Analyzing Repeated Measures Data

Module 2: Repeated Measures ANOVA

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Workshop Outline: Repeated Measures ANOVA

1. Teacher: One Within-Subjects Factor
    - 1.1 The Multivariate Model
        - Covariance Matrix of Residuals – Unstructured and Spherical
    - 1.2 The Univariate Model

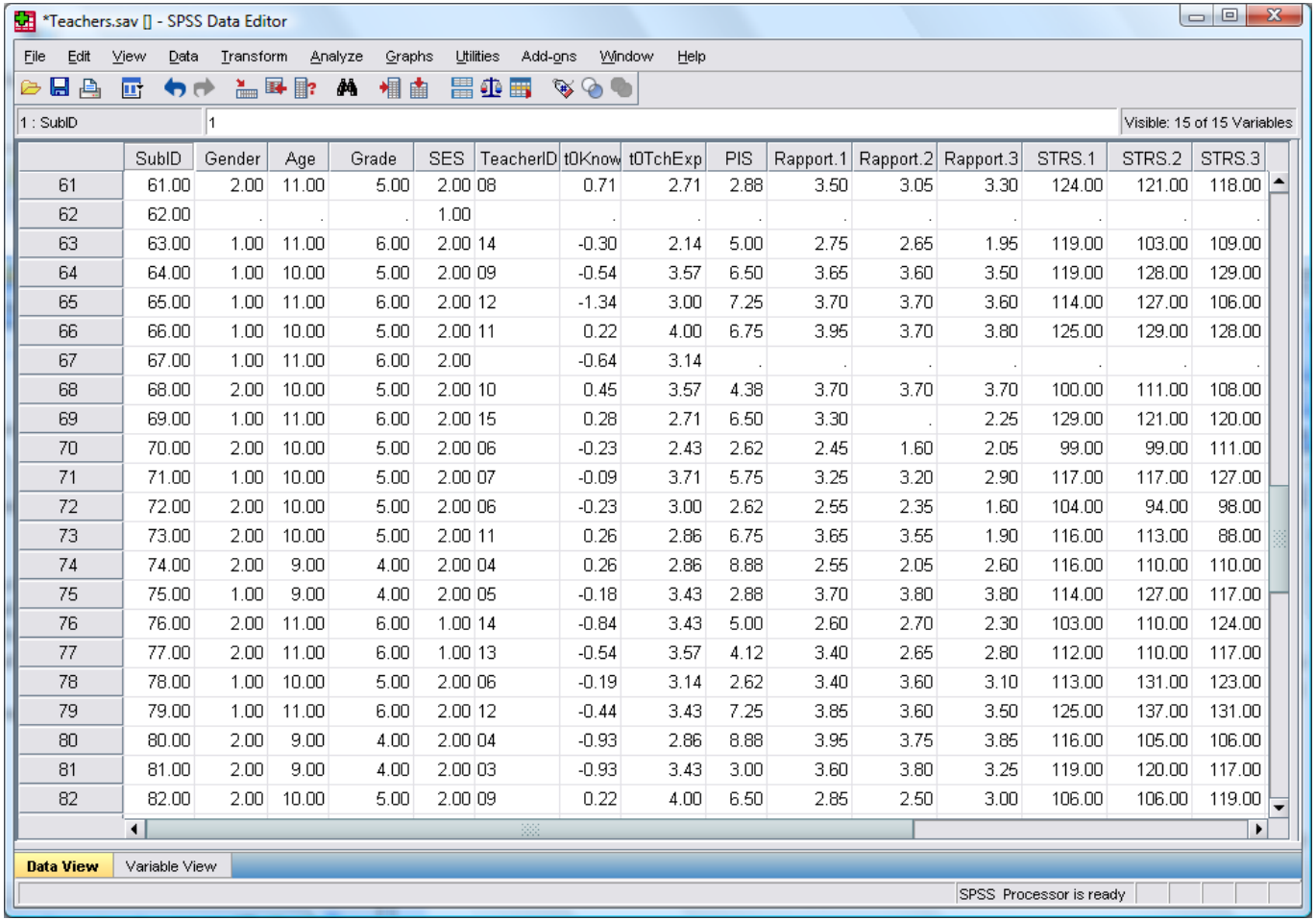
2. Physical Training: One Within- and One Between-Subjects Factor
3. Swallowing: Two Within-Subjects Factors

The Multivariate Model

Teacher Data Set: One Within-Subjects Factor

Research Question:

Do children's summertime expectancies of their teachers (t0TchExp) affect the quality of the teacher-student relationship as reported by children at any time point (t1Rapport, t2Rapport, t3Rapport)



