash + when.hashCode();
      hash = 31*hash + where.hashCode(); 
                                                                    for reference types,
      return hash;
                                                                    USE hashCode()
                        typically a small prime
```

Hash code design

"Standard" recipe for user-defined types.

- Combine each significant field using the 31x + y rule.
- If field is a primitive type, use wrapper type hashcode().
- If field is an array, apply to each element. or use Arrays. deepHashCode()
- If field is a reference type, use hashCode(). ← applies rule recursively

In practice. Recipe works reasonably well; used in Java libraries. In theory. Need a theorem for each type to ensure reliability.

Basic rule. Need to use the whole key to compute hash code; consult an expert for state-of-the-art hash codes.

Hash code. An int between -231 and 231-1.

Hash function. An int between 0 and M-1 (for use as array index).

typically a prime or power of 2

```
private int hash(Key key)
{ return key.hashCode() % M; }
bug
```

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```
private int hash(Key key)
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bug
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```
private int hash(Key key)
{ return Math.abs(key.hashCode()) % M; }
```

1-in-a-billion bug

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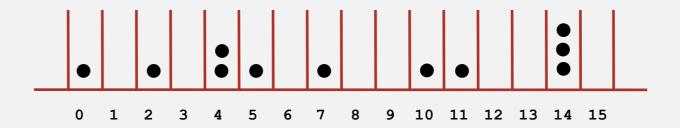
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```
private int hash(Key key)
     return key.hashCode() % M; }
bug
 private int hash(Key key)
     return Math.abs(key.hashCode()) % M; }
1-in-a-billion bug
                     hashCode() of "polygenelubricants" is -231
 private int hash(Key key)
     return (key.hashCode() & 0x7fffffff) % M; }
correct
```

Uniform hashing assumption

Uniform hashing assumption. Each key is equally likely to hash to an integer between 0 and M-1.

Bins and balls. Throw balls uniformly at random into M bins.



Birthday problem. Expect two balls in the same bin after $\sim \sqrt{\pi\,M/\,2}\,$ tosses.

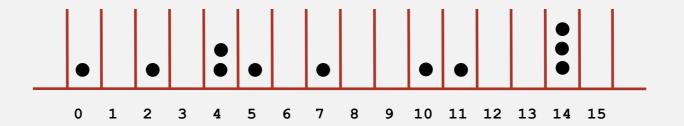
Coupon collector. Expect every bin has ≥ 1 ball after $\sim M \ln M$ tosses.

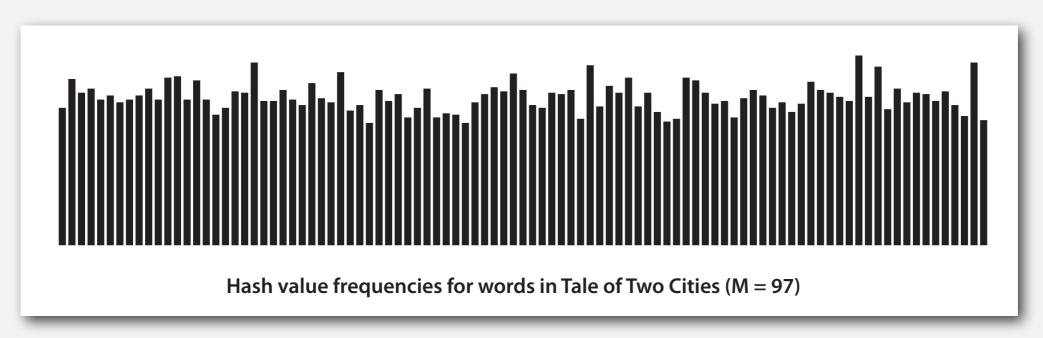
Load balancing. After M tosses, expect most loaded bin has $\Theta\left(\log M/\log\log M\right)$ balls.

Uniform hashing assumption

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Bins and balls. Throw balls uniformly at random into M bins.





Java's String data uniformly distribute the keys of Tale of Two Cities

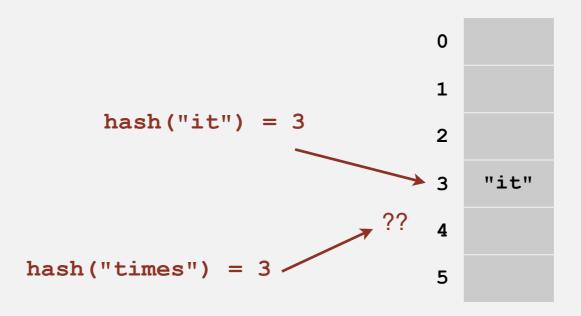
- hash functions
- separate chaining
- linear probing
- applications

Collisions

Collision. Two distinct keys hashing to same index.

