Notes on bottom-up parsing

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Bottom-up parsing

As the name suggests, bottom-up parsing begins with a string and tries to work backwards to the start symbol by applying the productions in reverse. Along the way, we look for substrings in the working string that match the right side of some production. When we find such substrings, we *reduce* them, i.e., substitute the left side non-terminal for the matching right side. The goal is to reduce our way up to the start symbol and report a successful parse.

Bottom-up parsing algorithms are in general more powerful than top-down methods, but not surprisingly, the constructions required in these algorithms are also more complex. It is difficult to write a bottom-up parser by hand for anything but the most trivial of grammars, but fortunately, there are excellent parser generator tools like `yacc` that build a parser from an input specification.

Shift-reduce parsing is the most commonly used and most powerful of the bottom-up techniques. It takes as input a stream of tokens and produces as output a list of productions to be used to build the parse tree, but the productions are discovered in reverse order of a top-down parser. Like a table-drive predictive parser, a bottom-up parser makes use of a stack to keep track of the position in the parse and a parsing table to determine what to do next.

To illustrate stack-based shift-reduce parsing, consider this simplified expression grammar:

```
S  –>  E
E  –>  T | E + T
T  –>  int | (E)
```

The shift-reduce strategy divides the string to parse into two parts: an undigested part and a semi-digested part. The undigested part is the input, and the semi-digested part is put on a stack. If parsing the string $\underline{v}$, it starts out completely undigested, so the input is initialized to $\underline{v}$, and the stack is initialized to empty. A shift-reduce parser proceeds by taking one of two actions at each step:

Reduce: If we can find a rule A –> $\underline{w}$, and if the contents of the stack are q$\underline{w}$ for some q (q may be empty), then we can reduce the stack to qA. We are applying the production backwards. For example, using the grammar above, if the stack contained `(int` we can use the rule T –> `int` to reduce the stack to `(T`.

There is also one special case: reducing the entire contents of the stack to the start symbol with no remaining input means you have successfully parsed all the input (e.g. the stack contains just $\underline{w}$, the input is empty, and you apply S –> $\underline{w}$).

The $\underline{w}$ being reduced is referred to as a *handle***.** Formally, a handle of a right sentential form $\underline{u}$ is a production A –> $\underline{w}$, and a position within $\underline{u}$ where the string $\underline{w}$ may be found and replaced by A to produce the previous right-sentential form in a rightmost derivation of $\underline{u}$. Recognizing valid handles is the difficult part of shift-reduce parsing.

Shift: If it is impossible to perform a reduction and there are tokens remaining in the undigested input, then we transfer a token from the input onto the stack. This is called a shift. For example, using the grammar above, suppose the stack contained `(` and the input contained `int+int)`. It is impossible to perform a reduction on `(` since it does not match the entire right side of any of our productions. So, we shift the first character of the input onto the stack, giving us `(int` on the stack and `+int)` remaining in the input.

The other possibility in shift-reduce parsing is we might encounter an error. This occurs when we reach a state where the current token cannot be part of a valid sentence. This might happen in the above grammar when we try to parse `int+)`.

The general idea is to read tokens from the input and push them onto the stack attempting to build sequences we recognize as the right side of a production. When we find a match, we replace that sequence with its non-terminal and continue working our way up the parse tree. If all goes well, we will end up moving everything from the input to the stack and will have built a sequence on the stack that we recognize as a right-hand side for the start symbol. This process builds the parse tree from the leaves upward, the inverse of the top-down parser.

