Analyzing Repeated Measures Data

Module 5: The Linear Mixed Model: Random Intercept Models

Karen Grace-Martin

Workshop Outline: Random Intercept Models

How Random Intercepts Account for Correlations among Observations

Intraclass Correlations

The Specification of Mixed Models

A Comparison to Marginal Models

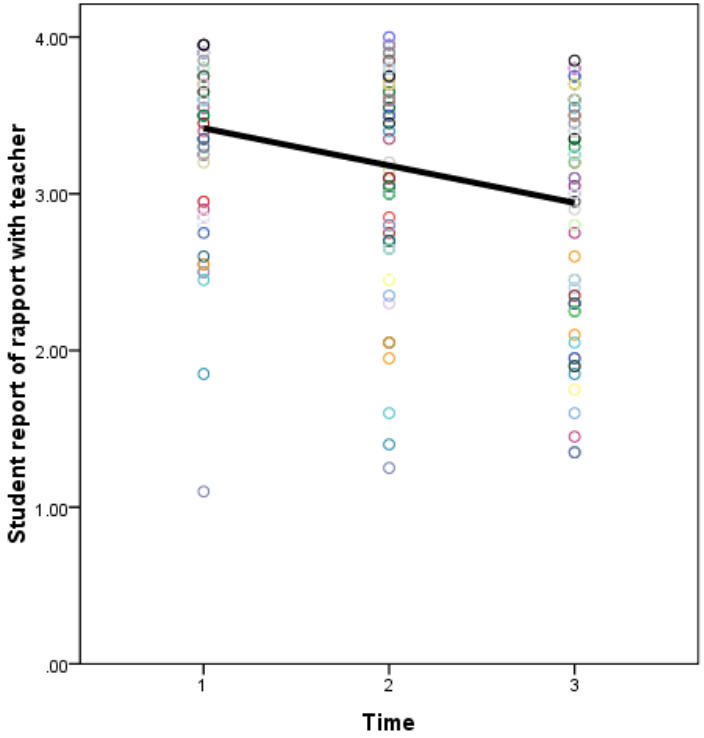
Physical Training Example

Swallowing Example

The Random Intercept Model

How Random Intercepts Account for Correlations Among Observations

Approach: Alter the Residual Structure



The Marginal Model

$$Rapport_{ij} = \beta_0 + \beta_1 Time + \varepsilon_{ij}$$

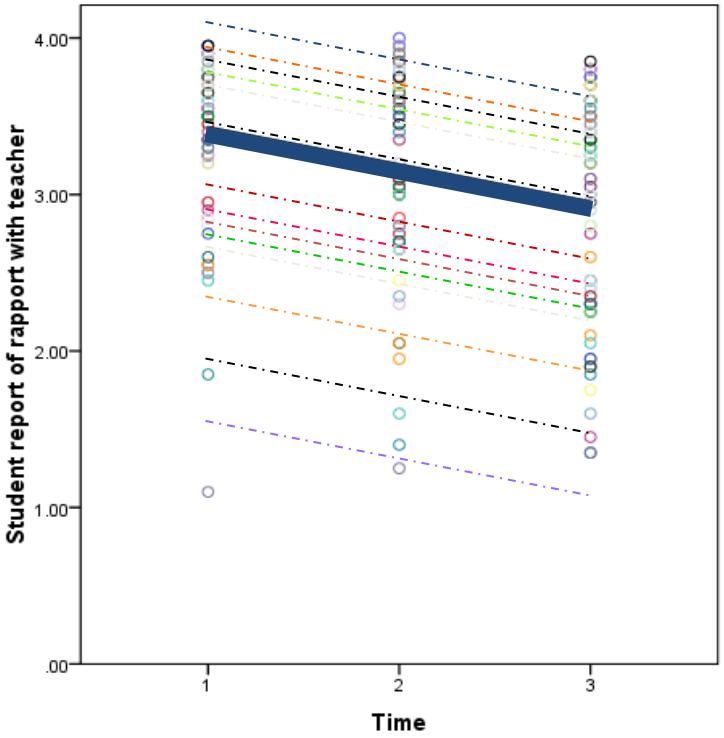
i = 1 to 82 subjects
j = 1 to 3 repeats per subject

$$R = \begin{bmatrix} [\Sigma] & & & & & & \\ 0 & [\Sigma] & & & & & \\ 0 & 0 & [\Sigma] & & & & \\ 0 & 0 & 0 & . & & & \\ 0 & 0 & 0 & 0 & . & & \\ 0 & 0 & 0 & 0 & 0 & . & \\ 0 & 0 & 0 & 0 & 0 & 0 & [\Sigma] \end{bmatrix}$$

$\varepsilon_{ij} \sim$ iid $N(0, \Sigma)$
for subject i and time j

$$\Sigma = \begin{bmatrix} Var_1 & Cov_{1,2} & Cov_{1,3} \\ Cov_{1,2} & Var_2 & Cov_{2,3} \\ Cov_{1,3} & Cov_{2,3} & Var_3 \end{bmatrix}$$

