Problem 1

Theorem:

The set F of all functions from N to N is uncountable.

Proof:

The proof is by contradiction. Suppose that F is countable, then there exists a correspondence between N and F. Each member f in F can be represented by a string of natural numbers. For example, f = "10, 4, 8, 55, ..." means that f(1) = 10, f(2) = 4, f(3) = 8, f(4) = 55, and so on. The following table shows a few entries of a hypothetical correspondence g between N and F:

n	**g(n)**				
1	**10**,	4,	8,	55,	...
2	2,	**7**,	99,	40,	...
3	4,	7,	**20**,	35,	...
4	32,	56,	126,	**24**,	...
...	...				

However, a function f* can be found in F that is not paired with any n in N. It can be constructed in a way that it is different from all the functions in the correspondence g. For example,

- f*(1) = 11 that is greater than (g(1))(1) by 1
- f*(2) = 8 that is greater than (g(2))(2) by 1
- f*(3) = 21 that is greater than (g(3))(3) by 1
- f*(4) = 25 that is greater than (g(4))(4) by 1

In other words, f*(i) is greater than (g(i))(i) by 1 for all i in N. It is different from all the functions in the correspondence g. A contradiction results and there does not exist a correspondence between N and F. Hence, the set F of all functions from N to N is uncountable.