Let's trace the operation of a general shift-reduce parser in terms of its actions (shift or reduce) and its data structure (a stack). The chart below traces a shift-reduce parsing of the string `(int+int)` using the example grammar from above:

PARSE STACK	REMAINING INPUT	PARSER ACTION
	`(int + int)$`	Shift (push next token from input onto stack, advance input)
`(`	`int + int)$`	Shift
`(int`	`+ int)$`	Reduce: `T –> int` (pop right-hand side of production off stack, push left-hand side, no change in input)
`(T`	`+ int)$`	Reduce: `E –> T`
`(E`	`+ int)$`	Shift
`(E +`	`int)$`	Shift
`(E + int`	`)$`	Reduce: `T –> int`
`(E + T`	`)$`	Reduce: `E –> E + T` (Ignore: `E –> T`)
`(E`	`)$`	Shift
`(E)`	`$`	Reduce: `T –> (E)`
`T`	`$`	Reduce: `E –> T`
`E`	`$`	Reduce: `S –> E`
`S`	`$`	

Notice in the above parse there is a step where there are two possible productions to use in a reduction. The reason we ignored the possibility of reducing `E –> T` was because that would have created the sequence `(E + E` on the stack which is not a *viable prefix* of a right sentential form. Formally, viable prefixes are the set of prefixes of right sentential forms that can appear on the stack of a shift-reduce parser, i.e. prefixes of right sentential forms that do not extend past the right end of the rightmost handle. Basically, a shift-reduce parser will only create sequences on the stack that can lead to an eventual reduction to the start symbol. Because there is no right-hand side that matches the sequence `(E + E` and no possible reduction that transform it to such, this is a dead end and is not considered. Later, we will see how the parser can determine which reductions are valid in a particular situation.

Not all type 2 grammars can be parsed with a shift-reduce parser. Ambiguous grammars are problematic because these grammars could yield more than one handle under some circumstances. These types of grammars create either *shift-reduce* or *reduce-reduce* conflicts**.** The former refers to a state where the parser cannot decide whether to shift or reduce. The latter refers to a state where the parser has more than one choice of production for reduction. An example of a shift-reduce conflict occurs with the if-then-else construct in programming languages. A typical production might be:

```
S  –> if E then S | if E then S else S
```

Consider what would happen to a shift-reduce parser deriving this string:

```
if E then if E then S else S
```

At some point the parser's stack would have:

```
if E then if E then S
```

with `else` as the next token. It could reduce because the contents of the stack match the right-hand side of the first production or shift the `else` trying to build the right-hand side of the second production. Reducing would close off the inner `if` and thus associate the `else` with the outer `if`. Shifting would continue building and later reduce the inner `if` with the `else`. Either is syntactically valid given the grammar, but two different parse trees result, showing the ambiguity. This quandary is commonly referred to as the *dangling else.* Does an `else` appearing within a nested `if` statement belong to the inner or the outer? The C and Java languages agree that an `else` is associated with its nearest unclosed `if`. Other languages, such as Ada and Modula, avoid the ambiguity by requiring a closing `endif` delimiter.

Reduce-reduce conflicts are not common and usually indicate a problem in the grammar definition.

Now that we have general idea of how a shift-reduce parser operates, we will look at how it recognizes a handle, and how it decides which production to use in a reduction. To deal with these two issues, we will look at a specific shift-reduce implementation called LR parsing.

LR parsing

LR parsers ("L" for left to right scan of input; "R" for rightmost derivation) are efficient, table-driven shift-reduce parsers. The class of grammars that can be parsed using LR methods is a proper superset of the class of grammars that can be parsed with predictive LL parsers. In fact, virtually all programming language constructs for which CFGs can be written can be parsed with LR techniques. As an added advantage, there is no need to apply transformations and or condition the grammar to make it acceptable for LR parsing the way that LL parsing requires.

The primary disadvantage is the amount of work it takes to build the tables by hand, which makes it infeasible to hand-code an LR parser for most grammars. Fortunately, there exist LR parser generator tools that create the parser from a CFG specification. The parser tool does all the tedious and complex work to build the necessary tables and can report any ambiguities or language constructs that interfere with the ability to parse it using LR techniques.

We begin by tracing how an LR parser works. Determining the handle to reduce in a sentential form depends on the sequence of tokens on the stack, not only the topmost ones that are to be reduced, but the context at which we are in the parse. Rather than reading and shifting tokens onto a stack, an LR parser pushes "states" onto the stack; these states describe what is on the stack so far. Think of each state as encoding the current left context. The state on top of the stack combined with possibly some tokens of lookahead enables us to figure out whether we have a handle to reduce, or whether we need to shift a new state on top of the stack for the next input token.

