2) For any integer a, $(a^2 - 2)$ is not divisible by 4.

Proof by contradiction: suppose there is an integer a such that $4 \mid (a^2 - 2)$.

Statement	Reason
$(a^2 - 2) = 4b$ for some integer b	definition of divisibility
$a^2 = 4b + 2 = 2(b + 1)$	subtraction and factoring
a^2 is an even number	definition of even
a is even	(see helper proof in ho#5)
$a = 2c$ for some integer c	definition of even
$(2c)^2 - 2 = 4c^2 - 2 = 4b$	substitution and mult
$4c^2 = 4b + 2$	addition
$2c^2 = 2b + 1$	division
c^2 is an integer so $2b + 1$ is even	contradiction: def. of odd

Since we have arrived at a contradiction (2b+1 is even), For any integer a, $(a^2 - 2)$ is not divisible by 4.

3) P(n) denotes: $n^2 - 1$ is divisible by 8 whenever n is an odd positive integer or a more precise statement: $(2n-1)^2 - 1$ is divisible by 8 using the definition of odd.

i) base case: prove that P(1) is true: $1^2 - 1 = 0$; 0 is divisible by 8

ii) induction: assume P(k): $(2k-1)^2 - 1$ is divisible by 8 and show P(k+1) is true:

$(2(k+1)-1)^2 - 1$ is divisible by 8

<u>PROOF:</u>

$(2k-1)^2 - 1 = 4k^2 - 4k$

$(2(k+1)-1)^2 - 1 = 4k^2 + 4k$

Therefore, $(2(k+1)-1)^2 - 1 = [(2k-1)^2 - 1] + 8k$

Both terms on the right side are divisible by 8.

P(k+1) is true when P(k) is true, and therefore P(n) is true for all natural numbers.

4) This maps to x1+x2+x3+x4 = 100 or C(1003, 1000).