

DATA1:=

26
27
28
30
34
41
52
69
61
58
65
78
72
73
94
97
105
99
71
49
40
35
31
29
28

First experiment in notebook

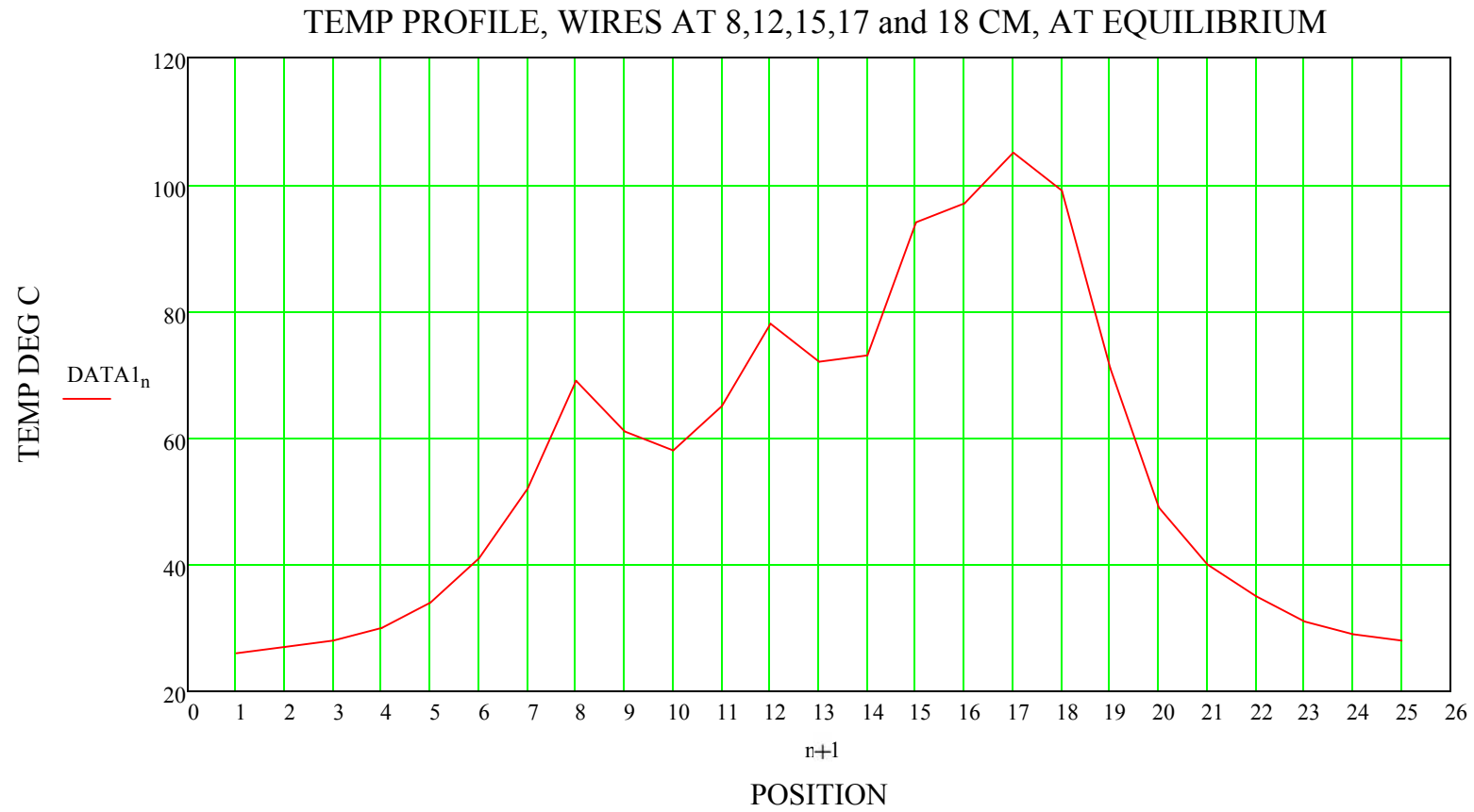
n:= 0..25

wires at 8,12,15,17,18

2.6amps 13V, 1500 mm wire, 280mm each leg

$$13 \cdot 2.6 \cdot \frac{280}{1500} = \blacksquare$$

watts per leg



assuming the wires are evenly spaced and equal amounts flow each direction, then between two wires we have

