

Lecture 3

Camera Models 2 & Camera Calibration



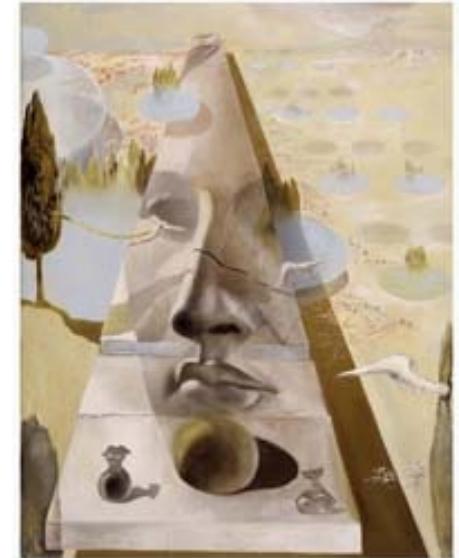
1891

Professor Silvio Savarese

Computational Vision and Geometry Lab

Lecture 3

Camera Models 2 & Camera Calibration



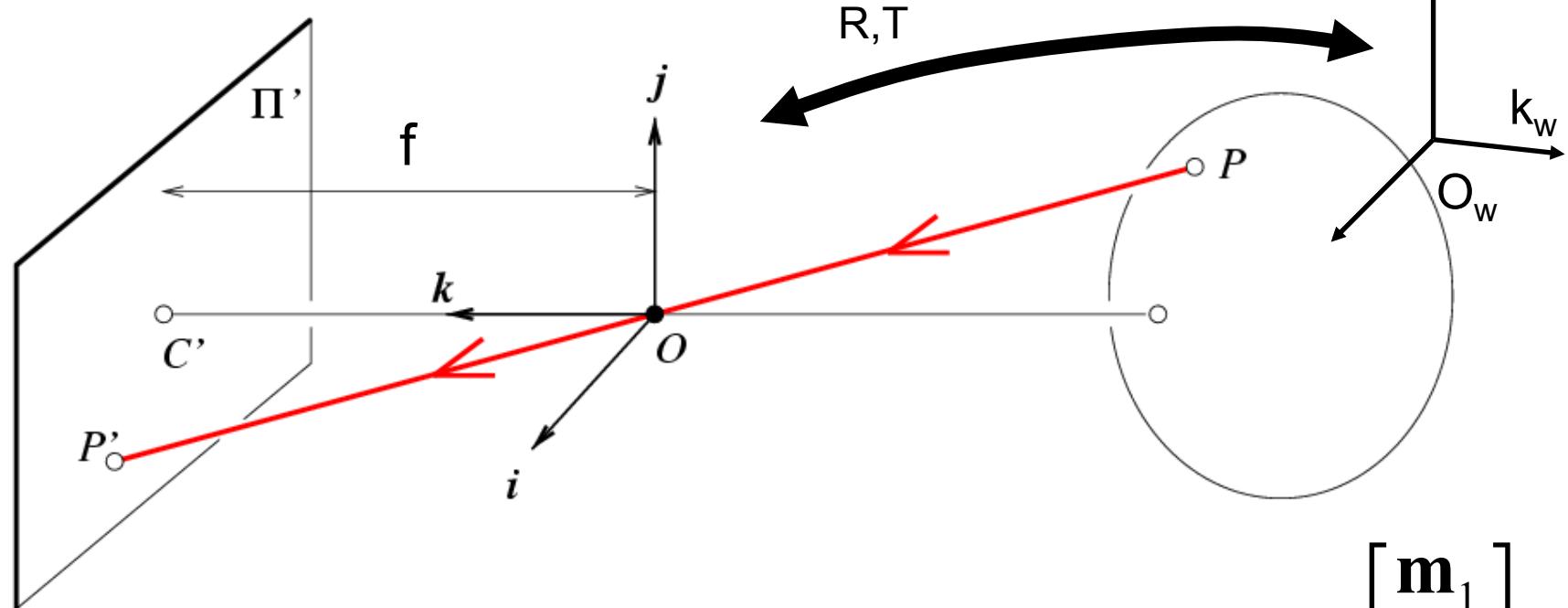
- Recap of camera models
- Camera calibration problem
- Camera calibration with radial distortion
- Example

Reading:

[FP] Chapter 1 “Geometric Camera Calibration”
[HZ] Chapter 7 “Computation of Camera Matrix P”

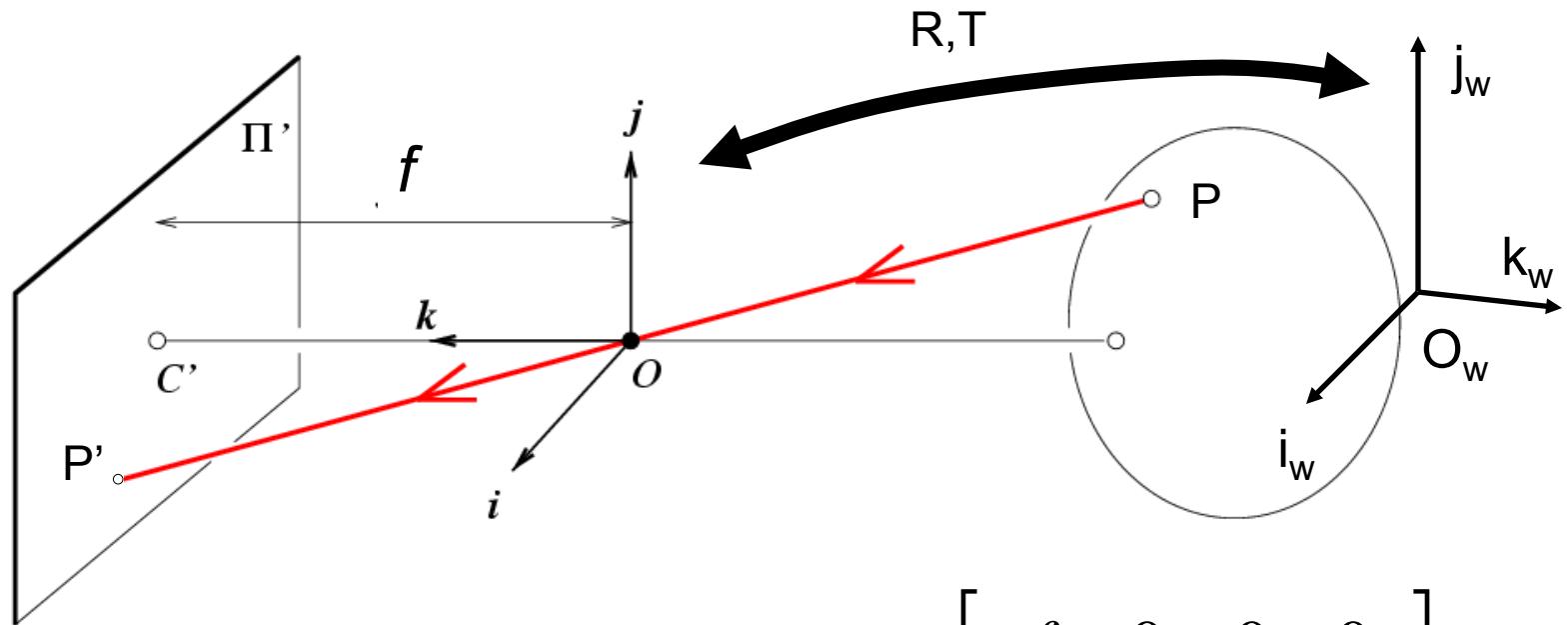
Some slides in this lecture are courtesy to Profs. J. Ponce, F-F Li

Projective camera



$$\begin{aligned}
 P'_{3 \times 1} &= M P_w = K_{3 \times 3} \begin{bmatrix} R & T \end{bmatrix}_{3 \times 4} P_w_{4 \times 1} & M = \begin{bmatrix} \mathbf{m}_1 \\ \mathbf{m}_2 \\ \mathbf{m}_3 \end{bmatrix} \\
 &= \begin{bmatrix} \mathbf{m}_1 \\ \mathbf{m}_2 \\ \mathbf{m}_3 \end{bmatrix} P_w = \begin{bmatrix} \mathbf{m}_1 P_w \\ \mathbf{m}_2 P_w \\ \mathbf{m}_3 P_w \end{bmatrix} & \xrightarrow{\text{E}} P'_E = \left(\frac{\mathbf{m}_1 P_w}{\mathbf{m}_3 P_w}, \frac{\mathbf{m}_2 P_w}{\mathbf{m}_3 P_w} \right)
 \end{aligned}$$

Exercise!

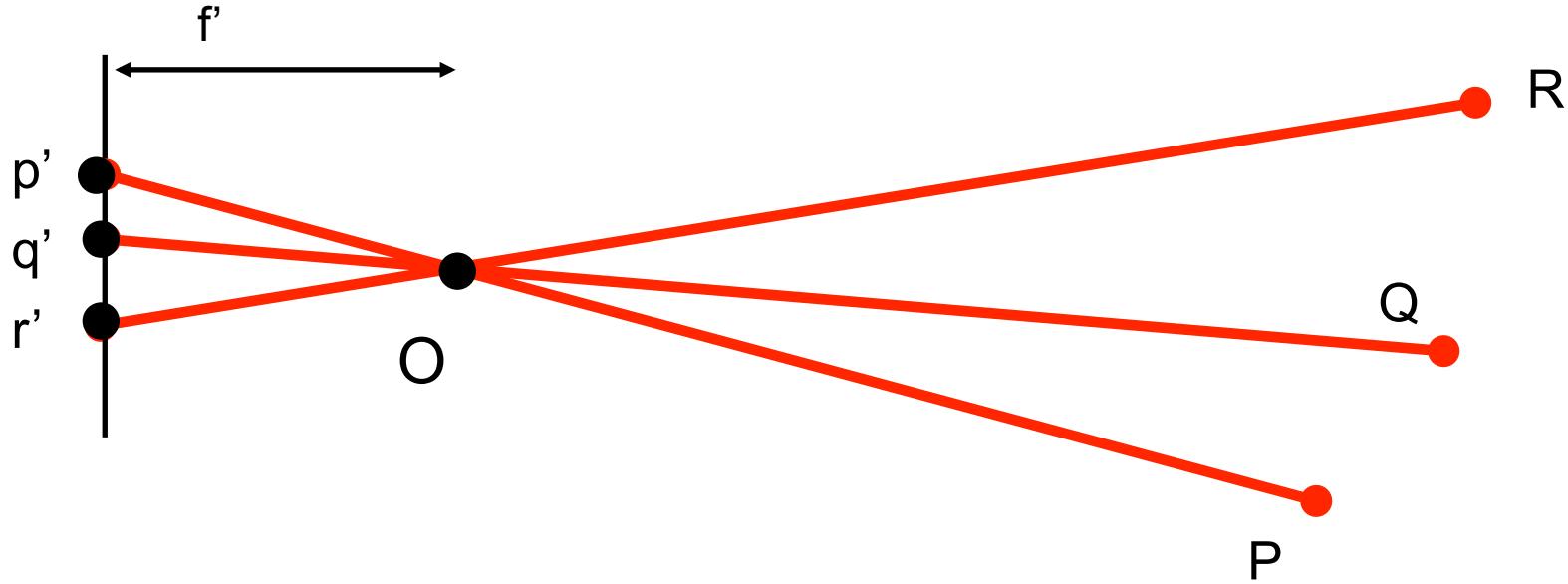


$$M = K \begin{bmatrix} R & T \end{bmatrix} = K \begin{bmatrix} I & 0 \end{bmatrix} = \begin{bmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$\rightarrow P'_E = \left(\frac{\mathbf{m}_1 P_w}{\mathbf{m}_3 P_w}, \frac{\mathbf{m}_2 P_w}{\mathbf{m}_3 P_w} \right) = \left(f \frac{x_w}{z_w}, f \frac{y_w}{z_w} \right)$$

