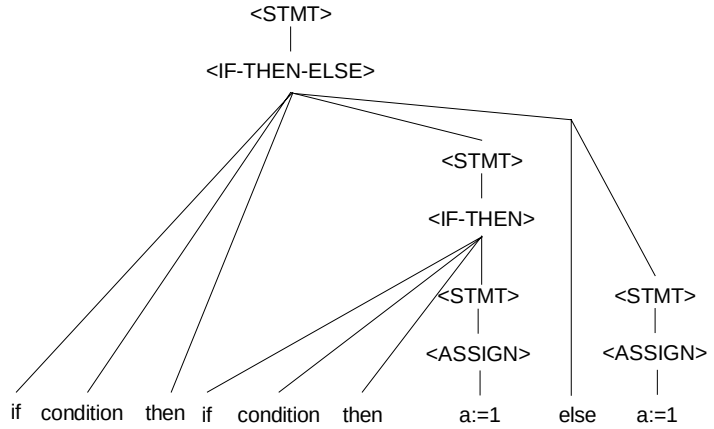
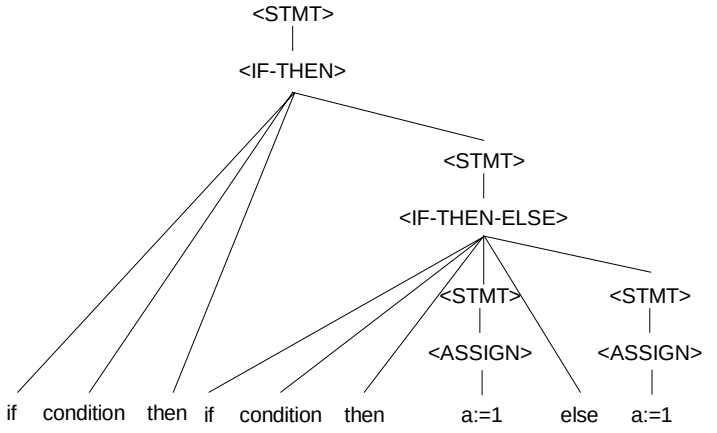
Problem 1

Solution:

a. For grammar G, we have the following string:
if condition then if condition then a:=1 else a:=1

To prove grammar G is ambiguous, we show the following two parse trees describing the above statement.





Or we may show two possible different leftmost derivations in grammar G for the above string producing the same result.

I.

$$\langle STMT\rangle \rightarrow \langle IF-THEN-ELSE\rangle$$
$$\rightarrow if\ \ condition\ then\ \langle STMT\rangle\ else\ \langle STMT\rangle$$
$$\rightarrow if\ \ condition\ then\ \langle IF-THEN\rangle\ else\ \langle STMT\rangle$$
$$\rightarrow if\ \ condition\ then\ if\ \ condition\ then\ \langle STMT\rangle\ else\ \langle STMT\rangle$$
$$\rightarrow if\ \ condition\ then\ if\ \ condition\ then\ \langle ASSIGN\rangle\ else\ \langle STMT\rangle$$
$$\rightarrow if\ \ condition\ then\ if\ \ condition\ then\ a:=1\ else\ \langle STMT\rangle$$
$$\rightarrow if\ \ condition\ then\ if\ \ condition\ then\ a:=1\ else\ \langle ASSIGN\rangle$$
$$\rightarrow if\ \ condition\ then\ if\ \ condition\ then\ a:=1\ else\ a:=1$$

II.

$$\begin{aligned}
\langle STMT\rangle &\to \langle IF-THEN\rangle \\
&\to \textit{if condition then } \langle STMT\rangle \\
&\to \textit{if condition then } \langle IF-THEN-ELSE\rangle \\
&\to \textit{if condition then if condition then } \langle STMT\rangle \textit{ else } \langle STMT\rangle \\
&\to \textit{if condition then if condition then } \langle ASSIGN\rangle \textit{ else } \langle STMT\rangle \\
&\to \textit{if condition then if condition then } a := 1 \textit{ else } \langle STMT\rangle \\
&\to \textit{if condition then if condition then } a := 1 \textit{ else } \langle ASSIGN\rangle \\
&\to \textit{if condition then if condition then } a := 1 \textit{ else } a := 1
\end{aligned}$$

b. The ambiguity is caused by the multiple selections in <STMT> rule. To remove the ambiguity, we choose the rule that match each ***else*** with the closest previous unmatched ***then***. Namely, only the second of the two parse trees given in a. is legal. The basic idea is that a statement appearing between a ***then*** and an ***else*** must be "matched", i.e., it must not end with an unmatched ***then*** followed by any statement, for the ***else*** would then be forced to match this unmatched ***then***. A matched statement is either an if-then-else statement containing no unmatched statements or it is any other kind of unconditional statement. Now we change the grammar G into G' as follow.

$$\begin{aligned}
\langle STMT\rangle &\to \langle MATCHED_STMT\rangle \mid \langle IF-THEN\rangle \mid \langle IF-THEN-ELSE\rangle \\
\langle MATCHED_STMT\rangle &\to \langle ASSIGN\rangle \mid \langle BEGIN-END\rangle \\
&\quad \mid \textit{if condition then } \langle MATCHED_STMT\rangle \textit{ else } \langle MATCHED_STMT\rangle \\
\langle IF-THEN\rangle &\to \textit{if condition then } \langle STMT\rangle \\
\langle IF-THEN-ELSE\rangle &\to \textit{if condition then } \langle MATCHED_STMT\rangle \textit{ else } \langle STMT\rangle \\
\langle BEGIN-END\rangle &\to \textit{begin } \langle STMT-LIST\rangle \textit{ end} \\
\langle STMT-LIST\rangle &\to \langle STMT-LIST\rangle\langle STMT\rangle \mid \langle STMT\rangle \\
\langle ASSIGN\rangle &\to a := 1
\end{aligned}$$

Alternatively, we can modify grammar G into G" so that only the first of the two parse trees given in a. is legal. For this case, a statement appearing after a ***then*** without any ***else*** must be "matched", i.e., it must not end with an unmatched ***then*** followed by any statement.

$$\begin{aligned}
\langle STMT\rangle &\to \langle MATCHED_STMT\rangle \mid \langle IF-THEN-ELSE\rangle \\
\langle MATCHED_STMT\rangle &\to \langle ASSIGN\rangle \mid \langle BEGIN-END\rangle \\
&\quad \mid \textit{if condition then } \langle MATCHED_STMT\rangle \\
\langle IF-THEN-ELSE\rangle &\to \textit{if condition then } \langle STMT\rangle \textit{ else } \langle STMT\rangle \\
\langle BEGIN-END\rangle &\to \textit{begin } \langle STMT-LIST\rangle \textit{ end} \\
\langle STMT-LIST\rangle &\to \langle STMT-LIST\rangle\langle STMT\rangle \mid \langle STMT\rangle \\
\langle ASSIGN\rangle &\to a := 1
\end{aligned}$$