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1. Show that it is undecidable whether a given context-free grammar generates at least one palindrome. *Hint*: Reduce PCP to this problem by constructing from each instance of PCP a grammar whose language contains a palindrome if and only if the PCP instance has a match.

Assume that the problem of determining whether a given context-free grammar generates at least one palindrome is decidable. Let the decider for this problem be denoted by *D*.

Use decider *D* in the construction of *S*, a decider for the PCP, as follows:

S: Given an instance $P = \{ [t_1, b_1], [t_2, b_2], \dots , [t_k, b_k] \}$,
of the PCP, construct a CFG **G** with the rule

$$T \rightarrow t_1T(b_1^R) \mid t_2T(b_2^R) \mid \dots \mid t_kT(b_k^R) \mid t_1(b_1^R) \mid t_2(b_2^R) \mid \dots \mid t_k(b_k^R)$$

[Where (b_i^R) is formed by reversing the characters of b_i in the standard way.
I.e. if $b_i = x_1 \dots x_n$ (where $|x_i| = 1$),
then $(b_i^R) = x_n \dots x_1$.]

Run *D* on input **G**: If *D* accepts, *accept*. If *D* rejects, *reject*.

Since we have constructed *S* to decide the PCP, an undecidable problem, we know our initial assumption—that decider *D* exists—must be false. We conclude that it is undecidable whether a given CFG generates at least one palindrome.