- Birthday problem \Rightarrow can't avoid collisions unless you have a ridiculous (quadratic) amount of memory.
- Coupon collector + load balancing \Rightarrow collisions will be evenly distributed.

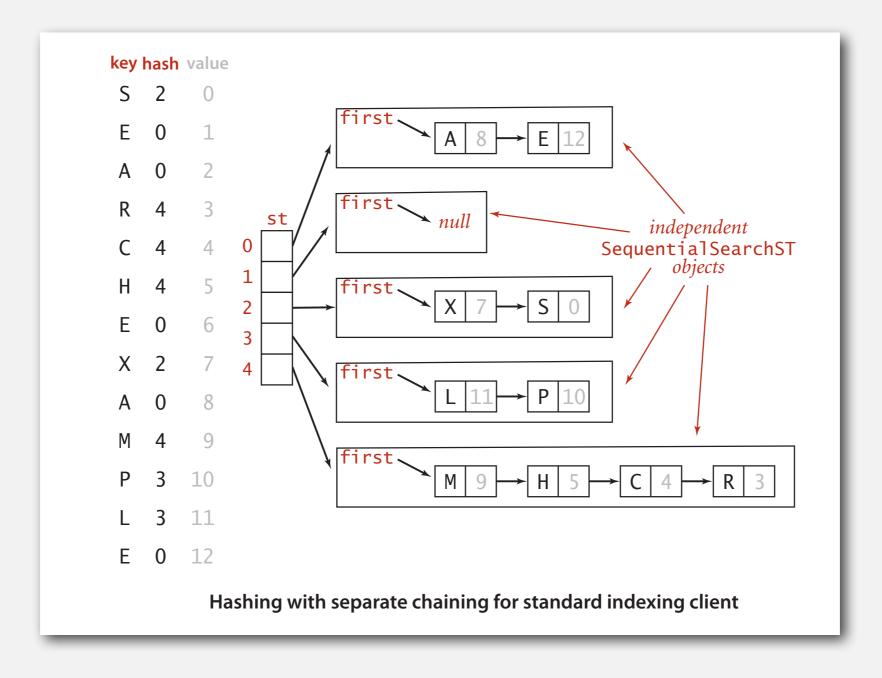
Challenge. Deal with collisions efficiently.



Separate chaining ST

Use an array of M < N linked lists. [H. P. Luhn, IBM 1953]

- Hash: map key to integer i between 0 and M-1.
- Insert: put at front of i^{th} chain (if not already there).
- Search: only need to search ith chain.



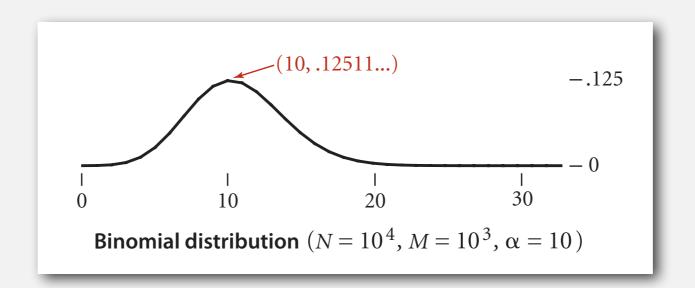
Separate chaining ST: Java implementation

```
public class SeparateChainingHashST<Key, Value>
  private int N;  // number of key-value pairs
  private int M;  // hash table size
  private SequentialSearchST<Key, Value> [] st; // array of STs
   public SeparateChainingHashST()
                                            array doubling and halving code omitted
     this(997); }
   public SeparateChainingHashST(int M)
      this.M = M;
      st = (SequentialSearchST<Key, Value>[]) new SequentialSearchST[M];
      for (int i = 0; i < M; i++)
         st[i] = new SequentialSearchST<Key, Value>();
   private int hash(Key key)
      return (key.hashCode() & 0x7fffffff) % M; }
   public Value get(Key key)
      return st[hash(key)].get(key); }
   public void put(Key key, Value val)
      st[hash(key)].put(key, val); }
```

Analysis of separate chaining

Proposition. Under uniform hashing assumption, probability that the number of keys in a list is within a constant factor of N/M is extremely close to 1.

Pf sketch. Distribution of list size obeys a binomial distribution.



equals() and hashCode()

Consequence. Number of probes for search/insert is proportional to N/M.

