Effect Size Statistics

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What is an effect size statistic?

"… an index that is (1) responsive to the strength of the association between an experimental manipulation and changes in behavior and (2) independent of sample size."

- Keppel

"…provide information about the magnitude and direction of the difference between two groups or the relationship between two variables."

- Durlak

Key uses

1. Sample size calculations
2. Equivalence testing
3. Reporting results
4. Comparing effects across studies
5. Meta analysis

Types of effect sizes

Measures of Association	Measures of Mean difference	Measures of Shared Variance
• Correlation Coefficients • Odds Ratios • Regression Coefficients	• Differences in means • Cohen's d • Hedge's g • Glass's delta	• Coefficient of Determination • Eta-squared • Omega Squared

Dimensions

Simple

In original units

- Meters
- Years
- Dollars
- kg

- Scale points?

Standardized

Original units removed

- Standard deviations

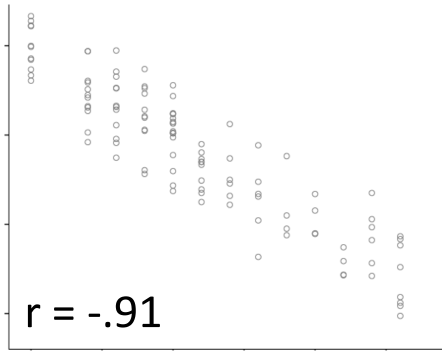
Types of effect sizes

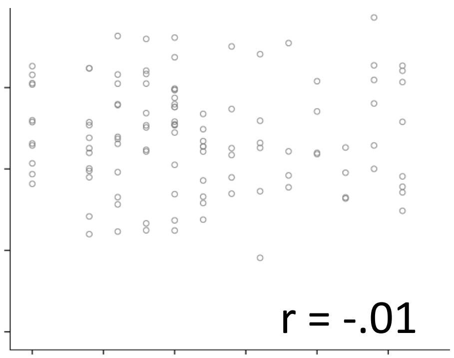
	Measures of Association	Measures of Mean difference	Measures of Shared Variance
Simple	• Odds Ratios • Regression Coefficients	• Differences in means	
Standardized	• Correlation Coefficients • Some Odds Ratios • Standardized Regression Coefficients	Standardized Differences in Means • Cohen's d • Hedge's g • Glass's delta	• Coefficient of Determination • Eta-squared • Omega Squared

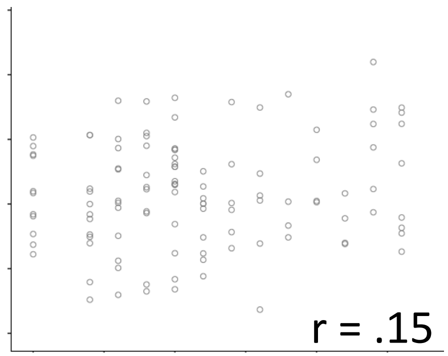
Measures of Association

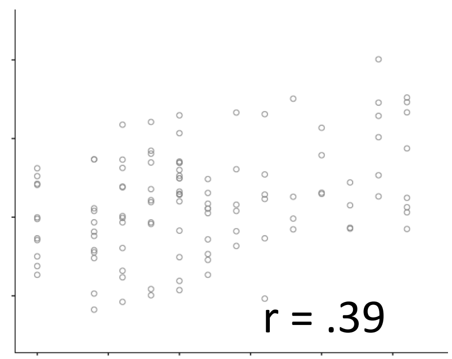
Pearson Correlation

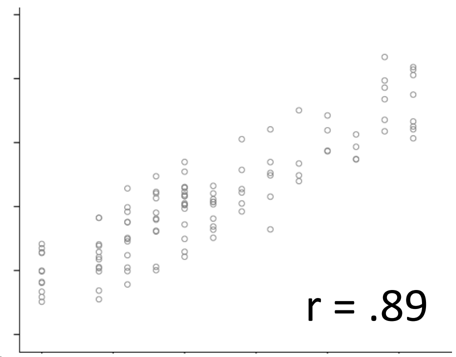
$$r_{xy} = \frac{\sum (x_i - \overline{x})(y_i - \overline{y})}{s_x s_y}$$











