CS154 Assignment 6

February 20, 2001

The homework should be done without collaboration!
Please submit every problem on a separate sheet.
Assignment due: 02/21/2001 at 3:15pm.

1. Show that the set of all functions from $\mathbb{N}$ to $\mathbb{N}$ (the natural numbers) is uncountable.

2. A **useless state** in a Turing machine is one that is never entered on any input string. Consider the problem of determining, given a Turning machine and one of its states, whether that state is useless. Show that this problem is undecidable.

3. Prove that a language L is recursive enumerable iff and only if there exists a decidable language D such that $L = \{x \mid \exists y \text{ such that } (x, y) \in D\}$.

4. Consider the variant of the Post Correspondence Problem, in which the dominos are colored red or green, and a valid matching must be an alternating sequence of red and green dominos. Show that this problem is undecidable.

5. Let a biNFA be an nondeterministic finite automaton which 2 heads reading the same input. The transition function has the form $\delta : Q \times \Sigma_\varepsilon \times \Sigma_\varepsilon \rightarrow \mathcal{P}(Q)$. If $\delta(q, a, b)$ contains r, this means that the machine can move from state q to state r by reading an a with its first head, and a b with its second head. Either a or b can be ε, and because of this it it possible that the two heads point at different positions in the string. The heads scan the input from left to right only, and the input is accepted it both heads are at the end of the input tape and the machine is in a final state.

   Show that it is undecidable whether a biNFA accepts the empty language.
   Hint: Use a reduction involving Post Correspondence Problem.

Extra credit problem (optional): A set S of languages can represented by a set of descriptions of Turing Machines that recognize exactly the languages in S. Let's say that *S has a decidable representation* if it can be represented by a decidable set of descriptions of Turning machines. Likewise *S has a recursive enumerable representation* if it can be represented by a recursive enumerable set of descriptions of Turning machines. Show that a set of languages has a decidable representation if and only if it has a recursive enumerable representation.