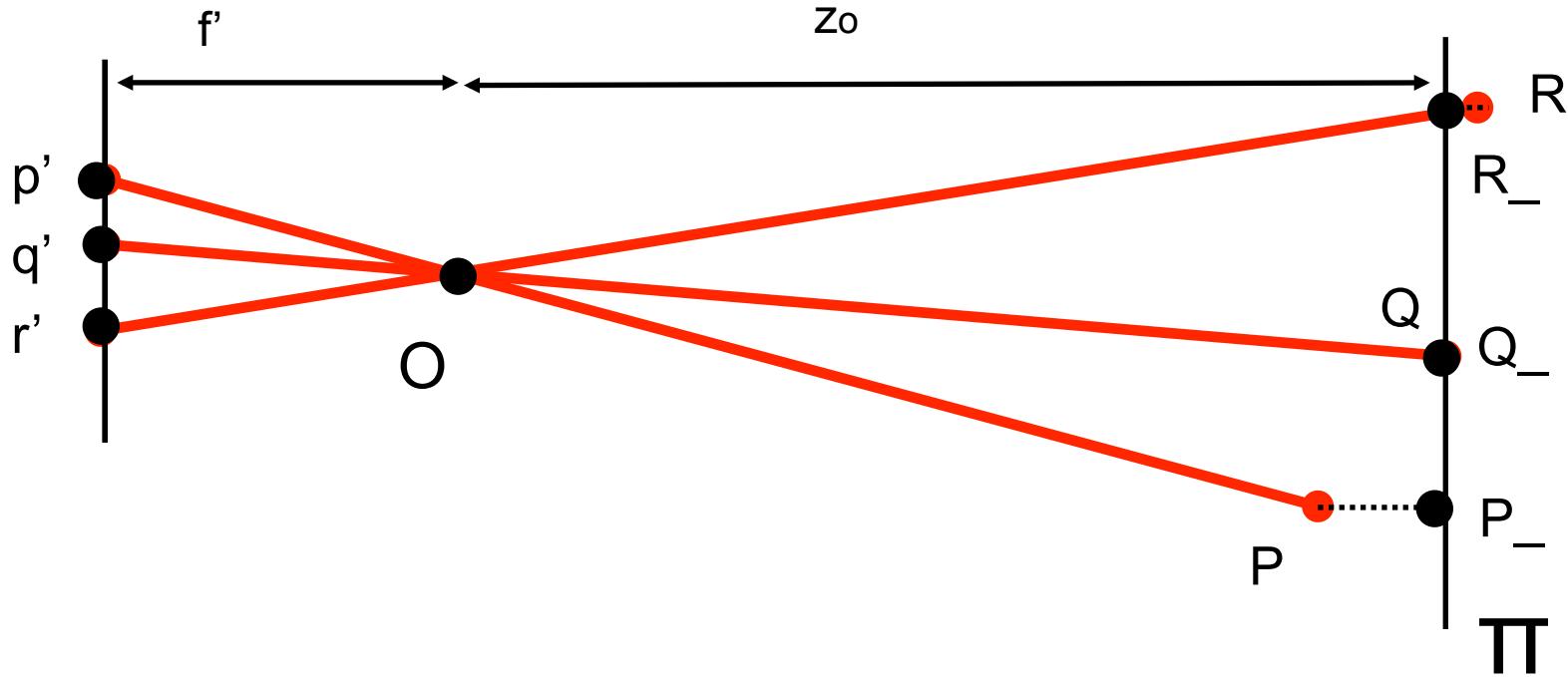
$$P_w = \begin{bmatrix} x_w \\ y_w \\ z_w \\ 1 \end{bmatrix}$$

Projective camera

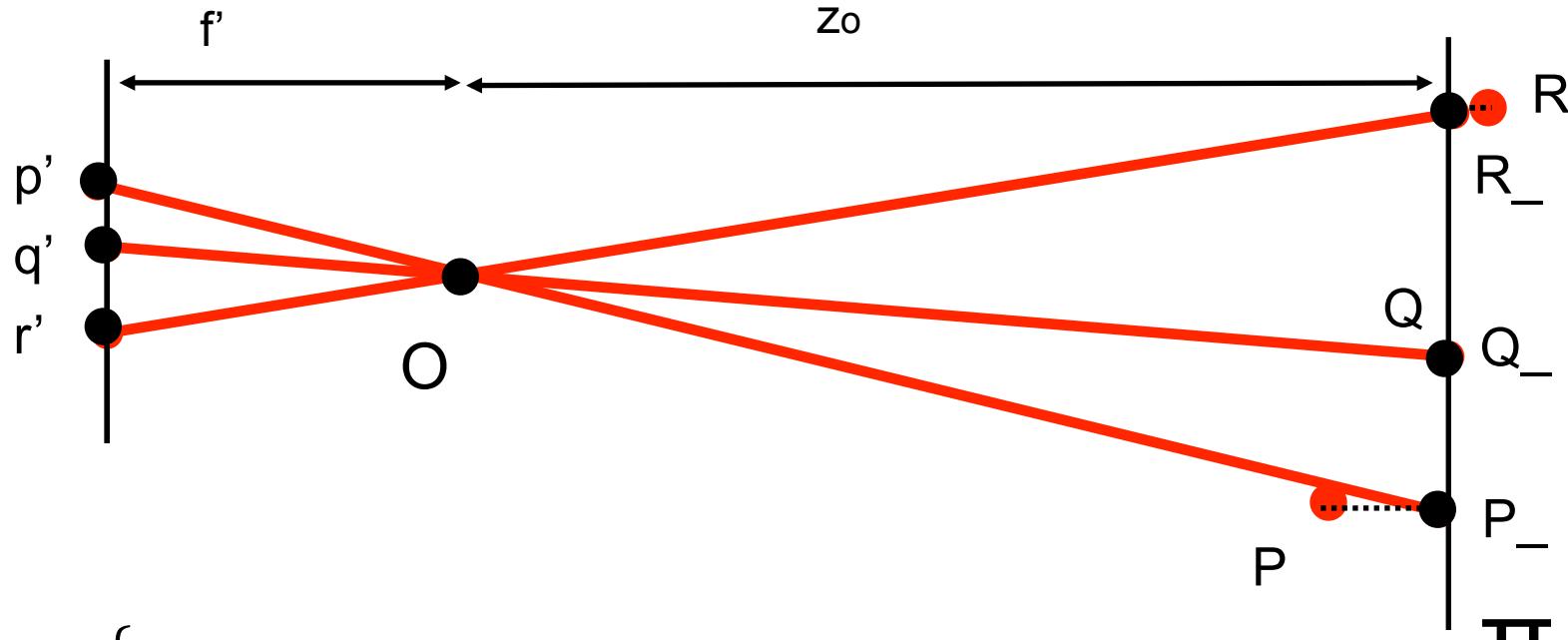


Weak perspective projection

When the relative scene depth is small compared to its distance from the camera



Weak perspective projection

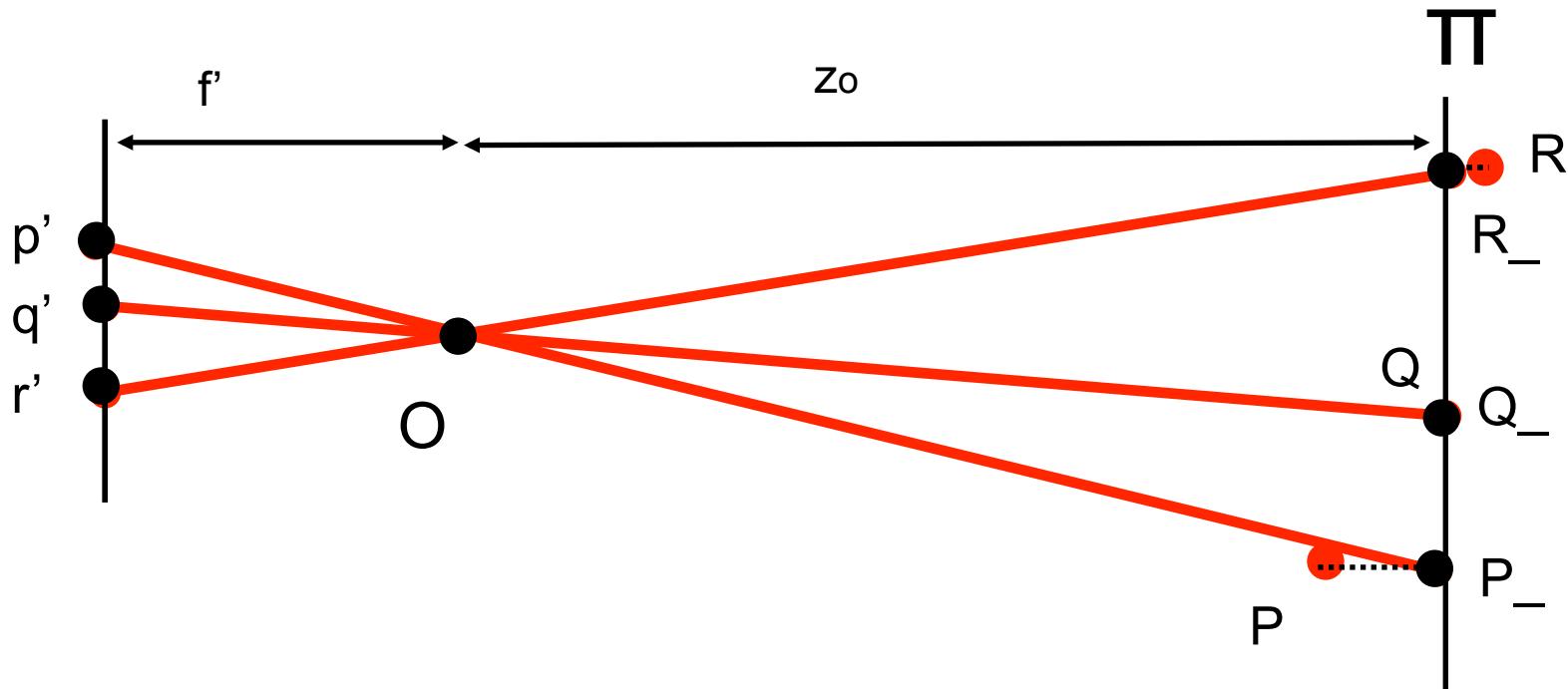


$$\left\{ \begin{array}{l} x' = \frac{f'}{z} x \\ y' = \frac{f'}{z} y \end{array} \right. \rightarrow \left\{ \begin{array}{l} x' = \frac{f'}{z_0} x \\ y' = \frac{f'}{z_0} y \end{array} \right.$$

Magnification m

The diagram shows a transformation from a general weak perspective projection equation to a specific one where the focal length f' is replaced by f'/z_0 . This replacement is highlighted with a red box around f'/z_0 . To the right of the equations, a wavy line indicates that the magnification factor $m = f'/z_0$ is constant across the image plane.

Weak perspective projection



Projective (perspective)

Weak perspective

$$M = K \begin{bmatrix} R & T \end{bmatrix} = \begin{bmatrix} A & b \\ v & 1 \end{bmatrix} \rightarrow M = \begin{bmatrix} A & b \\ 0 & 1 \end{bmatrix}$$

$$P' = M P_w = \begin{bmatrix} m_1 \\ m_2 \\ m_3 \end{bmatrix} P_w = \begin{bmatrix} m_1 P_w \\ m_2 P_w \\ m_3 P_w \end{bmatrix}$$

$$M = \begin{bmatrix} A & b \\ v & 1 \end{bmatrix} = \begin{bmatrix} m_1 \\ m_2 \\ m_3 \end{bmatrix}$$

E
 $\rightarrow \left(\frac{m_1 P_w}{m_3 P_w}, \frac{m_2 P_w}{m_3 P_w} \right)$

Perspective

$$P' = M P_w = \begin{bmatrix} m_1 \\ m_2 \\ m_3 \end{bmatrix} P_w = \begin{bmatrix} m_1 P_w \\ m_2 P_w \\ 1 \end{bmatrix}$$

E
 $\rightarrow (m_1 P_w, m_2 P_w)$

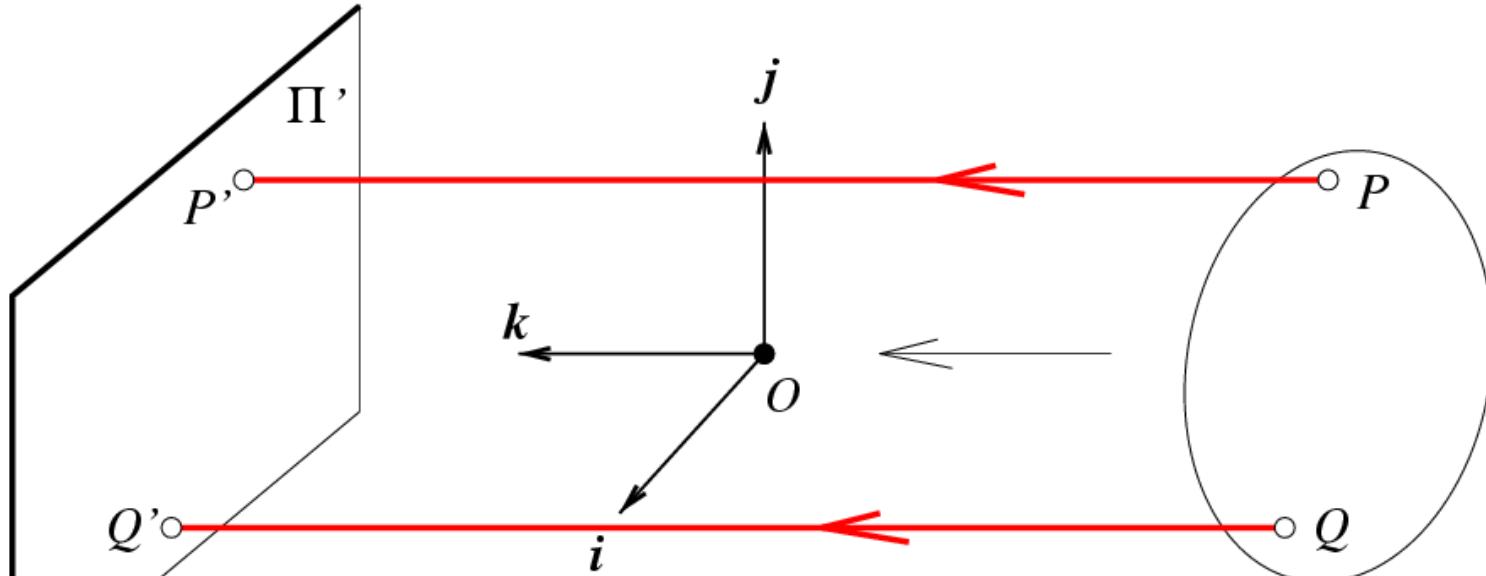
↑ ↑
magnification

$$M = \begin{bmatrix} A & b \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} m_1 & & m_1 \\ m_2 & & m_2 \\ m_3 & & 0 & 0 & 0 & 1 \end{bmatrix}$$

Weak perspective

Orthographic (affine) projection

Distance from center of projection to image plane is infinite



$$\left\{ \begin{array}{l} x' = \frac{f'}{z} x \\ y' = \frac{f'}{z} y \end{array} \right. \rightarrow \left\{ \begin{array}{l} x' = x \\ y' = y \end{array} \right.$$

Pros and Cons of These Models

- Weak perspective results in much simpler math.
 - Accurate when object is small and distant.
 - Most useful for recognition.
- Pinhole perspective is much more accurate for modeling the 3D-to-2D mapping.
 - Used in structure from motion or SLAM.