$$\text{heatdens}(\text{Pw}, \text{L}, \text{G}) := 2 \cdot \frac{\frac{\text{Pw}}{2}}{\text{L} \cdot \text{G}}$$

between wires 1 and 2

$$\text{heatdens}(6.309, 28, 4) = 0.056 \quad \frac{\text{W}}{\text{cm}^2}$$

between 2 and 3

$$\text{heatdens}(6.309, 28, 3) = 0.075$$

between wires 3 and 4

$$\text{heatdens}(6.309, 28, 2) = 0.113$$

between wires 4 and 5

$$\text{heatdens}(6.309, 28, 1) = 0.225$$

thermal\_wizzard.com predicts for heat up from a plate...

105 degC plate, 25 degC ambient, 100 cm^2 plate, 13.9W	.139	$\frac{\text{W}}{\text{cm}^2}$
280 cm^2, plate, 37.0W	.132	

## Second Experiment

Glass is a 14 inch round mirror, 0.187 thick

Two evenly spaced grids, 20 CM gap and 30 CM gap  
each section used 750mm 22 ga Nichrome wire  
30cm section was 5 wires of 126mm each  
20cm section was wires of 134mm each. The winding  
were in series and a ~12 supply was used

TimeData:=

0	25	25
1	32	31
2	38	35
3	44	41
4	48	44
5	53	46
6	58	50
7	62	52
8	64	55
9	68	57
10	70	59
11	72	61
12	74	62
15	80	67
20.5	87	72
25	90	77
30	92	78
45	95	79
60	95	80
75	95	81

ProfileData:=

1	53
2	68
3	85
4	89
5	96
6	93
7	96
8	92
9	94
10	87
11	85
12	63
13	49
19	36
22	45
23	55
24	73
25	73
26	73
27	82
28	76
29	72
30	81
31	79
32	75
33	84
34	80
35	73
36	79
37	72
38	53

