Confidence Indicators

Quiz!

A miracle drug was created that improved people's IQ by 10 points, with a 95 percent confidence interval of 5 to 15 points.

Which if any of the following are true:

1. With a 95% probability, the true mean for the entire population would fall within the 5 to 15 point confidence interval.

2. If you conducted the same experiment 100 times, the mean for each sample would fall within the 5 to 15 point range 95 times.

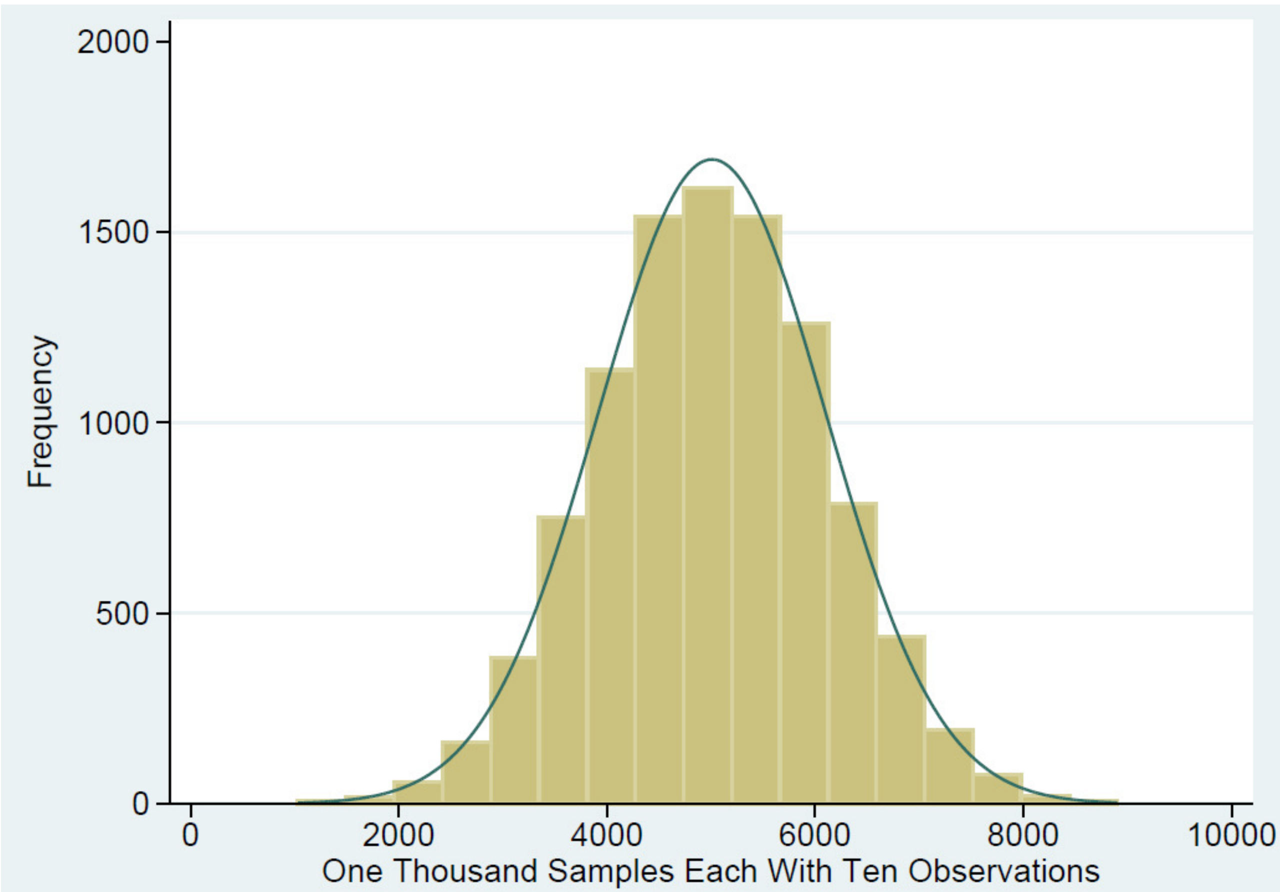
3. If you conducted the experiment 10,000 times, 9,500 times the confidence interval would contain the population's true mean.

4. 95% of the observations within the sample fall within the 5 to 15 point range.

