CS154 Assignment 6: Problem 3

First we show that if D is a decidable language, then the language $L = \{x \mid \exists y \ st \ (x, y) \in D\}$ is recursively enumerable.

We show that L can be enumerated in the following manner:

For every pair of strings u, v in the alphabet of D (in order of increasing $|u| + |v|$), if $(u, v) \in D$ then we output u (it is in L).

Clearly this will output any string in L in a finite (though unbounded) length of time, and thus is an enumerator for L, and thus L is recursively enumerable.

Next we show that if L is recursively enumerable, then there is some decidable language D st $L = \{x \mid \exists y \ st \ (x, y) \in D\}$.

Let D be the language accepting (u, v) where v is the decimal encoding of some integer i and u is the i^{th} string tested by some specific enumerator for L and u is output by this enumerator (at this stage).

Clearly D is decidable, because each of the requirements (integral value of v, the value of the i^{th} string tested by the enumerator, and the result of the enumeration at this stage) is decidable. Additionally, the construction of D provides that it satisfies the requirements established vis a vis its relationship with L. Thus, if L is recursively enumerable, there exists some decidable language satisfying the conditions set forth.

Therefore, L is recursively enumerable if and only if there exists some decidable language D such that $L = \{x \mid \exists y \ st \ (x, y) \in D\}$.