Regression Coefficients: Unstandardized and Standardized

Regression Coefficients
Dependent Variable: Physical Health Composite

	Unstandardized Coefficients	Standardized Coefficients		
Variable	B	β	t	p
Intercept	5931.907		32.497	.000
Depression Score	-84.633	-.451	-16.222	.000
Years of Education	30.748	.094	4.553	.000
Number of Children	34.881	.055	2.731	.006
Mental Health Score	-.169	-.178	-6.489	.000

Odds ratios

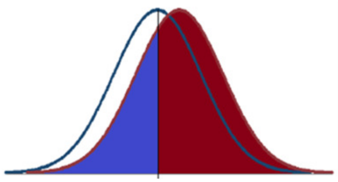
Logistic Regression Coefficients

Dependent Variable:
Successfully Navigates Asteroid Field

Variable	b	se	t	p	OR
Intercept	-8.221	2.581	3.185	.000	
Size of Ship	-.128	.041	3.12	.000	.88
Presence of R2 unit	.959	.327	2.93	.026	2.61

See: **Understanding Probability, Odds, and Odds Ratios in Logistic Regression**
https://thecraftofstatisticalanalysis.com/webinar-recording-signup/?cosid=605

Measures of Mean Difference



Mean Differences

Simple metric

$$\overline{y}_T - \overline{y}_C$$

T = Treatment
C = Control

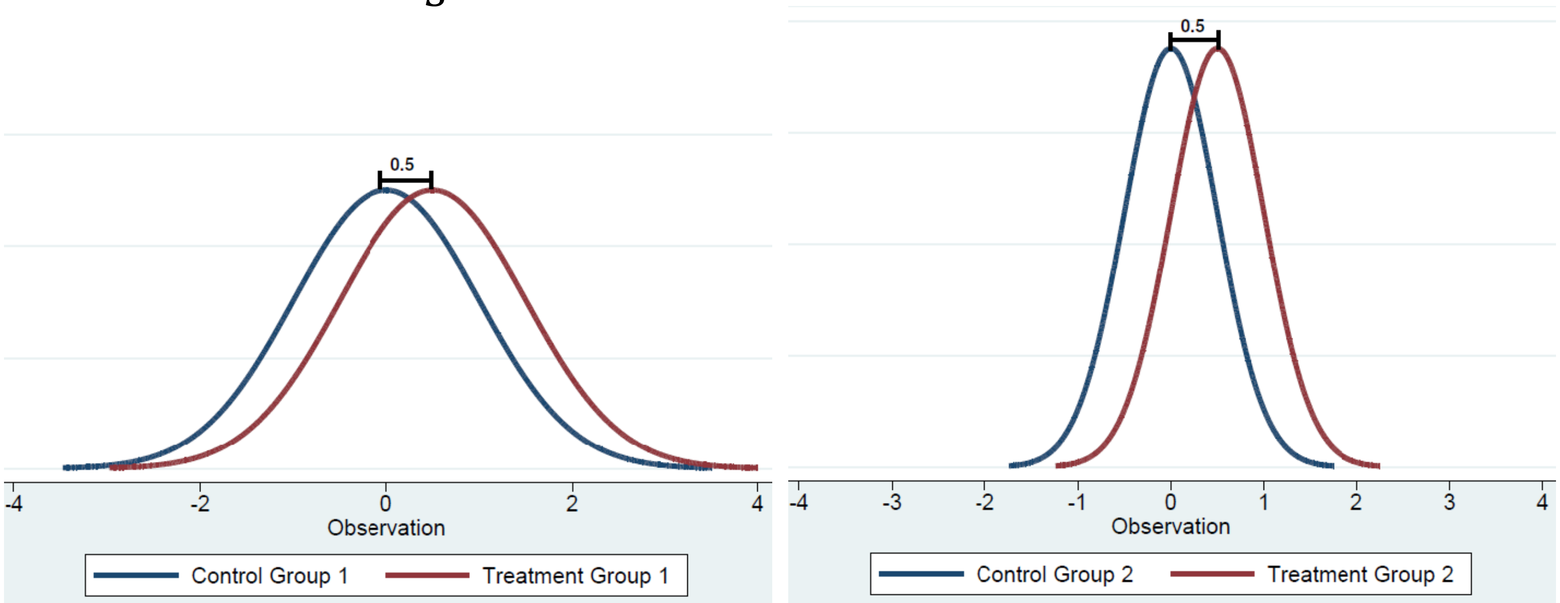
Readers may be unaware that a direct comparison of group means can serve as a useful ES.

