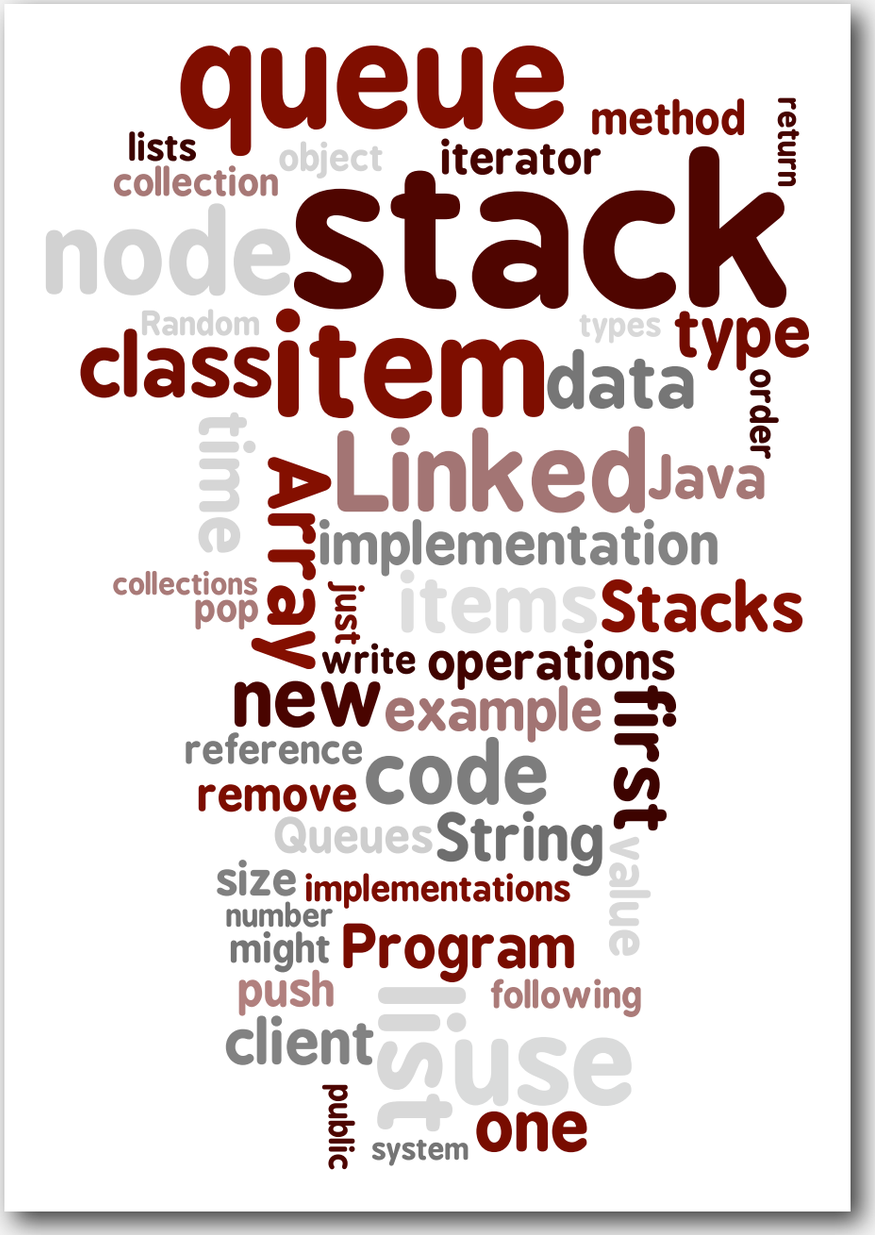
1.3 Stacks and Queues



- review: lists, linked lists
- stacks
- dynamic resizing
- queues
- doubly-linked lists
- generics
- iterators
- applications

Review: Arrays

- In Java, an array is an indexed collection of data values of the same type.
  - Array declaration and creation

```
<data type> [ ] <variable>
<variable> = new <data type> [ <size> ]
```

  - Example:

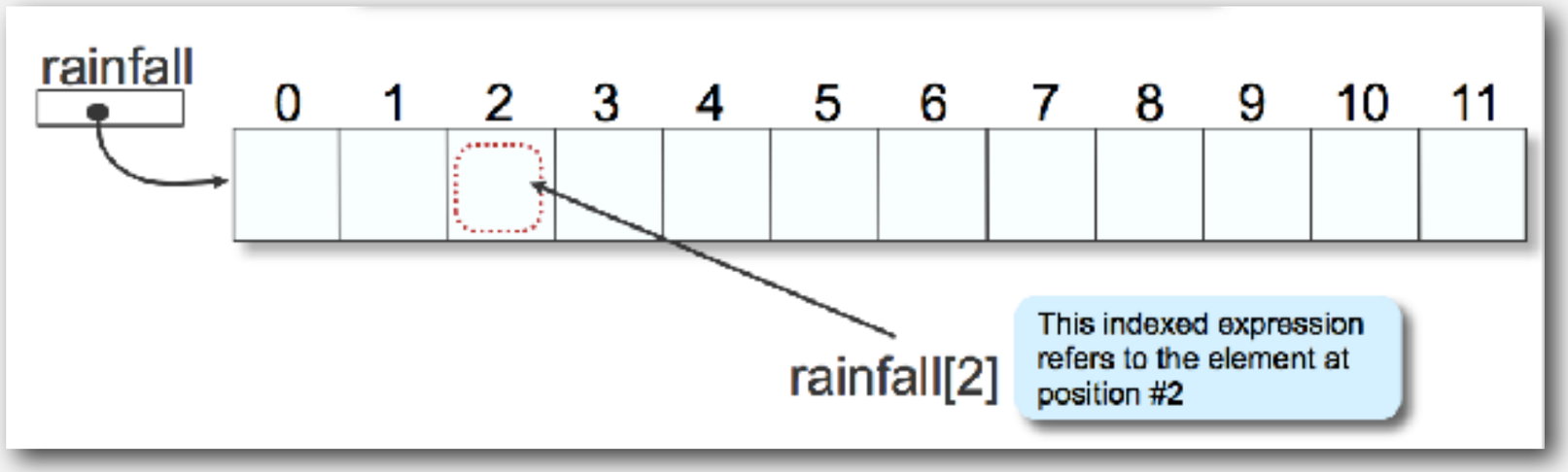
```
double[ ] rainfall;
rainfall = new double[12];
```

An array is like an object

  - Like other data types, it is possible to declare and initialize an array at the same time:

```
int[] number = { 2, 4, 6, 8 };
```

- Individual elements in an array are accessed with an indexed expression.



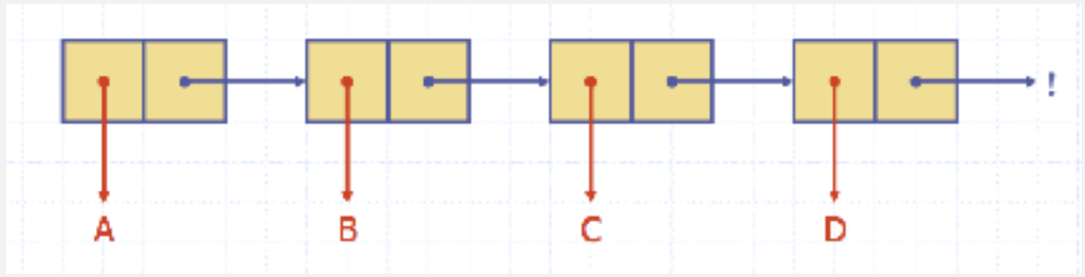
Review: Arrays

- Index out of bounds
  - The index for an array a, must evaluate to a value between 0 and a.length-1.
  - If it does not, then an "ArrayIndexOutOfBoundsException" is thrown
  - This run time exception does not need to be caught -- but will cause the program to terminate if not caught.

- Arrays of Objects
  - In Java, in addition to arrays of primitive data types, we can declare arrays of objects
  - The use of an array of objects allows us to model the application more cleanly and logically.

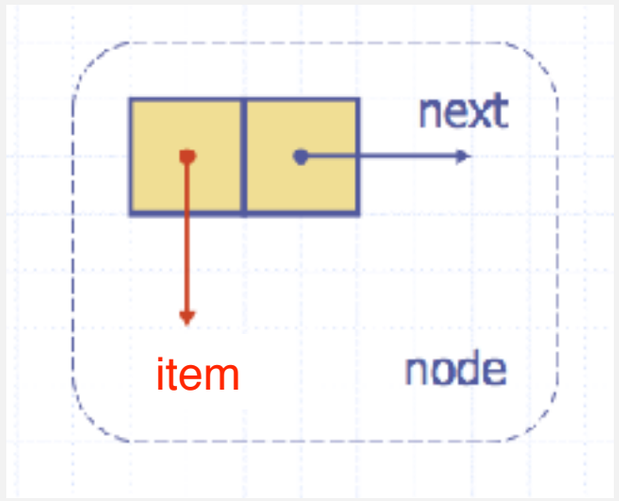
Review: Linked lists

·A singly linked list is a data structure consisting of a sequence of nodes



·Each node stores:

- Item
- Link to the next node



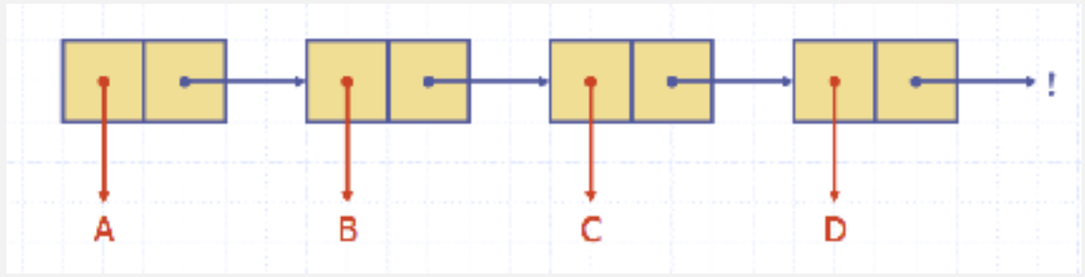

```
public class Node  {
private Object item;      // Instance variable
private Node next;        // Instance variable
/** Creates a node with null references. */
public Node(){
this(null, null);
}
/** Creates a node with specified contents. */
public Node(Object e, Node n){
item = e;
next = n;
}

// Accessor methods:
public Object getItem() {  return item; }
public Node getNext() {  return next;  }

// Modifier methods:
public void setItem(Object newItem){
item = newItem;
}
public void setNext(Node newNext){
next = newNext;
}
```

Review: Linked lists

·A singly linked list is a data structure consisting of a sequence of nodes



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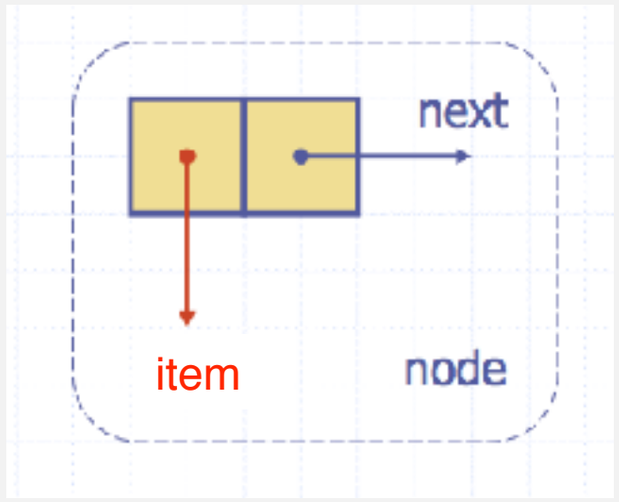


```
public class Node  {
private Object item;      // Instance variable
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/** Creates a node with null references. */
public Node(){
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item = e;
next = n;
}

// Accessor methods:
public Object getItem() {  return item; }
public Node getNext() {  return next;  }

// Modifier methods:
public void setItem(Object newItem){
item = newItem;
}
public void setNext(Node newNext){
next = newNext;
}
```

Review: Linked list

- Creating a linked list
  - Need a reference variable, **`first`**, that identifies the first node in the list

- Traversing a linked list
  - Once you are at the first node, you can use **`node.getNext()`** to get to the next node
  - Scan a linked list by assigning a variable **`curr`** to the value of **`first`**, then use the **`node.getNext()`** method of each node to proceed down the list
  - Conclude when **`curr == null`**

- Other details
  - If a linked list is empty, then **`first`** value is **`null`**
  - Inserting or deleting an element at the front of the list is easy, because the list maintains a reference that points to the first element

▸ stacks & queues

Stacks and queues

·Fundamental data types.

- Values: sets of objects.
- Operations: insert, remove, test if empty.
- Intent is clear when we insert.
- Which item do we remove?

LIFO = "last in first out"

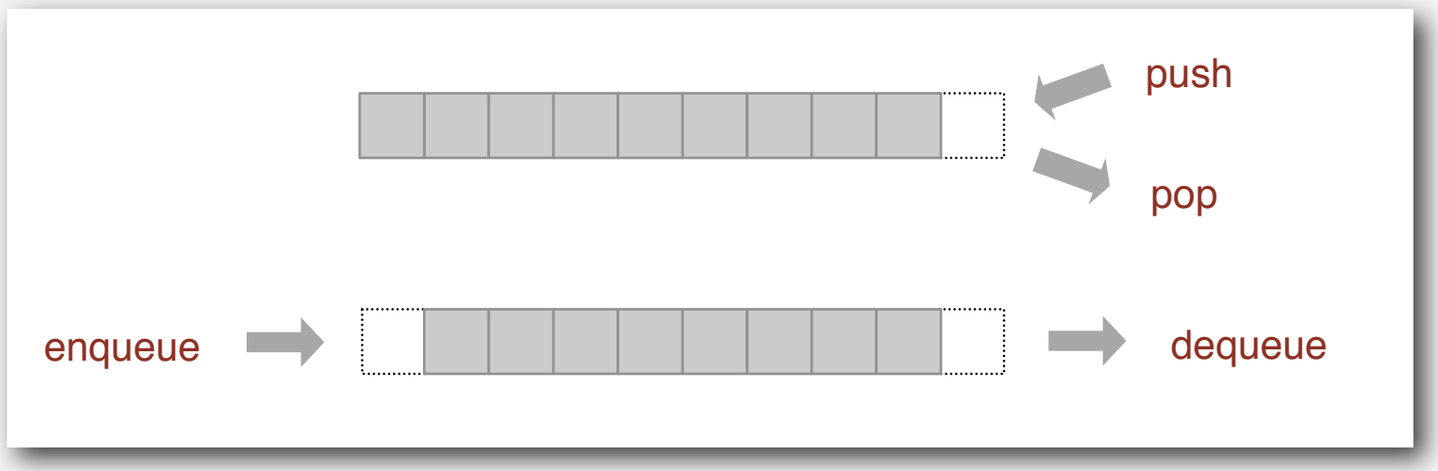
·Stack. Remove the item most recently added.

