



Interpreting (Even Tricky) Regression Coefficients

Multiplicative Terms for Continuous Predictor Variables

Karen Grace-Martin

Module 3: Multiplicative Terms for Continuous Predictor Variables

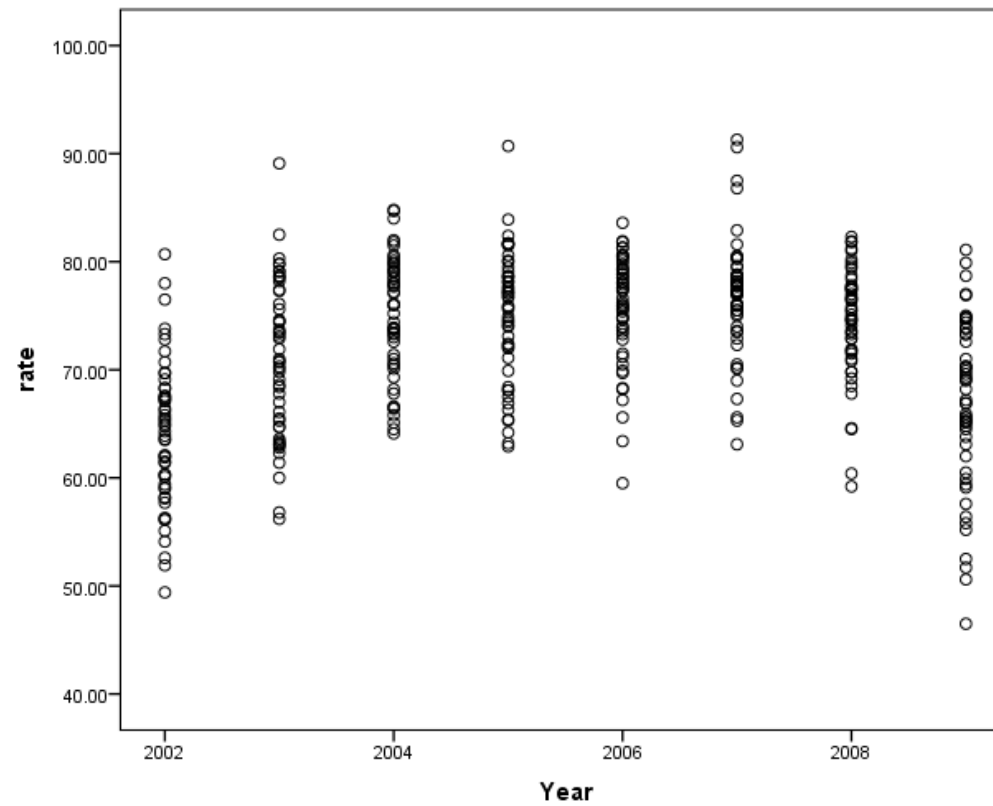


1. Quadratic Terms