- M too large \Rightarrow too many empty chains.
- M too small \Rightarrow chains too long.
- Typical choice: $M \sim N/5 \Rightarrow \text{constant-time ops.}$

M times faster than sequential search

ST implementations: summary

implementation		guarantee			average case	ordered	operations	
	search	insert	delete	search hit	insert	delete	iteration?	on keys
sequential search (linked list)	N	N	N	N/2	N	N/2	no	equals()
binary search (ordered array)	lg N	N	N	lg N	N/2	N/2	yes	compareTo()
BST	N	N	N	1.38 lg N	1.38 lg N	?	yes	compareTo()
red-black tree	2 lg N	2 lg N	2 lg N	1.00 lg N	1.00 lg N	1.00 lg N	yes	compareTo()
separate chaining	lg N *	lg N *	lg N *	3-5 *	3-5 *	3-5 *	no	equals()

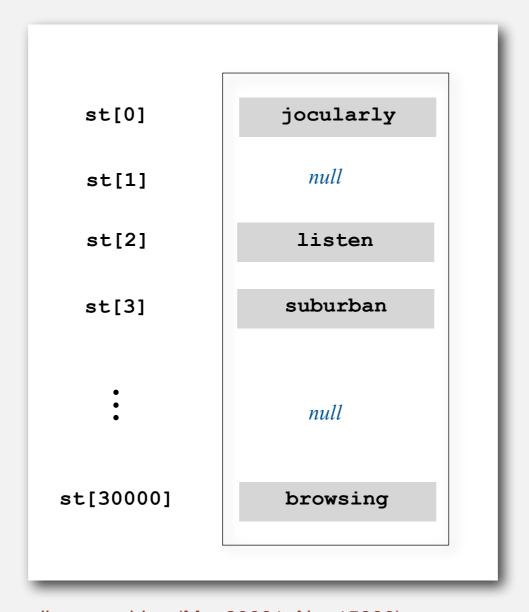
^{*} under uniform hashing assumption

- hash functions
- separate chaining
- linear probing
- applications

Collision resolution: open addressing

Open addressing. [Amdahl-Boehme-Rocherster-Samuel, IBM 1953]

When a new key collides, find next empty slot, and put it there.



linear probing (M = 30001, N = 15000)

Linear probing

Use an array of size M > N.

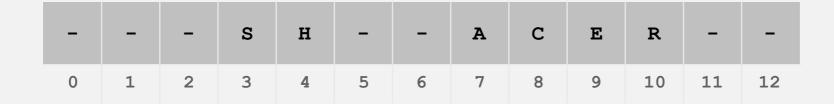
- Hash: map key to integer i between 0 and M-1.
- Insert: put at table index i if free; if not try i + 1, i + 2, etc.
- Search: search table index i; if occupied but no match, try i + 1, i + 2, etc.