Approach: Control for Subject

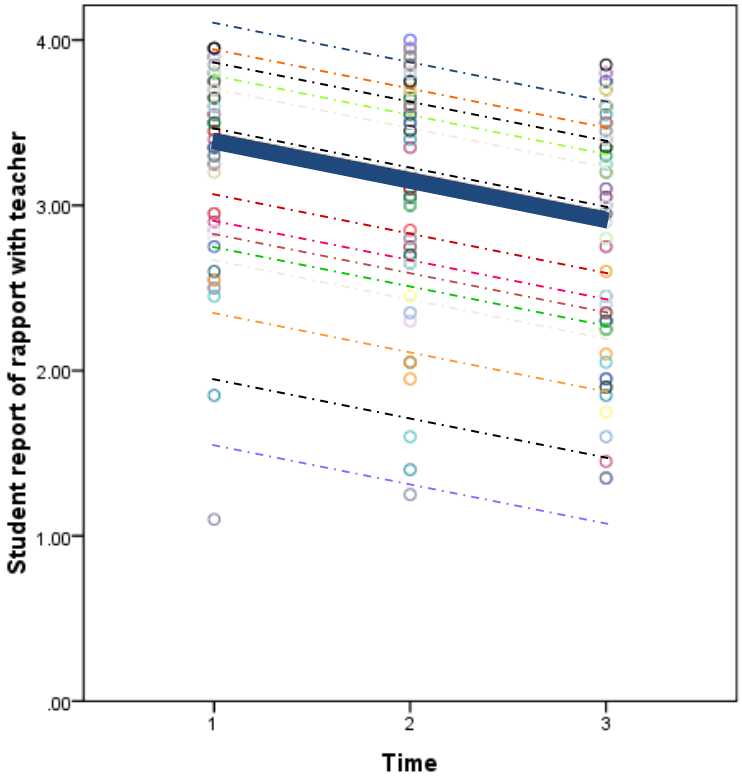


The Random Intercept Model

$$Rapport_{ij} = \beta_0 + \beta_1 Time + u_i + \varepsilon_{ij}$$

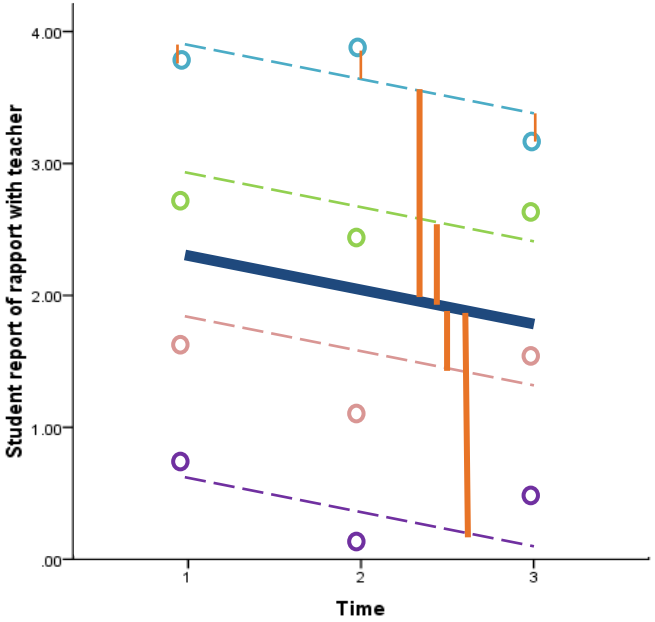
$u_i \sim$ iid $N(0, \sigma_0^2)$
for subject i