2. Interactions Between Two Continuous Variables
3. Interactions with Quadratic Terms

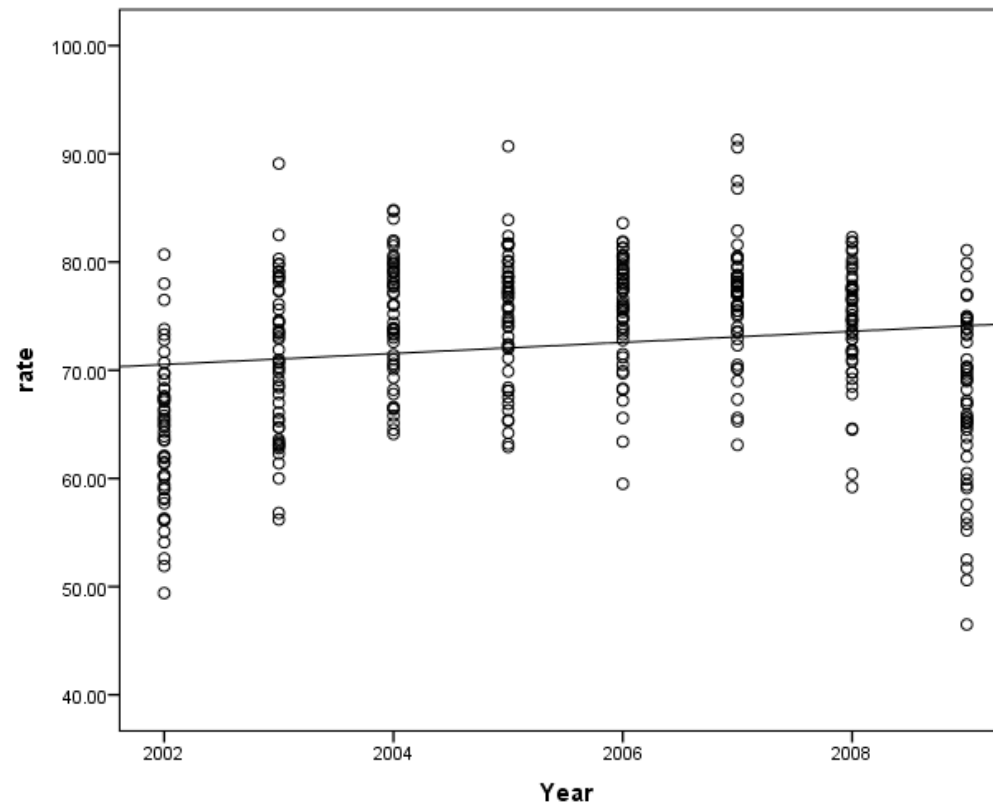


Quadratic Terms

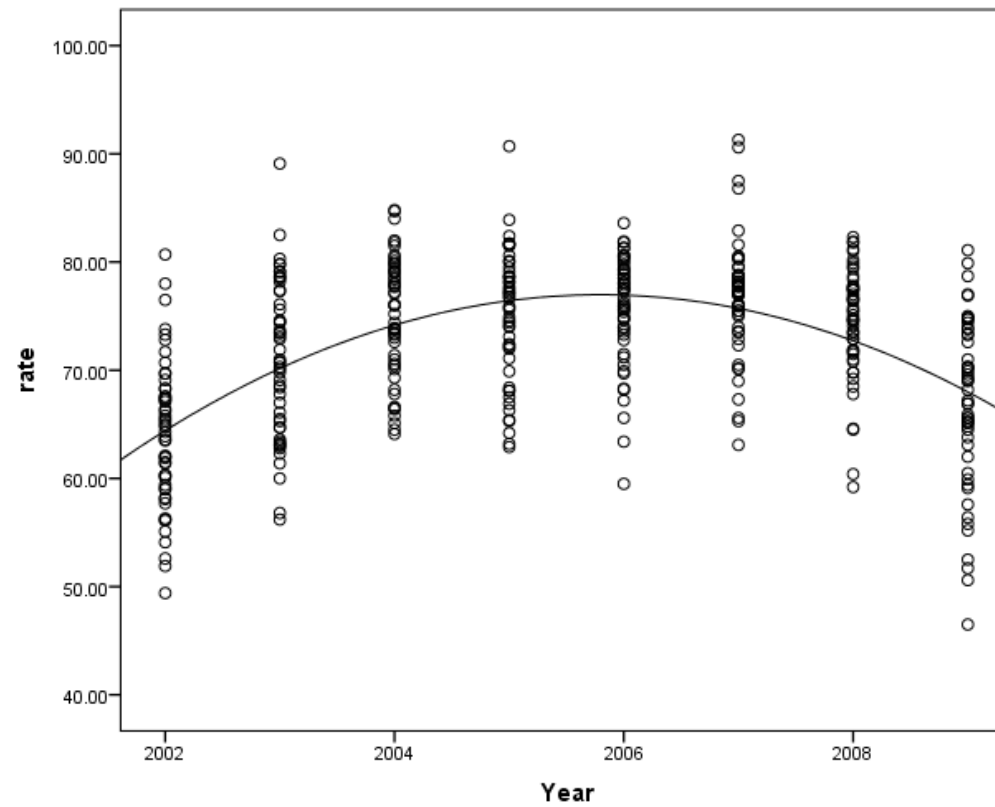
Quadratic Terms



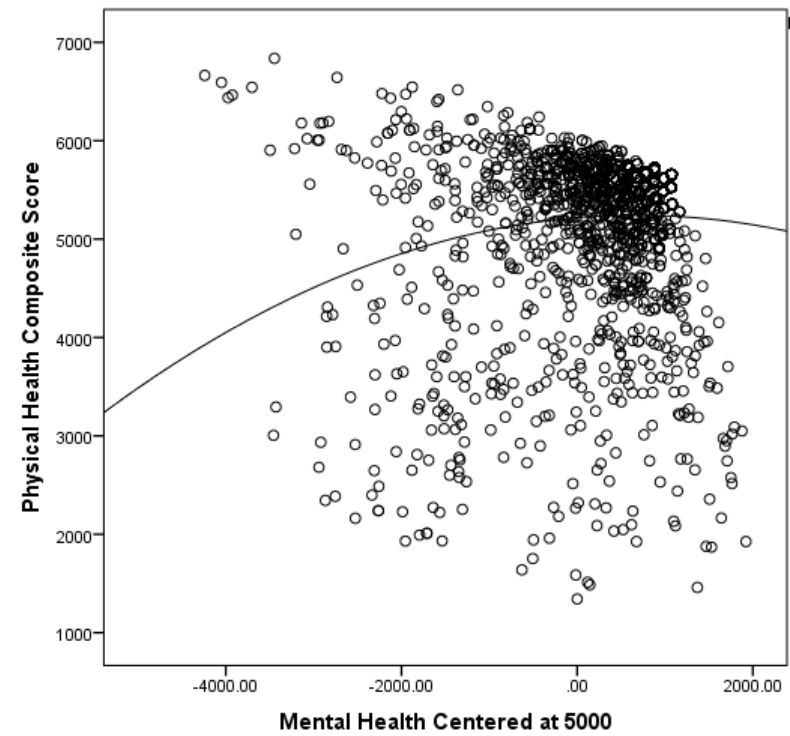
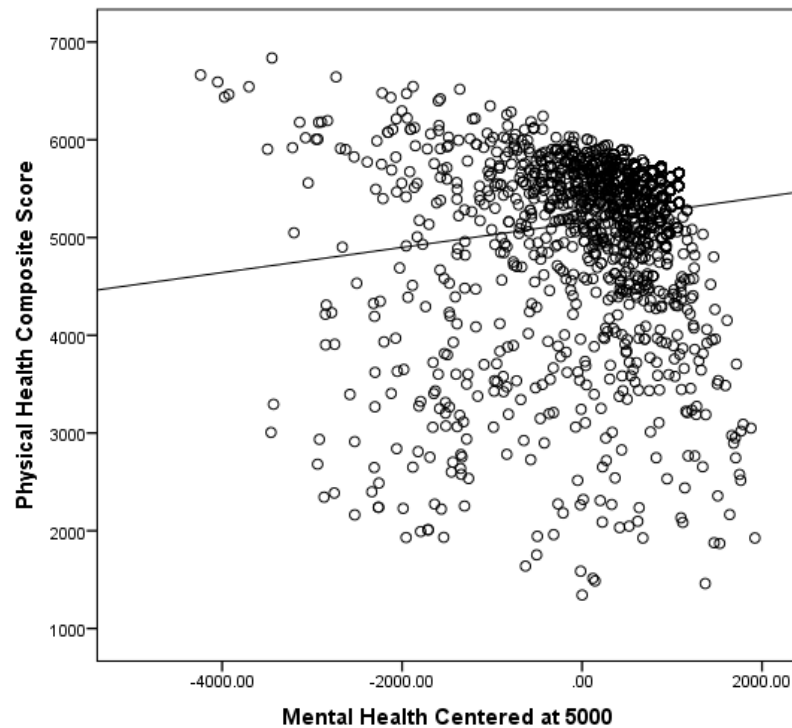
Quadratic Terms



