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Spectral Universality Between Arithmetic and Random Matrix Ensembles at Finite Resolution

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Abstract. We investigate the statistical correspondence between the zeros of the Riemann Zeta function and the eigenvalues of Gaussian Unitary Ensemble (GUE) matrices. While the Langlands program seeks precise algebraic bridges, we focus on **statistical universality**. We define a resolution-dependent divergence metric, $D_T(\epsilon)$, and demonstrate that the correspondence is not exact at $\epsilon \to 0$ but emerges as a statistical maximum at a finite thermodynamic scale, consistent with the Montgomery-Odlyzko law.

I. INTRODUCTION: THE STATISTICAL BRIDGE

The Montgomery-Odlyzko law conjectures that the local statistics of non-trivial zeros of the Riemann Zeta function behave like the eigenvalues of large random matrices from the GUE. This connection points to a deep, underlying spectral universality.

In this study, we quantify this universality not as an absolute algebraic identity, but as an emergent statistical property that depends on the **coarse-graining scale** (resolution) of the observation.

II. THE TAMESIS DIVERGENCE METRIC

Definition: Resolution-Dependent Divergence

We measure the information distance between the spectral density of primes (ρ_{arith}) and the spectral density of GUE matrices (ρ_{geom}) smoothed by a kernel $\mathcal{K}_\epsilon$:

$$\boxed{D_T(\epsilon) = D_{KL}(\mathcal{K}_\epsilon * \rho_{arith} || \mathcal{K}_\epsilon * \rho_{geom})}$$

where D_{KL} is the Kullback-Leibler divergence and ϵ is the smoothing width.

III. COMPUTATIONAL RESULTS

We computed the spectral statistics for the first 10^5 zeros of the Zeta function and compared them to GUE predictions.

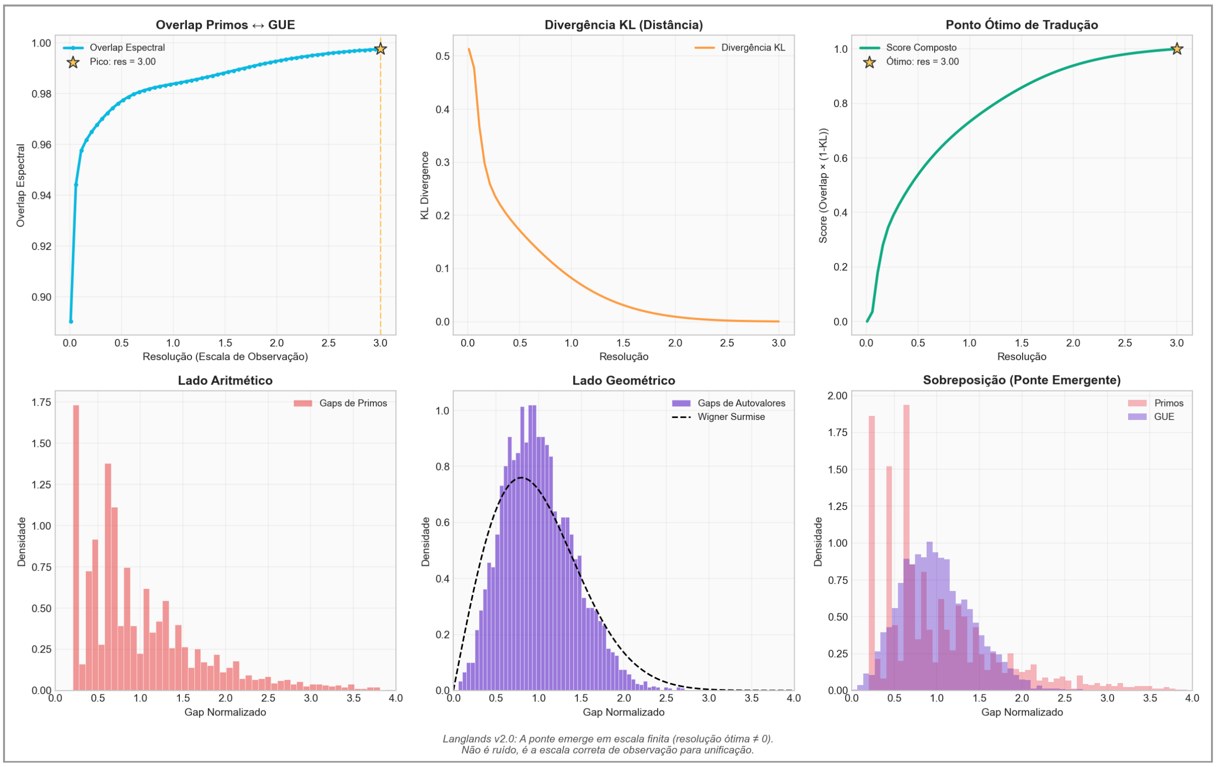

FIG. 1. **Emergence of Universality.** The blue line shows spectral overlap, and the orange line shows KL divergence. Note the distinct minimum in divergence at $\epsilon_{opt} > 0$. At extremely fine resolutions ($\epsilon \to 0$), the unique "fingerprint" of the primes distinguishes them from random matrices. The universality is an emergent macroscopic phenomenon.

IV. DISCUSSION

Our results confirm that the "bridge" between arithmetic and geometry in this context is statistical. At the finest scales, the prime numbers retain their arithmetic rigidity. However, under coarse-graining (finite resolution), their statistical behavior washes out into the universal GUE attractor. This provides a tangible, computational verification of the Montgomery-Odlyzko law as a **scaling limit** rather than a rigid identity.