Lecture 13: k-means and mean-shift clustering

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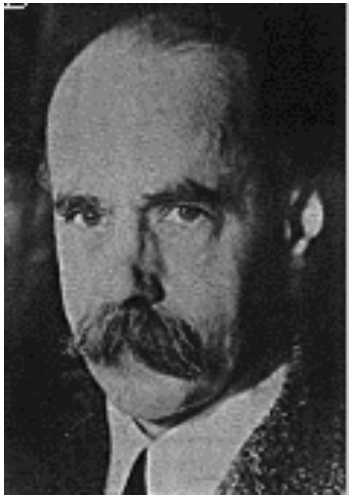
Recap: Gestalt Theory

- Gestalt: whole or group
  - Whole is greater than sum of its parts
  - Relationships among parts can yield new properties/features

- Psychologists identified series of factors that predispose set of elements to be grouped (by human visual system)

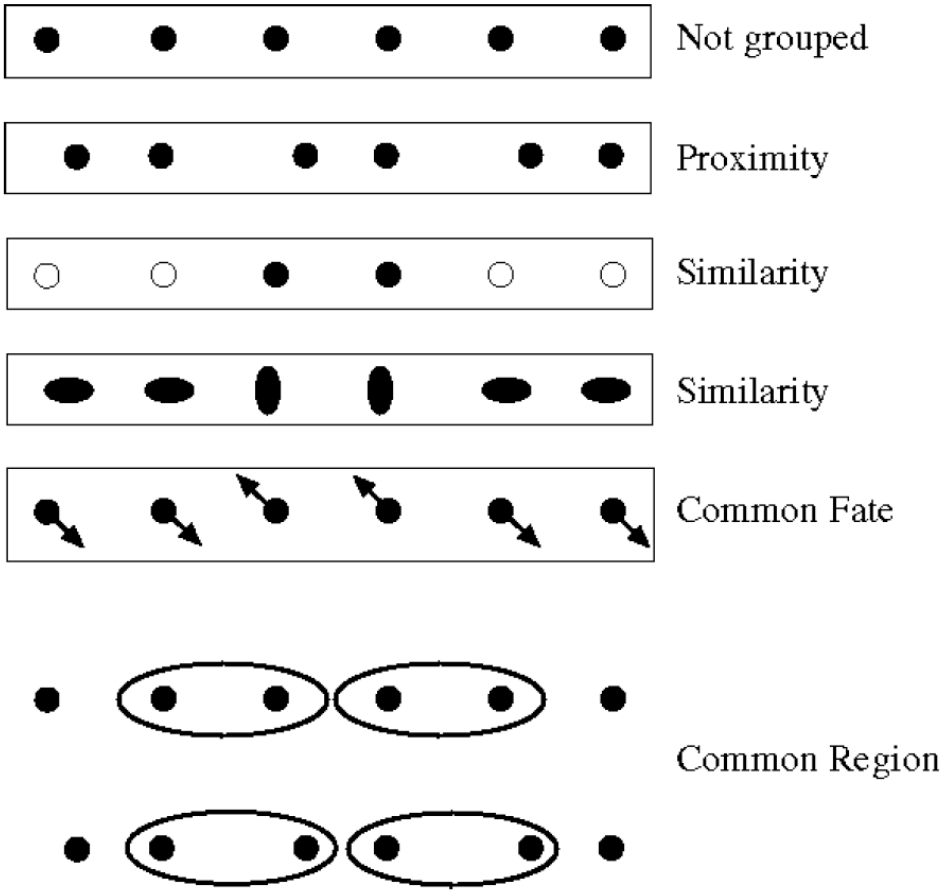
"I stand at the window and see a house, trees, sky. Theoretically I might say there were 327 brightnesses and nuances of colour. Do I have "327"? No. I have sky, house, and trees."

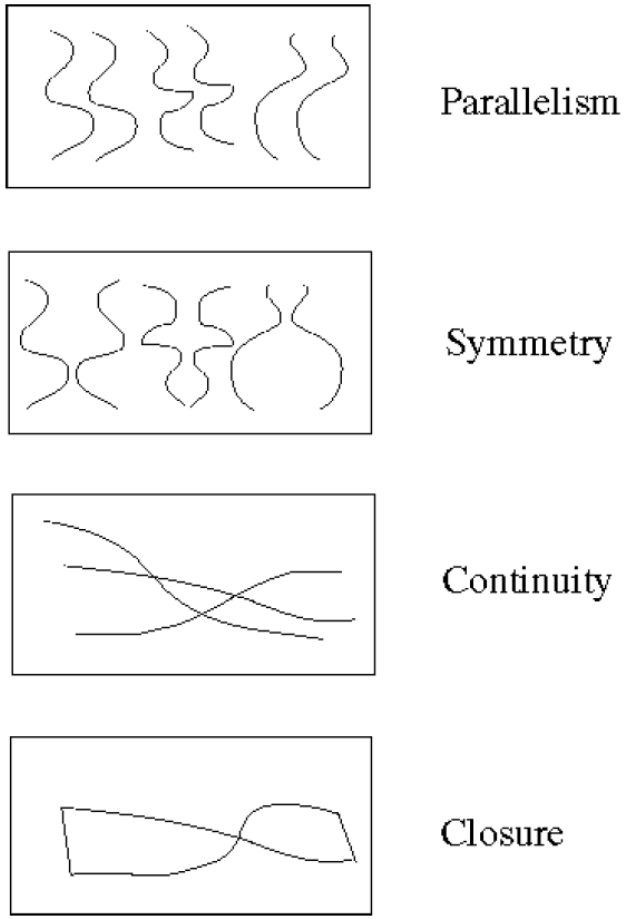
Max Wertheimer
(1880-1943)



Untersuchungen zur Lehre von der Gestalt,
***Psychologische Forschung*, Vol. 4, pp. 301-350, 1923**
http://psy.ed.asu.edu/~classics/Wertheimer/Forms/forms.htm

Recap: Gestalt Factors

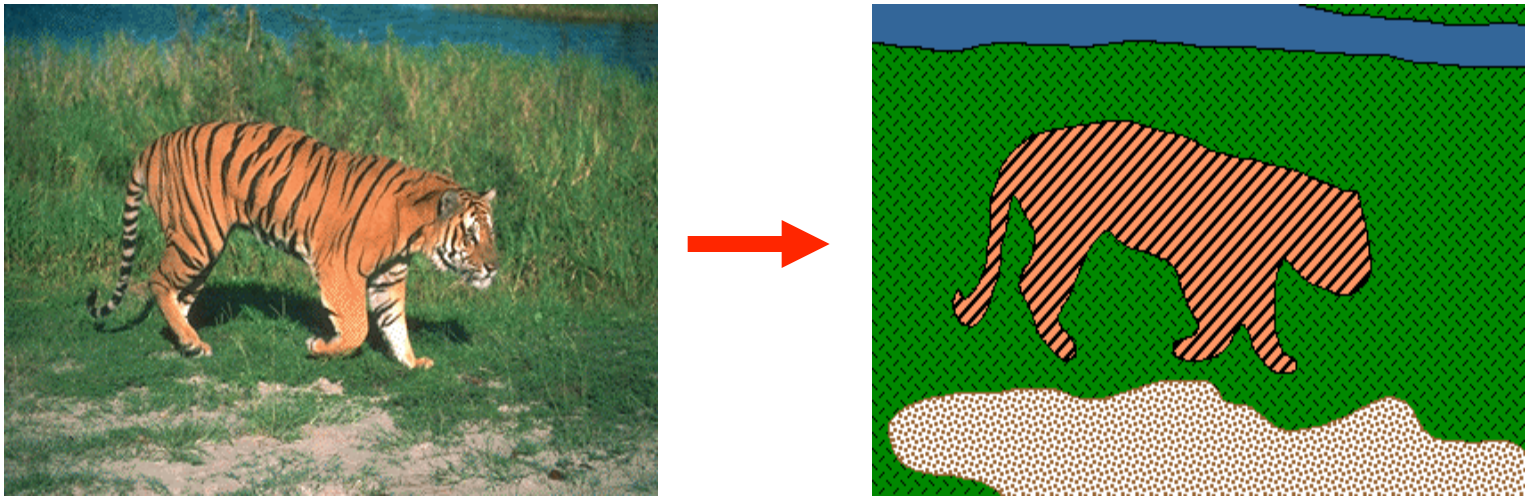




- These factors make intuitive sense, but are very difficult to translate into algorithms.

Recap: Image Segmentation

- Goal: identify groups of pixels that go together



What will we learn today?

- K-means clustering
- Mean-shift clustering

Reading: [FP] Chapters: 14.2, 14.4

D. Comaniciu and P. Meer,
Mean Shift: A Robust Approach toward Feature Space Analysis, PAMI 2002.

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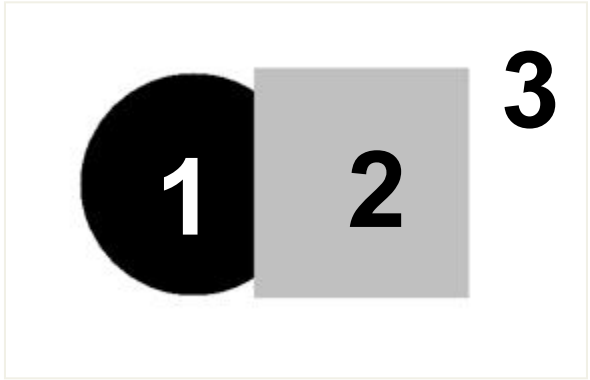
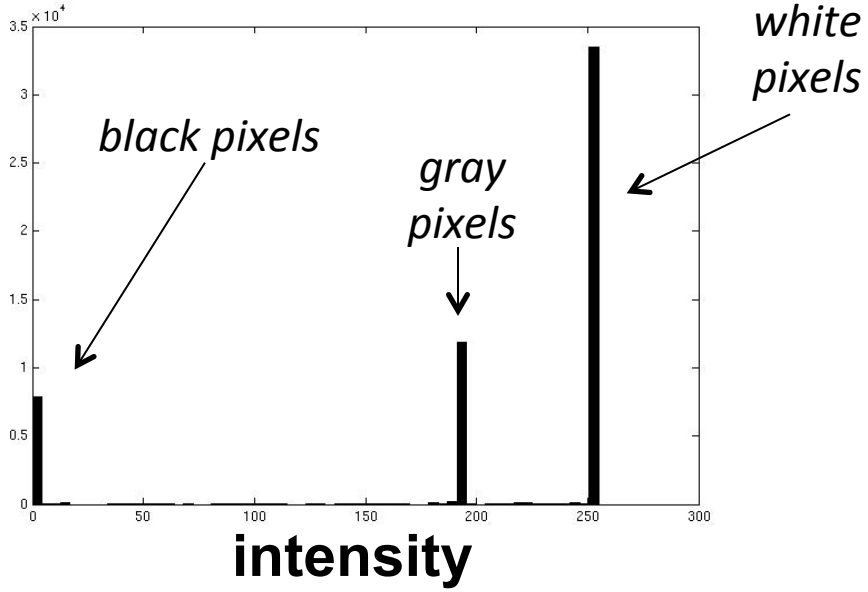
Image Segmentation: Toy Example

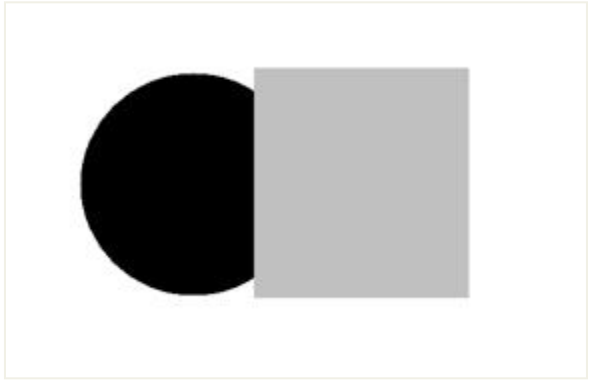


input image

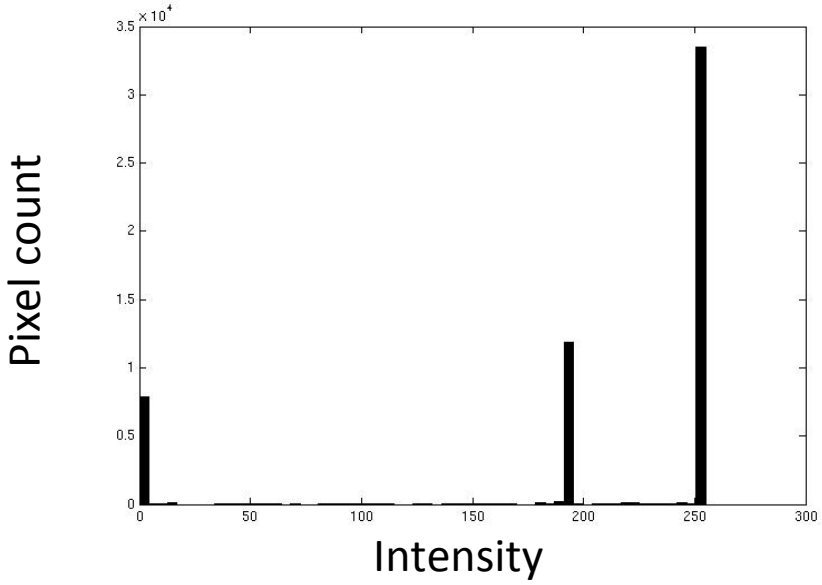


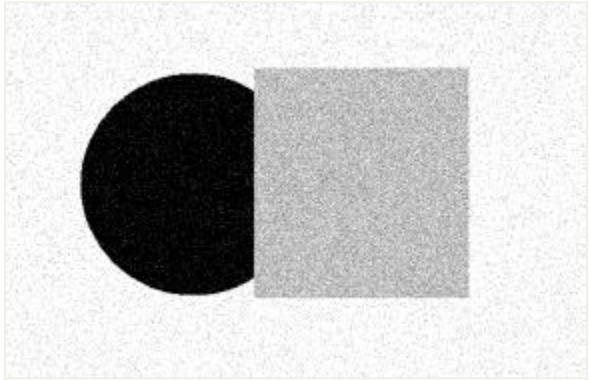
- These intensities define the three groups.
- We could label every pixel in the image according to which of these primary intensities it is.
  - i.e., segment the image based on the intensity feature.
- What if the image isn't quite so simple?

