CS154 Assignment 1

January 10, 2001

Please submit every problem on a separate sheet.
Assignment due: 01/17/2001 at 3:15pm.

1. Give a state diagram of a DFA over the alphabet $\{0, 1\}$ recognizing the language

$$\{w \mid w \text{ contains the substring } 0101, \text{ i.e. } w = x0101y \text{ for some } x \text{ and } y\}.$$

   Also give an NFA over the alphabet $\{0, 1\}$ with no more than 5 states, recognizing the same language.

2. Let $D = \{w \mid w \text{ contains an equal number of occurrences of the substrings } 01 \text{ and } 10\}$.
   Thus $101 \in D$ because 101 contains a single 01 and a single 10, but $1010 \notin D$ because 1010 contains two 10s and only one 01. Show that D is a regular language.

3. Prove by induction: The maximum number of nodes in a binary tree of height h is $2^h - 1$.

4. Prove : Given a connected undirected graph without any multiple edges and self-loops, we define the degree of a node to be the number of nodes it is connected to. Prove that any such graph has at least two nodes with the same degree. (*Hint*: You may need to use the *pigeon-hole* principle: If there are n pigeons and less than n holes to accommodate them, one of the holes must hold at least two pigeons.)

Extra credit problem (optional): Prove that in any group of 6 students taking this class, there is a group of three people who are all mutual strangers or all mutual acquaintances.

   You may assume that all pairs of students are either mutual strangers or mutual acquaintances.

   (Harder) Generalize the above to infinite populations: In a class of infinitely many students, prove that there are either infinitely many mutual strangers or infinitely many mutual acquaintances.