Weak perspective projection



The Kangxi Emperor's Southern Inspection Tour (1691-1698) By Wang Hui

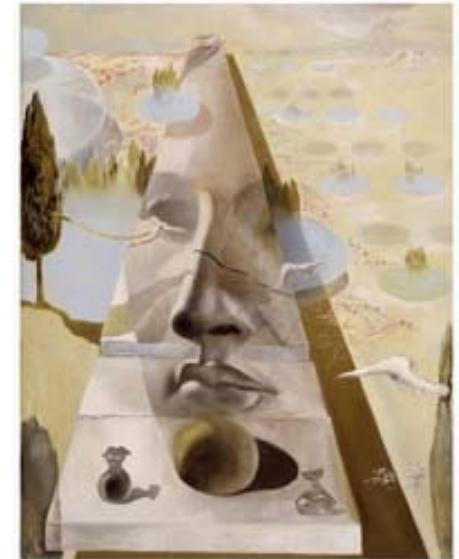
Weak perspective projection



The Kangxi Emperor's Southern Inspection Tour (1691-1698) By Wang Hui

Lecture 3

Camera Calibration



- Recap of camera models
- Camera calibration problem
- Camera calibration with radial distortion
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Reading:

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Projective camera

$$P' = M P_w = \boxed{K} \begin{bmatrix} R & T \end{bmatrix} P_w$$

Internal parameters External parameters

$$\mathcal{M} = \begin{pmatrix} \alpha \mathbf{r}_1^T - \alpha \cot \theta \mathbf{r}_2^T + u_0 \mathbf{r}_3^T & \alpha t_x - \alpha \cot \theta t_y + u_0 t_z \\ \frac{\beta}{\sin \theta} \mathbf{r}_2^T + v_0 \mathbf{r}_3^T & \frac{\beta}{\sin \theta} t_y + v_0 t_z \\ \mathbf{r}_3^T & t_z \end{pmatrix}_{3 \times 4}$$

$$K = \begin{bmatrix} \alpha & -\alpha \cot \theta & u_o \\ 0 & \frac{\beta}{\sin \theta} & v_o \\ 0 & 0 & 1 \end{bmatrix} \quad R = \begin{bmatrix} \mathbf{r}_1^T \\ \mathbf{r}_2^T \\ \mathbf{r}_3^T \end{bmatrix} \quad T = \begin{bmatrix} t_x \\ t_y \\ t_z \end{bmatrix}$$

Goal of calibration

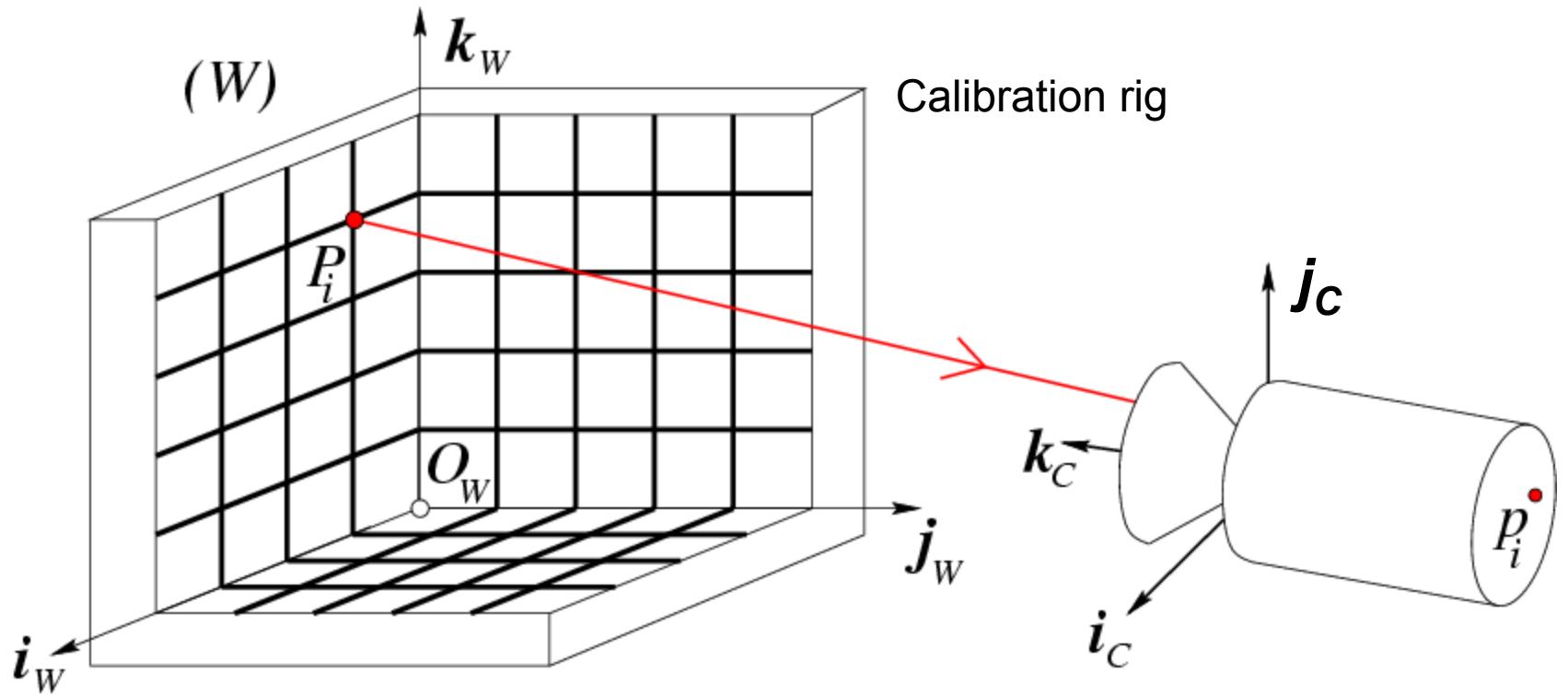
$$P' = M P_w = \begin{bmatrix} K & [R \quad T] \end{bmatrix} P_w$$

Internal parameters External parameters

Estimate intrinsic and extrinsic parameters from 1 or multiple images

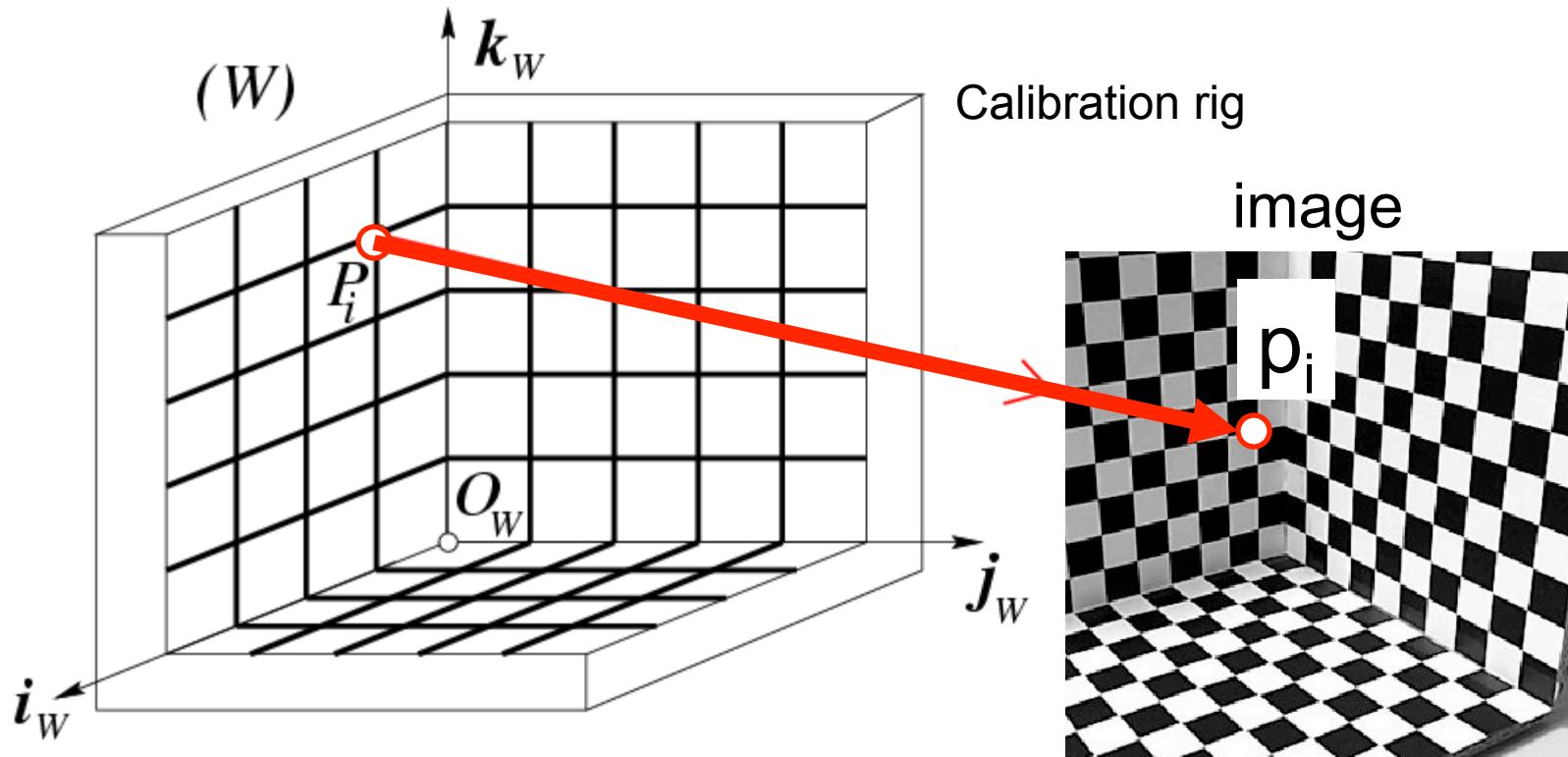
Change notation:
 $P = P_w$
 $p = P'$

Calibration Problem



- $P_1 \dots P_n$ with known positions in $[O_w, i_w, j_w, k_w]$

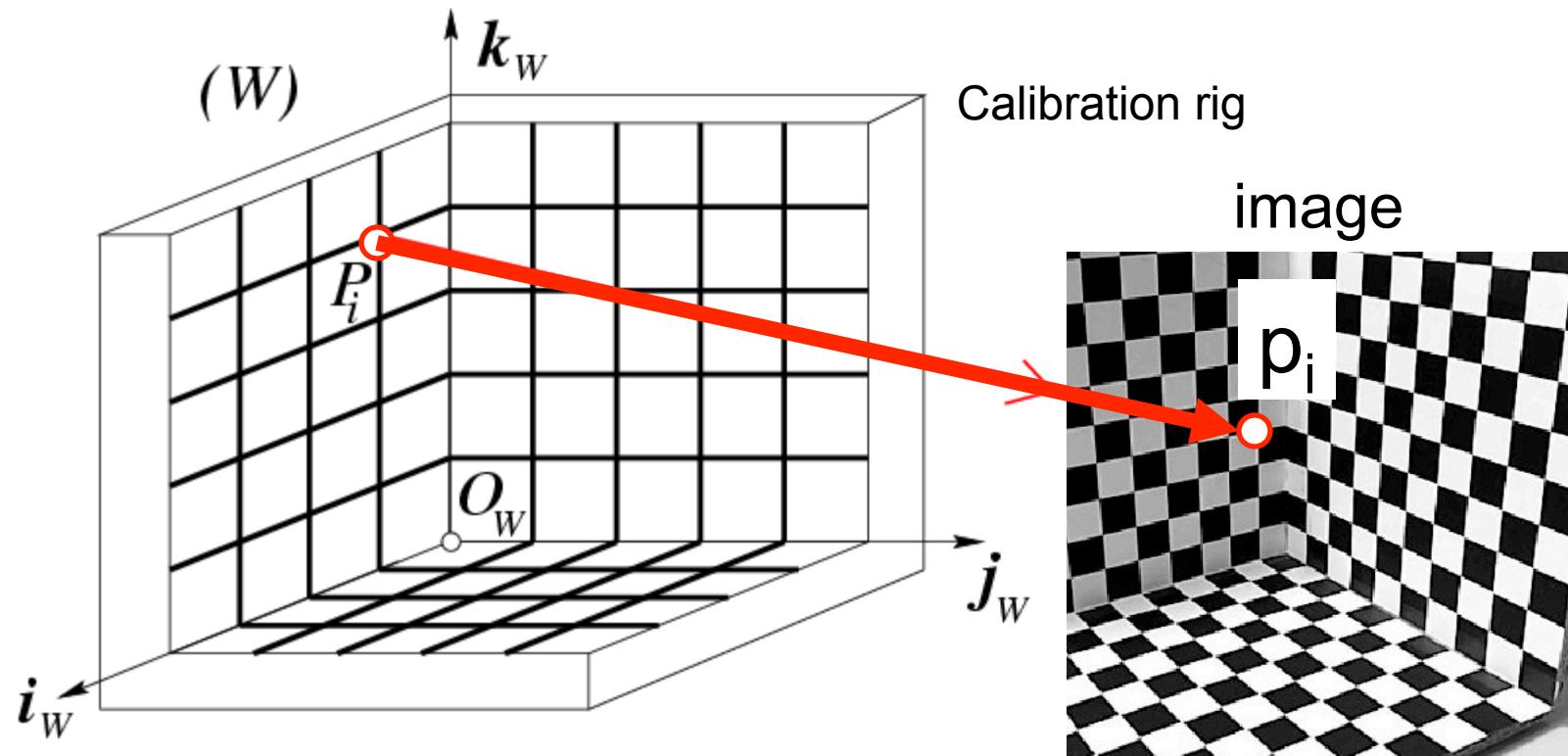
Calibration Problem



- $P_1 \dots P_n$ with **known** positions in $[O_w, i_w, j_w, k_w]$
- $p_1, \dots p_n$ **known** positions in the image