Slide credit: Kristen Grauman

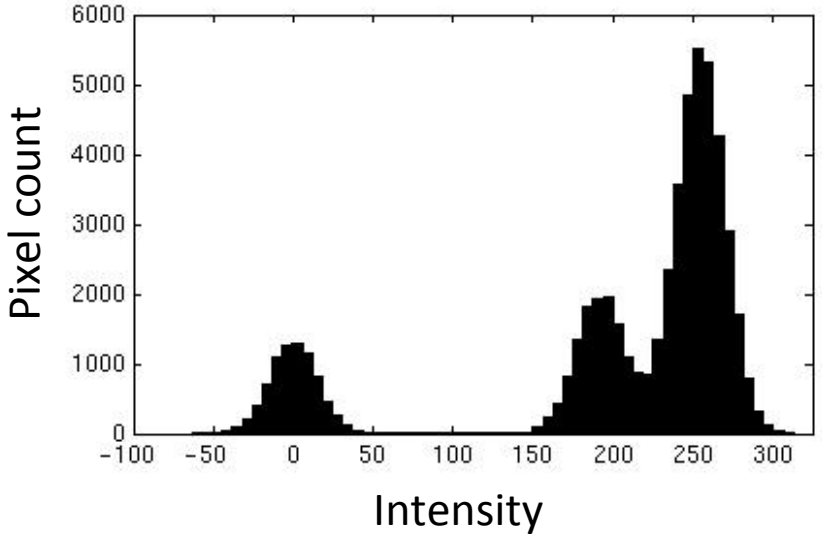


Input image

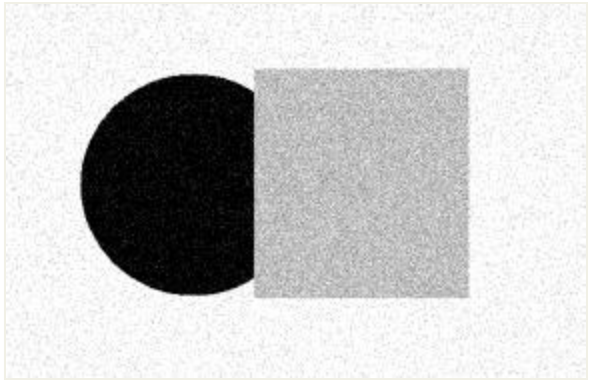




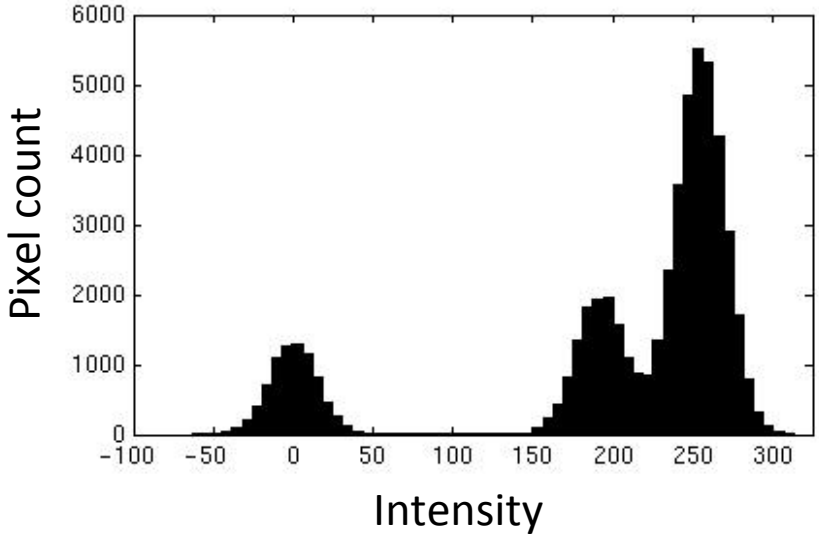
Input image



Slide credit: Kristen Grauman

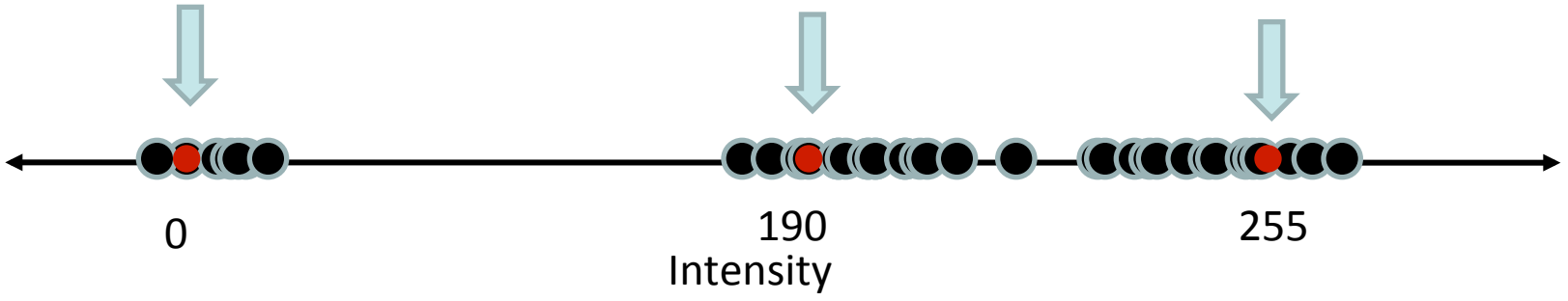


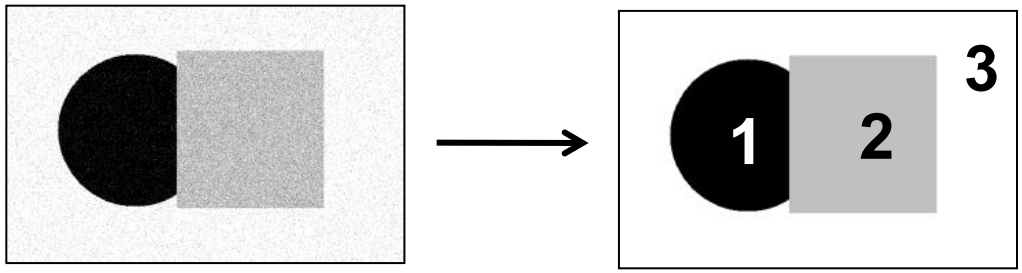
Input image



- Now how to determine the three main intensities that define our groups?
- We need to cluster.

Slide credit: Kristen Grauman





- Goal: choose three "centers" as the representative intensities, and label every pixel according to which of these centers it is nearest to.
- Best cluster centers are those that minimize Sum of Square Distance (SSD) between all points and their nearest cluster center c_i:

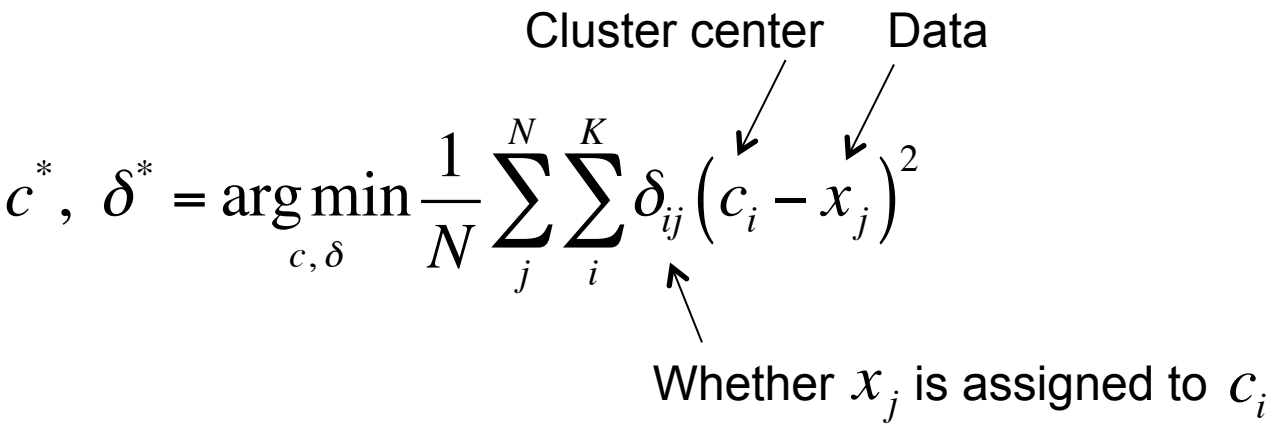
$$SSD = \sum_{cluster\,i} \sum_{x \in cluster\,i} \left(x - c_i\right)^2$$

Slide credit: Kristen Grauman

Clustering for Summarization

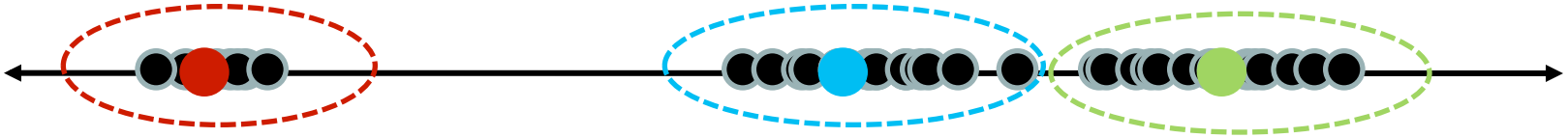
Goal: cluster to minimize variance in data given clusters

– Preserve information

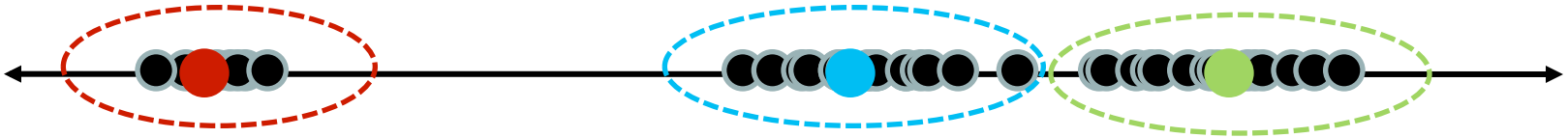


Clustering

- With this objective, it is a "chicken and egg" problem:
  - If we knew the *cluster centers*, we could allocate points to groups by assigning each to its closest center.



  - If we knew the *group memberships*, we could get the centers by computing the mean per group.



Slide credit: Kristen Grauman

K-means clustering

1. Initialize ($t = 0$): cluster centers $c_1, \ldots, c_K$

2. Compute δ^t: assign each point to the closest center
   - δ^t denotes the set of assignment for each x_j to cluster c_i at iteration t

$$\delta^t = \underset{\delta}{\operatorname{argmin}} \frac{1}{N} \sum_j^N \sum_i^K \delta_{ij}^{t-1} \left(c_i^{t-1} - x_j \right)^2$$

3. Computer c^t: update cluster centers as the mean of the points

$$c^t = \underset{c}{\operatorname{argmin}} \frac{1}{N} \sum_j^N \sum_i^K \delta_{ij}^{t} \left(c_i^{t-1} - x_j \right)^2$$

4. Update $t = t + 1$, Repeat Step 2-3 till stopped

Slide: Derek Hoiem

K-means clustering

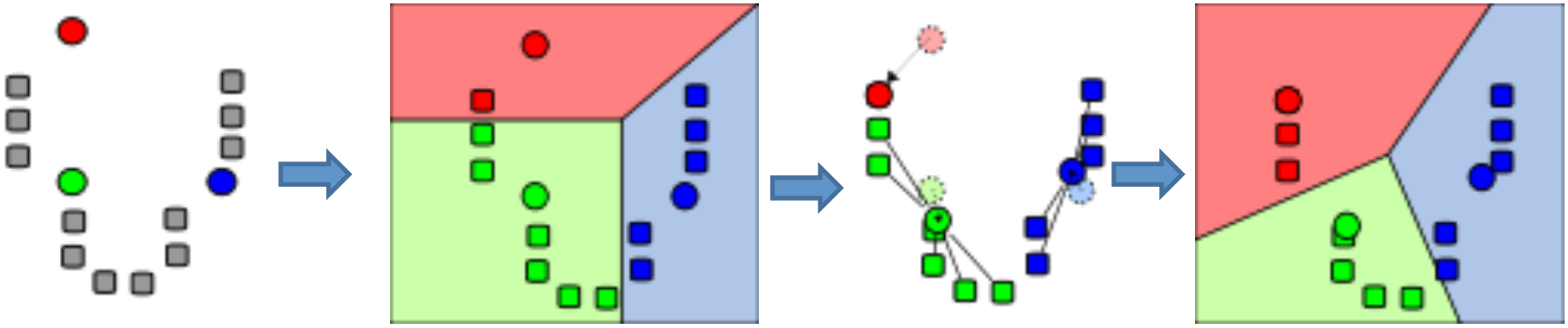
1. Initialize ($t = 0$): cluster centers $c_1, \ldots, c_K$
   - Commonly used: random initialization
   - Or greedily choose K to minimize residual
2. Compute δ^t: assign each point to the closest center
   - Typical distance measure:
     - Euclidean $sim(x, x') = x^T x'$
     - Cosine $sim(x, x') = x^T x' / (\|x\| \cdot \|x'\|)$
     - Others
3. Computer c^t: update cluster centers as the mean of the points

$$c^t = \underset{c}{\operatorname{argmin}} \frac{1}{N} \sum_j^N \sum_i^K \delta_{ij}^t \left(c_i^{t-1} - x_j \right)^2$$