5. There is a 95% probability that the 5 to 15 point range contains the population's true mean.

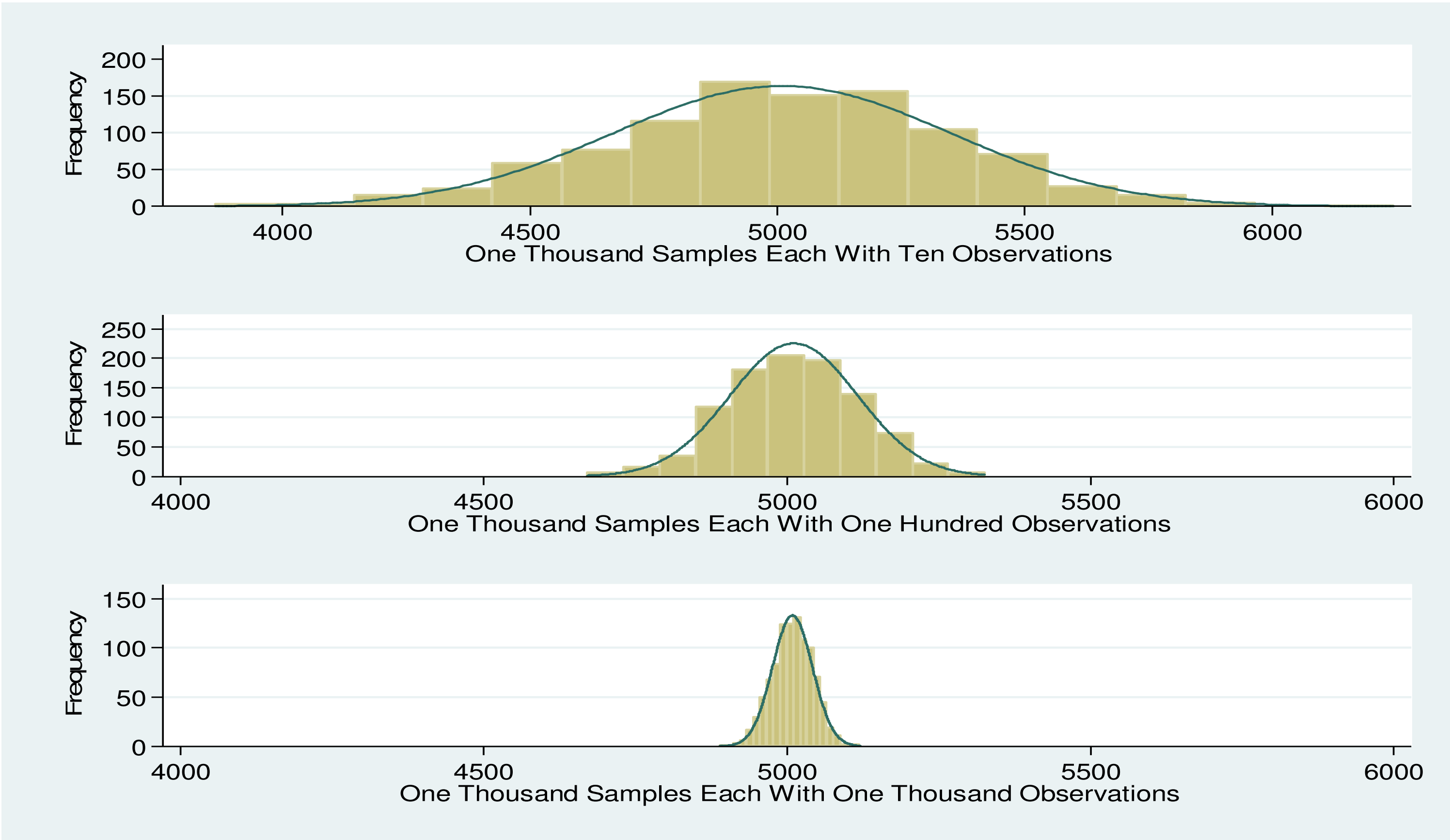
Let's simulate an experiment

Create a dataset (the population) of 10,000 Gaussian normal random numbers with a mean of 5,000 and a standard deviation of 1,100. The distribution of the observations is shown below in the histogram.



2) **Next draw ten observations and calculate the mean and standard deviation. Put the results into a new dataset. Repeat this action 999 times.**

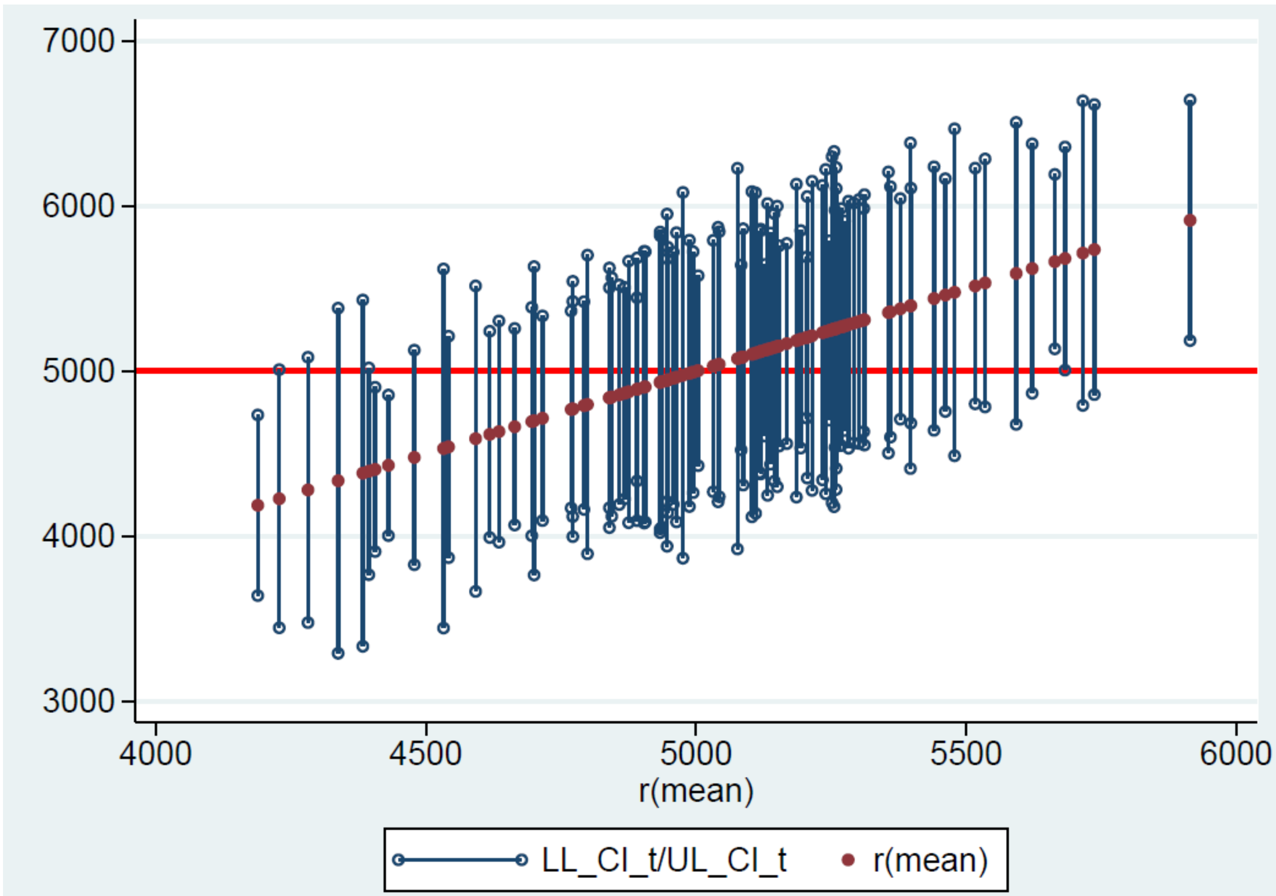
Repeat the same task but draw 100 observations 1,000 times. Do it once more taking 1,000 observations 1,000 times. The distribution of the means for each sample within each dataset is shown below in a histogram.



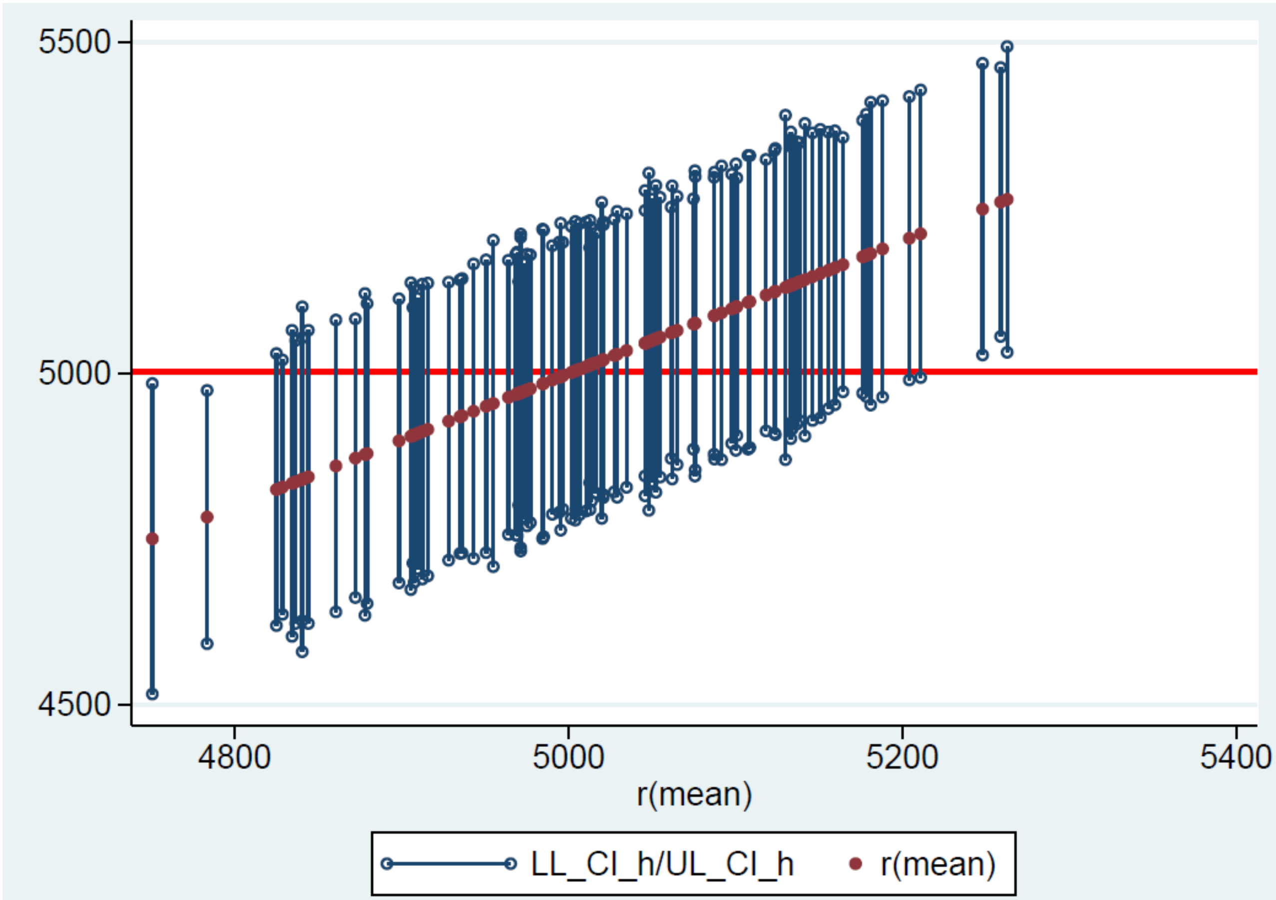
Distribution of the means for all 1,000 samples for the three datasets

