Analyzing Repeated Measures Data

Module 4: The Marginal Model

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Workshop Outline: The Marginal Model

1. The Model and the Concepts

2. Covariance Structures

3. Example: The Marginal Model in the Physical Training Data Set

4. Example: The Marginal Model in the Swallowing Data Set

The Marginal Model
Teacher Data Set

The Marginal Model

Do children's summertime expectancies of their teachers (t0TchExp) affect the quality of the teacher-student relationship as reported by children at any time point?

$$Rapport_{ij} = \beta_0 + \beta_1 Time1 + \beta_2 Time2 + \beta_3 t0TchExp + \beta_4 Time1*t0TchExp + \beta_5 Time2*t0TchExp + \varepsilon_{ij}$$

$$R = \begin{bmatrix} [\Sigma] & & & & & & \\ 0 & [\Sigma] & & & & & \\ 0 & 0 & [\Sigma] & & & & \\ 0 & 0 & 0 & . & & & \\ 0 & 0 & 0 & 0 & . & & \\ 0 & 0 & 0 & 0 & 0 & . & \\ 0 & 0 & 0 & 0 & 0 & 0 & [\Sigma] \end{bmatrix}$$

$\varepsilon_{ij} \sim$ iid $N(0, \Sigma)$
for subject i and time j

$$\Sigma = \begin{bmatrix} Var_1 & Cov_{1,2} & Cov_{1,3} \\ Cov_{1,2} & Var_2 & Cov_{2,3} \\ Cov_{1,3} & Cov_{2,3} & Var_3 \end{bmatrix}$$

The Marginal Model

We can now:

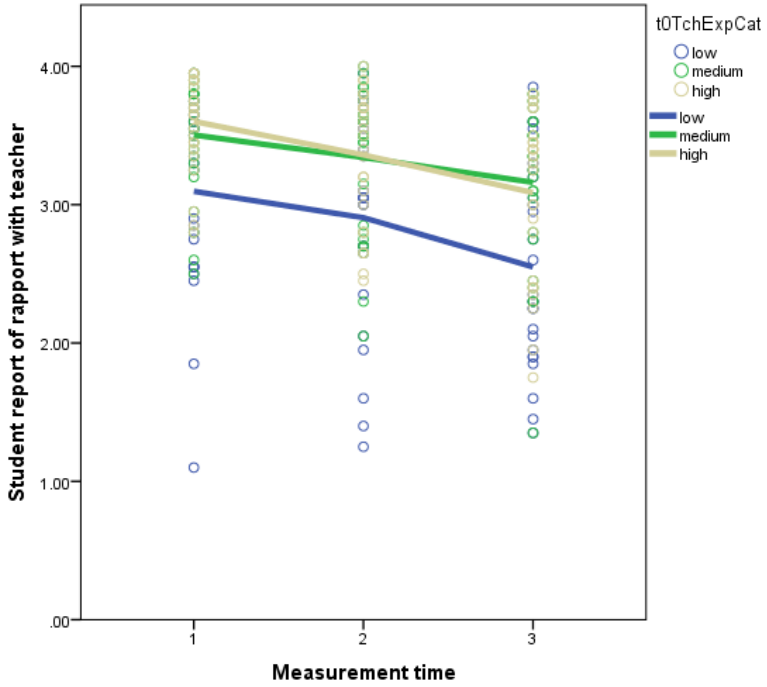
1. use long data format
2. choose the structure of Σ and measure model fit
3. get regression coefficients for the effect on univariate outcome
4. make Time categorical or continuous

We can still:

1. test the effects of between and within-subjects factors
2. test marginal means

We still cannot:

1. fit individual trajectories
2. fit repeats on more than one level of subject



$$Rapport_{ij} = \beta_0 + \beta_1 Time1 + \beta_2 Time2 + \beta_3 t0TchExp + \beta_4 Time1*t0TchExp + \beta_5 Time2*t0TchExp + \varepsilon_{ij}$$

Specifying the Marginal Model

Define single outcome variable

Define model fixed variables and whether each is categorical or continuous

$$Rapport_{ij} = \beta_0 + \beta_1 Time1 + \beta_2 Time2 + \beta_3 t0TchExp + \beta_4 Time1*t0TchExp + \beta_5 Time2*t0TchExp + \varepsilon_{ij}$$

$$R = \begin{bmatrix} [\Sigma] & & & & & & \\ 0 & [\Sigma] & & & & & \\ 0 & 0 & [\Sigma] & & & & \\ 0 & 0 & 0 & . & & & \\ 0 & 0 & 0 & 0 & . & & \\ 0 & 0 & 0 & 0 & 0 & . & \\ 0 & 0 & 0 & 0 & 0 & 0 & [\Sigma] \end{bmatrix}$$

$\varepsilon_{ij} \sim$ iid $N(0, \Sigma)$
for subject i and time j

Define the subject i: who gets a Σ block

$$\Sigma = \begin{bmatrix} Var_1 & Cov_{1,2} & Cov_{1,3} \\ Cov_{1,2} & Var_2 & Cov_{2,3} \\ Cov_{1,3} & Cov_{2,3} & Var_3 \end{bmatrix}$$

Define a covariance structure for Σ

Define the repeat Index j: What variable defines the rows and columns of Σ

Model Dimension[a]

		Number of Levels	Covariance Structure	Number of Parameters	Subject Variables	Number of Subjects
Fixed Effects	Intercept	1		1		
	Time	3		2		
	t0TchExp	1		1		
	Time * t0TchExp	3		2		
Repeated Effects	Time	3	Unstructured	6	SubID	80
Total		11		12		

a. Dependent Variable: Rapport Student report of rapport with teacher.

Information Criteria[a]

-2 Restricted Log Likelihood	301.159
Akaike's Information Criterion (AIC)	313.159
Hurvich and Tsai's Criterion (AICC)	313.534
Bozdogan's Criterion (CAIC)	339.814
Schwarz's Bayesian Criterion (BIC)	333.814

The information criteria are displayed in smaller-is-better forms.

a. Dependent Variable: Rapport Student report of rapport with teacher.

Fixed Effects

Type III Tests of Fixed Effects[a]

Source	Numerator df	Denominator df	F	Sig.
Intercept	1	78.523	16.623	.000
Time	2	78.591	1.138	.326
t0TchExp	1	78.423	9.796	.002
Time * t0TchExp	2	78.365	1.022	.365

a. Dependent Variable: Rapport Student report of rapport with teacher.

