MATH F111 - MATHEMATICS 1
Tutorial Sheet 14

November 11, 2024

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1. Evaluate the integral $\int_{-1}^{1} \int_{-\sqrt{1-x^2}}^{0} \cos(x^2 + y^2)\, dydx$ by first converting into polar coordinates.
2. Evaluate the integral $\iint_R 3x\, dA$, where $R = \{(r, \theta) : 1 \le r \le 2, 0 \le \theta \le \pi\}$.
3. Sketch the region of integration R and then evaluate the double integral using polar coordinates.

   (i) $\iint_R (x + y)\, dA$ where $R = \{(x, y) : 1 \le x^2 + y^2 \le 4, x \le 0\}$.

   (ii) $\iint_R \sqrt{1 + 4x^2 + 4y^2}\, dA$ where R is the below half of $x^2 + y^2 = 16$.

   (iii) $\iint_R (4xy - 7)\, dA$ where R is the portion of $x^2 + y^2 = 2$ in the first quadrant.
4. Using double integral, find the area enclosed by the curve $r = \sin 3\theta$ given in polar cordinates.
5. Compute $\lim_{a\to\infty} \iint_{D(a)} e^{-(x^2+y^2)}\, dxdy$, where

   (i) $D(a) = \{(x, y) : x^2 + y^2 \le a^2\}$

   (ii) $D(a) = \{(x, y) : 0 \le x \le a, 0 \le y \le a\}$.

   Hence, prove that (a) $\int_0^\infty e^{-x^2}\, dx = \frac{\sqrt{\pi}}{2}$ and (b) $\int_0^\infty x^2 e^{-x^2}\, dx = \frac{\sqrt{\pi}}{4}$.
6. Use a double integral to determine the area of the region that is inside $r = 4 + 2\sin\theta$ and outside $r = 3 - \sin\theta$.
7. Use polar coordinates to find the volume inside the cone $z = 2 - \sqrt{x^2 + y^2}$ and above the xy-plane.
8. Determine the volume of the region that lies under the sphere $x^2 + y^2 + z^2 = 9$, above the plane $z = 0$ and inside the cylinder $x^2 + y^2 = 5$.
9. Determine the volume of the region that lies behind the plane $x + y + z = 8$ and in front of the region in the yz-plane that is bounded by $z = \frac{3}{2}\sqrt{y}$ and $z = \frac{3}{4}y$.
10. Use a triple integral to find the volume of the tetrahedron bounded by the four planes $x = 0$, $z = 0$, $x = 2y$, and $x + 2y + z = 2$.
11. Reverse the order of integration in the integral $\int_0^1 \int_1^{x^2} \int_0^y f(x, y, z)\, dzdydx$. (Give all possibilities).
12. Find the volume of the region that is inside both the sphere $x^2 + y^2 + z^2 = 25$ and the cylinder $x^2 + y^2 = 9$.

13. A solid region W lies within the cylinder $x^2+y^2=1$, below the plane $z=4$, and above the paraboloid $z=1-x^2-y^2$. Evaluate $\iiint_W \sqrt{x^2+y^2}\,dV$.

14. Evaluate the integral $\int_{-2}^{2}\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}}\int_{\sqrt{x^2+y^2}}^{2}(x^2+y^2)\,dzdydx$.

15. Evaluate the integral $\iiint_W xy\,dV$, where W is the region bounded by $z=9-x^2-y^2$, $z=0$, $y=x^2$, $y=1$ and $y=0$.

16. Evaluate $\iiint_E z\,dV$, where E is the solid tetrahedron bounded by the four planes $x=0$, $y=0$, $z=0$, and $x+y+z=1$.