3) Next, calculate the confidence intervals for the 1,000 means in each of the three datasets.

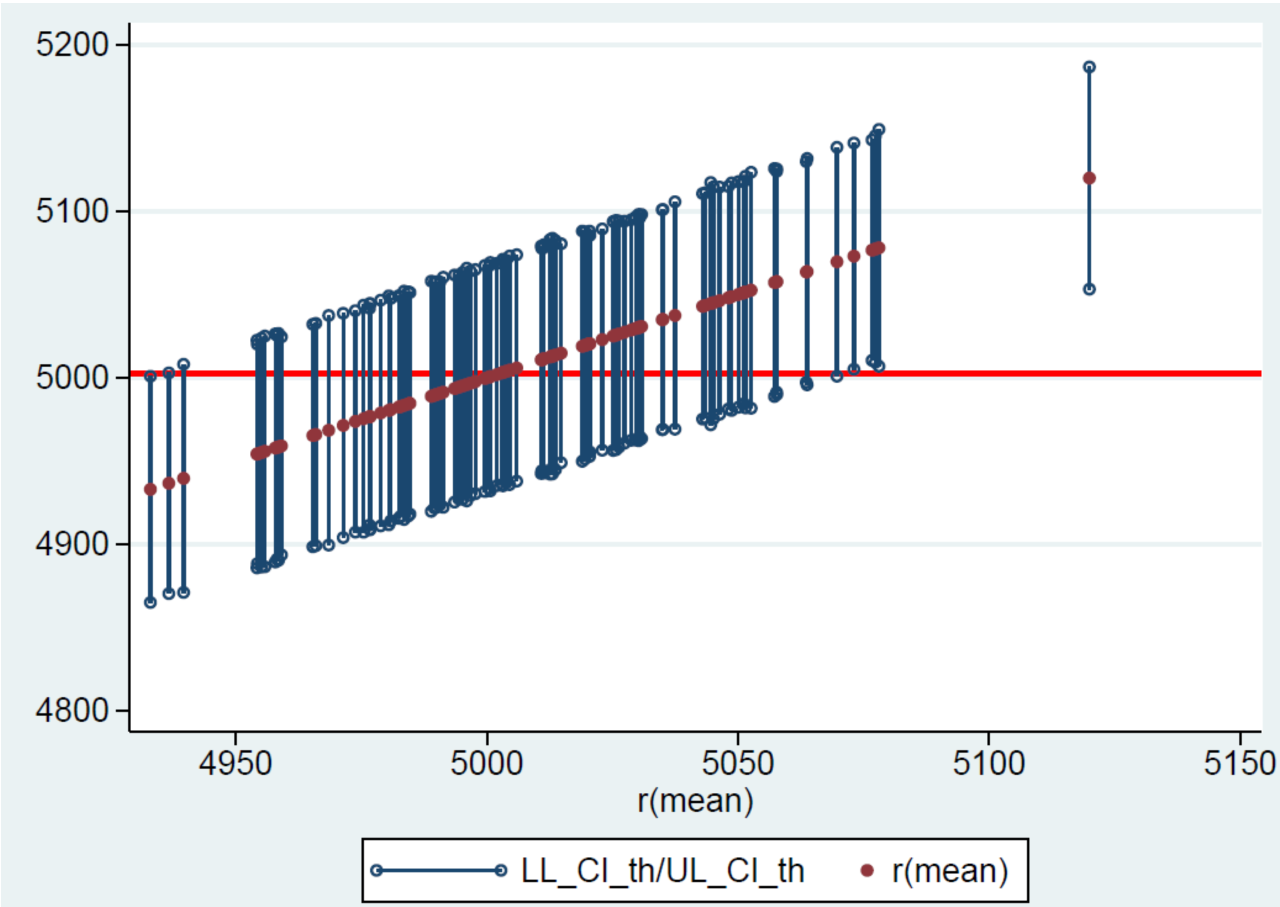
Mean and CI for a sample size of 10 observations, 100 times