Estimates of Fixed Effects[b]

Parameter	Estimate	Std. Error	df	t	Sig.	95% Confidence Interval	
						Lower Bound	Upper Bound
Intercept	1.450703	.556660	77.552	2.606	.011	.342377	2.559029
[Time=1]	.410283	.436608	77.195	.940	.350	-.459079	1.279644
[Time=2]	.667391	.447154	80.684	1.493	.139	-.222357	1.557140
[Time=3]	0[a]	0	.	.	.	.	.
t0TchExp	.457627	.169615	77.512	2.698	.009	.119917	.795338
[Time=1] * t0TchExp	.016616	.133012	77.175	.125	.901	-.248234	.281467
[Time=2] * t0TchExp	-.119492	.135864	80.221	-.879	.382	-.389859	.150876
[Time=3] * t0TchExp	0[a]	0	.	.	.	.	.

a. This parameter is set to zero because it is redundant.

b. Dependent Variable: Rapport Student report of rapport with teacher.

SPSS, SAS, Stata: specify structure

R: specify pattern of diagonals and off-diagonals separately

Estimates of Covariance Parameters[a]

Parameter		Estimate	Std. Error
Repeated Measures	UN (1,1)	.242387	.038813
	UN (2,1)	.242013	.044039
	UN (2,2)	.376709	.060968
	UN (3,1)	.205796	.043770
	UN (3,2)	.273762	.055699
	UN (3,3)	.439275	.070575

a. Dependent Variable: Rapport Student report of rapport with teacher.

Residual Covariance (R) Matrix[a]

	[Time = 1]	[Time = 2]	[Time = 3]
[Time = 1]	.242387	.242013	.205796
[Time = 2]	.242013	.376709	.273762
[Time = 3]	.205796	.273762	.439275

Unstructured

a. Dependent Variable: Rapport Student report of rapport with teacher.

UN *Unstructured.* This is a completely general covariance matrix.

$$\begin{bmatrix} \sigma_1^2 & \sigma_{21} & \sigma_{31} & \sigma_{41} \\ \sigma_{21} & \sigma_2^2 & \sigma_{32} & \sigma_{42} \\ \sigma_{31} & \sigma_{32} & \sigma_3^2 & \sigma_{43} \\ \sigma_{41} & \sigma_{42} & \sigma_{43} & \sigma_4^2 \end{bmatrix}$$

Model Dimension[a]

		Number of Levels	Covariance Structure	Number of Parameters	Subject Variables	Number of Subjects
Fixed Effects	Intercept	1		1		
	Time	3		2		
	t0TchExp	1		1		
	Time * t0TchExp	3		2		
Repeated Effects	Time	3	Compound Symmetry	2	SubID	80
Total		11		8		

a. Dependent Variable: Rapport Student report of rapport with teacher.

Information Criteria[a]

-2 Restricted Log Likelihood	326.558
Akaike's Information Criterion (AIC)	330.558
Hurvich and Tsai's Criterion (AICC)	330.611
Bozdogan's Criterion (CAIC)	339.443
Schwarz's Bayesian Criterion (BIC)	337.443

The information criteria are displayed in smaller-is-better forms.

a. Dependent Variable: Rapport Student report of rapport with teacher.

Type III Tests of Fixed Effects[a]

Source	Numerator df	Denominator df	F	Sig.
Intercept	1	78.804	16.204	.000
Time	2	154.057	1.086	.340
t0TchExp	1	78.678	10.186	.002
Time * t0TchExp	2	153.952	.463	.630

a. Dependent Variable: Rapport Student report of rapport with teacher.

Estimates of Fixed Effects[b]

Parameter	Estimate	Std. Error	df	t	Sig.	95% Confidence Interval	
						Lower Bound	Upper Bound
Intercept	1.450630	.497759	121.100	2.914	.004	.465192	2.436068
[Time=1]	.410356	.398108	153.032	1.031	.304	-.376141	1.196852
[Time=2]	.589257	.415363	154.633	1.419	.158	-.231262	1.409776
[Time=3]	0[a]	0	.	.	.	.	.
t0TchExp	.457642	.151673	121.014	3.017	.003	.157365	.757919
[Time=1] * t0TchExp	.016602	.121292	153.018	.137	.891	-.223022	.256226
[Time=2] * t0TchExp	-.096926	.126029	154.466	-.769	.443	-.345889	.152037
[Time=3] * t0TchExp	0[a]	0	.	.	.	.	.

a. This parameter is set to zero because it is redundant.

b. Dependent Variable: Rapport Student report of rapport with teacher.

SPSS, SAS

Estimates of Covariance Parameters[a]

Parameter		Estimate	Std. Error
Repeated Measures	CS diagonal offset	.112301	.012839
	CS covariance	.238979	.044581

a. Dependent Variable: Rapport Student report of rapport with teacher.

Residual Covariance (R) Matrix[a]

	[Time = 1]	[Time = 2]	[Time = 3]
[Time = 1]	.351281	.238979	.238979
[Time = 2]	.238979	.351281	.238979
[Time = 3]	.238979	.238979	.351281

Compound Symmetry

a. Dependent Variable: Rapport Student report of rapport with teacher.

CS *Compound symmetry.* This structure has constant variance and constant covariance.

SPSS manual:

$$\begin{bmatrix} \sigma^2+\sigma_1 & \sigma_1 & \sigma_1 & \sigma_1 \\ \sigma_1 & \sigma^2+\sigma_1 & \sigma_1 & \sigma_1 \\ \sigma_1 & \sigma_1 & \sigma^2+\sigma_1 & \sigma_1 \\ \sigma_1 & \sigma_1 & \sigma_1 & \sigma^2+\sigma_1 \end{bmatrix}$$

Stata

Random-effects Parameters		Estimate	Std. Err.	[95% Conf.	Interval]
SubID:	(empty)				
Residual: Exchangeable					
	var(e)	0.351281	0.045172	0.273021	0.451973
	cov(e)	0.238979	0.044581	0.151602	0.326357

$$\begin{bmatrix} \sigma^2 & \sigma_1 & \sigma_1 & \sigma_1 \\ \sigma_1 & \sigma^2 & \sigma_1 & \sigma_1 \\ \sigma_1 & \sigma_1 & \sigma^2 & \sigma_1 \\ \sigma_1 & \sigma_1 & \sigma_1 & \sigma^2 \end{bmatrix}$$

Covariance Structures for Sigma
Teacher Data Set

Common Covariance Structures

- Unstructured
- Compound Symmetry/Exchangeable
- Identity/Independence/Variance Components
- Diagonal/Banded Main Diagonal
- Autoregressive Lag 1
- Heterogeneous Autoregressive
- Heterogeneous Compound Symmetry
- Toeplitz

SPSS

Estimates of Covariance Parameters[a]

Parameter		Estimate	Std. Error
Repeated Measures	Variance	.351225	.032681

a. Dependent Variable: Rapport Student report of rapport with teacher.