Goal: compute intrinsic and extrinsic parameters

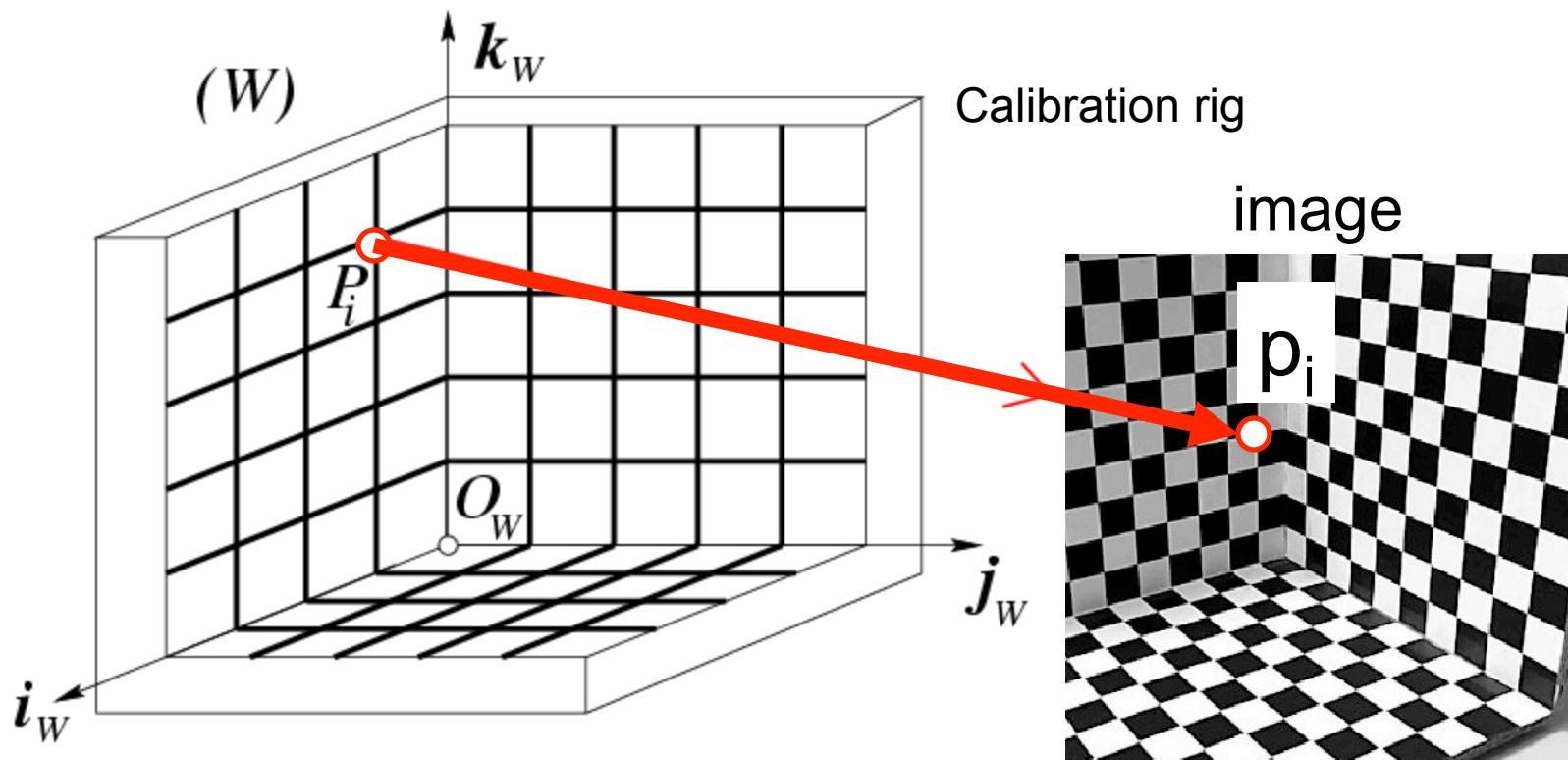
Calibration Problem



How many correspondences do we need?

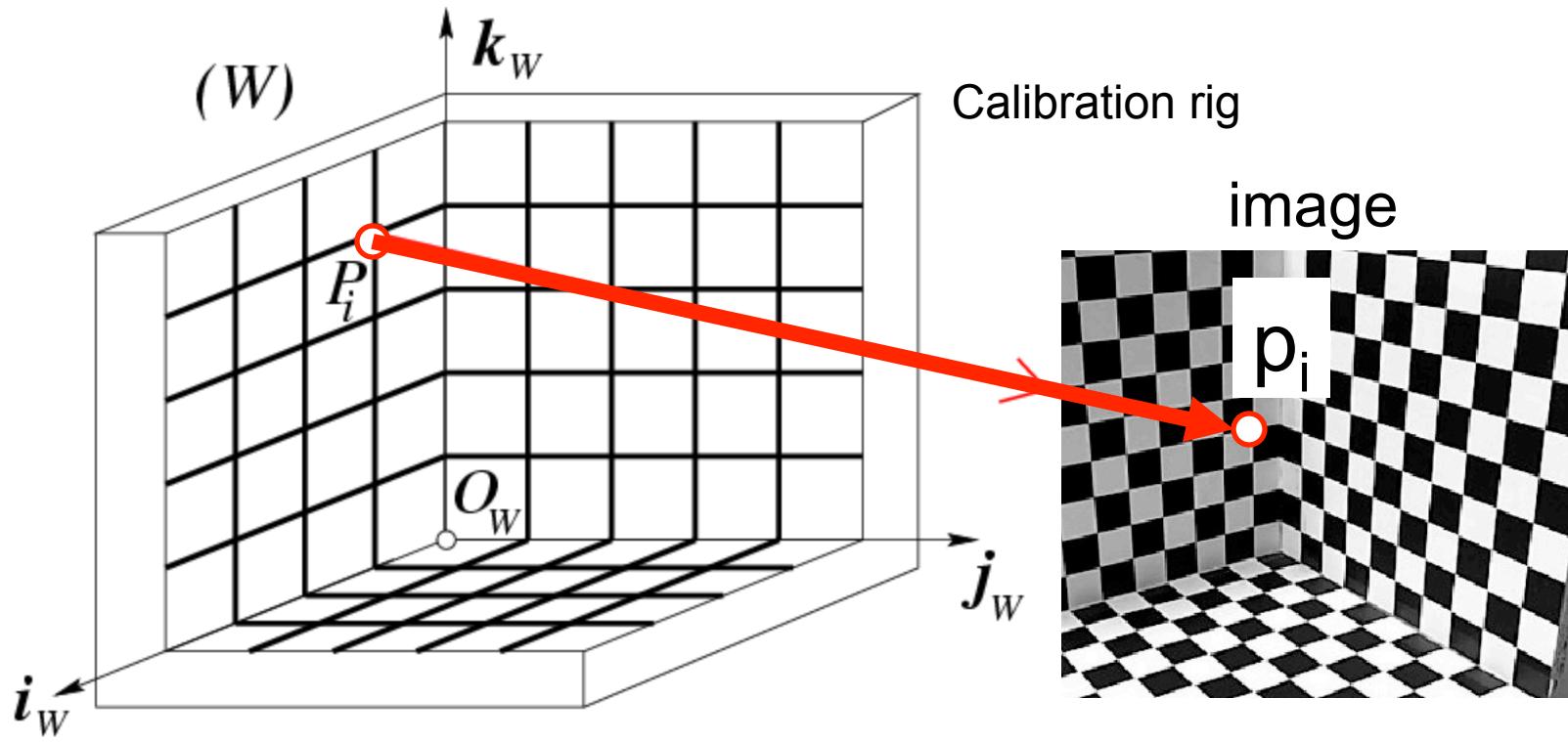
- M has 11 unknowns
- We need 11 equations
- 6 correspondences would do it

Calibration Problem



In practice, using more than 6 correspondences enables more robust results

Calibration Problem



$$p_i = \begin{bmatrix} u_i \\ v_i \end{bmatrix} = \begin{bmatrix} \frac{\mathbf{m}_1}{\mathbf{m}_3} P_i \\ \frac{\mathbf{m}_2}{\mathbf{m}_3} P_i \end{bmatrix} = M P_i$$

[Eq. 1]

$$\mathbf{M} = \begin{bmatrix} \mathbf{m}_1 \\ \mathbf{m}_2 \\ \mathbf{m}_3 \end{bmatrix}$$

in pixels

Calibration Problem

[Eq. 1]

$$\begin{bmatrix} u_i \\ v_i \end{bmatrix} = \begin{bmatrix} \frac{m_1 P_i}{m_3 P_i} \\ \frac{m_2 P_i}{m_3 P_i} \end{bmatrix}$$

$$u_i = \frac{m_1 P_i}{m_3 P_i} \rightarrow u_i(m_3 P_i) = m_1 P_i \rightarrow u_i(m_3 P_i) - m_1 P_i = 0$$

$$v_i = \frac{m_2 P_i}{m_3 P_i} \rightarrow v_i(m_3 P_i) = m_2 P_i \rightarrow v_i(m_3 P_i) - m_2 P_i = 0$$

[Eqs. 2]

Calibration Problem

$$\left\{ \begin{array}{l} u_1(\mathbf{m}_3 P_1) - \mathbf{m}_1 P_1 = 0 \\ v_1(\mathbf{m}_3 P_1) - \mathbf{m}_2 P_1 = 0 \\ \vdots \\ u_i(\mathbf{m}_3 P_i) - \mathbf{m}_1 P_i = 0 \quad [\text{Eqs. 3}] \\ v_i(\mathbf{m}_3 P_i) - \mathbf{m}_2 P_i = 0 \\ \vdots \\ u_n(\mathbf{m}_3 P_n) - \mathbf{m}_1 P_n = 0 \\ v_n(\mathbf{m}_3 P_n) - \mathbf{m}_2 P_n = 0 \end{array} \right.$$

Block Matrix Multiplication

$$A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \quad B = \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix}$$

What is AB ?

$$AB = \begin{bmatrix} A_{11}B_{11} + A_{12}B_{21} & A_{11}B_{12} + A_{12}B_{22} \\ A_{21}B_{11} + A_{22}B_{21} & A_{21}B_{12} + A_{22}B_{22} \end{bmatrix}$$

Calibration Problem

$$\left\{ \begin{array}{l} -u_1(\mathbf{m}_3 P_1) + \mathbf{m}_1 P_1 = 0 \\ -v_1(\mathbf{m}_3 P_1) + \mathbf{m}_2 P_1 = 0 \\ \vdots \\ -u_n(\mathbf{m}_3 P_n) + \mathbf{m}_1 P_n = 0 \\ -v_n(\mathbf{m}_3 P_n) + \mathbf{m}_2 P_n = 0 \end{array} \right.$$

→

$\boxed{\mathbf{P} \mathbf{m} = 0}$

[Eq. 4]

Homogenous linear system

$$\mathbf{P} \stackrel{\text{def}}{=} \begin{pmatrix} \mathbf{P}_1^T & \mathbf{0}^T & -u_1 \mathbf{P}_1^T \\ \mathbf{0}^T & \mathbf{P}_1^T & -v_1 \mathbf{P}_1^T \\ \vdots & & \\ \mathbf{P}_n^T & \mathbf{0}^T & -u_n \mathbf{P}_n^T \\ \mathbf{0}^T & \mathbf{P}_n^T & -v_n \mathbf{P}_n^T \end{pmatrix}_{2n \times 12}^{1 \times 4}$$

$$\mathbf{m} \stackrel{\text{def}}{=} \begin{pmatrix} \mathbf{m}_1^T \\ \mathbf{m}_2^T \\ \mathbf{m}_3^T \end{pmatrix}_{12 \times 1}^{4 \times 1}$$

Homogeneous $M \times N$ Linear Systems

$M = \text{number of equations} = 2n$

$N = \text{number of unknown} = 11$

The diagram illustrates a homogeneous linear system. On the left, there is a large rectangle labeled P with a red border. Above it, the letter N is written. To the left of P , the letter M is written. In the center, there is a smaller rectangle labeled m with a red border. To the right of the equals sign (=), there is another small rectangle labeled 0 with a red border. Dashed horizontal lines are drawn inside the rectangles P and 0 .

Rectangular system ($M > N$)

- 0 is always a solution
- To find non-zero solution

Minimize $|P m|^2$

under the constraint $|m|^2 = 1$

Calibration Problem

$$\mathbf{P} \mathbf{m} = 0$$

- How do we solve this homogenous linear system?
- Via SVD decomposition!