~ Durlak (2009)

Standardized Mean differences

Cohen's d: $d = \frac{\bar{y}_T - \bar{y}_C}{s}$

T = Treatment
C = Control



Standardized Mean differences

Cohen's d: $$d = \frac{\bar{y}_T - \bar{y}_C}{s}$$

T = Treatment
C = Control

$$s_{pooled} = \sqrt{\frac{\left((n_T - 1)s_T^2 + (n_C - 1)s_C^2\right)}{n_T + n_C - 2}}$$

Hedge's g
(bias correction for small samples): $$g \cong d(1 - \frac{3}{4(n_1 + n_2) - 9}$$

Glass's Δ
(for unequal variances): $$s = s_C$$

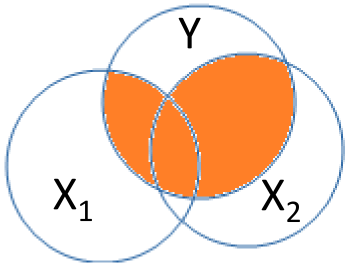
Interpretation, Power Calculations, and T-shirt sizes

"It is always important to think in terms of actual, absolute effect sizes, in the same units of measurement as where the inference is to be made.

There is really no honest way around addressing both the numerator and denominator of d separately."

~Lenth (2000)

Measures of Shared Variance



Coefficient of Determination: R^2

$$R^2 = \frac{SS_R}{SS_T}$$

ANOVA

Dependent Variable: Physical Health Composite Score

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	220398162.554[a]	4	55099540.638	93.760	.000
Intercept	620620814.804	1	620620814.804	1056.083	.000
Depression Score	154651793.207	1	154651793.207	263.164	.000
Years of Education	12182417.578	1	12182417.578	20.730	.000
Number of Children	4383254.842	1	4383254.842	7.459	.006
Mental Health Score	24744277.084	1	24744277.084	42.106	.000
Error	1224102367.476	2083	587663.162		
Total	57739934323.000	2088			
Corrected Total	1444500530.030	2087			

a. R Squared = .153 (Adjusted R Squared = .151)

Adjusted R^2

$$R^2_{adj} = 1 - \left[\frac{\left(1 - R^2\right)(n-1)}{n-k-1} \right]$$

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a. R Squared = .153 (Adjusted R Squared = .151)

Eta squared

$$\eta^2 = \frac{SS_{Effect}}{SS_{Total}}$$

$$\eta^2_{MHS} = \frac{24744277}{1444500530} = .017$$

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a. R Squared = .153 (Adjusted R Squared = .151)

Partial eta squared

$$\eta_p^2 = \frac{SS_{Effect}}{SS_{Effect} + SS_{Error}}$$

ANOVA

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Corrected Total	1444500530.030	2087			

a. R Squared = .153 (Adjusted R Squared = .151)

$$\omega_{MHS}^2 = \frac{24744277}{24744277 + 1224102367.476} = .020$$

Omega squared

$$\hat{\omega}_A^2 = \frac{SS_A - (a-1)(\text{MS}_{Error})}{SS_T + \text{MS}_{Error}}$$

ANOVA

Dependent Variable: Physical Health Composite Score

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a. R Squared = .153 (Adjusted R Squared = .151)

$$\omega_{MHS}^2 = \frac{24744277 - 1*587663}{1444500530 + 587663} = .017$$

Standardized ES are standard for:

Correlations
T-tests
Chi-square tests of independence
Linear Regression
Factorial ANOVA
Logistic Regression

…but not for

Mixed models
Nonparametric tests and models

References and Resources

- Effect Size Calculators https://www.psychometrica.de/effect_size.html
- Coe, R. (2002). It's the Effect Size, Stupid: what effect size is and why it is important http://www.leeds.ac.uk/educol/documents/00002182.htm
- Morris & DeShon (2003). Estimating Common Metric Effect Sizes From a Variety of Research Designs http://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.121.2394&rep=rep1&type=pdf
- Lenth (2001). Some Practical Guidelines for Effective Sample Size Determination https://stat.uiowa.edu/sites/stat.uiowa.edu/files/techrep/tr303.pdf
- Durlak (2009). How to Select, Calculate, and Interpret Effect Sizes http://jpepsy.oxfordjournals.org/content/34/9/917.full.pdf

References and Resources at The Analysis Factor

Resource Page: Effect Size Statistics, Power and Sample Size Calculations, and Statistical Inference

https://www.theanalysisfactor.com/resources/by-topic/effect-size-statistics-power-and-sample-size-calculations/