Residual Covariance (R) Matrix[a]

	[Time = 1]	[Time = 2]	[Time = 3]
[Time = 1]	0	0	0
[Time = 2]	0	0	0
[Time = 3]	0	0	0

Identity

a. Dependent Variable: Rapport Student report of rapport with teacher.

Stata: Independence

SAS: Variance Components

R: leave out correlation option

ID *Identity.* This is a scaled identity matrix.

$$\sigma^2 \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \sigma^2 & 0 & 0 & 0 \\ 0 & \sigma^2 & 0 & 0 \\ 0 & 0 & \sigma^2 & 0 \\ 0 & 0 & 0 & \sigma^2 \end{bmatrix}$$

SPSS

Estimates of Covariance Parameters[a]

Parameter		Estimate	Std. Error
Repeated Measures	Var: [Time=1]	.242387	.038813
	Var: [Time=2]	.372295	.060394
	Var: [Time=3]	.440680	.071022

a. Dependent Variable: Rapport Student report of rapport with teacher.

Residual Covariance (R) Matrix[a]

	[Time = 1]	[Time = 2]	[Time = 3]
[Time = 1]	0	0	0
[Time = 2]	0	0	0
[Time = 3]	0	0	0

Diagonal

DIAG *Diagonal.* This is a diagonal structure with heterogenous variance. This is the default covariance structure for repeated effects.

$$\begin{bmatrix} \sigma_1^2 & 0 & 0 & 0 \\ 0 & \sigma_2^2 & 0 & 0 \\ 0 & 0 & \sigma_3^2 & 0 \\ 0 & 0 & 0 & \sigma_4^2 \end{bmatrix}$$

SAS: Banded Main Diagonal: un(1)

$$\begin{bmatrix} \sigma_1^2 & \sigma_{12} & \sigma_{13} & 0 & 0 \\ \sigma_{12} & \sigma_2^2 & \sigma_{23} & \sigma_{24} & 0 \\ \sigma_{13} & \sigma_{23} & \sigma_3^2 & \sigma_{34} & \sigma_{35} \\ 0 & \sigma_{24} & \sigma_{34} & \sigma_4^2 & \sigma_{45} \\ 0 & 0 & \sigma_{35} & \sigma_{45} & \sigma_5^2 \end{bmatrix}$$

Estimated R Matrix for Subject 1

Row	Col1	Col2	Col3
1	0.2424		
2		0.3723	
3			0.4407

Stata: Banded q: ba 0

time	1	2	3
1	**0.24239**		
2	**0.00000**	**0.37229**	
3	**0.00000**	**0.00000**	**0.44068**

SPSS, SAS, Stata, R

Estimates of Covariance Parameters[a]

Parameter		Estimate	Std. Error
Repeated Measures	AR1 diagonal	.346905	.043019
	AR1 rho	.711913	.043053

a. Dependent Variable: Rapport Student report of rapport with teacher.

Residual Covariance (R) Matrix[a]

	[Time = 1]	[Time = 2]	[Time = 3]
[Time = 1]	.346905	.246966	.175818
[Time = 2]	.246966	.346905	.246966
[Time = 3]	.175818	.246966	.346905

First-Order Autoregressive

a. Dependent Variable: Rapport Student report of rapport with teacher.

AR1 *First-order autoregressive.*

$$\sigma^2 \begin{bmatrix} 1 & \rho & \rho^2 & \rho^3 \\ \rho & 1 & \rho & \rho^2 \\ \rho^2 & \rho & 1 & \rho \\ \rho^3 & \rho^2 & \rho & 1 \end{bmatrix}$$

SPSS, SAS

Estimates of Covariance Parameters[a]

Parameter		Estimate	Std. Error
Repeated Measures	Var: [Time=1]	.227855	.035078
	Var: [Time=2]	.376204	.060796
	Var: [Time=3]	.468555	.075400
	ARH1 rho	.738812	.041279

a. Dependent Variable: Rapport Student report of rapport with teacher.

Residual Covariance (R) Matrix[a]

	[Time = 1]	[Time = 2]	[Time = 3]
[Time = 1]	.227855	.216309	.178352
[Time = 2]	.216309	.376204	.310188
[Time = 3]	.178352	.310188	.468555

Heterogeneous First-Order Autoregressive

a. Dependent Variable: Rapport Student report of rapport with teacher.

Stata: not an option

R: corAR1 + weights

ARH1 *Heterogenous first-order autoregressive.*

$$\begin{bmatrix} \sigma_1^2 & \sigma_2\sigma_1\rho & \sigma_3\sigma_1\rho^2 & \sigma_4\sigma_1\rho^3 \\ \sigma_2\sigma_1\rho & \sigma_2^2 & \sigma_3\sigma_2\rho & \sigma_4\sigma_2\rho^2 \\ \sigma_3\sigma_1\rho^2 & \sigma_3\sigma_2\rho & \sigma_3^2 & \sigma_4\sigma_3\rho \\ \sigma_4\sigma_1\rho^3 & \sigma_4\sigma_2\rho^2 & \sigma_4\sigma_3\rho & \sigma_4^2 \end{bmatrix}$$

SPSS, SAS

Estimates of Covariance Parameters[a]

Parameter		Estimate	Std. Error
Repeated Measures	Var: [Time=1]	.239083	.037847
	Var: [Time=2]	.362114	.057484
	Var: [Time=3]	.461300	.074262
	CSH rho	.701432	.047035

a. Dependent Variable: Rapport Student report of rapport with teacher.