An LR parser uses two tables:

1. The *action table* `Action[s,a]` tells the parser what to do when the state on top of the stack is `s` and terminal `a` is the next input token. The possible actions are to shift a state onto the stack, to reduce the handle on top of the stack, to accept the input, or to report an error.
2. The *goto table* `Goto[s,X]` indicates the new state to place on top of the stack after a reduce of the non-terminal `X` while state `s` is on top of the stack.

The two tables are usually combined, with the action table specifying entries for terminals, and the goto table specifying entries for non-terminals.

Tracing an LR parser

We start with the initial state s_0 on the stack. The next input token is the terminal `a` and the current state is s_t. The action of the parser is as follows:

- If `Action[`s_t`,a]` is shift, we push the specified state onto the stack. We then call `yylex()` to get the next token `a` from the input.
- If `Action[`s_t`,a]` is reduce `Y –>` $X_1 \ldots X_k$ then we pop `k` states off the stack (one for each symbol in the right side of the production) leaving state s_u on top. `Goto[`s_u`,Y]` gives a new state s_V to push on the stack. The input token is still `a` (i.e. the input remains unchanged).
- If `Action[`s_t`,a]` is accept then the parse is successful and we are done.
- If `Action[`s_t`,a]` is error (the table location is blank) then we have a syntax error. With the current top of stack and next input we can never arrive at a sentential form with a handle to reduce.

As an example, consider the following simplified expression grammar. The productions have been sequentially numbered so we can refer to them in the action table:

```
1)  E –> E + T
2)  E –> T
3)  T –> (E)
4)  T –> id
```

Here is the combined action and goto table. In the action columns `sN` means shift state numbered `N` onto the stack number and `rN` action means reduce using production numbered `N`. The goto column entries are the number of the new state to push onto the stack after reducing the specified non-terminal. This is an LR(0) table (more details on the various table construction will come in a minute).

State on top of stack	Action					Goto	
	id	+	(	)	$	E	T
0	s4		s3			1	2
1		s5			accept		
2	r2	r2	r2	r2	r2		
3	s4		s3			6	2
4	r4	r4	r4	r4	r4		
5	s4		s3				8
6		s5		s7			
7	r3	r3	r3	r3	r3		
8	r1	r1	r1	r1	r1		

Here is a parse of `id + id + id` using the LR algorithm with the above action and goto table:

STACK STACK	REMAINING INPUT	PARSER ACTION
S_0	`id + (id)$`	Shift S_4 onto state stack, move ahead in input
S_0S_4	`+ (id)$`	Reduce `4) T –> id`, pop state stack, goto S_2, input unchanged
S_0S_2	`+ (id)$`	Reduce `2) E –> T`, goto S_1
S_0S_1	`+ (id)$`	Shift S_5
$S_0S_1S_5$	`(id)$`	Shift S_3
$S_0S_1S_5S_3$	`id)$`	Shift S_4
$S_0S_1S_5S_3S_4$	`)$`	Reduce `4) T –> id`, goto S_2
$S_0S_1S_5S_3S_2$	`)$`	Reduce `2) E –> T`, goto S_6
$S_0S_1S_5S_3S_6$	`)$`	Shift S_7
$S_0S_1S_5S_3S_6S_7$	`$`	Reduce `3) T –> (E)`, goto S_8
$S_0S_1S_5S_8$	`$`	Reduce `1) E –> E + T`, goto S_1
S_0S_1	`$`	Accept

Types of LR parsers

There are three types of LR parsers: *LR(k), simple LR(k),* and *lookahead LR(k)* (abbreviated to LR(k), SLR(k), LALR(k))). The k identifies the number of tokens of lookahead. We will usually only concern ourselves with 0 or 1 tokens of lookahead, but it does generalize to k > 1. The different classes of parsers all operate the same way (as shown above, being driven by their action and goto tables), but they differ in how their action and goto tables are constructed, and the size of those tables.

