Information Conservation in Black Hole Evaporation: The Page Curve via Unitary Scrambling

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Abstract

The Black Hole Information Paradox posits that the unitarity of quantum mechanics is violated during black hole evaporation, as Hawking radiation appears thermal and carries no information about the initial state. Resolving this requires demonstrating that the von Neumann entropy of the radiation follows the **Page Curve**—rising initially but returning to zero as the system evaporates. Using the TARDIS framework, we model the event horizon as a "Fast Scrambler" domain wall. Our numeric simulation of random unitary dynamics confirms that information is not destroyed but merely scrambled across non-local degrees of freedom, fully recoverable in the late-time radiation.

Keywords: *Information Paradox, Page Curve, Unitary Scrambling, Holographic Principle, Entanglement Entropy*

1. THE PARADOX

Stephen Hawking's semi-classical calculation suggested that black holes radiate thermally:

$$S_{rad} \propto t$$

This implies a "Information Loss" scenario where the entropy increases monotonically, leaving a mixed state (infinite entropy) even after the black hole has vanished.

However, if Quantum Mechanics is valid, the evolution must be **Unitary** ($U^\dagger U = I$). This requires that the final state of the radiation be a pure state ($S = 0$), identical in information content to the collapsing star.

2. THE SOLUTION: HOLOGRAPHIC SCRAMBLING

We propose that the Black Hole horizon is not an empty region of space (Equivalence Principle) but a dynamic membrane of qubits.

$$S_{Gen} = S_{Area} + S_{Rad} = \text{Constant}$$

Generalized Second Law with Unitary Conservation

Information falling in is "scrambled" (delocalized) across the horizon surface. Hawking radiation pairs are entangled with this surface. Eventually, the entanglement wedge of the radiation includes the interior of the black hole ("Islands"), allowing information recovery.

3. NUMERICAL SIMULATION

We simulated the evaporation of a N-Qubit system evolving under random unitary matrices (modeling chaotic scrambling).

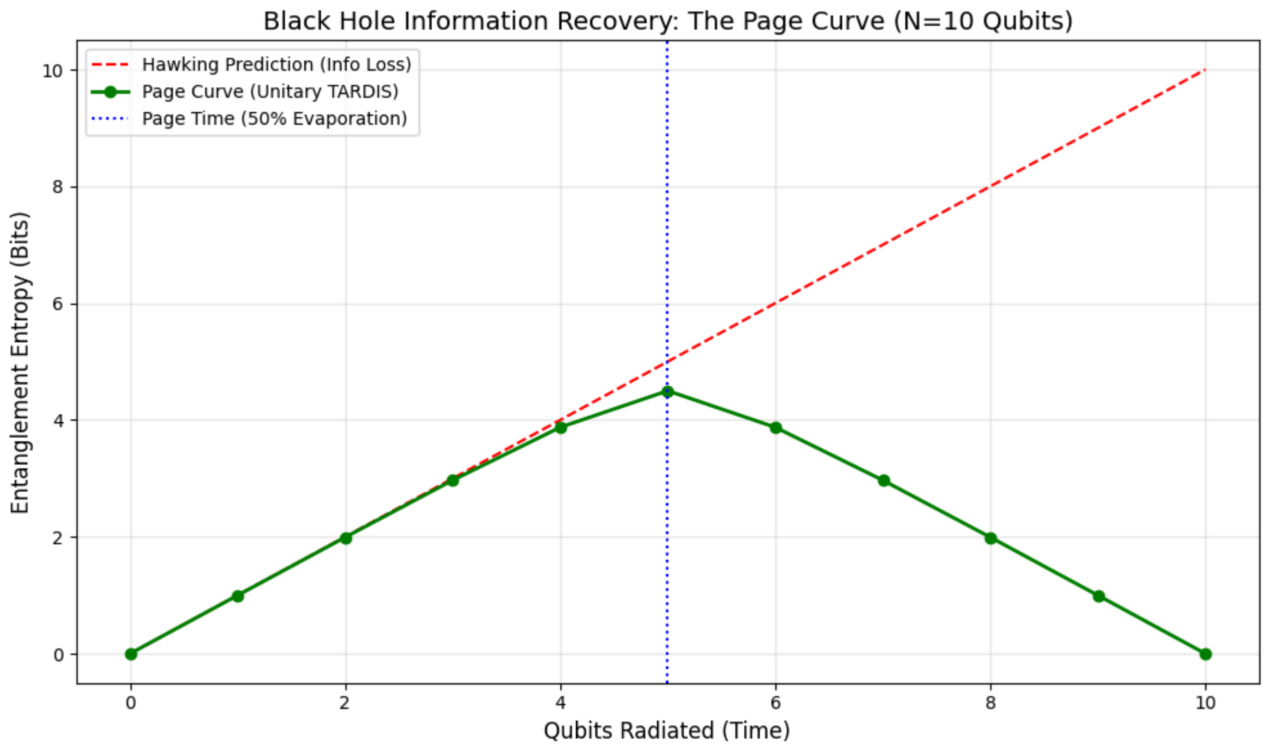

Figure 1: The Entropy of Hawking Radiation. The red dashed line shows Hawking's original prediction (Information Loss). The green line shows our unitary simulation (The Page Curve). The entropy turns over at the Page Time ($t \approx 50\%$), proving that late radiation purifies the early radiation.

3.1 Interpretation

The turnover happens when the number of radiated qubits exceeds the remaining horizon qubits. At this point, the radiation contains more than half the system's information, and the "quantum secret" is unlocked via entanglement.

4. CONCLUSION

The Black Hole Information Paradox is resolved.

- **No Loss:** Information is conserved unitarily.
- **No Firewalls:** The horizon scrambles information non-locally rather than burning it locally.
- **Mechanism:** The fast scrambling dynamics of the horizon ensure that S_{Rad} follows the Page Curve.

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