4. Update $t = t + 1$, Repeat Step 2-3 till stopped
   - c^t doesn't change anymore.

Slide: Derek Hoiem

K-means clustering



1. Initialize Cluster Centers

2. Assign Points to Clusters

3. Re-compute Means

Repeat (2) and (3)

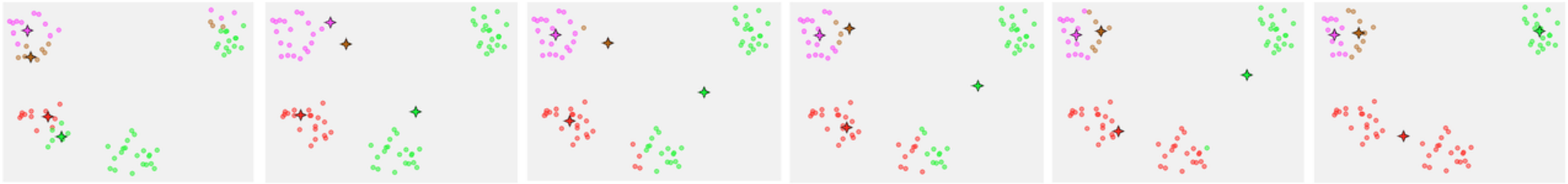
- Java demo:

http://home.dei.polimi.it/matteucc/Clustering/tutorial_html/AppletKM.html

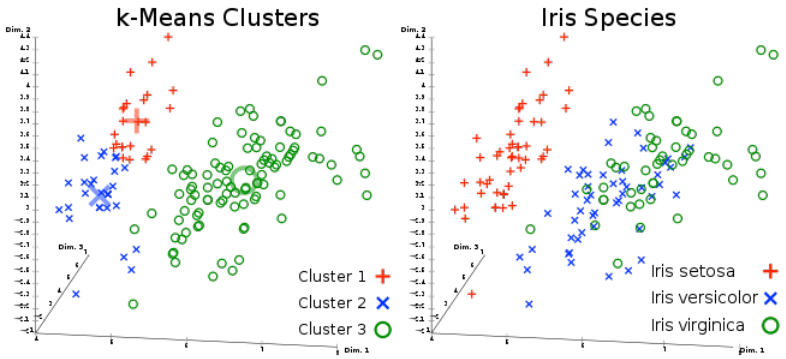
Illustration Source: wikipedia

K-means clustering

- Converges to a *local minimum* solution
  - Initialize multiple runs



- Better fit for spherical data

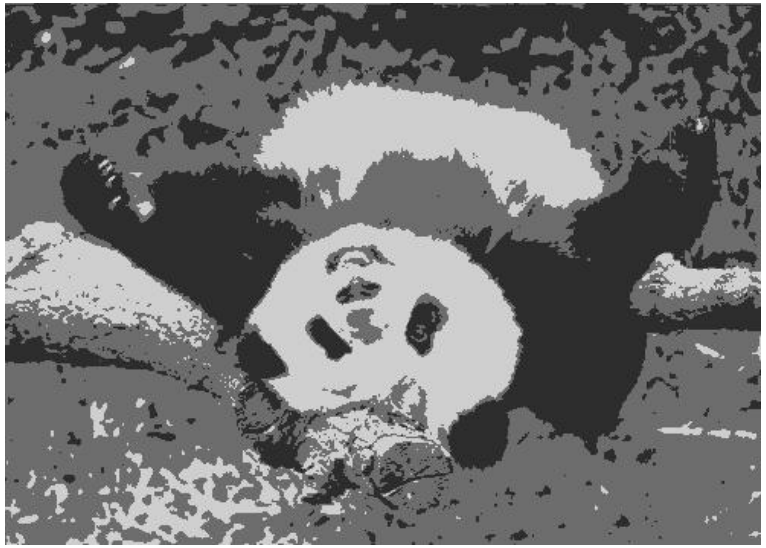


- Need to pick K (# of clusters)

Segmentation as Clustering

K=2

K=3



```
img_as_col = double(im(:));
cluster_membs = kmeans(img_as_col, K);

labelim = zeros(size(im));
for i=1:k
   inds = find(cluster_membs==i);
   meanval = mean(img_as_column(inds));
   labelim(inds) = meanval;
end
```

Slide credit: Kristen Grauman

K-Means++

- Can we prevent arbitrarily bad local minima?

1. Randomly choose first center.
2. Pick new center with prob. proportional to $\left(x - c_i\right)^2$
   - (Contribution of *x* to total error)
3. Repeat until *K* centers.

- Expected error = $O\left(\log K\right)$* optimal

Arthur & Vassilvitskii 2007

Slide credit: Steve Seitz

Feature Space

- Depending on what we choose as the *feature space*, we can group pixels in different ways.

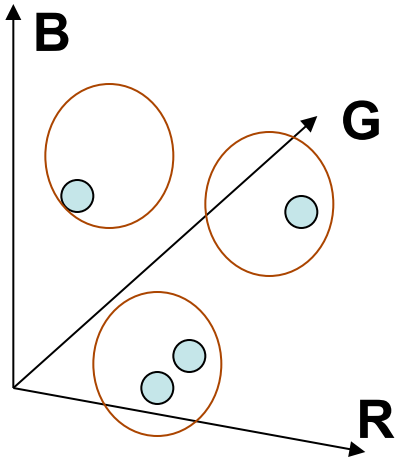
- Grouping pixels based on intensity similarity

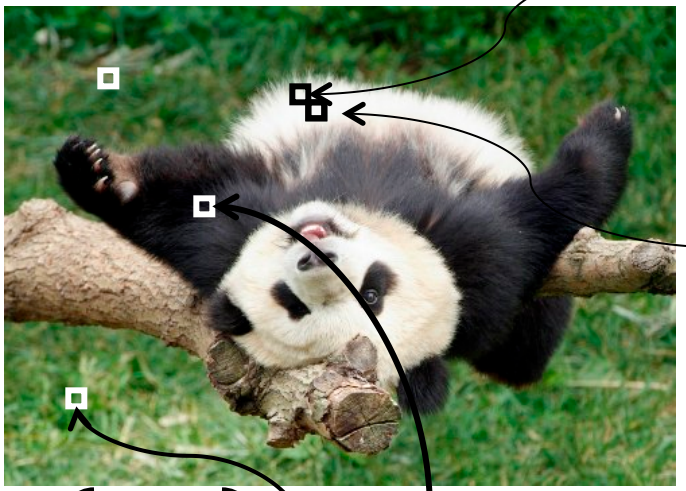
- Feature space: intensity value (1D)

Slide credit: Kristen Grauman

Feature Space

- Depending on what we choose as the *feature space*, we can group pixels in different ways.
- Grouping pixels based on color similarity



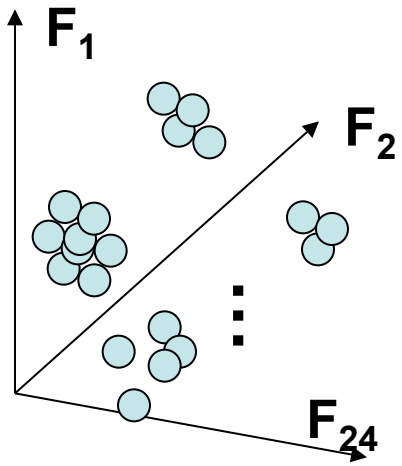


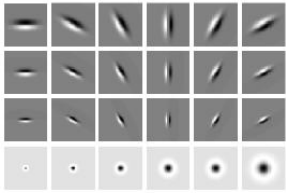
- Feature space: color value (3D)

Slide credit: Kristen Grauman

Feature Space

- Depending on what we choose as the *feature space*, we can group pixels in different ways.
- Grouping pixels based on texture similarity





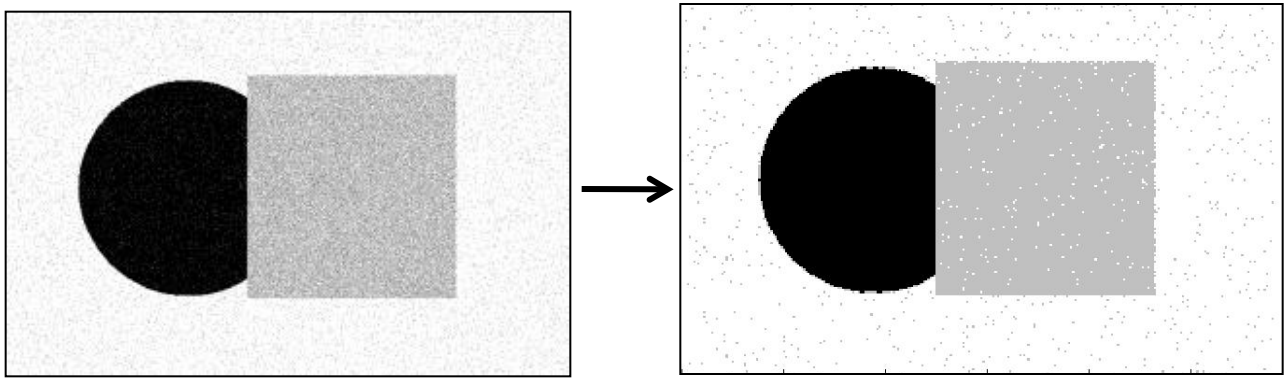
Filter bank of 24 filters

- Feature space: filter bank responses (e.g., 24D)

Slide credit: Kristen Grauman

Smoothing Out Cluster Assignments

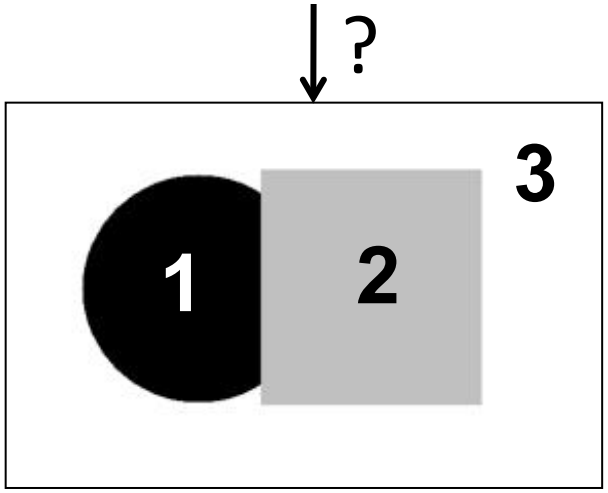
- Assigning a cluster label per pixel may yield outliers:



Original

Labeled by cluster center's intensity

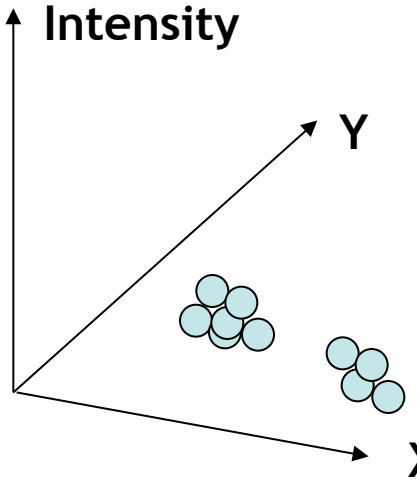
- How can we ensure they are spatially smooth?

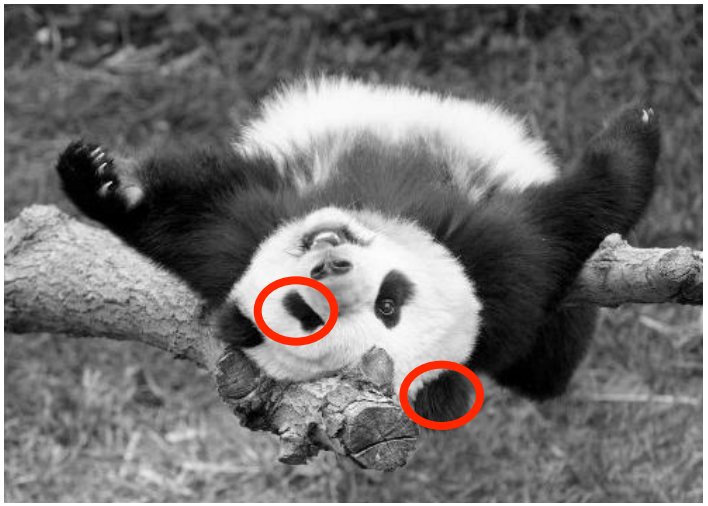


Slide credit: Kristen Grauman

Segmentation as Clustering

- Depending on what we choose as the *feature space*, we can group pixels in different ways.
- Grouping pixels based on *intensity+position* similarity





⇒ Way to encode both *similarity* and *proximity.*

Slide credit: Kristen Grauman

K-Means Clustering Results

- K-means clustering based on intensity or color is essentially vector quantization of the image attributes
  - Clusters don't have to be spatially coherent

Image

Intensity-based clusters

Color-based clusters

Image source: Forsyth & Ponce

K-Means Clustering Results

- K-means clustering based on intensity or color is essentially vector quantization of the image attributes
  - Clusters don't have to be spatially coherent
- Clustering based on (r,g,b,x,y) values enforces more spatial coherence

Image source: Forsyth & Ponce

How to evaluate clusters?

- Generative
  - How well are points reconstructed from the clusters?

