Lecture 2

Polar Coordinates: Curve sketching

Text book chapter: 11.4

Example-3

- Replace the following polar equation with equivalent cartesian equation.

  1. $r^2 = 4r\sin(\theta)$

  2. $r = 4\tan(\theta)\sec(\theta)$

  3. $r^2\sin(2\theta) = 2$

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2. Ans: $x^2 = 4y$

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  1. $r^2 = 4r\sin(\theta)$

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1. Ans: $x^2 + (y-2)^2 = 4$
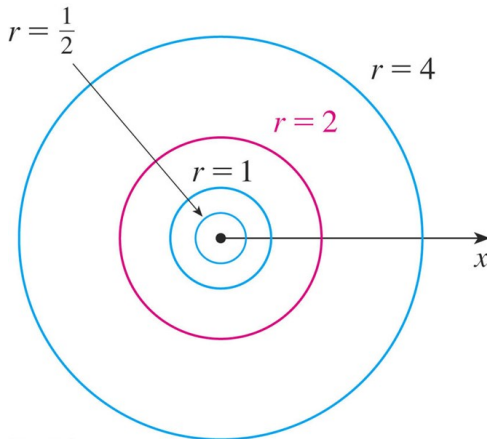2. Ans: $x^2 = 4y$
3. Ans $xy = 1$

Polar Curves

The graph of a polar equation $r = f(\theta)$ consists of all points that have at least one polar representation (r, θ), whose coordinates satisfy the equation.

1. What curve is represented by the polar equation $r = 4$?

2. Sketch the polar curve $\theta = 1$.

Polar curve

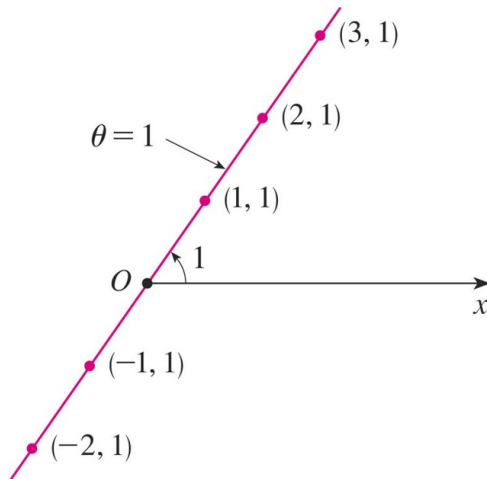The curve $r = 4$ represents the circle with center O and radius 4.

The curve $\theta = 1$ is the straight line that passes through O and makes an angle of 1 radian with the polar axis.

Example 1: Sketch the curve with polar equation
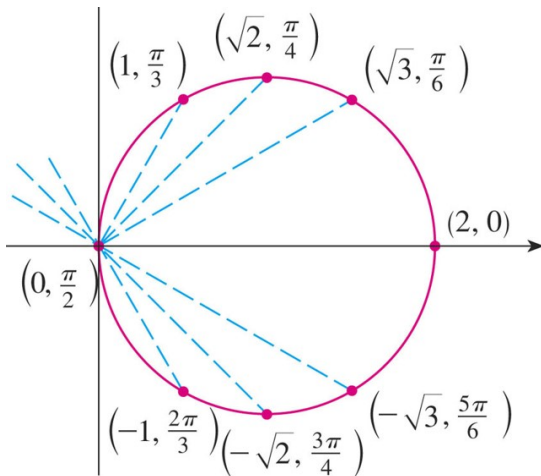$r = 2\cos(\theta)$. (b) Find a Cartesian equation for this curve.

Polar curve sketching

Example 1: Sketch the curve with polar equation
$r = 2\cos(\theta)$. (b) Find a Cartesian equation for this curve.

The table:

θ	$r = 2\cos\theta$
0	2
$\pi/6$	$\sqrt{3}$
$\pi/4$	$\sqrt{2}$
$\pi/3$	1
$\pi/2$	0
$2\pi/3$	-1
$3\pi/4$	$-\sqrt{2}$
$5\pi/6$	$-\sqrt{3}$
π	-2

Example 1

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Example 1b

To convert the given equation to a Cartesian equation, we use the following:

$$x = r\cos(\theta) = r^2/2$$
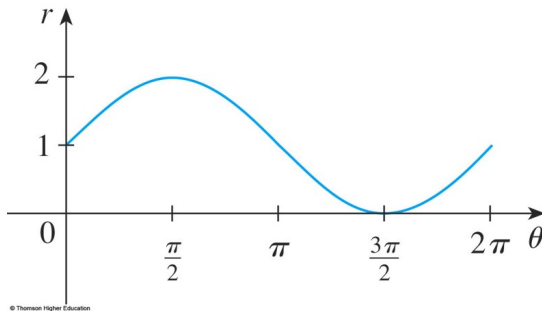
which gives: $2x = r^2 = x^2 + y^2$ or $x^2 + y^2 - 2x = 0$ Or the equation is

