



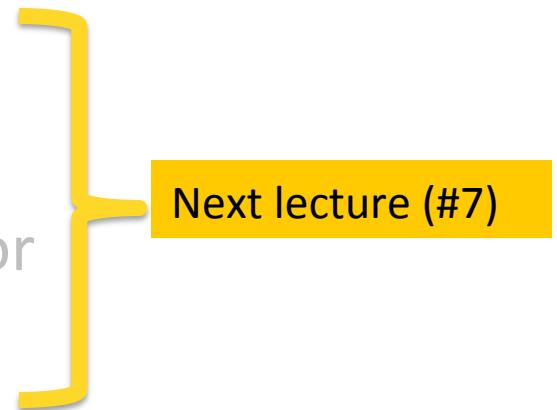
Lecture 6: Finding Features (part 1/2)

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Stanford AI Lab

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Stanford Vision Lab

What we will learn today?

- Local invariant features
 - Motivation
 - Requirements, invariances
- Keypoint localization
 - Harris corner detector
- Scale invariant region selection
 - Automatic scale selection
 - Difference-of-Gaussian (DoG) detector
- SIFT: an image region descriptor



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Some background reading:

Rick Szeliski, Chapter 4.1.1; David Lowe, IJCV 2004

Image matching: a challenging problem



Image matching: a challenging problem



by [Diva Sian](#)



by [swashford](#)

Harder Case

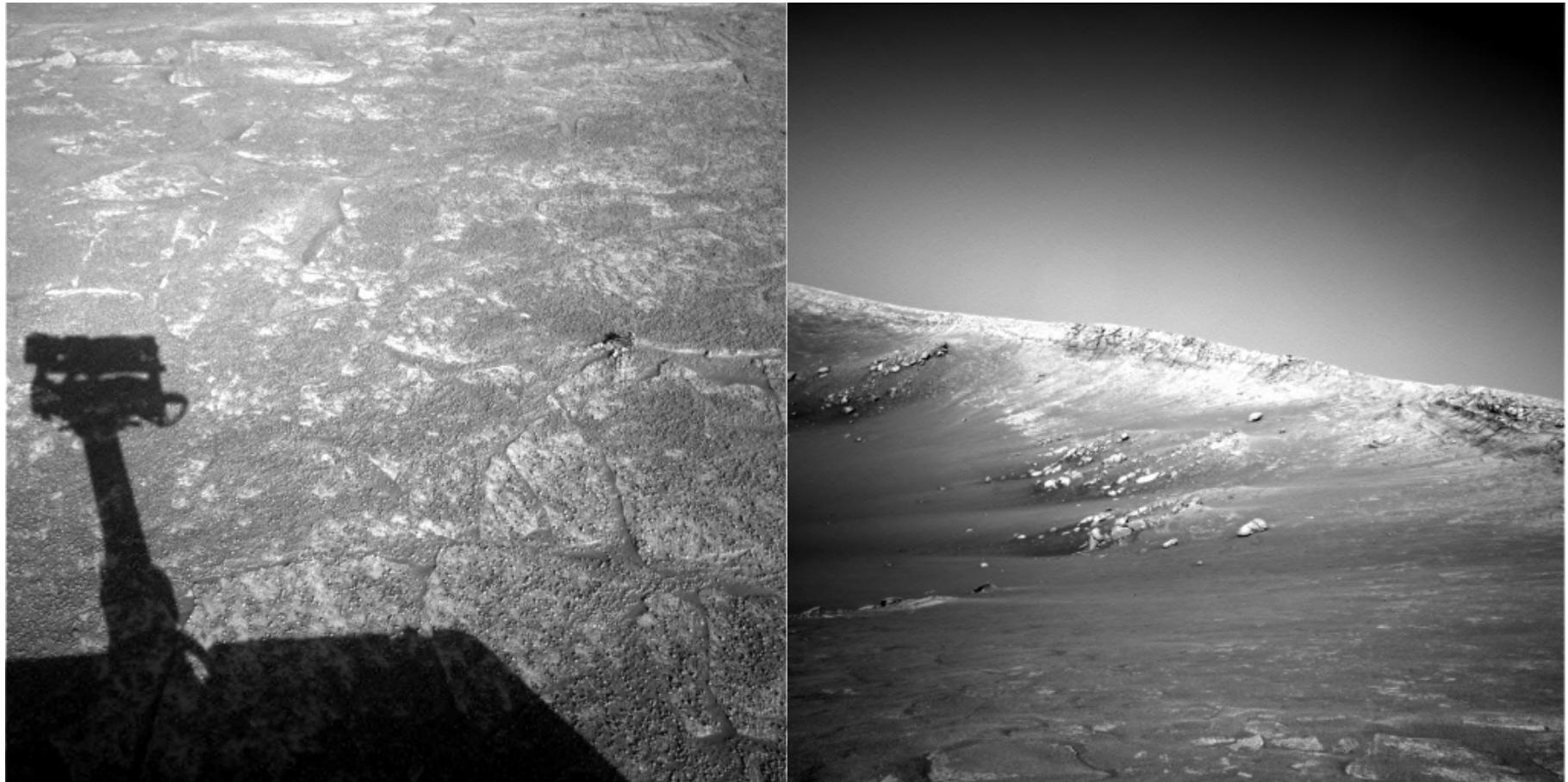


by [Diva Sian](#)



by [scgbt](#)

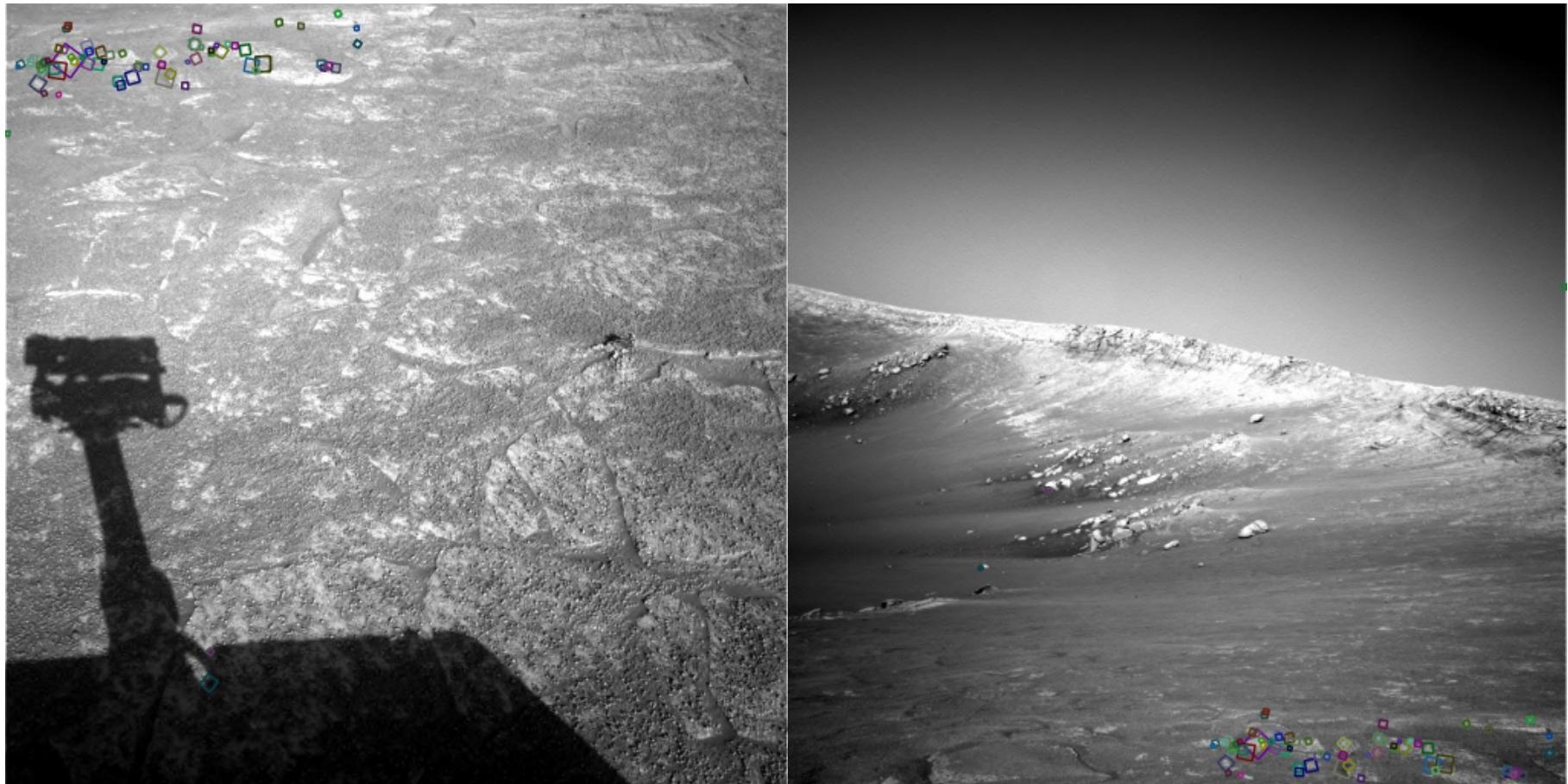
Harder Still?