	SubID	Gender	Age	Grade	SES	TeacherID	t0Know	t0TchExp	PIS	Rapport.1	Rapport.2	Rapport.3	STRS.1	STRS.2	STRS.3
61	61.00	2.00	11.00	5.00	2.00	08	0.71	2.71	2.88	3.50	3.05	3.30	124.00	121.00	118.00
62	62.00	.	.	.	1.00		.	.	.	.	.	.	.	.	.
63	63.00	1.00	11.00	6.00	2.00	14	-0.30	2.14	5.00	2.75	2.65	1.95	119.00	103.00	109.00
64	64.00	1.00	10.00	5.00	2.00	09	-0.54	3.57	6.50	3.65	3.60	3.50	119.00	128.00	129.00
65	65.00	1.00	11.00	6.00	2.00	12	-1.34	3.00	7.25	3.70	3.70	3.60	114.00	127.00	106.00
66	66.00	1.00	10.00	5.00	2.00	11	0.22	4.00	6.75	3.95	3.70	3.80	125.00	129.00	128.00
67	67.00	1.00	11.00	6.00	2.00		-0.64	3.14	.	.	.	.	.	.	.
68	68.00	2.00	10.00	5.00	2.00	10	0.45	3.57	4.38	3.70	3.70	3.70	100.00	111.00	108.00
69	69.00	1.00	11.00	6.00	2.00	15	0.28	2.71	6.50	3.30	.	2.25	129.00	121.00	120.00
70	70.00	2.00	10.00	5.00	2.00	06	-0.23	2.43	2.62	2.45	1.60	2.05	99.00	99.00	111.00
71	71.00	1.00	10.00	5.00	2.00	07	-0.09	3.71	5.75	3.25	3.20	2.90	117.00	117.00	127.00
72	72.00	2.00	10.00	5.00	2.00	06	-0.23	3.00	2.62	2.55	2.35	1.60	104.00	94.00	98.00
73	73.00	2.00	10.00	5.00	2.00	11	0.26	2.86	6.75	3.65	3.55	1.90	116.00	113.00	88.00
74	74.00	2.00	9.00	4.00	2.00	04	0.26	2.86	8.88	2.55	2.05	2.60	116.00	110.00	110.00
75	75.00	1.00	9.00	4.00	2.00	05	-0.18	3.43	2.88	3.70	3.80	3.80	114.00	127.00	117.00
76	76.00	2.00	11.00	6.00	1.00	14	-0.84	3.43	5.00	2.60	2.70	2.30	103.00	110.00	124.00
77	77.00	2.00	11.00	6.00	1.00	13	-0.54	3.57	4.12	3.40	2.65	2.80	112.00	110.00	117.00
78	78.00	1.00	10.00	5.00	2.00	06	-0.19	3.14	2.62	3.40	3.60	3.10	113.00	131.00	123.00
79	79.00	1.00	11.00	6.00	2.00	12	-0.44	3.43	7.25	3.85	3.60	3.50	125.00	137.00	131.00
80	80.00	2.00	9.00	4.00	2.00	04	-0.93	2.86	8.88	3.95	3.75	3.85	116.00	105.00	106.00
81	81.00	2.00	9.00	4.00	2.00	03	-0.93	3.43	3.00	3.60	3.80	3.25	119.00	120.00	117.00
82	82.00	2.00	10.00	5.00	2.00	09	0.22	4.00	6.50	2.85	2.50	3.00	106.00	106.00	119.00

The Repeated Measures Multivariate Model

$$Rapport_{ij} = \beta_0 + \beta_1 Time1 + \beta_2 Time2 + \beta_3 t0TchExp + \beta_4 t0TchExp*Time1 + \beta_5 t0TchExp*Time2 + \varepsilon_{ij}$$

$$[Rapport_{i1}\ Rapport_{i2}\ Rapport_{i3}] = \beta_0 + \beta_1 Time1 + \beta_2 Time2 + \beta_3 t0TchExp + \beta_4 t0TchExp*Time1 + \beta_5 t0TchExp*Time2 + [\varepsilon_{i1} + \varepsilon_{i2} + \varepsilon_{i3}]$$

$[\varepsilon_{i1} + \varepsilon_{i2} + \varepsilon_{i3}] \sim$ iid $N(0, \Sigma)$ for subject i

i = 1 to 82

$[\varepsilon_{i1} + \varepsilon_{i2} + \varepsilon_{i3}] \sim$ iid $N(0, \Sigma)$ for subject i

Unstructured

$$\Sigma = \begin{bmatrix} Var_1 & Cov_{1,2} & Cov_{1,3} \\ Cov_{1,2} & Var_2 & Cov_{2,3} \\ Cov_{1,3} & Cov_{2,3} & Var_3 \end{bmatrix} = \begin{bmatrix} \sigma_1^2 & \rho_{12} & \rho_{13} \\ \rho_{12} & \sigma_2^2 & \rho_{23} \\ \rho_{13} & \rho_{23} & \sigma_3^2 \end{bmatrix}$$

$[\varepsilon_{i1} + \varepsilon_{i2} + \varepsilon_{i3}] \sim$ iid N(0, Σ) for subject i

$$\begin{bmatrix} \sigma_1^2 & \rho_{12} & \rho_{13} \\ \rho_{12} & \sigma_2^2 & \rho_{23} \\ \rho_{13} & \rho_{23} & \sigma_3^2 \end{bmatrix} \quad \begin{matrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{matrix}$$

$$\begin{matrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{matrix} \quad \begin{bmatrix} \sigma_1^2 & \rho_{12} & \rho_{13} \\ \rho_{12} & \sigma_2^2 & \rho_{23} \\ \rho_{13} & \rho_{23} & \sigma_3^2 \end{bmatrix}$$

$$\bullet \bullet \bullet \quad \begin{bmatrix} \sigma_1^2 & \rho_{12} & \rho_{13} \\ \rho_{12} & \sigma_2^2 & \rho_{23} \\ \rho_{13} & \rho_{23} & \sigma_3^2 \end{bmatrix}$$

$[\varepsilon_{i1} + \varepsilon_{i2} + \varepsilon_{i3}] \sim$ iid N(0, Σ) for subject i

$$R = \begin{bmatrix} [\Sigma] & & & & & & \\ 0 & [\Sigma] & & & & & \\ 0 & 0 & [\Sigma] & & & & \\ 0 & 0 & 0 & . & & & \\ 0 & 0 & 0 & 0 & . & & \\ 0 & 0 & 0 & 0 & 0 & . & \\ 0 & 0 & 0 & 0 & 0 & 0 & [\Sigma] \end{bmatrix}$$