Quadratic Terms



Quadratic Terms



Quadratic Terms



$$\hat{Y} = \beta_0 + \beta_1 X_1 + \beta_2 X_1^2$$

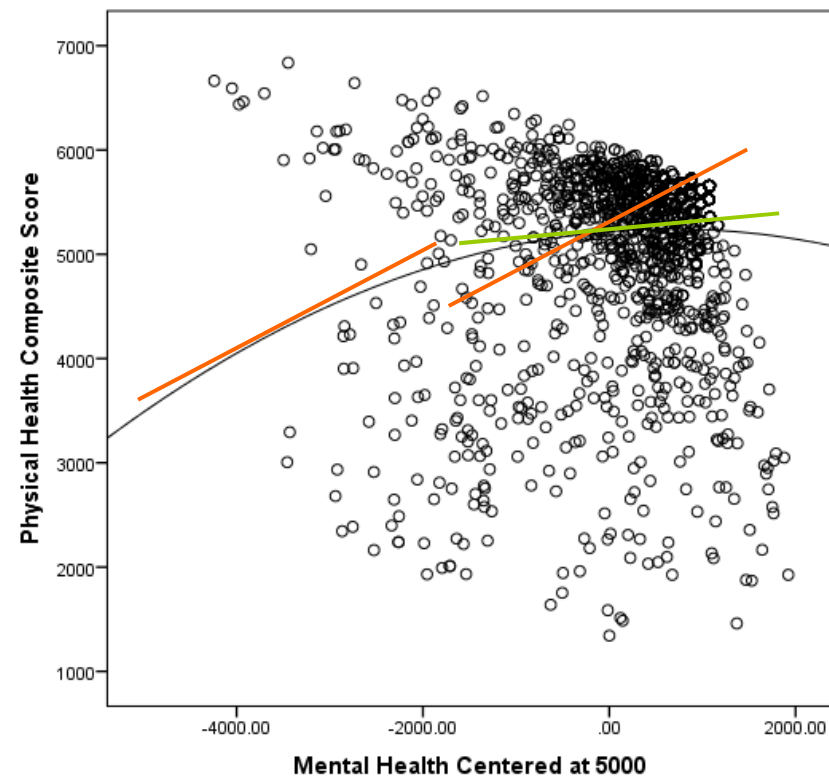
Interpretations:

β_0 = Average value of Y when all X = 0

β_1 = Linear Coefficient of X_1 : slope of the tangent line at $X_1 = 0$

β_2 = Quadratic Coefficient of X_1 : Curvilinear effect of X_1

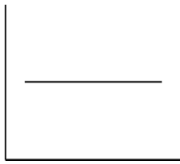

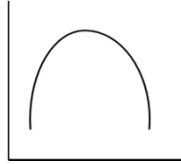
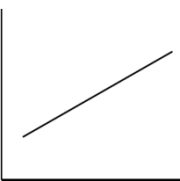
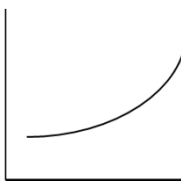
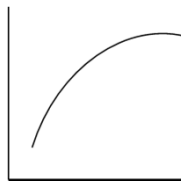
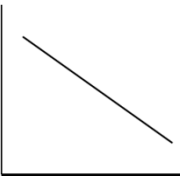
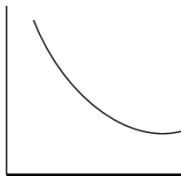
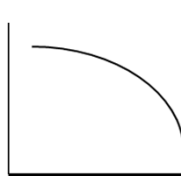
Quadratic Terms



Quadratic Terms

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_1^2$$

- b_1 is linear trend
- b_2 is curvature if X is centered

		Curvature		
		None $b_2 = 0$	Upward b_2 positive	Downward b_2 negative
Linear Trend	None $b_1 = 0$			
	Upward b_1 positive			
	Downward b_1 negative			



Quadratic Terms



Regression Coefficients

Dependent Variable: PCS2000

Variable	B	se	t	p
Intercept	5218.043	25.465	204.914	.000
MCS2000Cen	.073	.026	2.847	.004
MCS2000Sq	-.000054	1.518E-5	-3.599	.000

$$E(\text{Physical Health}) = 5218 + .07(\text{Mental Health}) - .00005(\text{Mental Health})^2$$

Quadratic Terms



$$E(Y|X) = \hat{Y} = \beta_0 + \beta_1 X_1 + \beta_2 X_1^2$$

What is the effect of X_1 on Y ?

- NOT β_1
- Depends on value of X_1
- Slope of the tangent line (derivative) = $\beta_1 + 2\beta_2 X_1$

Quadratic Terms



Regression Coefficients

Dependent Variable: PCS2000

Variable	B	se	t	p
Intercept	5218.043	25.465	204.914	.000
Mental Health Centered	.073	.026	2.847	.004
Mental Health Squared	-.00005	1.518E-5	-3.599	.000

Regression Coefficients

Dependent Variable: PCS2000

Variable	B	se	t	p
Intercept	5122.528	99.631	51.415	.000
Education	30.499	6.747	4.520	.000
NumberChildren	33.821	12.766	2.649	.008
Depression	-83.745	5.225	-16.026	.000
Mental Health Centered	-.200	.029	-6.853	.000
Mental Health Squared	-.00003	1.420E-5	-2.332	.020

Quadratic Terms



What is the effect of mental health on physical health (controlling for other covariates)?