NASA Mars Rover images

Slide credit: Steve Seitz

Answer Below (Look for tiny colored squares)



NASA Mars Rover images with SIFT feature matches
(Figure by Noah Snavely)

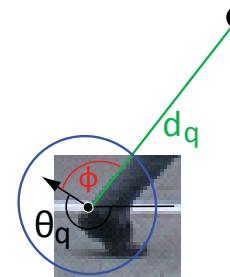
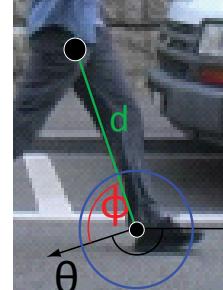
Slide credit: Steve Seitz

Motivation for using local features

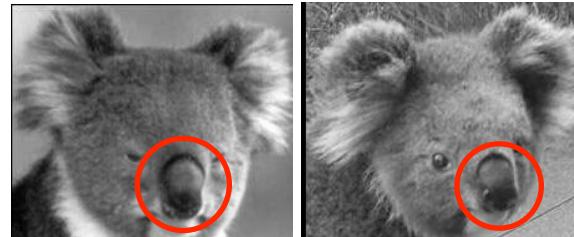
- Global representations have major limitations
- Instead, describe and match only local regions
- Increased robustness to
 - Occlusions



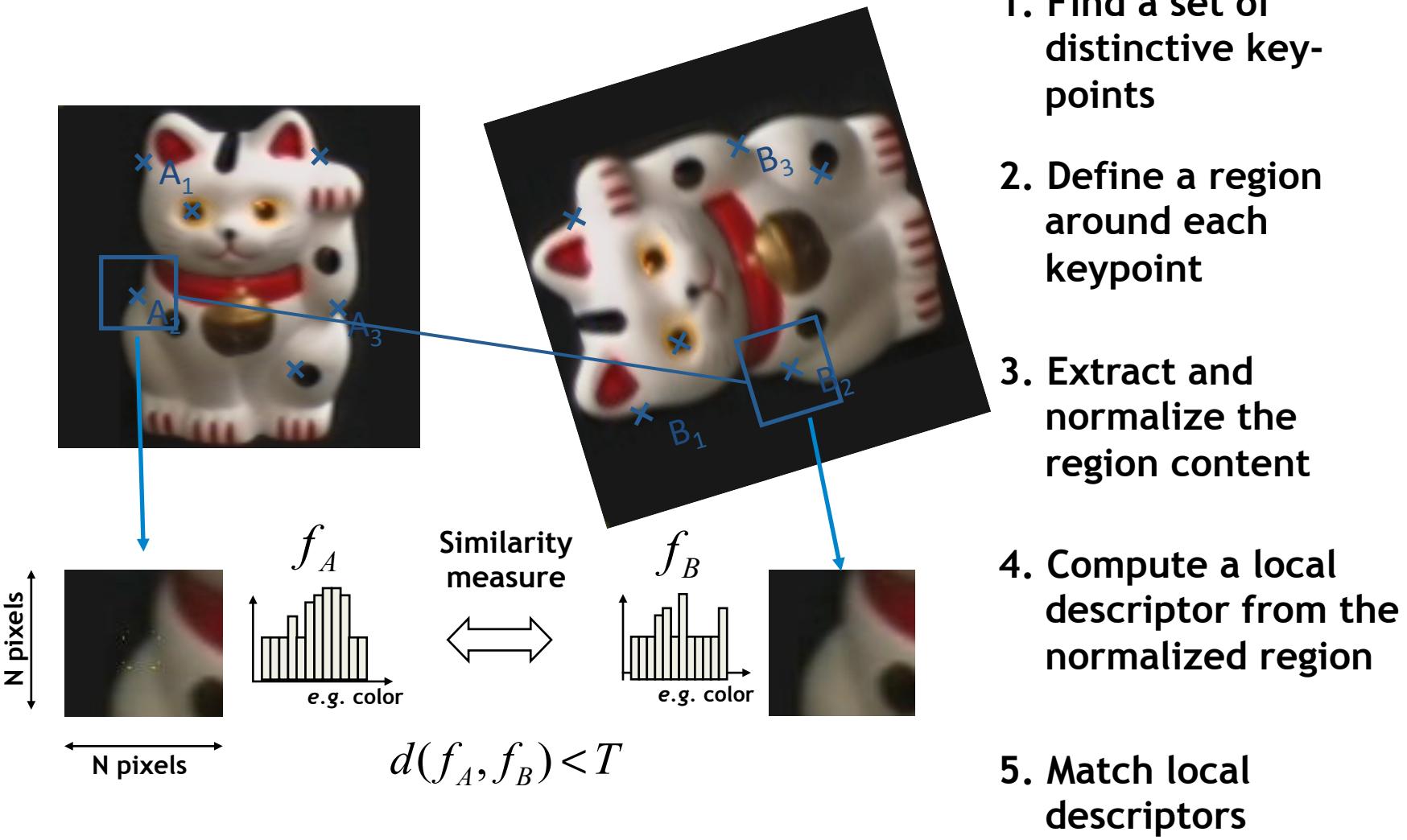
- Articulation



- Intra-category variations



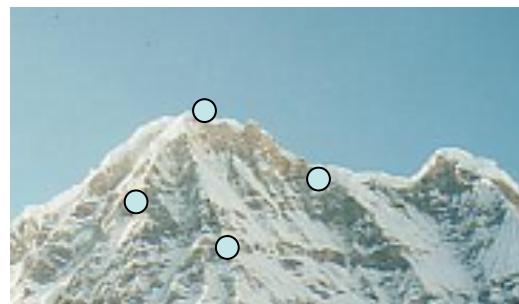
General Approach



Slide credit: Bastian Leibe

Common Requirements

- Problem 1:
 - Detect the same point *independently* in both images

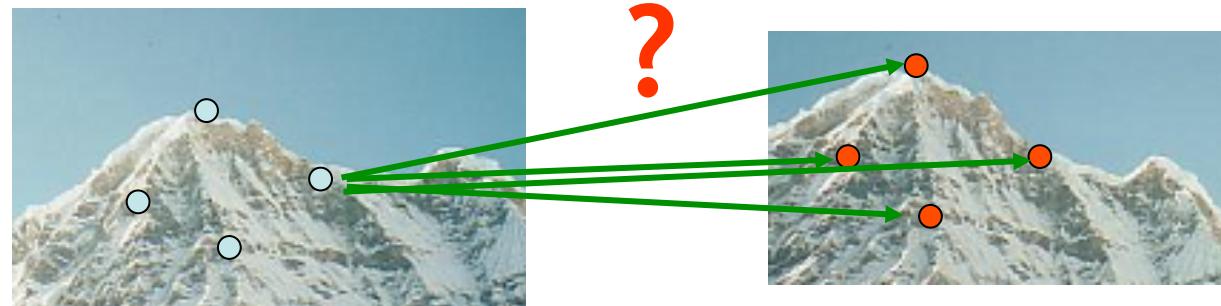


No chance to match!

We need a repeatable detector!

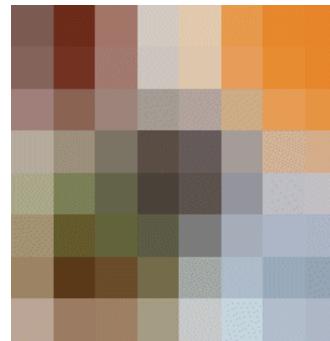
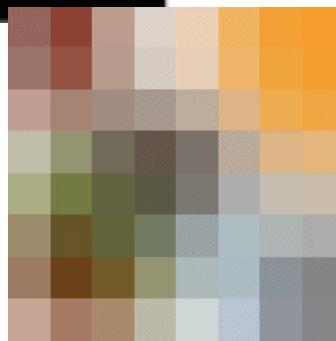
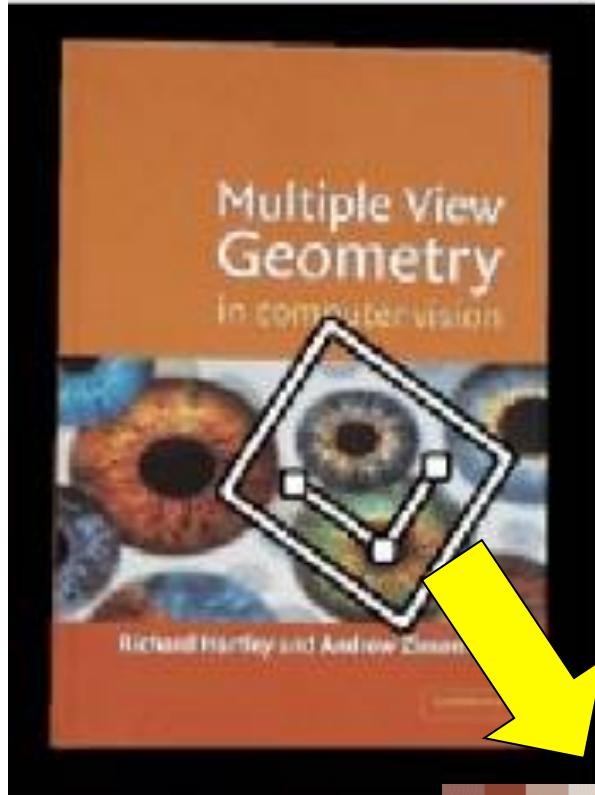
Common Requirements