$$V := 13.3$$

$$I_{\text{cold}} := 2.6$$

$$P_{\text{cold}} := I_{\text{cold}} \cdot V$$

$$P_{\text{cold}} = 34.58$$

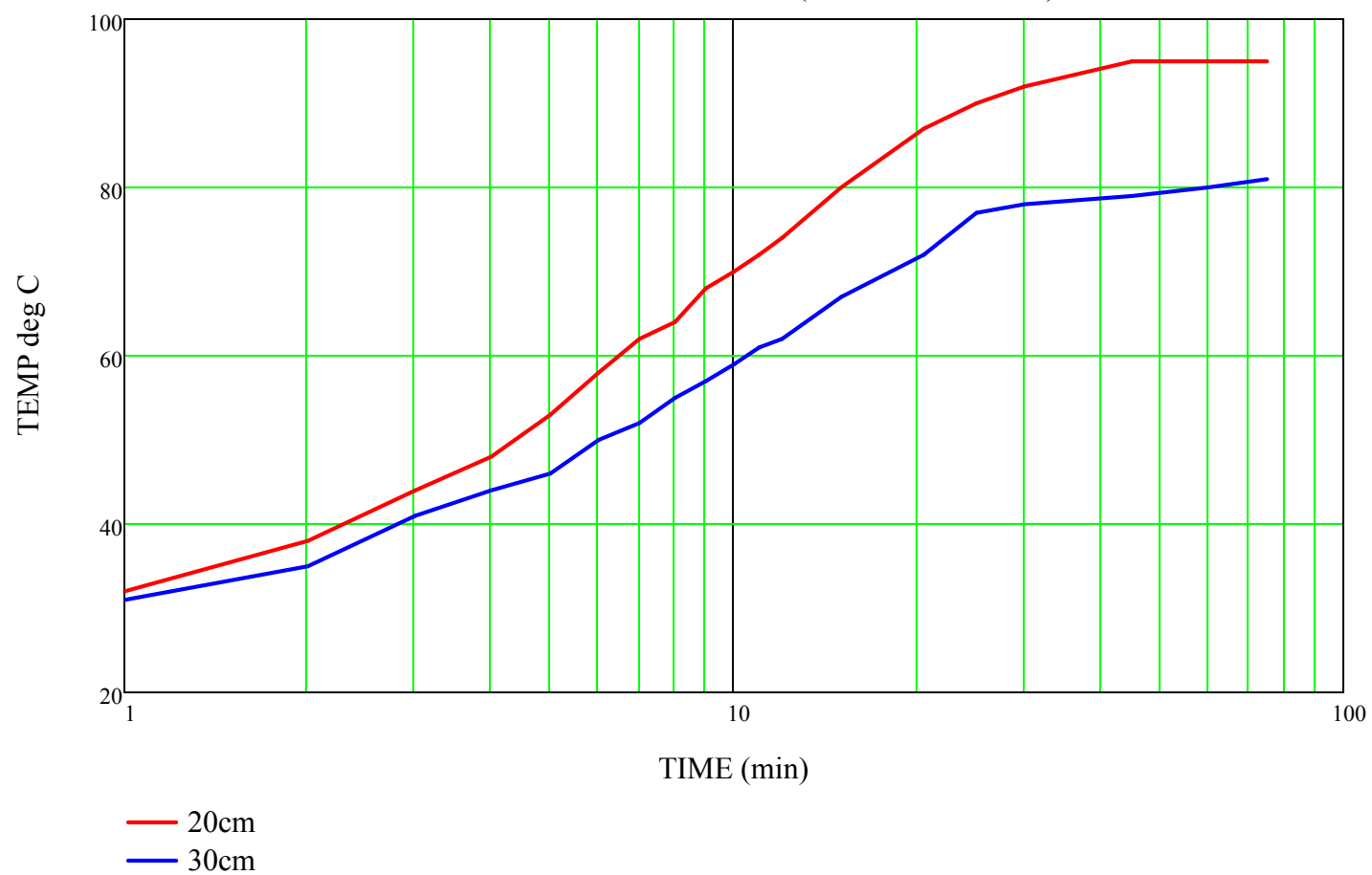
$$I_{\text{hot}} := 2.5$$

$$P_{\text{hot}} := I_{\text{hot}} \cdot V$$

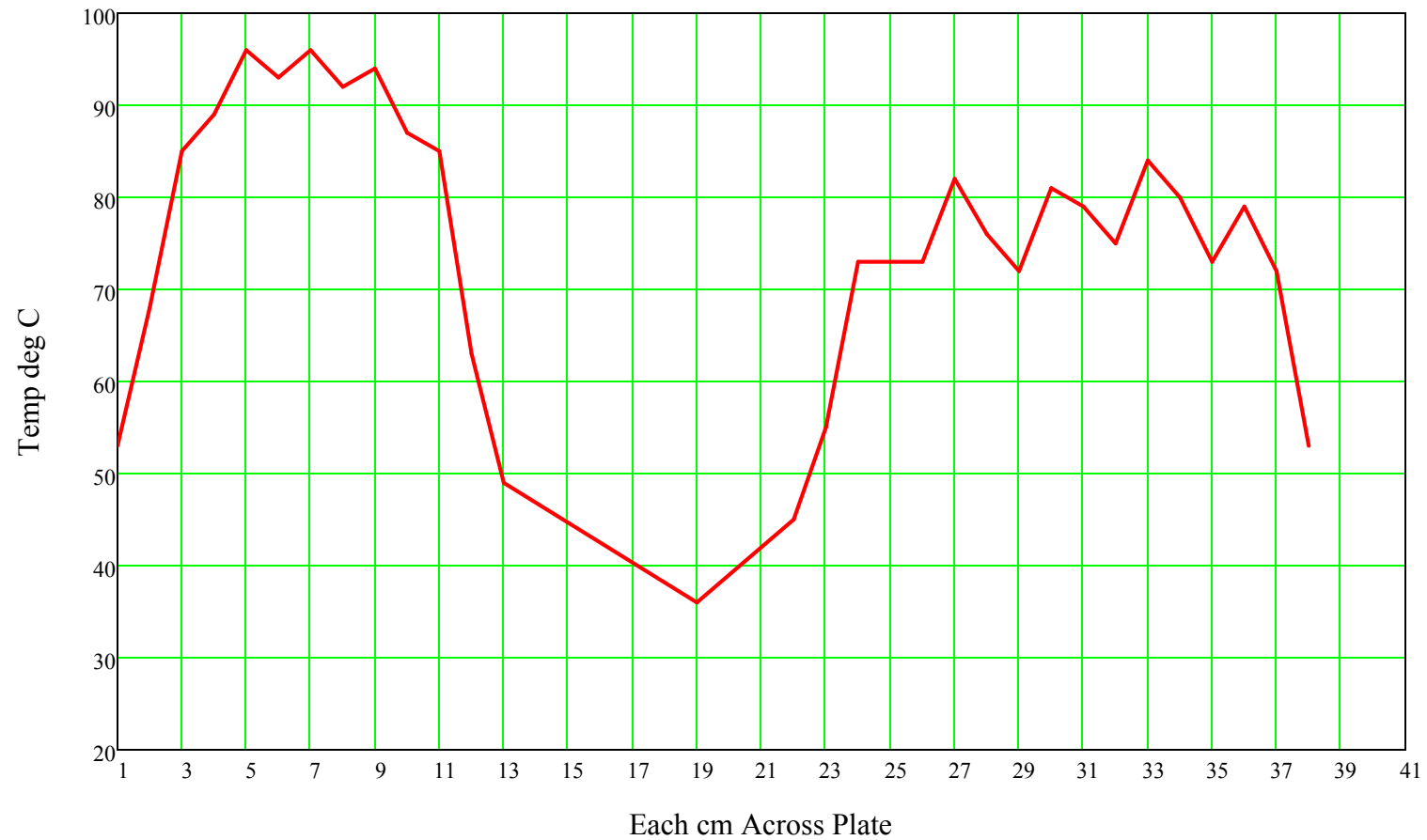
$$P_{\text{hot}} = 33.25$$

Second experiment in notebook

TEMPERATURE vs TIME (middle of section)



TEMPERATURE PROFILE wires at 3,5..11 (20 grid) 24,27...36 (30 grid)



TempProfile

## Transient Analysis

The temperature change of the plate as a function of time should follow an exponential expression

$$T(t) = T_A + T\Delta \cdot \left(1 - e^{-\frac{t}{\tau}}\right)$$

temperature as a function of time = initial temperature + final change x exponential expression, where  $\tau$  is the time constant of the thermal circuit

The thermal circuit looks like a parallel RC circuit, being charged by a current source.  $T\Delta$  is  $Q \cdot \text{thermal resistance}$ ,  $\tau$  is  $\text{heat capacity} \cdot \text{thermal resistance}$ .

estimate thermal resistance..

$$A_{20} := (13.4 + 1) \cdot (2.0 \cdot 4 + 1) \quad A_{20} = 129.6 \quad \text{cm}^2 \quad \text{approximate area that significant heat flows out of}$$

$$A_{30} := (12.6 + 1) \cdot (3.0 \cdot 4 + 1) \quad A_{30} = 176.8 \quad \text{cm}^2$$

$$Q := \frac{13.3 \cdot 2.5}{2} \quad Q = 16.625 \quad \text{watts per section}$$

$$THr_{20} := \frac{(90 - 25)}{Q} \quad THr_{20} = 3.91 \quad \text{degC per Watt}$$

$$THr_{30} := \frac{(75 - 25)}{Q} \quad THr_{30} = 3.008 \quad \text{degC per watt}$$