R is a 246*246 matrix

Σ is a 3*3 submatrix with an Unstructured Structure

Residual SSCP Matrix

		Rapport.1 Rapport.1: Time 1 Child report of relationship quality	Rapport.2 Rapport.2: Time 2 Child report of relationship quality	Rapport.3 Rapport.3: Time 3 Child report of relationship quality
Sum-of-Squares and Cross-Products	Rapport.1 Rapport.1: Time 1 Child report of relationship quality	17.799	17.858	15.240
	Rapport.2 Rapport.2: Time 2 Child report of relationship quality	17.858	28.177	20.519
	Rapport.3 Rapport.3: Time 3 Child report of relationship quality	15.240	20.519	33.109
Covariance	Rapport.1 Rapport.1: Time 1 Child report of relationship quality	.237	.238	.203
	Rapport.2 Rapport.2: Time 2 Child report of relationship quality	.238	.376	.274
	Rapport.3 Rapport.3: Time 3 Child report of relationship quality	.203	.274	.441
Correlation	Rapport.1 Rapport.1: Time 1 Child report of relationship quality	1.000	.797	.628
	Rapport.2 Rapport.2: Time 2 Child report of relationship quality	.797	1.000	.672
	Rapport.3 Rapport.3: Time 3 Child report of relationship quality	.628	.672	1.000

Based on Type III Sum of Squares

$$SS = \Sigma (X - M_X)^2$$

$$SCP = \Sigma (X - M_X)(Y - M_Y)$$

$$\text{cov}_{XY} = \frac{\sum (X - M_X)(Y - M_Y)}{n}$$

$$r_{XY} = \frac{\sum (X - M_X)(Y - M_Y)}{n \sigma_X \sigma_Y}$$

Multivariate Tests[b]

Effect		Value	F	Hypothesis df	Error df	Sig.
Time	Pillai's Trace	.022	.850[a]	2.000	74.000	.431
	Wilks' Lambda	.978	.850[a]	2.000	74.000	.431
	Hotelling's Trace	.023	.850[a]	2.000	74.000	.431
	Roy's Largest Root	.023	.850[a]	2.000	74.000	.431
Time * t0TchExp	Pillai's Trace	.027	1.023[a]	2.000	74.000	.365
	Wilks' Lambda	.973	1.023[a]	2.000	74.000	.365
	Hotelling's Trace	.028	1.023[a]	2.000	74.000	.365
	Roy's Largest Root	.028	1.023[a]	2.000	74.000	.365

a. Exact statistic

b. Design: Intercept + t0TchExp
Within Subjects Design: Time

Tests of Between-Subjects Effects

Measure:MEASURE_1
Transformed Variable:Average

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Intercept	9.295	1	9.295	11.225	.001
t0TchExp	9.963	1	9.963	12.031	.001
Error	62.106	75	.828		

Estimates

Measure:MEASURE_1

Time	Mean	Std. Error	95% Confidence Interval	
			Lower Bound	Upper Bound
1	3.395[a]	.056	3.284	3.505
2	3.210[a]	.070	3.071	3.349
3	2.941[a]	.076	2.790	3.092

a. Covariates appearing in the model are evaluated at the following values: t0TchExp Child expectancies of teacher = 3.2746.

Pairwise Comparisons

Measure:MEASURE_1

(I) Time	(J) Time	Mean Difference (I-J)	Std. Error	Sig.[a]	95% Confidence Interval for Difference[a]	
					Lower Bound	Upper Bound
1	2	.185*	.042	.000	.082	.288
	3	.454*	.059	.000	.309	.599
2	1	-.185*	.042	.000	-.288	-.082
	3	.269*	.059	.000	.124	.413
3	1	-.454*	.059	.000	-.599	-.309
	2	-.269*	.059	.000	-.413	-.124

Based on estimated marginal means

*. The mean difference is significant at the .05 level.

a. Adjustment for multiple comparisons: Sidak.

Parameter Estimates

Dependent Variable	Parameter	B	Std. Error	t	Sig.	95% Confidence Interval	
						Lower Bound	Upper Bound
Rapport.1 Rapport.1: Time 1 Child report of relationship quality	Intercept	1.571	.434	3.617	.001	.706	2.437
	t0TchExp	.557	.132	4.232	.000	.295	.819
Rapport.2 Rapport.2: Time 2 Child report of relationship quality	Intercept	1.854	.547	3.392	.001	.765	2.943
	t0TchExp	.414	.166	2.500	.015	.084	.744
Rapport.3 Rapport.3: Time 3 Child report of relationship quality	Intercept	1.284	.593	2.167	.033	.103	2.464
	t0TchExp	.506	.179	2.819	.006	.148	.864

Summary of the Multivariate Analysis

1. Repeated Measurements considered different outcome variables

2. Estimation with least squares

3. Estimates correlations among residuals to handle non-independence

4. Can test effects of between-subjects factors or covariates and time (within-factors)

5. It is less powerful than the univariate approach

Summary of the Multivariate Analysis

Therefore:

- Time (or WS factor) must be categorical, defined by multiple outcome variables
- Regression coefficients are estimated separately for each outcome
- No constraints on variances or covariances across measurements

The Univariate Model

Parameters to Estimate in the Multivariate Analysis

$[Rapport_{i1}\ Rapport_{i2}\ Rapport_{i3}] =$

$\beta_0 + \beta_1 Time1 + \beta_2 Time2 + \beta_3 t0TchExp$
$+ \beta_4 t0TchExp*Time1 + \beta_5 t0TchExp*Time2$
$+ [\varepsilon_{i1}\ \varepsilon_{i2}\ \varepsilon_{i3}]$

$[\varepsilon_{i1}\ \varepsilon_{i2}\ \varepsilon_{i3}]$ ~ iid $N(0, \Sigma)$ for subject i