- Problem 1:
 - Detect the same point *independently* in both images
- Problem 2:
 - For each point correctly recognize the corresponding one

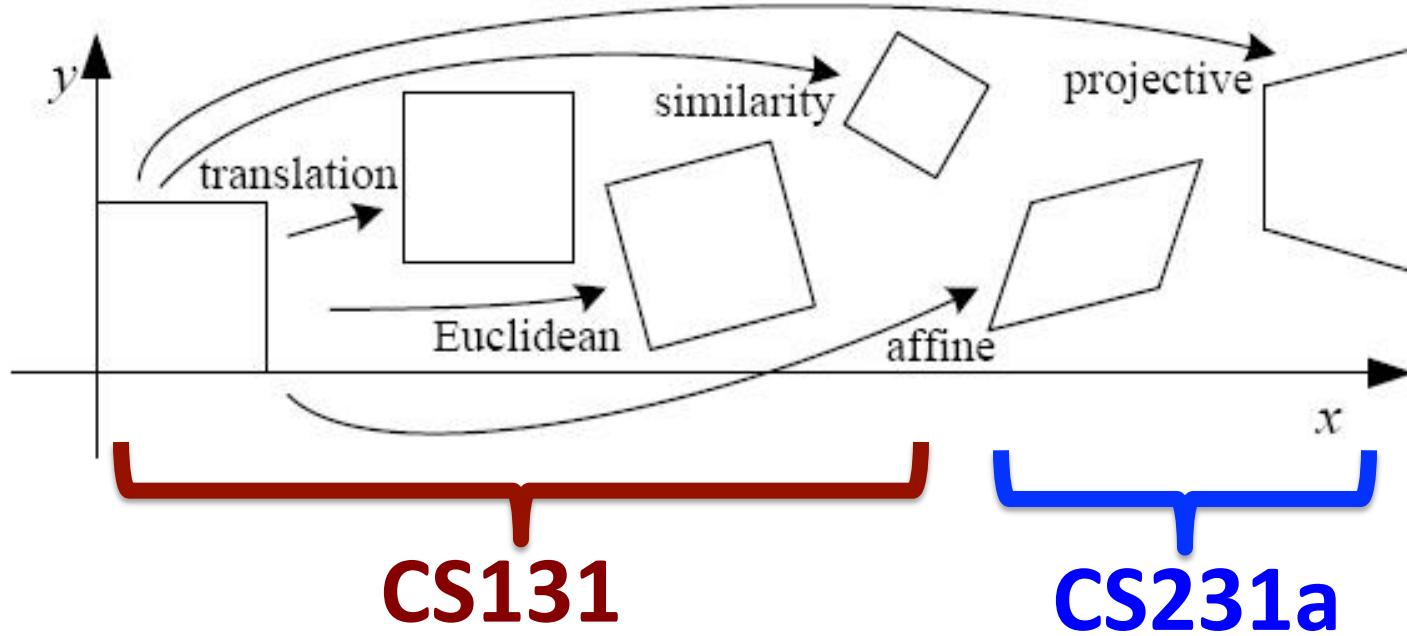


We need a reliable and distinctive descriptor!

Invariance: Geometric Transformations



Levels of Geometric Invariance



Invariance: Photometric Transformations



- Often modeled as a linear transformation:
 - Scaling + Offset

Requirements

- Region extraction needs to be **repeatable** and **accurate**
 - **Invariant** to translation, rotation, scale changes
 - **Robust** or **covariant** to out-of-plane (**?**affine) transformations
 - **Robust** to lighting variations, noise, blur, quantization
- **Locality**: Features are local, therefore robust to occlusion and clutter.
- **Quantity**: We need a sufficient number of regions to cover the object.
- **Distinctiveness** : The regions should contain “interesting” structure.
- **Efficiency**: Close to real-time performance.

Many Existing Detectors Available

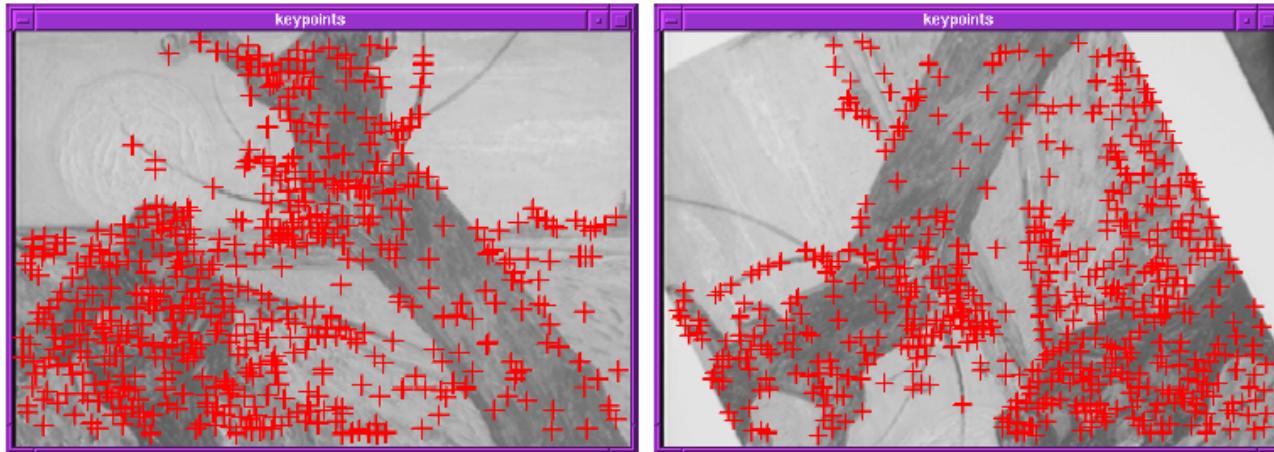
- Hessian & Harris [Beaudet '78], [Harris '88]
- Laplacian, DoG [Lindeberg '98], [Lowe '99]
- Harris-/Hessian-Laplace [Mikolajczyk & Schmid '01]
- Harris-/Hessian-Affine [Mikolajczyk & Schmid '04]
- EBR and IBR [Tuytelaars & Van Gool '04]
- MSER [Matas '02]
- Salient Regions [Kadir & Brady '01]
- Others...
- *Those detectors have become a basic building block for many recent applications in Computer Vision.*

Keypoint Localization



- Goals:
 - Repeatable detection
 - Precise localization
 - Interesting content
- ?** *Look for two-dimensional signal changes*

Finding Corners

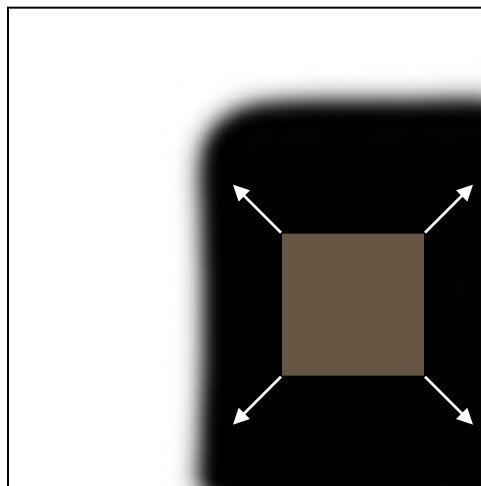


- Key property:
 - In the region around a corner, image gradient has two or more dominant directions
- Corners are *repeatable* and *distinctive*

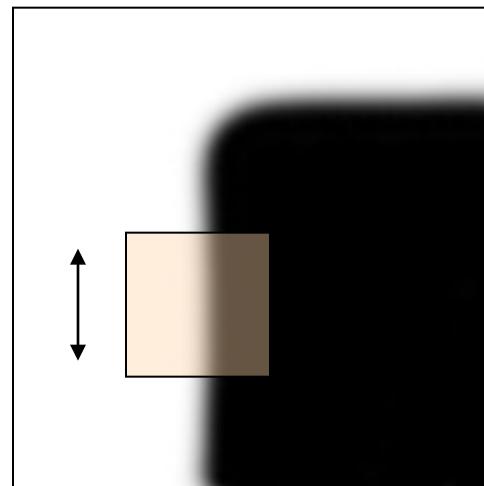
C.Harris and M.Stephens. "[A Combined Corner and Edge Detector.](#)"
Proceedings of the 4th Alvey Vision Conference, 1988.