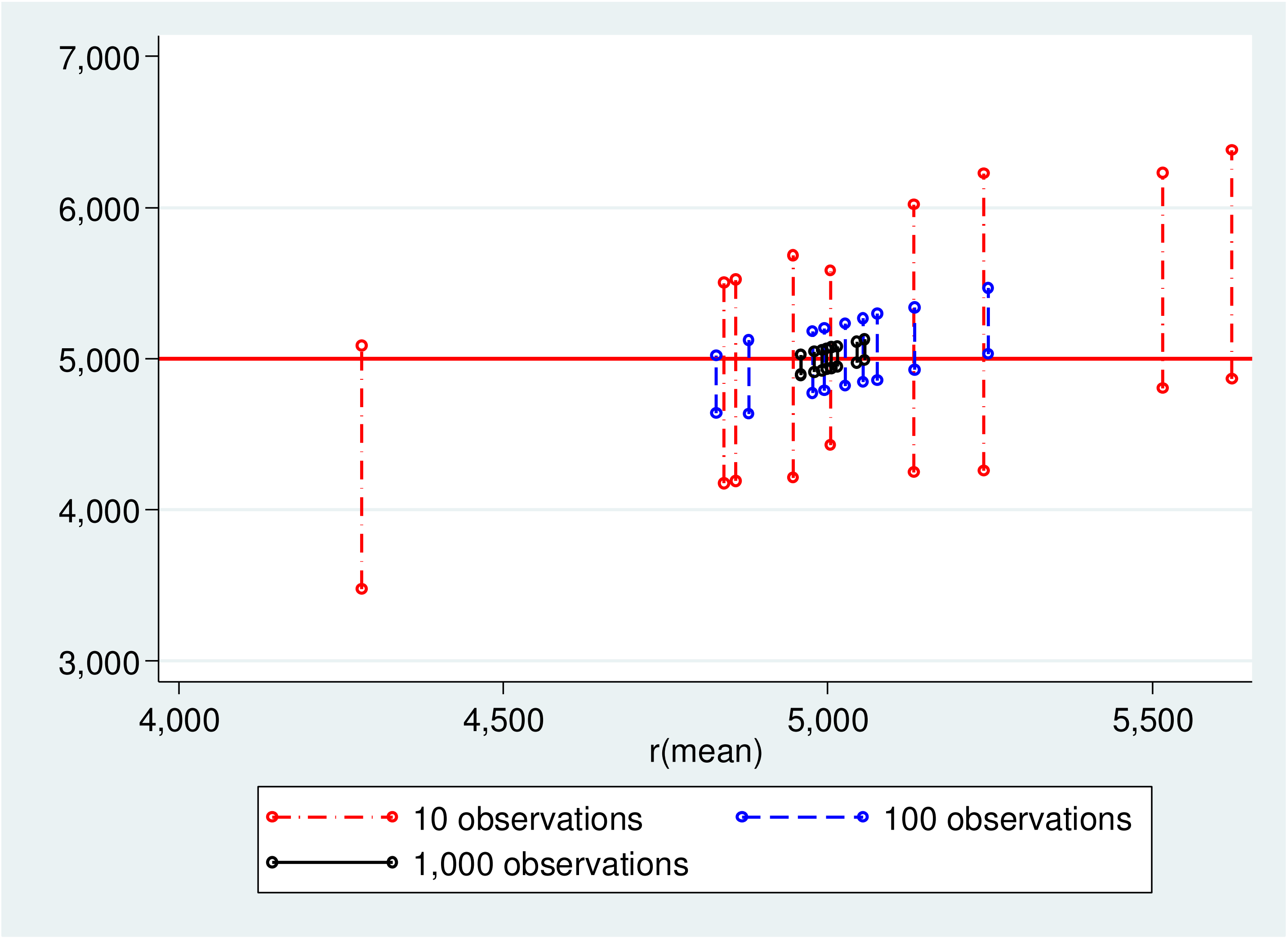
Mean and CI for a sample size of 100 observations, 100 times



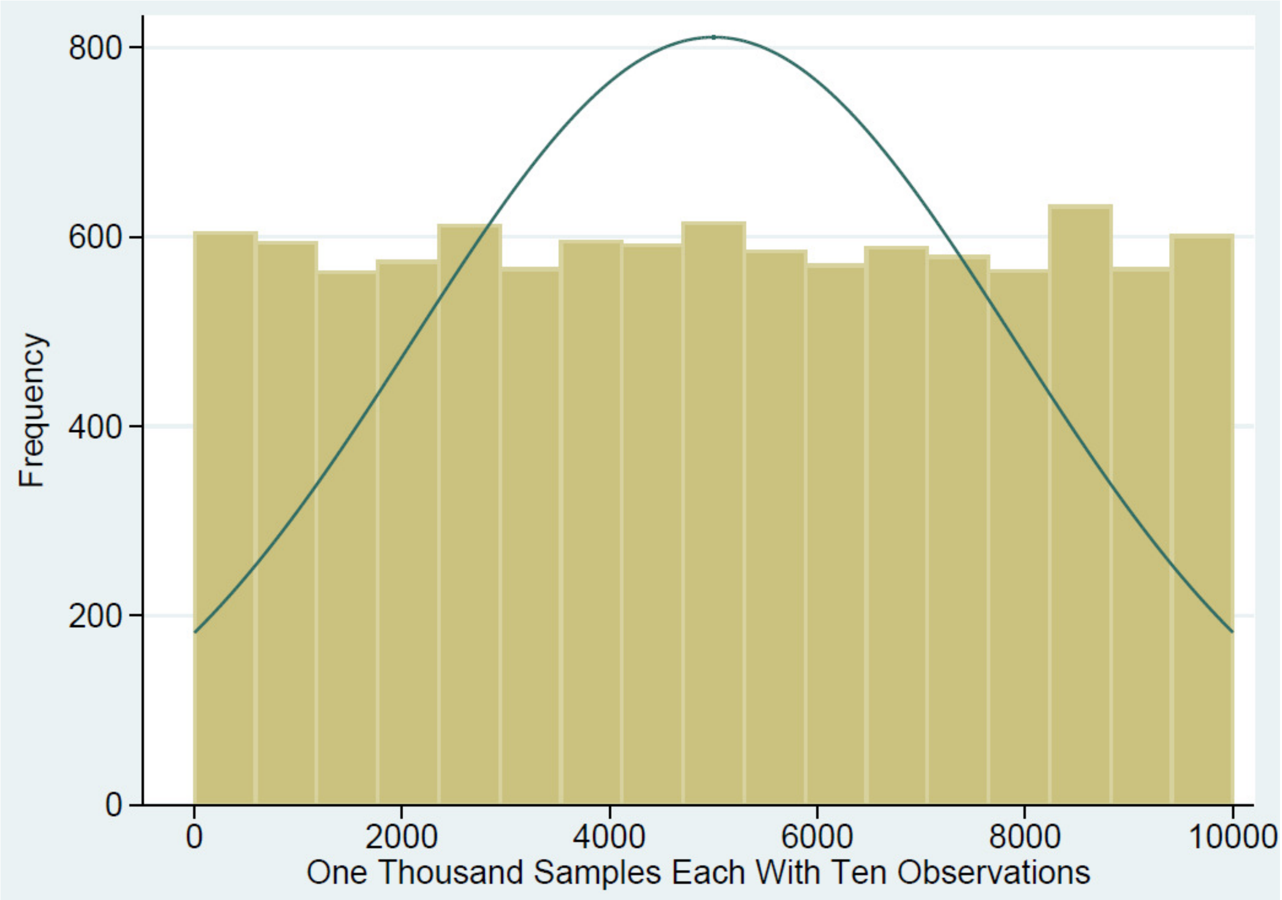
Mean and CI for a sample size of 1,000 observations, 100 times