5 Coefficients
3 Variances
3 Covariances

82 Subjects

$$\Sigma = \begin{bmatrix} \sigma_1^2 & \rho_{12} & \rho_{13} \\ \rho_{12} & \sigma_2^2 & \rho_{23} \\ \rho_{13} & \rho_{23} & \sigma_3^2 \end{bmatrix}$$

Parameters to Estimate with Unstructured Covariance Matrix

Repeats	2	3	4	5	6	…	11	12
Variances	2	3	4	5	6	…	11	12
Covariances	1	3	6	10	15	…	55	66
Number of Parameters in Σ	3	6	10	15	21	…	66	74

The Univariate Model

$$Rapport_{ij} = \beta_0 + \beta_1 Time1 + \beta_2 Time2 + \beta_3 t0TchExp + \beta_4 t0TchExp*Time1 + \beta_5 t0TchExp*Time2 + \varepsilon_{ij}$$

$$R = \begin{bmatrix} [\Sigma] & & & & & & \\ 0 & [\Sigma] & & & & & \\ 0 & 0 & [\Sigma] & & & & \\ 0 & 0 & 0 & . & & & \\ 0 & 0 & 0 & 0 & . & & \\ 0 & 0 & 0 & 0 & 0 & . & \\ 0 & 0 & 0 & 0 & 0 & 0 & [\Sigma] \end{bmatrix}$$

$$\Sigma = \begin{bmatrix} Var_1 & Cov_{1,2} & Cov_{1,3} \\ Cov_{1,2} & Var_2 & Cov_{2,3} \\ Cov_{1,3} & Cov_{2,3} & Var_3 \end{bmatrix}$$

Assumes Sphericity

$\varepsilon_{ij} \sim$ iid N(0, Σ)
for subject i and time j

i = 1 to 82
j=1 to 3

R is a 246*246 matrix
Σ is a 3*3 submatrix

What is Sphericity and Why is it Necessary?

1. To create valid F-tests for within-subject mean comparisons

2. To test any within-subject contrasts

What is Sphericity, Really?

Sphericity = Circularity

The Technical Definition

The variances of any set of orthonormal contrasts on the within-subjects factor have equal variances (and zero covariances). (Crowder & Hand, 1996).

The Intuitive Expression of this Definition

Variances of pairwise differences in means for each subject are equal. (Keppel, 1991).

$$Var(Y_1-Y_2) = Var(Y_1) + Var(Y_2) - 2Cov(Y_1, Y_2)$$

One Example of Sphericity: Compound Symmetry

Compound Symmetry:

$$\Sigma = \begin{bmatrix} Var_1 & Cov_{1,2} & Cov_{1,3} \\ Cov_{1,2} & Var_2 & Cov_{2,3} \\ Cov_{1,3} & Cov_{2,3} & Var_3 \end{bmatrix} = \begin{bmatrix} \sigma^2 & \rho & \rho \\ \rho & \sigma^2 & \rho \\ \rho & \rho & \sigma^2 \end{bmatrix}$$

$$Var(Y_1\text{-}Y_2) = Var(Y_1) + Var(Y_2) - 2Cov(Y_1, Y_2)$$

Parameters to Estimate in Compound Symmetry Covariance Matrix

Repeats	2	3	4	5	6	…	11	12
Variances	1	1	1	1	1	…	1	1
Covariances	1	1	1	1	1	…	1	1
Number of Parameters in Σ	2	2	2	2	2	…	2	2

Compound Symmetry:

$$\Sigma = \begin{bmatrix} Var_1 & Cov_{1,2} & Cov_{1,3} \\ Cov_{1,2} & Var_2 & Cov_{2,3} \\ Cov_{1,3} & Cov_{2,3} & Var_3 \end{bmatrix} = \begin{bmatrix} \sigma^2 & \rho & \rho \\ \rho & \sigma^2 & \rho \\ \rho & \rho & \sigma^2 \end{bmatrix}$$

Huynh – Feldt =

$$\Sigma = \begin{bmatrix} Var_1 & Cov_{1,2} & Cov_{1,3} \\ Cov_{1,2} & Var_2 & Cov_{2,3} \\ Cov_{1,3} & Cov_{2,3} & Var_3 \end{bmatrix} = \begin{bmatrix} \sigma_1^2 & \frac{\sigma_1^2 + \sigma_2^2}{2} - \lambda & \frac{\sigma_1^2 + \sigma_3^2}{2} - \lambda \\ \frac{\sigma_2^2 + \sigma_1^2}{2} - \lambda & \sigma_2^2 & \frac{\sigma_2^2 + \sigma_3^2}{2} - \lambda \\ \frac{\sigma_3^2 + \sigma_1^2}{2} - \lambda & \frac{\sigma_3^2 + \sigma_2^2}{2} - \lambda & \sigma_3^2 \end{bmatrix}$$