Residual Covariance (R) Matrix[a]

	[Time = 1]	[Time = 2]	[Time = 3]
[Time = 1]	.239083	.206387	.232944
[Time = 2]	.206387	.362114	.286682
[Time = 3]	.232944	.286682	.461300

Heterogeneous Compound Symmetry

a. Dependent Variable: Rapport Student report of rapport with teacher.

CSH *Heterogenous compound symmetry.* This structure has non-constant variance and constant correlation.

$$\begin{bmatrix} \sigma_1^2 & \sigma_2\sigma_1\rho & \sigma_3\sigma_1\rho & \sigma_4\sigma_1\rho \\ \sigma_2\sigma_1\rho & \sigma_2^2 & \sigma_3\sigma_2\rho & \sigma_4\sigma_2\rho \\ \sigma_3\sigma_1\rho & \sigma_3\sigma_2\rho & \sigma_3^2 & \sigma_4\sigma_3\rho \\ \sigma_4\sigma_1\rho & \sigma_4\sigma_2\rho & \sigma_4\sigma_3\rho & \sigma_4^2 \end{bmatrix}$$

Stata: not an option

R: corCompSymm + weights

SPSS

Estimates of Covariance Parameters[a]

Parameter		Estimate	Std. Error
Repeated Measures	TP diagonal	.348923	.044585
	TP rho 1	.713958	.045238
	TP rho 2	.602513	.074459

a. Dependent Variable: Rapport Student report of rapport with teacher.

Residual Covariance (R) Matrix[a]

	[Time = 1]	[Time = 2]	[Time = 3]
[Time = 1]	.348923	.249116	.210230
[Time = 2]	.249116	.348923	.249116
[Time = 3]	.210230	.249116	.348923

Toeplitz

a. Dependent Variable: Rapport Student report of rapport with teacher.

TP *Toeplitz*

$$\sigma^2 \begin{bmatrix} 1 & \rho_1 & \rho_2 & \rho_3 \\ \rho_1 & 1 & \rho_1 & \rho_2 \\ \rho_2 & \rho_1 & 1 & \rho_1 \\ \rho_3 & \rho_2 & \rho_1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \sigma^2 & \sigma_1 & \sigma_2 & \sigma_3 \\ \sigma_1 & \sigma^2 & \sigma_1 & \sigma_2 \\ \sigma_2 & \sigma_1 & \sigma^2 & \sigma_1 \\ \sigma_3 & \sigma_2 & \sigma_1 & \sigma^2 \end{bmatrix}$$

SAS, Stata

Estimated R Matrix for Subject 1

Row	Col1	Col2	Col3
1	0.3489	0.2491	0.2102
2	0.2491	0.3489	0.2491
3	0.2102	0.2491	0.3489

Covariance Parameter Estimates

Cov Parm	Subject	Estimate
TOEP(2)	SubID	0.2491
TOEP(3)	SubID	0.2102
Residual		0.3489

R: ARMA p=2, q=0

Unstructured

12 parameters

Information Criteria[a]

-2 Restricted Log Likelihood	301.159
Akaike's Information Criterion (AIC)	313.159
Hurvich and Tsai's Criterion (AICC)	313.534
Bozdogan's Criterion (CAIC)	339.814
Schwarz's Bayesian Criterion (BIC)	333.814

Residual Covariance (R) Matrix[a]

	[Time = 1]	[Time = 2]	[Time = 3]
[Time = 1]	.242387	.242013	.205796
[Time = 2]	.242013	.376709	.273762
[Time = 3]	.205796	.273762	.439275

Unstructured

Compound Symmetry

8 parameters

Information Criteria[a]

-2 Restricted Log Likelihood	326.558
Akaike's Information Criterion (AIC)	330.558
Hurvich and Tsai's Criterion (AICC)	330.611
Bozdogan's Criterion (CAIC)	339.443
Schwarz's Bayesian Criterion (BIC)	337.443

Residual Covariance (R) Matrix[a]

	[Time = 1]	[Time = 2]	[Time = 3]
[Time = 1]	.351281	.238979	.238979
[Time = 2]	.238979	.351281	.238979
[Time = 3]	.238979	.238979	.351281

Compound Symmetry

Identity

7 parameters

Information Criteria[a]

-2 Restricted Log Likelihood	435.026
Akaike's Information Criterion (AIC)	437.026
Hurvich and Tsai's Criterion (AICC)	437.044
Bozdogan's Criterion (CAIC)	441.469
Schwarz's Bayesian Criterion (BIC)	440.469

Estimates of Covariance Parameters[a]

Parameter		Estimate	Std. Error
Repeated Measures	Variance	.351225	.032681

Diagonal

9 parameters

Information Criteria[a]

-2 Restricted Log Likelihood	427.995
Akaike's Information Criterion (AIC)	433.995
Hurvich and Tsai's Criterion (AICC)	434.101
Bozdogan's Criterion (CAIC)	447.323
Schwarz's Bayesian Criterion (BIC)	444.323

Estimates of Covariance Parameters[a]

Parameter		Estimate	Std. Error
Repeated Measures	Var: [Time=1]	.242387	.038813
	Var: [Time=2]	.372295	.060394
	Var: [Time=3]	.440680	.071022

Autoregressive

8 parameters

Information Criteria[a]

-2 Restricted Log Likelihood	324.835
Akaike's Information Criterion (AIC)	328.835
Hurvich and Tsai's Criterion (AICC)	328.888
Bozdogan's Criterion (CAIC)	337.720
Schwarz's Bayesian Criterion (BIC)	335.720

Residual Covariance (R) Matrix[a]

	[Time = 1]	[Time = 2]	[Time = 3]
[Time = 1]	.346905	.246966	.175818
[Time = 2]	.246966	.346905	.246966
[Time = 3]	.175818	.246966	.346905

Heterogeneous Autoregressive

10 parameters

-2 Restricted Log Likelihood	308.796
Akaike's Information Criterion (AIC)	316.796
Hurvich and Tsai's Criterion (AICC)	316.973
Bozdogan's Criterion (CAIC)	334.565
Schwarz's Bayesian Criterion (BIC)	330.565

Residual Covariance (R) Matrix[a]

	[Time = 1]	[Time = 2]	[Time = 3]
[Time = 1]	.227855	.216309	.178352
[Time = 2]	.216309	.376204	.310188
[Time = 3]	.178352	.310188	.468555