Quadratic Terms



What is the effect of mental health on physical health (controlling for other covariates)?

$$E(\text{Physical Health}) = 5123 + 30.5(\text{Education}) + 33.8(\text{Number Children}) \\ - 83.7(\text{Depression}) - .2(\text{Mental Health Centered}) - .00003(\text{Mental Health Centered})^2$$

$$\text{Effect} = b_4 + 2b_5X_4$$



Quadratic Terms

What is the effect of mental health on physical health (controlling for other covariates)?

$$E(\text{Physical Health}) = 5123 + 30.5(\text{Education}) + 33.8(\text{Number Children}) \\ - 83.7(\text{Depression}) - .2(\text{Mental Health Centered}) - .00003(\text{Mental Health Centered})^2$$

$$\begin{aligned} \text{Effect} &= b_4 + 2b_5X_4 \\ &= -.2 - 2(.00003)*\text{MHC}_{\text{Cen}} \\ &= -.2 - .00006*\text{MHC}_{\text{Cen}} \end{aligned}$$



Quadratic Terms

What is the effect of mental health on physical health (controlling for other covariates)?

$$E(\text{Physical Health}) = 5123 + 30.5(\text{Education}) + 33.8(\text{Number Children}) - 83.7(\text{Depression}) - .2(\text{Mental Health Centered}) - .00003(\text{Mental Health Centered})^2$$

$$\begin{aligned}\text{Effect} &= b_4 + 2b_5X_4 \\ &= -.2 - 2(.00003)*\text{MHC}_{\text{Cen}} \\ &= -.2 - .00006*\text{MHC}_{\text{Cen}}\end{aligned}$$

$$\text{At } \text{MHC}_{\text{Cen}} = -4000$$

$$\text{Effect} = -.2 - .00006*(-4000) = .04$$

$$\text{At } \text{MHC}_{\text{Cen}} = 0$$

$$\text{Effect} = -.2 - .00006*0 = -.20$$

$$\text{At } \text{MHC}_{\text{Cen}} = 1500$$

$$\text{Effect} = -.2 - .00006*1500 = -.29$$

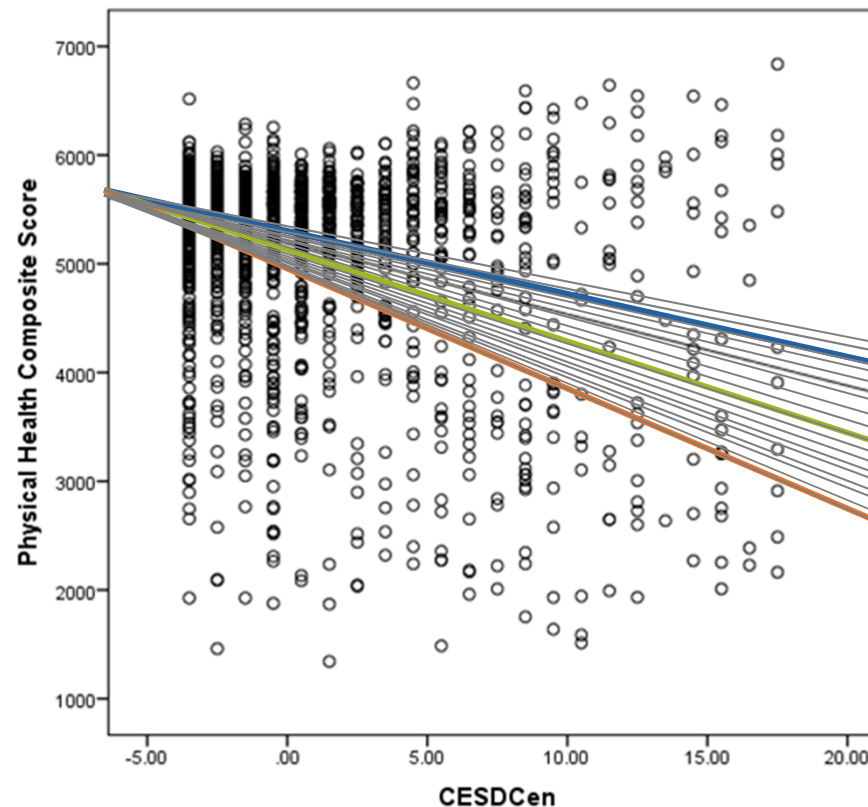


Interactions Between Two Continuous Variables

Interactions Between Two Continuous Variables



Effect of
Depression
at different
values of
Education



EducCen = 7

7 years above mean (20)

EducCen = 0

At mean (13)

EducCen = -7

7 years below mean (6)

Interactions Between Two Continuous Variables



$$E(Y|X) = b_0 + b_1X_1 + b_2X_2 + b_3X_1 * X_2$$

$$E(Y|X) = b_0 + b_1\text{Dep} + b_2\text{Educ} + b_3\text{Dep} * \text{Educ}$$

Definitions:

b_0 = Intercept

b_1 = First order effect of X_1

b_2 = First order effect of X_2

b_3 = Interaction term

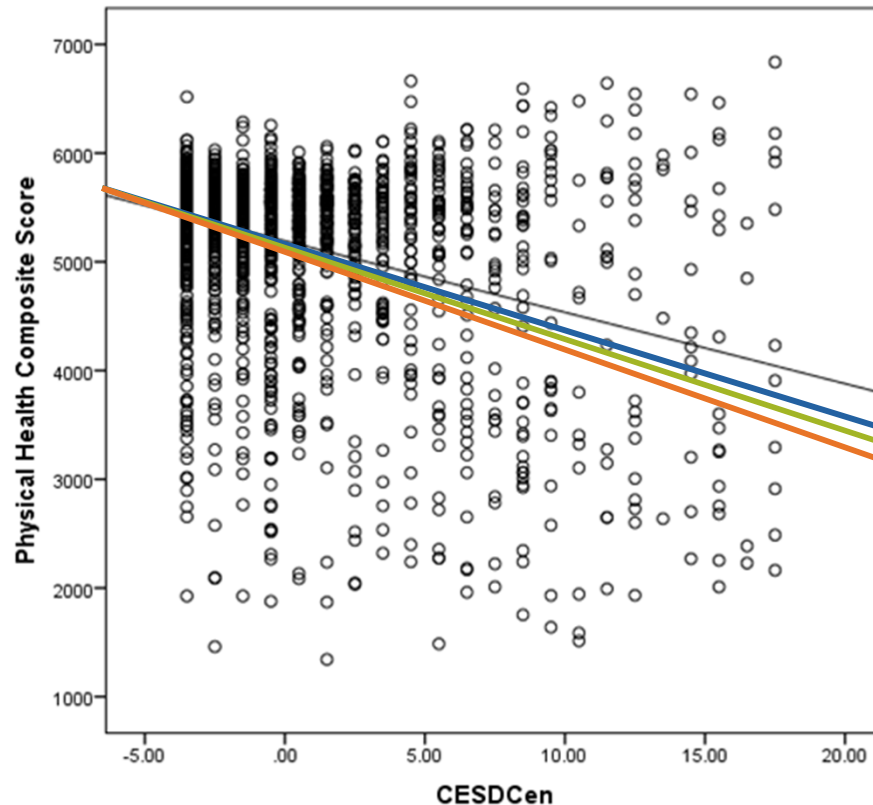
X_1 = Depression Centered at mean (Dep)