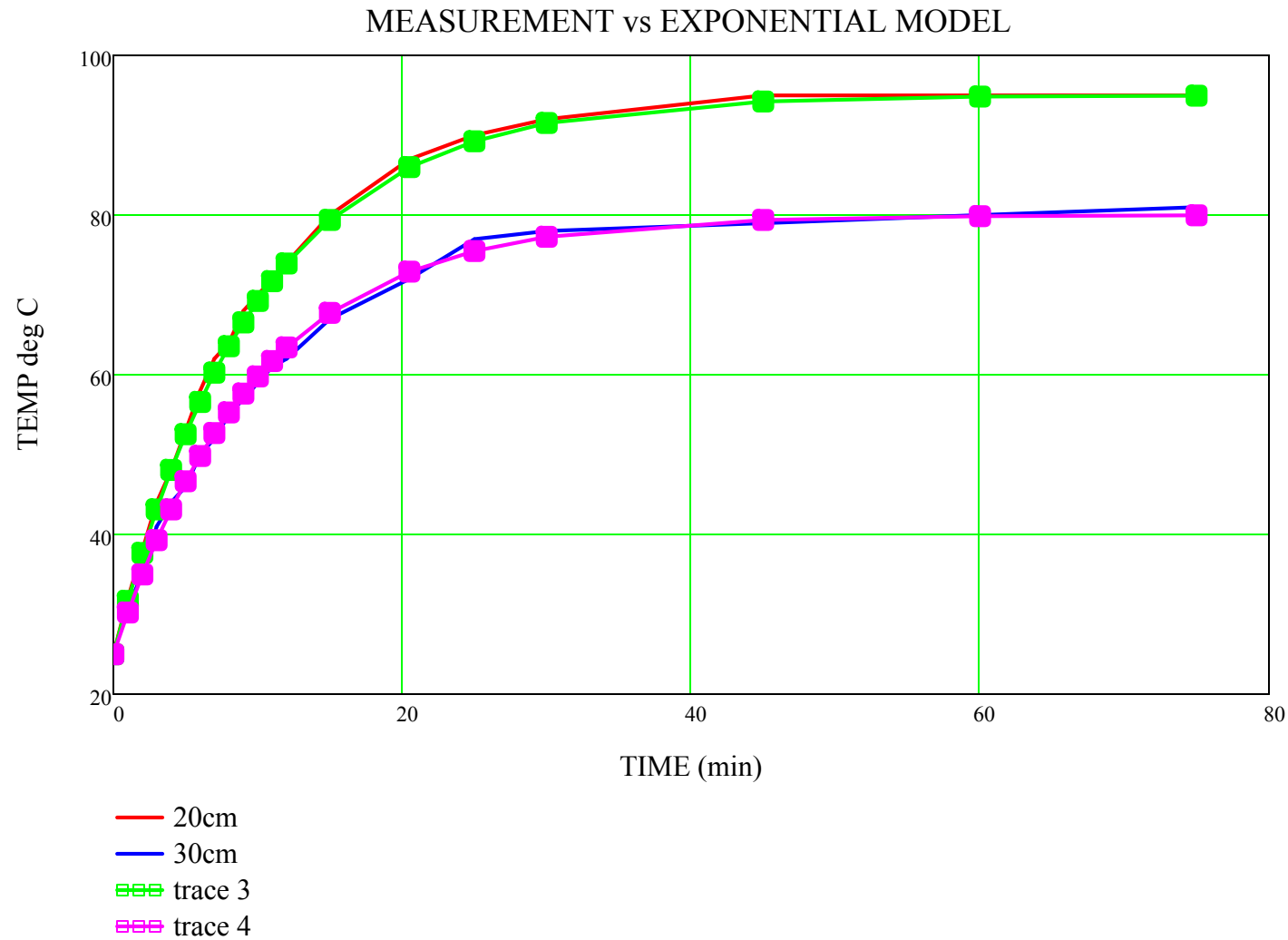
Based on eyeball averaging of the temperature across the sections profile