Heterogeneous Compound Symmetry

10 parameters

-2 Restricted Log Likelihood	310.743
Akaike's Information Criterion (AIC)	318.743
Hurvich and Tsai's Criterion (AICC)	318.920
Bozdogan's Criterion (CAIC)	336.512
Schwarz's Bayesian Criterion (BIC)	332.512

Residual Covariance (R) Matrix[a]

	[Time = 1]	[Time = 2]	[Time = 3]
[Time = 1]	.239083	.206387	.232944
[Time = 2]	.206387	.362114	.286682
[Time = 3]	.232944	.286682	.461300

Toeplitz

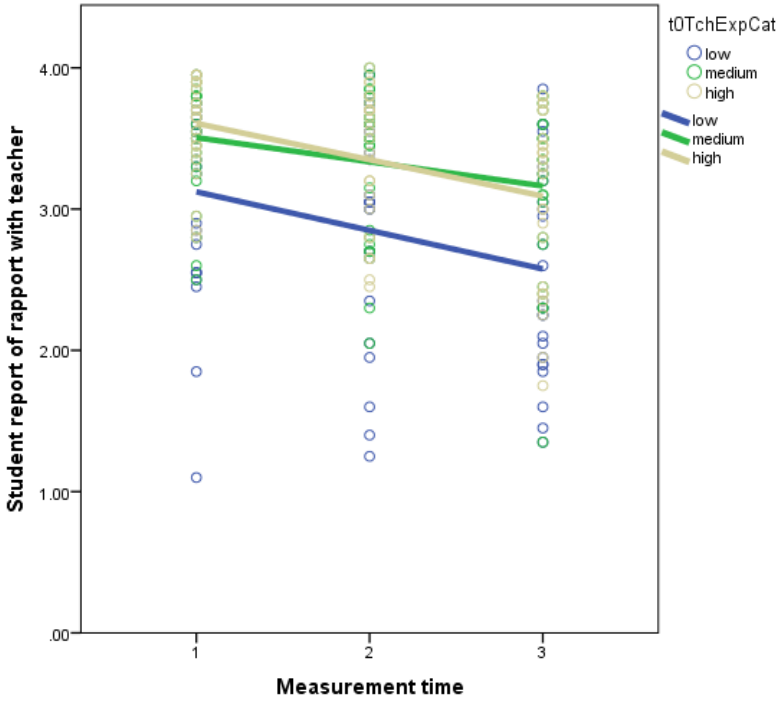
9 parameters

-2 Restricted Log Likelihood	322.236
Akaike's Information Criterion (AIC)	328.236
Hurvich and Tsai's Criterion (AICC)	328.341
Bozdogan's Criterion (CAIC)	341.563
Schwarz's Bayesian Criterion (BIC)	338.563

Residual Covariance (R) Matrix[a]

	[Time = 1]	[Time = 2]	[Time = 3]
[Time = 1]	.348923	.249116	.210230
[Time = 2]	.249116	.348923	.249116
[Time = 3]	.210230	.249116	.348923

Fit Time as Continuous



$$Rapport_{ij} = \beta_0 + \beta_1 Time + \beta_2\ tOTchExp + \beta_3\ Time*tOTchExp + \varepsilon_{ij}$$

Model Dimension[a]

		Number of Levels	Covariance Structure	Number of Parameters	Subject Variables	Number of Subjects
Fixed Effects	Intercept	1		1		
	Time	1		1		
	t0TchExp	1		1		
	t0TchExp * Time	1		1		
Repeated Effects	Time	3	Heterogeneous First-Order Autoregressive	4	SubID	80
Total		7		8		

a. Dependent Variable: Rapport Student report of rapport with teacher.

Information Criteria[a]

-2 Restricted Log Likelihood	306.995
Akaike's Information Criterion (AIC)	314.995
Hurvich and Tsai's Criterion (AICC)	315.171
Bozdogan's Criterion (CAIC)	332.799
Schwarz's Bayesian Criterion (BIC)	328.799

The information criteria are displayed in smaller-is-better forms.

a. Dependent Variable: Rapport Student report of rapport with teacher.

Fixed Effects

Type III Tests of Fixed Effects[a]

Source	Numerator df	Denominator df	F	Sig.
Intercept	1	93.199	15.425	.000
Time	1	145.936	.428	.514
t0TchExp	1	93.077	10.889	.001
t0TchExp * Time	1	146.119	.079	.779

a. Dependent Variable: Rapport Student report of rapport with teacher.

Estimates of Fixed Effects[a]

Parameter	Estimate	Std. Error	df	t	Sig.	95% Confidence Interval	
						Lower Bound	Upper Bound
Intercept	1.978532	.503763	93.199	3.928	.000	.978186	2.978877
Time	-.159397	.243615	145.936	-.654	.514	-.640866	.322073
t0TchExp	.506584	.153518	93.077	3.300	.001	.201731	.811436
t0TchExp * Time	-.020856	.074223	146.119	-.281	.779	-.167545	.125833

a. Dependent Variable: Rapport Student report of rapport with teacher.

Covariance Parameters

Estimates of Covariance Parameters[a]

Parameter		Estimate	Std. Error
Repeated Measures	Var: [Time=1]	.228355	.035177
	Var: [Time=2]	.373975	.060478
	Var: [Time=3]	.467885	.075302
	ARH1 rho	.735843	.041592

a. Dependent Variable: Rapport Student report of rapport with teacher.

Residual Covariance (R) Matrix[a]

	[Time = 1]	[Time = 2]	[Time = 3]
[Time = 1]	.228355	.215036	.176989
[Time = 2]	.215036	.373975	.307805
[Time = 3]	.176989	.307805	.467885