-	-	-	s	Н	-	-	A	С	E	R	-	-
0	1	2	3	4	5	6	7	8	9	10	11	12

Linear probing

Use an array of size M > N.

- Hash: map key to integer i between 0 and M-1.
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-	-	-	s	Н	-	-	A	С	E	R	I	-	
								8					

insert I hash(I) = 11

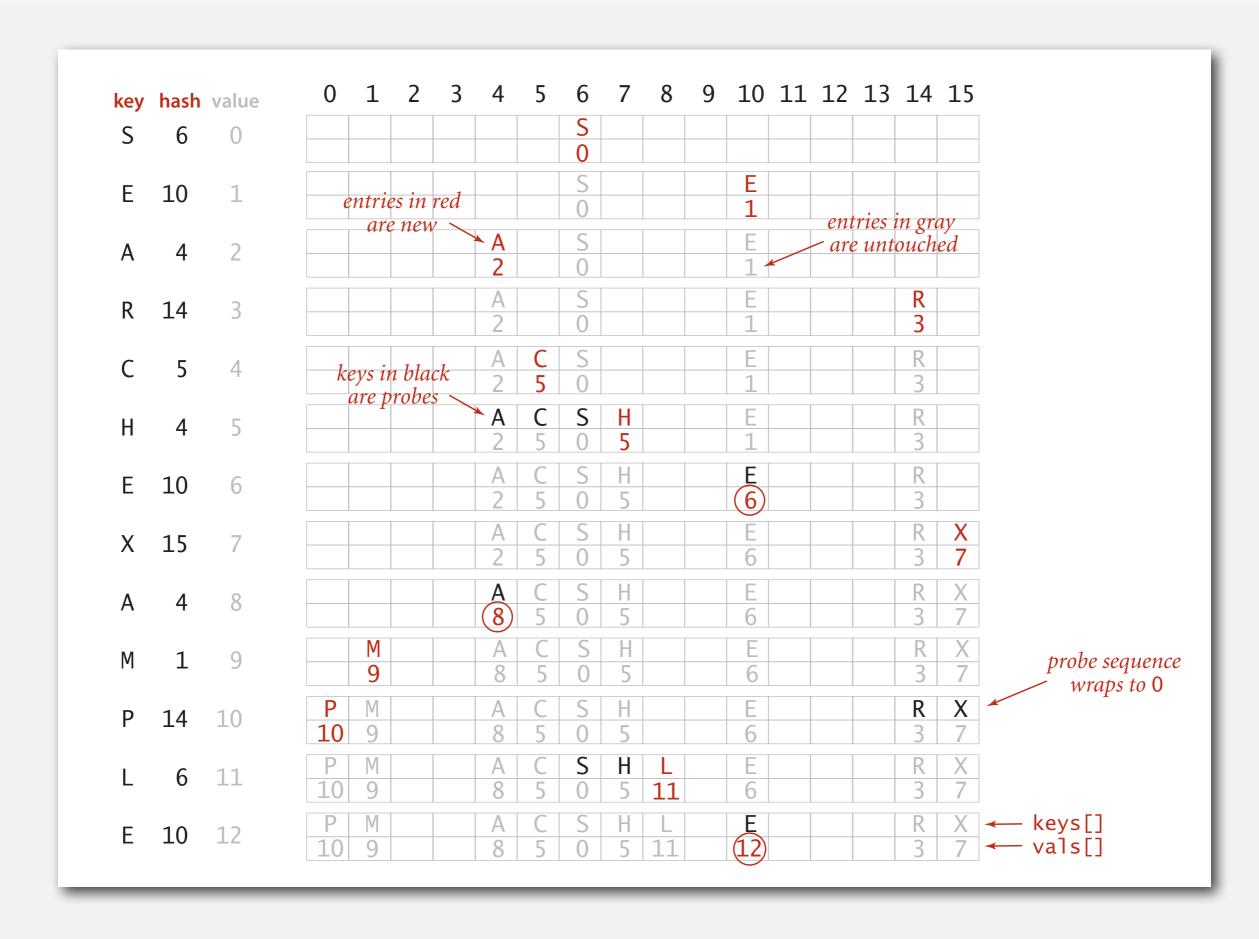
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- Insert: put at table index i if free; if not try i + 1, i + 2, etc.
- Search: search table index i; if occupied but no match, try i + 1, i + 2, etc.



Linear probing: trace of standard indexing client



Linear probing ST implementation

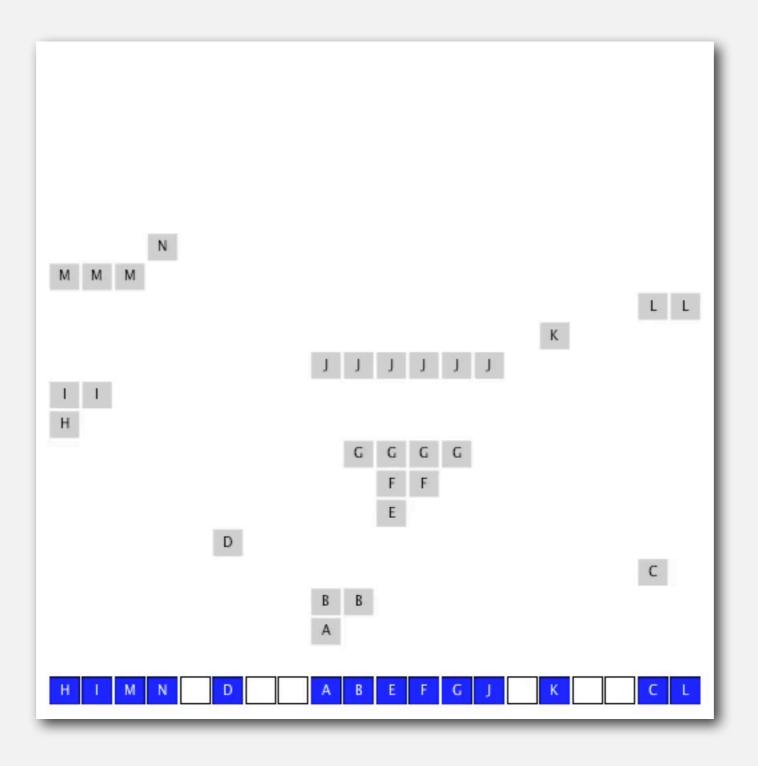
```
public class LinearProbingHashST<Key, Value>
   private int M = 30001;
   private Value[] vals = (Value[]) new Object[M];
   private Key[] keys = (Key[]) new Object[M];
   private int hash(Key key) { /* as before */ }
   public void put(Key key, Value val)
      int i;
      for (i = hash(key); keys[i] != null; i = (i+1) % M)
         if (keys[i].equals(key))
            break;
      keys[i] = key;
      vals[i] = val;
   public Value get(Key key)
      for (int i = hash(key); keys[i] != null; i = (i+1) % M)
         if (key.equals(keys[i]))
             return vals[i];
      return null;
```

array doubling and halving code omitted

Clustering

Cluster. A contiguous block of items.