$\varepsilon_{ij} \sim$ iid $N(0, \sigma^2)$
for outcome j

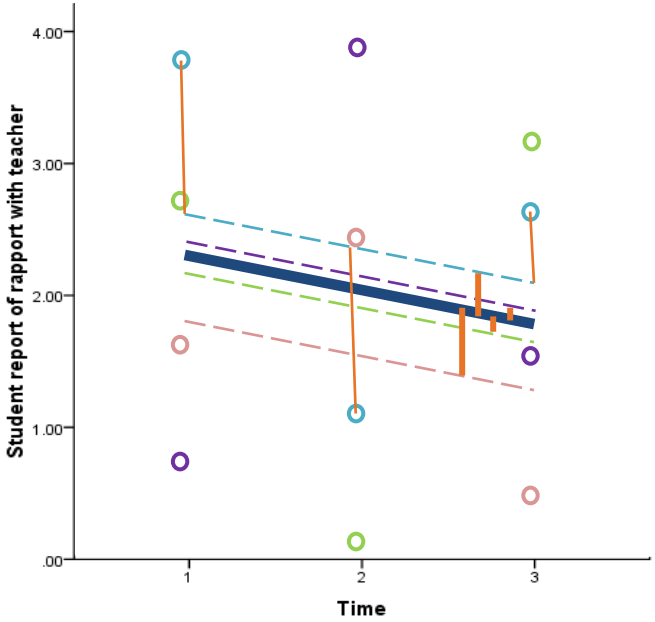


Two Scenarios, Same Data

β_0: Same β_1: Same
Var(Y_{ij}): Same



Corr ($Y_{ij,}$ $Y_{ij'}$): High
Var(u_i) : High Var(ε_{ij}): Low



Corr ($Y_{ij,}$ $Y_{ij'}$): Low
Var(u_i) : Low Var(ε_{ij}): High

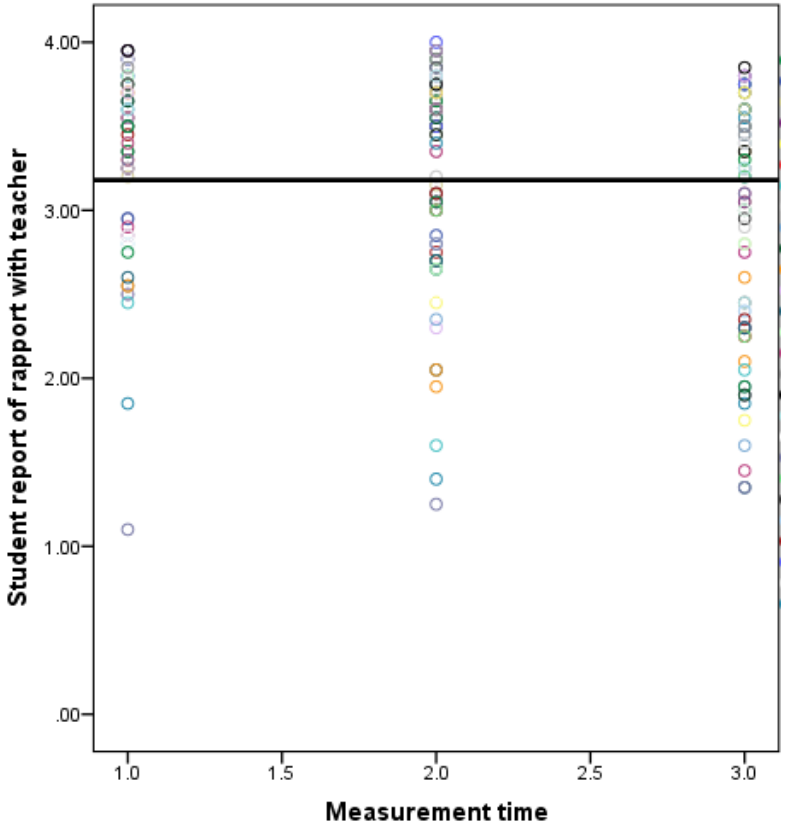
The Intraclass Correlation

The Empty Model

$Y_{ij} = \beta_0 + u_i + \varepsilon_{ij}$

$u_i \sim$ iid $N(0, \sigma_0^2)$
for subject i

$\varepsilon_{ij} \sim$ iid $N(0, \sigma^2)$
for outcome j



The Intraclass Correlation

$Y_{ij} = \beta_0 + u_i + \varepsilon_{ij}$

$Var(Y_{ij}) = Var(u_i) + Var(\varepsilon_{ij}) = \sigma_0^2 + \sigma^2$

$u_i \sim$ iid $N(0, \sigma_0^2)$
for subject i

$\varepsilon_{ij} \sim$ iid $N(0, \sigma^2)$
for outcome j

The Intraclass Correlation

$Y_{ij} = \beta_0 + u_i + \varepsilon_{ij}$

$u_i \sim$ iid $N(0, \sigma_0^2)$
for subject i

$\varepsilon_{ij} \sim$ iid $N(0, \sigma^2)$
for outcome j

$Var(Y_{ij}) = Var(u_i) + Var(\varepsilon_{ij}) = \sigma_0^2 + \sigma^2$

$Cov(Y_{ij}, Y_{ij'}) = u_i = \sigma_0^2$

The Intraclass Correlation

$Y_{ij} = \beta_0 + u_i + \varepsilon_{ij}$

$u_i \sim$ iid $N(0, \sigma_0^2)$
for subject i

$\varepsilon_{ij} \sim$ iid $N(0, \sigma^2)$
for outcome j

$Var(Y_{ij}) = Var(u_i) + Var(\varepsilon_{ij}) = \sigma_0^2 + \sigma^2$

$Cov(Y_{ij}, Y_{ij'}) = u_i = \sigma_0^2$

$$Corr(Y_{ij}, Y_{ij'}) = \frac{\sigma_0^2}{\sigma_0^2 + \sigma^2}$$

The Empty Model

Warnings

A(n) FIXED subcommand has no specifications and will therefore be ignored.

Model Dimension[b]

		Number of Levels	Covariance Structure	Number of Parameters	Subject Variables
Fixed Effects	Intercept	1		1	
Random Effects	Intercept[a]	1	Variance Components	1	SubID
Residual				1	
Total		2		3	

a. As of version 11.5, the syntax rules for the RANDOM subcommand have changed. Your command synta information.

b. Dependent Variable: Rapport Student report of rapport with teacher.

Information Criteria[a]

-2 Restricted Log Likelihood	389.563
Akaike's Information Criterion (AIC)	393.563
Hurvich and Tsai's Criterion (AICC)	393.614
Bozdogan's Criterion (CAIC)	402.508
Schwarz's Bayesian Criterion (BIC)	400.508

The information criteria are displayed in smaller-is-better forms.

a. Dependent Variable: Rapport Student report of rapport with teacher.

The Empty Model

Fixed Effects

Type III Tests of Fixed Effects[a]

Source	Numerator df	Denominator df	F	Sig.
Intercept	1	80.223	2632.826	.000

a. Dependent Variable: Rapport Student report of rapport with teacher.

Estimates of Fixed Effects[a]

Parameter	Estimate	Std. Error	df	t	Sig.	95% Confidence Interval	
						Lower Bound	Upper Bound
Intercept	3.179516	.061965	80.223	51.311	.000	3.056206	3.302826

a. Dependent Variable: Rapport Student report of rapport with teacher.

Covariance Parameters

Estimates of Covariance Parameters[a]

Parameter		Estimate	Std. Error
Residual		.165516	.018606
Intercept [subject = SubID]	Variance	.254590	.049485

a. Dependent Variable: Rapport Student report of rapport with teacher.

The Empty Model

$$Var(Y_{ij}) = Var(u_i) + Var(\varepsilon_{ij})$$
$$= \sigma_0^2 + \sigma^2$$
$$= .255 + .166 = .421$$

Covariance Parameters

Estimates of Covariance Parameters[a]

Parameter		Estimate	Std. Error
Residual		.165516	.018606
Intercept [subject = SubID]	Variance	.254590	.049485

a. Dependent Variable: Rapport Student report of rapport with teacher.

The Empty Model

$$Var(Y_{ij}) = Var(u_i) + Var(\varepsilon_{ij})$$
$$= \sigma_0^2 + \sigma^2$$
$$= .255 + .166 = .421$$

$$Cov(Y_{ij}, Y_{ij'}) = Var(u_i) = \sigma_0^2$$
$$= .255$$

Covariance Parameters

Estimates of Covariance Parameters[a]

Parameter		Estimate	Std. Error
Residual		.165516	.018606
Intercept [subject = SubID]	Variance	.254590	.049485

a. Dependent Variable: Rapport Student report of rapport with teacher.

The Empty Model

$$Var(Y_{ij}) = Var(u_i) + Var(\varepsilon_{ij})$$
$$= \sigma_0^2 + \sigma^2$$
$$= .255 + .166 = .421$$

$$Cov(Y_{ij,} Y_{ij'}) = Var(u_i) = \sigma_0^2$$
$$= .255$$

Covariance Parameters

Estimates of Covariance Parameters[a]

Parameter		Estimate	Std. Error
Residual		.165516	.018606
Intercept [subject = SubID]	Variance	.254590	.049485

a. Dependent Variable: Rapport Student report of rapport with teacher.

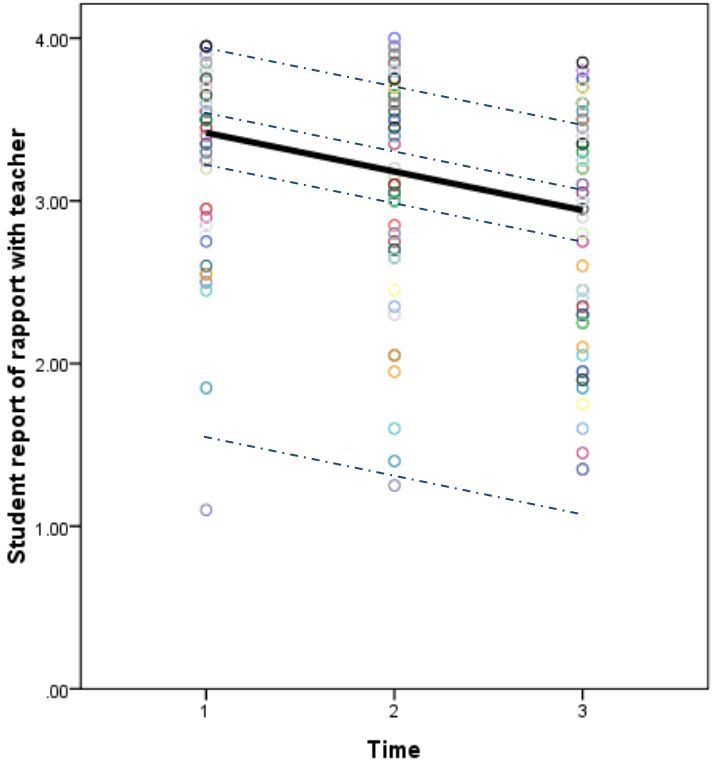
$$Corr\ (Y_{ij,} Y_{ij'}) = \frac{\sigma_0^2}{\sigma_0^2 + \sigma^2}$$
$$= .255/.421 = .606$$

