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Handout #11
CS109

due: 1/31, 90 pts

Problem Set #3

Combinatorics

Combinatoric problems can be very misleading. They seem simple, but may turn out to be surprisingly involved (of course, some of them *are* simple too). Many of the following problems will require careful thought and analysis. You may find that you will need to come back to a problem again and again to make sure you are including everything, and at the same time, not including too much.

As discussed in class, the best way to solve these problems is to: a) reduce the numbers so you can actually count what needs to be counted, and then watch for patterns and ways to generalize; and, b) map it to problems you have seen before. There are many examples of counting problems in the textbook and in the Combinatorics handout. These can serve as warm-ups and models for many of the problems below.

On all combinatoric problems, always show your work so we can award partial credit. Proofs are not required here; just show us your reasoning in deriving solutions. Also, it is not necessary to multiply formulas out to get exact values. Just show the values plugged into the appropriate formula.

1) Given 8 different English books, 7 different French books, and 5 different German books:

a) How many ways are there to select one book? (2 points)
b) How many ways are there to select three books, one of each language? (2 points)
c) How many ways are there to make a row of three books in which exactly one language is missing (the order of the three books makes a difference)? (6 points)

2) Rosen, Section 4.1

#40 (4 points)
#42 (6 points)

3) How many ways are there to place two identical queens on an 8 x 8 chessboard so that the queens are not in a common row, column or diagonal? (10 points)

4) A "scone shop" (only in Palo Alto....) has plain scones, raisin scones, blueberry scones, raspberry scones, apple scones, and cherry scones. The scones within a type are indistinguishable and the order of selection does not matter. You may assume that there are an infinite number of scones of each type (only in Palo Alto....). How many ways are there to choose:

a) 3 dozen scones? (3 points)
b) 2 dozen scones with at least two of each type? (5 points)

c) 2 dozen scones with at least one plain scone, at least two raisin scones, at least three blueberry scones, at least one raspberry scone, at least two apple scone, and no more than three cherry scones? (7 points)

5) Rosen, Section 4.3

#18 (5 points each)

6) Rosen, Section 4.3

#24 a (3 points)
#24 b (5 points)
#24 d (7 points)

7) A firm produces 4-digit house-number signs. The first digit cannot be 0. The company need not produce every number from 0000 to 9999 because some signs can be gotten from others by rotating them 180 degrees. For example, 6611 is 1199 turned around. How many different signs must the firm make? (7 points)

8) You have just been given the charge of 8 six-year-old boys for an afternoon. You could take them to a baseball game or Star Wars, but you decide to expand their horizons and take them to the ballet. Elliot, Brian, Cal, Jason, and Nik are fairly well-behaved boys (for six-year-olds). But, Josh, Sam and Garrett are little time bombs ready to go off when you least expect it. It is critical that you do not allow them to sit together. So, here you are at the ballet with 8 seats (assuming you are sitting in the row behind them (or in the next county!)). Find the number of ways of seating the 8 boys so that Josh, Garrett and Sam do not sit together. (8 points)