CS154 Assignment 4. Solutions to Problem 2

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Show that the complement of a CFL need not be context free
Solution : We make use of the *De-Morgan's* laws to provide a solution. It is easy to see that the intersection of two CFLs need not be a CFL. As an example consider the languages

$$L_1 = \{a^i\ b^i\ c^j \mid i, j > 0\} \text{ , and}$$

$$L_2 = \{a^i\ b^j\ c^j \mid i, j > 0\}$$

$$L_1 \bigcap L_2 = \{a^i\ b^i\ c^i \mid i > 0\}$$

It is easy to see that $L_1 \bigcap L_2$ is not a CFL by pumping lamma for CFLs. Let us assume (for sake of contradiction) that CFLs were closed under complementation. Then we know by deMorgan's theorems that

$$L_1 \bigcap L_2 = \overline{\overline{L_1} \bigcup \overline{L_2}}$$

Using this , we may prove that If CFLs are closed under complementation, they are also closed under intersection. But since they are not closed under intersection, they cannot be closed under complementation.