Adding Predictors to the Random Intercept Model

$Rapport_{ij} = \beta_0 + \beta_1 Time + u_i + \varepsilon_{ij}$

$u_i \sim$ iid $N(0, \sigma_0^2)$
for subject I

$\varepsilon_{ij} \sim$ iid $N(0, \sigma^2)$
for outcome j



Adding Predictors to the Random Intercept Model

Mixed Model Analysis

```
[DataSet1] C:
```

Model Dimension[b]

		Number of Levels	Covariance Structure	Number of Parameters	Subject Variables
Fixed Effects	Intercept	1		1	
	Time	1		1	
Random Effects	Intercept[a]	1	Variance Components	1	SubID
Residual				1	
Total		3		4	

a. As of version 11.5, the syntax rules for the RANDOM subcommand have changed. Your command syntax may you are using SPSS 11 syntax, please consult the current syntax reference guide for more information.

b. Dependent Variable: Rapport Student report of rapport with teacher.

Information Criteria[a]

-2 Restricted Log Likelihood	330.955
Akaike's Information Criterion (AIC)	334.955
Hurvich and Tsai's Criterion (AICC)	335.006
Bozdogan's Criterion (CAIC)	343.891
Schwarz's Bayesian Criterion (BIC)	341.891

The information criteria are displayed in smaller-is-better forms.

a. Dependent Variable: Rapport Student report of rapport with teacher.

Adding Predictors to the Random Intercept Model

Fixed Effects

Type III Tests of Fixed Effects[a]

Source	Numerator df	Denominator df	F	Sig.
Intercept	1	189.006	2012.731	.000
Time	1	157.570	78.144	.000

a. Dependent Variable: Rapport Student report of rapport with teacher.

Estimates of Fixed Effects[a]

Parameter	Estimate	Std. Error	df	t	Sig.	95% Confidence Interval	
						Lower Bound	Upper Bound
Intercept	3.647511	.081302	189.006	44.863	.000	3.487134	3.807888
Time	-.234027	.026474	157.570	-8.840	.000	-.286317	-.181737

a. Dependent Variable: Rapport Student report of rapport with teacher.

Covariance Parameters

Estimates of Covariance Parameters[a]

Parameter		Estimate	Std. Error
Residual		.111558	.012581
Intercept [subject = SubID]	Variance	.270281	.048856

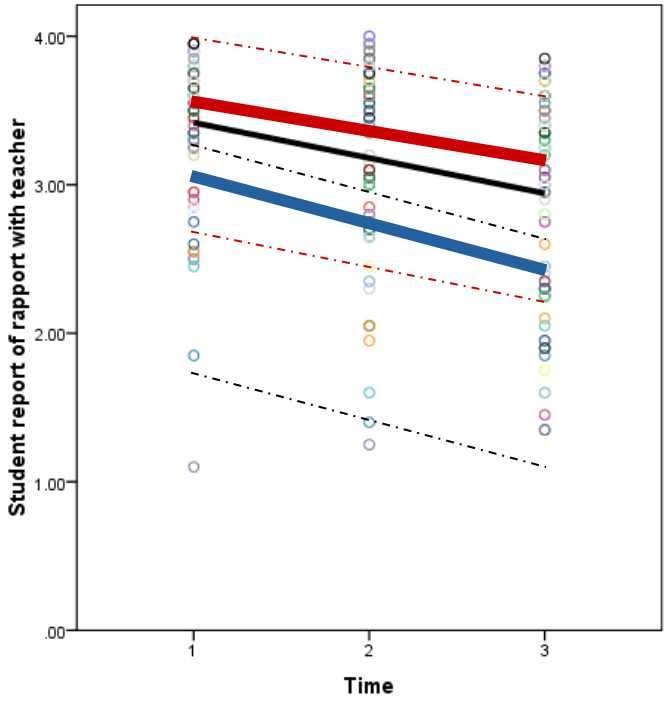
a. Dependent Variable: Rapport Student report of rapport with teacher.

The Random Intercept Model

$Rapport_{ij} = \beta_0 + \beta_1 Time + \beta_2 t0TchExp + \beta_3 Time*t0TchExp + u_i + \varepsilon_{ij}$

$u_i \sim$ iid $N(0, \sigma_0^2)$
for subject I

$\varepsilon_{ij} \sim$ iid $N(0, \sigma^2)$
for outcome j



The Random Intercept Model

Mixed Model Analysis

[DataSet1] C:

Model Dimension[b]

		Number of Levels	Covariance Structure	Number of Parameters	Subject Variables
Fixed Effects	Intercept	1		1	
	Time	1		1	
	t0TchExp	1		1	
	Time * t0TchExp	1		1	
Random Effects	Intercept[a]	1	Variance Components	1	SubID
Residual				1	
Total		5		6	

a. As of version 11.5, the syntax rules for the RANDOM subcommand have changed. Your command syntax ma you are using SPSS 11 syntax, please consult the current syntax reference guide for more information.

b. Dependent Variable: Rapport Student report of rapport with teacher.

Information Criteria[a]

-2 Restricted Log Likelihood	324.102
Akaike's Information Criterion (AIC)	328.102
Hurvich and Tsai's Criterion (AICC)	328.154
Bozdogan's Criterion (CAIC)	337.004
Schwarz's Bayesian Criterion (BIC)	335.004

The information criteria are displayed in smaller-is-better forms.

a. Dependent Variable: Rapport Student report of rapport with teacher.

The Random Intercept Model

Fixed Effects

Type III Tests of Fixed Effects[a]

Source	Numerator df	Denominator df	F	Sig.
Intercept	1	192.754	13.397	.000
Time	1	155.059	1.059	.305
t0TchExp	1	192.673	6.227	.013
Time * t0TchExp	1	155.045	.019	.890

a. Dependent Variable: Rapport Student report of rapport with teacher.

Estimates of Fixed Effects[a]

Parameter	Estimate	Std. Error	df	t	Sig.	95% Confidence Interval	
						Lower Bound	Upper Bound
Intercept	2.177248	.594854	192.754	3.660	.000	1.003989	3.350506
Time	-.204735	.198964	155.059	-1.029	.305	-.597765	.188295
t0TchExp	.452304	.181253	192.673	2.495	.013	.094809	.809800
Time * t0TchExp	-.008389	.060619	155.045	-.138	.890	-.128135	.111356

a. Dependent Variable: Rapport Student report of rapport with teacher.