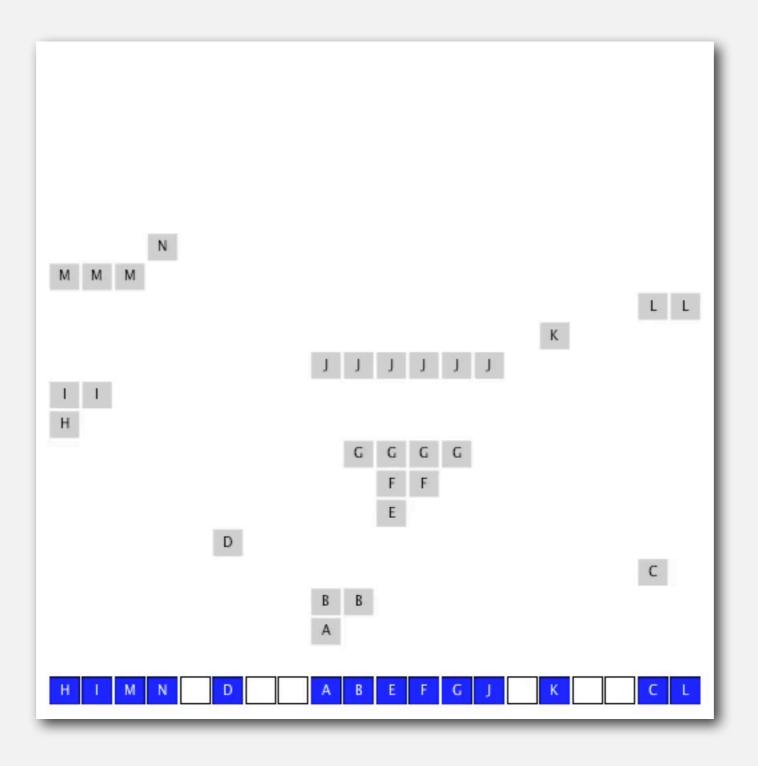
Observation. New keys likely to hash into middle of big clusters.



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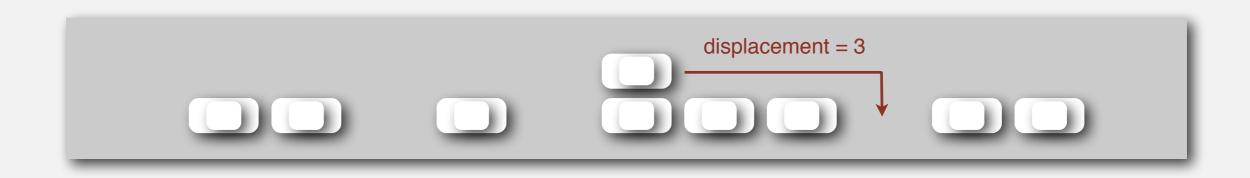


Knuth's parking problem

Model. Cars arrive at one-way street with M parking spaces.

Each desires a random space i: if space i is taken, try i + 1, i + 2, etc.

Q. What is mean displacement of a car?



Half-full. With M/2 cars, mean displacement is $\sim 3/2$.

Full. With M cars, mean displacement is $\sim \sqrt{\pi M/8}$

Analysis of linear probing

Proposition. Under uniform hashing assumption, the average number of probes in a hash table of size M that contains $N = \alpha M$ keys is:

$$\sim \frac{1}{2} \left(1 + \frac{1}{1 - \alpha} \right) \qquad \sim \frac{1}{2} \left(1 + \frac{1}{(1 - \alpha)^2} \right)$$
 search hit search miss / insert

Pf. [Knuth 1962] A landmark in analysis of algorithms.

Parameters.

- M too large \Rightarrow too many empty array entries.