X_2 = Education Centered at 13 (Educ)

X_3 = Dep*Educ



Interactions Between Two Continuous Variables



$$E(Y|X) = b_0 + b_1\text{Dep} + b_2\text{Educ} + b_3\text{Dep*Educ}$$

b_3 = difference in slopes of
green and blue lines

EducCen = 1

b_1 = slope of green line

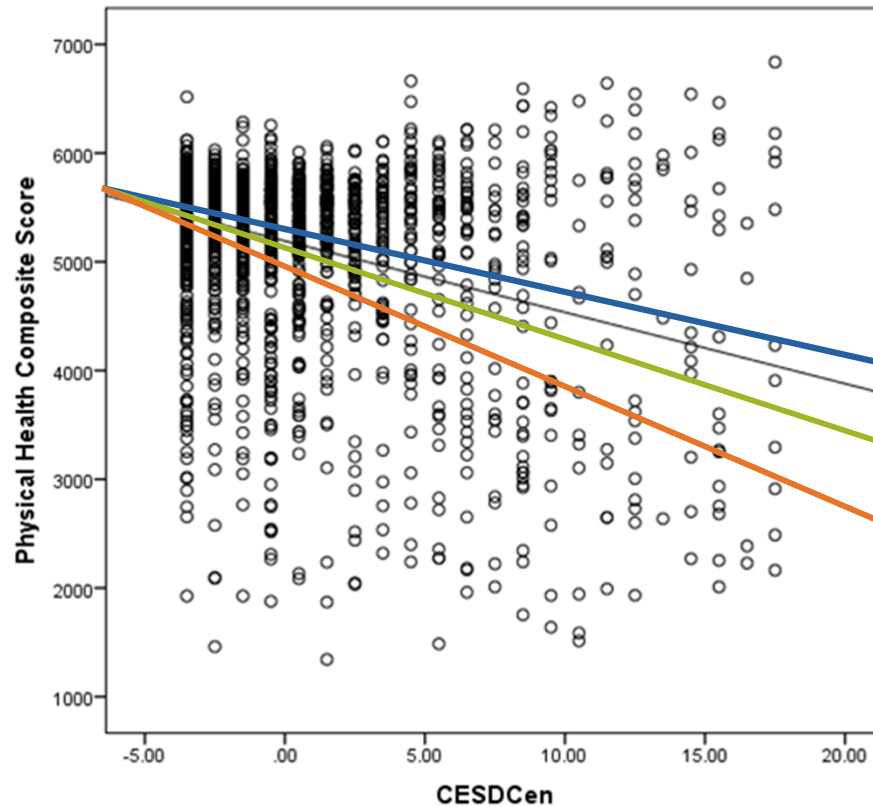
EducCen = 0

$-b_3$ = difference in slopes of
green and orange lines

EducCen = -1



Interactions Between Two Continuous Variables



$$E(Y|X) = b_0 + b_1\text{Dep} + b_2\text{Educ} + b_3\text{Dep}*\text{Educ}$$

$7*b_3$ = difference in slopes
of green and blue lines

EducCen = 7

b_1 = slope of green line

EducCen = 0

$-7*b_3$ = difference in slopes
of green and blue lines

EducCen = -7

Interactions Between Two Continuous Variables



Regression Coefficients

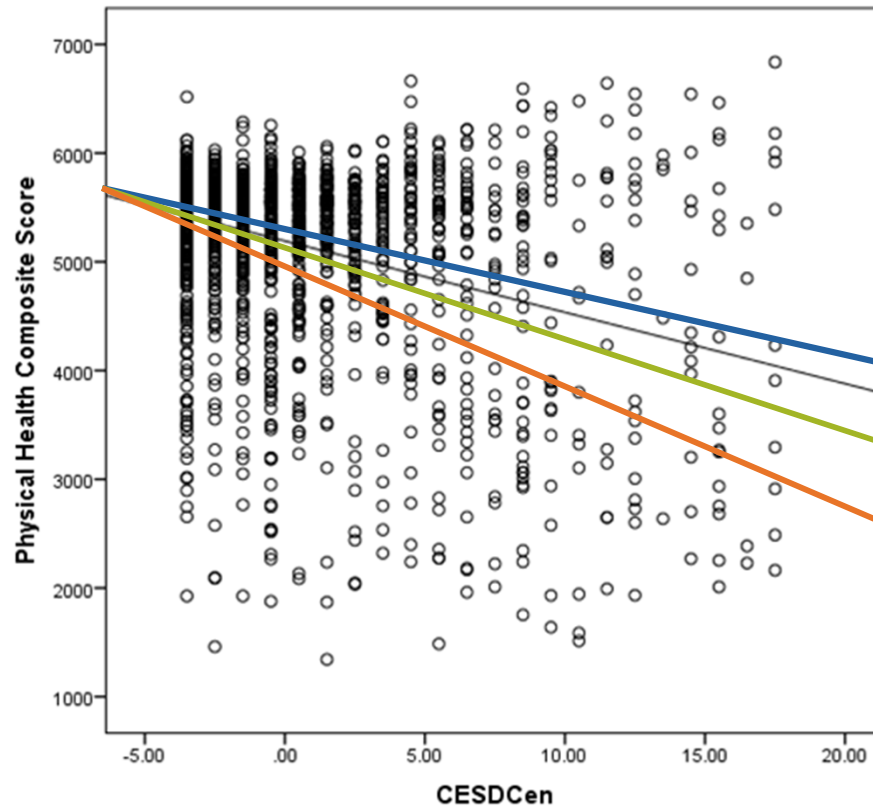
Dependent Variable: PCS2000

Variable	B	se	t	p
Intercept	5200.796	17.207	302.241	.000
Education Centered at 13	35.763	6.841	5.228	.000
Depression Centered at mean	-59.105	3.977	-14.862	.000
Education Centered * Depression Centered	5.672	1.573	3.606	.000

