CS154 Assignment 8

March 1, 2001

The homework should be done without collaboration!
Please submit every problem on a separate sheet.
Assignment due: 03/07/2001 at 3:15pm.

1. Give a transition diagram of a Turing machine with input alphabet $\Sigma = \{1, 2, 3, 4, 5, 6, 7, 8, 9, @\}$ that converts every input string from the 10-adic representation of a natural number (where the symbol @ has value 10) to the decimal representation of the same number. That is, the TM should accept any input string, and upon acceptance have the decimal representation (only) on its tape.

   Make your algorithm as efficient as possible and determine its time complexity.

   For those of you who forgot what a 10-adic number is, I put a paper handout in the handout hangout.

2. Show that the class of arithmetical languages is closed under union, concatenation and complement.

3. Show that the class P is closed under union, concatenation and complement.

4. Show that the class NTIME(n) is closed under concatenation.

Extra credit problem (optional): Two disjoint languages L_1 and L_2 are said to be *recursively separable* if there is a recursive language R such that $L_1 \cap R = \phi$ and $L_2 \subseteq R$. Drawing a *Venn-diagram* of L_1, L_2 and R illustrates the concept neatly. If there is no such recursive language R, the languages L_1 and L_2 are said to be *recursively inseparable.*

(i) (*trivial*) Show that the language $H = \{\langle M, w\rangle \mid M \text{ is a Turing machine, } w \text{ a string and } M \text{ halts on } w\}$ and $\overline{H}$ are recursively inseparable.

(ii) (*slightly harder*) Define $L_1 = \{\langle M\rangle \mid M \text{ accepts } \langle M\rangle\}$ and $L_2 = \{\langle M\rangle \mid M \text{ does not accept } \langle M\rangle\}$. Show that L_1 and L_2 are recursively inseparable.

(iii) Let $L = \{\langle M\rangle \mid M \text{ accepts } \varepsilon\}$ and $K = \{\langle M\rangle \mid M \text{ does not accept } \varepsilon\}$. Show that L and K are recursively inseparable.