Calibration Problem

$$\boxed{P} \mathbf{m} = 0$$

SVD decomposition of P

$$\boxed{U_{2n \times 12} \ D_{12 \times 12} V^T}_{12 \times 12}$$

Last column of V gives \mathbf{m}

Why? See pag 592 of HZ

$$\mathbf{m} \stackrel{\text{def}}{=} \begin{pmatrix} \mathbf{m}_1^T \\ \mathbf{m}_2^T \\ \mathbf{m}_3^T \end{pmatrix} \quad \downarrow \quad M$$

Extracting camera parameters

$$M = \begin{pmatrix} \alpha \mathbf{r}_1^T - \alpha \cot \theta \mathbf{r}_2^T + u_0 \mathbf{r}_3^T & \alpha t_x - \alpha \cot \theta t_y + u_0 t_z \\ \frac{\beta}{\sin \theta} \mathbf{r}_2^T + v_0 \mathbf{r}_3^T & \frac{\beta}{\sin \theta} t_y + v_0 t_z \\ \mathbf{r}_3^T & t_z \end{pmatrix} \rho$$

Extracting camera parameters

See [FP],
Sec. 1.3.1

$$\frac{M}{\rho} = \begin{pmatrix} \alpha \mathbf{r}_1^T - \alpha \cot \theta \mathbf{r}_2^T + u_0 \mathbf{r}_3^T \\ \frac{\beta}{\sin \theta} \mathbf{r}_2^T + v_0 \mathbf{r}_3^T \\ \mathbf{r}_3^T \end{pmatrix} = \frac{K}{\rho} \begin{bmatrix} R & T \end{bmatrix}$$

$$K = \begin{bmatrix} \alpha & -\alpha \cot \theta & u_o \\ 0 & \frac{\beta}{\sin \theta} & v_o \\ 0 & 0 & 1 \end{bmatrix}$$

Box 1

$$A = \begin{bmatrix} \mathbf{a}_1^T \\ \mathbf{a}_2^T \\ \mathbf{a}_3^T \end{bmatrix} \quad b = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

Estimated values

Intrinsic

$$\rho = \frac{\pm 1}{|\mathbf{a}_3|} \quad u_o = \rho^2 (\mathbf{a}_1 \cdot \mathbf{a}_3)$$

$$v_o = \rho^2 (\mathbf{a}_2 \cdot \mathbf{a}_3)$$

$$\cos \theta = \frac{(\mathbf{a}_1 \times \mathbf{a}_3) \cdot (\mathbf{a}_2 \times \mathbf{a}_3)}{|\mathbf{a}_1 \times \mathbf{a}_3| \cdot |\mathbf{a}_2 \times \mathbf{a}_3|}$$

Theorem (Faugeras, 1993)

Let $\mathcal{M} = (\mathcal{A} \quad \mathbf{b})$ be a 3×4 matrix and let \mathbf{a}_i^T ($i = 1, 2, 3$) denote the rows of the matrix \mathcal{A} formed by the three leftmost columns of \mathcal{M} .

- A necessary and sufficient condition for \mathcal{M} to be a perspective projection matrix is that $\text{Det}(\mathcal{A}) \neq 0$.
- A necessary and sufficient condition for \mathcal{M} to be a zero-skew perspective projection matrix is that $\text{Det}(\mathcal{A}) \neq 0$ and

$$(\mathbf{a}_1 \times \mathbf{a}_3) \cdot (\mathbf{a}_2 \times \mathbf{a}_3) = 0.$$

- A necessary and sufficient condition for \mathcal{M} to be a perspective projection matrix with zero skew and unit aspect-ratio is that $\text{Det}(\mathcal{A}) \neq 0$ and

$$\begin{cases} (\mathbf{a}_1 \times \mathbf{a}_3) \cdot (\mathbf{a}_2 \times \mathbf{a}_3) = 0, \\ (\mathbf{a}_1 \times \mathbf{a}_3) \cdot (\mathbf{a}_1 \times \mathbf{a}_3) = (\mathbf{a}_2 \times \mathbf{a}_3) \cdot (\mathbf{a}_2 \times \mathbf{a}_3). \end{cases}$$

Extracting camera parameters

$$\frac{\mathcal{M}}{\rho} = \begin{pmatrix} \alpha \mathbf{r}_1^T - \alpha \cot \theta \mathbf{r}_2^T + u_0 \mathbf{r}_3^T & \alpha t_x - \alpha \cot \theta t_y + u_0 t_z \\ \frac{\beta}{\sin \theta} \mathbf{r}_2^T + v_0 \mathbf{r}_3^T & \frac{\beta}{\sin \theta} t_y + v_0 t_z \\ \mathbf{r}_3^T & t_z \end{pmatrix} = \mathbf{K} [\mathbf{R} \quad \mathbf{T}]$$

A **b**

$$\mathbf{A} = \begin{bmatrix} \mathbf{a}_1^T \\ \mathbf{a}_2^T \\ \mathbf{a}_3^T \end{bmatrix} \quad \mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

Estimated values

Intrinsic

$$\alpha = \rho^2 |\mathbf{a}_1 \times \mathbf{a}_3| \sin \theta$$

$$\beta = \rho^2 |\mathbf{a}_2 \times \mathbf{a}_3| \sin \theta$$

Extracting camera parameters

$$\frac{\mathcal{M}}{\rho} = \begin{pmatrix} \alpha \mathbf{r}_1^T - \alpha \cot \theta \mathbf{r}_2^T + u_0 \mathbf{r}_3^T & \alpha t_x - \alpha \cot \theta t_y + u_0 t_z \\ \frac{\beta}{\sin \theta} \mathbf{r}_2^T + v_0 \mathbf{r}_3^T & \frac{\beta}{\sin \theta} t_y + v_0 t_z \\ \mathbf{r}_3^T & t_z \end{pmatrix} = \mathbf{K} [\mathbf{R} \quad \mathbf{T}]$$

A **b**

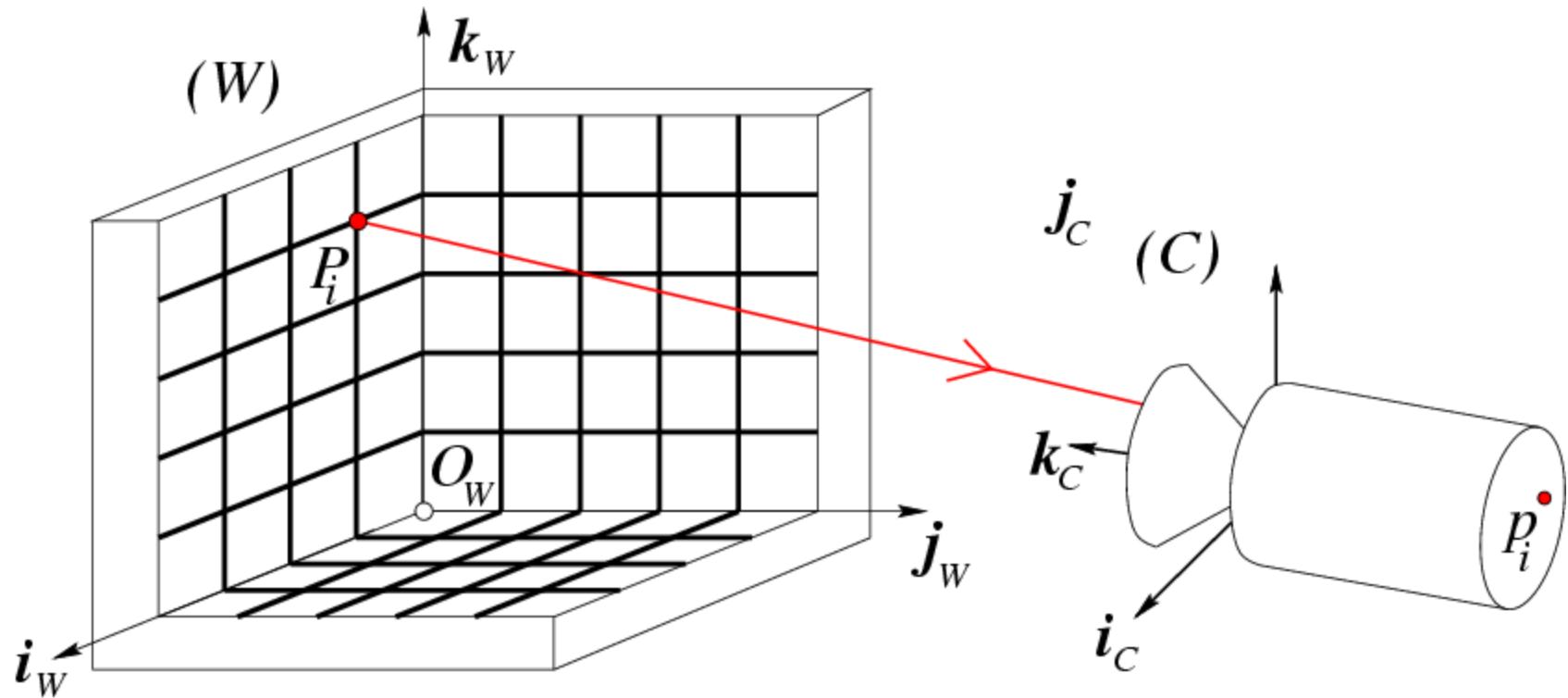
$$\mathbf{A} = \begin{bmatrix} \mathbf{a}_1^T \\ \mathbf{a}_2^T \\ \mathbf{a}_3^T \end{bmatrix} \quad \mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

Estimated values

Extrinsic

$$\mathbf{r}_1 = \frac{(\mathbf{a}_2 \times \mathbf{a}_3)}{|\mathbf{a}_2 \times \mathbf{a}_3|} \quad \mathbf{r}_3 = \frac{\pm \mathbf{a}_3}{|\mathbf{a}_3|}$$
$$\mathbf{r}_2 = \mathbf{r}_3 \times \mathbf{r}_1 \quad \mathbf{T} = \rho \mathbf{K}^{-1} \mathbf{b}$$

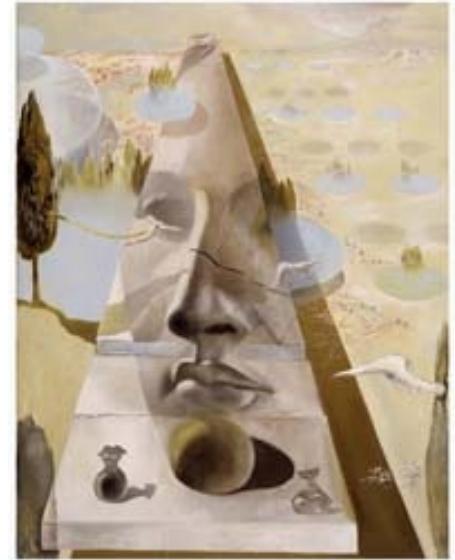
Degenerate cases



- P_i 's cannot lie on the same plane!
- Points cannot lie on the intersection curve of two quadric surfaces

Lecture 3

Camera Calibration



- Recap of projective cameras
- Camera calibration problem
- Camera calibration with radial distortion
- Example

Reading:

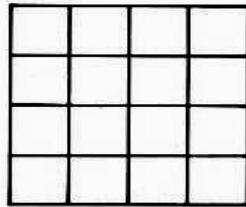
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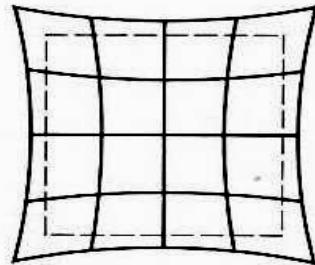
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Radial Distortion

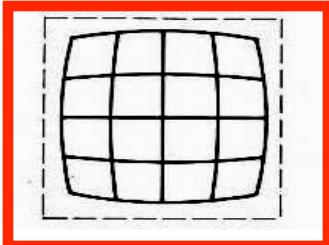
- Image magnification (in)decreases with distance from the optical axis
- Caused by imperfect lenses
- Deviations are most noticeable for rays that pass through the edge of the lens



No distortion



Pin cushion

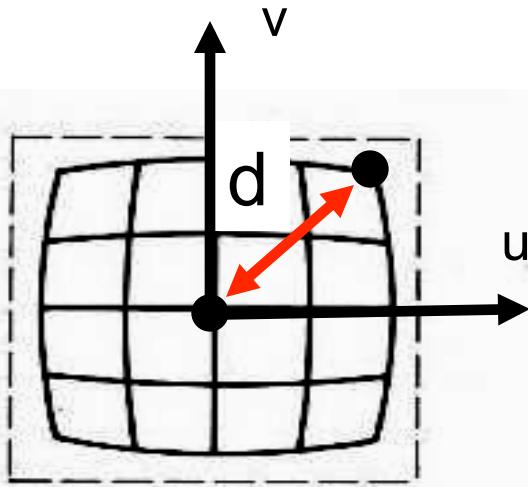


Barrel



Radial Distortion

Image magnification decreases with distance from the optical center



$$S_\lambda \begin{bmatrix} \frac{1}{\lambda} & 0 & 0 \\ 0 & \frac{1}{\lambda} & 0 \\ 0 & 0 & 1 \end{bmatrix} M P_i \rightarrow \begin{bmatrix} u_i \\ v_i \end{bmatrix} = p_i$$

Distortion coefficient

$$\lambda = 1 \pm \sum_{p=1}^3 \kappa_p d^{2p}$$

[Eq. 5] Polynomial function

$$d^2 = a u^2 + b v^2 + c u v$$

To model radial behavior

[Eq. 6]

Radial Distortion

$$\boxed{\begin{bmatrix} \frac{1}{\lambda} & 0 & 0 \\ 0 & \frac{1}{\lambda} & 0 \\ 0 & 0 & 1 \end{bmatrix} M P_i \rightarrow \begin{bmatrix} u_i \\ v_i \end{bmatrix} = p_i} \quad Q = \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix}$$

Q

$$p_i = \begin{bmatrix} u_i \\ v_i \end{bmatrix} = \begin{bmatrix} \frac{q_1 P_i}{q_3 P_i} \\ \frac{q_2 P_i}{q_3 P_i} \end{bmatrix}$$

Is this a linear system of equations?

$$\left\{ \begin{array}{l} u_i q_3 P_i = q_1 P_i \\ v_i q_3 P_i = q_2 P_i \end{array} \right.$$

No! why?

