CS154 HW1
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Problem 3: Prove by *induction*: The maximum number of nodes in a binary tree of height h is $2^h - 1$

Base case: A tree of height 1 can have at most one node (the root). $1 = 2^1 - 1$

Inductive step: Two binary trees of height h can be joined into a tree of height h+1 by joining their root nodes with a new root *(Figure 1)*. The two original trees can have

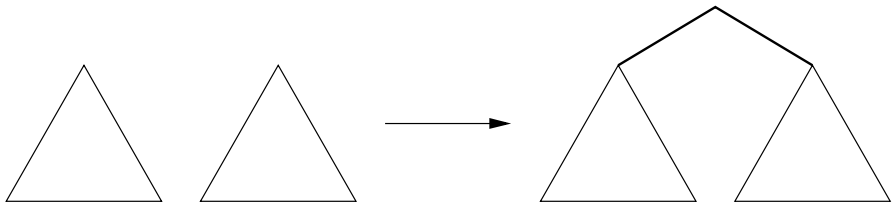

Figure 1: Joining two trees

at most $2^h - 1$ nodes (by our inductive hypothesis). The tree of height h+1 contains all of the nodes in the original two trees, plus the new root node. Thus, the tree of height h+1 will have at most $2 * (2^h - 1) + 1 = 2^{h+1} - 1$ nodes. *QED*