- Discriminative
  - How well do the clusters correspond to labels?
    - Purity
  - Note: unsupervised clustering does not aim to be discriminative

Slide: Derek Hoiem

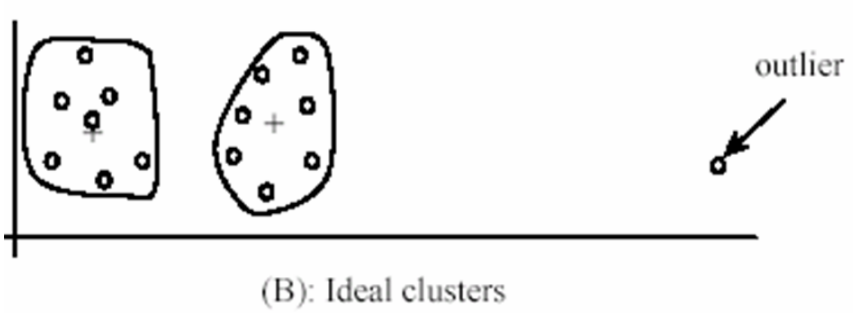
How to choose the number of clusters?

- Validation set
  - Try different numbers of clusters and look at performance
    - When building dictionaries (discussed later), more clusters typically work better

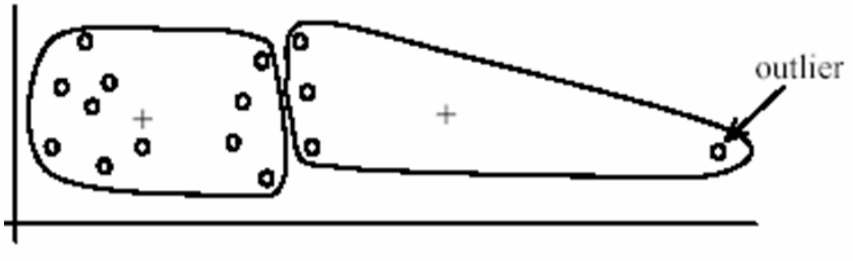
Slide: Derek Hoiem

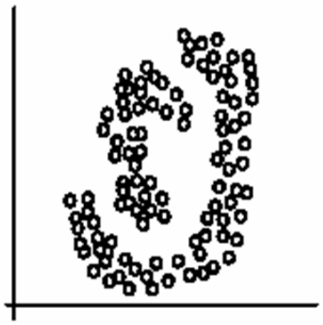
K-Means pros and cons

- Pros
  - Finds cluster centers that minimize conditional variance (good representation of data)
  - Simple and fast, Easy to implement
- Cons
  - Need to choose K
  - Sensitive to outliers
  - Prone to local minima
  - All clusters have the same parameters (e.g., distance measure is non-adaptive)
  - *Can be slow: each iteration is O(KNd) for N d-dimensional points
- Usage
  - Unsupervised clustering
  - Rarely used for pixel segmentation

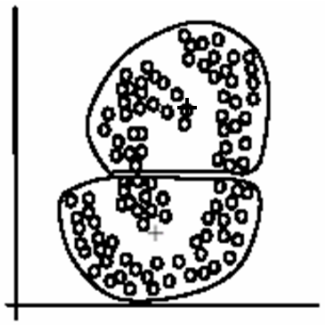


(B): Ideal clusters





(A): Two natural clusters



(B): *k*-means clusters

What will we learn today?

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- Mean-shift clustering

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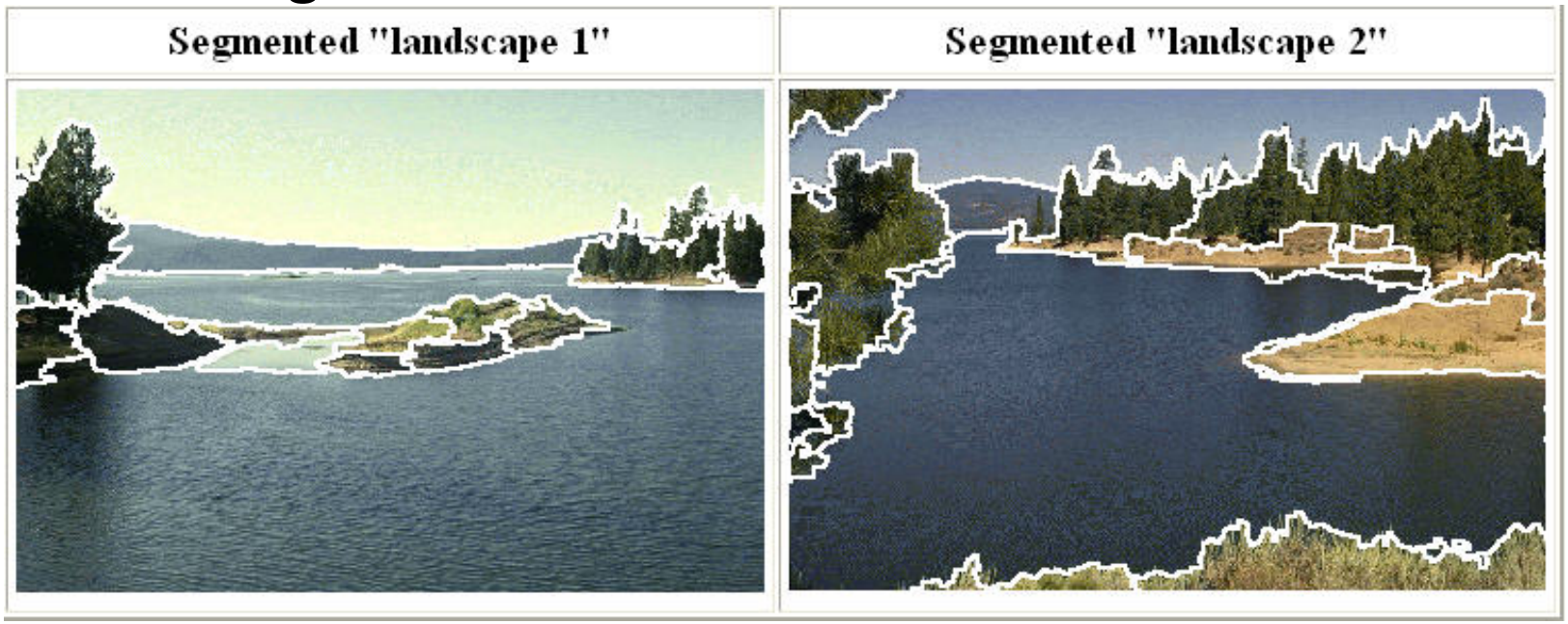
D. Comaniciu and P. Meer,
Mean Shift: A Robust Approach toward Feature Space Analysis, PAMI 2002.

Mean-Shift Segmentation

- An advanced and versatile technique for clustering-based segmentation

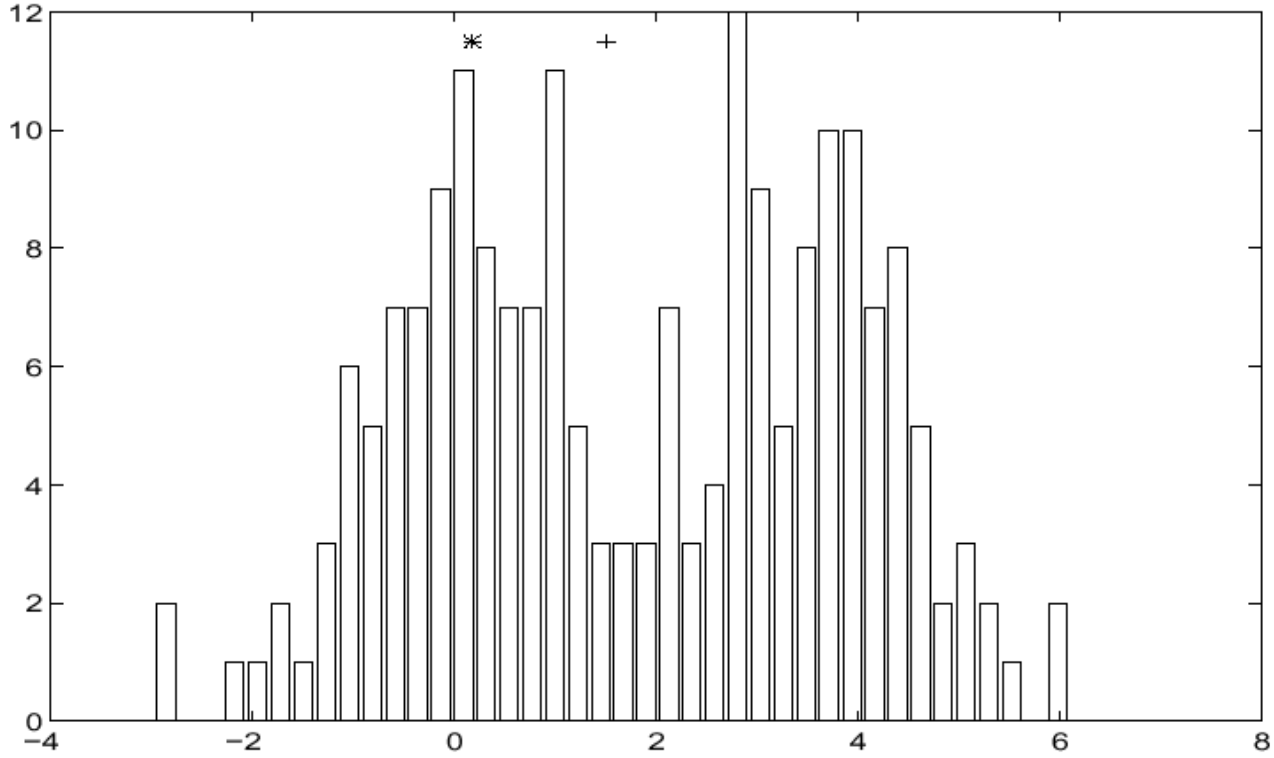


http://www.caip.rutgers.edu/~comanici/MSPAMI/msPamiResults.html

D. Comaniciu and P. Meer, Mean Shift: A Robust Approach toward Feature Space Analysis, PAMI 2002.

Slide credit: Svetlana Lazebnik

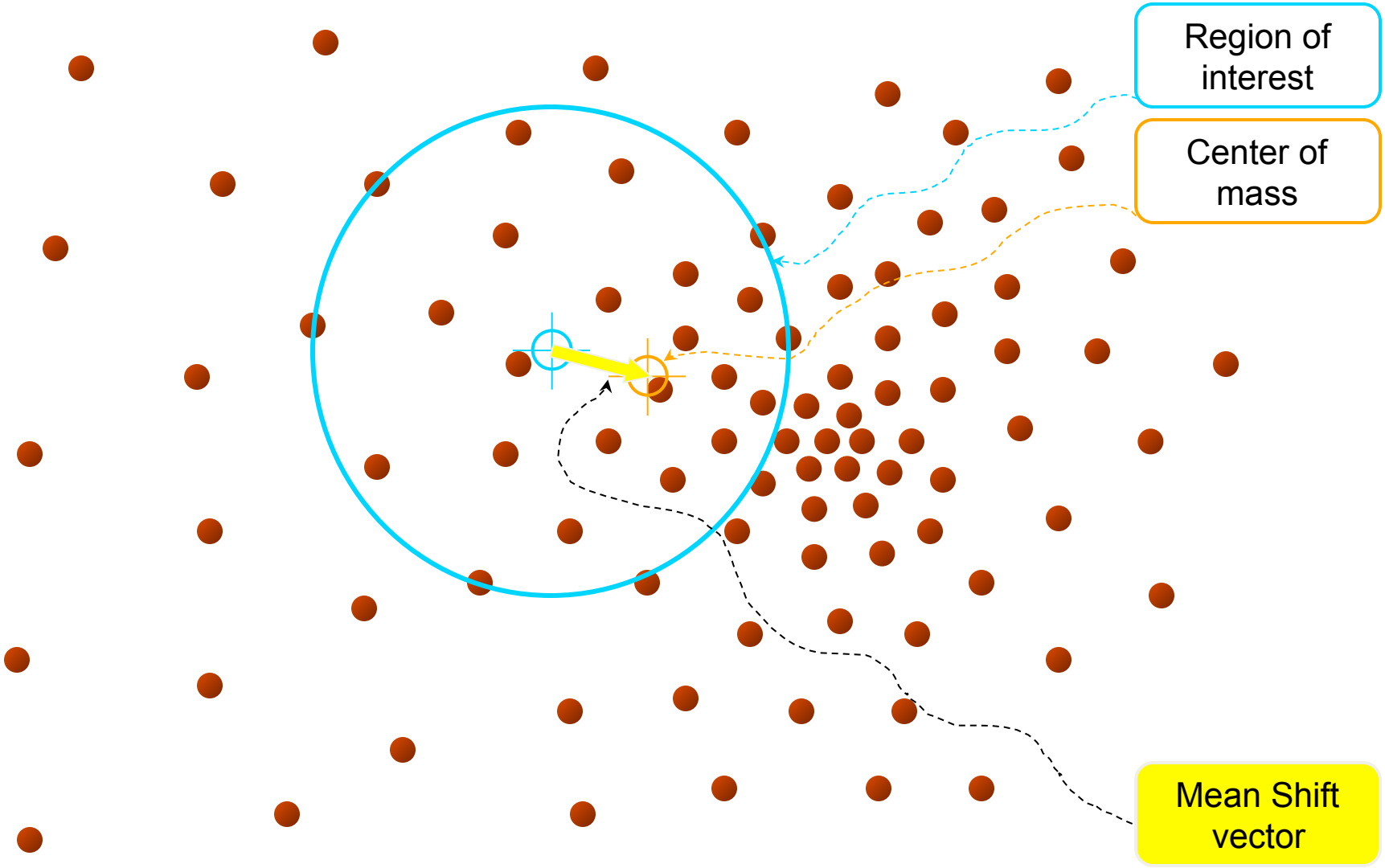
Mean-Shift Algorithm



- Iterative Mode Search
  1. Initialize random seed, and window W
  2. Calculate center of gravity (the "mean") of W: $\sum_{x \in W} xH(x)$
  3. Shift the search window to the mean
  4. Repeat Step 2 until convergence