Tests of Within-Subjects Effects

Measure:MEASURE_1

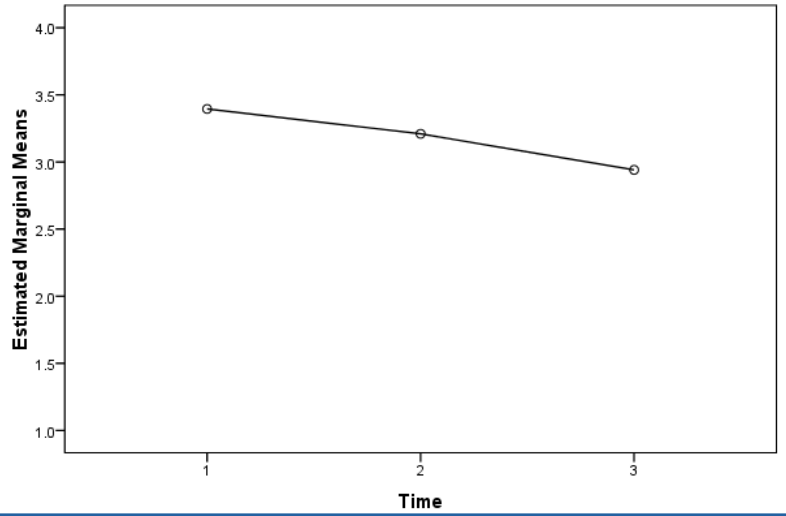
Source		Type III Sum of Squares	df	Mean Square	F	Sig.
Time	Sphericity Assumed	.205	2	.102	.904	.407
	Greenhouse-Geisser	.205	1.729	.118	.904	.395
	Huynh-Feldt	.205	1.790	.114	.904	.398
	Lower-bound	.205	1.000	.205	.904	.345
Time * t0TchExp	Sphericity Assumed	.144	2	.072	.635	.531
	Greenhouse-Geisser	.144	1.729	.083	.635	.509
	Huynh-Feldt	.144	1.790	.080	.635	.514
	Lower-bound	.144	1.000	.144	.635	.428
Error(Time)	Sphericity Assumed	16.979	150	.113		
	Greenhouse-Geisser	16.979	129.688	.131		
	Huynh-Feldt	16.979	134.241	.126		
	Lower-bound	16.979	75.000	.226		

Tests of Within-Subjects Contrasts

Measure:MEASURE_1

Source	Time	Type III Sum of Squares	df	Mean Square	F	Sig.
Time	Linear	.052	1	.052	.382	.539
	Quadratic	.153	1	.153	1.692	.197
Time * t0TchExp	Linear	.018	1	.018	.130	.720
	Quadratic	.126	1	.126	1.399	.241
Error(Time)	Linear	10.214	75	.136		
	Quadratic	6.765	75	.090		

Estimated Marginal Means of MEASURE_1



Mauchly's Test of Sphericity[b]

Measure:MEASURE_1

Within Subject...	Mauchly's W	Approx. Chi-Square	df	Sig.	Epsilon[a]		
					Greenhouse-Geisser	Huynh-Feldt	Lower-bound
Time	.843	12.605	2	.002	.865	.895	.500

Tests the null hypothesis that the error covariance matrix of the orthonormalized transformed dependent variables is proportional to an identity matrix.

a. May be used to adjust the degrees of freedom for the averaged tests of significance. Corrected tests are displayed in the Tests of Within-Subjects Effects table.

b. Design: Intercept + t0TchExp
Within Subjects Design: Time

Summary of the Univariate Model

1. Repeated Measurements considered a single outcome variable
2. Estimation with least squares
3. Estimates correlations among residuals to handle non-independence
4. Can test effects of time (within-subject factors) only if we assume sphericity of the residual covariance matrix
5. Time must be categorical. The only way to test order effects is through linear contrasts.

Example: Physical Training Data Set
One Between and One Within-Subjects Factor

2.2 Physical Training: One Within- and One Between-Subjects Factor

Research Question:

Do the three training regimens differ in their effect on BMI from pre-training to post-training measurements?

VIEWTABLE: Rm.Physicaltraining (Written by SAS

	Groups	Age (years)	Height (cm)	Body Mass	Body Mass	% Body Fat (pre)	% Body Fat (post)	Fat Mass	Fat Mass	Fat Free Mass	Fat Free Mass	Waist:Hip Ratio	Waist:Hip Ratio	Body Mass Index (pre)	Body Mass Index (post)
1	1	23	170	67	66	16.59229	10.67924	11.117	7.0483	55.88317	58.9517	0.83871	0.83607	23.183391	22.83737
2	1	25	169	75	74	24.4548	21.74261	18.341	16.09	56.6589	57.9105	0.94898	0.94792	26.259585	25.909457
3	1	23	184	65	64.5	10.87136	9.247213	7.0664	5.9645	57.93361	58.5355	0.90217	0.87432	19.19896	19.051276
4	1	25	181	83	83	21.33088	19.75213	17.705	16.394	65.29537	66.6057	0.9	0.90816	25.335002	25.335002
5	1	26	172	84	82	24.7628	22.306	20.801	18.291	63.19925	63.7091	0.85377	0.85784	28.393726	27.717685
6	1	25	184	76	73.5	13.28085	10.44621	10.093	7.678	65.90655	65.822	0.89474	0.90426	22.448015	21.709594
7	1	27	176	65	64	21.22852	21.74375	13.799	13.916	51.20146	50.084	0.94565	0.93333	20.983988	20.661157
8	1	23	172	58	59	15.13968	15.71584	8.781	9.2723	49.21898	49.7277	0.81461	0.85714	19.605192	19.943213
9	1	26	170	68	65	20.92189	19.72929	14.227	12.824	53.77311	52.176	0.87895	0.86842	23.529412	22.491349
10	2	24	169	80	81	10.99711	10.54735	8.7977	8.5434	71.20231	72.4566	0.89286	0.80347	28.010224	28.360352
11	2	27	175	70	70	24.22513	21.47502	16.958	15.033	53.04241	54.9675	0.87565	0.87368	22.857143	22.857143
12	2	24	169	55	55.5	7.728283	7.07739	4.2506	3.928	50.74944	51.572	0.78409	0.78161	19.257029	19.432093
13	2	26	174	57	58.5	10.92729	9.720373	6.2286	5.6864	50.77144	52.8136	0.83333	0.83333	18.826794	19.322235
14	2	26	185	62	64	11.66305	10.83051	7.2311	6.9315	54.76891	57.0685	0.87059	0.84884	18.115413	18.699781
15	2	26	174	65	67	18.4903	16.38867	12.019	10.98	52.9813	56.0196	0.87097	0.85106	21.46915	22.12974
16	2	25	183	68	67	16.79577	13.46378	11.421	9.0207	56.57887	57.9793	0.82632	0.8172	20.305175	20.006569
17	2	25	176	69	71	19.64908	19.49387	13.558	13.841	55.44213	57.1593	0.84574	0.84127	22.27531	22.920971
18	2	26	177	68	71	24.61252	23.50917	16.737	16.692	51.26348	54.3085	0.92021	0.90625	21.705129	22.662709
19	3	23	169	63	65	13.26987	12.53564	8.36	8.1482	54.63998	56.8518	0.8	0.79487	22.058051	22.758307
20	3	23	185	70	71	13.39107	10.32504	9.3737	7.3308	60.62625	63.6692	0.82105	0.84656	20.452885	20.745069
21	3	27	182	66	65	10.21696	6.892077	6.7432	4.4798	59.25681	60.5202	0.84615	0.82222	19.92513	19.623234
22	3	25	181	82	82	22.21923	20.58949	18.22	16.883	63.78023	65.1166	0.92611	0.94444	25.029761	25.029761