- M too small \Rightarrow search time blows up.
- Typical choice: $\alpha = N/M \sim \frac{1}{2}$.

probes for search hit is about 3/2
probes for search miss is about 5/2

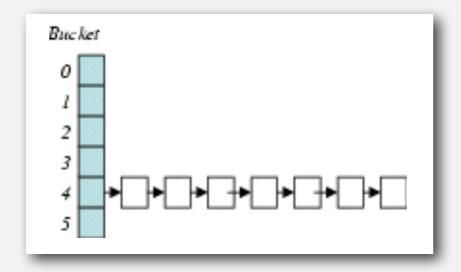
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separate chaining	lg N *	lg N *	lg N *	3-5 *	3-5 *	3-5 *	no	equals()
linear probing	lg N *	lg N *	lg N *	3-5 *	3-5 *	3-5 *	no	equals()

^{*} under uniform hashing assumption

War story: algorithmic complexity attacks

- Q. Is the uniform hashing assumption important in practice?
- A. Obvious situations: aircraft control, nuclear reactor, pacemaker.
- A. Surprising situations: denial-of-service attacks.



malicious adversary learns your hash function
(e.g., by reading Java API) and causes a big pile-up
in single slot that grinds performance to a halt

Real-world exploits. [Crosby-Wallach 2003]

- Bro server: send carefully chosen packets to DOS the server, using less bandwidth than a dial-up modem.
- Perl 5.8.0: insert carefully chosen strings into associative array.
- Linux 2.4.20 kernel: save files with carefully chosen names.

Algorithmic complexity attack on Java

Goal. Find family of strings with the same hash code.

Solution. The base-31 hash code is part of Java's string API.

key	hashCode()			
"Aa"	2112			
"BB"	2112			

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Solution. The base-31 hash code is part of Java's string API.

key	hashCode()			
"Aa"	2112			
"BB"	2112			

key	hashCode()
"АаАаАаАа"	-540425984
"AaAaAaBB"	-540425984
"AaAaBBAa"	-540425984
"AaAaBBBB"	-540425984
"AaBBAaAa"	-540425984
"AaBBAaBB"	-540425984
"AaBBBBAa"	-540425984
"AaBBBBBB"	-540425984

key	hashCode()
"BBAaAaAa"	-540425984
"BBAaAaBB"	-540425984
"BBAaBBAa"	-540425984
"BBAaBBBB"	-540425984
"BBBBAaAa"	-540425984
"BBBBAaBB"	-540425984
"BBBBBBAa"	-540425984
"BBBBBBBB"	-540425984

2^N strings of length 2N that hash to same value!

Diversion: one-way hash functions

known to be insecure

One-way hash function. "Hard" to find a key that will hash to a desired value (or two keys that hash to same value).

Ex. MD4, MD5, SHA-0, SHA-1, SHA-2, WHIRLPOOL, RIPEMD-160,

```
String password = args[0];
MessageDigest sha1 = MessageDigest.getInstance("SHA1");
byte[] bytes = sha1.digest(password);

/* prints bytes as hex string */
```

Applications. Digital fingerprint, message digest, storing passwords. Caveat. Too expensive for use in ST implementations.

Separate chaining vs. linear probing

Separate chaining.

- Easier to implement delete.
- Performance degrades gracefully.
- Clustering less sensitive to poorly-designed hash function.

Linear probing.

- Less wasted space.
- Better cache performance.

Hashing: variations on the theme

Many improved versions have been studied.

Two-probe hashing. (separate-chaining variant)

- Hash to two positions, put key in shorter of the two chains.
- Reduces expected length of the longest chain to $\log \log N$.

Double hashing. (linear-probing variant)

- Use linear probing, but skip a variable amount, not just 1 each time.
- Effectively eliminates clustering.
- Can allow table to become nearly full.
- Difficult to implement delete.

Hashing vs. balanced search trees

Hashing.

- Simpler to code.
- No effective alternative for unordered keys.
- Faster for simple keys (a few arithmetic ops versus $\log N$ compares).
- Better system support in Java for strings (e.g., cached hash code).

Balanced search trees.

- Stronger performance guarantee.
- Support for ordered ST operations.
- Easier to implement compare To () correctly than equals () and hashcode ().

Java system includes both.

- Red-black trees: java.util.TreeMap, java.util.TreeSet.
- Hashing: java.util.HashMap, java.util.IdentityHashMap.

