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CS109

Handout #18

due: 2/18, 92 points

Problem Set #5

Sets and relations are by nature, not too difficult to understand conceptually, but there are some challenging areas of particular relevance to us as computer scientists. They are important data models, and they also provide us with excellent material for continuing to develop skills in doing proofs.

On this problem set, if we ask for a proof, be formal and precise (i.e., use a statement and reason chart format, or a *well-organized, detailed* paragraph). If we ask for an inductive proof, include the six steps. On all questions, always show your work.

Sets and Paradoxes

1) (10 points) Rosen Section 1.7 #40 (hint: look at the solution to Exercise 37 in 1.7).

2) (10 points) Prove or disprove: There exists a book that refers to all those books and only those books that do not refer to themselves.

Relations

3) (8 points) Rosen, Section 6.3, #18. Please explain in general terms, not using specific examples.

4) (10 points) Rosen, p. 433 #2.

5) (6 points) Rosen, Section 6.5, #10

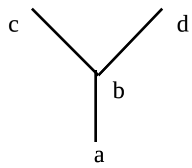
6) (12 points) Rosen, Section 6.5, #38

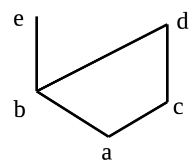
7) (6 points) Rosen, Section 6.1, #6

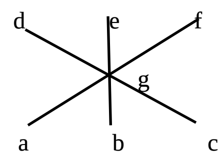
8) (4 points) Let A be the set of all lines in a plane. A binary relation is defined on A as follows: For all l_1 and l_2 in A, $l_1 R l_2$ iff l_1 is perpendicular to l_2. Is R reflexive, symmetric, transitive, or none of these?

9) A subset of a partial order such that every two elements of this subset are comparable is called a *chain.* A *maximal chain* is a chain that is not a subset of a larger chain. A subset of a partial order is called an *antichain* if every two elements of this subset are noncomparable.

a) For each of the following Hasse diagrams, find all the maximal chains and antichains (12 points):







b) (4 points) Find an antichain with the greatest number of elements in the poset of Exercise 24, section 6.6 in Rosen.

Induction Practice

Don't forget the six steps!

10) (10 points) Prove that a set with n elements has n(n-1)(n-2)/6 subsets containing exactly three elements whenever n is an integer greater than or equal to 3.