Heterogeneous First-Order Autoregressive

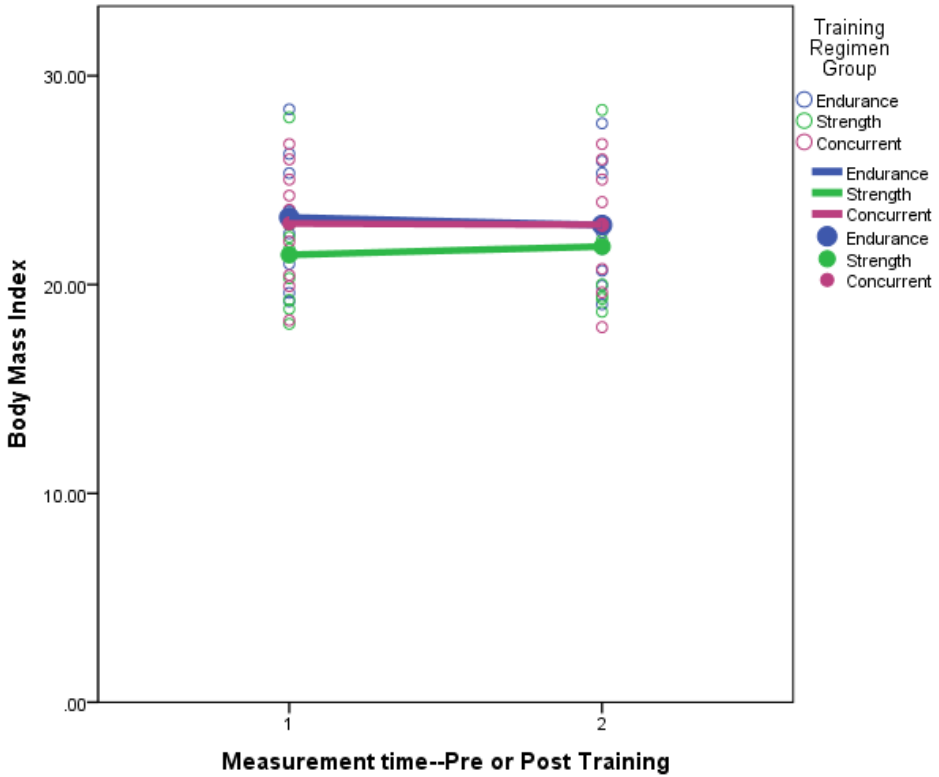
a. Dependent Variable: Rapport Student report of rapport with teacher.

The Marginal Model
Physical Training Data Set

2.2 Physical Training

Research Question:

Do the three training regimens differ in their effect on BMI from pre-training to post-training measurements?



$$BMI_{ij} = \beta_0 + \beta_1 Time + \beta_2 Endurance + \beta_3 Strength + \beta_4 Time*Endurance + \beta_4 Time*Strength + \varepsilon_{ij}$$

The SAS System

The Mixed Procedure

Model Information	
Data Set	WORK.TRAINING
Dependent Variable	BMI
Covariance Structure	Unstructured
Subject Effect	id
Estimation Method	REML
Residual Variance Method	None
Fixed Effects SE Method	Model-Based
Degrees of Freedom Method	Between-Within

Class Level Information		
Class	Levels	Values
time	2	0 1
group	3	1 2 3

Dimensions	
Covariance Parameters	3
Columns in X	12
Columns in Z	0
Subjects	27
Max Obs Per Subject	2

Number of Observations	
Number of Observations Read	54
Number of Observations Used	54
Number of Observations Not Used	0

Iteration History			
Iteration	Evaluations	-2 Res Log Like	Criterion
0	1	253.74989020	
1	1	163.92176446	0.00000000

Convergence criteria met.

Estimated R Matrix for Subject 1		
Row	Col1	Col2
1	8.8836	8.6875
2	8.6875	8.7019

Covariance Parameter Estimates		
Cov Parm	Subject	Estimate
UN(1,1)	id	8.8836
UN(2,1)	id	8.6875
UN(2,2)	id	8.7019

Fit Statistics	
-2 Res Log Likelihood	163.9
AIC (smaller is better)	169.9
AICC (smaller is better)	170.5
BIC (smaller is better)	173.8

Null Model Likelihood Ratio Test		
DF	Chi-Square	Pr > ChiSq
2	89.83	<.0001

Type 3 Tests of Fixed Effects				
Effect	Num DF	Den DF	F Value	Pr > F
time	1	24	0.01	0.9138
group	2	24	0.62	0.5454
time*group	2	24	6.28	0.0064

Same Model with Compound Symmetry

The SAS System

The Mixed Procedure

Model Information	
Data Set	WORK.TRAINING
Dependent Variable	BMI
Covariance Structure	Compound Symmetry
Subject Effect	id
Estimation Method	REML
Residual Variance Method	Profile
Fixed Effects SE Method	Model-Based
Degrees of Freedom Method	Between-Within

Estimated R Matrix for Subject 1		
Row	Col1	Col2
1	8.7927	8.6875
2	8.6875	8.7927

Covariance Parameter Estimates		
Cov Parm	Subject	Estimate
CS	id	8.6875
Residual		0.1052

Fit Statistics	
-2 Res Log Likelihood	164.0
AIC (smaller is better)	168.0
AICC (smaller is better)	168.3
BIC (smaller is better)	170.6

Type 3 Tests of Fixed Effects				
Effect	Num DF	Den DF	F Value	Pr > F
time	1	24	0.01	0.9138
group	2	24	0.62	0.5454
time*group	2	24	6.28	0.0064

Least Squares Means							
Effect	time	Groups	Estimate	Standard Error	DF	t Value	Pr > |t|
time	0		22.5215	0.5707	24	39.47	<.0001
time	1		22.5118	0.5707	24	39.45	<.0001
group		1	23.0330	0.9855	24	23.37	<.0001
group		2	21.6229	0.9855	24	21.94	<.0001
group		3	22.8940	0.9855	24	23.23	<.0001
time*group	0	1	23.2153	0.9884	24	23.49	<.0001
time*group	0	2	21.4246	0.9884	24	21.68	<.0001
time*group	0	3	22.9245	0.9884	24	23.19	<.0001
time*group	1	1	22.8507	0.9884	24	23.12	<.0001
time*group	1	2	21.8213	0.9884	24	22.08	<.0001
time*group	1	3	22.8634	0.9884	24	23.13	<.0001