Confidence Interval for all three sample sizes, 10 times each

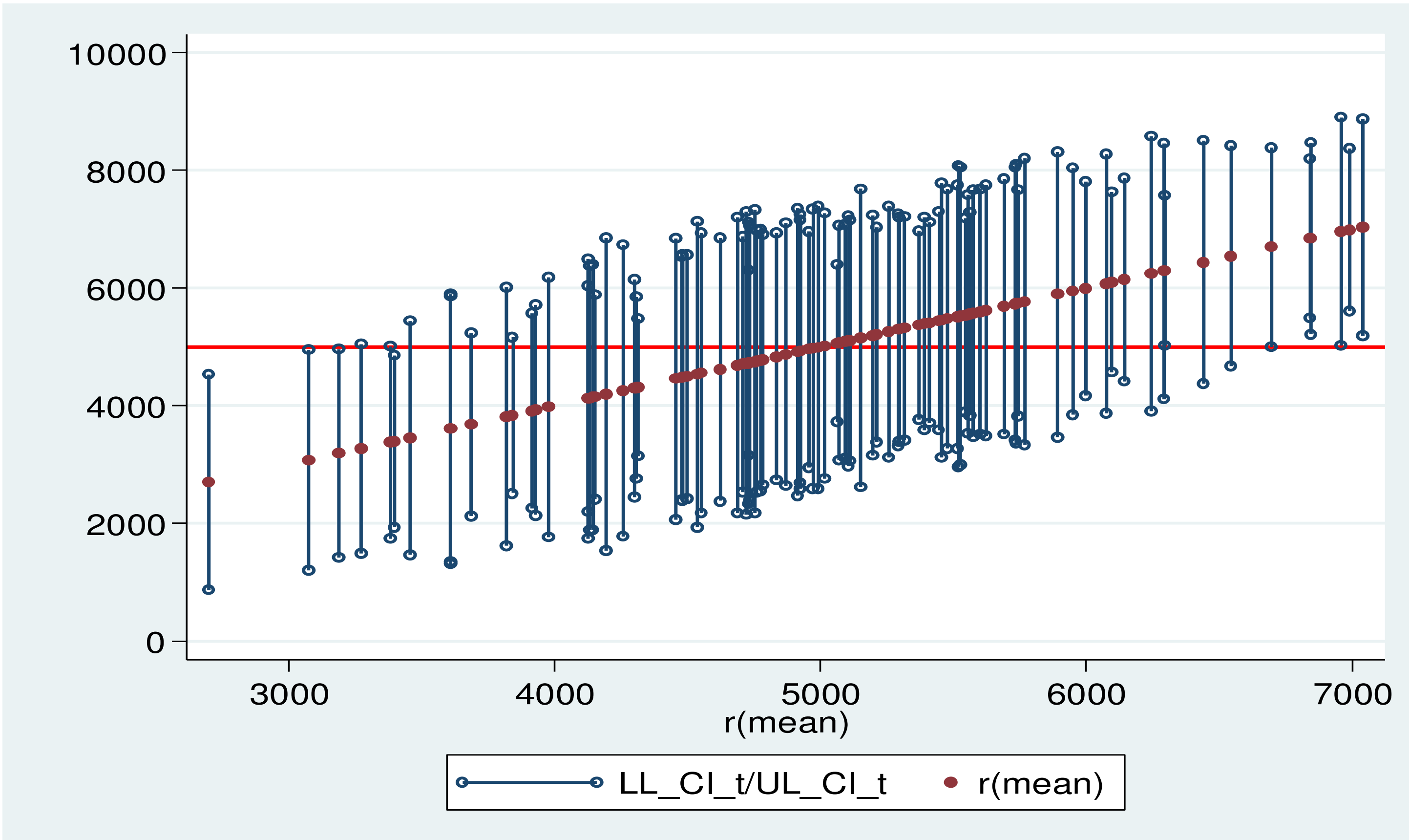


1. **Create a dataset (the population) of 10,000 rectangularly (uniformly) distributed random numbers between 1 and 10,000 with replacement. Mean = 5,002, sd=2,892 with 6,408 unique numbers**.