$$E(\text{Physical Health}) = 5201 + 35.8(\text{Years of Education}) - 59.1(\text{Depression}) + 5.7(\text{Education} * \text{Depression})$$



Interactions Between Two Continuous Variables



$E(\text{Physical Health}) = 5201 + 35.8(\text{Years of Education}) - 59.1(\text{Depression}) + 5.7(\text{Education} * \text{Depression})$

$7 * b_3 = 7 * 5.7 = 39.9$

$b_1 = -59.1$

$-7 * b_3 = -7 * 5.7 = -39.9$

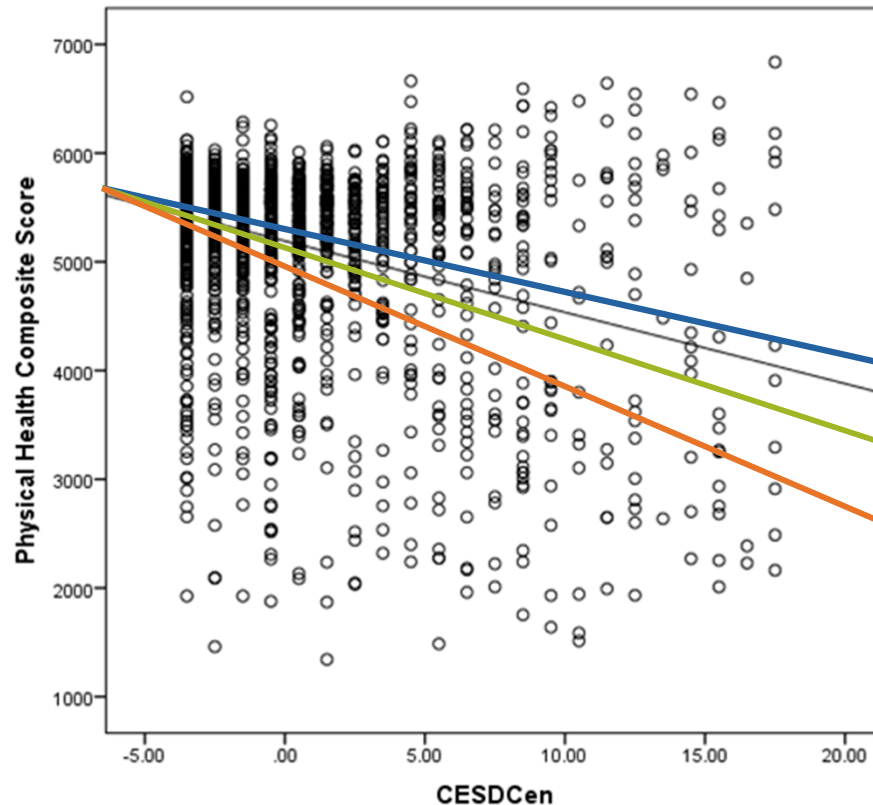
EducCen = 7

EducCen = 0

EducCen = -7



Interactions Between Two Continuous Variables



$$E(\text{Physical Health}) = 5201 + 35.8(\text{Years of Education}) - 59.1(\text{Depression}) + 5.7(\text{Education} * \text{Depression})$$

Effect of Depression at 20 years of Education
 $= -59.1 + 39.9 = -19.2$

Effect of Depression at 13 years of Education
 $= -59.1$

Effect of Depression at 6 years of Education
 $= -59.1 - 39.9 = -99$

EducCen = 7

EducCen = 0

EducCen = -7

Interactions Between Two Continuous Variables



$$E(Y|X) = b_0 + b_1X_1 + b_2X_2 + b_3X_1 * X_2$$

What is the effect of X_1 on Y ?

- NOT b_1
- Depends on value of X_2
- Derivative = $b_1 + b_3X_2$

Interactions Between Two Continuous Variables



Regression Coefficients

Dependent Variable: PCS2000

Variable	B	se	t	p
Intercept	5196.382	25.773	201.618	.000
Education Centered at 13	33.537	6.778	4.948	.000
Number Children	36.405	12.742	2.857	.004
Depression Centered at mean	-81.209	5.288	-15.358	.000
Mental Health Centered	-.168	.026	-6.434	.000
Education Centered *	5.623	1.556	3.613	.000
Depression Centered				

$$E(\text{Physical Health}) = 5196 + 33.5(\text{Years of Education}) + 36.4(\text{Number of Children}) - 81.2(\text{Depression}) - .17(\text{Mental Health}) + 5.6(\text{Education} * \text{Depression})$$



Interactions Between Two Continuous Variables

What is the effect of depression on physical health (controlling for other covariates)?