Covariance Parameters

Estimates of Covariance Parameters[a]

Parameter		Estimate	Std. Error
Residual		.112200	.012744
Intercept [subject = SubID]	Variance	.238225	.044439

a. Dependent Variable: Rapport Student report of rapport with teacher.

Random Intercept Model Comparison

	Model 1 Empty Model	Model 2 Time as Fixed	Model 3: Time, Exp, Time*Exp as Fixed
-2LL	389.6	331.0	324.1
Residual Variance	.166	.112	.112
Subject Variance	.255	.270	.238
Subject effect is	Height variation around the overall intercept=grand mean	Height variation around the overall intercept=mean at time=0	Height variation around intercept for subjects' expectancy level
ICC	.61	.71	.68

The Random Intercept Model: Specification

General Specification of a Linear Mixed Model

$Rapport_{ij} = \beta_0 + \beta_1 Time + \beta_2\ t0TchExp + \beta_3\ Time*t0TchExp + u_i + \varepsilon_{ij}$

$\mathbf{Y_i} = \mathbf{X_i\beta} + Z_i u_i + \varepsilon_i$

$$E\begin{bmatrix} u_i \\ \varepsilon_i \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \qquad Var\begin{bmatrix} u_i \\ \varepsilon_i \end{bmatrix} = \begin{bmatrix} G & 0 \\ 0 & \Sigma \end{bmatrix}$$

$u_i \sim N(0, G)$

$$G = Var(u_i) = \begin{bmatrix} \sigma_0^{\ 2} \end{bmatrix}$$

$\varepsilon_{ij} \sim N(0, \Sigma)$

$$\mathbf{\Sigma} = Var(\varepsilon_i) = \begin{bmatrix} \sigma^2 & 0 & 0 \\ 0 & \sigma^2 & 0 \\ 0 & 0 & \sigma^2 \end{bmatrix}$$

Specify a Random Intercept Model

Define Single Outcome Variable

Define Model Fixed Effects and whether each is categorical or continuous

$$Rapport_{ij} = \beta_0 + \beta_1 Time + \beta_2\, t0TchExp + \beta_3\, Time*t0TchExp + u_i + \varepsilon_{ij}$$

Define the random effects: which aspects of a subjects' trajectory vary—intercept

Define the Subject i: who gets a unique intercept

$$\mathbf{u}_i = \begin{bmatrix} u_{0i} \end{bmatrix} \quad \sim N(0, G) \qquad \varepsilon_{ij} \sim N(0, \Sigma)$$

$$\mathbf{G} = Var(u_i) = \begin{bmatrix} \sigma_o^2 \end{bmatrix}$$

Define a covariance structure for G

$$\mathbf{\Sigma} = Var(\varepsilon_i) = \begin{bmatrix} \sigma^2 & 0 & 0 \\ 0 & \sigma^2 & 0 \\ 0 & 0 & \sigma^2 \end{bmatrix}$$

Define a covariance structure for Σ

The Random Intercept Model
Comparison to the Marginal Model

Variance of Y in a Random Intercept Model

$$\mathbf{Y} = \mathbf{X}\beta + \mathbf{Z}u + \varepsilon$$

$$\mathbf{G} = Var(u_i) = \left[\sigma_o^2\right]$$

$$\mathbf{\Sigma} = Var(\varepsilon_i) = \begin{bmatrix} \sigma^2 & 0 & 0 \\ 0 & \sigma^2 & 0 \\ 0 & 0 & \sigma^2 \end{bmatrix}$$

$$Var(Y_{ij}) = \mathbf{ZGZ'} + \mathbf{\Sigma}$$

Variance of Y in a Random Intercept Model

$$\mathbf{Y} = \mathbf{X}\beta + \mathbf{Z}u + \varepsilon$$

$$\mathbf{G} = Var(u_i) = \left[\sigma_o^2\right]$$

$$\mathbf{\Sigma} = Var(\varepsilon_i) = \begin{bmatrix} \sigma^2 & 0 & 0 \\ 0 & \sigma^2 & 0 \\ 0 & 0 & \sigma^2 \end{bmatrix}$$

$$Var(Y_{ij}) = \mathbf{ZGZ'} + \mathbf{\Sigma}$$

$$ZGZ' = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \left[\sigma_0^2\right] \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} \sigma_0^2 & \sigma_0^2 & \sigma_0^2 \\ \sigma_0^2 & \sigma_0^2 & \sigma_0^2 \\ \sigma_0^2 & \sigma_0^2 & \sigma_0^2 \end{bmatrix}$$

Variance of Y in a Random Intercept Model

$$\mathbf{Y} = \mathbf{X}\beta + \mathbf{Z}u + \varepsilon$$

$$\mathbf{G} = Var(u_i) = \left[\sigma_o^2\right]$$

$$\mathbf{\Sigma} = Var(\varepsilon_i) = \begin{bmatrix} \sigma^2 & 0 & 0 \\ 0 & \sigma^2 & 0 \\ 0 & 0 & \sigma^2 \end{bmatrix}$$

$$Var(Y_{ij}) = \mathbf{ZGZ'} + \mathbf{\Sigma}$$

$$ZGZ'+\Sigma = \begin{bmatrix} \sigma_0^2 & \sigma_0^2 & \sigma_0^2 \\ \sigma_0^2 & \sigma_0^2 & \sigma_0^2 \\ \sigma_0^2 & \sigma_0^2 & \sigma_0^2 \end{bmatrix} + \begin{bmatrix} \sigma^2 & 0 & 0 \\ 0 & \sigma^2 & 0 \\ 0 & 0 & \sigma^2 \end{bmatrix}$$

Variance of Y in a Random Intercept Model

$$\mathbf{Y} = \mathbf{X}\beta + \mathbf{Z}u + \varepsilon$$

$$\mathbf{G} = Var(u_i) = \left[\sigma_o^2\right]$$

$$\mathbf{\Sigma} = Var(\varepsilon_i) = \begin{bmatrix} \sigma^2 & 0 & 0 \\ 0 & \sigma^2 & 0 \\ 0 & 0 & \sigma^2 \end{bmatrix}$$

$$Var(Y_{ij}) = \mathbf{ZGZ'} + \mathbf{\Sigma}$$

$$ZGZ'+\Sigma = \begin{bmatrix} \sigma_0^2 & \sigma_0^2 & \sigma_0^2 \\ \sigma_0^2 & \sigma_0^2 & \sigma_0^2 \\ \sigma_0^2 & \sigma_0^2 & \sigma_0^2 \end{bmatrix} + \begin{bmatrix} \sigma^2 & 0 & 0 \\ 0 & \sigma^2 & 0 \\ 0 & 0 & \sigma^2 \end{bmatrix}$$

$$= \begin{bmatrix} \sigma_0^2 + \sigma^2 & \sigma_0^2 & \sigma_0^2 \\ \sigma_0^2 & \sigma_0^2 + \sigma^2 & \sigma_0^2 \\ \sigma_0^2 & \sigma_0^2 & \sigma_0^2 + \sigma^2 \end{bmatrix}$$