[Eqs.7]

General Calibration Problem

$$\begin{bmatrix} u_i \\ v_i \end{bmatrix} = \begin{bmatrix} \frac{\mathbf{q}_1}{\mathbf{q}_3} P_i \\ \frac{\mathbf{q}_2}{\mathbf{q}_3} P_i \\ \frac{\mathbf{q}_3}{\mathbf{q}_3} P_i \end{bmatrix} \xrightarrow{\text{red arrow}} X = f(Q) \quad [\text{Eq .8}]$$

i=1...n measurements parameters

$f()$ is the nonlinear mapping

-Newton Method

-Levenberg-Marquardt Algorithm

- Iterative, starts from initial solution
- May be slow if initial solution far from real solution
- Estimated solution may be function of the initial solution (because of local minima)
- Newton requires the computation of J, H
- Levenberg-Marquardt doesn't require the computation of H

General Calibration Problem

$$\begin{bmatrix} u_i \\ v_i \end{bmatrix} = \begin{bmatrix} \frac{\mathbf{q}_1}{\mathbf{q}_3} P_i \\ \frac{\mathbf{q}_2}{\mathbf{q}_3} P_i \end{bmatrix} \xrightarrow{\text{red arrow}} X = f(Q) \quad [\text{Eq .8}]$$

i=1...n measurements parameters

$f()$ is the nonlinear mapping

A possible algorithm

1. Solve linear part of the system to find approximated solution
2. Use this solution as initial condition for the full system
3. Solve full system using Newton or L.M.

General Calibration Problem

$$\begin{bmatrix} u_i \\ v_i \end{bmatrix} = \begin{bmatrix} \frac{\mathbf{q}_1}{\mathbf{q}_3} P_i \\ \frac{\mathbf{q}_2}{\mathbf{q}_3} P_i \end{bmatrix} \xrightarrow{\text{red arrow}} X = f(Q) \quad [\text{Eq .8}]$$

↑
measurements ↓
parameters

$f()$ is the nonlinear mapping

Typical assumptions:

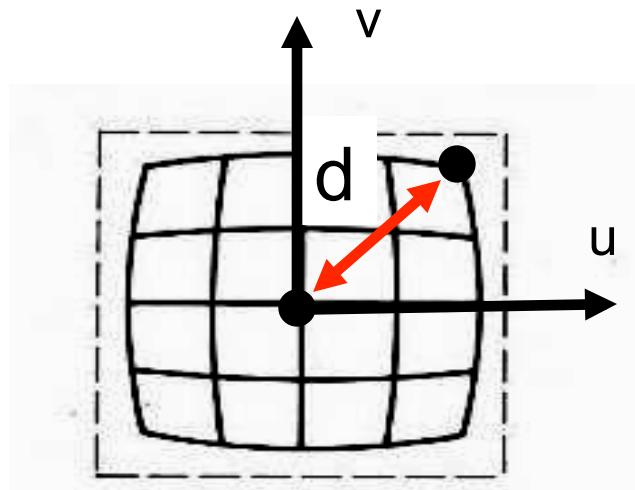
- zero-skew, square pixel
- u_o, v_o = known center of the image

Radial Distortion

$$p_i = \begin{bmatrix} u_i \\ v_i \end{bmatrix} = \begin{bmatrix} \frac{q_1 P_i}{q_3 P_i} \\ \frac{q_2 P_i}{q_3 P_i} \end{bmatrix} = \frac{1}{\lambda} \begin{bmatrix} \frac{m_1 P_i}{m_3 P_i} \\ \frac{m_2 P_i}{m_3 P_i} \end{bmatrix}$$

Can we estimate m_1 and m_2 and ignore the radial distortion?

Hint:



$$\frac{u_i}{v_i} = \text{slope}$$

Radial Distortion

Tsai [87]

Estimating \mathbf{m}_1 and \mathbf{m}_2 ...

$$\mathbf{p}_i = \begin{bmatrix} u_i \\ v_i \end{bmatrix} = \frac{1}{\lambda} \begin{bmatrix} \frac{\mathbf{m}_1 P_i}{\mathbf{m}_3 P_i} \\ \frac{\mathbf{m}_3 P_i}{\mathbf{m}_2 P_i} \\ \frac{\mathbf{m}_2 P_i}{\mathbf{m}_3 P_i} \\ \frac{\mathbf{m}_3 P_i}{\mathbf{m}_1 P_i} \end{bmatrix} \quad \xrightarrow{\text{Eq .9}} \quad \frac{u_i}{v_i} = \frac{\frac{(\mathbf{m}_1 P_i)}{(\mathbf{m}_3 P_i)}}{\frac{(\mathbf{m}_2 P_i)}{(\mathbf{m}_3 P_i)}} = \frac{\mathbf{m}_1 P_i}{\mathbf{m}_2 P_i}$$

[Eq .10]

$$\begin{cases} v_1(\mathbf{m}_1 P_1) - u_1(\mathbf{m}_2 P_1) = 0 \\ v_i(\mathbf{m}_1 P_i) - u_i(\mathbf{m}_2 P_i) = 0 \\ \vdots \\ v_n(\mathbf{m}_1 P_n) - u_n(\mathbf{m}_2 P_n) = 0 \end{cases}$$

[Eq .11]

$$L \mathbf{n} = 0$$



Get \mathbf{m}_1 and
 \mathbf{m}_2 by SVD

$$\mathbf{L} \stackrel{\text{def}}{=} \begin{pmatrix} v_1 \mathbf{P}_1^T & -u_1 \mathbf{P}_1^T \\ v_2 \mathbf{P}_2^T & -u_2 \mathbf{P}_2^T \\ \vdots & \vdots \\ v_n \mathbf{P}_n^T & -u_n \mathbf{P}_n^T \end{pmatrix}$$
$$\mathbf{n} = \begin{bmatrix} \mathbf{m}_1^T \\ \mathbf{m}_2^T \end{bmatrix}$$

Radial Distortion

Once that \mathbf{m}_1 and \mathbf{m}_2 are estimated...

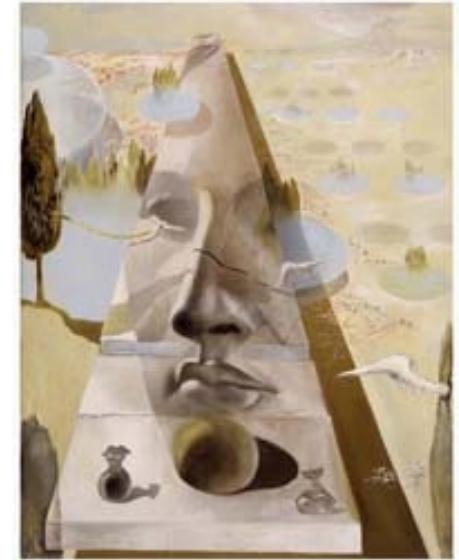
$$\mathbf{p}_i = \begin{bmatrix} u_i \\ v_i \end{bmatrix} = \frac{1}{\lambda} \begin{bmatrix} \frac{\mathbf{m}_1 P_i}{\mathbf{m}_3 P_i} \\ \frac{\mathbf{m}_2 P_i}{\mathbf{m}_3 P_i} \end{bmatrix}$$

\mathbf{m}_3 is non linear function of \mathbf{m}_1 , \mathbf{m}_2 , λ

There are some degenerate configurations for which \mathbf{m}_1 and \mathbf{m}_2 cannot be computed

Lecture 3

Camera Calibration



- Recap of projective cameras
- Camera calibration problem
- Camera calibration with radial distortion
- Example

Reading:

[FP] Chapter 1 “Geometric Camera Calibration”

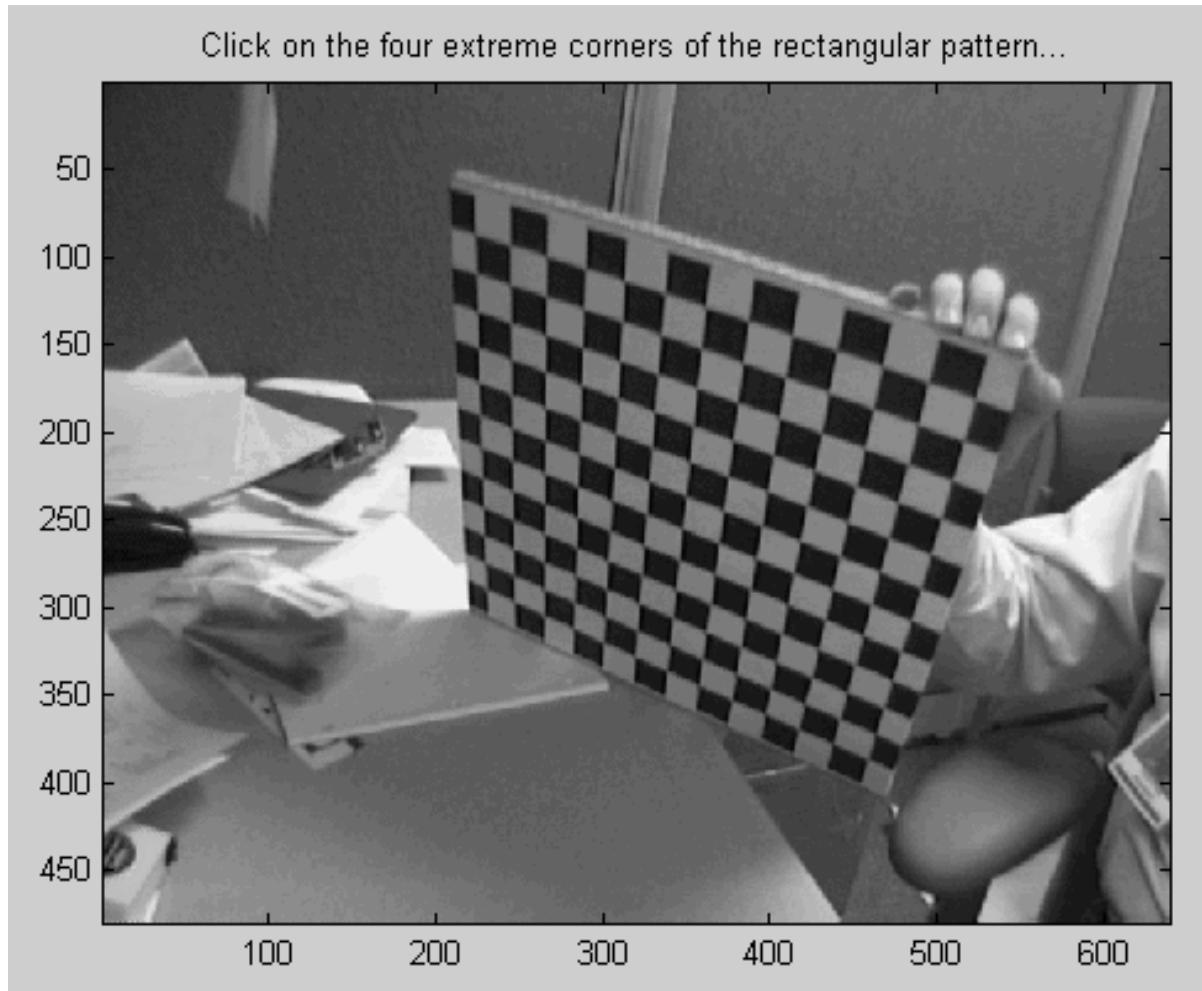
[HZ] Chapter 7 “Computation of Camera Matrix P”