·Analogy. Cafeteria trays, Web surfing.

FIFO = "first in first out"

·Queue. Remove the item least recently added.

·Analogy. Registrar's line.



Client, implementation, interface

·Separate interface and implementation.

·Ex: stack, queue, priority queue, symbol table, union-find, ...

·Benefits.

- Client can't know details of implementation ⇒
  can easily substitute new and improved implementation.
- Implementation can't know details of client needs ⇒
  many clients can re-use the same implementation.
- Design: creates modular, reusable libraries.
- Performance: use optimized implementation where it matters.

Client: refers to a program that calls a method in another library.

Implementation: refers to the Java code that implements the methods in an API.

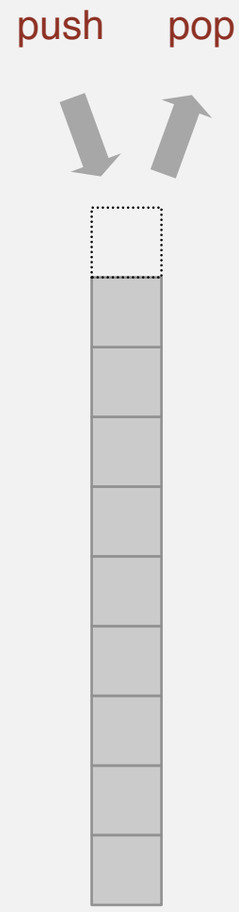
Interface: refers to a list of methods with names, signatures, basic descriptions.

- **stacks**
- **dynamic resizing**
- **queues**
- **generics**
- **iterators**
- **applications**

Stack API

·Warmup. Stack of strings objects.

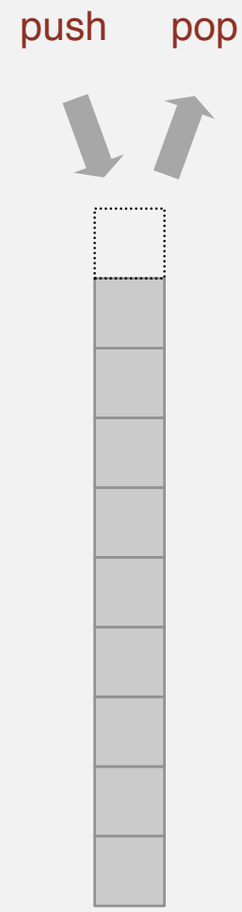
public class StackOfStrings	
StackOfStrings()	*create an empty stack*
void push(String s)	*insert a new item onto stack*
String pop()	*remove and return the item most recently added*
boolean isEmpty()	*is the stack empty?*
int size()	*number of items on the stack*



·Challenge. Reverse sequence of strings from standard input.

Stack test client

```
public static void main(String[] args)
{
   StackOfStrings stack = new StackOfStrings();
   while (!StdIn.isEmpty())
   {
      String item = StdIn.readString();
      if (item.equals("-")) StdOut.print(stack.pop());
      else                  stack.push(item);
   }
}
```




```
% more tobe.txt
to be or not to - be - - that - - - is

% java StackOfStrings < tobe.txt
to be not that or be
```

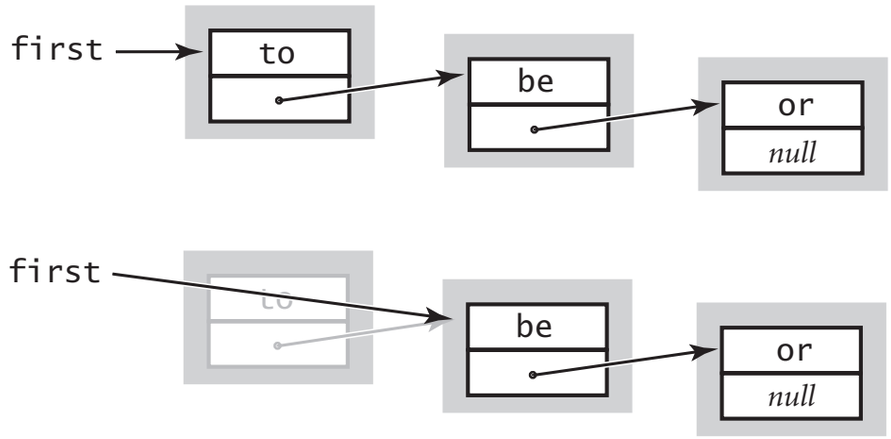
Stack pop: linked-list implementation

save item to return

```
String item = first.item;
```

change top-of-stack pointer

```
first = first.next;
```



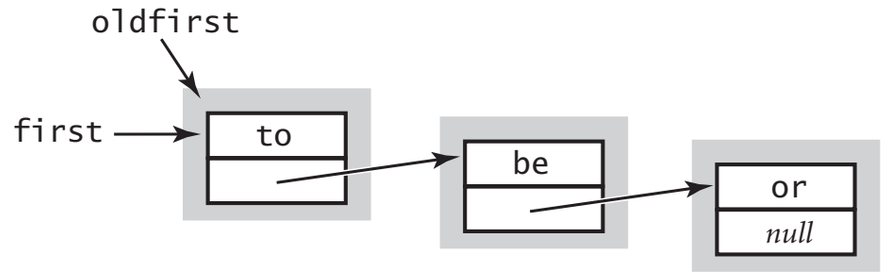
return saved item

```
return item;
```

Stack push: linked-list implementation

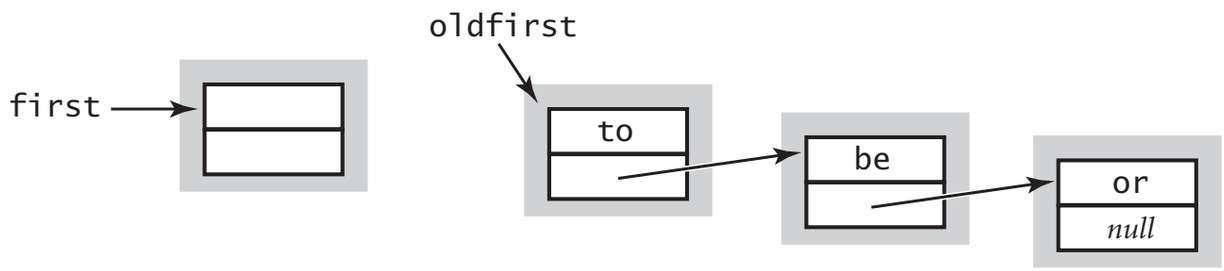
save a link to the list

```
Node oldfirst = first;
```



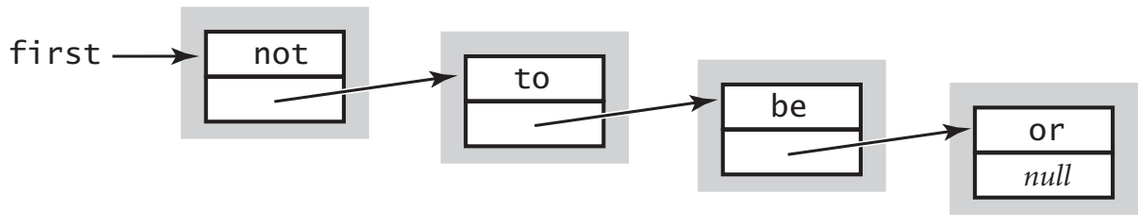
create a new node for the beginning

```
first = new Node();
```



set the instance variables in the new node

```
first.item = "not";
first.next = oldfirst;
```



Stack: linked-list implementation in Java

```
public class StackOfStrings
{
   private Node first = null;

   private class Node
   {
      String item;
      Node next;
   }

   public boolean isEmpty()
   {  return first == null;  }

   public void push(String item)
   {
      Node oldfirst = first;
      first = new Node();
      first.item = item;
      first.next = oldfirst;
   }

   public String pop()
   {
      if (isEmpty()) throw new RuntimeException();
      String item = first.item;
      first = first.next;
      return item;
   }
}
```

inner class

stack underflow

Stack: linked-list implementation performance

·Proposition. Using a linked-list implementation of a stack, every operation takes constant time in the worst case.

·Proposition. Uses ~ $16\,N$ bytes to represent a stack with N items.

```
private class Node
{
   String item;
   Node next;
}
```

8 bytes (object overhead)

4 bytes (reference to String)

4 bytes (reference to Node)

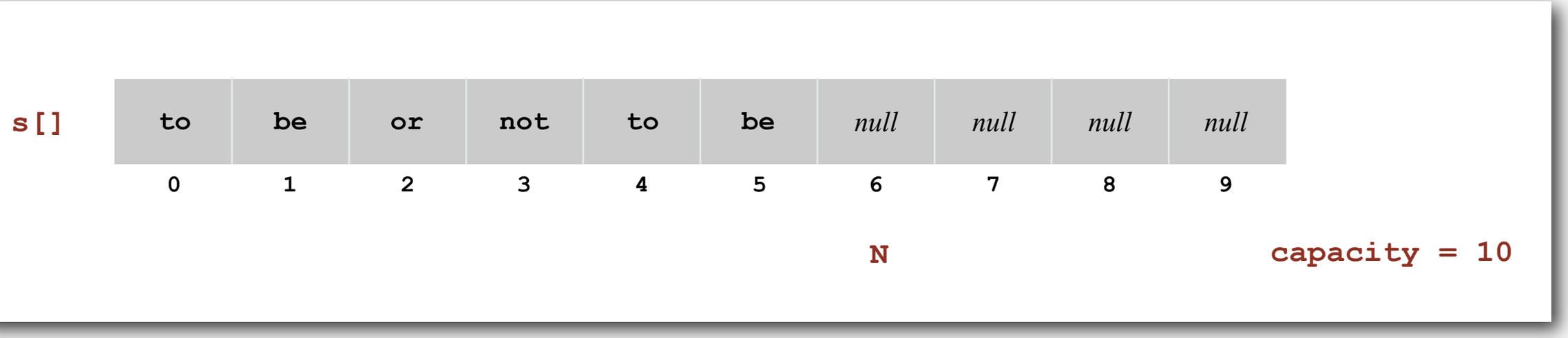
16 bytes per stack item

·Remark. Analysis includes memory for the stack (but not the strings themselves, which the client owns).

Stack: array implementation

·Array implementation of a stack.

- Use array `s[]` to store `N` items on stack.
- `push()`: add new item at `s[N]`.
- `pop()`: remove item from `s[N-1]`.



·Defect. Stack overflows when `N` exceeds capacity. [stay tuned]

Stack: array implementation

```
public class StackOfStrings
{
   private String[] s;
   private int N = 0;

   public StackOfStrings(int capacity)
   {  s = new String[capacity];  }

   public boolean isEmpty()
   {  return N == 0;  }

   public void push(String item)
   {  s[N++] = item;  }

   public String pop()
   {  return s[--N];  }
}
```

a cheat (stay tuned)

decrement N;

then use to index into array

Stack: array implementation

```
public class StackOfStrings
{
   private String[] s;
   private int N = 0;

   public StackOfStrings(int capacity)
   {  s = new String[capacity];  }

   public boolean isEmpty()
   {  return N == 0;  }

   public void push(String item)
   {  s[N++] = item;  }

   public String pop()
   {  return s[--N];  }
}
```

a cheat (stay tuned)

decrement N;

then use to index into array

```
public String pop()
{
   String item = s[--N];
   s[N] = null;
   return item;
}
```

this version avoids "loitering":

garbage collector reclaims memory

only if no outstanding references

- stacks
- **dynamic resizing**
- queues
- generics
- iterators
- applications

Stack: dynamic-array implementation

- Problem. Requiring client to provide capacity does not implement API!
- Q. How to grow and shrink array?

- First try.
  - `push()`: increase size of `s[]` by 1.
  - `pop()`: decrease size of `s[]` by 1.

- Too expensive.
  - Need to copy all item to a new array.
  - Inserting first N items takes time proportional to $1 + 2 + \ldots + N \sim N^2 / 2$.

infeasible for large N

- Challenge. Ensure that array resizing happens infrequently.

Stack: dynamic-array implementation

"repeated doubling"

- Q. How to grow array?
- A. If array is full, create a new array of twice the size, and copy items.

```
public StackOfStrings() {  s = new String[1];  }

public void push(String item)
{
   if (N == s.length) resize(2 * s.length);
   s[N++] = item;
}

private void resize(int capacity)
{
   String[] copy = new String[capacity];
   for (int i = 0; i < N; i++)
      copy[i] = s[i];
   s = copy;
}
```

cost of array resizing is now

$2 + 4 + 8 + \ldots + N \sim 2N$

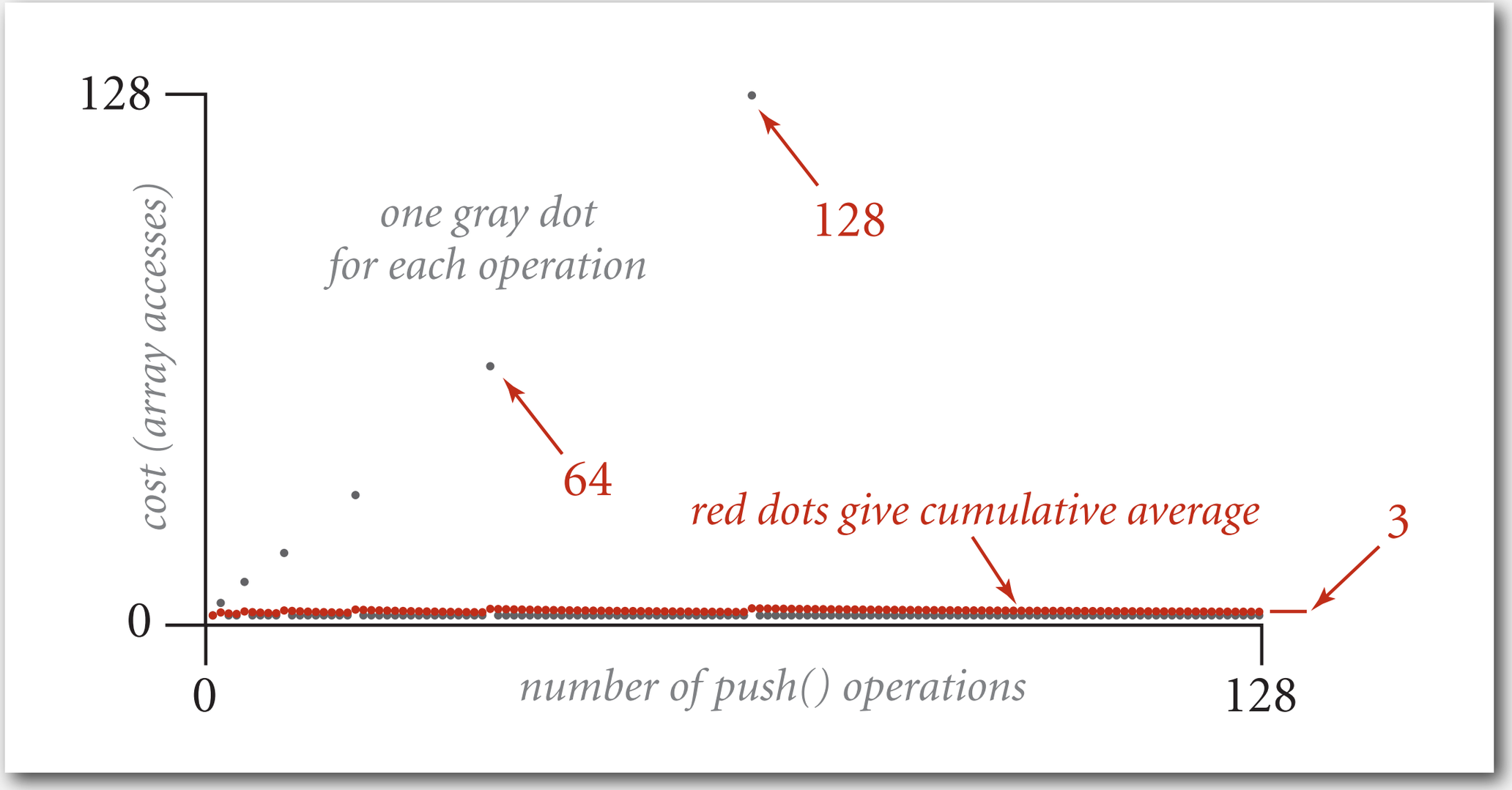
- Consequence. Inserting first N items takes time proportional to N (not N^2).