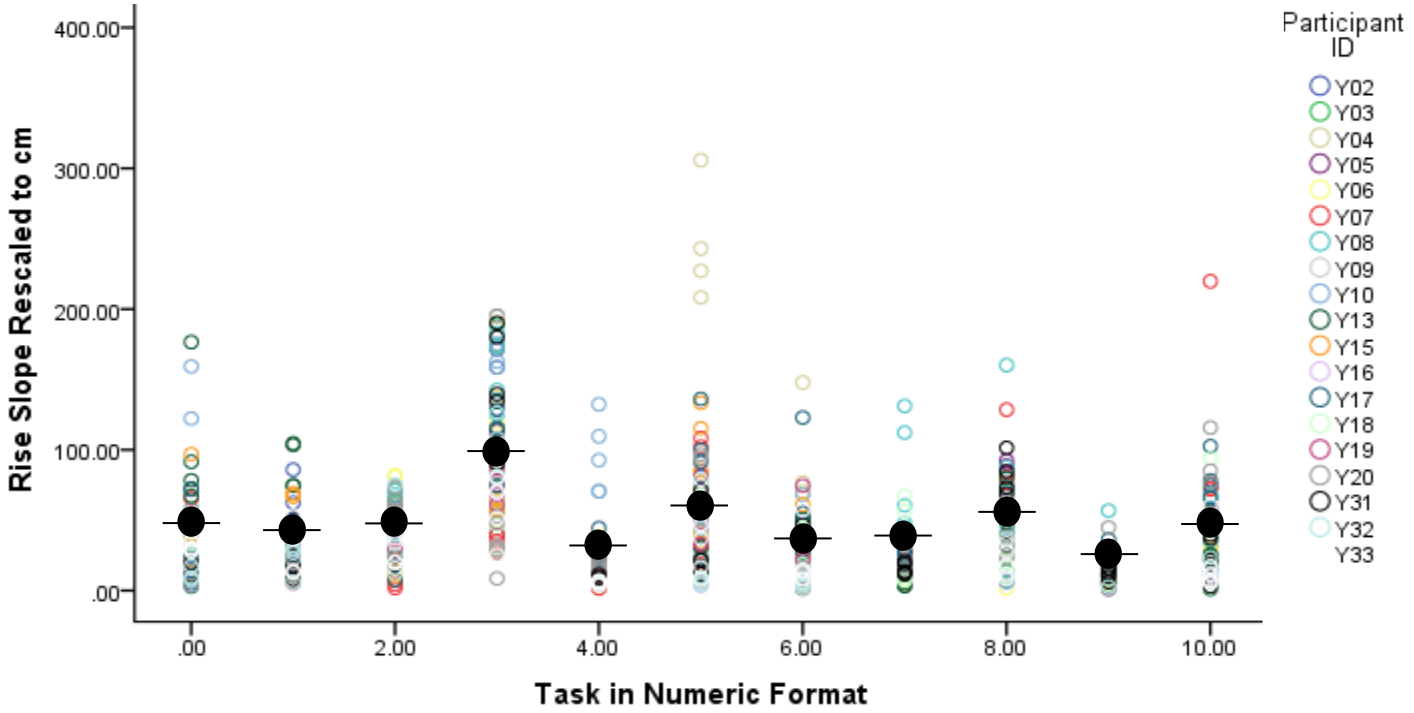
Differences of Least Squares Means											
Effect	time	Groups	_time	Groups	Estimate	Standard Error	DF	t Value	Pr > |t|	Adjustment	Adj P
time	0		1		0.009659	0.08829	24	0.11	0.9138	Tukey-Kramer	0.9138
group		1		2	1.4100	1.3936	24	1.01	0.3217	Tukey	0.5768
group		1		3	0.1390	1.3936	24	0.10	0.9214	Tukey	0.9945
group		2		3	-1.2711	1.3936	24	-0.91	0.3708	Tukey	0.6381
time*group	0	1	0	2	1.7907	1.3978	24	1.28	0.2124	Tukey-Kramer	0.7921
time*group	0	1	0	3	0.2907	1.3978	24	0.21	0.8370	Tukey-Kramer	0.9999
time*group	0	1	1	1	0.3646	0.1529	24	2.38	0.0254	Tukey-Kramer	0.2014
time*group	0	1	1	2	1.3940	1.3978	24	1.00	0.3286	Tukey-Kramer	0.9143
time*group	0	1	1	3	0.3518	1.3978	24	0.25	0.8034	Tukey-Kramer	0.9998
time*group	0	2	0	3	-1.4999	1.3978	24	-1.07	0.2939	Tukey-Kramer	0.8872
time*group	0	2	1	1	-1.4261	1.3978	24	-1.02	0.3178	Tukey-Kramer	0.9066
time*group	0	2	1	2	-0.3967	0.1529	24	-2.59	0.0159	Tukey-Kramer	0.1374
time*group	0	2	1	3	-1.4389	1.3978	24	-1.03	0.3136	Tukey-Kramer	0.9034
time*group	0	3	1	1	0.07387	1.3978	24	0.05	0.9583	Tukey-Kramer	1.0000
time*group	0	3	1	2	1.1033	1.3978	24	0.79	0.4377	Tukey-Kramer	0.9667
time*group	0	3	1	3	0.06110	0.1529	24	0.40	0.6930	Tukey-Kramer	0.9985
time*group	1	1	1	2	1.0294	1.3978	24	0.74	0.4686	Tukey-Kramer	0.9752
time*group	1	1	1	3	-0.01277	1.3978	24	-0.01	0.9928	Tukey-Kramer	1.0000
time*group	1	2	1	3	-1.0422	1.3978	24	-0.75	0.4632	Tukey-Kramer	0.9739

The Marginal Model
Swallowing Data Set

2.3 Swallowing

Research Question:

Does the mean pressure rise slope in the anterior bulb differ across the eleven swallowing tasks, and do the nine experimental tasks differ from water and nectar-thick apple juice?



$$RiseSlope_{ij} = \beta_0 + \beta_1 Task1 + \beta_2 Task2 + \ldots + \beta_{10} Task10 + \varepsilon_{ij}$$

