CS154 Assignment 7, Problem 2

Is it decidable whether a given Turing machine

1. recognizes the empty language?

   No, by Rice's theorem. The property is certainly about the languages recognized by TMs. And it is non-trivial because there exists at least one TM at accepts the empty language and at least one that doesn't.

2. recognizes a non-regular language?

   No, by Rice's theorem (as above).

3. recognizes a recursive language?

   No, by Rice's theorem. The property is certainly about the languages recognized by TMs. And it is non-trivial because there exists at least one TM that accepts a recursive language: take for instance a TM that never accepts anything, and at least one that doesn't: for instance a TM that recognizes the acceptance problem (which is known to be recursive enumerable but not recursive).

4. recognizes a recursive enumerable language?

   Yes. This is a trivial property, so Rice's theorem doesn't apply. Any TM accepts a recursive enumerable language, so this is trivially decidable.

5. accepts exactly 3 words?

   No, by Rice's theorem. The property is certainly about the languages recognized by TMs. And it is non-trivial because there exists at least one TM at accepts exactly 3 words and at least one that doesn't.

6. accepts the empty string?

   No, by Rice's theorem (as above).

7. halts in less than 7 steps, on every input?

   Yes, this is decidable. In less than 7 steps a TM will at most read the first 6 characters on the input tape. Thus it suffices to simulate a given TM for 6 steps on any input string of length ≤ 6 (of which there are only finitely many) and check whether it halts all the time.

8. will ever write the letter @ on its tape when given input ah?

   No, because the question whether a given TM accepts the input string ah can be reduced to it, and that problem is undecidable by Rice's theorem, as above. Given a TM M, let M' be the same TM in which all occurrences of @ are replaced by a different symbol that wasn't in the alphabet beforehand. Furthermore, whenever the M would accept, M' will write the symbol @ on its tape. As a result, M' will write @ on its tape exactly then when M would accept. Thus M' will ever write the letter @ on its tape when given input ah, exactly when M would accept the input ah. Thus, the reduction that maps the description of M to that of M' has the required property. If there would be a way to decide whether a given TM M' ever writes the letter @ on its tape when given input ah, this could be used to decide whether a given TM M would accept the string ah.