CEGEP CHAMPLAIN - ST. LAWRENCE
201-510-LW: Business Statistics
Patrice Camiré

Problem Sheet #17

Hypothesis Testing & Confidence Intervals
Additional Problems

1. An insurance company used to set its policy rates according to the belief that claims are in average \$1270 with a standard deviation of \$453. The accuracy of this information is crucial as errors have the potential to be extremely costly. The company decides to randomly sample 41 claims and finds a sample mean of \$1469 and a sample standard deviation of \$582.

   (a) Formulate an appropriate hypothesis about the true standard deviation of claims and test it at the 1% level of significance.

   (b) Formulate an appropriate hypothesis about the true mean of claims and test it at the 1% level of significance.

   (c) If you change the 1% to 2.5% in part (b), does your conclusion change? Justify.

   (d) Construct a 95% confidence interval for the true mean of claims.

   (e) How large large should your sample be if you want to be 95% confident that the true mean is within \$100 of the sample mean?

   (f) Construct a 99% confidence interval for the true mean of claims.

   (g) How large large should your sample be if you want to be 99% confident that the true mean is within \$100 of the sample mean?

   (h) Construct a 98% confidence interval for the true standard deviation of claims.

2. It was recently discovered that a company sold contaminated beef. The company claims that only 30% of its packages do contain contaminated beef. Health inspectors randomly sampled 120 packages and found that 48 contained contaminated beef.

   (a) Formulate a proper hypothesis about the proportion of contaminated packages and test it at the 4% level of significance.

   (b) Construct a 95% confidence interval for the true proportion of contaminated packages.

   (c) Is your confidence interval in part (b) consistent with your conclusion in part (a)? Justify.

   (d) How large should the sample be so that we are 90% confident that the true proportion of contaminated packages is within 2% of the sample proportion?

   (e) Repeat part (d), but this time assume that no data was previously obtained by health inspectors.

Answers

1. (a) $H_0 : \sigma = 453$, $H_1 : \sigma > 453$, $\alpha = 0.01$, $n = 41$, $df = 40$, $\chi^2_{obs} \approx 66.03$, $R = [63.69, \infty)$. We reject H_0.

   (b) $H_0 : \mu = 1270$, $H_1 : \mu > 1270$, $\alpha = 0.01$, $n = 41$, $df = 40$, $t_{obs} \approx 2.19$, $R = [2.423, \infty)$. We accept H_0.

   (c) Yes, since $R = [2.021, \infty)$. We reject H_0.

   (d) $1285.3 \leq \mu \leq 1652.7$

   (e) $n \geq 131$

   (f) $1223.22 \leq \mu \leq 1714.78$

   (g) $n \geq 226$

   (h) $461.23 \leq \sigma \leq 781.93$

2. (a) $H_0 : p = 0.3$, $H_1 : p > 0.3$, $\alpha = 0.04$, $n = 120$, $np = 36 > 5$, $nq = 84 > 5$, $\hat{p} = 0.4$, $\hat{q} = 0.6$, $z_{obs} \approx 2.39$, $R = [1.75, \infty)$. We reject H_0.

   (b) $z_c = 1.96$, $0.3123 \leq p \leq 0.4877$

   (c) Yes, since all values inside the confidence interval are greater than 0.3.

   (d) $z_c \approx 1.65$, $E = 0.02$, $n \geq 1634$

   (e) $z_c \approx 1.65$, $E = 0.02$, $n \geq 1702$