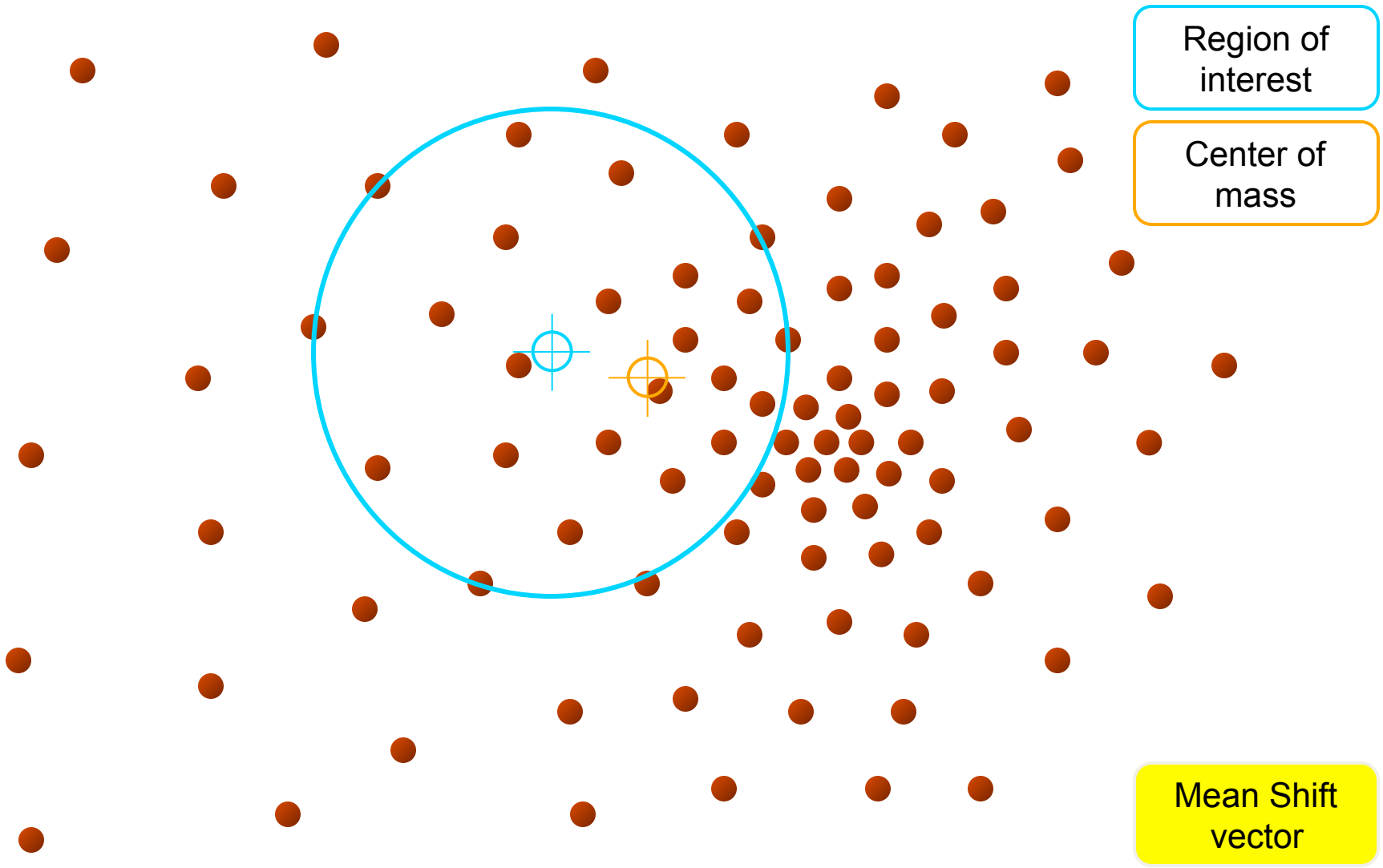
Slide credit: Steve Seitz

Mean-Shift



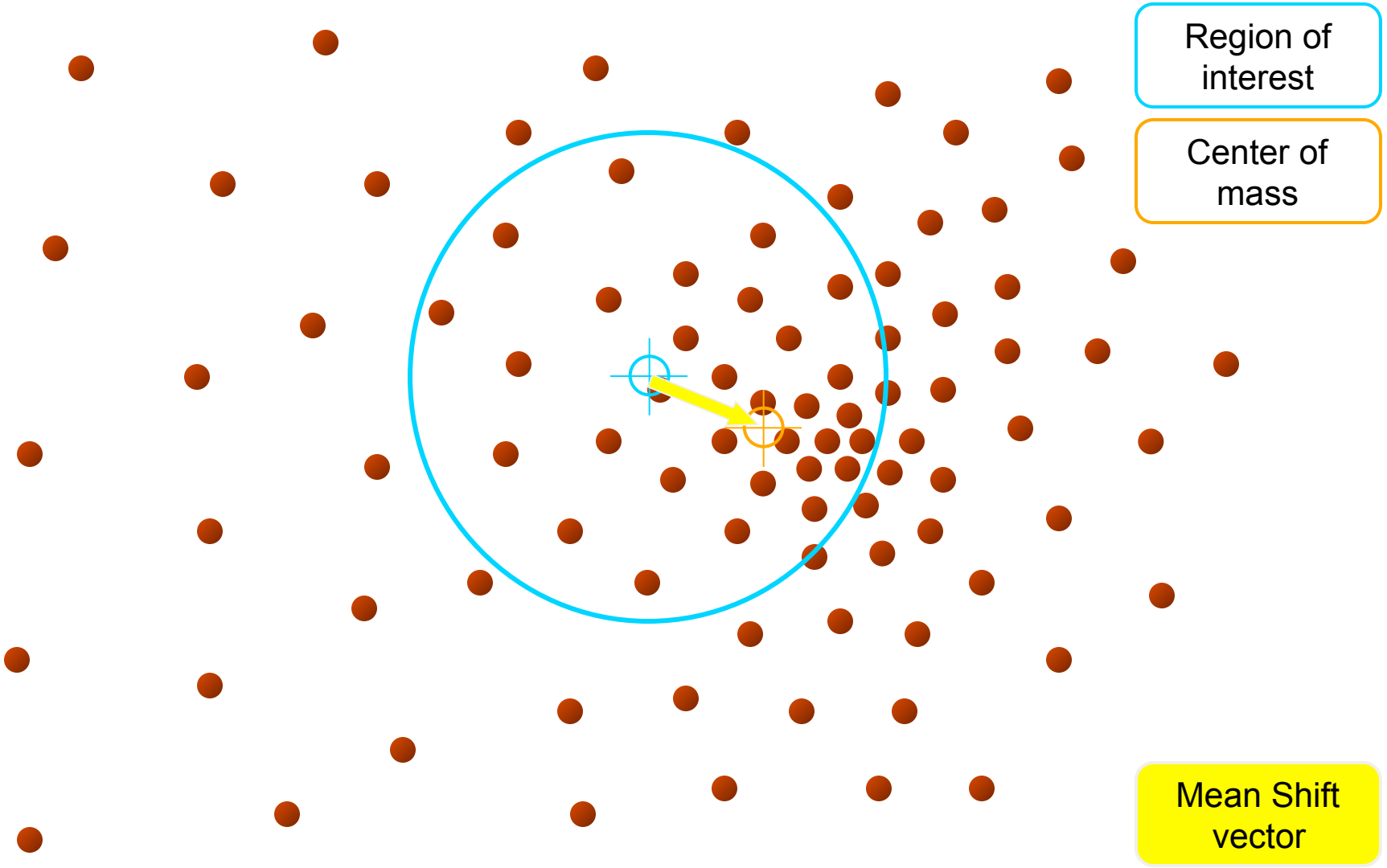
Slide by Y. Ukrainitz & B. Sarel

Mean-Shift



Slide by Y. Ukrainitz & B. Sarel

Mean-Shift



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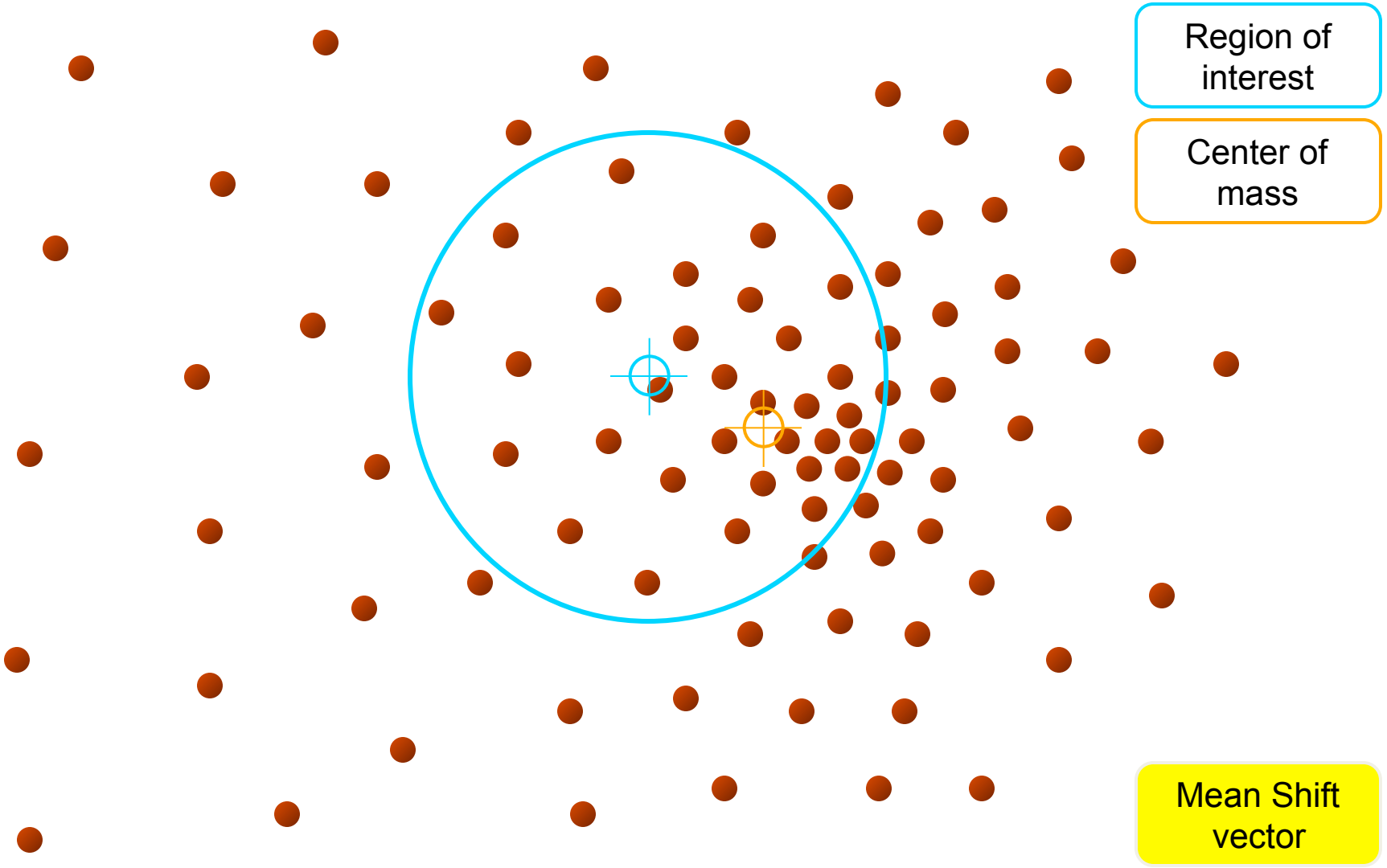
Mean-Shift



