# Review for Problem Set 1

## CS 231a Winter 2015-16

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# Today's Agenda

- Camera Model
- Rotation Matrices
- Homogeneous Coordinates
- Vanishing Points
- Problem Discussion
  - Constrained Optimization

# Camera Model

- $x$: 2D point in the image frame (homogeneous)
- $X$: 3D point in the world frame (homogeneous)
- $[\ R \mid t\ ]$: Camera rotation and translation (extrinsics)
- $K$: Camera calibration matrix (intrinsics)

# Extrinsics

$$P_b = [\ R \mid t\ ]\ P_a$$

What are the coordinate systems of the points $P_a$ and $P_b$?

World to Camera: $P_{camera} = [\ R \mid t\ ]\ P_{world}$

- Let $R = \mathbf{I_3}$, $t = [\ 0 \quad 0 \quad 0\ ]^T$

What are the world coordinates of the camera center?

$[\ 0 \quad 0 \quad 0\ ]^T$

For any arbitrary [ R | t ] ?

$-R^T t$

# Rotation Matrices

- The columns of a rotation matrix are the original coordinate system's basis vectors represented in the rotated coordinate system.

$$R \,.\, [1 \quad 0 \quad 0]^T = [R_1 \;|\; R_2 \;|\; R_3] \,.\, [1 \quad 0 \quad 0]^T = R_1$$

- What about the rows?

Hint: Rows of R are columns of $R^T$

The rotated coordinate system's basis vectors represented in the original coordinate system

# Homogeneous Coordinates: Conversion

- *To* homogeneous: $[x\ y]^T$ becomes $[x\ y\ 1]^T$
- *From* homogeneous: $[x\ y\ w]^T$ becomes $[x/w\ y/w]^T$
- Is mapping from $\mathbf{R}^n$ (Euclidean) to $\mathbf{P}^n$ (Projective) unique?

  In projective space, all scalar multiples of a point are equivalent.

  [2 3 1] and [4 6 2] both map back to [2 3] in $\mathbf{R}^2$
- Cartesian:Euclidean as Homogeneous:Projective

# Homogeneous Coordinates: What’s the point?

- We would like to use the powerful tools of linear algebra
- Is perspective projection a linear transformation?

  No. Involves division by depth.

- Is translation a linear transformation?

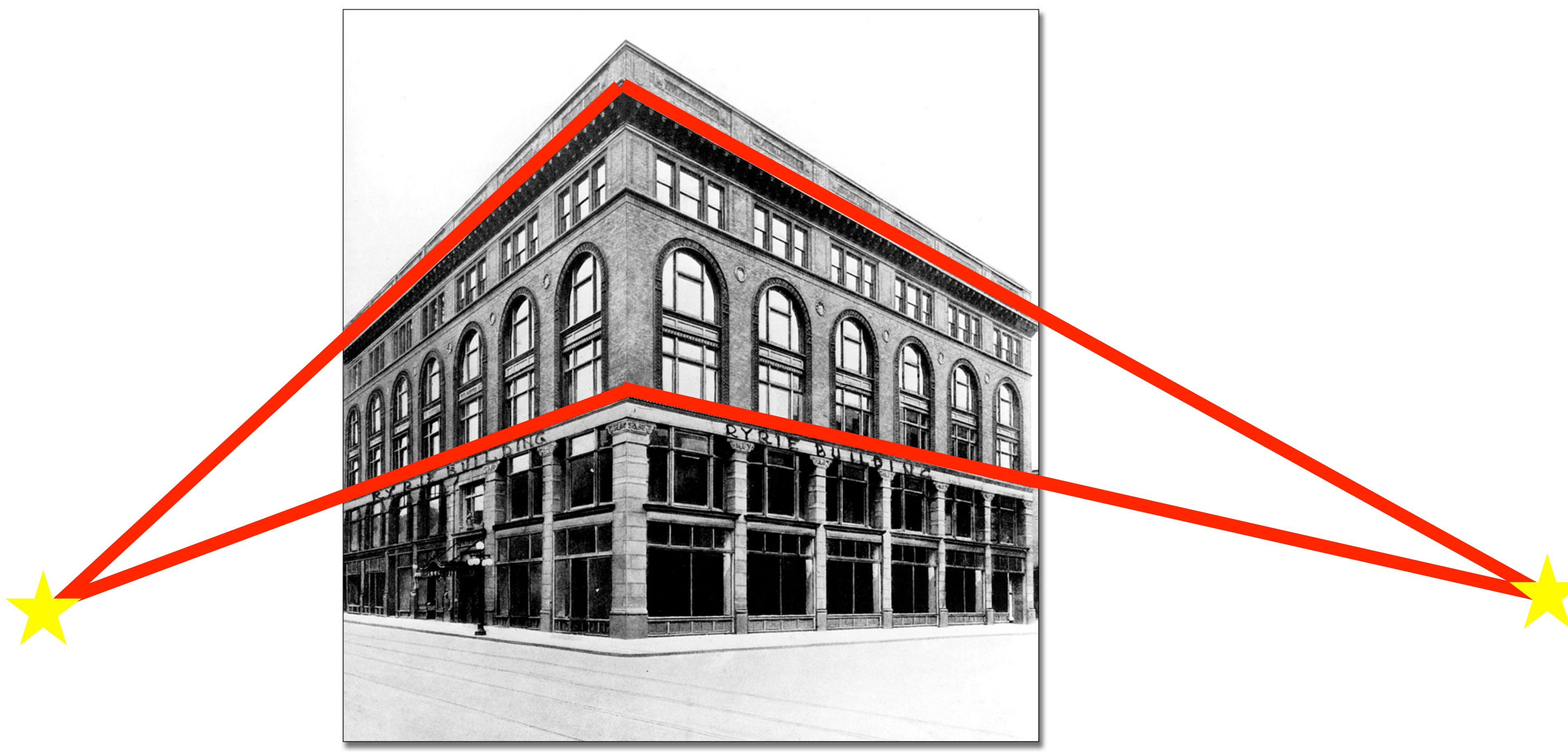
  No. Doesn’t preserve the origin.