The Random Intercept Model

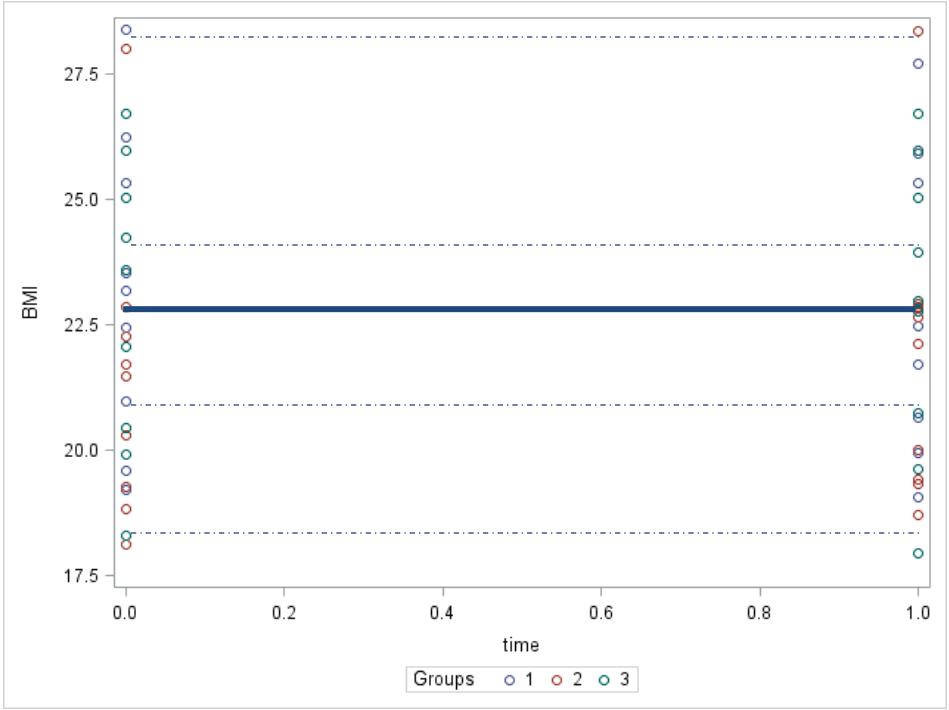
Example: Physical Training

The Empty Model: Physical Training

$$BMI_{ij} = \beta_0 + u_i + \varepsilon_{ij}$$

$u_i \sim$ iid $N(0, \sigma_0^2)$
for subject i, i = 1 to 27

$\varepsilon_{ij} \sim$ iid $N(0, \sigma^2)$
for outcome j, j = 1 to 2



The Empty Model: Physical Training

The SAS System

The Mixed Procedure

Model Information	
Data Set	WORK.TRAINING
Dependent Variable	BMI
Covariance Structure	Variance Components
Subject Effect	id
Estimation Method	REML
Residual Variance Method	Profile
Fixed Effects SE Method	Model-Based
Degrees of Freedom Method	Containment

Class Level Information		
Class	Levels	Values
time	2	0 1
group	3	1 2 3

Dimensions	
Covariance Parameters	2
Columns in X	1
Columns in Z Per Subject	1
Subjects	27
Max Obs Per Subject	2

Number of Observations	
Number of Observations Read	54
Number of Observations Used	54
Number of Observations Not Used	0

Iteration History			
Iteration	Evaluations	-2 Res Log Like	Criterion
0	1	267.18235847	
1	1	175.41787688	0.00000000

Convergence criteria met.

The Empty Model: Physical Training

Covariance Parameter Estimates		
Cov Parm	Subject	Estimate
Intercept	id	8.4146
Residual		0.1425

Fit Statistics	
-2 Res Log Likelihood	175.4
AIC (smaller is better)	179.4
AICC (smaller is better)	179.7
BIC (smaller is better)	182.0

Solution for Fixed Effects					
Effect	Estimate	Standard Error	DF	t Value	Pr > |t|
Intercept	22.5166	0.5606	26	40.16	<.0001