Stack: amortized cost of adding to a stack

- Cost of inserting first N items. $N + (2 + 4 + 8 + \ldots + N) \sim 3N.$

1 array accesses per push

k array accesses to double to size k



Stack: dynamic-array implementation

·Q. How to shrink array?

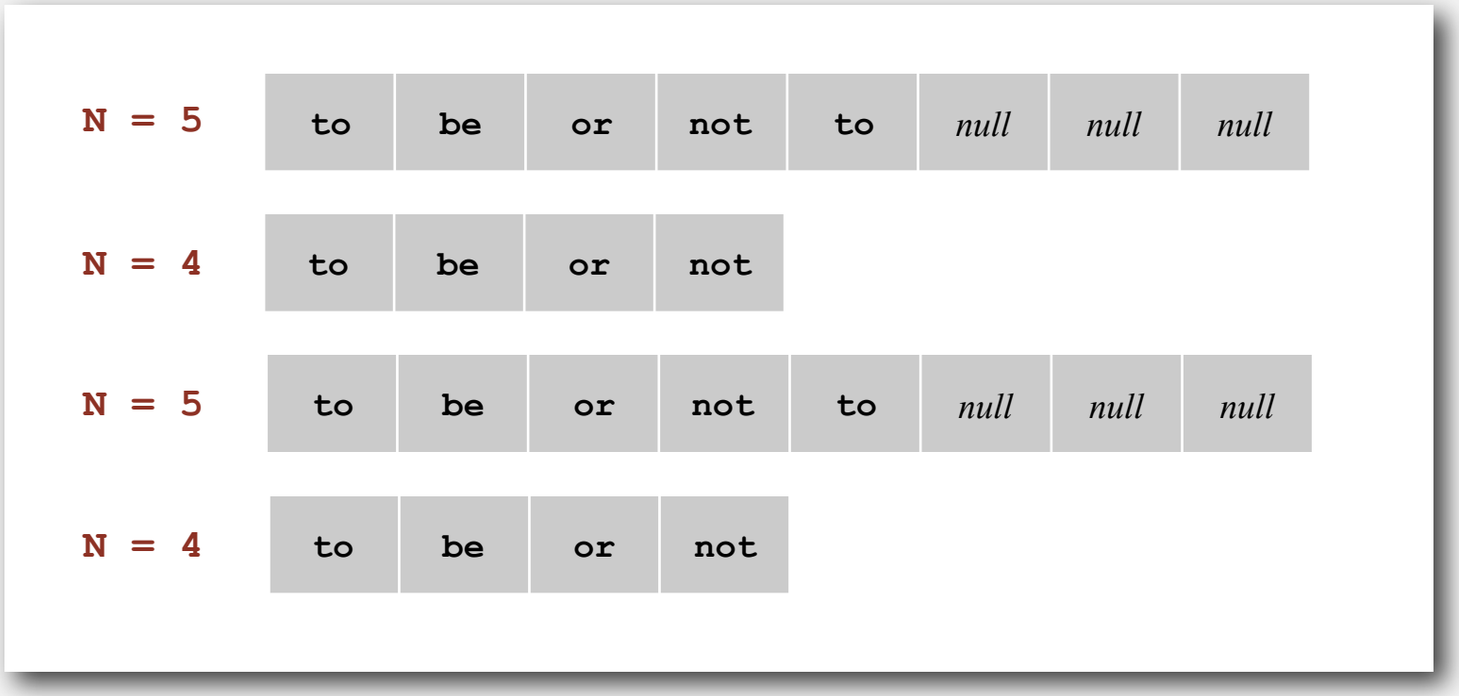
·First try.

- `push()`: double size of `s[]` when array is full.
- `pop()`: halve size of `s[]` when array is one-half full.

·Too expensive.

"thrashing"

- Consider push-pop-push-pop-... sequence when array is full.
- Takes time proportional to N per operation in worst case.



Stack: dynamic-array implementation

- Q. How to shrink array?

- Efficient solution.
  - `push()`: double size of `s[]` when array is full.
  - `pop()`: halve size of `s[]` when array is one-quarter full.

```
public String pop()
{
   String item = s[--N];
   s[N] = null;
   if (N > 0 && N == s.length/4) resize(s.length / 2);
   return item;
}
```

- Invariant. Array is between 25% and 100% full.

Stack: dynamic-array implementation trace

StdIn	StdOut	N	a.length	a							
				0	1	2	3	4	5	6	7
		0	1	*null*							
to		1	1	to							
be		2	2	to	be						
or		3	4	to	be	or	*null*				
not		4	4	to	be	or	not				
to		5	8	to	be	or	not	to	*null*	*null*	*null*
-	to	4	8	to	be	or	not	*null*	*null*	*null*	*null*
be		5	8	to	be	or	not	be	*null*	*null*	*null*
-	be	4	8	to	be	or	not	*null*	*null*	*null*	*null*
-	not	3	8	to	be	or	*null*	*null*	*null*	*null*	*null*
that		4	8	to	be	or	that	*null*	*null*	*null*	*null*
-	that	3	8	to	be	or	*null*	*null*	*null*	*null*	*null*
-	or	2	4	to	be	*null*	*null*				
-	be	1	2	to	*null*						
is		2	2	to	is						

Stack dynamic-array implementation: performance

·Amortized analysis. Average running time per operation over a worst-case sequence of operations. [stay tuned]

·Proposition. Starting from empty stack (with dynamic resizing), any sequence of M push and pop operations takes time proportional to M.

	best	worst	amortized
construct	1	1	1
push	1	N	1
pop	1	N	1
size	1	1	1

doubling and shrinking

running time for doubling stack with N items

Stack dynamic array implementation: memory usage

Proposition. Uses between ~ $4N$ and ~ $16N$ bytes to represent a stack with N items.

- ~ $4N$ when full.
- ~ $16N$ when one-quarter full.

```
public class DoublingStackOfStrings
{
   private String[] s;
   private int N = 0;
   …
}
```

4 bytes × array size

4 bytes

Remark. Analysis includes memory for the stack (but not the strings themselves, which the client owns).

Stack implementations: dynamic array vs. linked List

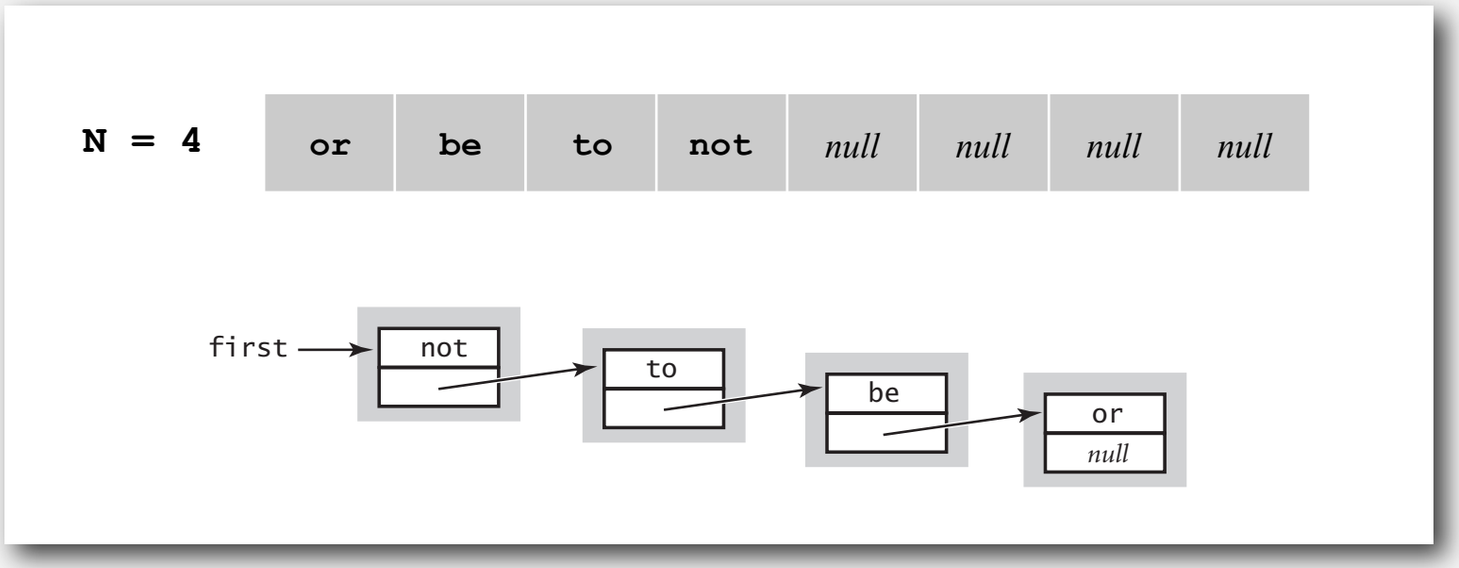
·Tradeoffs. Can implement a stack with either dynamic array or linked list; client can use interchangeably. Which one is better?

·Linked-list implementation.

- Every operation takes constant time in the worst case.
- Uses extra time and space to deal with the links.

·Dynamic-array implementation.

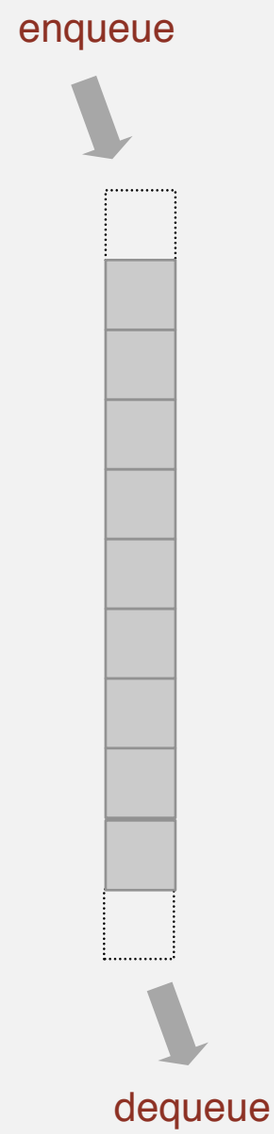
- Every operation takes constant amortized time.
- Less wasted space.



- stacks
- dynamic resizing
- **queues**
- generics
- iterators
- applications

Queue API

public class QueueOfStrings	
QueueOfStrings()	*create an empty queue*
void enqueue(String s)	*insert a new item onto queue*
String dequeue()	*remove and return the item least recently added*
boolean isEmpty()	*is the queue empty?*
int size()	*number of items on the queue*



Queue test client

```
public static void main(String[] args)
{
   QueueOfStrings q = new QueueOfStrings();
   while (!StdIn.isEmpty())
   {
      String item = StdIn.readString();
      if (item.equals("-")) StdOut.print(q.dequeue());
      else                  q.enqueue(item);
   }
}
```

```
% more tobe.txt
to be or not to - be - - that - - - is

% java QueueOfStrings < tobe.txt
to be or not to be
```

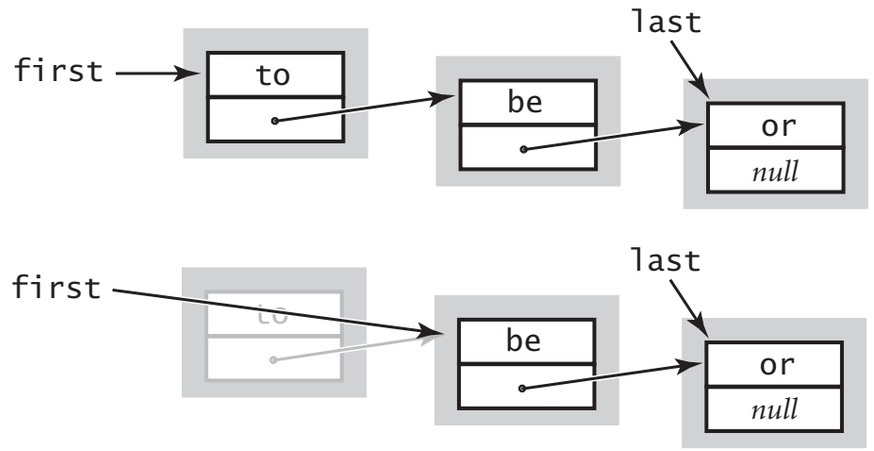
Queue dequeue: linked-list implementation

save item to return

```
String item = first.item;
```

change head-of-queue pointer

```
first = first.next;
```



return saved item

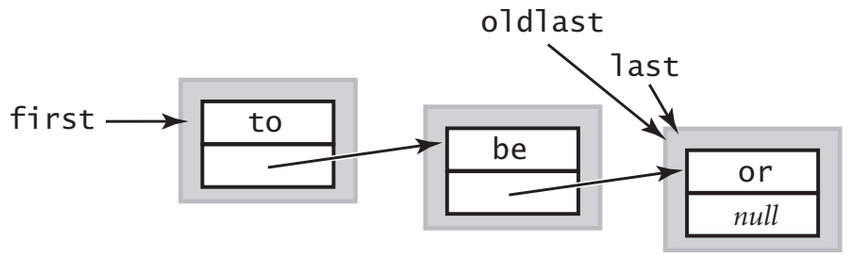
```
return item;
```

- Remark. Identical code to linked-list stack `pop()`.

