Maggie Johnson Handout #12a
CS109

Problem Set #3 Solutions

1a) The sum rule applies: 8 + 7 + 5 = 20 ways.
1b) The product rule applies: 8 * 7 * 5 = 280 ways.
1c) The way to select 3 books from 2 of the sets at a time is: P(8+7, 3) + P(7+5, 3) + P(8+5, 3) which comes to 2730 + 1320 + 1716 = 5766. This includes too many sequences of 3 books, however, because each sequence of 3 must have exactly one language missing (not two). 5766 includes sequences of 3 books of all the same language. We need to subtract out P(8,3) + P(5,3) + P(7,3) twice, since the sequences of 3 books of the same language occur twice in our calculation of 5766. 2 * (P(8,3) + P(5,3) + P(7,3)) = 1212. 5766 - 1212 = 4554.

2a) There are 2^5 strings that begin with two 0's (since there are two choices for the last 5 bits). There are 2^4 strings that end with three 1's (since there are two choices for the first 4 bits). But there are 2^2 strings that both begin with two 0's and end with three 1's which must be excluded. Thus, we get: $2^5 + 2^4 - 2^2 = 44$.
2b) First, we count the number of bit strings of length 10 with five consecutive 0's. If the five 0's begins in the first bit, there is a free choice for the last 5 bits only: $2^5 = 32$ such strings. If it starts in the second bit, then the first bit must be 1, the next 5 are 0's giving $2^4 = 16$ such strings. If it starts in the third bit, the second bit must be a 1 but the 1st and last three bits can be freely chosen. This gives $2^4 = 16$ such strings. Similarly, there are 16 strings that have the consecutive 0's starting in each of positions four, five and six. This gives a total of 32 + 5 * 16 = 112 strings containing 5 consecutive 0's. Symmetrically, there are 112 strings containing 5 consecutive 1's. There are exactly two strings containing both which must be excluded: 112+112 - 2 = 222 strings.

3) There are 44 - 2i ways to place the second queen if the first queen is i squares from the edge of the board, for i = 1, 2, 3, 4. When i = 1, there are 28 positions for the first queen, and 42 for the second queen giving 28 * 42. When i = 2, there are 20 positions for the first queen, and 40 for the second queen giving 20 * 40, and so forth. Thus, we get 28 * 42 + 20 * 40 + 12 * 38 + 4 * 36, but this is double counting, so we need to divide this result by 2: (28 * 42 + 20 * 40 + 12 * 38 + 4 * 36) / 2 = 1288.

4a) C(6 + 36 - 1, 36) = C(41,36)
4b) If we first pick the two of each type, we have picked 2 * 6 = 12 scones. This leaves one dozen to choose without restriction: C(6+12-1, 12) = C(17, 12)
4c) First we include the lower bound restrictions. If we choose the required 9 scones then there are 24-9=15 scones left to choose. If there were no restrictions on the cherry scones, we would have C(6+15-1, 15) ways to make the selection. If we were to violate the cherry scone restriction and get four of them, there would be C(6+11-1,11) choices. Therefore the number of ways to make the selection without violating the restriction is C(6+15-1, 15) - C(6+11-1,11) = 15504-4368 = 11136.

5a) This is just a matter of choosing 10 players from the group of 13, since we are not told to worry about what positions they play. C(13,10) = 286.
5b) Same as part (a) except we are concerned with order in which the choices are made since there are 10 distinct positions to be filled. P(13,10) = 13!/3!.
5c) There is only one way to choose the 10 players without choosing a woman since there are exactly 10 men. Therefore, using part (a) there are 286-1=285 ways to choose the players if at least one of them must be a woman.

6a) Take all possible strings and subtract out those without the letter a: $26^6 - 25^6$.
6b) If our string is to contain both these letters, then we need to subtract from the total number of strings, the number that fail to contain one or the other (or both) of these letters. We know that 25^6 strings fail to contain an a, and similarly 25^6 strings fail to contain b. This is overcounting since 24^6 fail to contain both of these letters. So there are $25^6 + 25^6 - 24^6$ strings that fail to contain at least one of these letters. Thus, the answer is $26^6 - (25^6 + 25^6 - 24^6)$.
6d) First choose the positions for a and b; this can be done in C(6,2) ways since once we pick two positions, we put the a in the leftmost and the b in the other. There are four remaining positions, and these can be filled in P(24,4) ways since there are 24 letters left. Therefore, the answer is C(6,2) * P(24,4).

7) 0, 1, 6, 8, 9 can be flipped and still make sense. The number of usable signs (first digit nonzero) that flip to a different usable sign is 4 * 5 * 5 * 4 (note that 0 cannot be in the first or last position since we are flipping the sign). We need to subtract symmetric signs from this group also, i.e., signs like 1001, 1881, etc. There are 4 * 5 symmetric signs. So 9000 - (380/2) = 8810 different signs must be made.

8) First we seat the 5 “good” boys (5!). Then we consider the spaces between them and at the ends. There are 6 possible spaces; once we use one for Josh, we have 5 left to choose from for Garrett. Then once we place Garrett, we have 4 left to choose from for Sam: 5! * 6 * 5 * 4.