$$RiseSlope_{ijk} = \mu + \alpha_k + \varepsilon_{ijk}$$

```
## Generalized least squares fit by REML
##   Model: RiseSlopecm ~ Task
##   Data: swallowing
##        AIC      BIC    logLik
##   8209.968 8271.409 -4091.984
##
## Correlation Structure: Compound symmetry
##  Formula: ~Task | ParticipantID
##  Parameter estimate(s):
##       Rho
## 0.0920018
```

```
## Marginal variance covariance matrix
##          [,1]     [,2]     [,3]     [,4]     [,5]     [,6]     [,7]
##  [1,] 1073.200   98.736   98.736   98.736   98.736   98.736   98.736
##  [2,]   98.736 1073.200   98.736   98.736   98.736   98.736   98.736
##  [3,]   98.736   98.736 1073.200   98.736   98.736   98.736   98.736
##  [4,]   98.736   98.736   98.736 1073.200   98.736   98.736   98.736
##  [5,]   98.736   98.736   98.736   98.736 1073.200   98.736   98.736
##  [6,]   98.736   98.736   98.736   98.736   98.736 1073.200   98.736
##  [7,]   98.736   98.736   98.736   98.736   98.736   98.736 1073.200
##  [8,]   98.736   98.736   98.736   98.736   98.736   98.736   98.736
##  [9,]   98.736   98.736   98.736   98.736   98.736   98.736   98.736
## [10,]   98.736   98.736   98.736   98.736   98.736   98.736   98.736
## [11,]   98.736   98.736   98.736   98.736   98.736   98.736   98.736
## [12,]   98.736   98.736   98.736   98.736   98.736   98.736   98.736
## [13,]   98.736   98.736   98.736   98.736   98.736   98.736   98.736
## [14,]   98.736   98.736   98.736   98.736   98.736   98.736   98.736
## [15,]   98.736   98.736   98.736   98.736   98.736   98.736   98.736
## [16,]   98.736   98.736   98.736   98.736   98.736   98.736   98.736
```

```
## Denom. DF: 834
##              numDF  F-value p-value
## (Intercept)      1 92.10662  <.0001
## Task            10 41.03174  <.0001
```

```
## Coefficients:
##                     Value Std.Error   t-value p-value
## (Intercept)      37.38596  3.931170  9.510136  0.0000
## TaskAHMAXFAST    55.96546  4.529343 12.356200  0.0000
## TaskAHMAXSLOW   -17.68107  4.681060 -3.777150  0.0002
## TaskANEC         -5.29267  4.803780 -1.101771  0.2709
## TaskDSW          -1.53781  4.907124 -0.313382  0.7541
## TaskESS          13.30188  4.541684  2.928843  0.0035
## TaskNESS        -13.00216  4.593454 -2.830585  0.0048
## TaskPHMAX       -13.36429  5.543519 -2.410795  0.0161
## TaskPHMAXFAST     7.15288  5.176211  1.381875  0.1674
## TaskPHMAXSLOW   -26.82823  5.420434 -4.949462  0.0000
## TaskPMAXTP       -1.40369  4.871752 -0.288128  0.7733
```

```
lsmeans(model11,"Task",df=94.475) ###for example

##  Task         lsmean       SE    df  lower.CL  upper.CL
##  AHMAX     37.38596 3.931170 94.47 29.581045  45.19088
##  AHMAXFAST 93.35143 3.931170 94.47 85.546507 101.15635
##  AHMAXSLOW 19.70490 4.105056 94.47 11.554749  27.85505
##  ANEC      32.09330 4.244462 94.47 23.666372  40.52022
##  DSW       35.84816 4.361081 94.47 27.189699  44.50662
##  ESS       50.68784 3.945383 94.47 42.854705  58.52098
##  NESS      24.38380 4.004869 94.47 16.432566  32.33504
##  PHMAX     24.02168 5.066534 94.47 13.962615  34.08074
##  PHMAXFAST 44.53884 4.661793 94.47 35.283348  53.79433
##  PHMAXSLOW 10.55773 4.931558 94.47  0.766649  20.34881
##  PMAXTP    35.98228 4.321241 94.47 27.402913  44.56164
##
## Confidence level used: 0.95
```

```
##   Simultaneous Confidence Intervals
##
## Multiple Comparisons of Means: Tukey Contrasts
```

```
## Linear Hypotheses:
##                                Estimate   lwr        upr
## AHMAXFAST - AHMAX == 0          55.96546   41.41327   70.51765
## AHMAXSLOW - AHMAX == 0         -17.68107  -32.72070   -2.64143
## ANEC - AHMAX == 0               -5.29267  -20.72659   10.14125
## DSW - AHMAX == 0                -1.53781  -17.30376   14.22815
## ESS - AHMAX == 0                13.30188   -1.28996   27.89372
## NESS - AHMAX == 0              -13.00216  -27.76033    1.75601
## PHMAX - AHMAX == 0             -13.36429  -31.17489    4.44632
## PHMAXFAST - AHMAX == 0           7.15288   -9.47762   23.78337
## PHMAXSLOW - AHMAX == 0         -26.82823  -44.24338   -9.41308
## PMAXTP - AHMAX == 0             -1.40369  -17.05599   14.24862
## AHMAXSLOW - AHMAXFAST == 0     -73.64653  -88.68616  -58.60689
## ANEC - AHMAXFAST == 0          -61.25813  -76.69205  -45.82421
## DSW - AHMAXFAST == 0           -57.50327  -73.26922  -41.73731
## ESS - AHMAXFAST == 0           -42.66358  -57.25542  -28.07174
## NESS - AHMAXFAST == 0          -68.96762  -83.72579  -54.20945
## PHMAX - AHMAXFAST == 0         -69.32975  -87.14036  -51.51914
## PHMAXFAST - AHMAXFAST == 0     -48.81258  -65.44308  -32.18209
## PHMAXSLOW - AHMAXFAST == 0     -82.79369 -100.20885  -65.37854
## PMAXTP - AHMAXFAST == 0        -57.36915  -73.02146  -41.71684
## ANEC - AHMAXSLOW == 0           12.38840   -3.51551   28.29230
## DSW - AHMAXSLOW == 0            16.14326   -0.07293   32.35945
## ESS - AHMAXSLOW == 0            30.98294   15.90587   46.06002
## NESS - AHMAXSLOW == 0            4.67891  -10.56003   19.91784
## PHMAX - AHMAXSLOW == 0           4.31678  -13.87356   22.50711
## PHMAXFAST - AHMAXSLOW == 0      24.83394    7.76385   41.90403
## PHMAXSLOW - AHMAXSLOW == 0      -9.14717  -26.95753    8.66319
## PMAXTP - AHMAXSLOW == 0         16.27738    0.17774   32.37701
## DSW - ANEC == 0                  3.75486  -12.74130   20.25102
## ESS - ANEC == 0                 18.59454    3.13053   34.05856
## NESS - ANEC == 0                -7.70949  -23.31940    7.90041
```

When Do Marginal Models Work?

1. When there are relatively few repeats or when a relatively simple covariance structure fits
2. When subjects are not clustered in some higher level
3. When the focus is on the average, not individual trajectories
4. When mixed models can't be calculated
5. When a random slope model can't be fit but you need more refinement in the covariance structure of Sigma