Queue enqueue: linked-list implementation

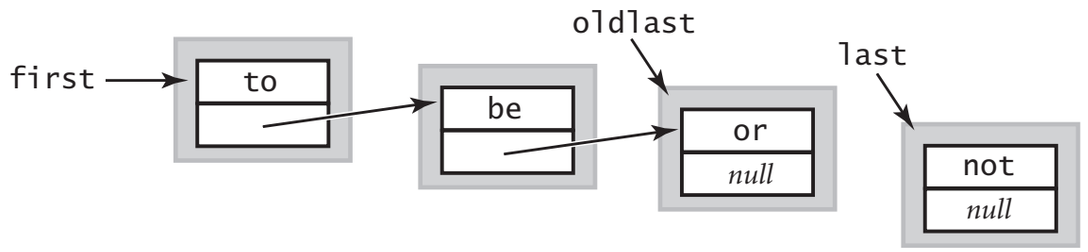
save a link to the last node

```
Node oldlast = last;
```



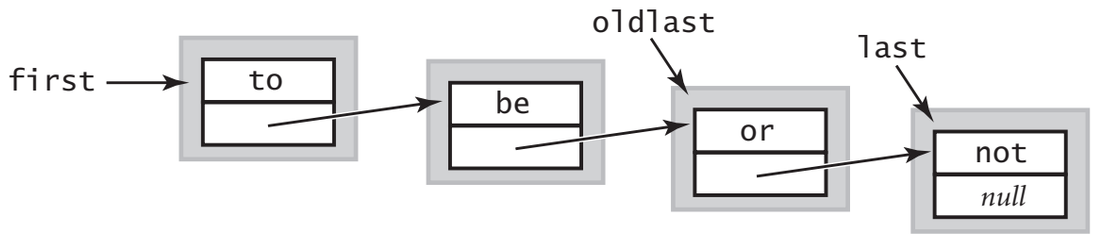
create a new node for the end

```
Node last = new Node();
last.item = "not";
last.next = null;
```



link the new node to the end of the list

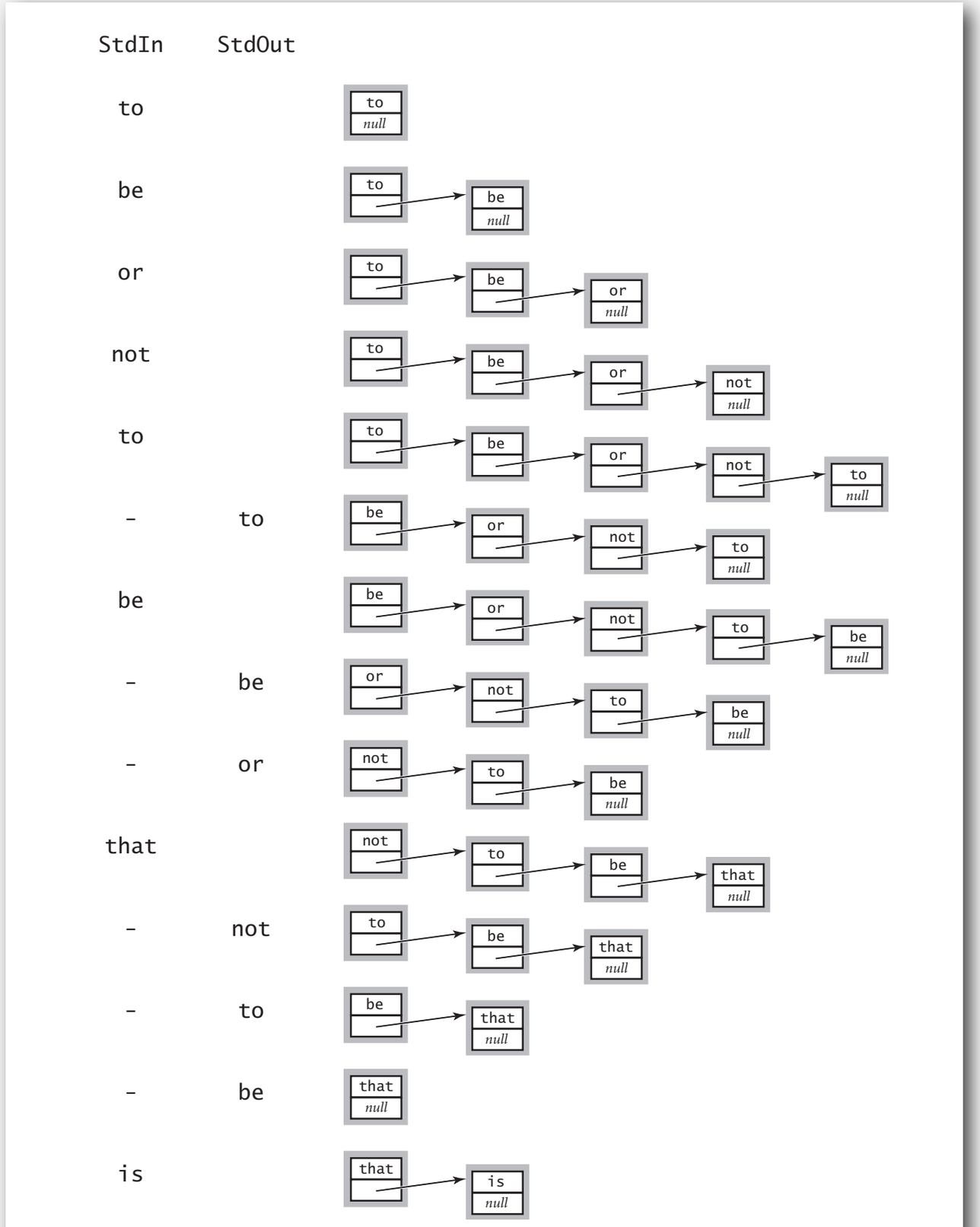
```
oldlast.next = last;
```



Queue: linked-list implementation in Java

```
public class QueueOfStrings
{
   private Node first, last;

   private class Node
   {  /* same as in StackOfStrings */  }

   public boolean isEmpty()
   {  return first == null;  }

   public void enqueue(String item)
   {
      Node oldlast = last;
      last = new Node();
      last.item = item;
      last.next = null;
      if (isEmpty()) first = last;
      else           oldlast.next = last;
   }

   public String dequeue()
   {
      String item = first.item;
      first       = first.next;
      if (isEmpty()) last = null;
      return item;
   }
}
```

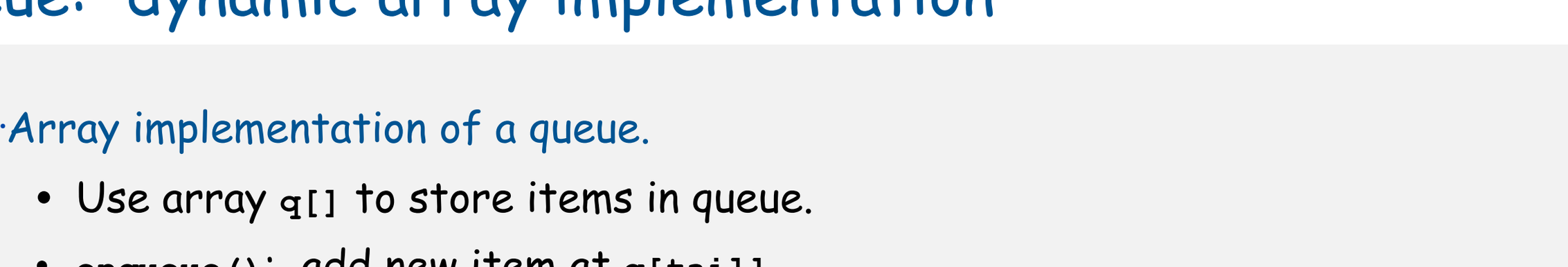
Queue: linked-list trace



Queue: dynamic array implementation

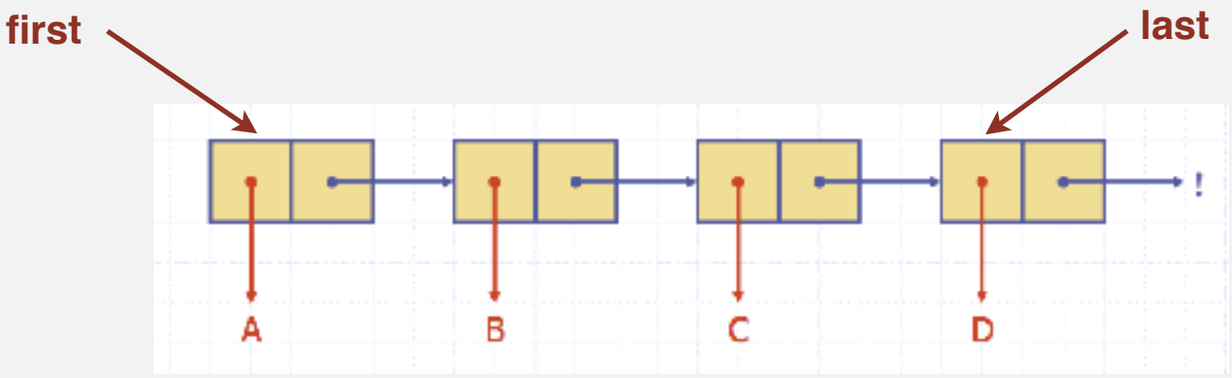
·Array implementation of a queue.

- Use array `q[]` to store items in queue.
- `enqueue()`: add new item at `q[tail]`.
- `dequeue()`: remove item from `q[head]`.
- Update `head` and `tail` modulo the `capacity`.
- Add dynamic resizing.



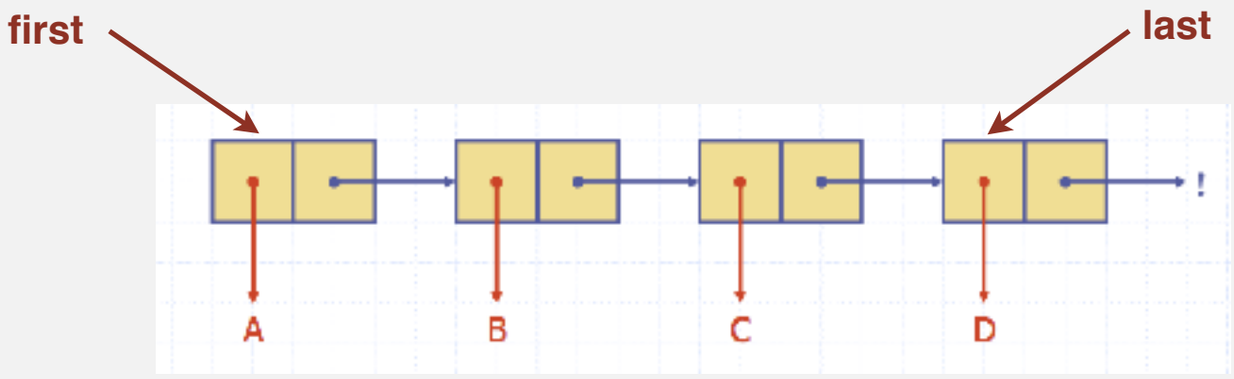
doubly-linked lists

More on singly-linked lists



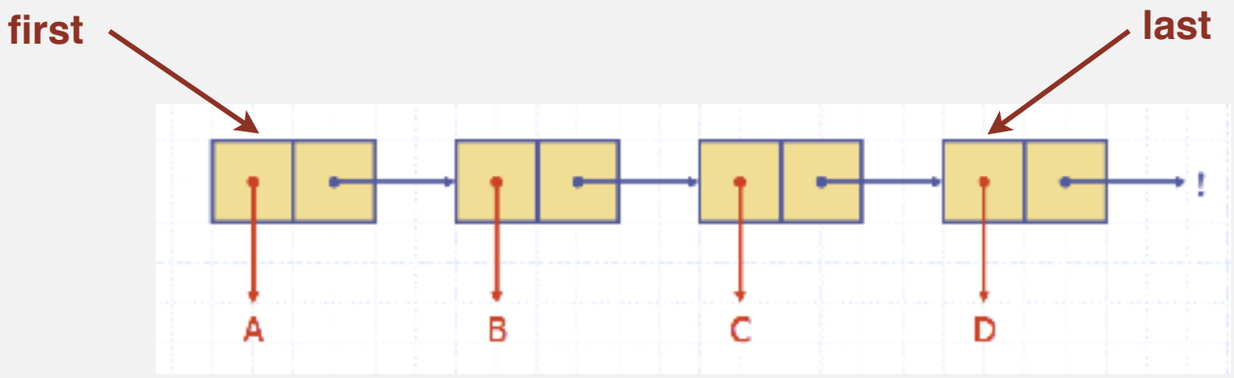
More on singly-linked lists

- Note that the queue implementation needed a pointer to the end of the list (last) as well as the beginning of the list (first)
  - It is easy to maintain the pointers to both ends of the list if all we do is remove from the front and insert at the end



More on singly-linked lists

- Note that the queue implementation needed a pointer to the end of the list (last) as well as the beginning of the list (first)
  - It is easy to maintain the pointers to both ends of the list if all we do is remove from the front and insert at the end



- But what if we want to delete at the end of the list?