Corners as Distinctive Interest Points

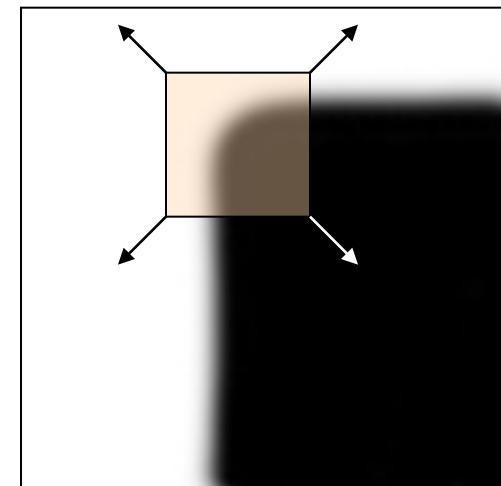
- Design criteria
 - We should easily recognize the point by looking through a small window (*locality*)
 - Shifting the window in *any direction* should give *a large change* in intensity (*good localization*)



“flat” region:
no change in all
directions



“edge”:
no change along
the edge direction



“corner”:
significant change
in all directions

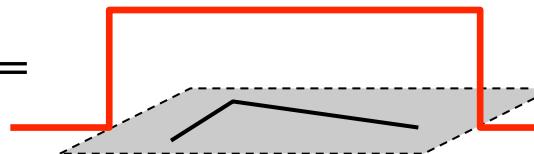
Harris Detector Formulation

- Change of intensity for the shift $[u,v]$:

$$E(u, v) = \sum w(x, y) [I(x + u, y + v) - I(x, y)]^2$$

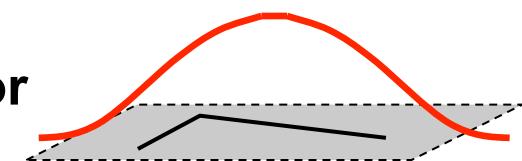
Window function Shifted intensity Intensity

Window function $w(x, y) =$



1 in window, 0 outside

or



Gaussian

Harris Detector Formulation

- This measure of change can be approximated by:

$$E(u, v) \approx [u \ v] M \begin{bmatrix} u \\ v \end{bmatrix}$$

where M is a 2×2 matrix computed from image derivatives:

$$M = \sum_{x,y} w(x, y) \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$

↑
Sum over image region – the area we are
checking for corner

**Gradient with
respect to x ,
times gradient
with respect to y**

$$M = \begin{bmatrix} \sum I_x I_x & \sum I_x I_y \\ \sum I_x I_y & \sum I_y I_y \end{bmatrix} = \sum \begin{bmatrix} I_x \\ I_y \end{bmatrix} [I_x \ I_y]$$

Harris Detector Formulation

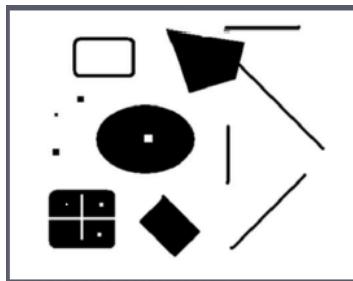
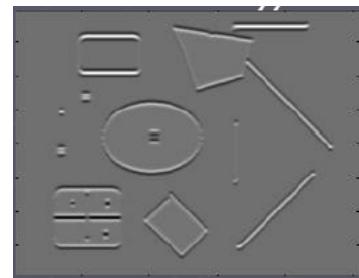


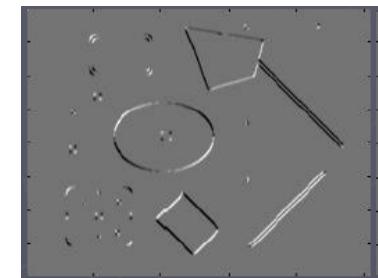
Image I



I_x



I_y



$I_x I_y$

where M is a 2×2 matrix computed from image derivatives:

$$M = \sum_{x,y} w(x,y) \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$

Sum over image region – the area we are checking for corner

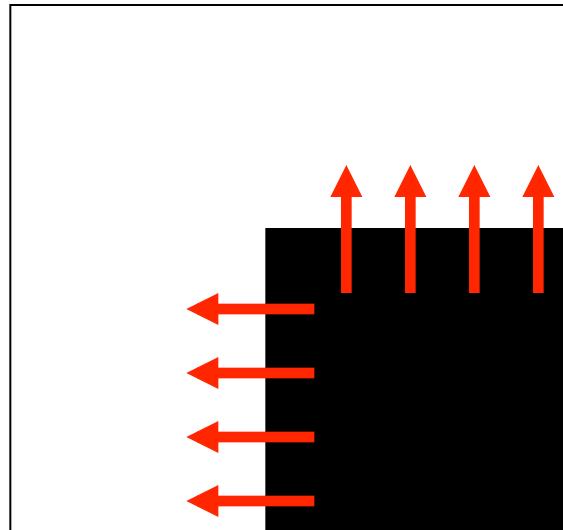
Gradient with respect to x , times gradient with respect to y

$$M = \begin{bmatrix} \sum I_x I_x & \sum I_x I_y \\ \sum I_x I_y & \sum I_y I_y \end{bmatrix} = \sum \begin{bmatrix} I_x \\ I_y \end{bmatrix} [I_x \ I_y]$$

what Does This Matrix Reveal?

- First, let's consider an axis-aligned corner:

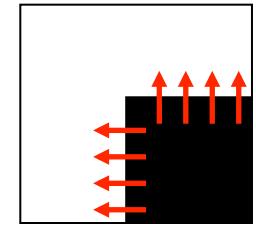
$$M = \begin{bmatrix} \sum I_x^2 & \sum I_x I_y \\ \sum I_x I_y & \sum I_y^2 \end{bmatrix} = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}$$



what Does This Matrix Reveal?

- First, let's consider an axis-aligned corner:

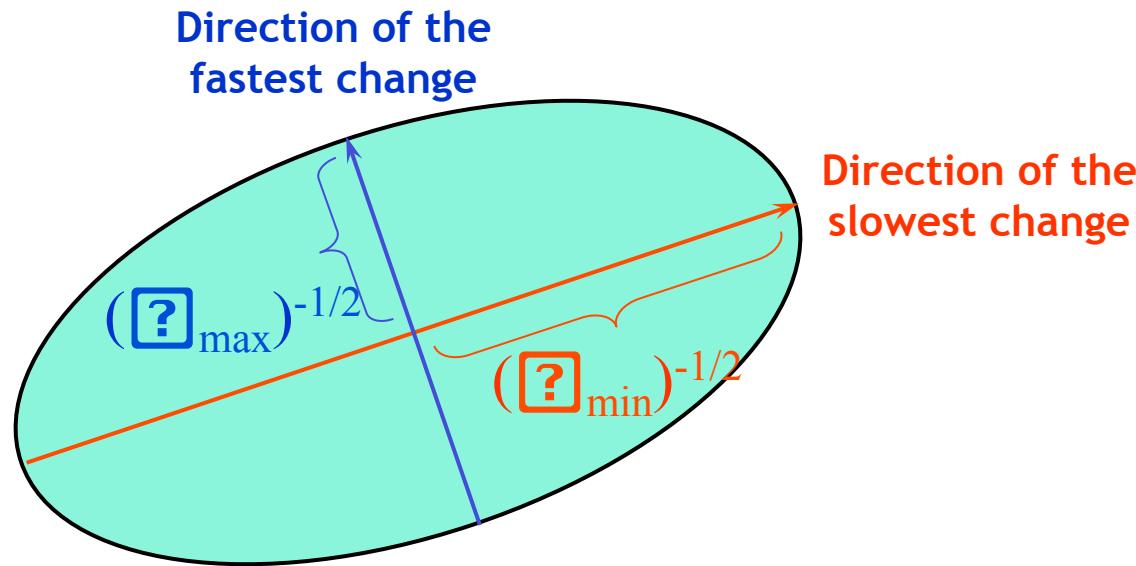
$$M = \begin{bmatrix} \sum I_x^2 & \sum I_x I_y \\ \sum I_x I_y & \sum I_y^2 \end{bmatrix} = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}$$



- This means:
 - Dominant gradient directions align with x or y axis
 - If either λ is close to 0, then this is not a corner, so look for locations where both are large.
- What if we have a corner that is not aligned with the image axes?