- Homogeneous coordinates provide a solution for the above.

# Vanishing Points

# Vanishing Points

- Can there be multiple vanishing points?



- Maximum number of vanishing points? Infinite!

# Vanishing Points

- Under perspective projection, lines that are parallel in the world frame are no longer parallel in the image frame
    - Exception: Lines parallel to the image plane remain parallel
- In image space, parallel lines meet at the vanishing point
- In the projective space, parallel lines meet at points at infinity (also known as ideal points)
    - Homogeneous coordinates of such points, e.g., [x y 0]

# Problem Discussion

- Problem Set 1

# Constrained Optimization using Lagrange Multipliers

- A constrained optimization problem is a problem of the form maximize (or minimize) the function $F(x,y)$ subject to the condition $g(x, y) = 0$.

# Constrained Optimization

## Subject to equality constraints

Given $f : \mathbb{R}^n \to \mathbb{R}$ and $g : \mathbb{R}^n \to \mathbb{R}^m$,

$$\begin{aligned} \text{Minimize:} \quad & f(\mathbf{x}) \\ \text{Subject to:} \quad & g(\mathbf{x}) = 0 \end{aligned}$$

We can solve optimization problems of this form using the method of Lagrange multipliers.

# Constrained Optimization

## Lagrange Multilpliers

- Define the Lagrangian as:

$$\Lambda(\mathbf{x}, \lambda) = f(\mathbf{x}) + \lambda^\top g(\mathbf{x})$$

  where $\lambda \in \mathbb{R}^m$

- Each $\lambda_i \in \lambda$ is known as a Lagrange Multiplier.

- The necessary conditions for optimality are:

  1. $\nabla_{\mathbf{x}} \Lambda(\mathbf{x}, \lambda) = 0$
  2. $\nabla_{\lambda} \Lambda(\mathbf{x}, \lambda) = 0$

# Constrained Optimization

## The Lagrange Optimality Conditions

- $\nabla_{\mathbf{x}} \Lambda(\mathbf{x}, \lambda) = 0$
  - A consequence of $\nabla_{\mathbf{x}} f(\mathbf{x}) = -\lambda \nabla_{\mathbf{x}} g(\mathbf{x})$
  - Gives us $n$ equations (recall that $\mathbf{x} \in \mathbb{R}^n$)
- $\nabla_{\lambda} \Lambda(\mathbf{x}, \lambda) = 0$
  - The equality constraints in disguise.
  - Gives us $m$ equations (recall that $\lambda \in \mathbb{R}^m$)
- We have $(n + m)$ unknowns and $(n + m)$ equations.

Solve simultaneously.