More on singly-linked lists

- Note that the queue implementation needed a pointer to the end of the list (last) as well as the beginning of the list (first)
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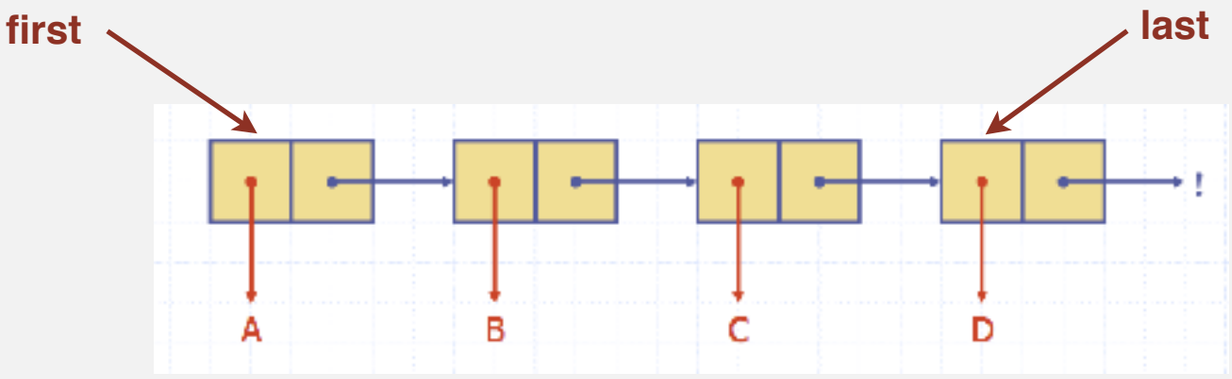
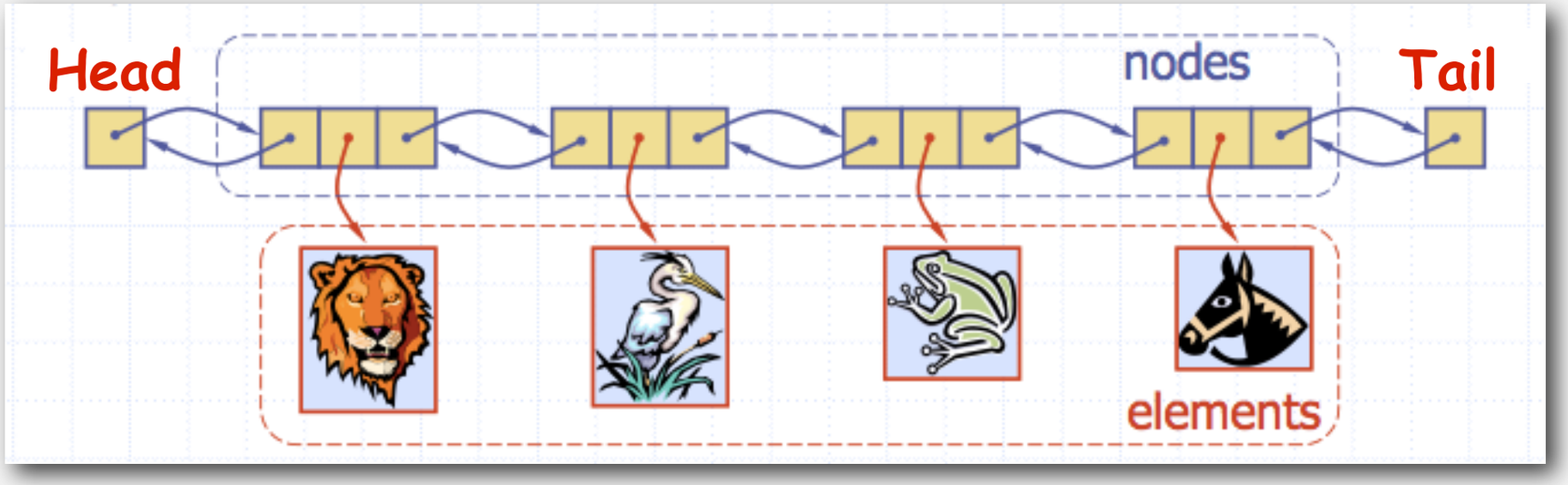


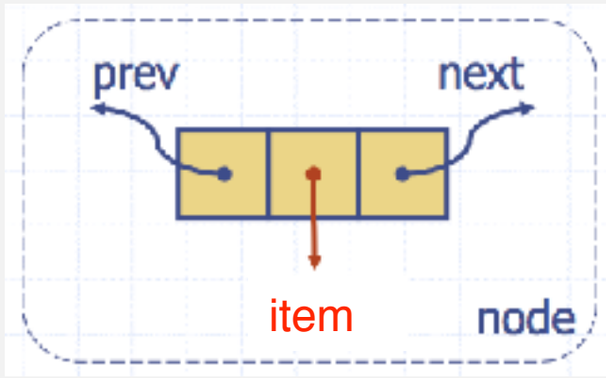
- But what if we want to delete at the end of the list?
  - There is no constant time way to update the tail to point to the previous!

Doubly-linked lists

- A doubly linked list is a data structure consisting of a sequence of nodes
  - Special head and tail nodes

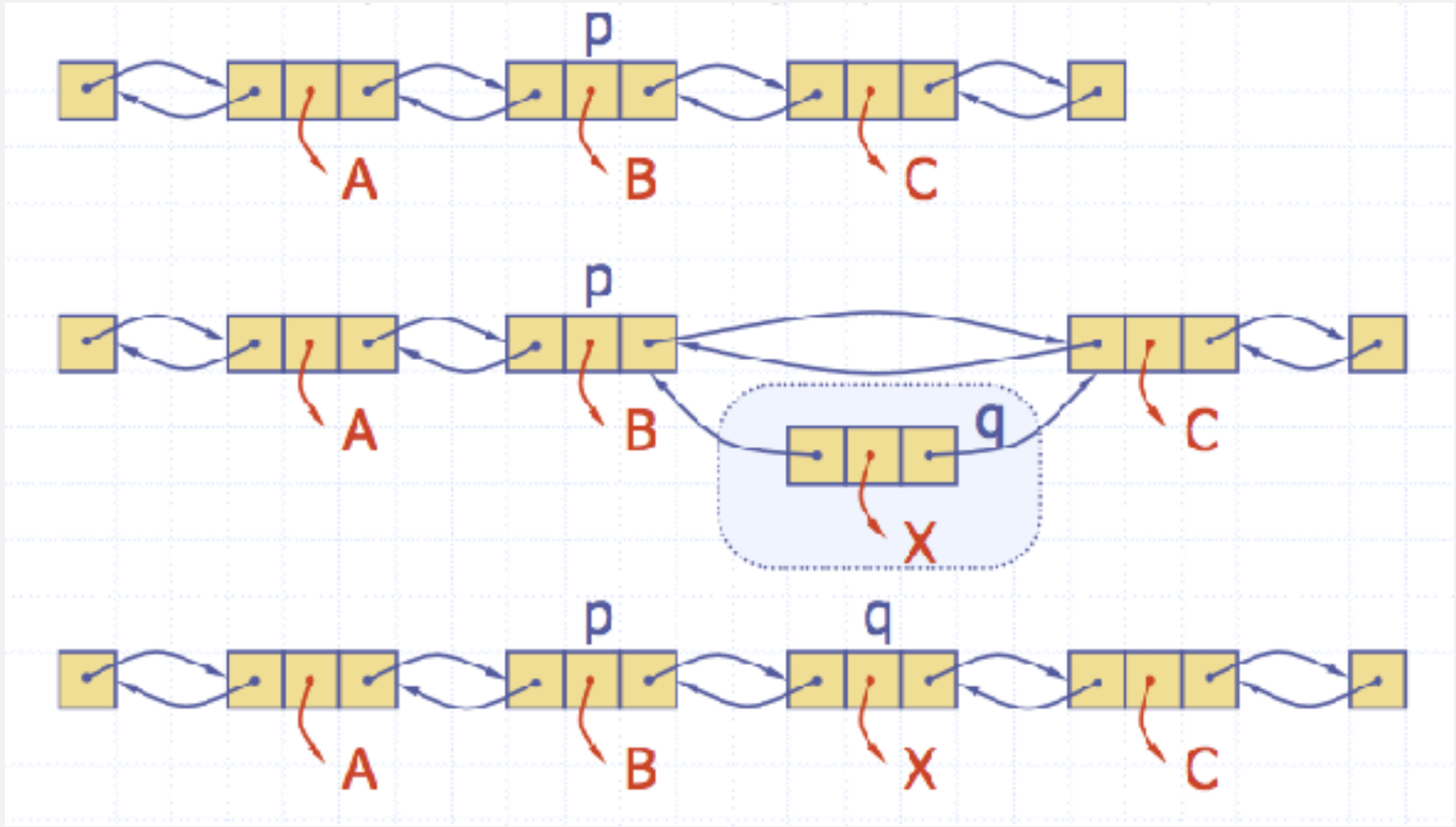


- Each node stores:
  - Item
  - Link to the next node
  - Link to the previous node



Insertion

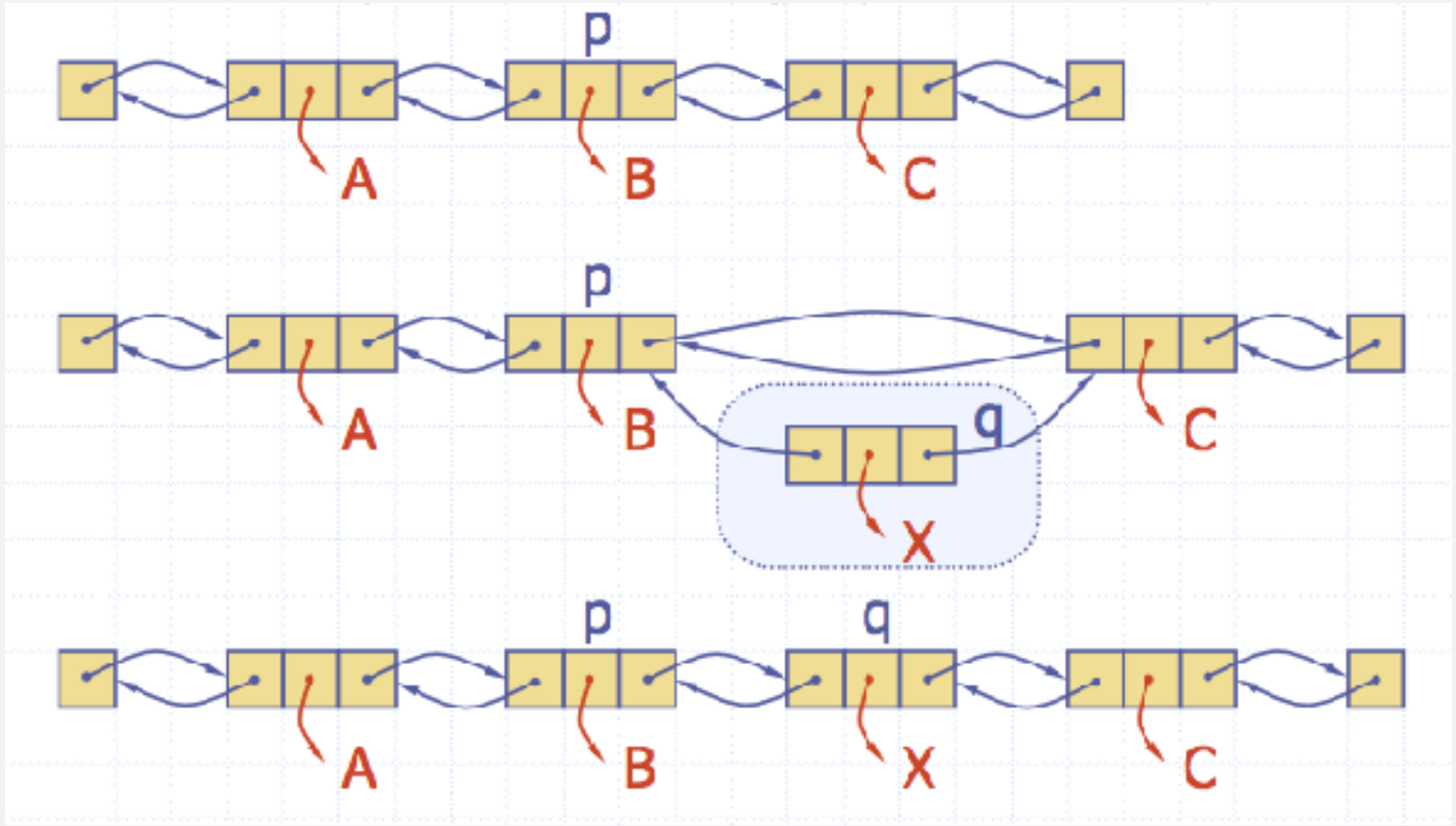
- Below we visualize the operation **insertAfter(p,X)**




```
public static void insertAfter(p,x):
  Node v = new Node()      //Create a new node v
  v.setItem(x)
  v.setPrev(p)             //link v to its predecessor
  v.setNext(p.getNext())   //link v to its successor
 (p.getNext()).setPrev(v) //link p's old successor to v
  p.setNext(v)            //link p to its new successor
```

Insertion

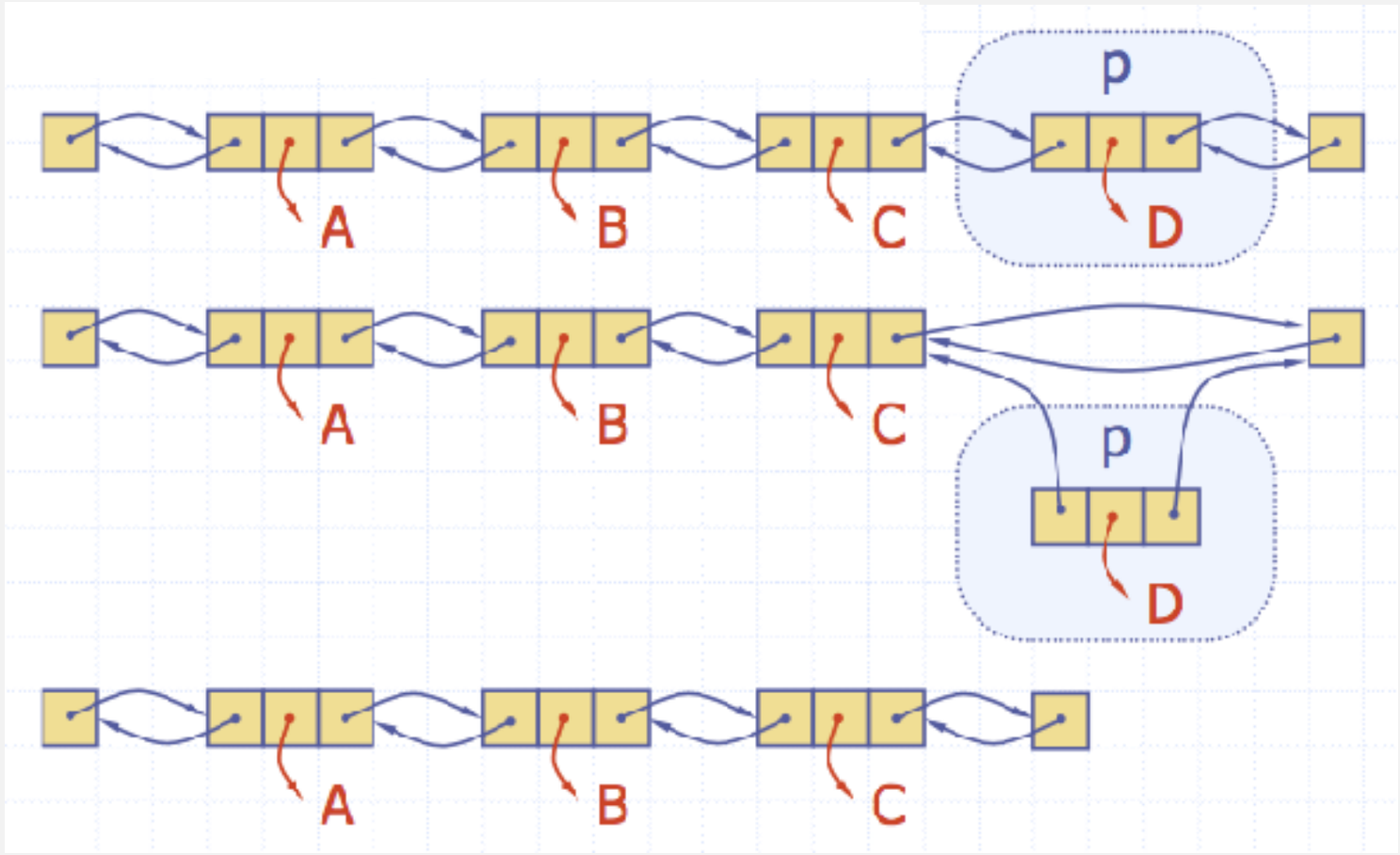
- Below we visualize the operation **insertAfter(p,X)**




```
public static void insertAfter(p,x):
  Node v = new Node()      //Create a new node v
  v.setItem(x)
  v.setPrev(p)             //link v to its predecessor
  v.setNext(p.getNext())   //link v to its successor
 (p.getNext()).setPrev(v) //link p's old successor to v
  p.setNext(v)            //link p to its new successor
```