$Var(Y_{ij}) = \sigma_0^2 + \sigma^2$

$= 8.41 + .14 = 8.55$

$Cov(Y_{ij}, Y_{ij'}) = Var(u_i) =$

$= 8.41$

$ICC = 8.41/(8.55)$

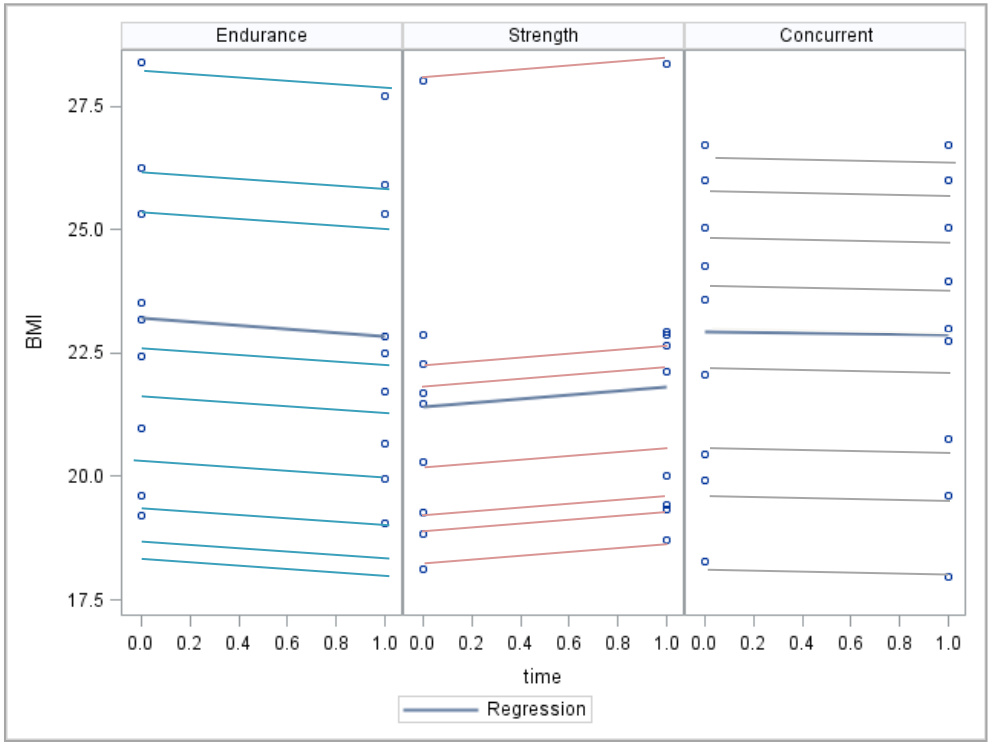
$= .983$

Adding Fixed Effects: Physical Training

$BMI_{ij} = \beta_0 + \beta_1 Time + \beta_2 Group_i + \beta_3 Time*Group_i + u_i + \varepsilon_{ij}$

$u_i \sim$ iid $N(0, \sigma_0^2)$
for subject i, i = 1 to 27

$\varepsilon_{ij} \sim$ iid $N(0, \sigma^2)$
for outcome j, j = 1 to 2



Adding Fixed Effects: Physical Training

The SAS System

The Mixed Procedure

Model Information	
Data Set	WORK._SGSRT2_
Dependent Variable	BMI
Covariance Structure	Variance Components
Subject Effect	id
Estimation Method	REML
Residual Variance Method	Profile
Fixed Effects SE Method	Model-Based
Degrees of Freedom Method	Containment

Class Level Information		
Class	Levels	Values
time	2	0 1
group	3	Concurrent Endurance Strength

Dimensions	
Covariance Parameters	2
Columns in X	12
Columns in Z Per Subject	1
Subjects	27
Max Obs Per Subject	2

Number of Observations	
Number of Observations Read	54
Number of Observations Used	54
Number of Observations Not Used	0

Iteration History			
Iteration	Evaluations	-2 Res Log Like	Criterion
0	1	253.74989020	
1	1	164.02970987	0.00000000

Convergence criteria met.

Adding Fixed Effects: Physical Training

Covariance Parameter Estimates		
Cov Parm	Subject	Estimate
Intercept	id	8.6875
Residual		0.1052

Fit Statistics	
-2 Res Log Likelihood	164.0
AIC (smaller is better)	168.0
AICC (smaller is better)	168.3
BIC (smaller is better)	170.6

ICC= 8.6875/(.1052 + 8.6875)

= .988

Adding Fixed Effects: Physical Training

Type 3 Tests of Fixed Effects				
Effect	Num DF	Den DF	F Value	Pr > F
group	2	24	0.62	0.5454
time	1	24	0.01	0.9138
time*group	2	24	6.28	0.0064

Least Squares Means							
Effect	time	Groups	Estimate	Standard Error	DF	t Value	Pr > |t|
time*group	0	Concurrent	22.9245	0.9884	24	23.19	<.0001
time*group	0	Endurance	23.2153	0.9884	24	23.49	<.0001
time*group	0	Strength	21.4246	0.9884	24	21.68	<.0001
time*group	1	Concurrent	22.8634	0.9884	24	23.13	<.0001
time*group	1	Endurance	22.8507	0.9884	24	23.12	<.0001
time*group	1	Strength	21.8213	0.9884	24	22.08	<.0001

Random Intercept Model Comparison

Model 2: Full Model

Covariance Parameter Estimates		
Cov Parm	Subject	Estimate
Intercept	id	8.6875
Residual		0.1052

Fit Statistics	
-2 Res Log Likelihood	164.0
AIC (smaller is better)	168.0
AICC (smaller is better)	168.3
BIC (smaller is better)	170.6

Model 3: Repeated Model

Estimated R Matrix for Subject 1		
Row	Col1	Col2
1	8.7927	8.6875
2	8.6875	8.7927

Covariance Parameter Estimates		
Cov Parm	Subject	Estimate
CS	id	8.6875
Residual		0.1052

Fit Statistics	
-2 Res Log Likelihood	164.0
AIC (smaller is better)	168.0
AICC (smaller is better)	168.3
BIC (smaller is better)	170.6

Random Intercept Model Comparison

Model 2: Full Model

Covariance Parameter Estimates		
Cov Parm	Subject	Estimate
Intercept	id	8.6875
Residual		0.1052

Model 3: Full Marginal Model

Estimated R Matrix for Subject 1		
Row	Col1	Col2
1	8.7927	8.6875
2	8.6875	8.7927

Covariance Parameter Estimates		
Cov Parm	Subject	Estimate
CS	id	8.6875
Residual		0.1052

$Var(Y_{ij}) = \sigma_0^2 + \sigma^2$

$= 8.6875 + .1052 = 8.7927$

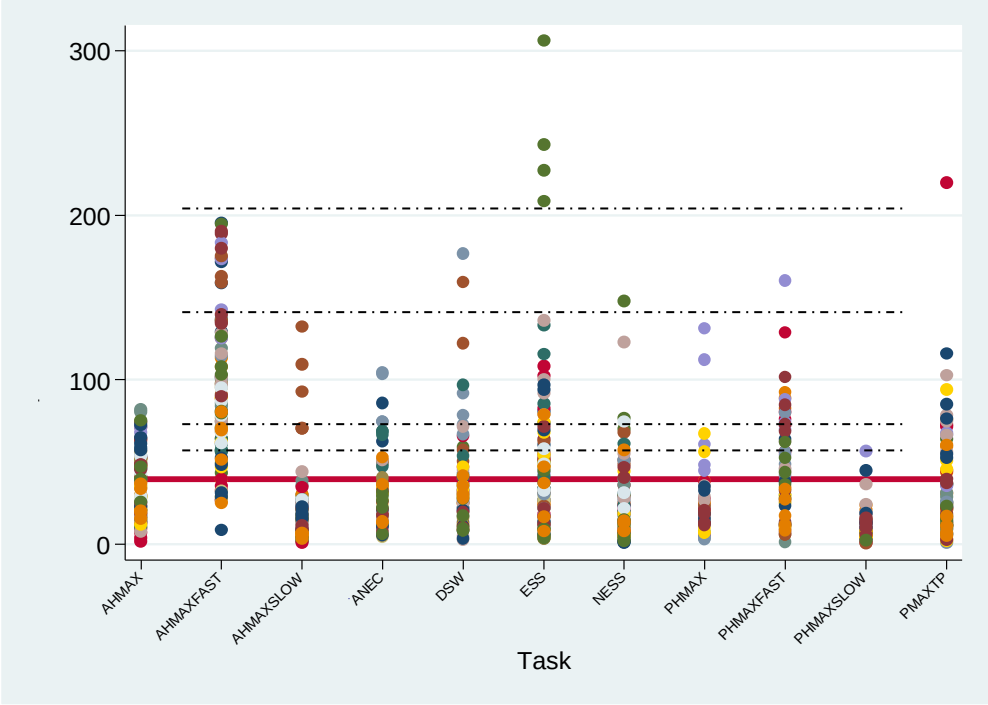
$Cov(Y_{ij}, Y_{ij'}) = Var(u_i) = 8.6875$

