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CS109

Handout #21

due: 3/3, 145 points

Problem Set #6

Trees and graphs are the most important data models that we will learn about in CS109. Many of the following problems will help you to develop skills in using these data models.

Structural induction is a key proof technique in proving properties of trees and graphs, as well as proving that algorithms that operate on them have certain properties. Be sure to use structural induction when asked (this is the sub-tree type of induction), and not node induction. If we do not specify, use whatever form you like. Remember that node induction can be very tricky because it is often difficult to define all the ways one might add a node to a graph for the inductive step.

On this problem set, if we ask for a proof, be formal and precise (i.e., use a statement and reason chart format, or a well-organized, detailed paragraph). If we ask for an inductive proof, include the six steps. On all questions, always show your work.

Trees

1) (10 points) Rosen, Section 8.1 #18

2) (5 points each) Rosen, Section 8.1 #22 and #24 (assume m >= 2)

3) (25 points) Rosen, Section 8.1, #40 (Define a recurrence relation for the number of vertices in a rooted Fibonacci tree T_n. Then, define closed form formulas for the number of vertices, leaves, internal vertices and the height - note that you can use the Fibonacci numbers in your solutions. Prove using induction that your closed form formula for the number of vertices is correct.)

Graphs

4) (4 points) Is it possible to have a group of seven people such that each person knows exactly three other people in the group?

5) (6 points) Suppose all vertices of a graph G have degree p, where p is an odd number. Prove that the number of edges in G is a multiple of p.

6) (10 points) Rosen, Section 7.2, #36

7) (12 points) Rosen, Section 7.5, #36

8) (12 points) Rosen, Section 7.5, #54 (a-c)

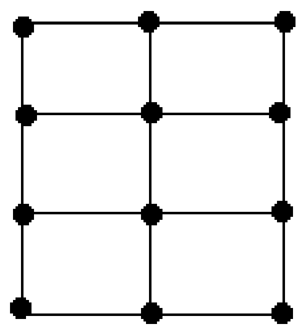
9) (10 points) Rosen, Section 7.5, #70

10) (15 points) The incidence matrix of a directed graph $G = (V,E)$ is a $|V| \times |E|$ matrix $B = (b_{ij})$ such that:

$$b_{ij} = \begin{cases} -1 & \text{if edge } j \text{ leaves vertex } i \\ 1 & \text{if edge } j \text{ enters vertex } i \\ 0 & \text{otherwise} \end{cases}$$

The transpose of a directed graph $G = (V,E)$ is $G^T = (V,E^T)$. G^T is the graph G with all its edges reversed (which is E^T). Describe what the entries of the matrix product $B * B^T$ represent where B is the incidence matrix and B^T is the transpose of B. Note that the incidence matrix of the original graph is V x E, but the transpose matrix is E^T X V (otherwise you cannot do the matrix multiplication).

11) (10 points) A plotter plots the following figure. It takes the plotter 0.05 seconds to plot each horizantal edge and 0.02 seconds to plot each vertical edge. Assuming the time required to trace a line remains the same whether the plotter is plotting or deadheading, and that the plotter must follow the position of the lines , what is the minimum length of time required for the plotter to plot this figure? (NOTE: the Chinese Postman Problem is defined in terms of circuits, not paths so compute your answer based on starting and ending at the same vertex.)



12) (10 points) In the game of dominoes, two dominoes can be put end to end if the ends have the same number of dots on them. A standard set of dominoes has one piece for each (unordered) pair of distinct integers from 0 to 6 inclusive, and one piece of the form (j, j) for each j from 0 to 6. Is it possible to arrange all the dominoes into one big circle? If all the pieces with no dots on one or both ends are excluded, is it possible to arrange the remaining pieces into a circle? Model this situation using a graph.

13) (10 points) Solve the following instant insanity puzzle. Show the underlying graph, the subgraphs and give a solution.