$E(\text{Physical Health}) = 5196 - .17(\text{Mental Health}) + 36.4(\text{Number of Children}) + 33.5(\text{Years of Education}) - 81.2(\text{Depression}) + 5.6(\text{Education} * \text{Depression})$

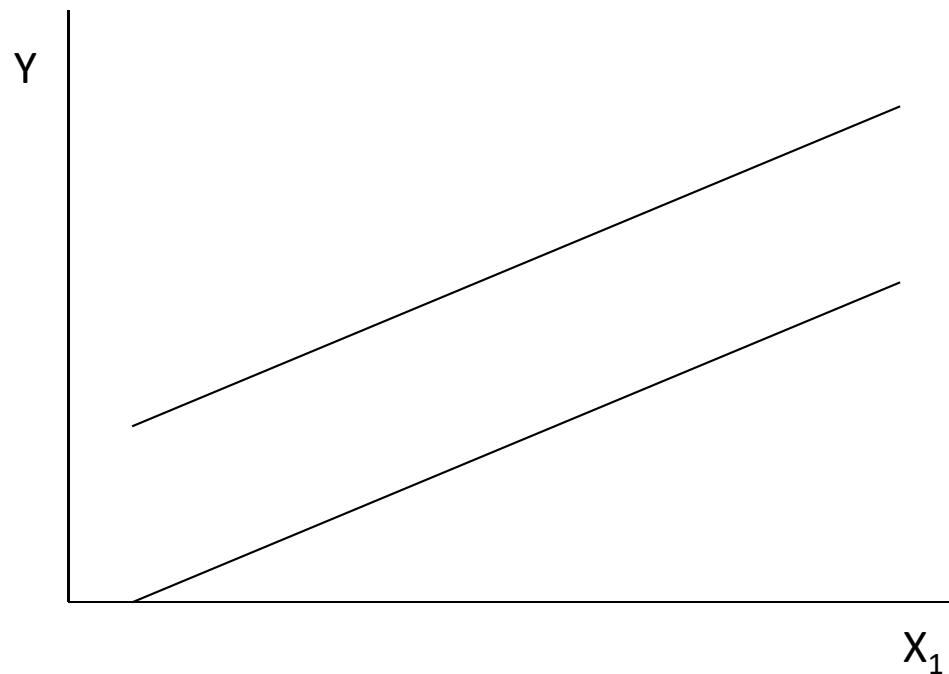
$$\begin{aligned}\text{Effect} &= b_1 + b_3X_2 \\ &= -81.2 + 5.6(\text{Education})\end{aligned}$$

Education = -7	= -81.2 + 5.6(Education)	= - 120.4
Education = 0	= -81.2 + 5.6(Education)	= - 81.2
Education = 7	= -81.2 + 5.6(Education)	= - 42.0



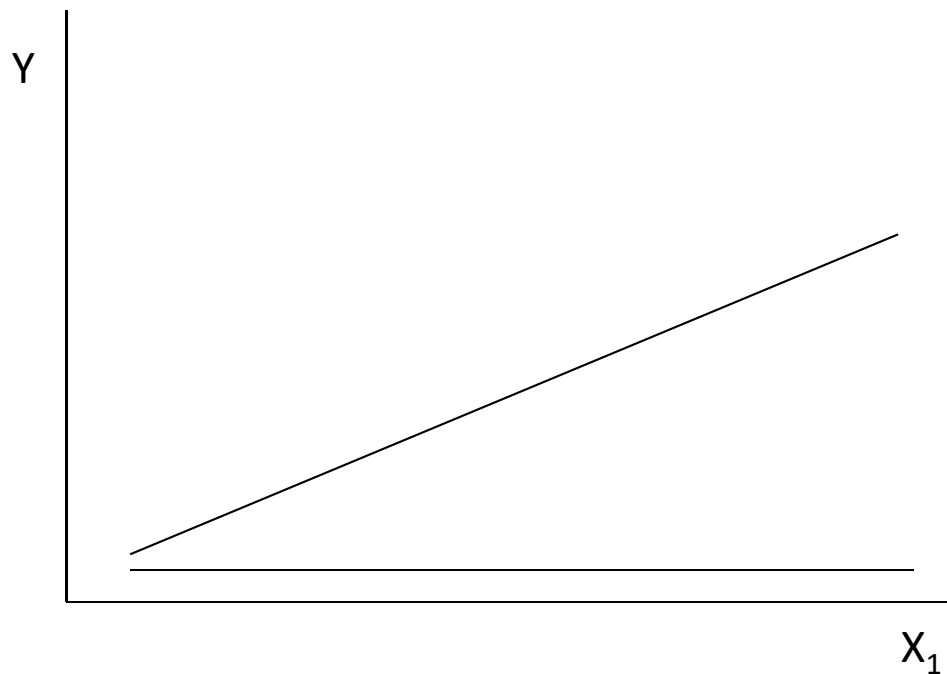
Interactions with Quadratic Terms

Interactions with Quadratic Terms



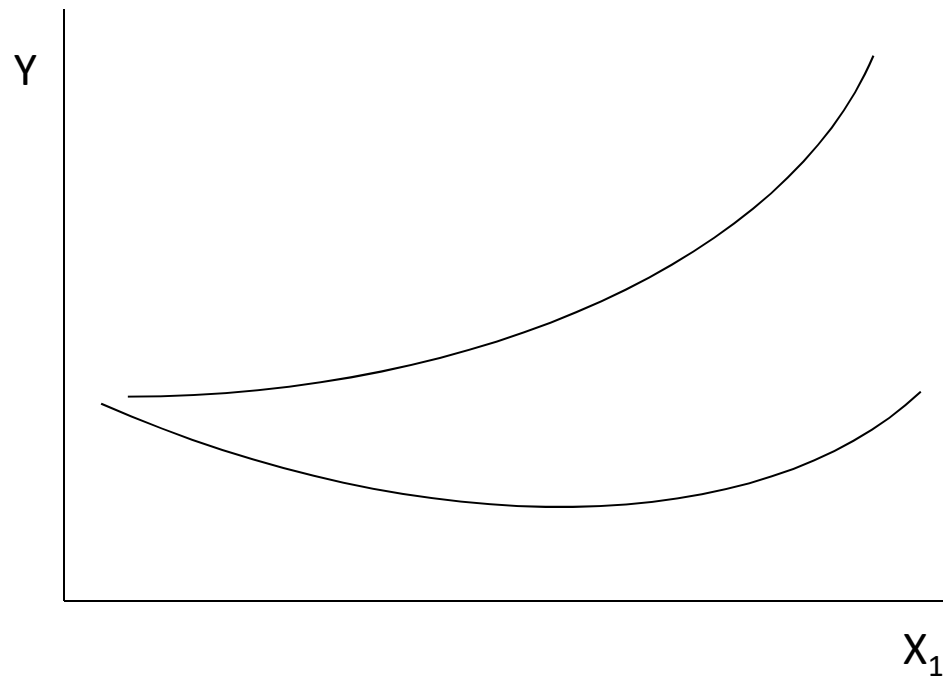
$$E(Y|X) = \beta_0 + \beta_1 X_1 + \beta_2 X_1^2 + \beta_3 X_2 + \beta_4 X_1 * X_2 + \beta_5 X_1^2 * X_2$$