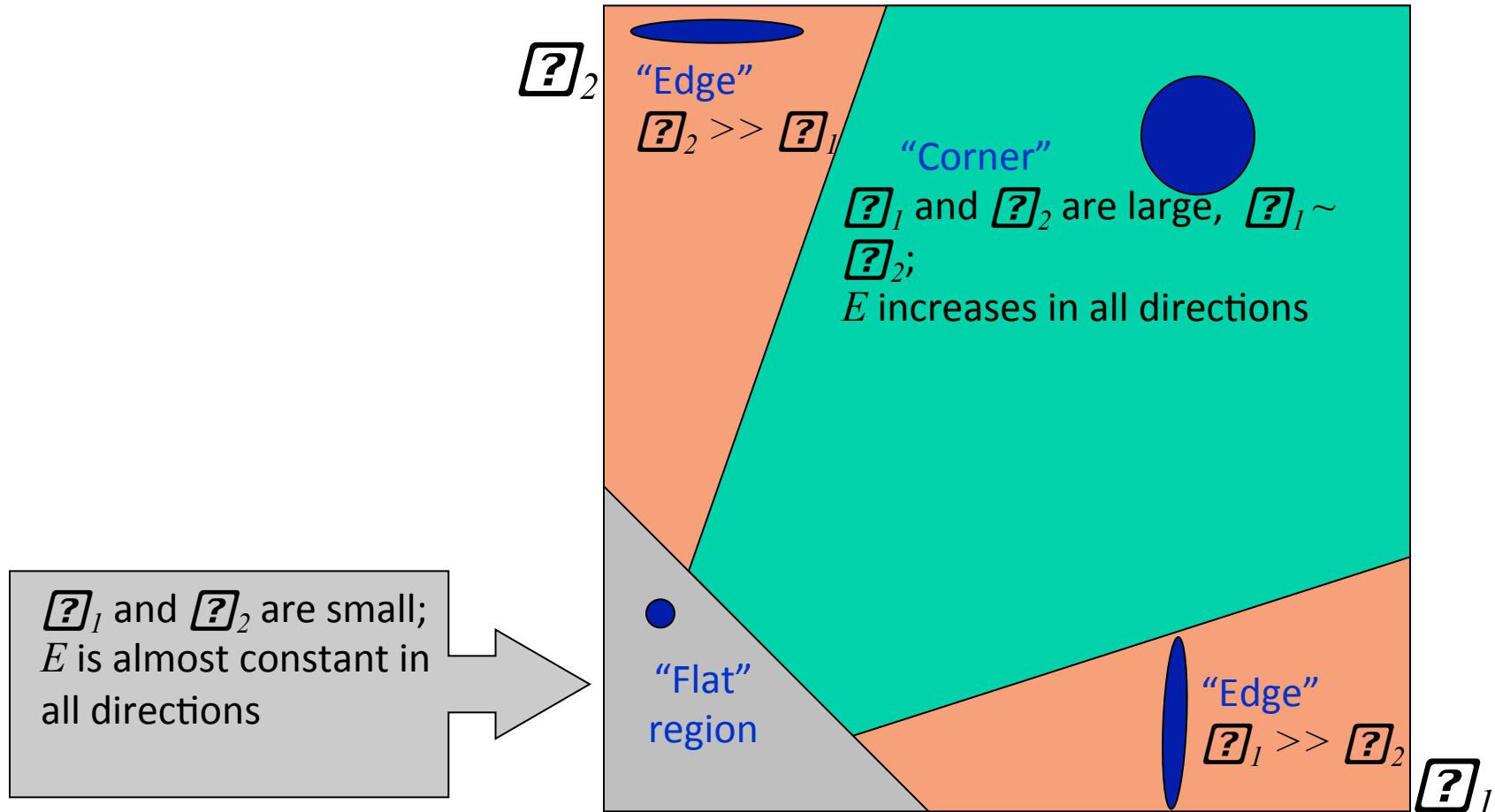
General Case

- Since M is symmetric, we have $M = R^{-1} \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} R$
(Eigenvalue decomposition)
- We can visualize M as an ellipse with axis lengths determined by the eigenvalues and orientation determined by R



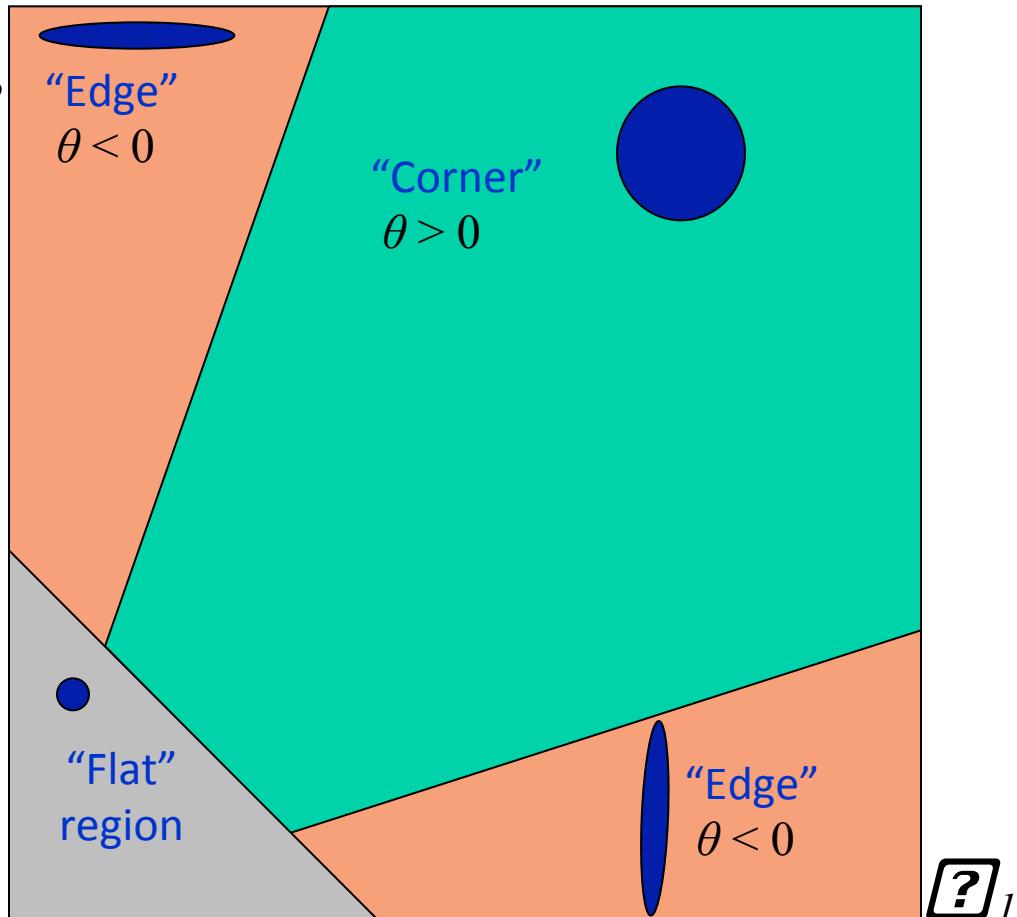
Interpreting the Eigenvalues

- Classification of image points using eigenvalues of M :



Corner Response Function

$$\theta = \det(M) - \alpha \operatorname{trace}(M)^2 = \lambda_1 \lambda_2 - \alpha(\lambda_1 + \lambda_2)^2$$



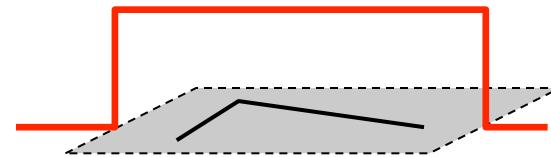
- Fast approximation
 - Avoid computing the eigenvalues
 - α : constant (0.04 to 0.06)

Window Function $w(x,y)$

$$M = \sum_{x,y} w(x,y) \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$

- Option 1: uniform window
 - Sum over square window
 - Problem: not rotation invariant

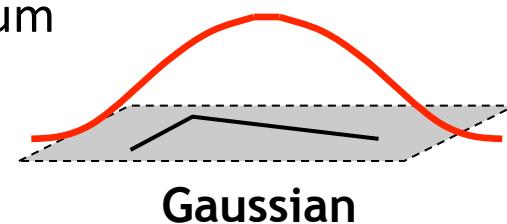
$$M = \sum_{x,y} \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$



1 in window, 0 outside

- Option 2: Smooth with Gaussian
 - Gaussian already performs weighted sum
 - Result is rotation invariant

$$M = g(\sigma) * \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$



Gaussian

Summary: Harris Detector [Harris88]

