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Handout #4a
CS109

Problem Set #1 Solutions

Propositional Logic

Rosen, Section 1.1:

8a) r ^ ~q
8b) p ^ q ^ r
8e) (p ^ q) -> r
8f) r <-> (q v p)

24e) (p v q) ^ ~r

p	q	r	(pvq)	~r	(pvq)^~r
T	T	T	T	F	F
T	F	T	T	F	F
F	T	T	T	F	F
F	F	T	F	F	F
T	T	F	T	T	T
T	F	F	T	T	T
F	T	F	T	T	T
F	F	F	F	T	F

Rosen, Section 1.2:

8d) [(p v q) ^ (p -> r) ^ (q -> r)] -> r

p	q	r	(pvq)	(p->r)	(q->r)	^	^	->
T	T	T	T	T	T	T	T	T
T	F	T	T	T	T	T	T	T
F	T	T	T	T	T	T	T	T
F	F	T	F	T	T	F	T	T
T	T	F	T	F	F	F	F	T
T	F	F	T	F	T	T	F	T
F	T	F	T	T	F	T	F	T
F	F	F	F	T	T	F	F	T
		*					*	

18) ~(p   q) and p <-> q are logically equivalent: The first of these expressions is T when p and q both have the same truth value (both T or both F); remember we are taking the "~" of XOR. The exact same thing is true for p <-> q.

36) Show that {|} forms a functionally complete set: The truth table for the Sheffer stroke is

p	q	p | q
T	T	F
T	F	T
F	T	T
F	F	T

From this truth table, we can see that p | p is logically equivalent to ~p.   We can also see that ((p | p) | (q | q)) is logically equivalent to (p v q). The solution to exercise 29 (in the Student Solutions Guide) proves that any compound proposition is logically equivalent to one that uses only ~ and v. Our observations above indicate that we can get rid of all the negations and disjunctions by using NANDs. Thus, every compound proposition can be converted into a logically equivalent compound proposition involving only NANDs.

40) Show that if p <=> q and q <=> r, then p <=> r.

To say that p <=> q is to say that the truth tables are identical; similarly to say q <=> r is to say that the truth tables are identical. If the truth tables for p <=> q and the truth tables for q <=> r are identical, then the truth tables for p and r are identical (this is a fundamental axiom of the notion of equality). Therefore, p <=> r.

Propositional Logic Proofs

1) L: Fred lives in France
S: Fred speaks French
D: Fred drives a Miata
B: Fred rides a bike.

~L -> ~S
~D
L -> B
S v D

B

1) ~L -> ~S	given
2) S -> L	contrapositive
3) L -> B	given
4) S -> B	syllogism
5) S v D	given
6) ~D	given
7) S	disjunctive syllogism
8) B	S is true (see (4))

2) not valid: Let the statements have the following truth values. Then the premises are true, but the conclusion is false.

p = T
q = F
w = T
r = F
t = T
~t = F

Predicate Logic

Rosen, Section 1.3:

12a) x F(x, Fred)

12b) y F(Evelyn, y)

12c) x y F(x, y)

12d) ~ x y F(x, y)

12e) y x F(x, y)

12f) ~ x [F(x, Fred) ^ F(x, Jerry)]

12g) y1 y2 [F(Nancy, y1) ^ F(Nancy, y2) ^ y1 != y2 ^ y (F(Nancy, y) -> (y = y1 v y = y2))]

12h) y [x F(x, y) ^ z (x F(x, z) -> z = y)]

12i) ~ x F(x, x)

12j) x y [x != y ^ F(x, y) ^ z ((F(x, z) ^ z != x) -> z = y)]

18) Let P(s,c,m) be the statement that student s has class standing c and is majoring in m. The variable s ranges over students in the class; c ranges over the four class standings; and, m ranges over all possible majors.

18a) s m P(s, junior, m)	true
18b) s c P(s, c, CS)	false (there are some math majors)
18c) s c m (P(s,c,m) ^ (c != junior) ^ (m != math))	true (there is a sophomore CS major)
18d) s (c P(s, c, CS) v m P(s, sophomore, m))	false (there is a freshman math major)
18e) m c s P(s, c, m)	false (It cannot be that m is math since there is no senior math major; it cannot be that m is CS since there is no freshman CS major. Nor, can m be any other major.)

42) We need to show that each of these propositions implies the other. Suppose that x P(x) v x Q(x) is true. We want to show that x y(P(x) v Q(y)) is true. By our hypothesis, one of two things must be true: either P is universally true or Q is universally true. In the first case, x y(P(x) v Q(y)) is true since the first expression in the disjunction is true, no matter what x and y are; in the second case, x y(P(x) v Q(y)) is true since the second expression in the disjunction is true, no matter what x and y are. Now we need to prove the converse. Suppose x y(P(x) v Q(y)) is true. Wa want to show that x P(x) v x Q(x) is true. If x P(x) is true then we are done. Otherwise, P(x0) must be false for some x0 in the domain of discourse. For this x0, the hypothesis tells us that P(x0) v Q(y) is true, no matter what y is. Since P(x0) is false, Q(y) must be true for each y. In other words, y Q(y) is true or to change the name of the meaningless quantified variable, x Q(x) is true. This implies that x P(x) v x Q(x) is true, as desired.

Resolution

1) (q -> r), (p -> q), (p v q) |- r

1) ~q v r	
2) ~p v q	
3) p v q	
4) ~r	negation of the conclusion
5) q	resolution of (2) and (3)
6) r	resolution of (1) and (5)
7) false	resolution of (4) and (6)

2) [(p ^ q) -> r], (r -> ~p), (~q -> ~p), (~r -> p) |- r

1) ~p v ~q v r	
2) ~r v ~p	
3) q v ~p	
4) r v p	
5) ~r	negation of the conclusion
6) r v ~q	resolution of (1) and (4)
7) ~p v r	resolution of (6) and (3)
8) ~p	resolution of (7) and (2)
9) r	resolution of (8) and (4)
10) false	resolution of (9) and (5)