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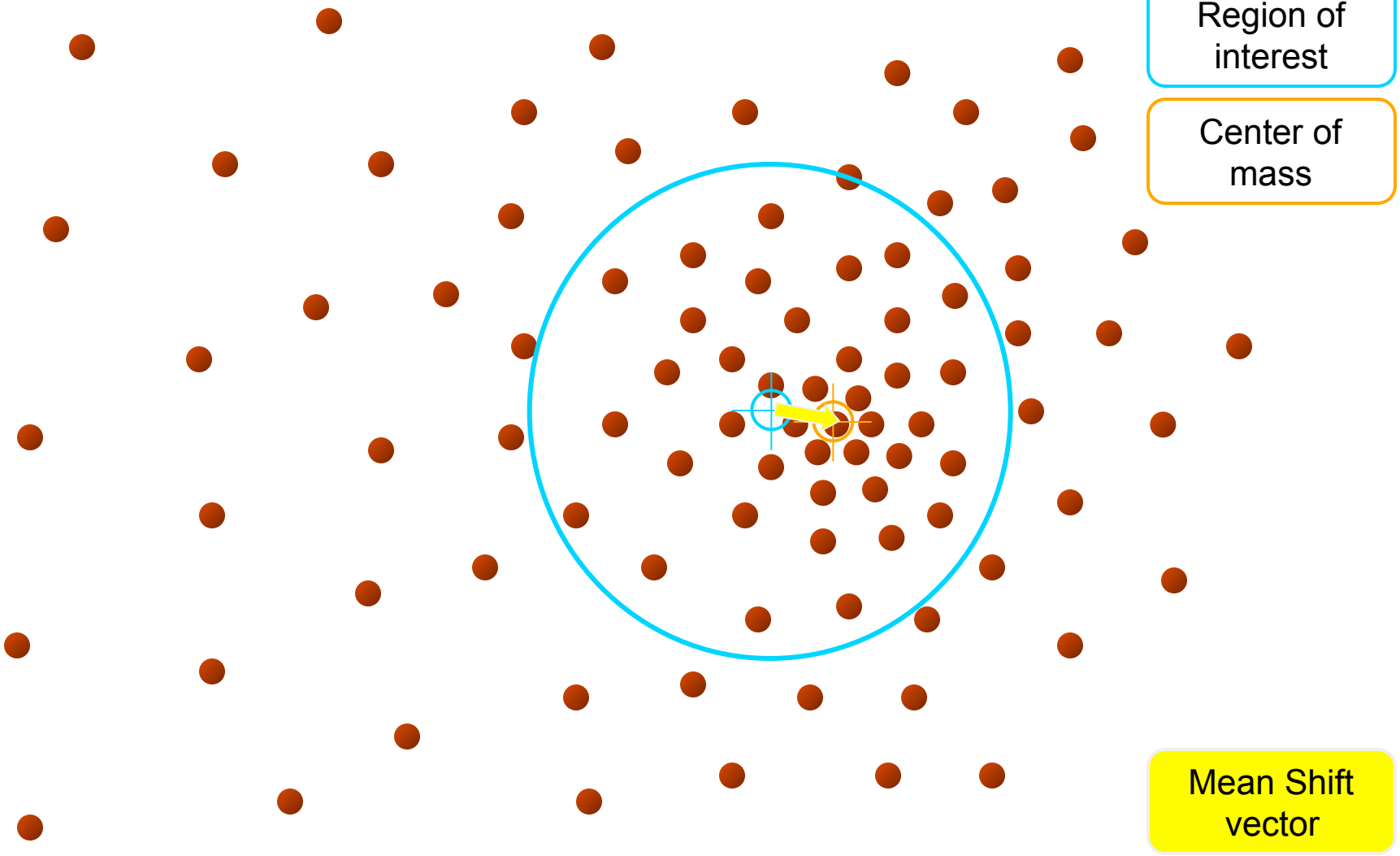
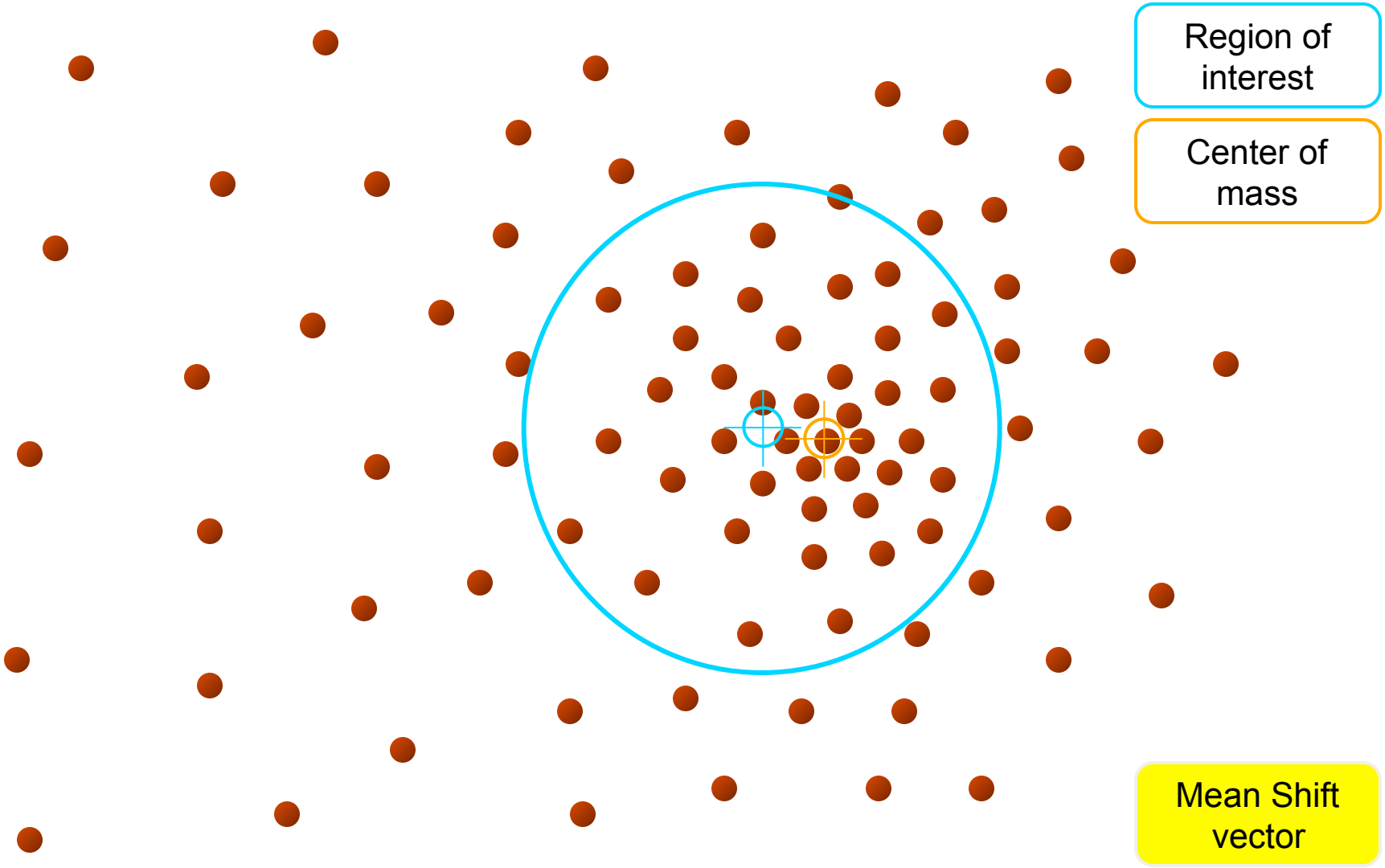
Mean-Shift



Slide by Y. Ukrainitz & B. Sarel

Mean-Shift



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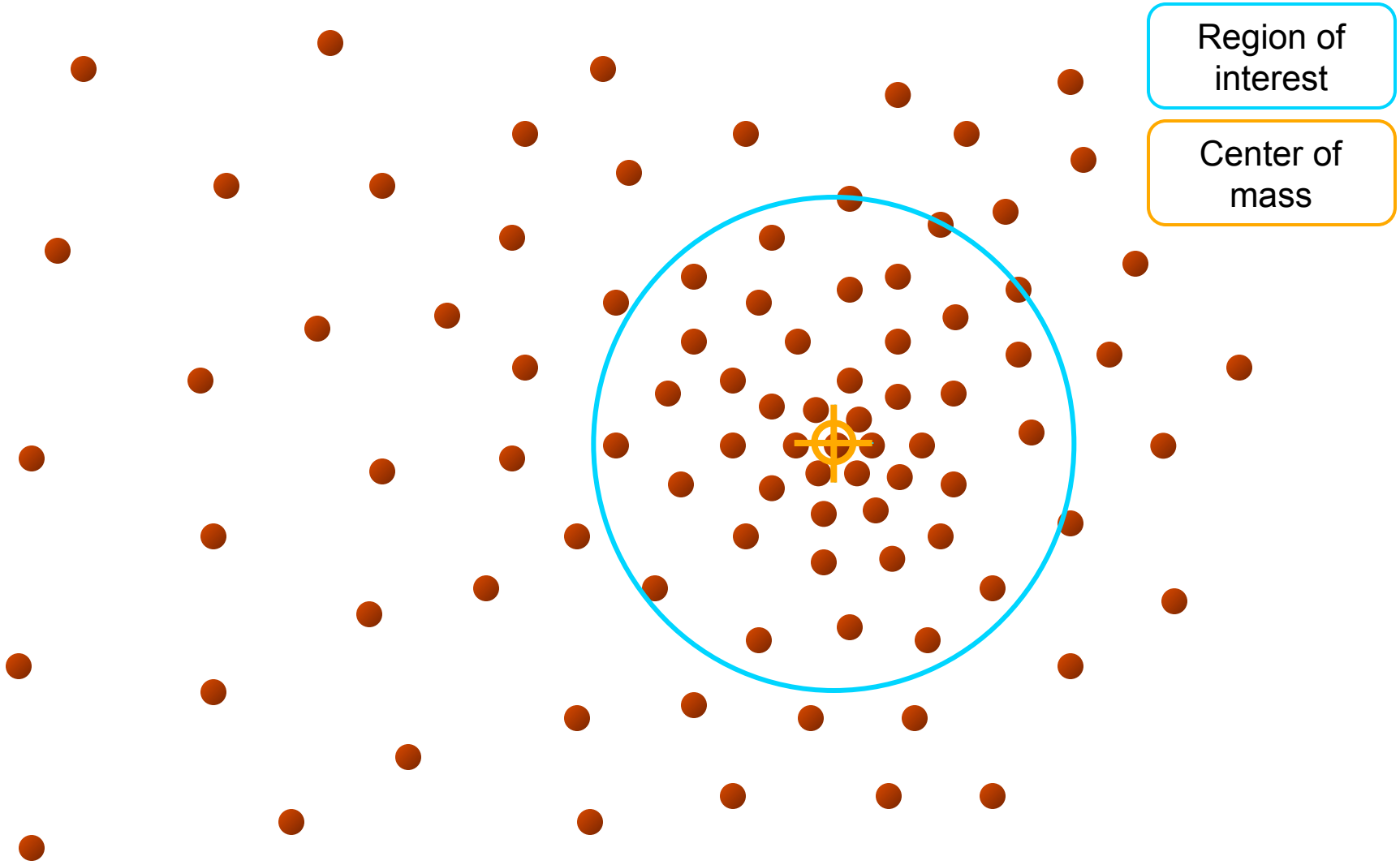
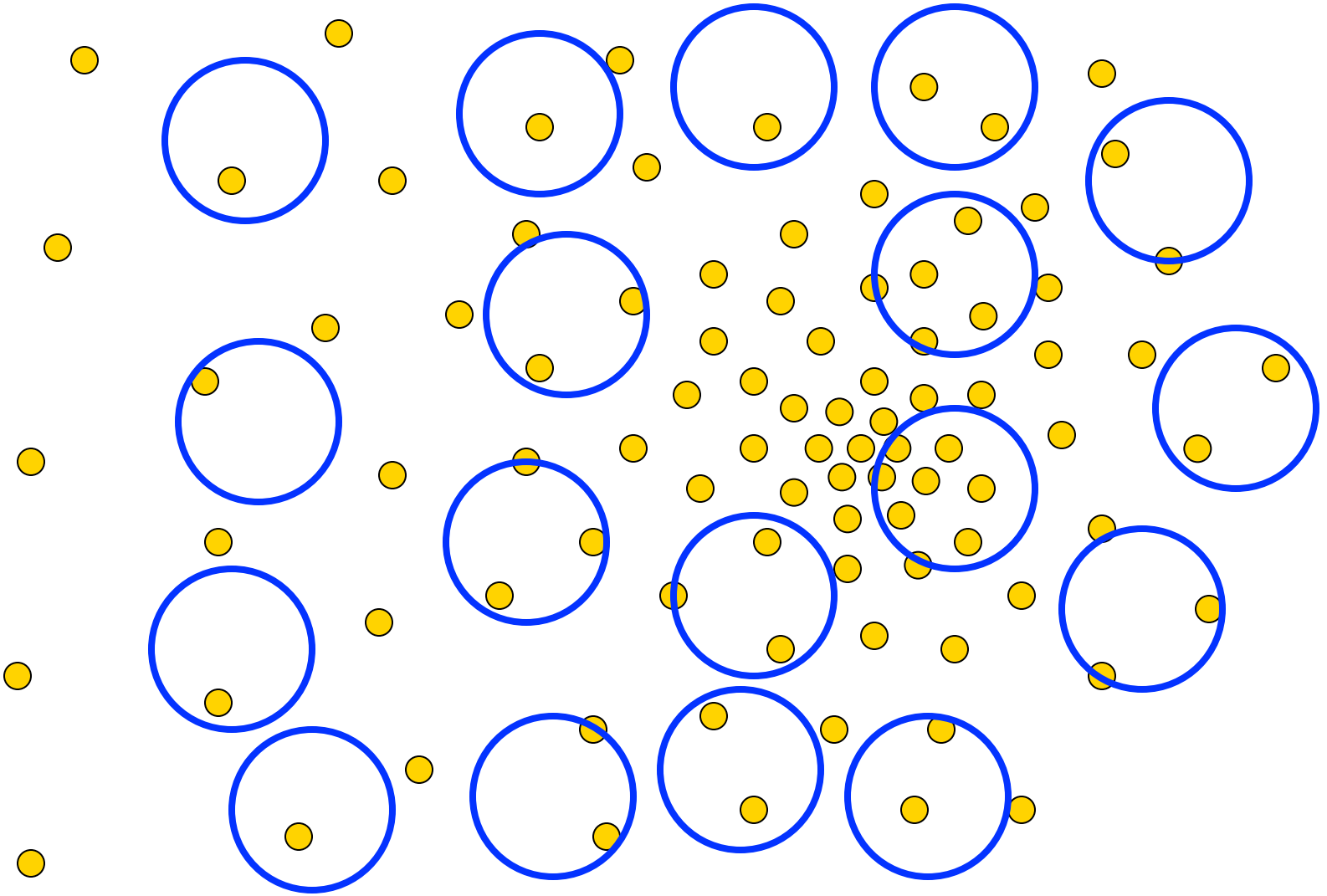
Mean-Shift