Deletion

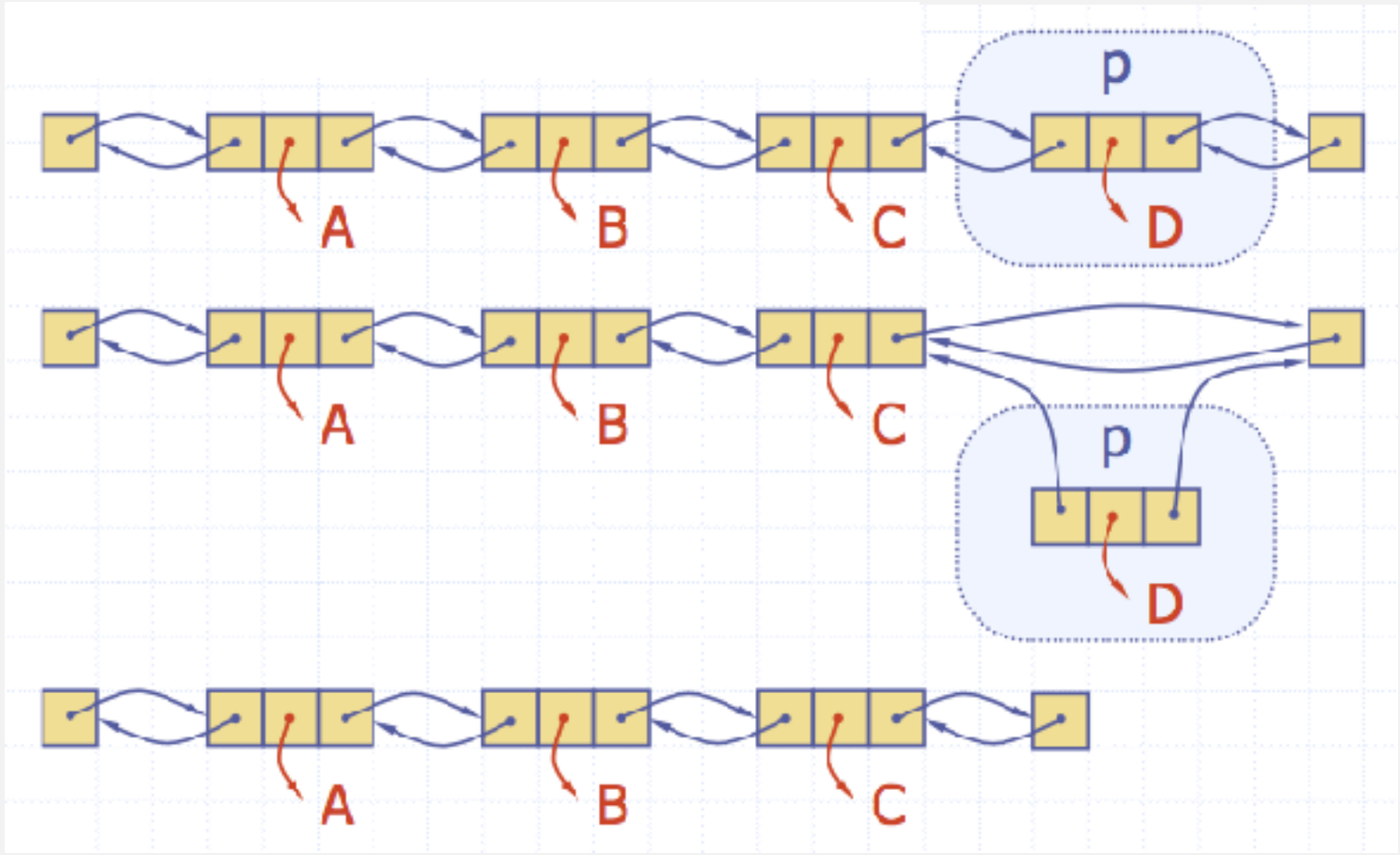
- Below we visualize the operation **remove(p)**, where p = last()




```
public static Object remove(p):
  t = p.item                              //tmp variable to hold the return value
 (p.getPrev()).setNext(p.getNext())       //linking over p
 (p.getNext()).setPrev(p.getPrev())
  p.setPrev(null)                         //invalidating the position p
  p.setNext(null)
  return t
```

Deletion

- Below we visualize the operation **remove(p)**, where p = last()




```
public static Object remove(p):
  t = p.item                              //tmp variable to hold the return value
 (p.getPrev()).setNext(p.getNext())       //linking over p
 (p.getNext()).setPrev(p.getPrev())
  p.setPrev(null)                         //invalidating the position p
  p.setNext(null)
  return t
```

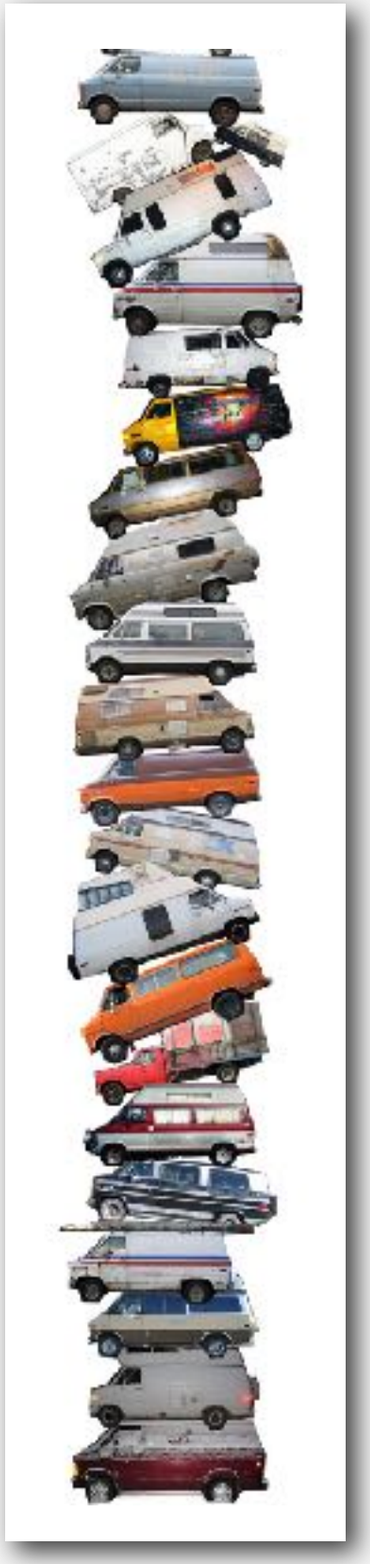
- stacks
- dynamic resizing
- queues
- **generics**
- iterators
- applications

Parameterized stack

- We implemented: `StackOfStrings`.
- We also want: `StackOfURLs`, `StackOfInts`, `StackOfVans`, etc.?

- Attempt 1. Implement a separate stack class for each type.
  - Rewriting code is tedious and error-prone.
  - Maintaining cut-and-pasted code is tedious and error-prone.

- @#$*! most reasonable approach until Java 1.5.



Parameterized stack

- We implemented: `StackOfStrings`.
- We also want: `StackOfURLs`, `StackOfInts`, `StackOfVans`, etc.?

- Attempt 2. Implement a stack with items of type `Object`.
  - Casting is required in client.
  - Casting is error-prone: run-time error if types mismatch.

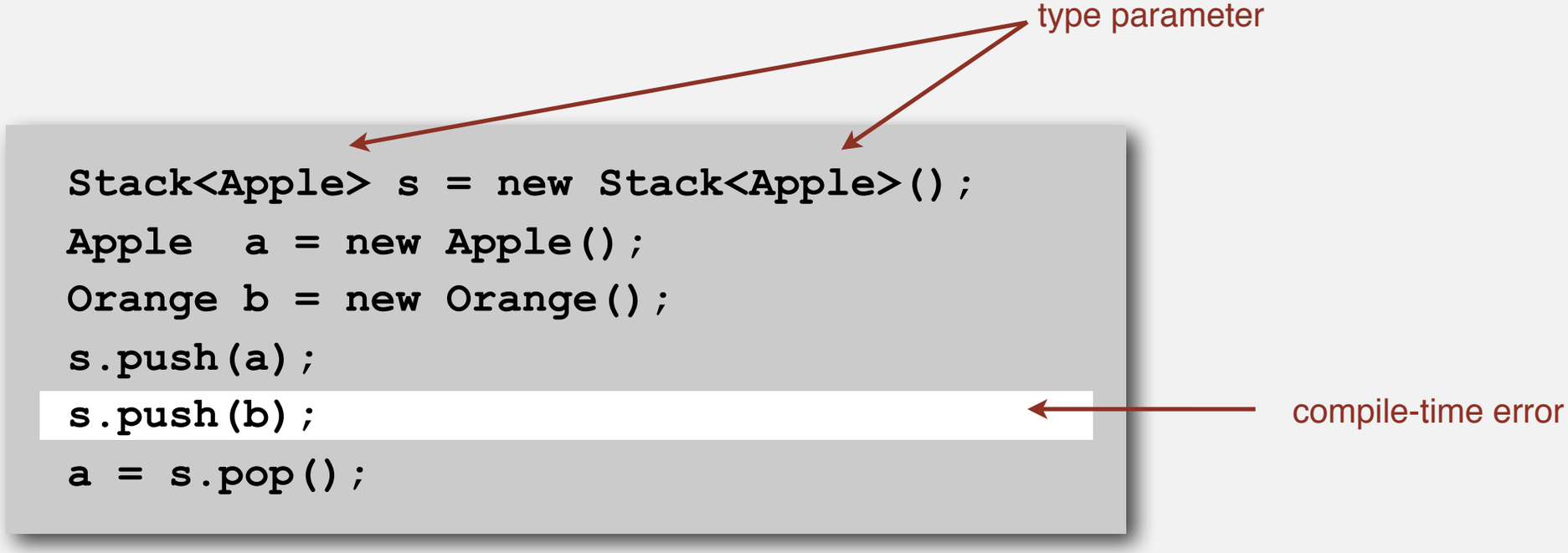
```
StackOfObjects s = new StackOfObjects();
Apple  a = new Apple();
Orange b = new Orange();
s.push(a);
s.push(b);
a = (Apple) (s.pop());
```

run-time error

Parameterized stack

- We implemented: `StackOfStrings`.
- We also want: `StackOfURLs`, `StackOfInts`, `StackOfVans`, etc.?

- Attempt 3. Java generics.
  - Avoid casting in client.
  - Discover type mismatch errors at compile-time instead of run-time.



- Guiding principles. Welcome compile-time errors; avoid run-time errors.

Generic stack: linked-list implementation

```
public class LinkedStackOfStrings
{
   private Node first = null;

   private class Node
   {
      String item;
      Node next;
   }

   public boolean isEmpty()
   {  return first == null;  }

   public void push(String item)
   {
      Node oldfirst = first;
      first = new Node();
      first.item = item;
      first.next = oldfirst;
   }

   public String pop()
   {
      String item = first.item;
      first = first.next;
      return item;
   }
}
```

```
public class Stack<Item>
{
   private Node first = null;

   private class Node
   {
      Item item;
      Node next;
   }

   public boolean isEmpty()
   {  return first == null;  }

   public void push(Item item)
   {
      Node oldfirst = first;
      first = new Node();
      first.item = item;
      first.next = oldfirst;
   }

   public Item pop()
   {
      Item item = first.item;
      first = first.next;
      return item;
   }
}
```

generic type name

Generic stack: array implementation

```
public class ArrayStackOfStrings
{
   private String[] s;
   private int N = 0;

   public StackOfStrings(int capacity)
   {  s = new String[capacity];  }

   public boolean isEmpty()
   {  return N == 0;  }

   public void push(String item)
   {  s[N++] = item;  }

   public String pop()
   {  return s[--N];  }
}
```

the way it should be

```
public class ArrayStack<Item>
{
   private Item[] s;
   private int N = 0;

   public Stack(int capacity)
   {  s = new Item[capacity];  }

   public boolean isEmpty()
   {  return N == 0;  }

   public void push(Item item)
   {  s[N++] = item;  }

   public Item pop()
   {  return s[--N];  }
}
```

Generic stack: array implementation

```
public class ArrayStackOfStrings
{
   private String[] s;
   private int N = 0;

   public StackOfStrings(int capacity)
   {  s = new String[capacity];  }

   public boolean isEmpty()
   {  return N == 0;  }

   public void push(String item)
   {  s[N++] = item;  }

   public String pop()
   {  return s[--N];  }
}
```

the way it should be

```
public class ArrayStack<Item>
{
   private Item[] s;
   private int N = 0;

   public Stack(int capacity)
   {  s = new Item[capacity];  }

   public boolean isEmpty()
   {  return N == 0;  }

   public void push(Item item)
   {  s[N++] = item;  }

   public Item pop()
   {  return s[--N];  }
}
```

@#$*! generic array creation not allowed in Java

Generic stack: array implementation

```
public class ArrayStackOfStrings
{
   private String[] s;
   private int N = 0;

   public StackOfStrings(int capacity)
   {  s = new String[capacity];  }

   public boolean isEmpty()
   {  return N == 0;  }

   public void push(String item)
   {  s[N++] = item;  }

   public String pop()
   {  return s[--N];  }
}
```

the way it is

```
public class ArrayStack<Item>
{
   private Item[] s;
   private int N = 0;

   public Stack(int capacity)
   {  s = (Item[]) new Object[capacity]; }

   public boolean isEmpty()
   {  return N == 0;  }

   public void push(Item item)
   {  s[N++] = item;  }

   public Item pop()
   {  return s[--N];  }
}
```

the ugly cast

Generic data types: autoboxing

- Q. What to do about primitive types?