We will consider LR(0) parsing first, which is the simplest of all the LR parsing methods. It is also the weakest and although of theoretical importance, it is not used much in practice because of its extreme limitations. Adding just one token of lookahead to get LR(1) vastly increases the parsing power, but the problem with LR(1) (and LR(k) in general) is these parsers have many thousands of states for a typical programming language. SLR(k) parsing is a variant with only a few hundred states for a typical programming language, but it much weaker than full LR(k) in terms of the number of grammars for which it is applicable. LALR(k) parses a larger set of languages than SLR(k) but not quite as many as LR(k). In particular, LALR(k) parsers can parse all common programming language constructs with the same number of states as the equivalent SLR parser (but LALR is harder to construct). LALR(1) is the method used by the yacc parser generator.

In order to begin to understand how LR parsers work, we need to delve into how their tables are derived. The tables contain all the information that drives the parser. As an example, we will show how to derive LR(0) parsing tables since they are the simplest and then discuss how to do SLR(1), LR(1), and LALR(1).

The essence of LR parsing is building the tables that tell us where there is a handle on the top of the stack that can be reduced. Recognizing a handle is actually easier than predicting a production was in top-down parsing. The weakness of LL(k) parsing techniques is that they must be able to predict which product to use, having seen only k symbols of the right-hand side. For LL(1), this means just one symbol has to tell all. In contrast, for an LR(k) grammar is able to postpone the decision until it has seen tokens corresponding to the entire right-hand side (plus k more tokens of lookahead). This doesn't mean the task is trivial. More than one production may have the same right-hand side and what looks like a right-hand side may not really be because of its context. The addition of nullable non-terminals is another complication because they can be matched in any parsing context.

Constructing LR(0) parsing tables

Generating an LR parsing table consists largely of identifying what states are necessary and arranging the transitions among them. At the heart of the table construction is the notion of an LR(0) *configuration* or *item*. A configuration is a production of the grammar with a dot at some position on its right side. For example, A –> XYZ gives four items:

A –> •XYZ
A –> X•YZ
A –> XY•Z
A –> XYZ•

This dot marks how far we have gotten in parsing the production. Everything to the left of the dot has been shifted and next input token is in the `First()` set of the symbol after the dot (or in the follow set if that symbol is nullable). The state we push on the stack will correspond to specific sets of these configurations, indicating what we have read thus far. A dot at the right end of a configuration indicates that we have that entire configuration on the stack i.e., we have a handle that we can reduce. A dot in the middle of the configuration indicates that we need to shift a token that could start the symbol following the dot. For example, if we are currently in this position:

A –> X•YZ

We want to shift something from `First(Y)` (something that matches the next input token). Say we have a production Y –> $\underline{u}$ | $\underline{w}$. Given that, these three productions all correspond to the same state of the shift-reduce parser:

A –> X•YZ
Y –> •$\underline{u}$
Y –> •$\underline{w}$

In other words, at some point as we parse a string in this language, we expect to see a substring derivable from YZ as input. Because of the expansions for Y, we also could expect to see a string derivable from either $\underline{u}$ or $\underline{w}$. We can put these three items into a set and call it a *configurating set* of the LR parser. The action of adding equivalent configurations to create a configurating set is called *closure*. Our parsing tables will have one state corresponding to each configurating set.

These configurating sets represent states that the parser can be in as it parses a string. We could model this as a finite automaton where we move from one state to another via transitions marked with a symbol of the CFG. For example,

$$\begin{tikzcd}\boxed{A \to X \bullet YZ} \arrow[r, "Y"] & \boxed{A \to XY \bullet Z}\end{tikzcd}$$

Recall that we push states onto the stack in a LR parser. These states describe what is on the stack so far. The state on top of the stack (potentially combined with some lookahead) enables us to figure out whether we have a handle to reduce, or whether we need to read the next input token and shift a new state on top of the stack. We shift until we reach a state where the dot is at the end of a production, at which point we reduce. This finite automaton is the basis for a LR parser: each time we perform a shift we are following a transition to a new state.