- sets
- dictionary clients
- indexing clients
- sparse vectors
- ▶ challenges

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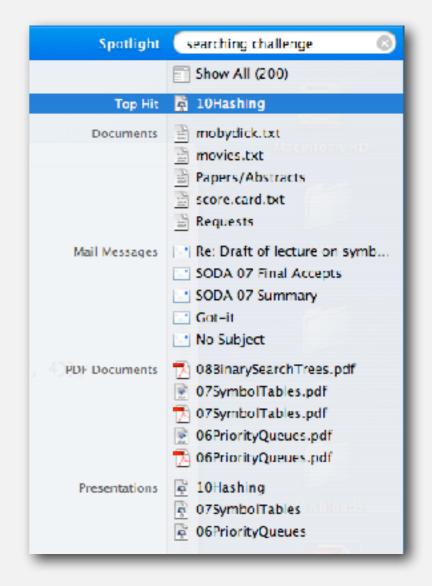
^{*} under uniform hashing assumption

Problem. Index for a PC or the web.

Assumptions. 1 billion++ words to index.

Which searching method to use?

- Hashing
- Red-black-trees
- Doesn't matter much.



Problem. Index for a PC or the web.

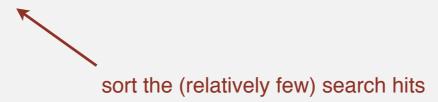
Assumptions. 1 billion++ words to index.

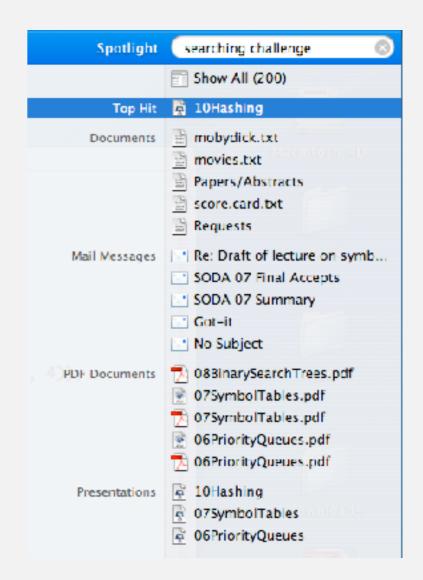
Which searching method to use?

- Hashing
- Red-black-trees ← too much space
- Doesn't matter much.

Solution. Symbol table with:

- Key = query string.
- Value = set of pointers to files.





Problem. Index for an e-book.

Assumptions. Book has 100,000+ words.

Which searching method to use?

- 1. Hashing
- 2. Red-black-tree
- 3. Doesn't matter much.

Index

Address, 34-85 Adjacency list, 120-123. Abstract data type (ADT), 127abetturat classes, 16.5 Ajan, M., 464 chases, 129-136 collections of sens, 137-158 creating, 157-154 defined, 128 Algorithm, 4-8, 27-64 analysis of, 6 duplicaminants, 173-176 equivelence relations, 159-162. mentee, 35, 60-52 FIFO queux, 165-171 first class, 177 186 Milery Search, 56-55 generic operations, 273 index imms, 177 essentiawosy operatione, 139efficiency, 6, 30, 32 modular programming, 135 polynomic, 188-192 exponential-time, 219 priority queena, 375-376 makingstown, 25-90 poshdown stack, 1 56-156 mulo, 135 symbol rahk, 497-806. primary naturety, 34 ALIT interfores probroilistic, EL erre; (nykerny), 274 rocumentes, 49-52, 57 complex number (Complex), 181. existence take 12TL 665 common, 195 renting time, 34-40. ful praint quest (72ful 1). search, 55-56, 498 indicat priority quere (PQL), steps in, 22-23 itan (ngl and, 273, 495 key (mottey), 458 polynomial (Poly), 188 Array, 12, 85 point (Point), 134 privately queue frqu. 375. bilings search, 57 quere of int hint locate, 166.