- Wrapper type.
  - Each primitive type has a wrapper object type.
  - Ex: `Integer` is wrapper type for `int`.

- Autoboxing. Automatic cast between a primitive type and its wrapper.

- Syntactic sugar. Behind-the-scenes casting.

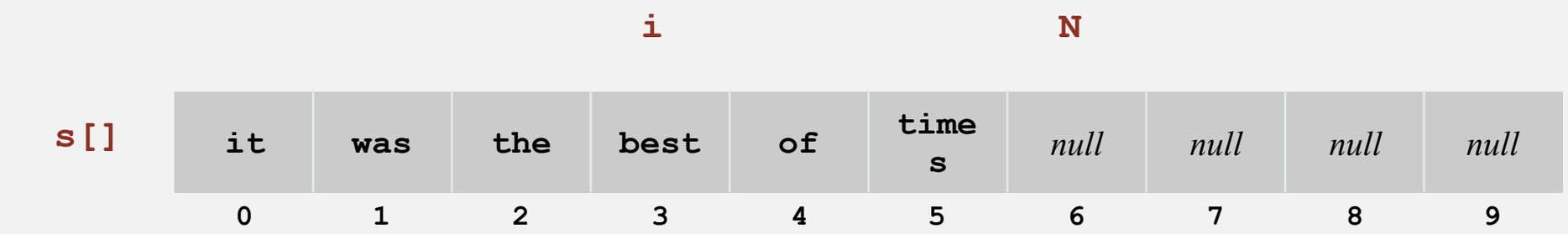
```
Stack<Integer> s = new Stack<Integer>();
s.push(17);         // s.push(new Integer(17));
int a = s.pop();    // int a = s.pop().intValue();
```

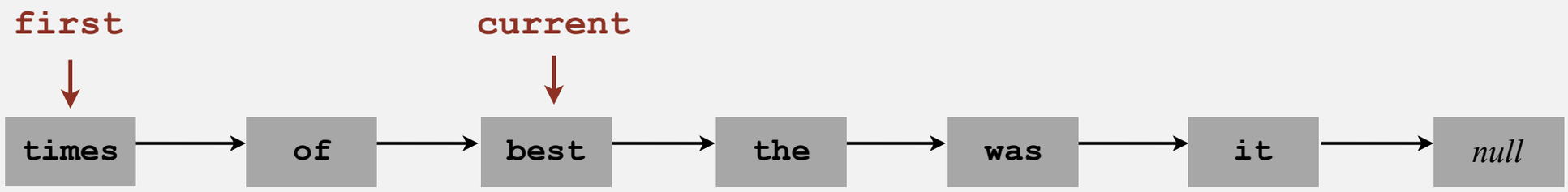
- Bottom line. Client code can use generic stack for any type of data.

- stacks
- dynamic resizing
- queues
- generics
- **iterators**
- applications

Iteration

- Design challenge. Support iteration over stack items by client, without revealing the internal representation of the stack.





- Java solution. Make stack implement the `Iterable` interface.

Iterators

- Q. What is an `Iterable` ?
- A. Has a method that returns an `Iterator`.

- Q. What is an `Iterator` ?
- A. Has methods `hasNext()` and `next()`.

- Q. Why make data structures `Iterable` ?
- A. Java supports elegant client code.

Iterable interface

```
public interface Iterable<Item>
{
   Iterator<Item> iterator();
}
```

Iterator interface

```
public interface Iterator<Item>
{
   boolean hasNext();
   Item next();
   void remove();      <-- optional; use at your own risk
}
```

"foreach" statement

```
for (String s : stack)
   StdOut.println(s);
```

equivalent code

```
Iterator<String> i = stack.iterator();
while (i.hasNext())
{
   String s = i.next();
   StdOut.println(s);
}
```

Stack iterator: linked-list implementation

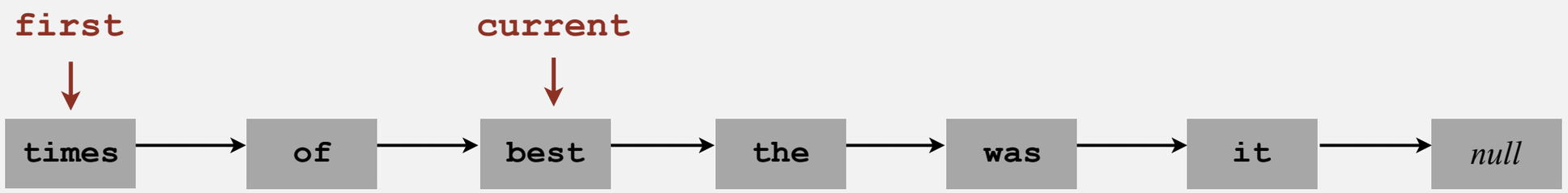
```
import java.util.Iterator;

public class Stack<Item> implements Iterable<Item>
{
    ...

    public Iterator<Item> iterator() { return new ListIterator();  }

    private class ListIterator implements Iterator<Item>
    {
        private Node current = first;

        public boolean hasNext() {  return current != null;  }
        public void remove()     {  /* not supported */     }
        public Item next()
        {
            Item item = current.item;
            current   = current.next;
            return item;
        }
    }
}
```



Stack iterator: array implementation

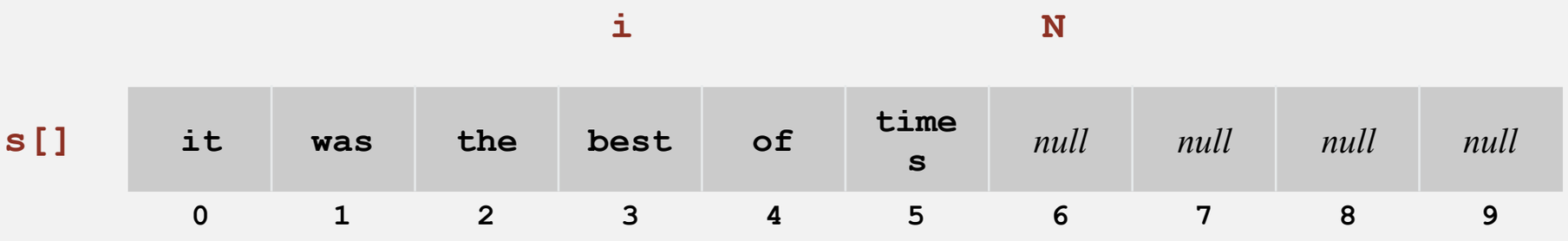
```
import java.util.Iterator;

public class Stack<Item> implements Iterable<Item>
{
    …

    public Iterator<Item> iterator() { return new ArrayIterator(); }

    private class ArrayIterator implements Iterator<Item>
    {
        private int i = N;

        public boolean hasNext() {  return i > 0;          }
        public void remove()     {  /* not supported */   }
        public Item next()       {  return s[--i];         }
    }
}
```



Iteration: concurrent modification

- Q. What if client modifies the data structure while iterating?
- A. A fail-fast iterator throws a `ConcurrentModificationException`.

concurrent modification

```
for (String s : stack)
   stack.push(s);
```

- To detect:
  - Count total number of `push()` and `pop()` operations in `Stack`.
  - Save current count in `*Iterator` subclass upon creation.
  - Check that two values are still equal when calling `next()` and `hasNext()`.

Bag API

- When order doesn't matter:

public class Bag<Item> implements Iterable<Item>	
Bag()	*create an empty bag*
void add(Item x)	*insert a new item onto bag*
int size()	*number of items in bag*
Iterable<Item> iterator()	*iterator for all items in bag*

- stacks
- dynamic resizing
- queues
- generics
- iterators
- **applications**

Java collections library

List interface. `java.util.List` is API for ordered collection of items.

`public interface List<Item> implements Iterable<Item>`		
	`List()`	*create an empty list*
`boolean`	`isEmpty()`	*is the list empty?*
`int`	`size()`	*number of items*
`void`	`add(Item item)`	*append item to the end*
`Item`	`get(int index)`	*return item at given index*
`Item`	`remove(int index)`	*return and delete item at given index*
`boolean`	`contains(Item item)`	*does the list contain the given item?*
`Iterator<Item>`	`iterator()`	*iterator over all items in the list*
	`...`	

Implementations. `java.util.ArrayList` uses dynamic array;
`java.util.LinkedList` uses linked list.

Java collections library

`java.util.Stack`.

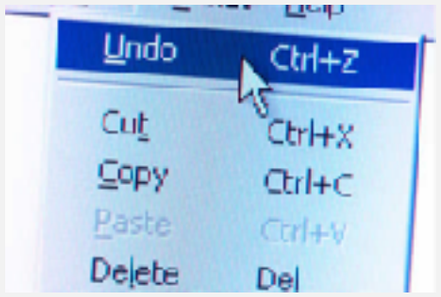
- Supports `push()`, `pop()`, `size()`, `isEmpty()`, and iteration.
- Also implements `java.util.List` interface from previous slide, including, `get()`, `remove()`, and `contains()`.
- Bloated and poorly-designed API ⇒ don't use.

`java.util.Queue`. An interface, not an implementation of a queue.

Lesson. Don't use a library until you understand its API!

Stack applications

- Parsing in a compiler.
- Java virtual machine.
- Undo in a word processor.
- Back button in a Web browser.
- PostScript language for printers.
- Implementing function calls in a compiler.
- ...



Function calls

- How a compiler implements a function.
  - Function call: push local environment and return address.
  - Return: pop return address and local environment.

- Recursive function. Function that calls itself.
- Note. Can always use an explicit stack to remove recursion.

p = 216, q = 192

gcd (216, 192)

```
static int gcd(int p, int q) {
   if (q == 0) return p;
   else
}
```

p = 192, q = 24

gcd (192, 24)

```
static int gcd(int p, int q) {
   if (q == 0) return p;
   else
}
```

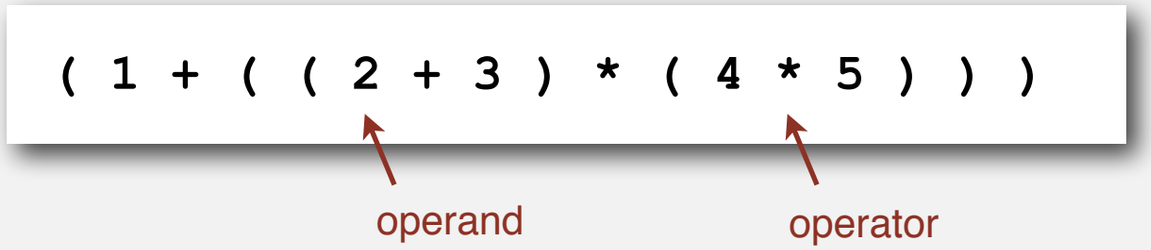
p = 24, q = 0

gcd (24, 0)

```
static int gcd(int p, int q) {
   if (q == 0) return p;
   else return gcd(q, p % q);
}
```

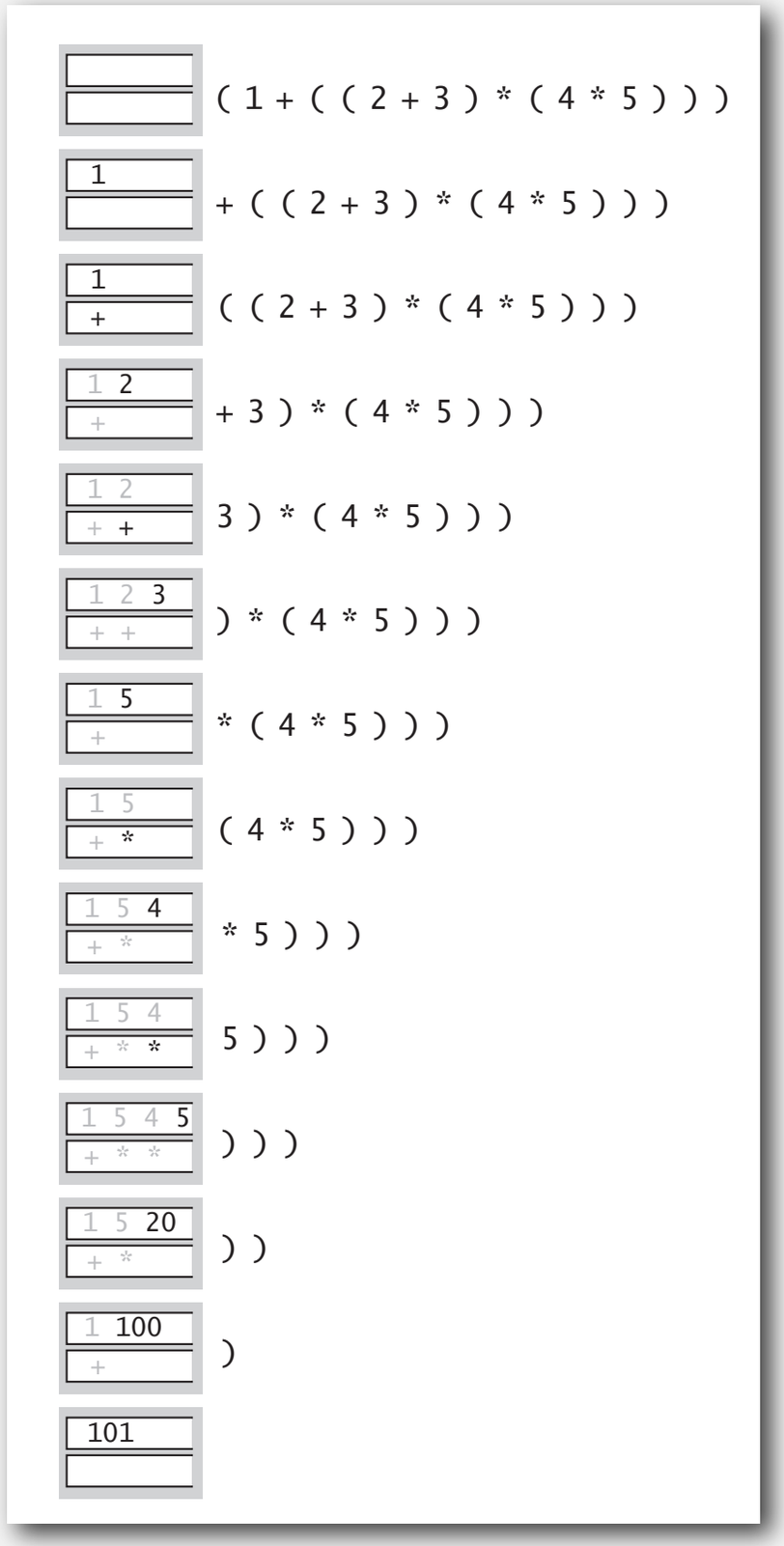
Arithmetic expression evaluation

- Goal. Evaluate infix expressions.