Now for the formal rule for what to put in a configurating set. We start with a configuration:

$$A \to X_1 \ldots X_i \bullet X_{i+1} \ldots X_j$$

which we place in the configurating set. We then perform the closure operation on the items in the configurating set. For each item in the configurating set where the dot precedes a non-terminal, we add configurations derived from the productions defining that non-terminal with the dot at the start of the right side of those productions. So, if we have

$$X_{i+1} \to Y_1 \ldots Y_g \mid Z_1 \ldots Z_h$$

in the above example, we would add the following to the configurating set.

$$X_{i+1} \to \bullet\, Y_1 \ldots Y_g$$
$$X_{i+1} \to \bullet\, Z_1 \ldots Z_h$$

We repeat this operation for all configurations in the configurating set where a dot precedes a non-terminal until no more configurations can be added. So, if Y_1 and Z_1 are terminals in the above example, we would just have the three productions in our configurating set. If they are non-terminals, we would need to add the Y_1 and Z_1 productions as well.

In summary, to create a configurating set for the starting configuration $A \to \bullet \underline{u}$, we follow the closure operation:

1. $A \to \bullet \underline{u}$ is in the configurating set
2 If $\underline{u}$ begins with a terminal, we are done with this production
3. If $\underline{u}$ begins with a non-terminal B, add all productions with B on the left side, with the dot at the start of the right side: $B \to \bullet \underline{v}$
4. Repeat steps 2 and 3 for any productions added in step 3. Continue until you reach a fixed point.

The other information we need to build our tables is the transitions between configurating sets. For this, we define the *successor* function. Given a configurating set C and a grammar symbol X, the successor function computes the successor configurating set C' = `succ(C,X)`. The successor function describes what set you move to upon recognizing a given symbol.

The successor function is quite simple to compute, We take all the configurations in C where there is a dot preceding X, move the dot past X and put the new configurations in C'. The successor configurating set is the closure of C'. The successor configurating set C' represents the state we move to when encountering symbol X in state C.

The successor function is defined to only recognize viable prefixes. There is a transition from $A \to \underline{u} \bullet x \underline{v}$, to $A \to \underline{ux} \bullet \underline{v}$ on the input x. If what was already being recognized as a viable prefix and we've just seen an x, then we can extend the prefix by adding this symbol without destroying viability.

Here is an example how building a configurating set, performing closure, and computing the successor function. Consider the following item from our example expression grammar:

$$E \to E \bullet + T$$

To obtain the successor configurating set on + we first put the following configuration in C':

```
E –> E +•T
```

We then perform a closure on this set:

```
E –> E +•T
T –> •(E)
T –> • id
```

Now, to create the action and goto tables, we need to construct all the configurating sets and successor functions for the expression grammar. At the highest level, we want to start with a configuration with a dot before the start symbol and move to a configuration with a dot after the start symbol. This represents shifting and reducing an entire sentence of the grammar. To do this, we need the start symbol to appear on the right side of a production. This may not happen in the grammar so we have to create a special production. We can create an *augmented grammar* by just adding the production:

```
S' –> • S
```

where `S` is the start symbol. So we start with the initial configurating set C_0 which is the closure of `S' –> •S`. The augmented grammar for the example expression grammar:

```
0) E' –> E
1) E –> E + T
2) E –> T
3) T –> (E)
4) T –> id
```

We create the complete family `F` of configurating sets as follows:

1. Start with `F` containing the configurating set C_0, derived from the configuration `S' –> • S`
2. For each configurating set `C` in `F` and each grammar symbol `X` such that `successor(C,X)` is not empty, add `successor(C,X)` to `F`
3. Repeat step 2 until no more configurating sets can be added to `F`

