Notes on LALR parsing

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The motivation for LALR

Because a canonical LR parser splits states based on differing lookahead sets, it typically has many more states that the corresponding SLR parser. Potentially it could require the Cartesian product of all possible LR(0) configurations and the power set of the terminals— yikes! It never actually gets that bad in practice, but a canonical LR parser for an ordinary programming language might have an order of magnitude more states than an SLR parser. Is there something in between?

With LALR (*lookahead LR*) parsing, we attempt to reduce the number of states in an LR(1) parser by merging similar states. This will reduce the number of states down the same number as in SLR(1), but with still retaining some of the power of the LR(1) lookaheas. Let's examine the LR(1) configurating sets from an example given in the LR parsing handout.

S' → S
S → XX
X → aX
X → b

I_0:
S' → •S, $
S → •XX, $
X → •aX, a/b
X → •b, a/b

I_1:
S' → S•, $

I_2:
S → X•X, $
X → •aX, $
X → •b, $

I_3:
X → a•X, a/b
X → •aX, a/b
X → •b, a/b

I_4:
X → b•, a/b

I_5:
S → XX•, $

I_6:
X → a•X, $
X → •aX, $
X → •b, $

I_7:
X → b•, $

I_8:
X → aX•, a/b

I_9:
X → aX•, $

Notice that some of the LR(1) states look suspiciously similar. Take I_3 and I_6 for example. These two states are virtually identical— they have the same number of items, the core of each item is identical, and they differ only in their lookahead sets. This observation may make you wonder if it possible to merge them into one state. The same is true of I_4 and I_7 and I_8 and I_9. If we did merge, we would end up removing those six states and replacing with just these three:

I_{36}:
X → a•X, a/b/$
X → •aX, a/b/$
X → •b, a/b/$

I_{47}:
X → b•, a/b/$

I_{89}:
X → aX•, a/b/$

But isn't this just SLR(1) all over again? In the above example, yes it is, since after the merging we did end up with the complete follow sets as the lookahead. This is not always the case however:

```
S'  ← S
S   ← Bbb | aab | bBa
B   ← a
```

I_0:
S' ← •S, $
S ← •Bbb, $
S ← •aab, $
S ← •bBa, $
B ← •a, b

I_1:
S' ← S•, $

I_2:
S ← B•bb, $

I_3:
S ← a•ab, $
B ← a•, b
....

In SLR(1) parsing, a shift-reduce conflict would occur in state 3 if the next input token was anything in `Follow(B)` which includes `a` and `b`. In LALR(1), state 3 will shift on `a` and reduce on `b`. Intuitively, this is because the LALR(1) state "remembers" that we got to state 3 after first seeing an `a`. Thus we are trying to parse either `Bbb` or `aab`. In order for that first `a` to be a valid reduction to `B`, the next input token has to be exactly `b` since that is the only symbol that can follow `B` in this particular context. Although elsewhere an expansion of `B` can be followed by an `a`, we consider only the subset of the follow set that can appear here, we are able to avoid the conflict that an SLR(1) parser would have.

Conflicts in LALR mergings

An important question arises: can merging states ever introduce new conflicts? A shift-reduce conflict cannot exist in a merged set unless the conflict were already present in the LR(1) configurating sets themselves. When merging, the two sets must have the same core items. If the merged set has a configuration that shifts on `a` and another that reduces on `a`, both configurations must have been present in the original sets, and at least one of those sets had a conflict already.

Reduce-reduce conflicts, however, are another story. Consider the following grammar:

```
S'  ← S
S   ← aBc | bCc | aCd | bBd
B   ← e
C   ← e
```

The LR(1) configurating sets are as follows:

I_0:
S' ← •S, $
S ← •aBc, $
S ← •bCc, $
S ← •aCd, $
S ← •bBd, $

I_1:
S' ← S•, $

I_2:
S ← a•Bc, $
S ← a•Cd, $
B ← •e, c
C ← •e, d

I_3:
S ← b•Cc, $
S ← b•Bd, $
B ← •e, d
C ← •e, c

I_4:
S ← aB•c, $

I_5:
S ← aC•d, $

I_6:
B ← e•, c
C ← e•, d

I_7:
S ← bC•c, $

I_8:
S ← bB•d, $

I_9:
B ← e•, d
C ← e•, c

I_{10}:
S ← aBc•, $

I_3: S ← b • Cc, $
S ← b • Bd, $
C ← e, c
B ← e, d

I_4: S ← aB • c, $

I_5: S ← aC • d, $

I_{11}: S ← aCd • , $

I_{12}: S ← bCc • , $

I_{13}: S ← bBd • , $

We try to merge I_6 and I_9 since they have the same core items and they only differ in lookahead:

I_{69}: C ← e, c/d
B ← e, d/c

However, this creates a problem. The merged configurating set allows a reduction to either `B` or `C` when next token is `c` or `d`. This is a reduce-reduce conflict and can be an unintended consequence of merging LR states. When such a conflict arises in doing a merging, we say the grammar is not LALR(1).

LALR table construction

A LALR parsing table is of the same style as LR and SLR (with action and goto sections) and drives the parser in the same way. A LALR table should have fewer states (rows) than the corresponding LR table. The LALR table will have the same number of rows as the SLR, but there may be fewer action entries— some reductions are not valid if we are more precise about the lookahead. This might mean some conflicts are avoided so an action cell that has conflicting actions in SLR table may have a unique entry in an LALR.

The "brute-force" method

There are two ways to construct LALR(1) parsing tables. The first and most obvious way is to construct LR(1) table and merge the sets manually. This is sometimes referred as the "brute-force" way. If you don't mind first finding all the multitude of states required by the canonical parser, compressing the LR table into the LALR version is straightforward.

1. Construct all canonical LR(1) states.
2. Merge those states that are identical if the lookaheads are ignored, i.e. two states being merged must have the same number of items and the items have the same core (i.e. the same productions, differing only in lookahead). The lookahead on merged items is the union of the lookahead from the states being merged.
3. The successor function for the new LALR(1) state is the union of the successors of the merged states. If the two configurations have the same core, then the original successors must have the same core as well, and thus the new state has the same successors.
4. The action and goto entries are constructed from the LALR(1) states as for the canonical LR(1) parser.

Let's do an example to make this more clear. Consider the LR(1) table for the grammar given on page 1 of this handout. There are nine states.

State on top of stack	Action			Goto	
	a	b	$	S	X
0	s3	s4		1	2
1			acc		
2	s6	s7			5
3	s3	s4			8
4	r3	r3			
5			r1		
6	s6	s7			9
7			r3		
8	r2	r2			
9			r2		

Looking at the configurating sets, we saw that states 3 and 6 can be merged, so can 4 and 7, and 8 and 9. Now we build this LALR(1) table with the six remaining states:

State on top of stack	Action			Goto	
	a	b	$	S	X
0	s36	s47		1	2
1			acc		
2	s36	s47			5
36	s36	s47			89
47	r3	r3	r3		
5			r1		
89	r2	r2	r2		

A more efficient method

Having to compute the LR(1) configurating sets first means we won't save any time or effort in building a LALR parser. However, the work wasn't all for naught, because when the parser is executing, it will only need the compressed table which saves us memory. The savings can be an order of magnitude in the number of states.

However there is a more efficient strategy for building the LALR(1) states called *step-by-step merging*. The idea is that you merge the configurating sets as you go, rather than waiting until the end to find the identical ones. Sets of states are constructed as in the LR(1) method, but at each point where a new set is spawned, you first check to see whether it may be merged with an existing set. This means examining the other states to see if one with the same core already exists. If so, you merge the new set with the existing one, otherwise you add it normally.