$$(x-1)^2 + y^2 = 1$$
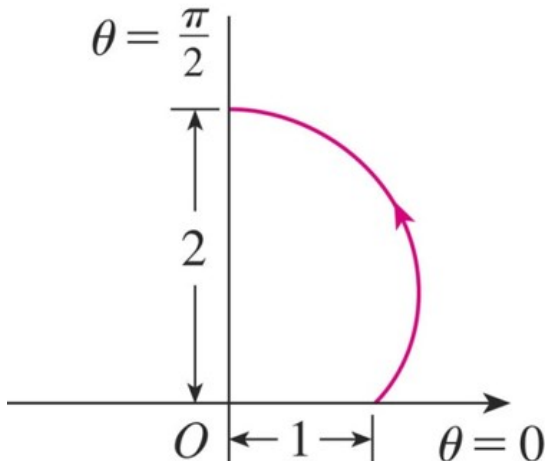
which represent a circle with center $(1,0)$ and radius 1.

Example 2

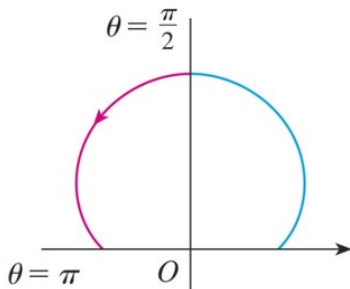Sketch the curve $r = 1 + \sin(\theta)$. We first sketch the graph in cartesian coordinates.

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Example 2

We see that, as θ increases from 0 to $\pi/2$, r (the distance from O) increases from 1 to 2. So, we sketch the corresponding part of the polar curve as follows:
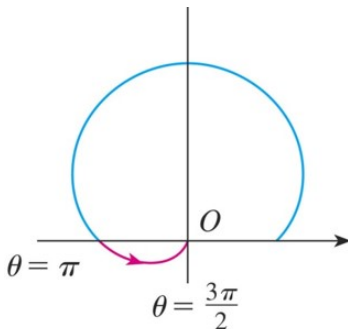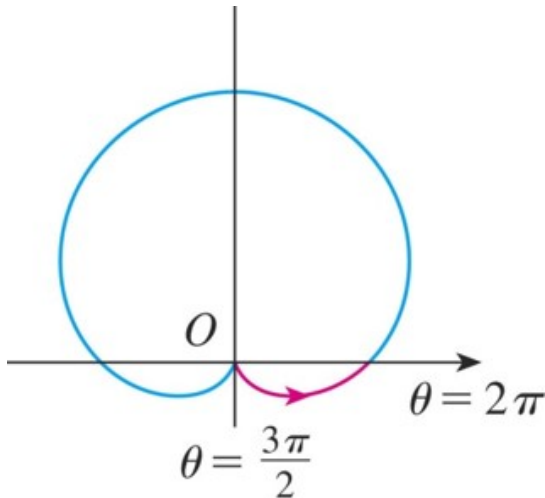
Example 2 Cont.

(b)   (c)
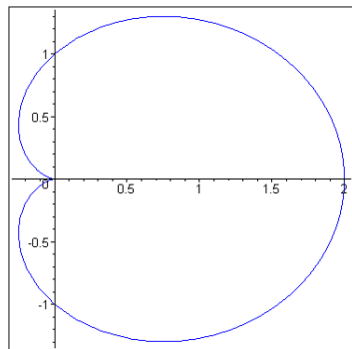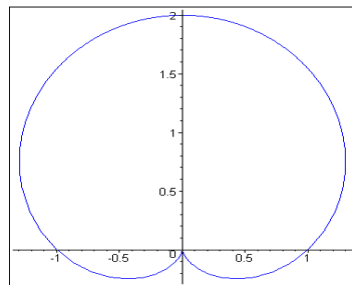
Example 2

Putting together the various parts of the curve, we sketch the complete curve as shown next. It's called the cardioid.
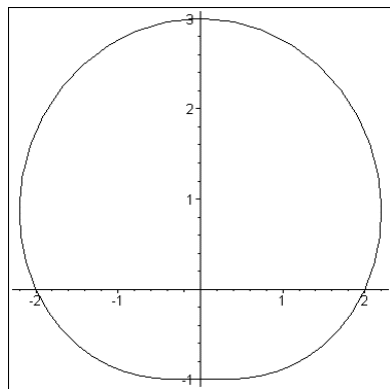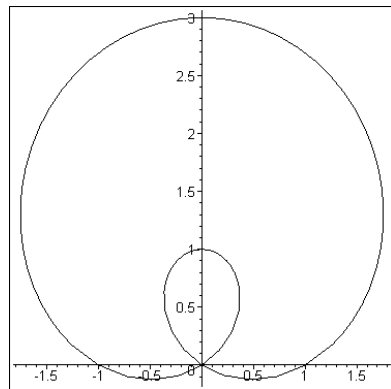
$A : 1 + \cos\theta, \quad B : 1 + \sin\theta$

Limacons

$A : c + d \sin\theta (c > d)$ $B : c + d \cos\theta (c < d)$