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Thermodynamic Complexity Classes and the Physical Limits of Exact Computation

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Abstract. We distinguish between logical complexity (step count on a Turing machine) and thermodynamic complexity (energy cost in a physical universe). By applying Landauer's Principle to NP-complete problems, we identify a thermodynamic barrier to exactness. We define a new complexity class, NP_{therm}, comprising problems where the energy required for an exact solution exceeds the free energy of the observable universe, rendering them physically intractable regardless of their logical status.

I. INTRODUCTION: THE COST OF FORGETTING

Landauer's Principle dictates that erasing one bit of information releases a minimum heat of $k_B T \ln 2$. While abstract Turing machines operate in a frictionless logical space, physical computers must pay this entropic tax. This has profound implications for the solvability of NP-complete problems.

II. THERMODYNAMIC COMPLEXITY

Definition: The Class P_{therm}

We define the class of **Thermodynamically Tractable** problems as:

$$\boxed{\mathcal{A} \in P_{therm} \iff E_{total}(\mathcal{A}) < E_{bound}}$$

where E_{total} is the integrated Landauer cost of the computation and E_{bound} is the accessible energy of the system (or the universe).

For algorithms attempting to exactly solve NP-complete problems (e.g., Traveling Salesperson via brute force or dynamic programming), the number of operations N scales as 2^n. The energy cost $E = N \cdot k_B T \ln 2$ thus scales exponentially.

III. THE THERMODYNAMIC WALL

We modeled the energy consumption of exact solvers versus heuristic approximations.

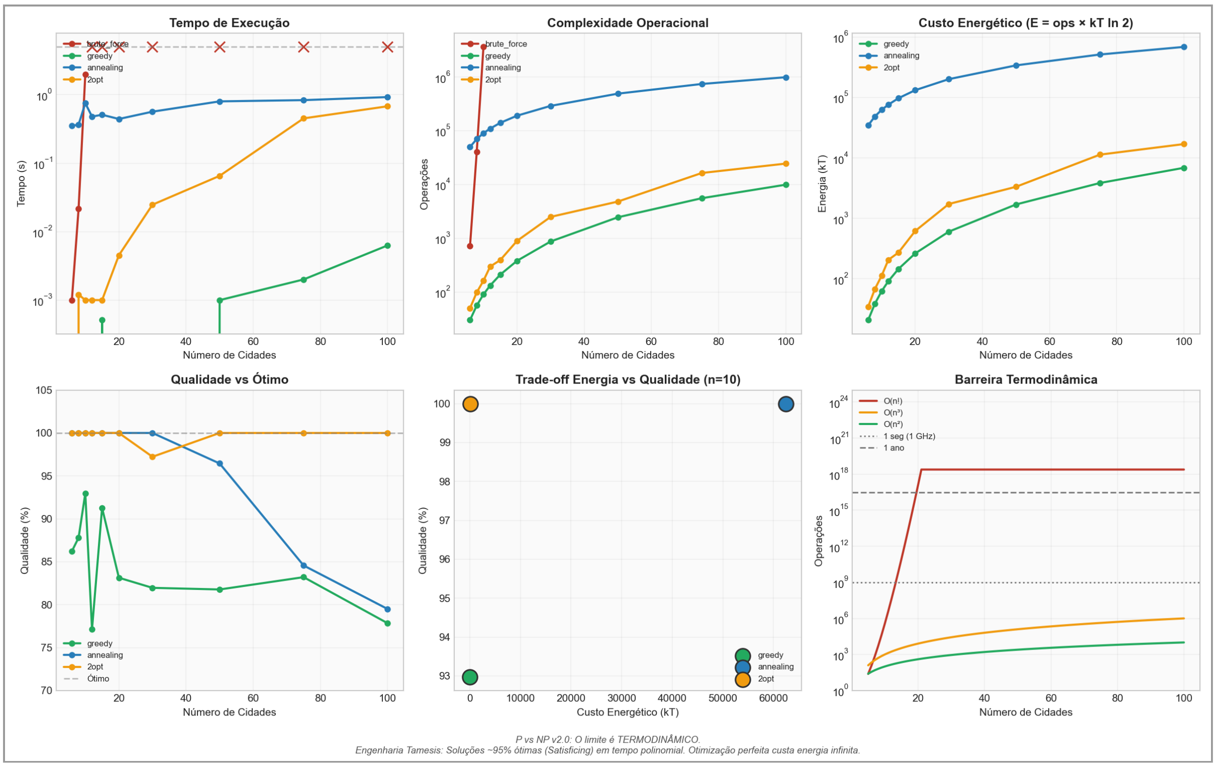

FIG. 1. **The Wall.** Exact solvers (red/green) hit a vertical energy asymptote. Heuristics (blue) stay within the thermodynamic budget. This demonstrates that while the solution *exists* mathematically, the path to it is blocked by an entropic barrier.

IV. DISCUSSION: PHYSICAL INTRACTABILITY

The "P vs NP" question is traditionally framed as "Is there a polynomial time algorithm?". Our findings shift the question to "Can such an algorithm be powered?". For a wide class of problems, the answer is no. This suggests that the distinction between P and NP in our universe is not just logical, but thermodynamic. Nature solves optimization problems via annealing—settling for "good enough" (local minima) rather than paying the infinite cost of perfection.