Slide by Y. Ukrainitz & B. Sarel

Real Modality Analysis



Tessellate the space with windows

Run the procedure in parallel

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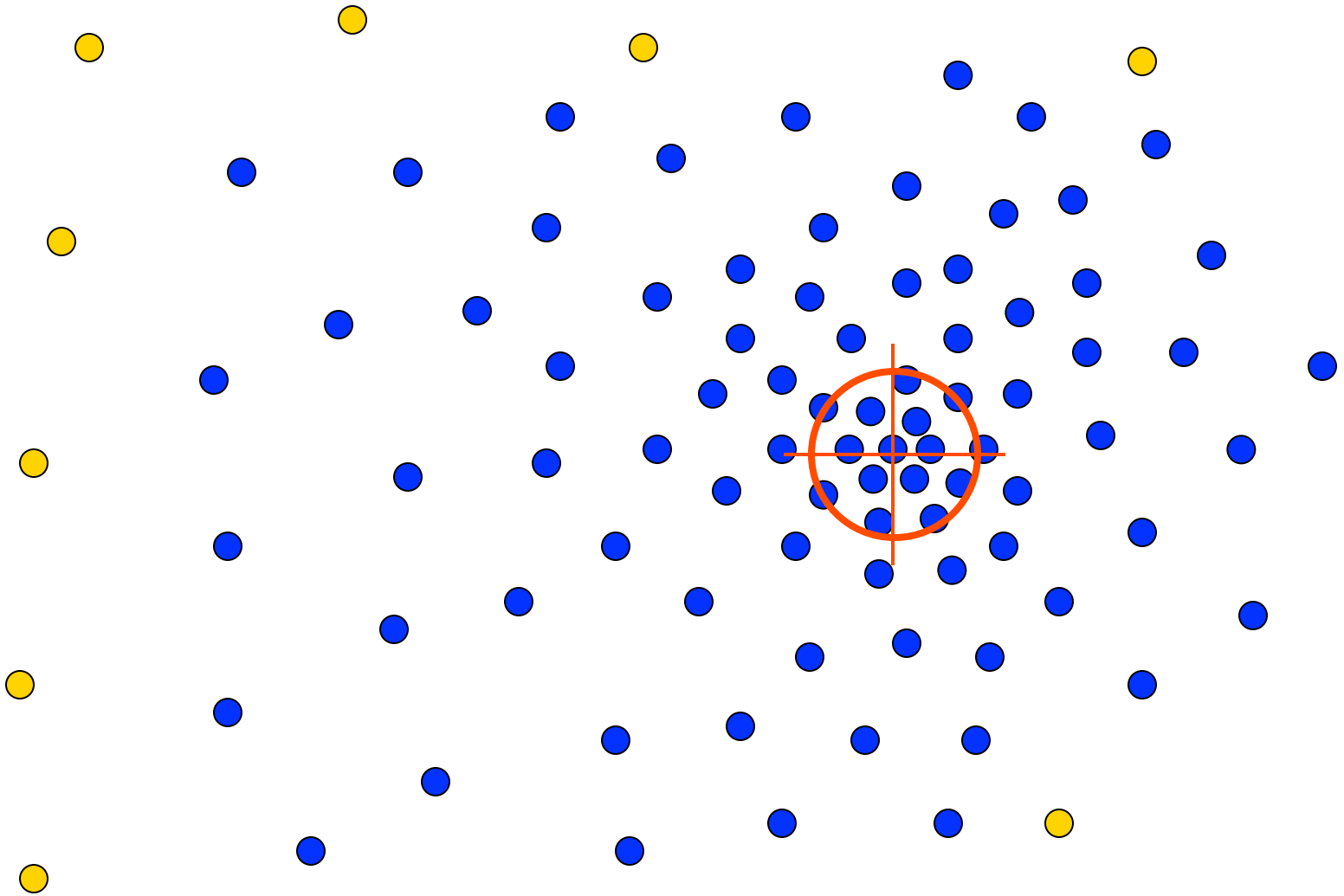
Real Modality Analysis

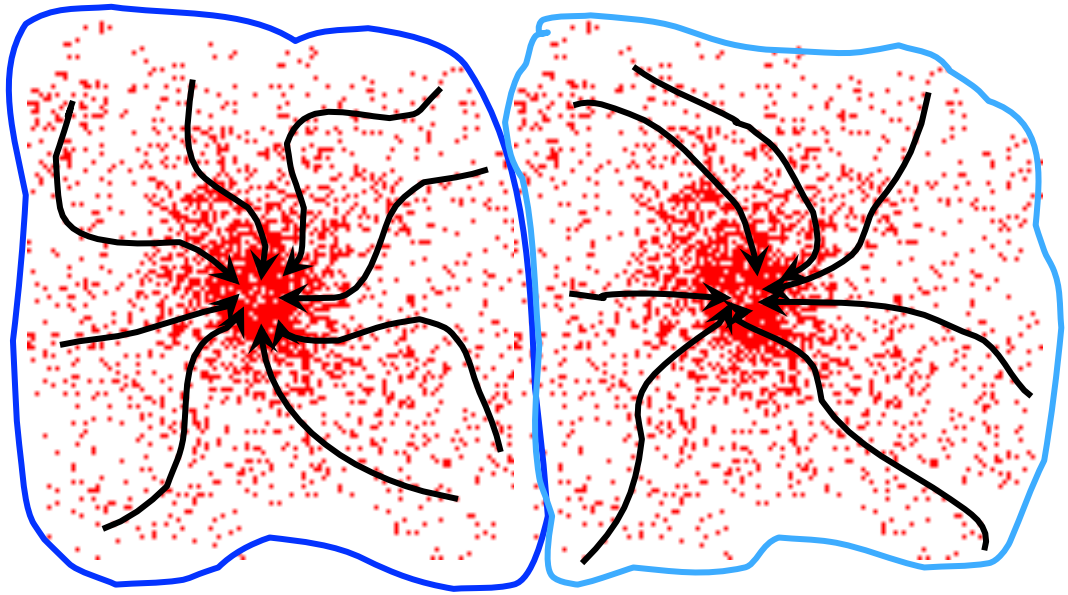


The blue data points were traversed by the windows towards the mode.

Slide by Y. Ukrainitz & B. Sarel

Mean-Shift Clustering

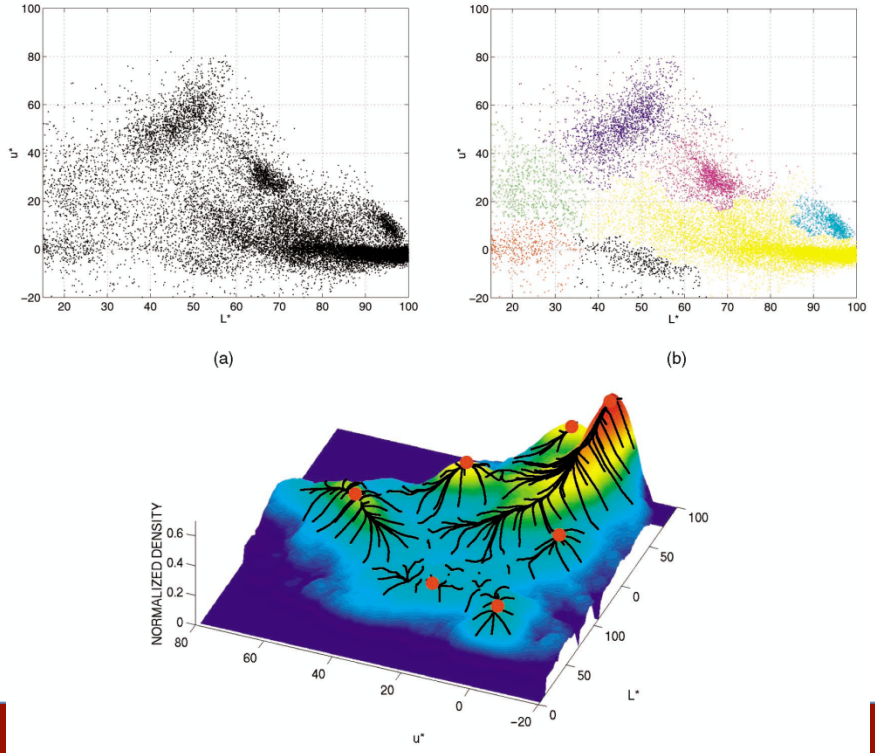
- Cluster: all data points in the attraction basin of a mode
- Attraction basin: the region for which all trajectories lead to the same mode



Slide by Y. Ukrainitz & B. Sarel

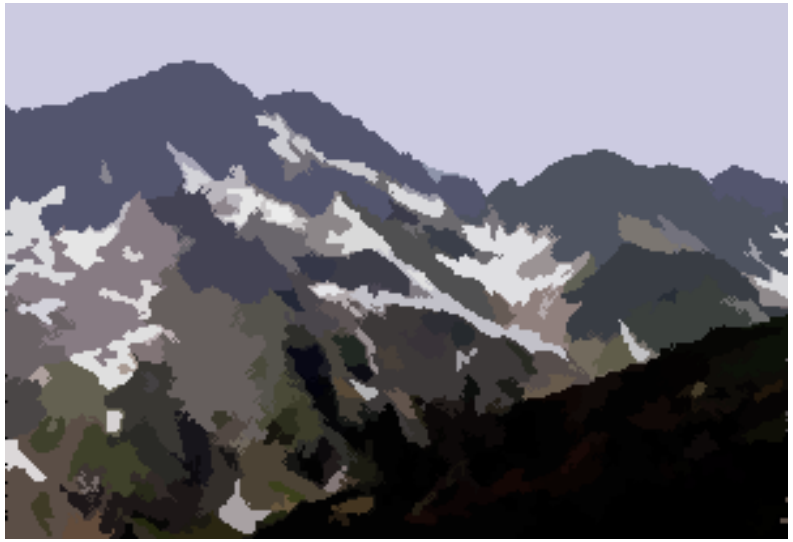
Mean-Shift Clustering/Segmentation

- Find features (color, gradients, texture, etc)
- Initialize windows at individual pixel locations
- Perform mean shift for each window until convergence
- Merge windows that end up near the same “peak” or mode