- Two-stack algorithm. [E. W. Dijkstra]
  - Value: push onto the value stack.
  - Operator: push onto the operator stack.
  - Left parens: ignore.
  - Right parens: pop operator and two values; push the result of applying that operator to those values onto the operand stack.

- Context. An interpreter!



Arithmetic expression evaluation

```
public class Evaluate
{
   public static void main(String[] args)
   {
      Stack<String> ops  = new Stack<String>();
      Stack<Double> vals = new Stack<Double>();
      while (!StdIn.isEmpty()) {
         String s = StdIn.readString();
         if      (s.equals("("))                ;
         else if (s.equals("+"))    ops.push(s);
         else if (s.equals("*"))    ops.push(s);
         else if (s.equals(")"))
         {
            String op = ops.pop();
            if      (op.equals("+")) vals.push(vals.pop() + vals.pop());
            else if (op.equals("*")) vals.push(vals.pop() * vals.pop());
         }
         else vals.push(Double.parseDouble(s));
      }
      StdOut.println(vals.pop());
   }
}
```

```
% java Evaluate
( 1 + ( ( 2 + 3 ) * ( 4 * 5 ) ) )
101.0
```

Correctness

- Q. Why correct?
- A. When algorithm encounters an operator surrounded by two values within parentheses, it leaves the result on the value stack.

```
( 1 + ( ( 2 + 3 ) * ( 4 * 5 ) ) )
```

- as if the original input were:

```
( 1 + ( 5 * ( 4 * 5 ) ) )
```

- Repeating the argument:

```
( 1 + ( 5 * 20 ) )
( 1 + 100 )
101
```

- Extensions. More ops, precedence order, associativity.

Stack-based programming languages

- Observation 1. The 2-stack algorithm computes the same value if the operator occurs after the two values.

```
( 1 ( ( 2 3 + ) ( 4 5 * ) * ) + )
```

- Observation 2. All of the parentheses are redundant!

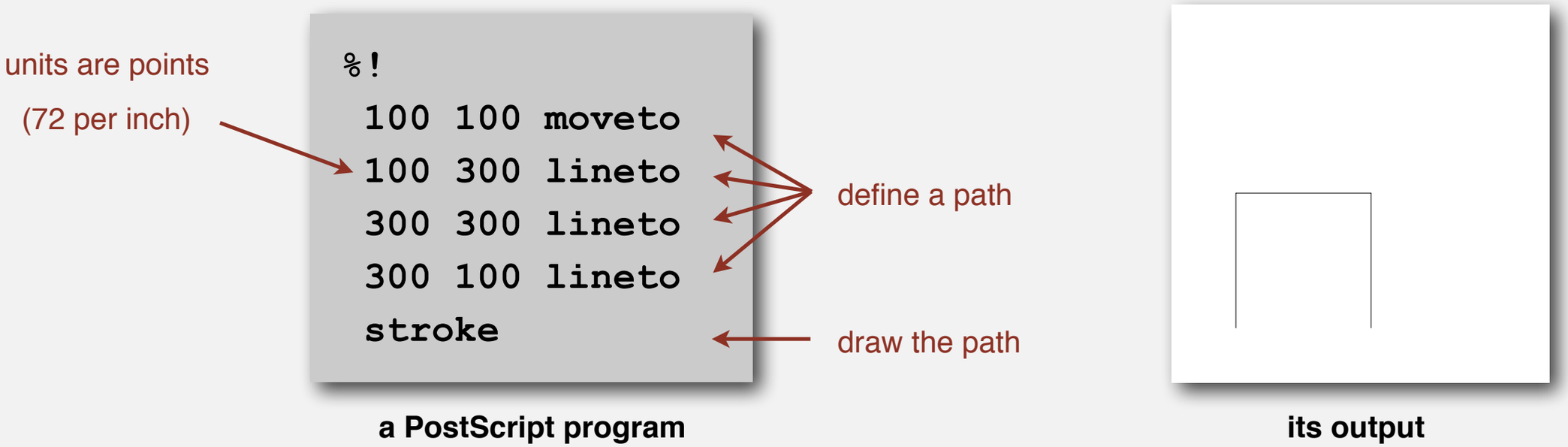
```
1 2 3 + 4 5 * * +
```

Jan Lukasiewicz

- Bottom line. Postfix or "reverse Polish" notation.
- Applications. Postscript, Forth, calculators, Java virtual machine, ...

PostScript

- PostScript. [Warnock-Geschke 1980s]
  - Postfix program code.
  - Turtle graphics commands.
  - Variables, types, text, loops, conditionals, functions, ...



- Simple virtual machine, but not a toy.
  - Easy to specify published page.
  - Easy to implement in printers.
  - Revolutionized the publishing world.

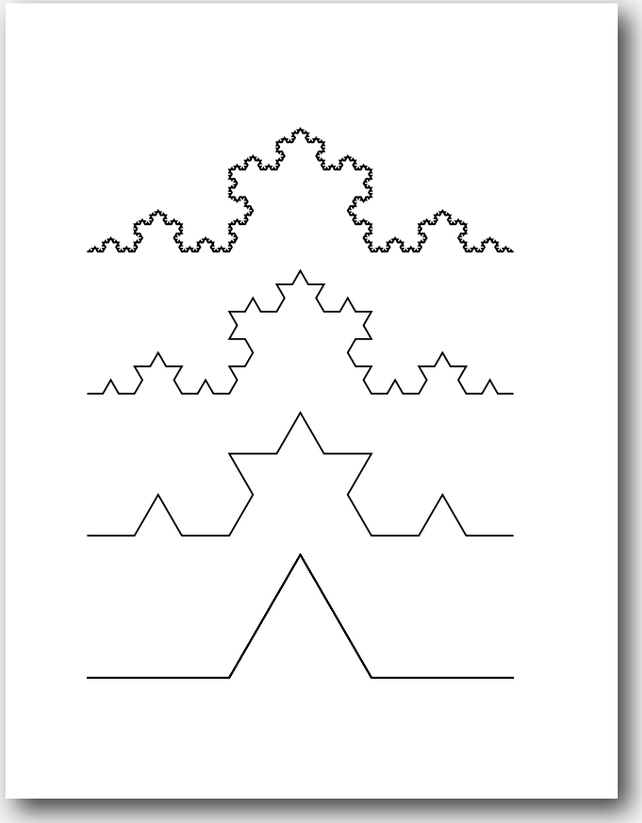
PostScript applications

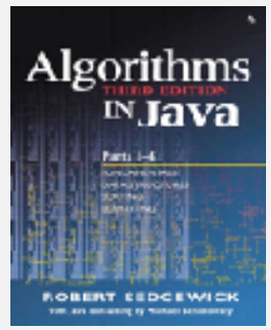
- Algorithms, 3rd edition. Figures created directly in PostScript.

```
%!
72 72 translate

/kochR
  {
    2 copy ge { dup 0 rlineto }
      {
        3 div
        2 copy kochR 60 rotate
        2 copy kochR -120 rotate
        2 copy kochR 60 rotate
        2 copy kochR
      } ifelse
    pop pop
  } def

0   0 moveto   81 243 kochR
0  81 moveto   27 243 kochR
0 162 moveto    9 243 kochR
0 243 moveto    1 243 kochR
stroke
```





see page 218

- Algorithms, 4th edition. Figures created using enhanced version of `stdDraw` that saves to PostScript for vector graphics.

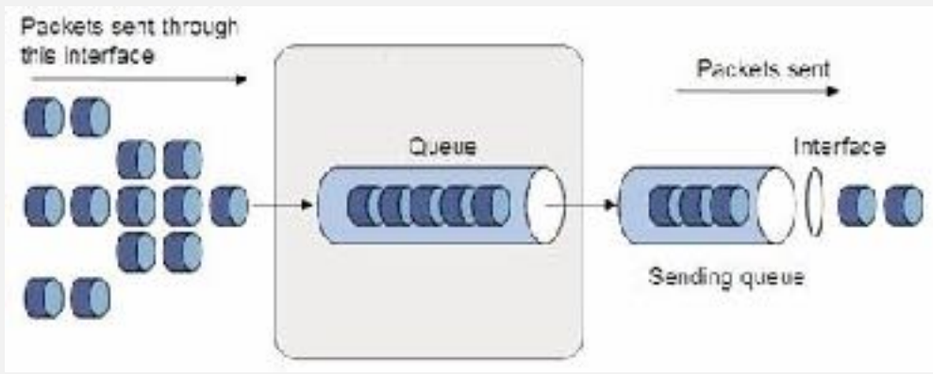
Queue applications

·Familiar applications.

- iTunes playlist.
- Data buffers (iPod, TiVo).
- Asynchronous data transfer (file IO, pipes, sockets).
- Dispensing requests on a shared resource (printer, processor).

·Simulations of the real world.

- Traffic analysis.
- Waiting times of customers at call center.
- Determining number of cashiers to have at a supermarket.



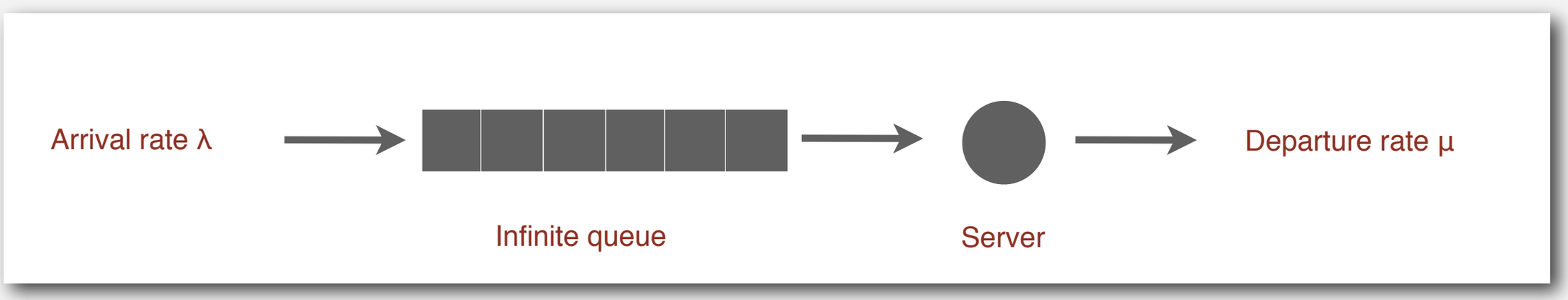
M/M/1 queuing model

·M/M/1 queue.

- Customers arrive according to Poisson process at rate of λ per minute.
- Customers are serviced with rate of μ per minute.

interarrival time has exponential distribution $\Pr[X \leq x] = 1 - e^{-\lambda x}$

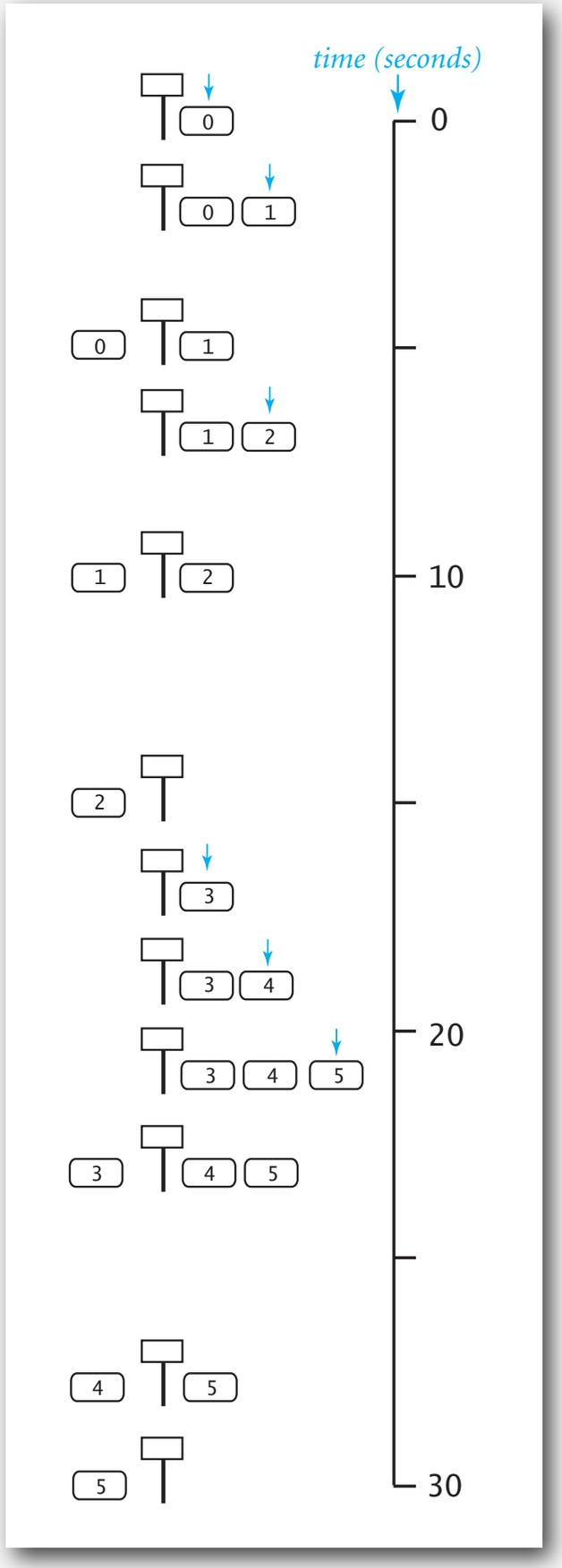
service time has exponential distribution $\Pr[X \leq x] = 1 - e^{-\mu x}$



·Q. What is average wait time W of a customer in system?

·Q. What is average number of customers L in system?

M/M/1 queuing model: example simulation



	arrival	*departure*	*wait*
0	0	5	5
1	2	10	8
2	7	15	8
3	17	23	6
4	19	28	9
5	21	30	9