Probability Rules and Applications

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THE ANALYSIS FACTOR

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Empirical Probability

Empirical probability is the relative frequency of a frequency distribution based upon observation

P(Event) = Frequency of Event / Number of Trials

Presence of suspected water source	**Presence of diarrhea**		**Total**
	Yes (1)	No (2)	
Yes (1)	78 (a)	1,422 (b)	1,500
No (2)	50 (c)	950 (d)	1,000
Total	128 (a+c)	2,372 (b+d)	2,500

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Probability Rules

1. All probabilities are between 0 and 1 inclusive

The probability of an impossible event is 0

The probability of a sure event is 1

2. The sum of all the probabilities for all possible events is equal to one

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3. The probability of an event not occurring is one minus the probability of it occurring.

$$P(A) = 1-P(\text{not } A)$$

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4. Addition Rule

General Addition Rule

P(A or B) = P(A) + P(B) - P(A and B)

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Mutually Exclusive Events

Two events are mutually exclusive if they cannot occur at the same time.

P(A and B) = 0

Specific Addition Rule

Only valid when the events are mutually exclusive.

P(A or B) = P(A) + P(B)

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Conditional Probability

The probability that event B occurs, given that event A has already occurred is

P(B|A) =

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5. Multiplication Rule

General Multiplication Rule

P(A and B) = P(A) * P(B|A) and

P(A and B) = P(B) * P(A|B)

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Independent Events

Two events are independent if one occurring does not affect or influence the probability of the other occurring

Specific Multiplication Rule

If A and B are Independent:

$$P(A \text{ and } B) = P(A) * P(B)$$

Independence Revisited

The following four statements are equivalent:

- A and B are independent events
- P(A and B) = P(A) * P(B)
- $P(A|B) = P(A)$
- $P(B|A) = P(B)$

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Chi-Square Test of Independence

			Experienced Joint Pain		Total
			No	**Yes**	
Runs more that 25km/week	**No**	Count	215	75	290
		% of Non-runners	74%	**26%**	100%
	Yes	Count	785	380	1165
		% of Runners	67%	**33%**	100%
Total		Count	1000	455	1455

H_0: Joint Pain and Running are Independent

A = Experienced Joint Pain

B = Runs a lot

If A and B are independent, then P(A|B) = P(A)

P(A|B) =

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Logistic Regression

$$Ln\left(\frac{P}{1-P}\right) = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \ldots + \beta_k X_k$$

$$Ln\left(\frac{\hat{P}}{1-\hat{P}}\right) = 4.32 - 6.59AL - 6.18FL - 7.11LA - 6.14MS$$

$$Ln\left(\frac{\hat{P}}{1-\hat{P}} \mid AL\right) = 4.32 - 6.59(1) - 6.18(0) - 7.11(0) - 6.14(0)$$

Analysis of Maximum Likelihood Estimates						
Parameter		DF	Estimate	Standard Error	Wald Chi-Square	Pr > ChiSq
Intercept		1	4.3197	0.4109	110.4913	<.0001
State	AL	1	-6.5884	0.5107	166.4152	<.0001
State	FL	1	-6.1804	0.5779	114.3788	<.0001
State	LA	1	-7.1100	0.5661	157.7236	<.0001
State	MS	1	-6.1382	0.4426	192.3230	<.0001

Sampling

Simple Random Sampling: Each individual has an equal probability of being sampled

P(sampled) = 1/N

Two-Stage Cluster Sampling: N_c Clusters are sampled, then samples of size n are randomly taken from each sample

Sampling Weight = 1/P(sampled)

P(1st person in cluster 1 is sampled)

= P(sampled|cluster is sampled)*P(cluster is sampled)

= $(1/n)*(1/N_c)$

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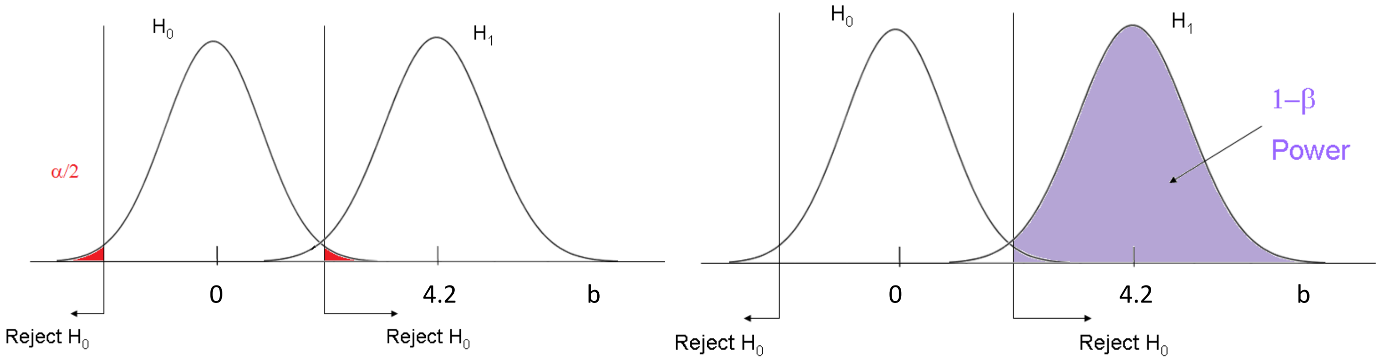
P values and Power



		Truth	
		H_0 True	H_0 False
Decision	Reject H_0	Type I error	**Correct**
	Accept H_0	Correct	Type II error

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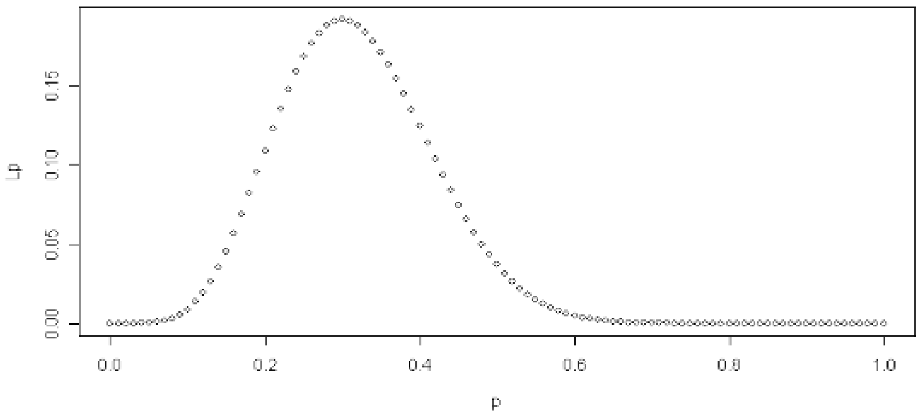
Maximum Likelihood Estimation



$$L(p) = P(p \mid x_i)$$

$$f(x) = \left(\frac{n!}{x!(n-x)!}\right) p^x (1-p)^{n-x}$$

$$L(p) = \prod_{i=1}^{n} f(x_i) = \prod_{i=1}^{n} \left(\frac{n!}{x_i!(n-x_i)!}\right) p^{x_i} (1-p)^{n-x_i}$$

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