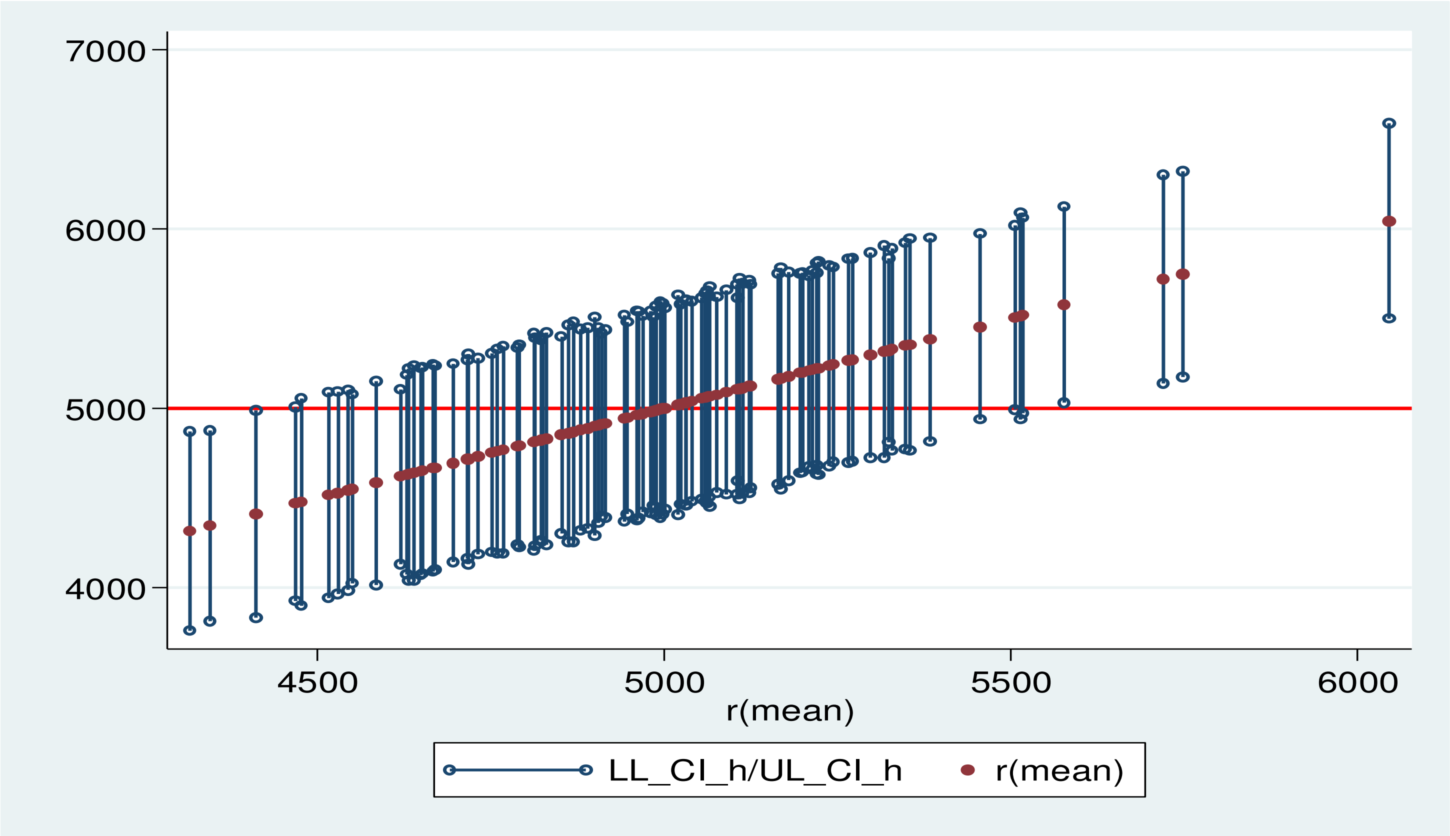


2. Repeat same steps as previously done with the Gaussian normal distributed population.

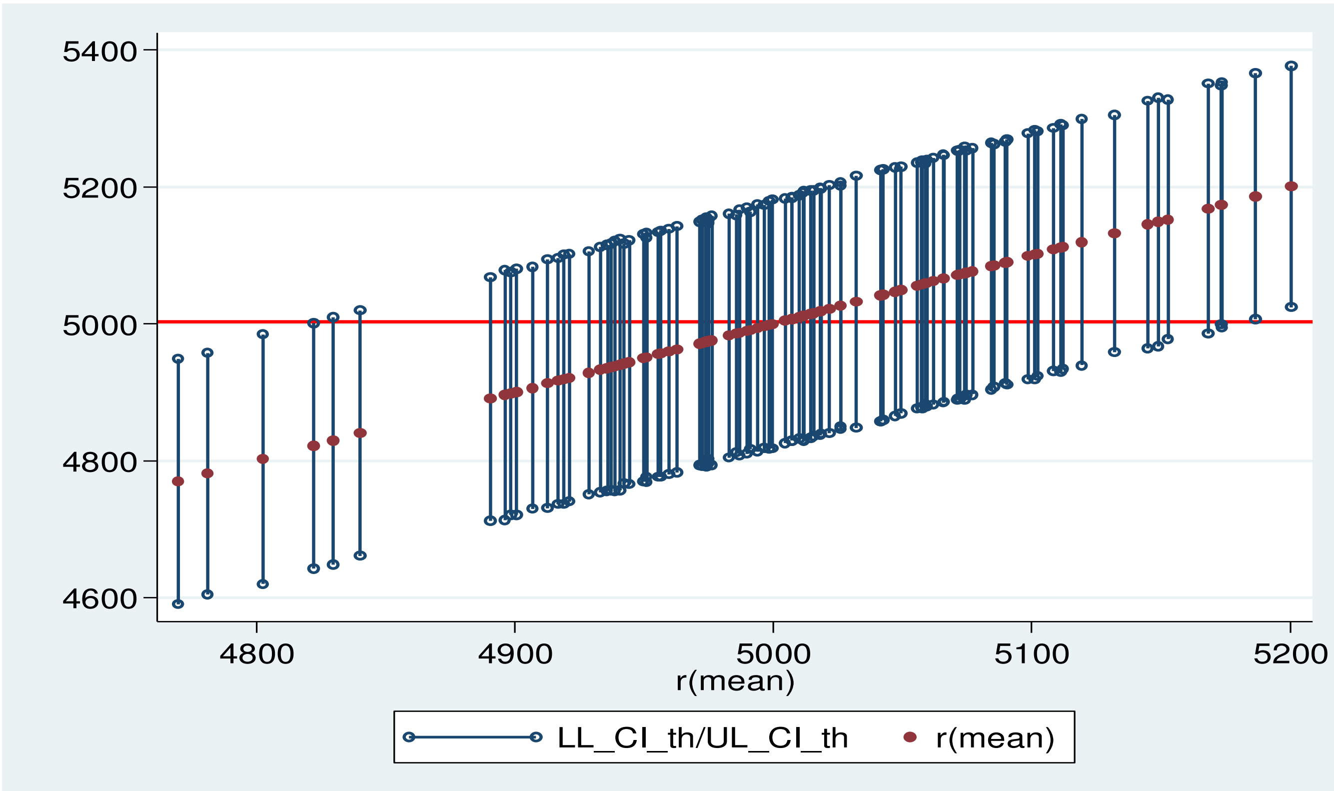
Mean and CI for a sample size of 10 observations, 100 times



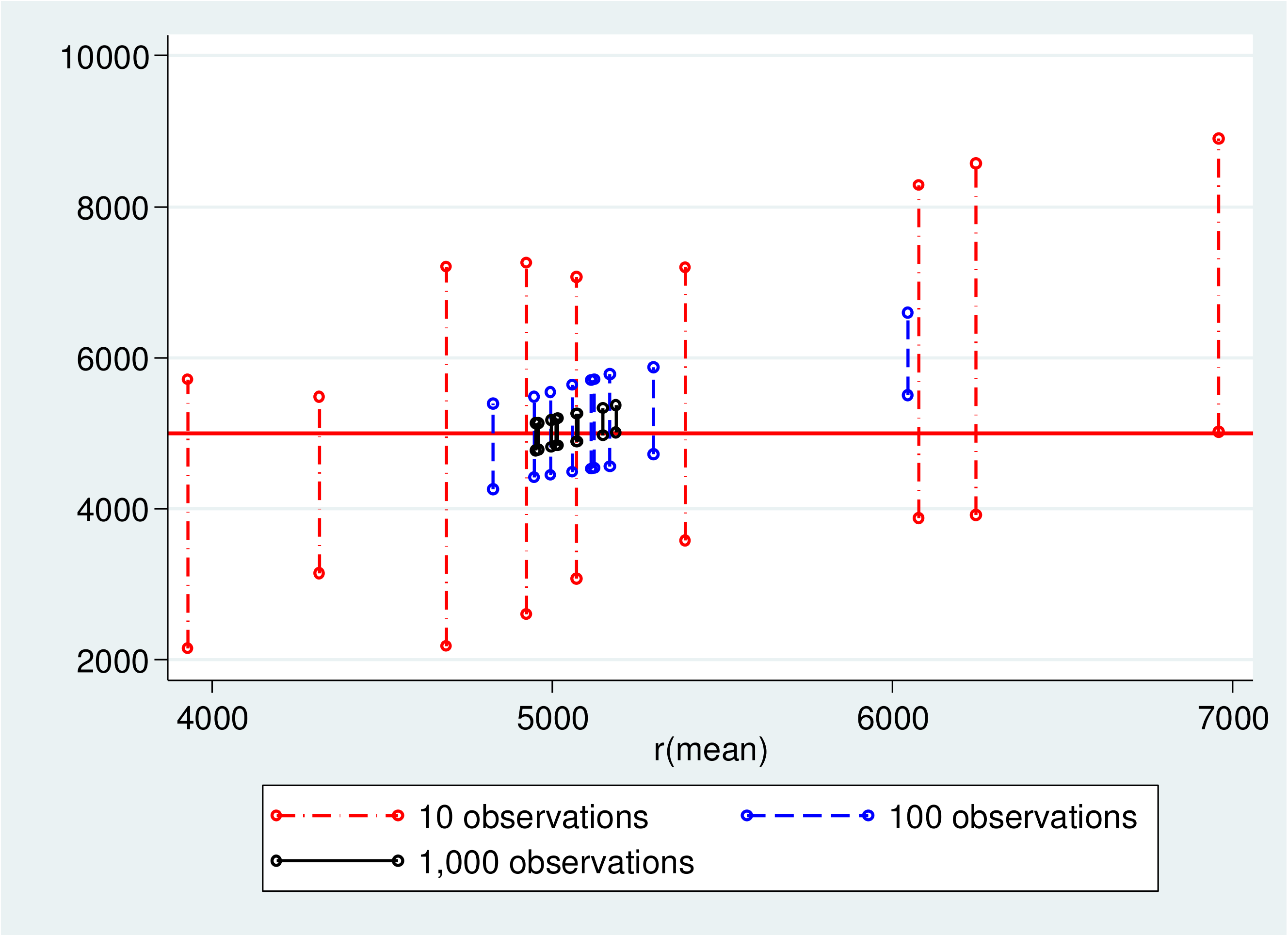
Mean and CI for a sample size of 100 observations, 100 times