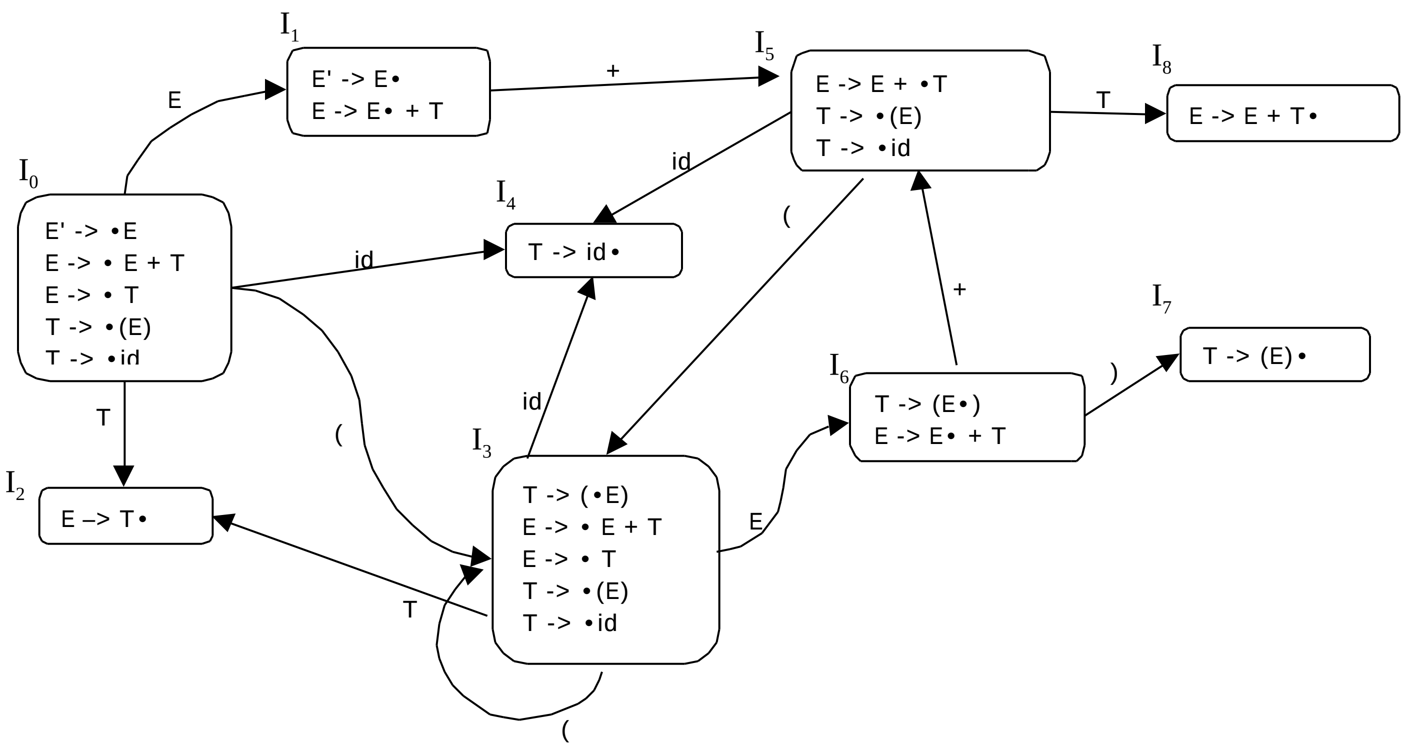
Here is the full family of configurating sets for the grammar given above.

Configurating set		Successor
I_0:	`E' –> •E`	I_1
	`E –> •E+T`	I_1
	`E –> •T`	I_2
	`T –> •(E)`	I_3
	`T –> •id`	I_4
I_1:	`E' –> E•`	Accept
	`E –> E•+T`	I_5
I_2:	`E –> T•`	Reduce 2
I_3:	`T –> (•E)`	I_6
	`E –> •E+T`	I_6
	`E –> •T`	I_2

	T –> •(E)	I_3
	T –> •id	I_4
I_4:	T –> id•	Reduce 4
I_5:	E –> E+•T	I_8
	T –> •(E)	I_3
	T –> •id	I_4
I_6:	T –> (E•)	I_7
	E –> E•+T	I_5
I_7:	T –> (E)•	Reduce 3
I_8:	E –> E+T•	Reduce 1

Note that the order of defining and numbering the sets is not important; what is important is that all the sets are included.