ICC= 8.6875/8.7927 = .988

The Random Intercept Model

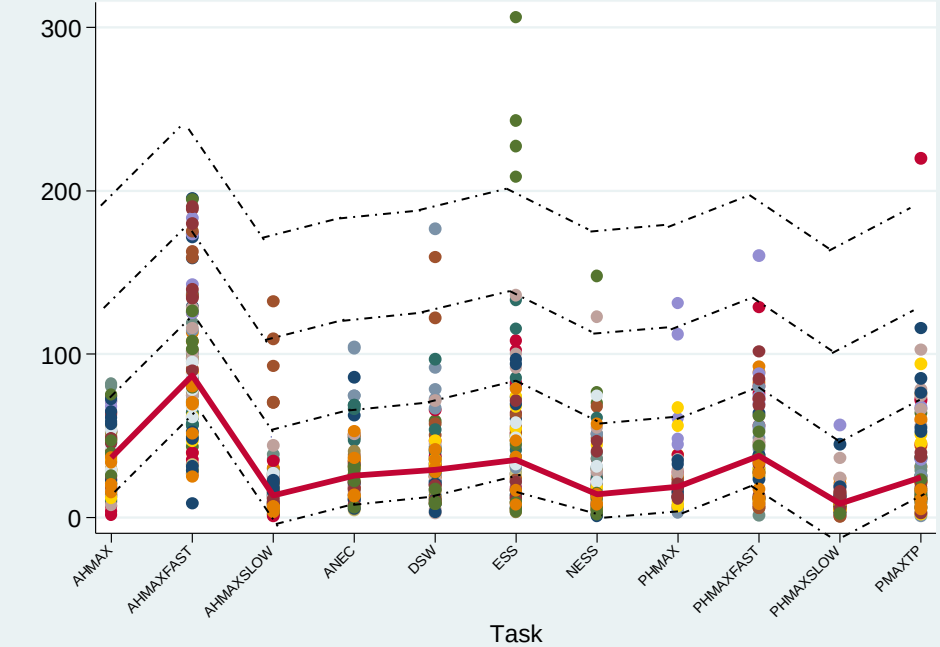
Example: Swallowing

Swallowing Data



$$RiseSlope_{ij} = \beta_0 + u_i + \varepsilon_{ij}$$

Swallowing Data



$$RiseSlope_{ij} = \beta_0 + \beta_1 Task1 + \beta_2 Task2 + \ldots + \beta_{10} Task10 + u_i + \varepsilon_{ij}$$

Swallowing Data

Empty Model

Akaike's information criterion and Bayesian information criterion

Model	Obs	ll(null)	ll(model)	df	AIC	BIC
.	845	.	-4283.515	3	8573.029	8587.247

Note: N=Obs used in calculating BIC; see [R] BIC note

Full Model

Akaike's information criterion and Bayesian information criterion

Model	Obs	ll(null)	ll(model)	df	AIC	BIC
.	845	.	-4091.984	13	8209.968	8271.579

Note: N=Obs used in calculating BIC; see [R] BIC note

Swallowing Data

Empty Model

Random-effects Parameters	Estimate	Std. Err.	[95% Conf.	Interval]
Participan~D: Identity var(_cons)	81.152	40.455	30.547	215.592
var(Residual)	1448.569	71.362	1315.242	1595.410

LR test vs. linear regression: chibar2(01) = 16.71 Prob >= chibar2 = 0.0000

Intraclass correlation

Level	ICC	Std. Err.	[95% Conf.	Interval]
ParticipantID	.0530501	.0253036	.0204475	.1307003

Full Model

Random-effects Parameters	Estimate	Std. Err.	[95% Conf.	Interval]
Participan~D: Identity var(_cons)	98.736	41.972	42.918	227.149
var(Residual)	974.460	48.276	884.288	1073.826

LR test vs. linear regression: chibar2(01) = 37.66 Prob >= chibar2 = 0.0000

Residual intraclass correlation

Level	ICC	Std. Err.	[95% Conf.	Interval]
ParticipantID	.0920015	.0359013	.041816	.1904459

Swallowing Data

Empty Model

```
Mixed-effects REML regression                   Number of obs      =       845
Group variable: ParticipantID                   Number of groups   =        19

                                                Obs per group: min =        27
                                                               avg =      44.5
                                                               max =        55

                                                Wald chi2(0)       =         .
Log restricted-likelihood = -4283.5145          Prob > chi2        =         .
```

RiseSlopecm	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
_cons	39.805	2.462	16.17	0.000	34.979	44.631

Full Model

```
Mixed-effects REML regression                   Number of obs      =       845
Group variable: ParticipantID                   Number of groups   =        19

                                                Obs per group: min =        27
                                                               avg =      44.5
                                                               max =        55

                                                Wald chi2(10)      =    410.76
Log restricted-likelihood = -4091.9838          Prob > chi2        =    0.0000
```

RiseSlopecm	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
Task						
AHMAXFAST	55.965	4.529	12.36	0.000	47.088	64.843
AHMAXSLOW	-17.681	4.681	-3.78	0.000	-26.856	-8.506
ANEC	-5.293	4.804	-1.10	0.271	-14.708	4.123
DSW	-1.538	4.907	-0.31	0.754	-11.156	8.080
ESS	13.302	4.542	2.93	0.003	4.400	22.203
NESS	-13.002	4.593	-2.83	0.005	-22.005	-3.999
PHMAX	-13.364	5.544	-2.41	0.016	-24.229	-2.499
PHMAXFAST	7.153	5.176	1.38	0.167	-2.992	17.298
PHMAXSLOW	-26.828	5.420	-4.95	0.000	-37.452	-16.204
PMAXTP	-1.404	4.872	-0.29	0.773	-10.952	8.145
_cons	37.386	3.931	9.51	0.000	29.681	45.091