The GLM Procedure

Repeated Measures Analysis of Variance

Tests of Hypotheses for Between Subjects Effects

Source	DF	Type III SS	Mean Square	F Value	Pr > F
group	2	21.7383569	10.8691785	0.62	0.5454
Error	24	419.5256816	17.4802367		

Repeated Measures Level Information

Dependent Variable	BMI_pre	BMI_post
Level of Time	1	2

Partial Correlation Coefficients from the Error SSCP Matrix / Prob > |r|

DF = 24	BMI_pre	BMI_post
BMI_pre	1.000000	0.988084
		<.0001
BMI_post	0.988084	1.000000
	<.0001	

E = Error SSCP Matrix

Time_N represents the contrast between the nth level of Time and the last

	Time_1
Time_1	5.0513

MANOVA Test Criteria and Exact F Statistics for the Hypothesis of no Time Effect

H = Type III SSCP Matrix for Time

E = Error SSCP Matrix

S=1 M=-0.5 N=11

Statistic	Value	F Value	Num DF	Den DF	Pr > F
Wilks' Lambda	0.99950152	0.01	1	24	0.9138
Pillai's Trace	0.00049848	0.01	1	24	0.9138
Hotelling-Lawley Trace	0.00049873	0.01	1	24	0.9138
Roy's Greatest Root	0.00049873	0.01	1	24	0.9138

Univariate Tests of Hypotheses for Within Subject Effects

Source	DF	Type III SS	Mean Square	F Value	Pr > F
Time	1	0.00125960	0.00125960	0.01	0.9138
Time*group	2	1.32179169	0.66089585	6.28	0.0064
Error(Time)	24	2.52563846	0.10523494		

Means of Within Subjects Effects

Level of Time	N	Mean	Std Dev
1	27	22.52146474	2.97314046
2	27	22.51180535	2.87751762

Least Squares Means

group	BMI_pre LSMEAN	BMI_post LSMEAN
1	23.2152523	22.8506781
2	21.4245963	21.8212880
3	22.9245457	22.8634499

Example: Swallowing Data Set
Two Within-Subjects Factors

2.3 Swallowing: Two Within-Subjects Factors

Research Question:

Does the mean pressure rise slope in the anterior bulb differ across the four swallowing tasks, and are there any order effects of trial?

RM_02_Examples_Syntax-R.R* | swallow.small.wide | training | RM-02-Create-small-wide-data-for-S...

Filter

	ParticipantID	Sex	RiseSlope.ANEC.1	RiseSlope.DSW.1	RiseSlope.ESS.1	RiseSlope.NESS.1	RiseSlope.ANEC.2	RiseSlope.DSW.2	RiseSlope.ESS.2	RiseSlope.NESS.2
41	Y02	F	490.99199	136.48882	601.40711	86.57551	347.87129	255.19969	734.53960	52.01120
161	Y03	F	503.17479	NA	639.32237	48.87984	NA	NA	470.93531	143.76423
281	Y04	M	NA	359.55056	3058.22268	704.34345	NA	416.61406	2082.61644	NA
401	Y05	F	510.00668	246.91358	323.24649	98.69737	503.36460	86.41975	235.92336	118.20331
521	Y06	F	114.25525	401.06952	468.73073	394.18060	304.83674	272.72727	271.64071	174.78368
641	Y07	M	315.84488	374.43445	1084.59302	237.85182	194.07379	325.03909	826.22212	88.54782
761	Y08	M	109.58056	227.00304	434.60765	181.08652	265.59356	216.68472	216.97002	NA
881	Y09	F	NA	NA	297.74436	27.06767	NA	NA	93.73297	42.00000
1001	Y10	F	383.83838	501.70580	539.64301	189.68133	183.65473	1592.85159	234.20323	250.03206
1121	Y13	F	486.39259	726.46688	330.39910	272.09864	1034.66494	1765.43894	305.90010	515.45497
1241	Y15	M	687.57117	411.78334	1153.06122	196.92087	667.18995	217.39721	854.83929	238.09524
1361	Y16	M	54.32937	173.17487	56.27516	55.24592	366.72326	NA	36.59913	296.83266
1481	Y17	F	111.48539	51.93089	1002.04120	553.32626	138.65546	245.57507	463.47770	190.98549
1601	Y18	M	342.31802	358.10998	743.44178	187.84820	124.85544	111.82949	680.93001	55.92972
1721	Y19	F	205.15656	NA	487.60331	115.21423	228.78019	NA	456.58060	81.91276
1841	Y20	F	115.38461	36.39893	66.53438	16.66872	57.69231	109.89011	138.46154	133.13610
1961	Y31	M	78.89280	NA	716.27370	NA	215.70977	91.51324	123.53328	405.73599

```
Multivariate Tests: task
                 Df test stat approx F num Df den Df   Pr(>F)
Pillai            1 0.7233233  4.35721      3      5 0.073428 .
Wilks             1 0.2766767  4.35721      3      5 0.073428 .
Hotelling-Lawley  1 2.6143263  4.35721      3      5 0.073428 .
Roy               1 2.6143263  4.35721      3      5 0.073428 .
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