Slide credit: Svetlana Lazebnik

Mean-Shift Segmentation Results



http://www.caip.rutgers.edu/~comanici/MSPAMI/msPamiResults.html

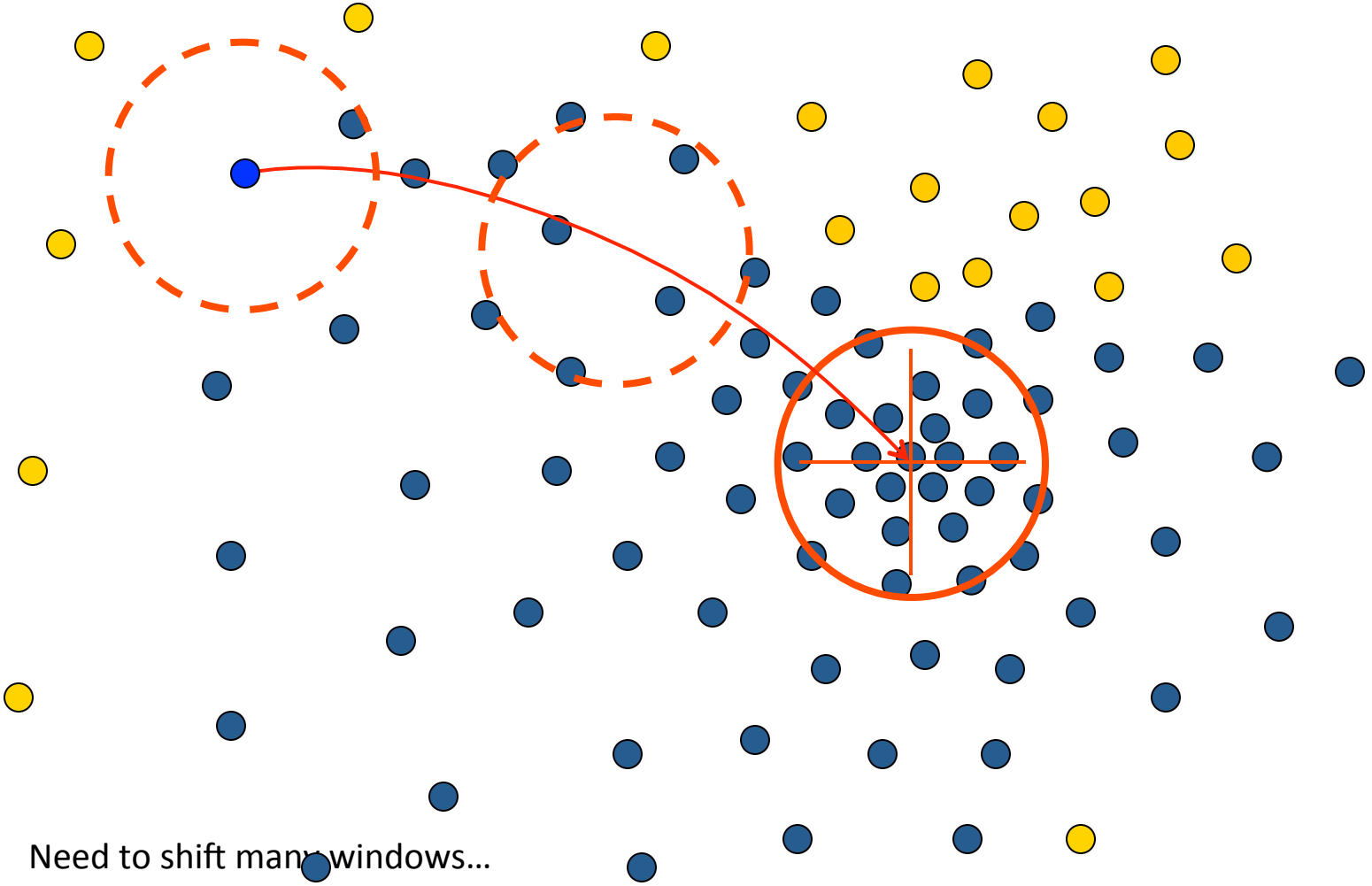
Slide credit: Svetlana Lazebnik

More Results

Slide credit: Svetlana Lazebnik

More Results

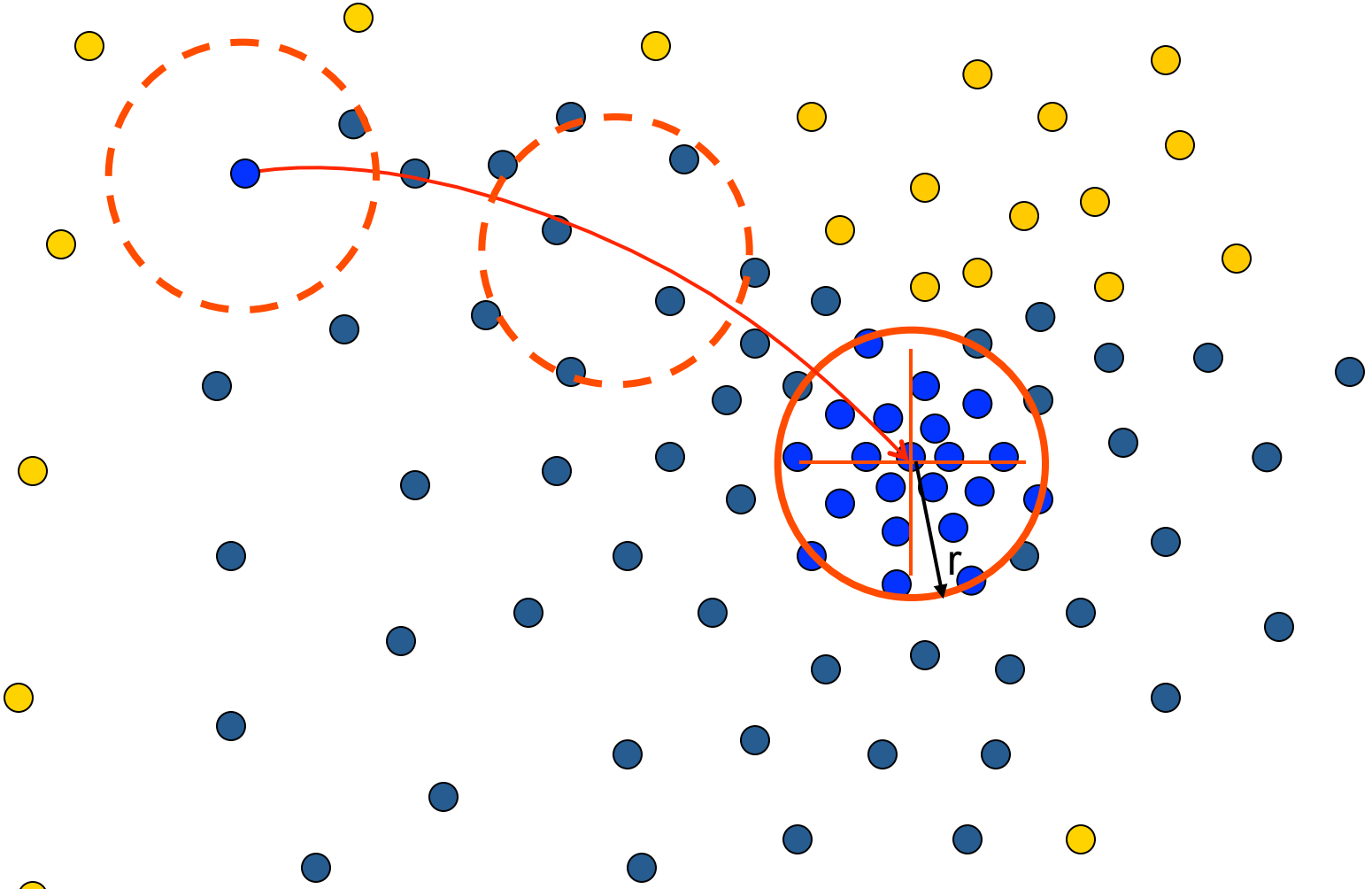
Problem: Computational Complexity



- Need to shift many windows…
- Many computations will be redundant.

Slide credit: Bastian Leibe

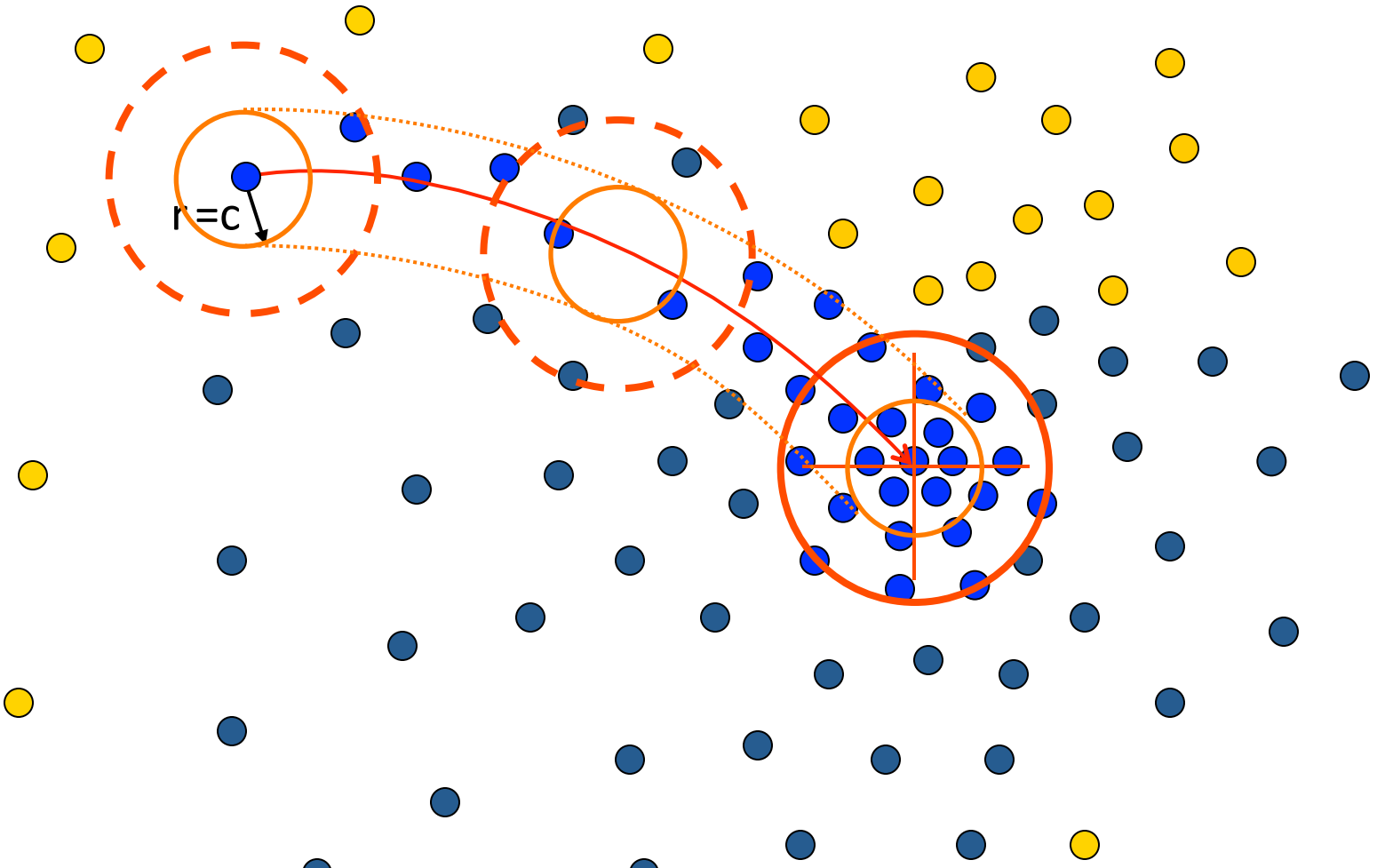
Speedups: Basin of Attraction



1. Assign all points within radius r of end point to the mode.

Slide credit: Bastian Leibe

Speedups



2. Assign all points within radius r/c of the search path to the mode -> reduce the number of data points to search.

Slide credit: Bastian Leibe

Summary Mean-Shift

- Pros
  - General, application-independent tool
  - Model-free, does not assume any prior shape (spherical, elliptical, etc.) on data clusters
  - Just a single parameter (window size h)
    - h has a physical meaning (unlike k-means)
  - Finds variable number of modes
  - Robust to outliers
- Cons
  - Output depends on window size
  - Window size (bandwidth) selection is not trivial
  - Computationally (relatively) expensive (~2s/image)
  - Does not scale well with dimension of feature space

Slide credit: Svetlana Lazebnik

technical note

Mean-shift Algorithm

Comaniciu & Meer, 2002

Given n data points $\mathbf{x}_i \in \mathbb{R}^d$, the multivariate kernel density estimate using a radially symmetric kernel[1] (e.g., Epanechnikov and Gaussian kernels), $K(\mathbf{x})$, is given by,

$$\hat{f}_K = \frac{1}{nh^d} \sum_{i=1}^{n} K\left(\frac{\mathbf{x} - \mathbf{x}_i}{h}\right), \tag{1}$$

where h (termed the *bandwidth* parameter) defines the radius of kernel. The radially symmetric kernel is defined as,

$$K(\mathbf{x}) = c_k k(\|\mathbf{x}\|^2), \tag{2}$$

where c_k represents a normalization constant. Taking the gradient of the density estimator (1) and some further algebraic manipulation yields,

$$\nabla \hat{f}(\mathbf{x}) = \underbrace{\frac{2c_{k,d}}{nh^{d+2}} \left[\sum_{i=1}^{n} g\left(\left\|\frac{\mathbf{x} - \mathbf{x}_i}{h}\right\|^2\right)\right]}_{\text{term 1}} \underbrace{\left[\frac{\sum_{i=1}^{n} \mathbf{x}_i g\left(\left\|\frac{\mathbf{x}-\mathbf{x}_i}{h}\right\|^2\right)}{\sum_{i=1}^{n} g\left(\left\|\frac{\mathbf{x}-\mathbf{x}_i}{h}\right\|^2\right)} - \mathbf{x}\right]}_{\text{term 2}}, \tag{3}$$

where $g(x) = -k'(x)$ denotes the derivative of the selected kernel profile. The first term is proportional to the density estimate at $\mathbf{x}$ (computed with the kernel $G = c_g g(\|\mathbf{x}\|^2)$). The second term, called the *mean shift* vector, $\mathbf{m}$, points toward the direction of maximum increase in density and is proportional to the density gradient estimate at point $\mathbf{x}$ obtained with kernel K. The mean shift procedure for a given point $\mathbf{x}_i$ is as follows: (see Fig. 1):

1. Compute the mean shift vector $\mathbf{m}(\mathbf{x}_i^t)$.
2. Translate density estimation window: $\mathbf{x}_i^{t+1} = \mathbf{x}_i^t + \mathbf{m}(\mathbf{x}_i^t)$.
3. Iterate steps 1. and 2. until convergence, i.e., $\nabla f(\mathbf{x}_i) = 0$.

technical note

Mean-shift Algorithm

Comaniciu & Meer, 2002

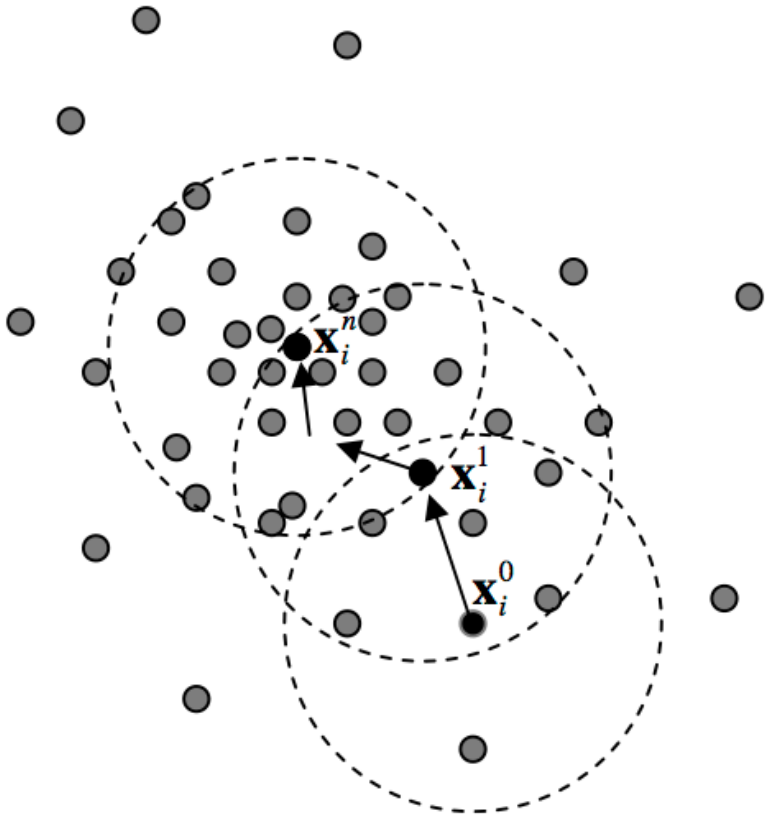


Figure 1: Mean shift procedure. Starting at data point $\mathbf{x}_i$, run the mean shift procedure to find the stationary points of the density function. Superscripts denote the mean shift iteration, the shaded and black dots denote the input data points and successive window centres, respectively, and the dotted circles denote the density estimation windows.

What will we have learned today

- K-means clustering
- Mean-shift clustering

Reading: [FP] Chapters: 14.2, 14.4

D. Comaniciu and P. Meer,
Mean Shift: A Robust Approach toward Feature Space Analysis, PAMI 2002.