Midterm exam CS154

February 9, 2001

1. (24 points) **For each of the questions below, state *true* or *false*.** We give 3 points for each correct answer and take away 5 for a wrong answer. No penalty for not answering a question.

   (a) A CFL without any inherent ambiguity is regular.
   (b) CFLs are closed under union operation.
   (c) CFLs are closed under intersection.
   (d) Languages accepted by PDAs are closed under intersection.
   (e) Any subset of a regular language is regular.
   (f) There is a CFL whose complement is regular.
   (g) If L is a CFL so is L^R. (Here L^R is the language obtained by *reversing* each string in L.)
   (h) If L is regular, the language LL^R is context-free.

2. (40 points) Consider the following CFG:

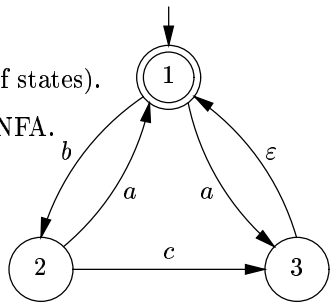
$$
\begin{aligned}
S &\to ABC \\
A &\to DE \\
B &\to FGH \mid b \\
C &\to cd \\
D &\to CD \mid \varepsilon \\
E &\to Dc \\
F &\to dC \\
G &\to BH \\
H &\to ab
\end{aligned}
$$

   Here $V = \{S, A, B, C, D, E, F, G, H\}$, $\Sigma = \{a, b, c, d\}$ and S is the start variable.

   (a) (10 points) Construct a parse tree for *cdcdcdbababcd*.
   (b) (15 points) Explain why the grammar is ambiguous, and construct an unambiguous CFG generating the same language.
   (c) (15 points) Write *cdcdcdbababcd* as $uvxyz$ such that $|vy|$ is an odd number, and for $i \geq 0$ also uv^ixy^iz is in the language generated by the CFG.

3. (20 points) Provide a CFG and a 3-state PDA recognizing the langauge of all strings over the alphabet $\{a, b\}$ with an equal number of a's and b's.

4. (20 points) Convert the following NFA to a DFA and minimize it (i.e. find a DFA recognizing the same language with a minimal number of states).



5. (13 pts.) Give a regular expression describing the same language as this NFA.

6. (58 points) Are the following languages regular? Are they context-free? Prove your answers succinctly!

   (a) (7 points) The language accepted by the NFA on the right.
   (b) (7 points) $\{0^n \mid n \text{ is a multiple of } 5\}$.
   (c) (14 points) $\{0^k1^m2^m3^k \mid k, m \geq 0\}$.
   (d) (10 points) The infinite set of all strings over the alphabet $\{0, 1\}$ that do not contain a substring of the form xxx with $|x| \geq 1$.
   (e) (20 points) The set of all strings over the alphabet $\{0, 1\}$ of the form $xxxw$ with $|x| \geq 1$.

7. (25 points) Give an algorithm to decide, given two regular expressions, whether or not their languages have at least one string in common.