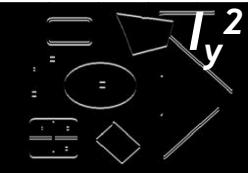
- Compute second moment matrix (autocorrelation matrix)

$$M(\sigma_I, \sigma_D) = g(\sigma_I) * \begin{bmatrix} I_x^2(\sigma_D) & I_x I_y(\sigma_D) \\ I_x I_y(\sigma_D) & I_y^2(\sigma_D) \end{bmatrix}$$

1. Image derivatives



2. Square of derivatives



3. Gaussian filter $g(s_p)$



4. Cornerness function - two strong eigenvalues

$$\begin{aligned}\theta &= \det[M(\sigma_I, \sigma_D)] - \alpha[\text{trace}(M(\sigma_I, \sigma_D))]^2 \\ &= g(I_x^2)g(I_y^2) - [g(I_x I_y)]^2 - \alpha[g(I_x^2) + g(I_y^2)]^2\end{aligned}$$

5. Perform non-maximum suppression



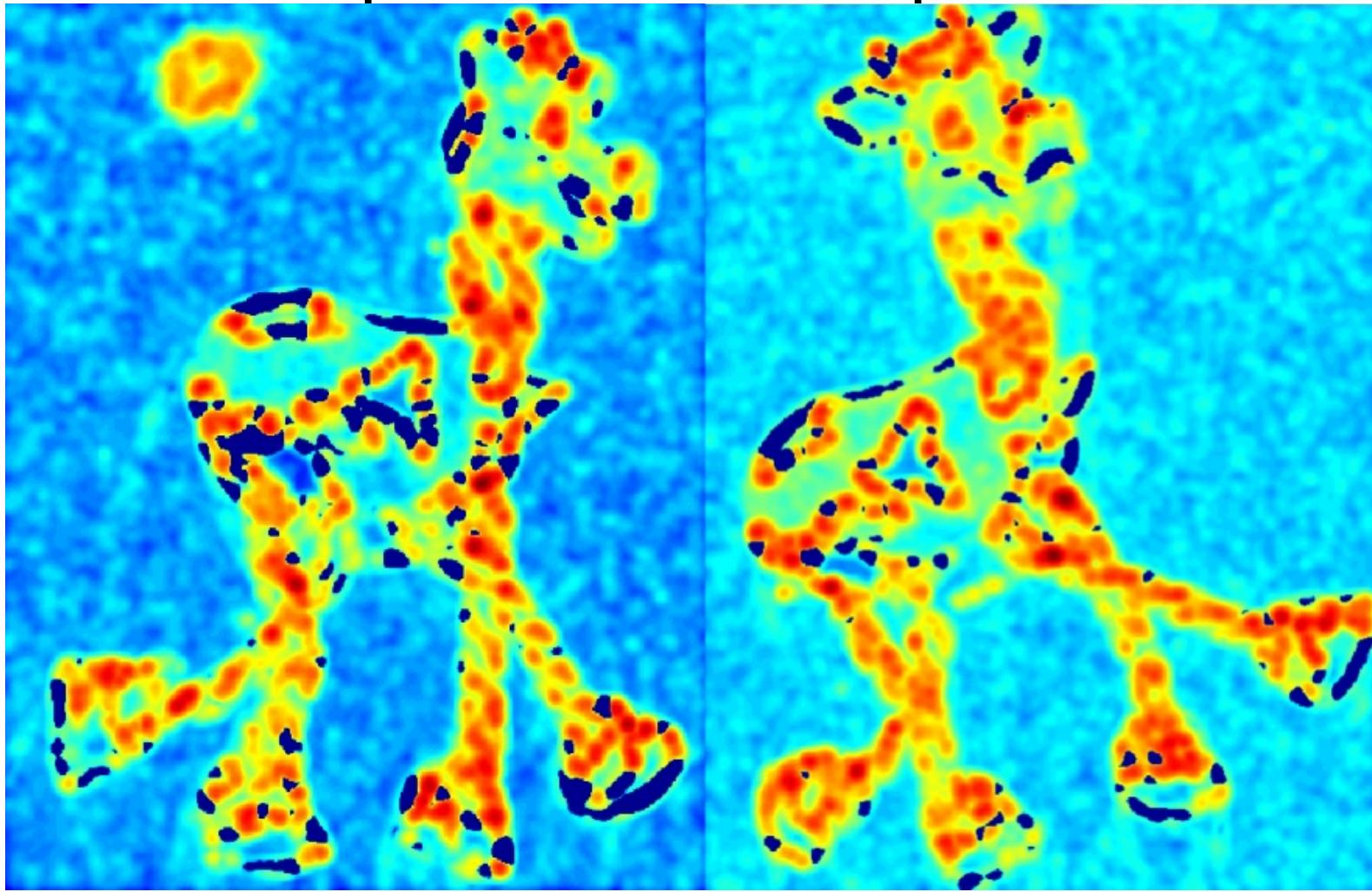
Harris Detector: Workflow



Slide adapted from Darya Frolova, Denis Simakov

Harris Detector: Workflow