Interactions with Quadratic Terms



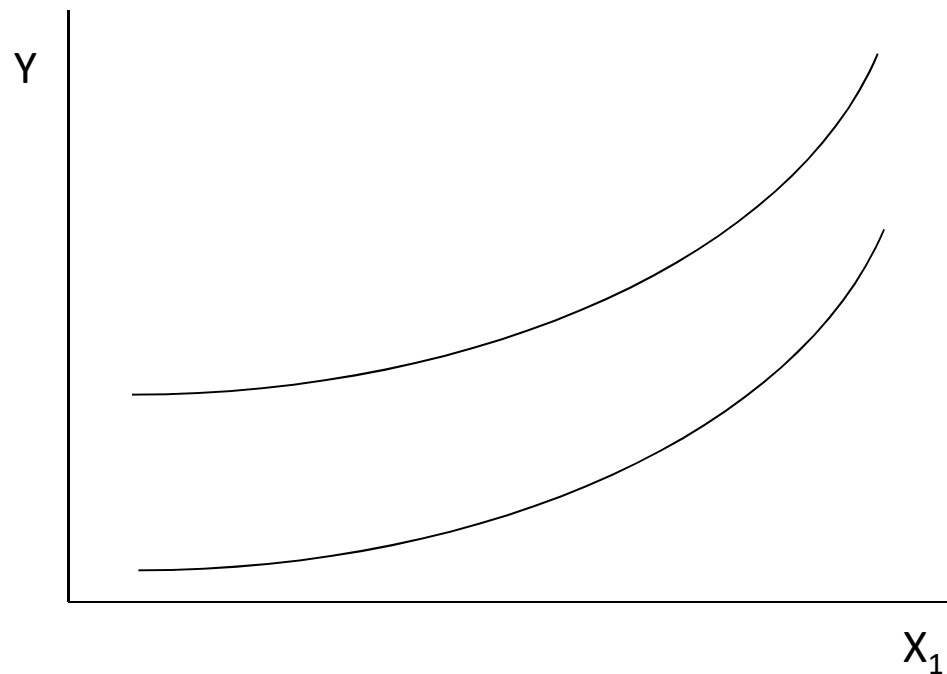
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Interactions with Quadratic Terms



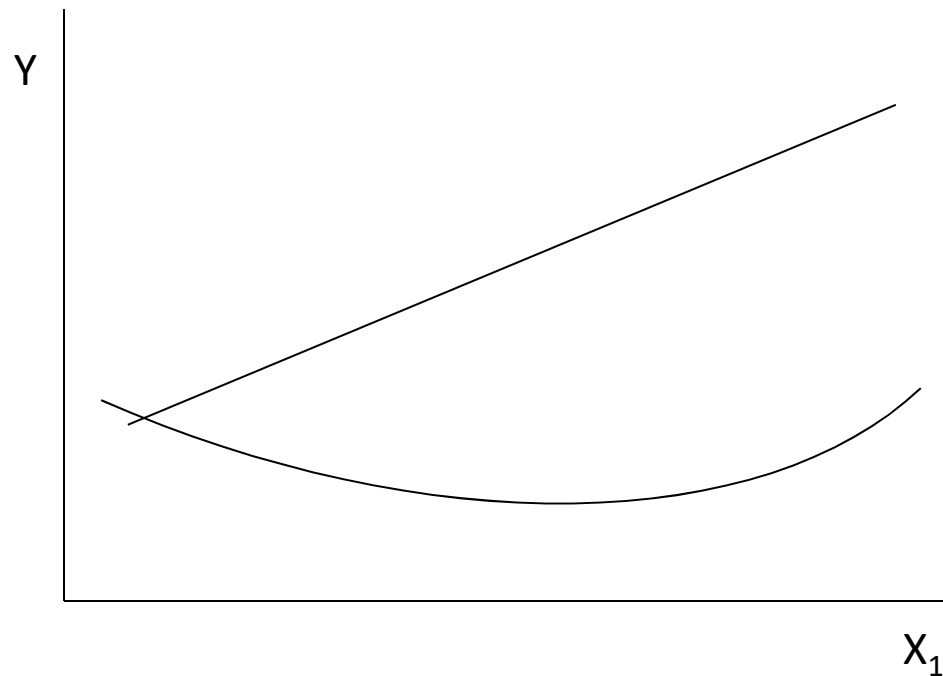
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Interactions with Quadratic Terms



$$E(Y|X) = \beta_0 + \beta_1 X_1 + \beta_2 X_1^2 + \beta_3 X_2 + \beta_4 X_1 * X_2 + \beta_5 X_1^2 * X_2$$

Interactions with Quadratic Terms



$$E(Y|X) = \beta_0 + \beta_1 X_1 + \beta_2 X_1^2 + \beta_3 X_2 + \beta_4 X_1 * X_2 + \beta_5 X_1^2 * X_2$$

Interactions with Quadratic Terms



Regression Coefficients

Dependent Variable: Maternal weight gain

Variable	B	se	t	p
Intercept	27.992	.904	30.952	.000
Female	.325	1.332	.244	.807
OrderCen	-2.078	.447	-4.648	.000
Order_sq	-.086	.202	-.428	.669
Female * OrderCen	1.206	.695	1.736	.083
Female * Order_sq	.860	.335	2.571	.010