stack of ant (antStack), 140 and linked lists, 92, 94-95 embel tack (41), 208 post index (TC), 525 union-find (IF), 153 Abstract in place trenging, 351and arring, 119 Abstract operation, 10 Agons control state, Lill. Vectors, 57. Actual data, 31. Adamer class, 135-157 Adaptive con, 253 Many ros. 38 depth-franseurch, 251-256 Afficerry matrix, 120-122 Inked Ess, 110 acerum operations, 10, 31, 34. average forces-case perfecbig-Oh norange, 44-47 computational complexity, 62-601 hrz, 583 empirical analysis, 70-72, 5% logarithm touction, 40 47. mothematical analysis, 10-16, advallent, 701 Barra, 584 See also Randomized algorithm Americainet apprenat. \$17,427 Anthinetic operator, 177-179, 158, 191 595 554 md-Nack, 177-151 #iz lists, 597-594 spley, 566-571 denomic allocation, 37

manging, 349-350 mundmenderal, 117-118 wheness, 36-10, 39 onling, 265 267, 273 276 pyo-dimensional, 117-118, 120visualizations, 295 Surabo indes, army Array representation FLIU quere, 163-169 polynomial ADT, 191-132 prior ty queue, 177-378, 403. publicara stack, 148 150 random queue, 170 symbol table, 50%, 511-512, Asymptotic expression, 45-46. iverage deviation, 80-81 heстру-сых ренотапос, 33, 40-Bitries, 584, 892, 704 esternal internal pages, 695 4-5-6-7-8 etta, (0.5-704) Markov chain, 701 misora, 201-203 aurobinant, 897-701 Galanced mee, 233, 353-528 bottom rp. 576, 584-535 beighs-balancee, 383 indexed sequential access, 650personners, 525-576, 581-582. randomical, 559-564

727

Problem. Index for an e-book.

Assumptions. Book has 100,000+ words.

Which searching method to use?

- 1. Hashing
- ✓ 2. Red-black-tree ← need ordered iteration
 - 3. Doesn't matter much.

Solution. Symbol table with:

- Key = index term.
- Value = ordered set of pages on which term appears.

stack of ant (antStack), 140 and linked lists, 92, 94-95 symbol tack (\$1, 208) manging, 349-350 post index (TC), 525 multidimensional, 117-118 union-find (IF), 153 wferences, 86-97, 39 Abstract in place mening, 351anting, 265 267, 273 276 and arring, 119 Index Abstract operation, 10 pyo-dimensional, 117-118, 120-Access countrel gazes, 1.11 Vectors, 57. Actual data, 31. Adamer class, 185-197 visualizations, 295 Adaptive com, 258 Surabo links, army Address, 34-85 Array representation Adjacency list, 120-123. binary mos. 38 Abstract data type (ADT), 127depth-franseurch, 251-256 FifU quere, 163-169 Adjusticy matrix, 120-T22 Inked Ess, 110 abetturat classes, 16.5 polynomial ADT, 191-132 Ajan, M., 464 chases, 129-136 Algorithm, 4-8, 27-64 prior ty queue, 177-378, 403. collections of terms, 137-158 creating, 157-154 defined, 128 acerum operacione, 10, 31, 34. publicova stack, 148 150 analysis of, 6 random queue, 170 duplicate items, 173-176 symbol table, 508, 511-512, average forces-case perfecequivelence relations, 159-162. mentee, 35, 60-52 FIFO guenes, 165-171 Asymptotic expression, 45-46. big-Oh norange, 44-47 first class, 177 186 iverage deviation, 80-81 Miles send, 56 55 generic operations, 273 heстру-сых ренотапос, 33, 40computational complexity, 62index imms, 177 essentiawous operations, 139-601 hrz, 583 efficience, 6, 30, 32 empirical analysis, 70-32, 55 modular programming, 135 polynomic, 188-192 experiential-time, 21.9 Bitzer, 584, 692, 704 priority queens, 375-376 makingstown, 25-90 estimatimental pages, 695 poshdown stack, 1 56-156 logarithm touction, 40 47. 4-5-6-7-8 etta, (0.5-704) mothematical analysis, 10-16, Markov chain, 701 mulo, 135 symbol rahk, 497-806. tersons, 201-706 primary naturety, 34 aurobinant, 897-701 AUT interfores erie; (nykurny), 274 probroilistic, EL advallent, 701 recumences, 49-52, 57 Galanced mee, 233, 353-528 complex number (Complex), 181. existence take 1871, 665 examine, 195 B prez, 584 renting time, 34-40 bottom rp. 576, 584-535 ful praint quest (72ful 1). search, 55-56, 498 beigh-balanced, 383 indicat priority quere (PQ:1, steps in, 22-23 indexed sequential access, 650-See also Randomized algorithm item (ngl and, 273, 495 Americanien apprenal. \$17,427 personners, 525-576, 581-582. Anthinetic operator, 177-179, 158, 191 key (myttey), 452 595 554 randomical, 559-564 polynomial (Poly), 188 point (Point), Lie-Array, 12, 35 md-Nack, 177-151 bilings search, 57 Air lists, 597-554 priority queue HQL 375. spley, 566-571 quere of int tint locnet, 166. denomic allocation, 37 727