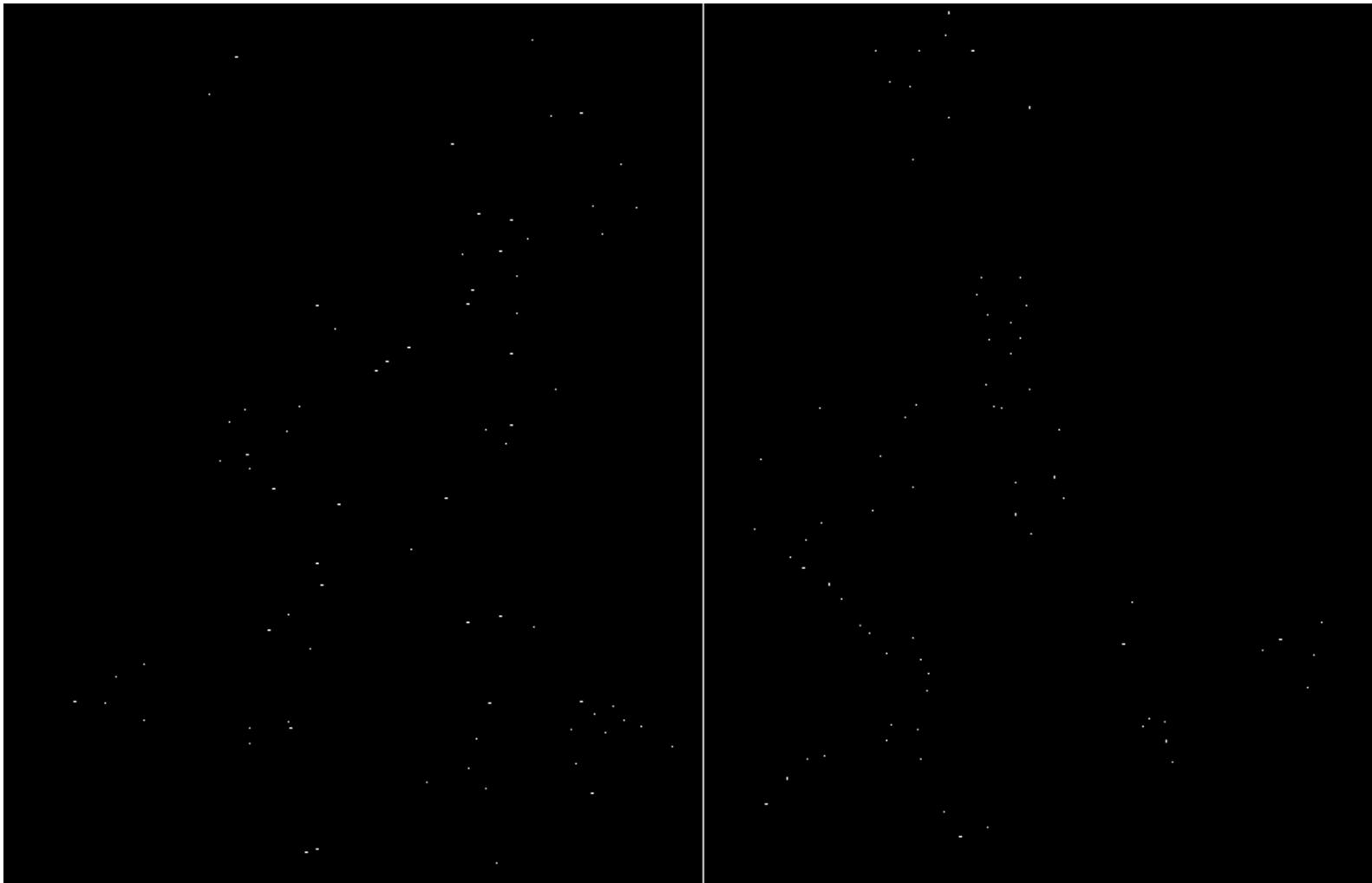
- computer corner responses θ



Slide adapted from Darya Frolova, Denis Simakov

Harris Detector: Workflow

- Take only the local maxima of θ , where $\theta > \text{threshold}$



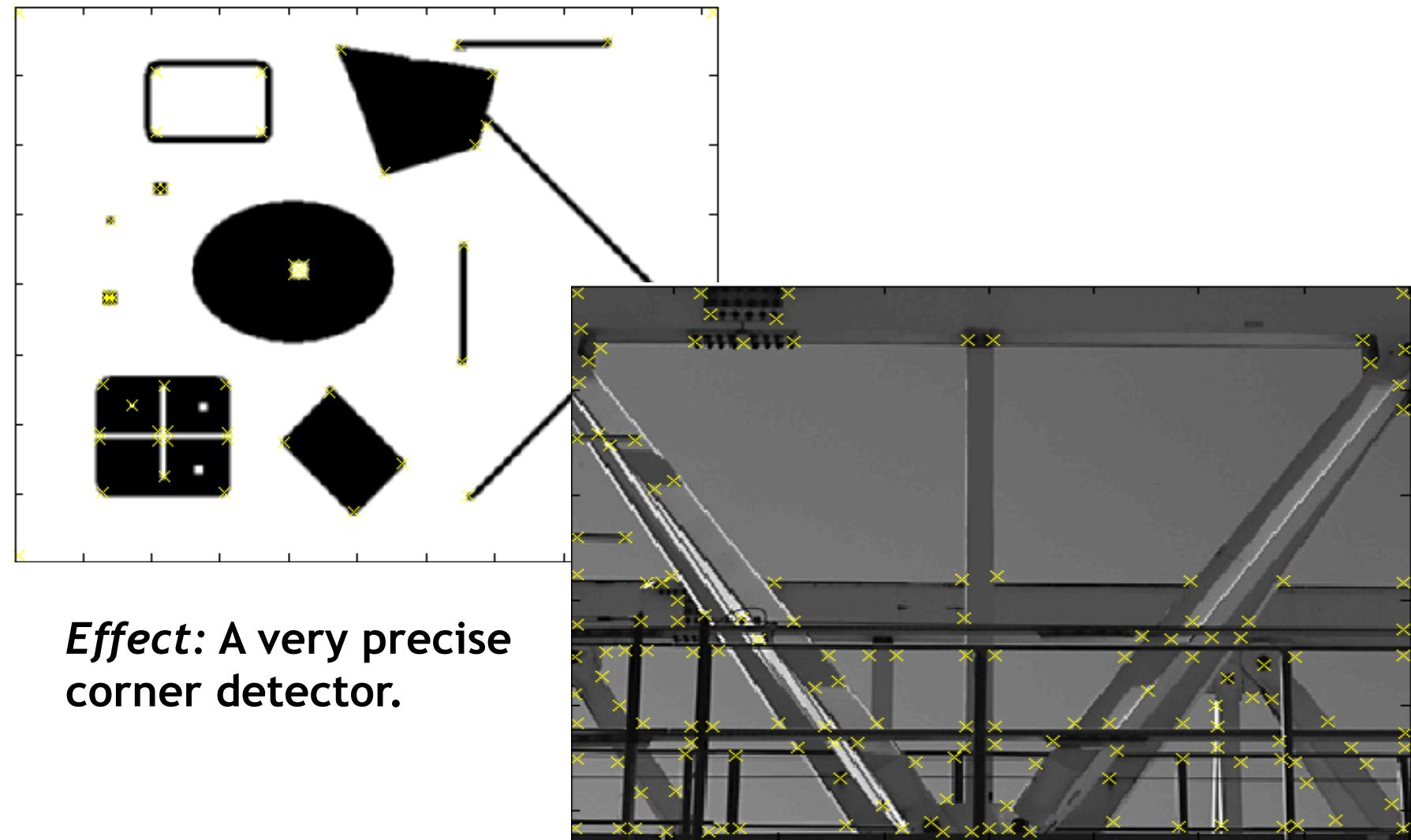
Harris Detector: Workflow

- Resulting Harris points



Slide adapted from Darya Frolova, Denis Simakov

Harris Detector – Responses [Harris88]



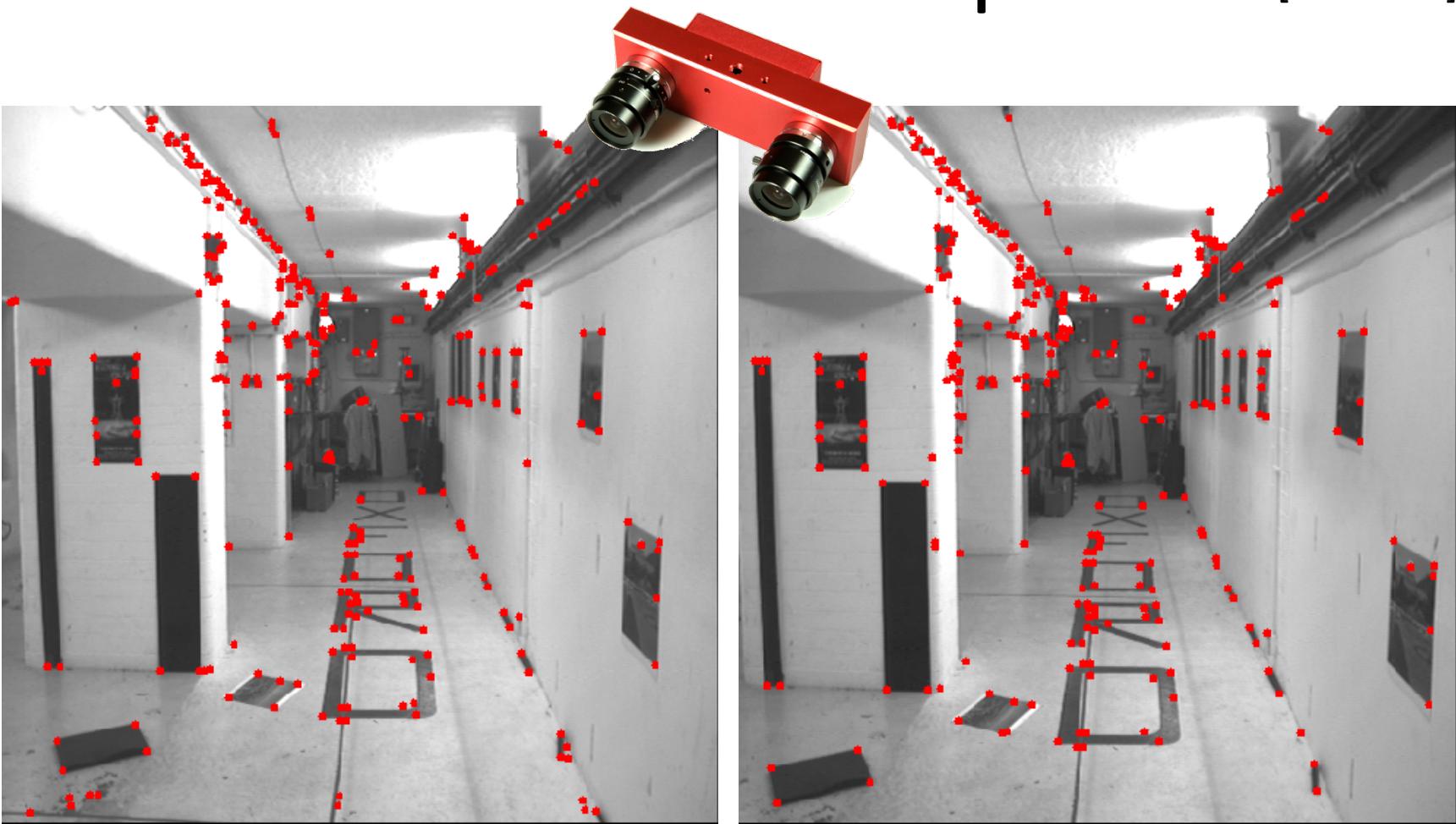
Effect: A very precise corner detector.

Harris Detector – Responses [Harris88]



Slide credit: Krystian Mikolajczyk

Harris Detector – Responses [Harris88]



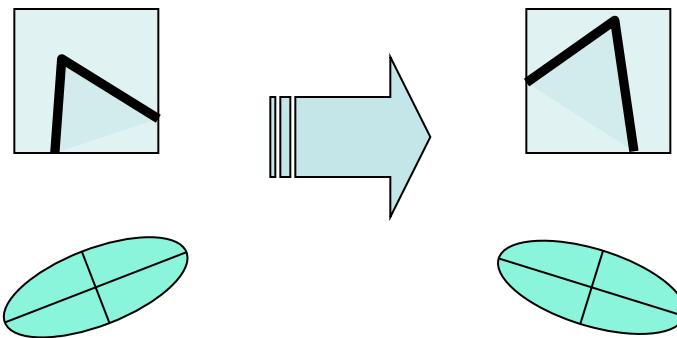
- Results are well suited for finding stereo correspondences

Harris Detector: Properties

- Translation invariance?

