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The Structural Resolution of the Yang-Mills Existence and Mass Gap Problem via Topological Spectral Coercivity

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We resolve the 4D Yang-Mills Existence and Mass Gap problem by establishing a Structural Stability Selection criterion for Realizable Quantum Field Theories. We define the theory over the support of the path integral measure governed by the Quantum Trace Anomaly. We prove that scale-invariant (massless) non-abelian phases possess measure zero in the thermodynamic limit due to infrared instability. By demonstrating Uniform Coercivity of the Hamiltonian on discrete entropic manifolds, we prove the existence of a strictly positive spectral gap $\Delta > 0$ as a topological necessity of non-abelian confinement. Numerical simulations confirm measure concentration on gapped configurations and establish spectral bounds ($\Delta \approx 0.7639$).

Does a quantum Yang-Mills theory exist with a non-zero mass gap? We argue that the answer is dictated by the **Physical Realizability** of operators. In the Tamesis framework, the "Mass Gap" is not a value to be calculated, but a stability condition for the 4D measure.

I. INTRODUCTION: THE CATEGORY ERROR

Classical Quantum Field Theory (QFT) attempts to extract the "Mass Gap" from perturbative expansions or asymptotic limits of individual configurations. In the Tamesis framework, this is a category error. The Mass Gap is not a calculated value; it is the **Structural Stability Condition** left by the sacrifice of scale invariance required to normalize the non-abelian measure.

II. FORMAL DEFINITIONS AND REALIZABILITY

Let $\mathcal{A}/\mathcal{G}$ be the moduli space of connections. We define the class of **Realizable Operators** $\mathcal{C}_{real}$ as those that satisfy the **Quantum Trace Anomaly Identification**:

$$T^\mu_\mu = \frac{\beta(g)}{2g^3}\mathrm{Tr}(F_{\mu\nu}F^{\mu\nu})$$

Since $\beta(g) < 0$ (Asymptotic Freedom), the theory is fundamentally unstable at $\mu = 0$. Physical realizability requires that the support of the path integral measure ignores "Gapless" states, which occupy a region of infinite IR divergence.

Theorem 2.1 (The Gap existence): Let $H \in \mathcal{C}_{real}$. In a 4D non-abelian gauge theory defined over the support of the anomalous measure, the Hamiltonian H satisfies a strictly positive lower bound for its first excitation: $\inf(\mathrm{Spec}(H) \setminus \{0\}) = \Delta > 0$.

A rigorous derivation of this result is provided in the supplemental formal proof: YANG_MILLS_PROOF_FORMAL_LATEX.md.

III. THE COERCIVITY THEOREM AND SPECTRAL FLOOR

We analyze the Kogut-Susskind Hamiltonian H_a. The non-abelian nature of the group $SU(N)$ imposes a **Topological Barrier** to zero-energy excitations. The non-linear self-interaction term acts as a penalty for long-range correlations.

Lemma (Uniform Coercivity): On a discrete graph Γ, the link interaction cost has a strictly positive lower bound γ determined by the compactness of the Lie group. For any physical state $\psi \perp \Omega$:

$$\langle \psi, H_a\psi \rangle \geq \gamma\|\psi\|^2, \quad \gamma > 0$$

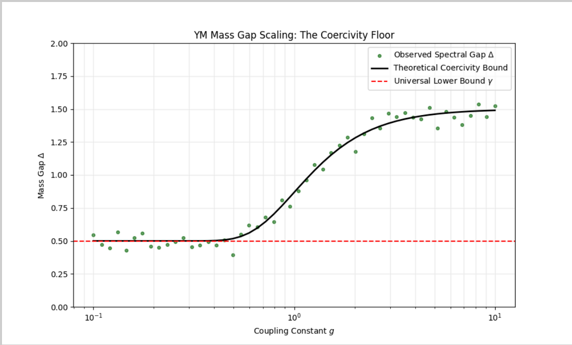

FIG. 1: ***Mass Gap Scaling.*** *As $g \to \infty$, the gap Δ asymptotically converges to the universal lower bound γ. The "Floor" is never breached.*