A useful means to visualize the configurating sets and successors is with a diagram like this below. The transitions mark the successor relationship between sets. We call this a *goto-graph* or *transition diagram.*



To construct the LR(0) table, we use the following algorithm. The input is an augmented grammar G' and the output is the action/goto tables:

1. Construct `F = {`I_0, I_1, ... I_n`}`, the collection of configurating sets for G'.
2. State i is determined from I_i. The parsing actions for the state are determined as follows:
   a) If `A –>` u`•` is in I_i then set `Action[i,a]` to reduce `A –>` u for all input. (`A` not equal to `S'`).
   b) If `S' –> S•` is in I_i then set `Action[i,$]` to accept.
   c) If `A –>` u`•a`v is in I_i and `succ(`I_i`, a) =` I_j, then set `Action[i,a]` to shift j (`a` is a terminal).
3. The goto transitions for state i are constructed for all non-terminals `A` using the rule: If `succ(`$I_{i,}$ `A) =` $I_{j,}$ then `Goto [i, A] = j`.
4. All entries not defined by rules 2 and 3 are errors.
5. The initial state is the one constructed from the configurating set containing `S' –> •S`.

Notice how the shifts in the action table and the goto table are just transitions to new states. The reductions are where we have a handle on the stack that we pop off and replace with the non-terminal for the handle; this occurs in the states where the • is at the end of a production.

At this point, we should go back and look at the parse of `id + (id)` from before and trace what the states mean. (Refer to the action and goto tables and the parse diagrammed on page 4 and 5).

First of all, here is the actual parse (notice it is an reverse rightmost derivation, if you read from the bottom upwards, it is always the rightmost non-terminal that operated on).

```
id + (id)
T + (id)
E + (id)
E + (T)
E + (E)
E + T
E
E'
```

We start by pushing s_0 on the stack. The first token we read is an `id`. In configurating set I_0, the successor of `id` is set $I_{4,}$ this means pushing s_4 onto the stack. This is a final state for `id` (the • is at the end of the production) so we reduce the production `T –> id`. We pop s_4 to match the `id` being reduced and we are back in state s_0. We reduced the handle into a `T`, so we use the goto part of the table, and `Goto[0, T]` tells us to push s_2 on the stack. (In set I_0 the successor for `T` was set I_2). In set I_2 the action is to reduce `E –> T`, so we pop off the s_2 state and are back in s_0. `Goto[0, E]` tells us to push s_1. From set I_1 seeing a `+` takes us to set I_5 (push s_5 on the stack).

From set I_5 we read an open `(` which that takes us to set I_3 (push s_3 on the stack). We have an `id` coming up and so we shift state s_4. Set 4 reduces `T –> id`, so we pop s_4 to remove right side and we are back in state s_3. We use the goto table `Goto[3, T]` to get to set I_2. From here we reduce `E –> T`, pop s_2 to get back to state s_3 now we goto s_6. . `Action[6, )]` tells us to shift s_7. Now in s_7 we reduce `T –> (E)`. We pop the top three states off (one for each symbol in the right-hand side of the production being reduced) and we are back in s_5 again. `Goto[5,T]` tells us to push s_8. We reduce by `E –> E + T` which pops off three states to return to s_0. Because we just reduced `E` we goto $s_{1.}$ The next input symbol is `$` means we completed the production `E' –> E` and the parse is successful.

The stack allows us to keep track of what we have seen so far and what we are in the middle of processing. We shift states that represent the amalgamation of the possible options onto the stack until we reach the end of a production in one of the states. Then we reduce. After a reduce, states are popped off the stack to match the symbols of the matching right-side. What's left on the stack is what we have yet to process.

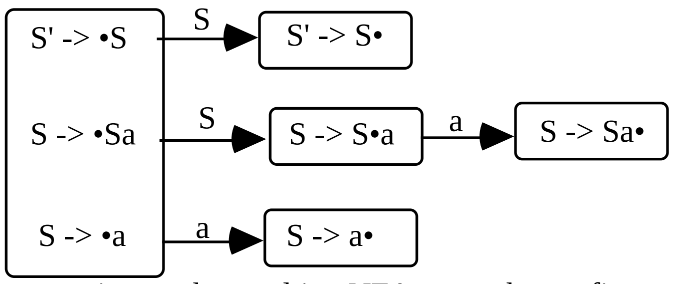
Consider what happens when we try to parse `id++`. We start in s_0 and do the same as above to parse the id to T and then to E. Now we are in set I5 and we encounter another + which is an error because the action table is empty for that transition. This means there was no successor for + from that configurating set, because there was no viable prefix that begins E++.