Here is an example of this method in action:

```
S'  ← S
S   ← V = E
E   ← F | E + F
F   ← V | int | (E)
V   ← id
```

Start building the LR(1) collection of configurating sets as you would normally:

I_0: S' ← •S, $
S ← •V = E, $
V ← •id, =

I_1: S' ← S•, $

I_2: S' ← V• = E, $

I_3: V ← id•, =

I_4: S ← V = •E, $
E ← •F, $/+
E ← •E + F, $/+
F ← •V, $/+
F ← •int, $/+
F ← •(E), $/+
V ← •id, $/+

I_5: S ← V = E•, $
E ← E• + F, $/+

I_6: E ← F•, $/+

I_7: F ← V•, $/+

I_8: F ← int•, $/+

I_9: F ← (•E), $/+
E ← •F,)/+
E ← •E + F,)/+
F ← •V,)/+
F ← •int,)/+
F ← •(E),)/+
V ← •id)/+

I_{10}: F ← (E•), $/+
E ← E• + F,)/+

When we construct state I_{11}, we get something we've seen before:

I_{11}: E ← F•,)/+

It has the same core as I_6 so rather than add a new state, we go ahead and merge with that one to get:

I_{611}: E ← F•, $/+/)

We have a similar situation on state I_{12} which can be merged with state I_7. The algorithm continues like this, merging into existing states where possible and only adding new states when necessary. When we finish creating the sets, we construct the table just as in LR(1).

Error handling

As in LL(1) parsing tables, we implement error processing any of the variations of LR parsing by placing appropriate actions in the parse table. Here is a parse table with error messages inserted. The definitions of some of these error messages are given below.

E ← E + E | E * E | (E) | id

State on top of stack	Action						Goto
	id	+	*	(	)	$	E
0	s3	**e1**	**e1**	s2	**e2**	**e1**	1
1	**e3**	s4	s5	**e3**	**e2**	acc	
2	s3	**e1**	**e1**	s2	**e2**	**e1**	6
3	**e3**	r4	r4	**e3**	r4	r4	
4	s3	**e1**	**e1**	s2	**e2**	**e1**	7
5	s3	**e1**	**e1**	s2	**e2**	**e1**	8
6	**e3**	s4	s5	**e3**	s9	**e4**	
7	**e3**	r1	s5	**e3**	r1	r1	
8	**e3**	r2	r2	**e3**	r2	r2	
9	**e3**	r3	r3	**e3**	r3	r3	

Error `e1` is called from states 0, 2, 4, 5 when we encounter an operator. All of these states expect to see the beginning of an expression, i.e., an id or a left parenthesis. One way to fix: act as though id was seen in the input and push state 3 on the stack (the goto for id in these states). The message printed might be something like “missing operand”.

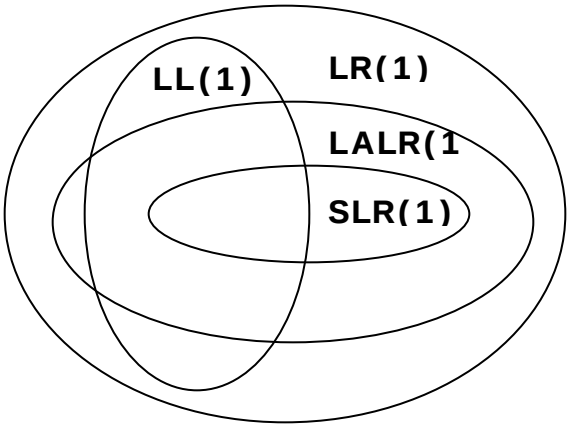
Error `e2` is called from states 0, 1, 2, 4, 5 on finding a right parenthesis where we were expecting either the beginning of a new expression (or potentially the end of input for state 1). A possible fix: remove right parenthesis from the input and discard it. The message printed: "unbalanced right parenthesis"

Error `e3` is called from state 1, 3, 6, 7, 8, 9 on finding id or left parenthesis. What were these states expecting? What might be a good fix? How should you report the error to the user?

Error `e4` is called from state 6 on finding $. What is a reasonable fix? What do you tell the user?

Relationships between LL(1) and the various LR(1) grammars

A picture is worth a thousand words:



The hierarchy of LR variants is clear: every SLR(1) grammar is LALR(1) which in turn is LR(1). There are grammars that don’t meet the requirements for the weaker forms that can be parsed by the more powerful variations.

We’ve seen several examples of grammars that are not LL(1) that are LR(1). A not immediately obvious fact is that every LL(1) grammar is LR(1). Proving this rigorously is a little tricky, but your intuition should tell you that an LR(1) parser uses more information the LL(1) parser since it postpones the decision about which production is being expanded until it sees the entire right side rather than attempting to predict after seeing just the first terminal.

Bibliography

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