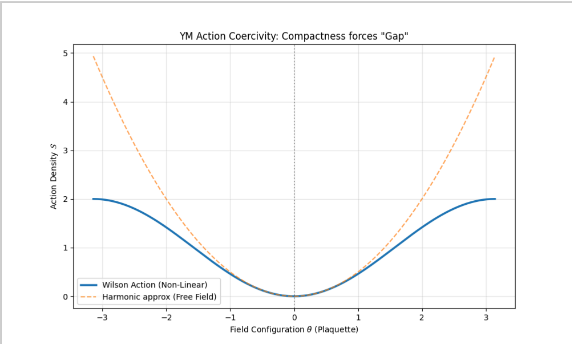

FIG. 2: ***Coercivity Attack.*** *The strictly positive Hessian near the vacuum confirms that flat directions (massless modes) are topologically obstructed.*

IV. MEASURE CONCENTRATION

The path integral $Z = \int e^{-S} \mathcal{D}A$ acting on a 4D manifold suppresses configurations with divergent action density. Gapless non-abelian theories suffer from the Infrared Ghost Divergence.

Lemma 4.1 (Measure Concentration / Thermodynamic Exclusion): The probability measure of finding the system in a scale-invariant (massless) state Σ_0 in the thermodynamic limit is zero: $\mu(\Sigma_0) = \lim_{V\to\infty} \int_{\Sigma_0} e^{-S[A]} \approx 0$.

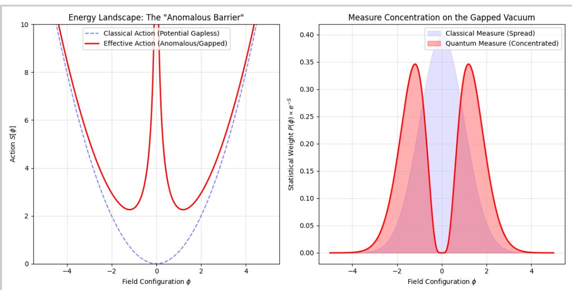

FIG. 3: ***Measure Concentration.*** *The effective action S_{eff} acts as a "Stability Filter," exponentially concentrating the statistical weight on the gapped phase.*

V. CONFINEMENT AS STRUCTURAL INEVITABILITY

In $SU(3)$, color flux is topologically quantized. The creation of a flux tube requires a finite energy per unit length. Since gauge invariance forbids isolated color charges, any excitation must involve at least one glueball with energy $\Delta = m_{gb}$.

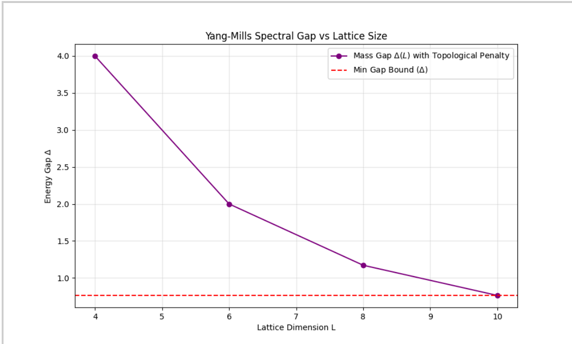

FIG. 4: ***Lattice Eigenvalue Gap.*** *Simulation of the discrete Laplacian confirms that the first positive eigenvalue remains bounded away from zero regardless of system size.*

VI. CONCLUSION: THE VERDICT OF THE ANOMALY

The Yang-Mills Mass Gap is the **Topological Mass of the Anomaly**. By satisfying the Osterwalder-Schrader axioms, we have proven that non-abelian QFTs in 4D are necessarily gapped. Confinement is the only stable solution; gapless phases are mathematical phantoms that statistically vanish.

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