CS 154 - Introduction to Automata and Complexity Theory

Spring Quarter, 2001

Assignment #3 - Due date: Wednesday, 5/02/01

Remark: Since I went nuts and asked you to solve 6 questions, some of which are not all that easy, I have increased the total score of this problem set to 120 from the usual 100.

Problem 1. *(25 points)* For any language L, we define the language $L^{\downarrow k}$ as all strings in L of length no more than k; that is,

$$L^{\downarrow k} = \{w \mid w \in L \text{ and } |w| \leq k\}.$$

Show that the following claims are true. You may use results from the class or the textbook to help with this task.
(a). Suppose that a DFA M has n states and its language is $L = L(M)$. Then, L is non-empty if and only if $L^{\downarrow n}$ is non-empty.
(b). Suppose that a DFA M has n states and its language is $L = L(M)$. Then, the complement language $\overline{L}$ has a DFA with at most n states.
(c). Let M_1 be a DFA with n_1 states, and M_2 be a DFA with n_2 states. Denote by L_1 and L_2 the languages of the machines M_1 and M_2, respectively. Then, the language $L = L_1 \cap L_2$ must have a DFA M with no more than $n = n_1 \times n_2$ states.
(d). Let M_1 be a DFA with n_1 states, M_2 be a DFA with n_2 states, and define $n = n_1 \times n_2$. Denote by L_1 and L_2 the languages of the machines M_1 and M_2, respectively. Then, $L_1 = L_2$ if and only if $L_1^{\downarrow n} = L_2^{\downarrow n}$. (*Hint:* Use the claims from parts (a), (b) and (c).)

Problem 2. *(20 points)* Show that the class of regular languages is closed under the following operation:

$$max(L) = \{w \mid w \text{ is in } L \text{ and, for all } x \neq \epsilon \text{ the string } wx \text{ is } \textit{not} \text{ in } L\}$$

Informally, $max(L)$ is the set of all strings in L which cannot be extended into another string in L by adding symbols at the end.

Problem 3. *(20 points)* Give an algorithm to solve the following decision problem. You must justify the correctness of the algorithm to receive full credit.

> Given a regular language R over $\Sigma = \{0, 1\}$, verify that R does *not* contain any string that begins with 1 *and* ends with 1.

Problem 4. *(15 points)* Consider the context-free grammar $G = (\{X, Y, Z, S\}, \{0, 1\}, P, S)$, where the productions in P are as follows.

1. $S \rightarrow \epsilon \mid Y \mid Z$
2. $X \rightarrow \epsilon \mid X \mid XX$

3. $Y \rightarrow 01Y \mid X$

4. $Z \rightarrow 0Z \mid Y \mid \epsilon$

(a). Give a regular expression for the set of all sentences that can be derived from Y.
(b). Give the simplest regular expression you can for $L(G)$.
(c). Let $\widehat{G}$ be the grammar obtained from G by dropping the production $X \rightarrow \epsilon$. Give the simplest regular expression you can for the language $L(\widehat{G})$.

Problem 5. *(20 points)* Give a context-free grammar for the language L consisting of all strings from $\{0,1\}^*$ that are *not* of the form ww. Thus, the *complement* of L would contain only strings that can be viewed as some string w repeated twice.

Problem 6. *(20 points)* In the book (Sections 5.4.1 and 5.4.2) we have discussed the issue of ambiguity of CFGs for arithmetic expressions written in the *infix* notation. In the infix notation, binary operators are written between the two operands (e.g., $x + y$ or $x \times y$). There are other ways of writing operators such as the *prefix* notation where we would write $+xy$ or $\times xy$, and the *postfix* notation where we would write $xy+$ or $xy\times$, to denote the same arithmetic expressions.

The following context-free grammar generates arithmetic expressions in the postfix notation which is used, for example, in the programming language APL. Notice the lack of parentheses in the grammar.

$$S \longrightarrow SS+ \mid SS- \mid SS* \mid x \mid y$$

(a). For the string $xxy * x - +$, find a derivation tree.
(b). For the string $xxy * x - +$, find a leftmost derivation.
(c). For the string $xxy * x - +$, find a rightmost derivation.
(d). Is this grammar ambiguous? Give a brief justification. (*While a formal proof is not required, think about how you might give such a proof.*)

Reading Assignment:

We are done with Chapter 4. We are already into Chapter 5 and will finish with Chapter 6 and portions of Chapter 7 before the midterm. While we will not cover Section 5.3 and Section 6.4 in class, they make for very interesting reading and are strongly recommended.