CS154 Assignment 1 Problem 4

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Proof

To prove that a connected , undirected graph with no multiple edges or self-loops has at least two nodes with the same degree, we will show that it is impossible for every node of a graph to have a different degree. The least degree any node could have is 1 , because the graph is connected. If the graph has n nodes, the highest degree any node could have is (n-1) , the case where it has an edge to every other node. So for a graph with n nodes, the degrees that the nodes have fall into the range $1 \cdots$ n-1. Since there are less than n possibilities in the range 1 ... n-1, it is impossible for every node to have a different degree. By pigeon principle, at least two nodes have the same degree. That completes the proof.