Subset construction and closure

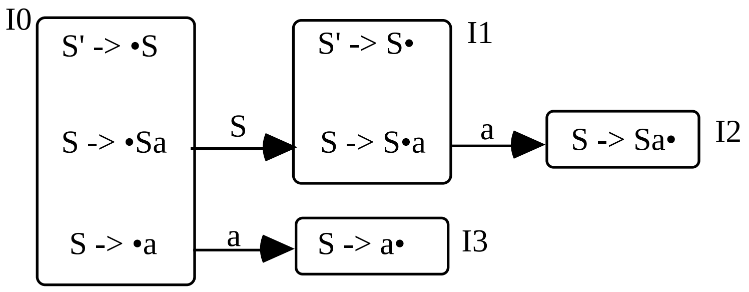
You may have noticed a similarity between subset construction and the closure operation. If you think back to a few lectures, we explored the subset construction algorithm for converting an NFA into a DFA. The basic idea was create new states that represent the non-determinism by grouping the possibilities that look the same at that stage and only diverging when you get more information. The same idea applies to creating the configurating sets for the grammar and the successor function for the transitions. We create a NFA whose states are all the different individual configurations. We put all the initial configurations into one start state. Then draw all the transitions from this state to the other states where all the other states have only one configuration each. This is the NFA we do subset construction on to convert into a DFA. Here is a simple example starting from the grammar consisting of strings with one or more a's:

1) S' –> S
2) S –> Sa
3) S –> a

Close on the augmented production and put all those configurations in a set:



Do subset construction on the resulting NFA to get the configurating sets:



Interesting, isn't it, to see the parallels between the two processes? They both are grouping the possibilities into states that only diverge once we get further along and can be sure of which path to follow.

Limitations of LR(0) parsing

The LR(0) method may appear to be a strategy for creating a parser for any grammar, but in fact, the grammars we have used as examples were specifically selected to fit the criteria needed for LR(0) parsing. Remember that LR(0) means we are parsing with zero tokens of lookahead. The parser must be able to determine what action to take in each state without looking at any further input symbols, i.e. by only considering what the parsing stack contains so far. In an LR(0) table, each state must only shift or reduce. Thus an LR(0) configurating set cannot have both shift and reduce items, and can only have exactly one reduce item. This turns out to be a rather limiting constraint.

To be precise, a grammar is LR(0) if the following two conditions hold for each configurating set:

1. For any configurating set containing the item A –> u•xv there is no complete item B –> w• in that set. In the tables, this translates to no shift-reduce conflict on any state. This means the successor function from that set either shifts to a new state or reduces, but not both.
2. There is at most one complete item A –> u• in each configurating set. This translates to no reduce-reduce conflict on any state. The successor function has at most one reduction.

Very few grammars are LR(0). For example, any grammar with an ∂-rule is usually not LR(0). If the grammar contains the production A –> ∂, then the item A –> •∂ will create a shift-reduce conflict for any production with A on the right side, such as P –> uAv. And ∂-rules are very frequent in grammars.

Even modest extensions to our example grammar cause trouble. Suppose we extend it to allow array elements, by adding the production rule T–>id[E]. When we construct the configurating sets, we will have one containing the items T–>id• and T–>id•[E] which will be a shift-reduce conflict. Or suppose we allow assignments by adding the productions E –> V = E and V –> id. One of the configurating sets for this grammar contains the items V–>id• and T–>id•, leading to a reduce-reduce conflict.

The above examples show that the LR(0) method is just too weak to be useful. This is caused by the fact that we try to decide what action to take only by considering what we have seen so far, without using any information about the upcoming input. By adding just a single token lookahead, we can vastly increase the power of the LR parsing technique and not be so plagued by conflicts. There are three ways to use a one token lookaheads: SLR(1), LR(1) and LALR(1), each of which we will consider in turn in the next few lectures.

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