CS154 Assignment 7

February 22, 2001

The homework should be done without collaboration!
Please submit every problem on a separate sheet.
Assignment due: 02/28/2001 at 3:15pm.

1. Show that it is undecidable whether a given context-free grammar generates at least one palindrome. *Hint:* Reduce PCP to this problem by constructing from each instance of PCP a grammar whose language contains a palindrome if and only if the PCP instance has a match.

2. Is it decidable whether a given Turing machine

   (a) recognizes the empty language?

   (b) recognizes a non-regular language?

   (c) recognizes a recursive language?

   (d) recognizes a recursive enumerable language?

   (e) accepts exactly 3 words?

   (f) accepts the empty string?

   (g) halts in less than 7 steps, on every input?

   (h) will ever write the letter @ on its tape when given input ah?

   Explain your answers **briefly.**

3. Consider the equivalence problem for Turing machines: given descriptions of Turing machines M and N, do M and N recognize the same language? Is this problem decidable? Is it recursive enumerable? Prove your answers succinctly.

4. Let us call a Turing machine *stupid* if for some input it ever tries to move its head left when it is already on the leftmost position of the input tape. Is the set of stupid Turing machines decidable? Does there exist an algorithm to convert any stupid Turing machine into a non-stupid one that accepts the same language? Prove your answers succinctly.

5. Consider the fragment of first order logic in which no universal quantifiers ("$\forall$", "forall") occur and in which all existential quantifiers ("$\exists$", "there exists") occur at the outside of formulas only, i.e. there are only formulas $\exists x_1 \exists x_2 \cdots \exists x_n \varphi$ in which φ is quantifier-free. Show that this fragment is undecidable.

Extra credit problem (optional): Prove that every infinite recursive enumerable language contains an infinite recursive sublanguage.