$$T \Delta(t, Q, T_{Hr}, H_c) := \frac{H_c \cdot Q \cdot t^2}{H_c \cdot T_{Hr} \cdot t^2 + 2}$$

$$T_{m20}(t) := 25 + 70 \cdot \left( 1 - e^{-\frac{t}{\frac{40}{4}}} \right)$$

$$T_{m30}(t) := 25 + 55 \cdot \left( 1 - e^{-\frac{t}{\frac{40}{4}}} \right)$$

These models are just eyeball fits based on the assumption that it follows an exponential model, and it shows a very good fit. A good rule of thumb is "nearly up in 4 time constants". These plots are nearly up in 40 minutes, hence 40/4, or 10





heat capacity of glass

$$Q_{cg} := 0.84 \frac{\text{J}}{\text{g} \cdot \text{K}} \quad \text{Joules per gram-Kelvin}$$

from <http://hyperphysics.phy-astr.gsu.edu/hbase/tables/sphtt.html>

$$M_p := 930 \quad \text{g (by measurement)}$$

$$V_p := 35^2 \cdot \frac{\pi}{4} \cdot .39 \quad \text{cm}^2 \text{ by measurement}$$

$$A_p := 35^2 \cdot \frac{\pi}{4} \quad A_p = 962.113$$

$$Q_{c20} := Q_{cg} \cdot M_p \cdot \frac{A_{20}}{A_p} \quad Q_{c20} = 105.23 \quad \frac{\text{J}}{\text{K}} \quad \text{Joules per degree K, (a Joule is a W-s. so a W is a J/s)}$$

$$Q_{c30} := Q_{cg} \cdot M_p \cdot \frac{A_{30}}{A_p} \quad Q_{c30} = 143.555 \quad \frac{\text{J}}{\text{K}}$$

$$Q_{c20} \cdot \text{THr20} = 411.427 \quad \text{J/K} \times \text{K/J/s, or s}$$

$$Q_{c30} \cdot \text{THr30} = 431.745$$

These time constants are in seconds, whereas the "by inspection" time constants above are minutes

$$\frac{Q_{c20} \cdot \text{THr20}}{60} = 6.857 \quad \text{Minutes} \quad \text{So the calculated time constants are about 7 minutes and the measured time constant is about 10 minutes.}$$

$$\frac{Q_{c30} \cdot \text{THr30}}{60} = 7.196 \quad \text{Minutes}$$

Estimate the time constant of the plate

from thermal wizard, 90 deg C rise, 310mm square plate (962 cm<sup>2</sup>) requires 132 W, for .682 degC/W

$$Q_{cg} \cdot M_p = 781.2 \quad \frac{J}{K} \quad \text{heat capacity of plate}$$

$$\frac{Q_{cg} \cdot M_p \cdot 0.682}{60} = 8.88 \quad \text{estimated time constant of plate.}$$

$$8.88 \cdot \frac{10}{7} = 12.686 \quad \text{fudge it up by the ratio of measured to calculated}$$

$$\tau_p := 12.7$$

Now we want to solve for the final temp that will heat the plate in 10 minutes. The way we heat the plate faster is to provide extra heat flow over that required to keep it hot. Then when the plate reaches the desired temperature the controller starts running the heater with a duty cycle..  
If the controller didn't, the plate would get much hotter.

$$110 = 20 + TP \cdot \left( 1 - e^{-\frac{t}{\tau_p}} \right)$$

$$TP(\tau_p, t) := -\frac{90}{e^{-\frac{t}{\tau_p}} - 1}$$

$$TP(\tau_p, 10) = 165.145$$

Range of maximum temperatures

$$TP(8.88, 10) = 133.193$$

$$\text{Wheat}(\tau_p, t) := \frac{TP(\tau_p, t)}{.688}$$

$$\text{Wheat}(\tau_p, 10) = 240.037$$

$$\text{Wheat}(10, 10) = 206.945$$

$$\text{Wheat}(8.88, 10) = 193.594$$

3 estimates of the power to heat the plate in 10 minutes, calculated, fudged time constant, experimental time constant, calculated time constant