```
Multivariate Tests: trial
                 Df test stat  approx F num Df den Df  Pr(>F)
Pillai            1 0.4182684 0.7190059      4      4 0.62151
Wilks             1 0.5817316 0.7190059      4      4 0.62151
Hotelling-Lawley  1 0.7190059 0.7190059      4      4 0.62151
Roy               1 0.7190059 0.7190059      4      4 0.62151
```

```
Error in eigen(qr.coef(SSPE.qr, x$SSPH), symmetric = FALSE) :
  infinite or missing values in 'x'
In addition: Warning message:
In summary.Anova.mlm(swallowing.results) : Singular error SSP matrix:
non-sphericity test and corrections not available
```

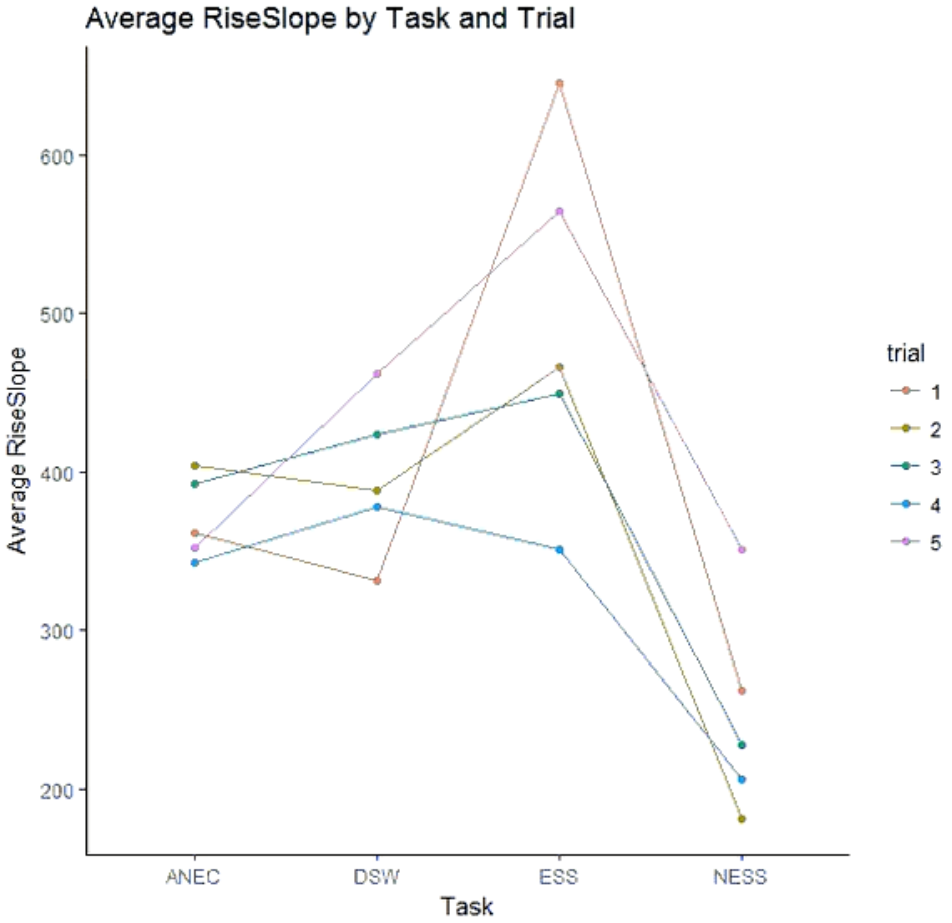
Sum of squares and products for error:

	task1:trial1	task2:trial1	task3:trial1	task1:trial2	task2:trial2	task3:trial2	task1:trial3	task2:trial3
task1:trial1	774385.22	300724.24	251340.4	578809.8	-137768.79	73610.71	621836.9	722591.39
task2:trial1	300724.24	208071.84	184909.3	275371.0	53969.71	68444.89	301347.4	309545.14
task3:trial1	251340.41	184909.31	581862.7	458705.2	310159.51	513508.78	462339.3	330031.15
task1:trial2	578809.80	275371.03	458705.2	687263.8	231943.57	255924.40	644536.2	556777.10
task2:trial2	-137768.79	53969.71	310159.5	231943.6	817789.13	83623.23	286786.8	-116152.41
task3:trial2	73610.71	68444.89	513508.8	255924.4	83623.23	626661.04	210270.6	203690.97
task1:trial3	621836.92	301347.44	462339.3	644536.2	286786.83	210270.59	726119.6	601231.95
task2:trial3	722591.39	309545.14	330031.1	556777.1	-116152.41	203690.97	601232.0	890201.40
task3:trial3	-69850.19	-40920.04	1023365.4	459021.1	475271.83	1196362.10	393429.0	-45542.42
task1:trial4	557318.08	229673.13	521572.4	764265.4	344035.65	306437.22	683166.0	550582.00
task2:trial4	374891.83	241547.83	659022.9	714574.1	574363.72	438702.34	669802.5	298202.55
task3:trial4	-155255.62	-98183.52	691050.2	412242.5	508235.32	831019.62	204752.2	-262442.34

	task3:trial3	task1:trial4	task2:trial4	task3:trial4
task1:trial1	-69850.19	557318.1	374891.8	-155255.62
task2:trial1	-40920.04	229673.1	241547.8	-98183.52
task3:trial1	1023365.37	521572.4	659022.9	691050.21
task1:trial2	459021.10	764265.4	714574.1	412242.52
task2:trial2	475271.83	344035.6	574363.7	508235.32
task3:trial2	1196362.10	306437.2	438702.3	831019.62
task1:trial3	393429.05	683166.0	669802.5	204752.18
task2:trial3	-45542.42	550582.0	298202.6	-262442.34
task3:trial3	2915402.60	672138.3	1099215.2	2208671.83
task1:trial4	672138.33	926469.3	838198.4	616635.41
task2:trial4	1099215.19	838198.4	1024612.1	998239.48
task3:trial4	2208671.83	616635.4	998239.5	2252329.23