Harris Detector: Properties

- Translation invariance
- Rotation invariance?

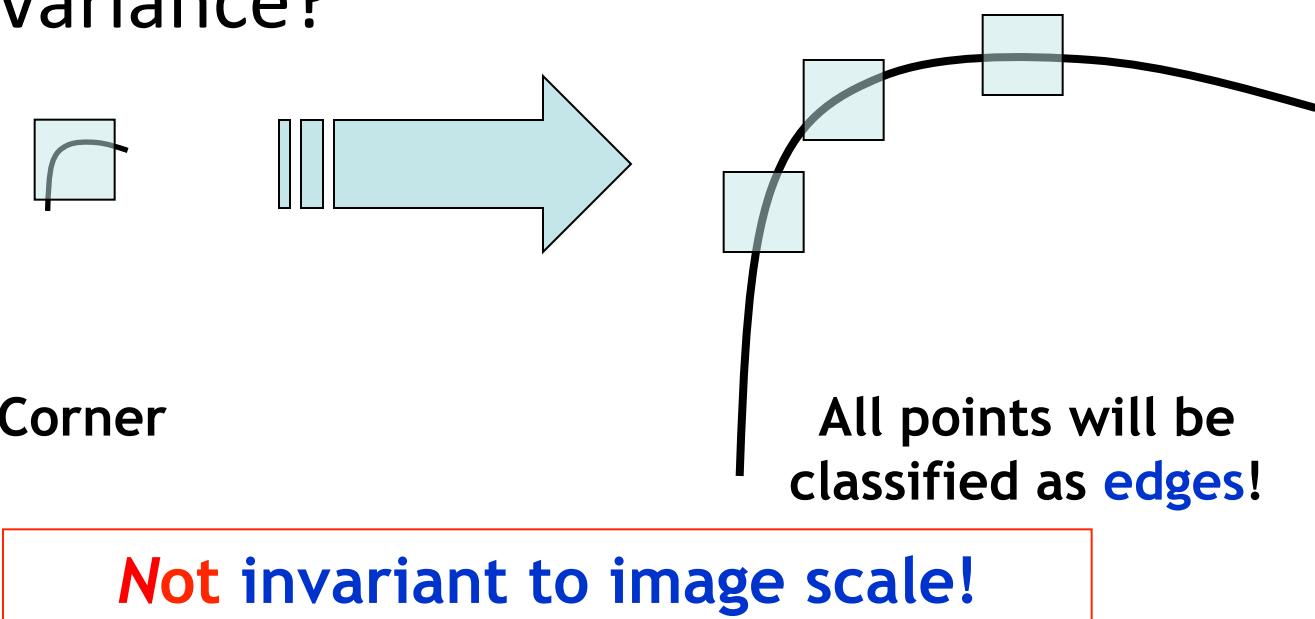


Ellipse rotates but its shape (i.e. eigenvalues) remains the same

Corner response θ is invariant to image rotation

Harris Detector: Properties

- Translation invariance
- Rotation invariance
- Scale invariance?

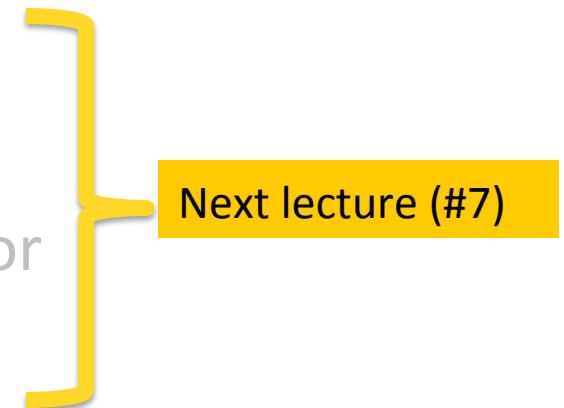


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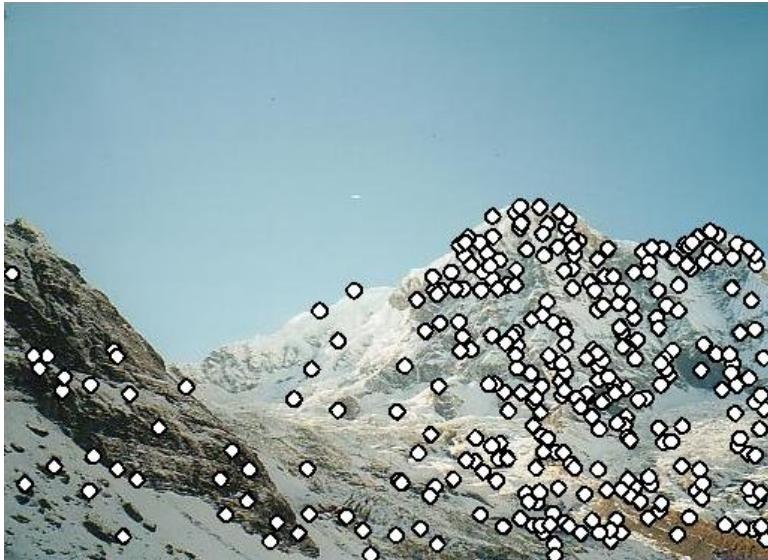
Next lecture (#7)

Application: Image Stitching



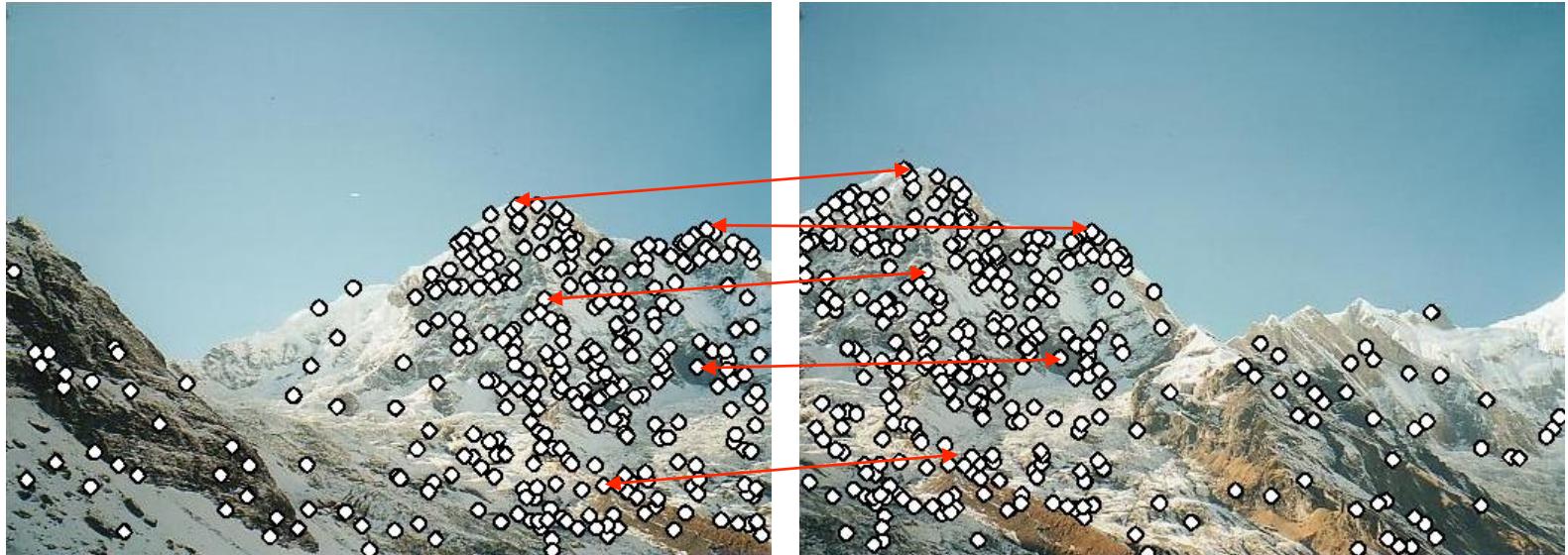
Slide credit: Darya Frolova, Denis Simakov

Application: Image Stitching



- Procedure:
 - Detect feature points in both images

Application: Image Stitching



- Procedure:
 - Detect feature points in both images
 - Find corresponding pairs

Application: Image Stitching



- Procedure:
 - Detect feature points in both images
 - Find corresponding pairs
 - Use these pairs to align the images