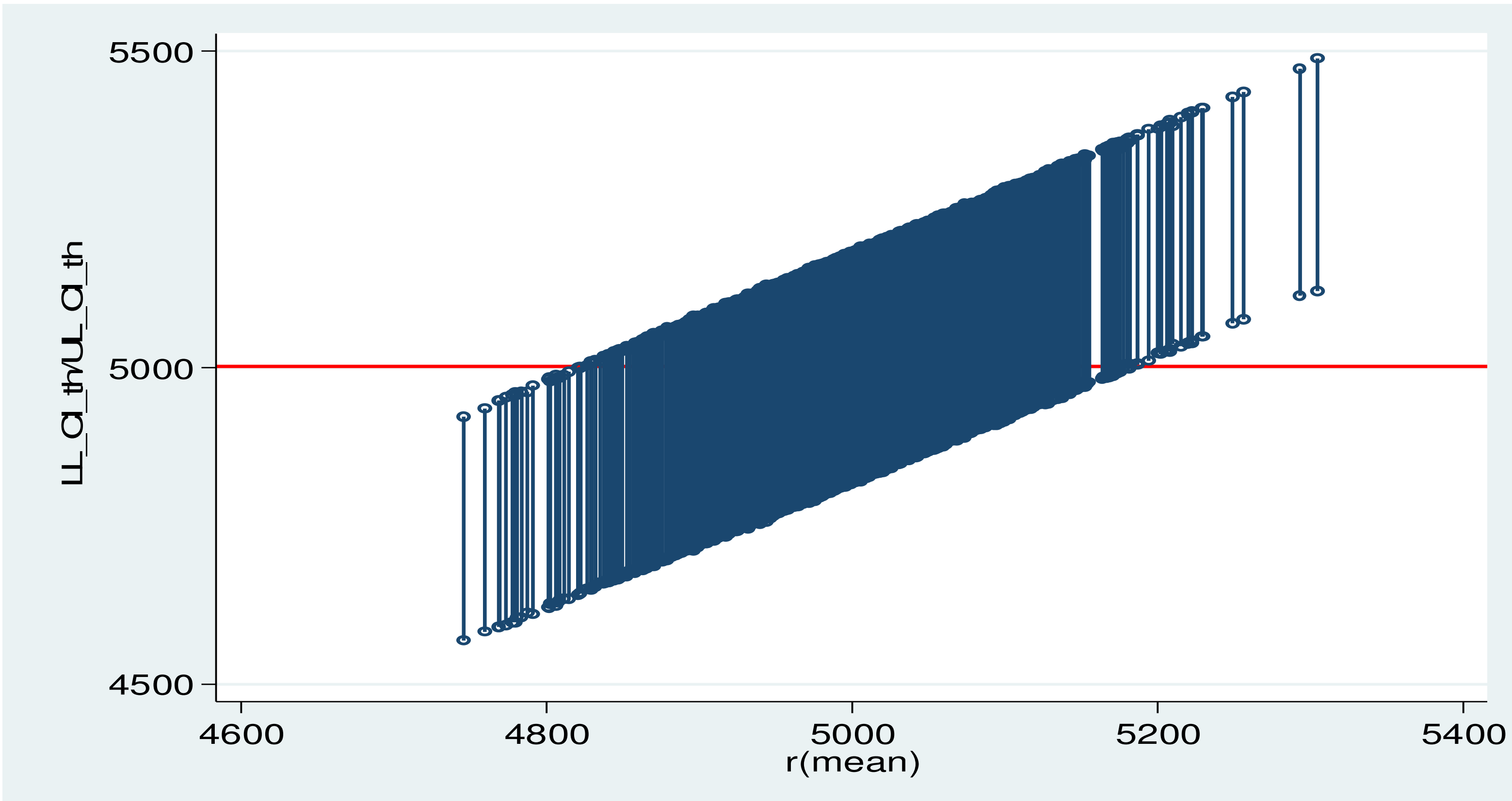
Mean and CI for a sample size of 1,000 observations, 100 times



Confidence Interval for all three sample sizes, 10 times each



Mean and CI for a sample size of 1,000 observations, 1,000 times



Comparison of Samples From Normally Distributed Population

Variable	Obs	Mean	Std. Dev.	Min	Max
mean_t	1000	5,009	342	3,863	6,246
mean_h	1000	5,011	105	4,671	5,326
mean_th	1000	5,009	33	4,890	5,120
LL_CI_t	1000	4,247	377	2,908	5,559
LL_CI_h	1000	4,797	107	4,469	5,125
LL_CI_th	1000	4,941	33	4,821	5,053
UL_CI_t	1000	5,771	388	4,477	7,007
UL_CI_h	1000	5,225	105	4,870	5,575
UL_CI_th	1000	5,076	33	4,958	5,187
CI_width_t	1000	1,523	343	632	2,674
CI_width_h	1000	428	29	337	521
CI_width_th	1000	135	3	126	146

Comparison of Samples From Uniformly Distributed Population

Variable	Obs	Mean	Std. Dev.	Min	Max
mean_t	1000	5,013	927	2,579	8,196
mean_h	1000	5,016	302	4,083	6,046
mean_th	1000	5,002	90	4,746	5,305
LL_CI_t	1000	2,966	997	675	7,043
LL_CI_h	1000	4,449	303	3,551	5,500
LL_CI_th	1000	4,822	90	4,569	5,121
UL_CI_t	1000	7,060	981	3,669	9,593
UL_CI_h	1000	5,583	303	4,615	6,591
UL_CI_th	1000	5,181	90	4,923	5,488
CI_width_t	1000	4,094	689	1,884	6,008
CI_width_h	1000	1,134	50	974	1,274
CI_width_th	1000	359	5	340	374

Formulas

(Note: for all samples less than 30 or where the population standard deviation is unknown use the "student t table rather than the Z table)

Continuous Outcomes

1) Confidence Interval for One Sample:

Equation: sample mean ± number of standard deviations to get to required confidence level (Z or T score) times the <u>standard deviation divided by the square root of the number of observations</u> (standard error)

$$X \pm Z_{(1-\propto)} \frac{s}{\sqrt{n}}$$

2) Matched Samples:

Equation: the difference of the two sample means ± Z or T score times the standard error

$$(\bar{X}_1 - \bar{X}_2) \pm Z_{(1-\propto)} \frac{s}{\sqrt{n}}$$

3) Two Independent Samples:

Equation: the difference of the two sample means ± Z or T score times the pooled estimate of the common standard deviation times the square root of 1 over the number of observations in sample 1 plus 1 over the number of observations in sample 2. "Sp" is the pooled estimated of the common standard deviation

$$(\bar{X}_1 - \bar{X}_2) \pm Z_{(1-\propto)} Sp \sqrt{\frac{1}{n_1} + \frac{1}{n_2}} \qquad \mathrm{Sp} = \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}}$$

Dichotomous Outcomes

4) Confidence Interval for One Sample:

Equation: the sample proportion ± Z or T score times the standard error

$$\widehat{p \pm} Z_{(1-\propto)}\sqrt{\frac{p(1-p)}{n}}$$

5) Two Independent Samples:

Equation: the difference of the two samples' proportion ± Z or T score times the pooled standard error

$$(\hat{p}_1 - \hat{p}_2) \pm Z_{(1-\propto)}\sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$$

6) Relative Risk (RR=p1/p2):

Equation: The natural log of the risk ratio ± Z or T score times the pooled standard error. The pooled standard error: n_1 and n_2 are the two sample sizes, x_1 and x_2 are the number of observed events from the two samples

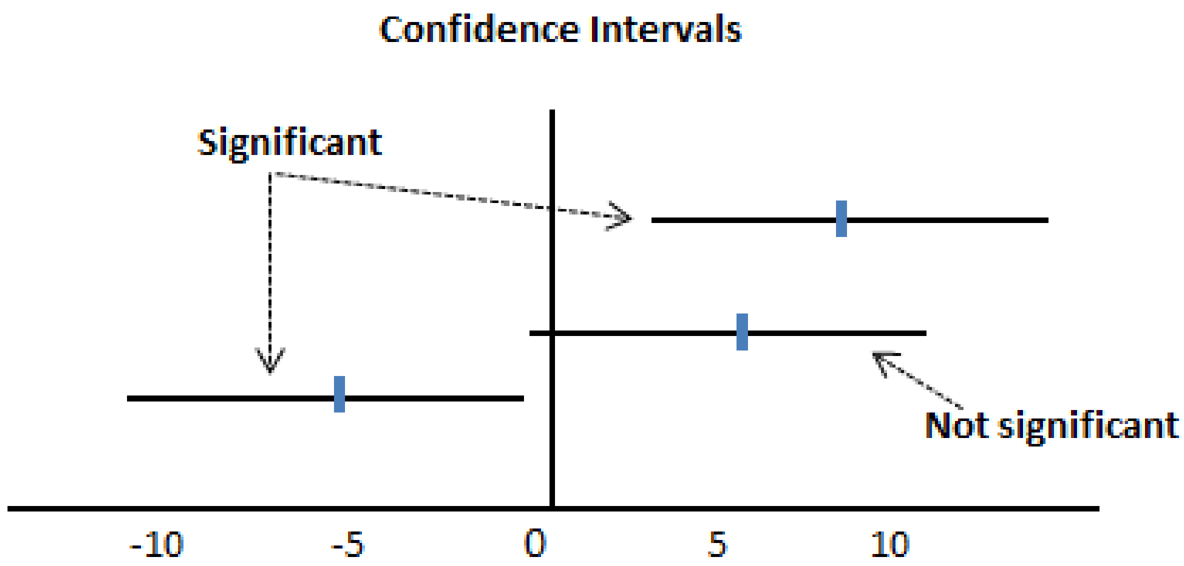
$$\mathrm{Ln(RR)} \pm Z_{(1-\propto)}\sqrt{\frac{(n_1 - x_1)/x_1}{n_1} + \frac{(n_2 - x_2)/x_2}{n_2}}$$

The antilog of the lower and upper limits must be taken in order to determine the CI.

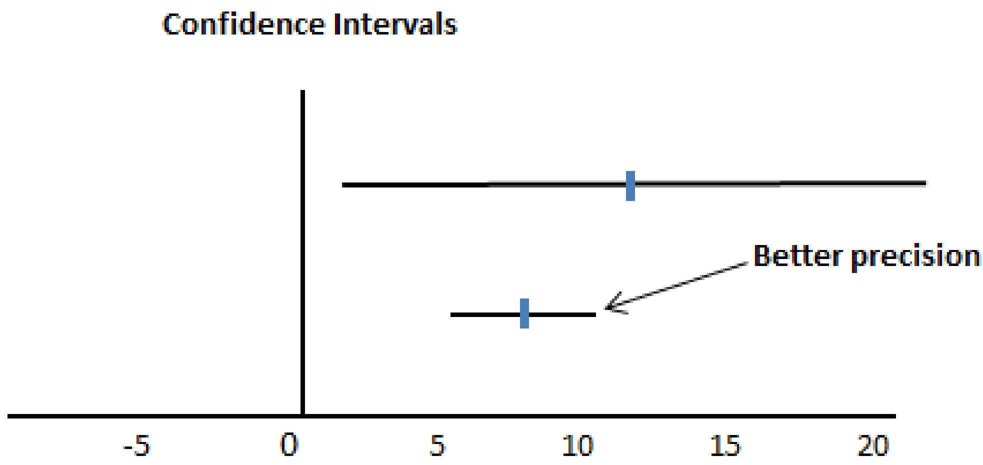
Confidence Intervals and p-Values

Null Hypothesis Significance Testing

$$H_0: x_1 = 0$$
$$H_a: x_1 \neq 0$$



Confidence Intervals and Precision

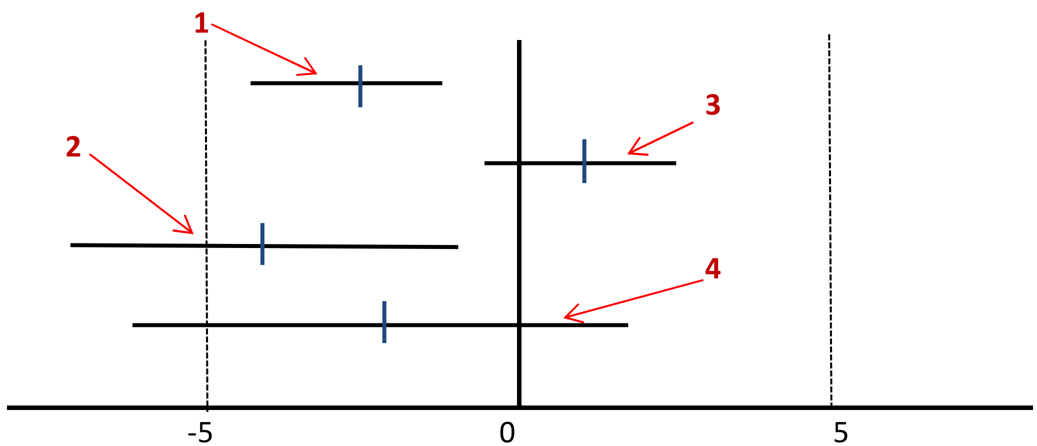


Equivalence Testing

- **Objective is to determine whether two treatments have equal results**
- **This is not the same as showing** $H_0: x_1 - x_2 = 0$
- **The researcher must determine an acceptable range where the results of the two therapies are "close enough" to be considered similar**
- **This range is known as the "equivalence margin" δ**

Example: The results from two independent therapies were analysed by two independent researchers. One researcher wanted to determine whether therapy A was statistically different from therapy B. The other researcher wanted to determine whether the two therapies' results were "close enough" to be considered equivalent. This researcher chose an equivalence margin of 5 units. The table and graph below show four "possible" confidence intervals to make the point that a "null hypothesis significance test" and an "equivalence test" do not necessarily reach the same conclusions.

Possible Outcome	Statistically Different	Statistically Equivalent
1	**Yes**	**Yes**
2	**Yes**	**No**
3	**No**	**Yes**
4	**No**	**No**



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