```
Univariate Type III Repeated-Measures ANOVA Assuming Sphericity

                  SS num Df Error SS den Df       F    Pr(>F)
(Intercept) 22739784      1  3500731      7 45.4701 0.0002667 ***
task         1268149      3  4303815     21  2.0626 0.1358539
trial         229898      4  1662137     28  0.9682 0.4404243
task:trial    417052     12  3285410     84  0.8886 0.5615358
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
Warning message:
In summary.Anova.mlm(swallowing.results, multivariate = F) :
  Singular error SSP matrix:
non-sphericity test and corrections not available
```

```
                Mauchly's test of sphericity
                Contrasts orthogonal to ~1

                Contrasts spanned by
                ~task

  data: SSD matrix from
  lm(formula = cbind(RiseSlope.ANEC.1, RiseSlope.ANEC.2, RiseSlope.ANEC.3,
  SSD matrix from RiseSlope.ANEC.4, RiseSlope.ANEC.5, RiseSlope.DSW.1, RiseSlope.DSW.2,
  SSD matrix from RiseSlope.DSW.3, RiseSlope.DSW.4, RiseSlope.DSW.5, RiseSlope.ESS.1,
  SSD matrix from RiseSlope.ESS.2, RiseSlope.ESS.3, RiseSlope.ESS.4, RiseSlope.ESS.5,
  SSD matrix from RiseSlope.NESS.1, RiseSlope.NESS.2, RiseSlope.NESS.3, RiseSlope.NESS.4,
  SSD matrix from RiseSlope.NESS.5) ~ 1, data = swallow.small.wide)
  W = 0.1745, p-value = 0.08104

            Mauchly's test of sphericity
            Contrasts orthogonal to
            ~1

            Contrasts spanned by
            ~trial

data: SSD matrix from
lm(formula = cbind(RiseSlope.ANEC.1, RiseSlope.ANEC.2, RiseSlope.ANEC.3,
SSD matrix from RiseSlope.ANEC.4, RiseSlope.ANEC.5, RiseSlope.DSW.1, RiseSlope.DSW.2,
SSD matrix from RiseSlope.DSW.3, RiseSlope.DSW.4, RiseSlope.DSW.5, RiseSlope.ESS.1,
SSD matrix from RiseSlope.ESS.2, RiseSlope.ESS.3, RiseSlope.ESS.4, RiseSlope.ESS.5,
SSD matrix from RiseSlope.NESS.1, RiseSlope.NESS.2, RiseSlope.NESS.3, RiseSlope.NESS.4,
SSD matrix from RiseSlope.NESS.5) ~ 1, data = swallow.small.wide)
W = 0.071449, p-value = 0.1306
```



tasktrial	lsmean	SE	df	lower.CL	upper.CL
RiseSlope.ANEC.1	360.9373	69.66956	7	196.19494	525.6796
RiseSlope.ANEC.2	404.1588	111.66425	7	140.11478	668.2027
RiseSlope.ANEC.3	391.9662	118.29173	7	112.25076	671.6817
RiseSlope.ANEC.4	342.8246	69.23862	7	179.10126	506.5479
RiseSlope.ANEC.5	352.5043	66.20210	7	195.96115	509.0474
RiseSlope.DSW.1	331.5708	75.29355	7	153.52982	509.6117
RiseSlope.DSW.2	388.4858	199.28193	7	-82.74113	859.7126
RiseSlope.DSW.3	423.6458	114.95794	7	151.81350	695.4782
RiseSlope.DSW.4	378.4171	86.82966	7	173.09758	583.7366
RiseSlope.DSW.5	461.5151	72.34957	7	290.43554	632.5946
RiseSlope.ESS.1	645.5018	144.30167	7	304.28257	986.7210
RiseSlope.ESS.2	466.1113	102.19977	7	224.44720	707.7753
RiseSlope.ESS.3	449.4168	143.33286	7	110.48845	788.3452
RiseSlope.ESS.4	350.8382	81.42037	7	158.30967	543.3668
RiseSlope.ESS.5	564.3925	164.60158	7	175.17158	953.6133
RiseSlope.NESS.1	261.3658	52.09360	7	138.18398	384.5475
RiseSlope.NESS.2	180.9973	52.88493	7	55.94434	306.0503
RiseSlope.NESS.3	228.0733	52.71525	7	103.42160	352.7251
RiseSlope.NESS.4	206.2511	43.02241	7	104.51931	307.9830
RiseSlope.NESS.5	350.8842	142.04418	7	15.00312	686.7653

Confidence level used: 0.95

Key Takeaways:

When Does Repeated Measures ANOVA Work?

1. Compound Symmetry (or other sphericity) is a reasonable assumption
2. The repeating variable (the within-subject factor, or time) is nominal
3. When all interaction effects are of interest
4. When subjects are not clustered in some higher level
5. There is only one measurement per repeat
6. No data are missing
7. The within subject factor has an equal number of observations