Some slides in this lecture are courtesy to Profs. J. Ponce, F-F Li

Calibration Procedure

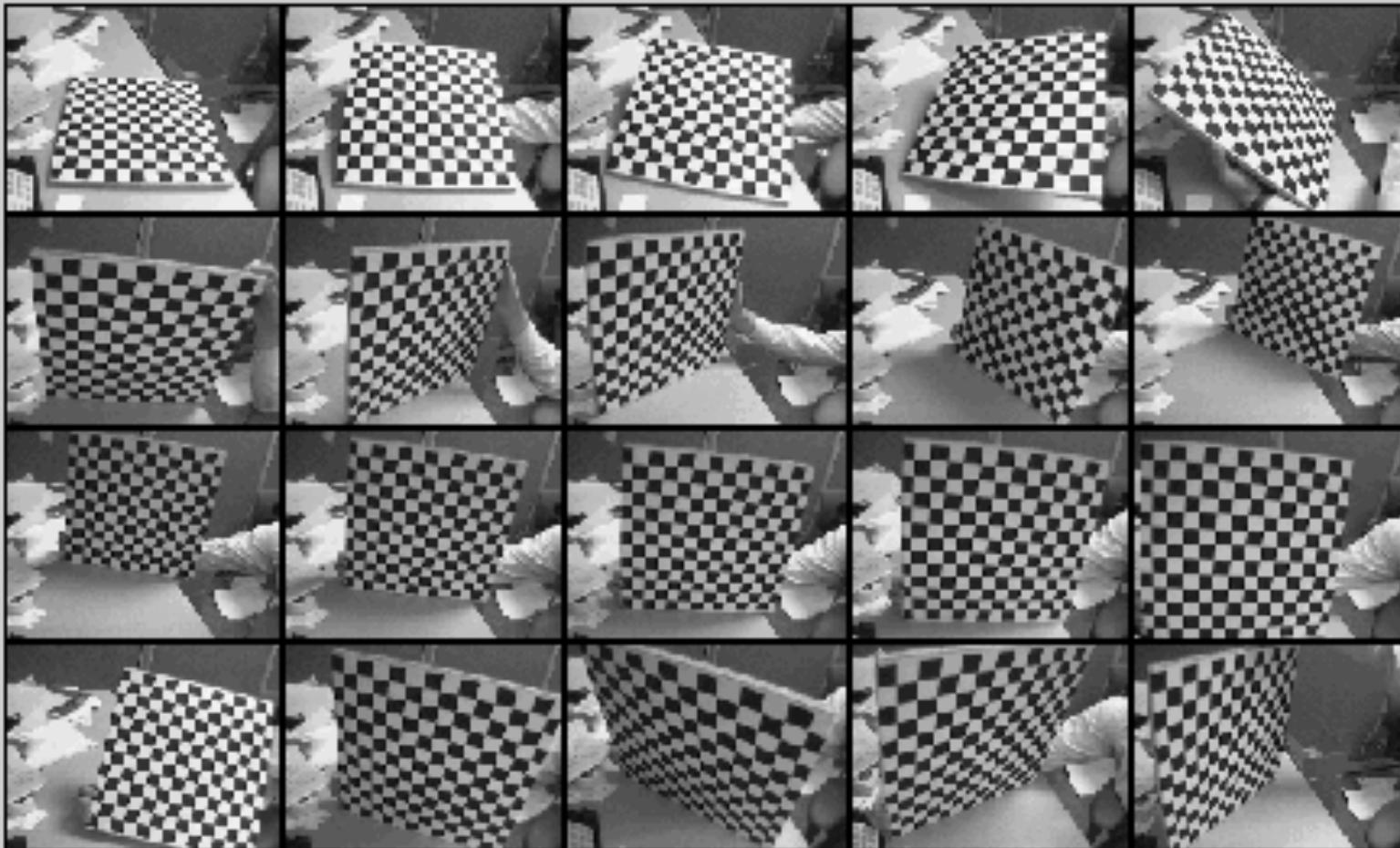
Camera Calibration Toolbox for Matlab
J. Bouguet – [1998-2000]

http://www.vision.caltech.edu/bouguetj/calib_doc/index.html#examples



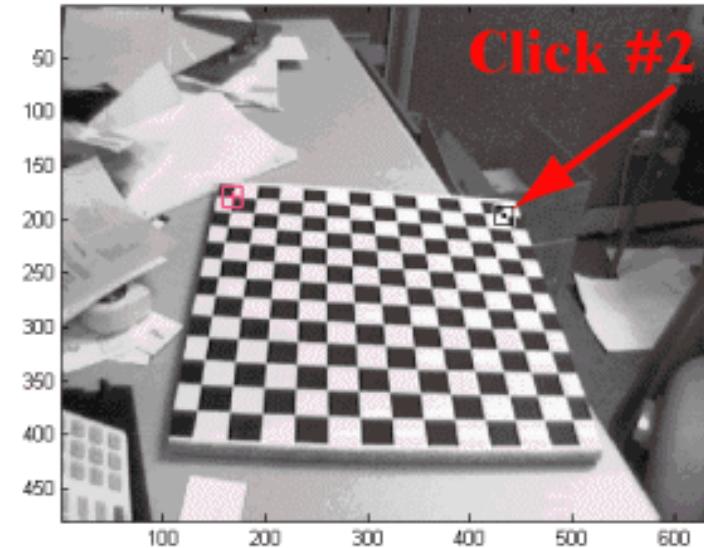
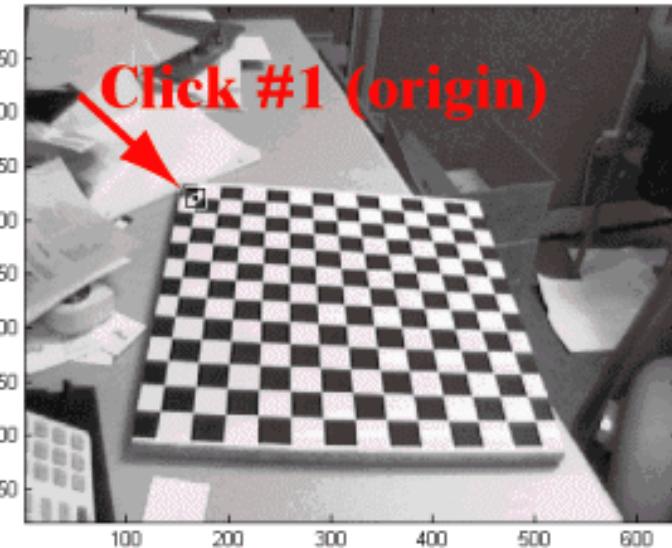
Calibration Procedure

Calibration images

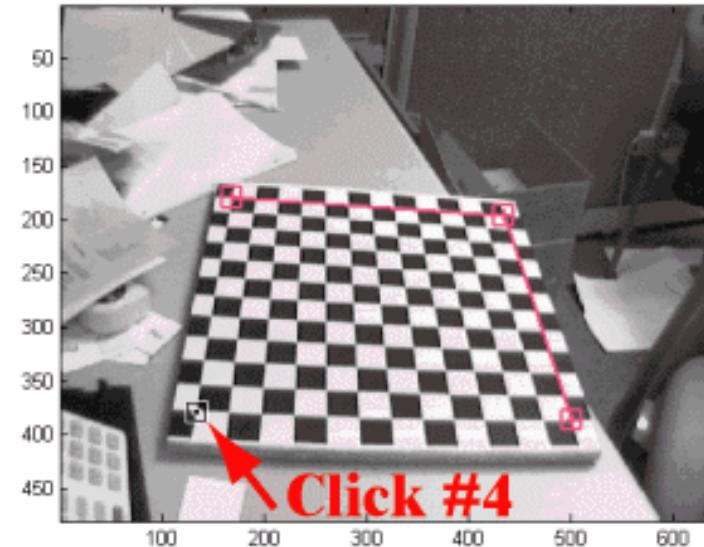
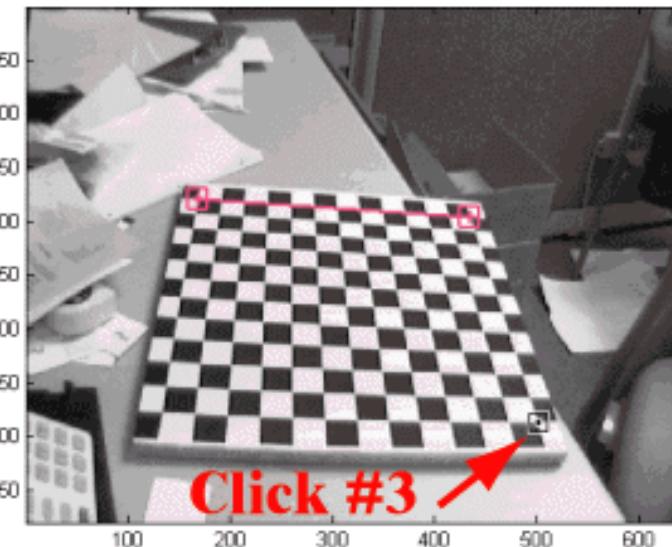


Calibration Procedure

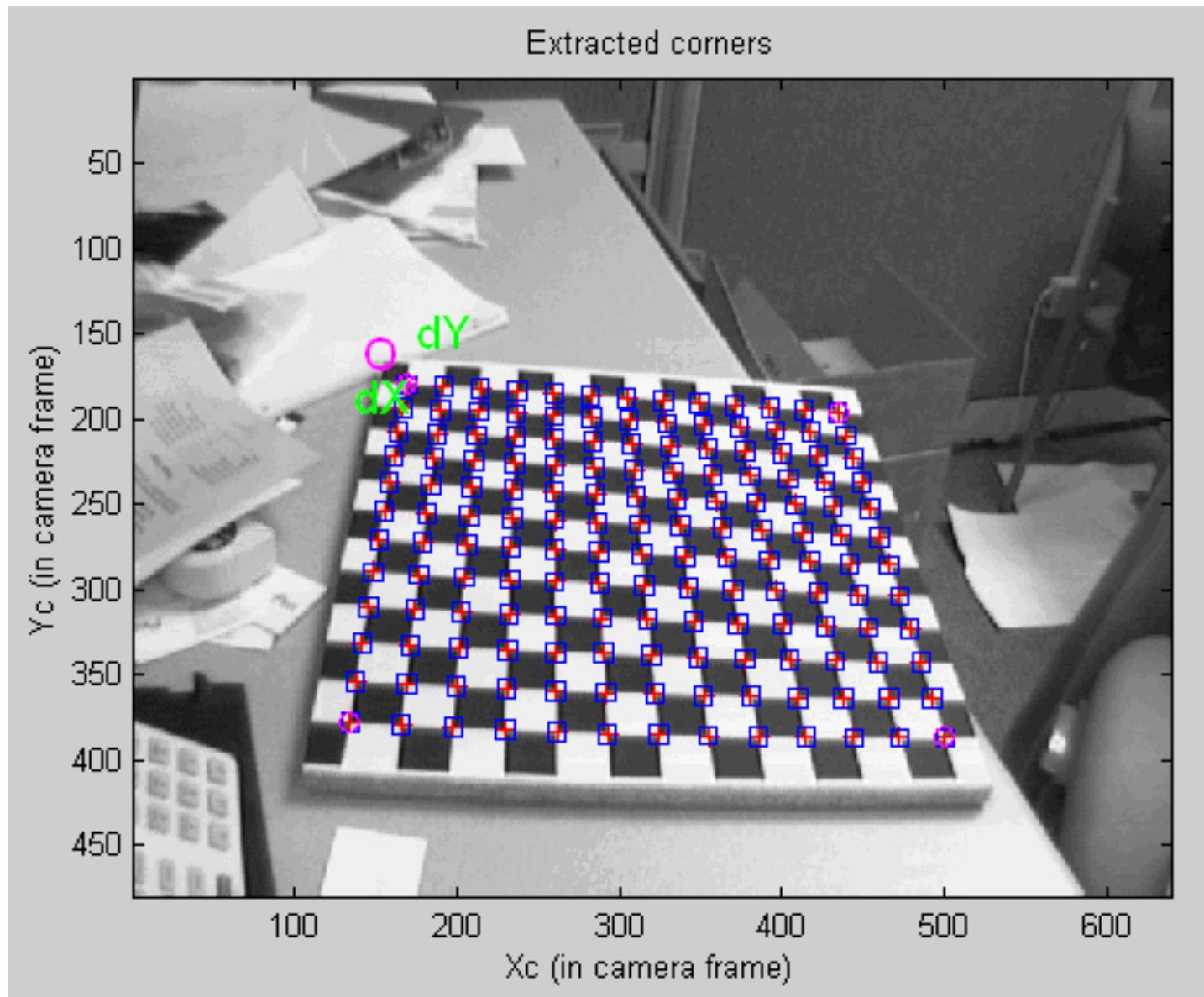
Click on the four extreme corners of the rectangular pattern (first corner = origin)... Image 1 Click on the four extreme corners of the rectangular pattern (first corner = origin)... Image 1



