# SAMPLE SOLUTION

EXERCISE 2

We show that the weighted graph problem (WEIGH hereafter), is NP-complete by polynomially reducing HAMILTONIAN PATH for simple graphs and without designated vertices to this problem. We assume that the latter problem is NP-complete.

Let $G = (V, E)$ ($V$ is the set of nodes, and $E$ the set of edges of $G$) be a simple graph. We convert $G$ into a weighted graph $G'$ by assigning weights to elements of $E$. More particularly, we assign 1 to each $e \in E$. This conversion is polynomial (in fact, we go through the edge set once).

Now suppose we are interested in the question of whether G has a Hamiltonian Path. If WEIGH is polynomially decidable, the we can ask whether $G'$ has a simple path of weight exactly $n - 1$ where $n$ is the number of nodes in $G$.

To see that this works, note that a simple path that contains $n - 1$ edges visits exactly $n$ nodes (since no node can be repeated). Furthermore, since each edge is assigned the weight 1, a simple path of weight $n - 1$ through $G'$ has to visit all nodes exactly once.

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