

CS154 Assignment 9

March 7, 2001

The homework should be done without collaboration!

Please submit every problem on a separate sheet.

Assignment due: **Monday** 03/12/2001 at 3:15pm.

1. Consider the following algorithm to test whether a given number n is prime

```
Algorithm PRIME-TEST ( $n$ )
  For  $i=2$  to  $n/2$  do
    if ( $n$  is a multiple of  $i$ ) ,
      return "non-prime";
  end-for
  return "prime".
```

So far, computer scientists have held the unproven view that primality testing is a problem that is not likely to be in **P**. Does this algorithm qualify as a polynomial time procedure? Explain your answer.

2. A weighted graph is defined as a graph whose edges have integer numbers or weights associated with them. Any path in the weighted graph has a weight given by the sum of the weights along its edges. Prove that given a weighted graph G and a number k , the problem of whether G has a simple path of weight k is NP-complete. (A simple path is one where no nodes are repeated along the path). You may assume that the **HAMILTONIAN PATH** problem is NP-complete. The definition of that problem and the proof of its NP-completeness are in the book.
3. Show that every problem in the class **P** is polynomial time reducible to every other problem in **P**.¹
4. A 2-way NFA is a NFA that has the additional ability to move its head to the left. It can be shown that 2-way NFAs recognize the same class of languages as (1-way) NFAs, the regular languages. Show that a language is regular iff it is in $\text{SPACE}(k)$ for some constant k .
5. Sipser 7.26 (on page 274). A copy of this problem is in the handout hang out.

Extra credit problem (optional): Show that the following problem is NP-complete: Given a family of sets and a number k , are there k pairwise disjoint sets in the family?

¹If you follow the definition of reducibility from the book there is one exception to this statement. Which one?