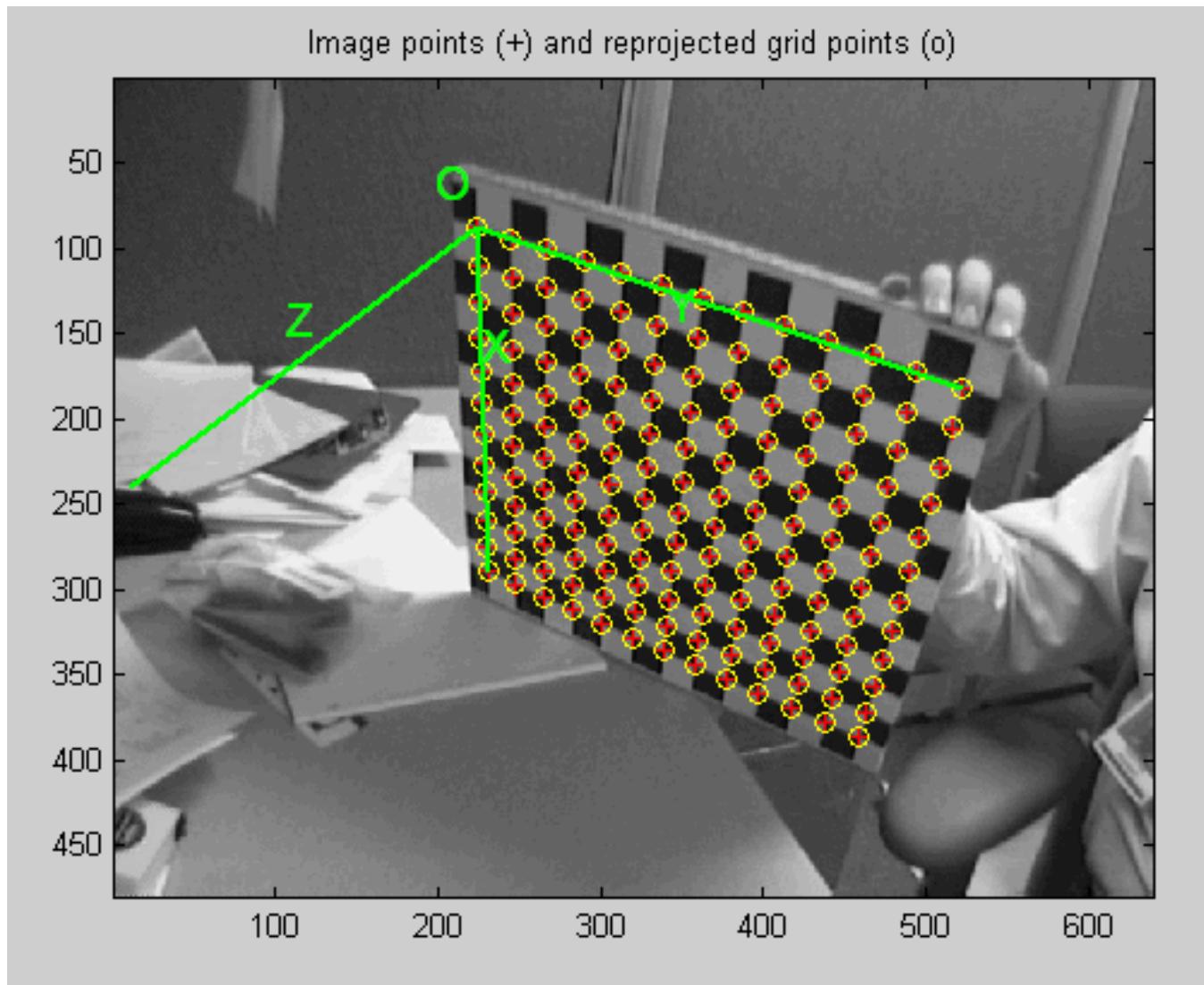
Click on the four extreme corners of the rectangular pattern (first corner = origin)... Image 1 Click on the four extreme corners of the rectangular pattern (first corner = origin)... Image 1



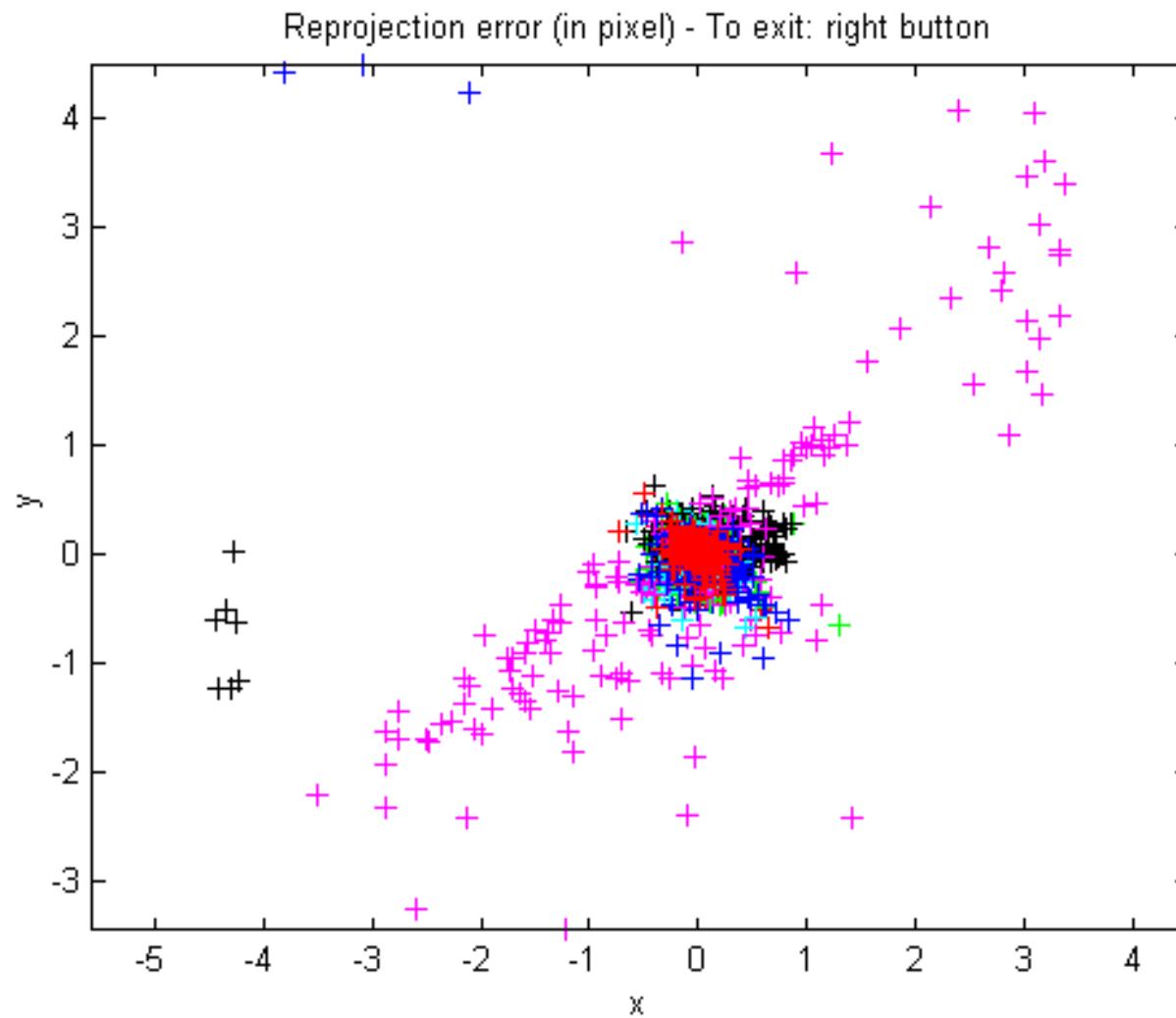
Calibration Procedure



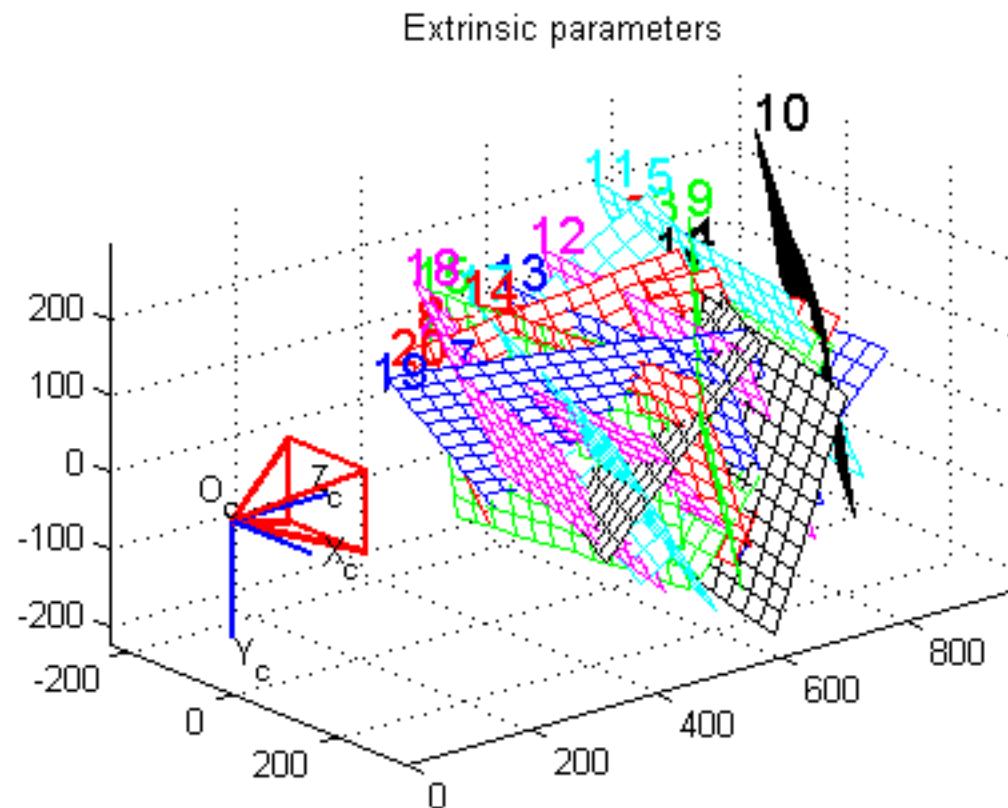
Calibration Procedure



Calibration Procedure

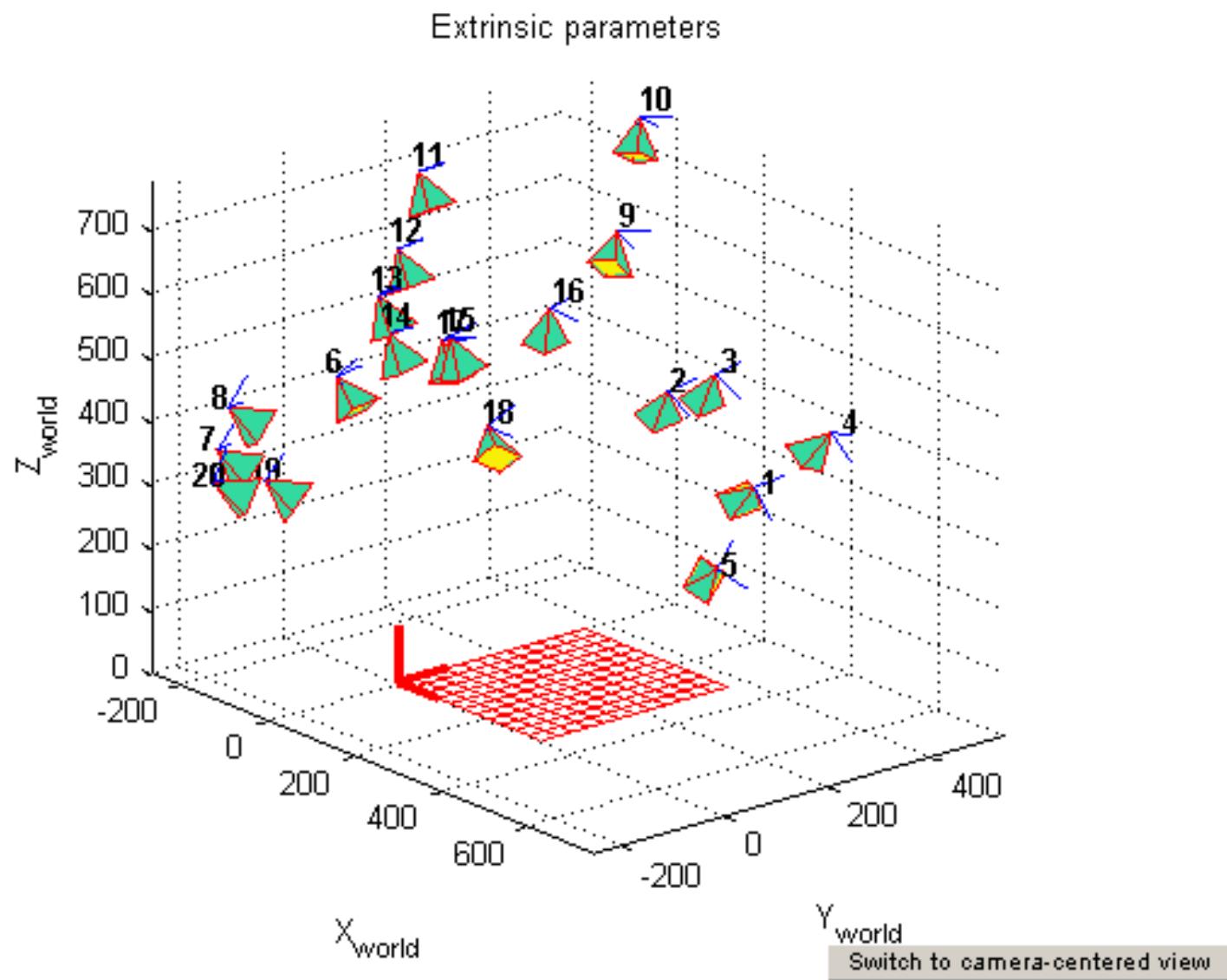


Calibration Procedure



[Switch to world-centered view](#)

Calibration Procedure



Next lecture

- Single view reconstruction

Eigenvalues and Eigenvectors

Eigendecomposition

$$A = S\Lambda S^{-1} = S \begin{bmatrix} \lambda_1 & & & \\ & \lambda_2 & & \\ & & \ddots & \\ & & & \lambda_N \end{bmatrix} S^{-1}$$

Eigenvectors of A are
columns of S

$$S = [\mathbf{v}_1 \quad \mathbf{v}_N]$$

Singular Value decomposition

$$A = U \Sigma V^{-1} \quad \Sigma = \begin{bmatrix} \sigma_1 & & & \\ & \sigma_2 & & \\ & & \ddots & \\ & & & \sigma_N \end{bmatrix}$$

U, V = orthogonal matrix

$$\sigma_i = \sqrt{\lambda_i}$$

σ = singular value
 λ = eigenvalue of $A^t A$