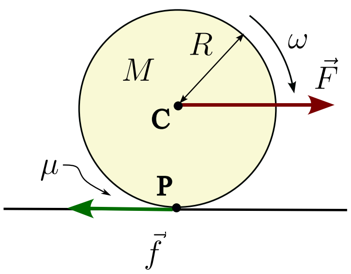
Tutorial 4 **Rolling Dynamics, Angular momentum** 20 August 2024

P1.



Assume friction has the direction shown.
If the disc is only rotating with angular speed ω then the velocity of the bottom-most point on the disc is $v_P = -R\omega$. In addition suppose the disc is translating with velocity v_C. If the disc now touches the surface so that the point P on the disc is at rest,

$$v = v_C - R\omega = 0, \implies v_C = R\omega \implies a = R\alpha.$$

Now the disc is subjected to forces and torques about the center as well.

(a) Rolling without slipping: α is the angular acceleration caused by the torque about the center: $\alpha = Rf$.

The acceleration of the center, a is caused by the net force on the disc:

$$F - f = Ma = MR\alpha = MR\frac{Rf}{\frac{1}{2}MR^2} = 2f \implies F = 3f.$$

The value of $f \leq \mu Mg$, which decides the upper bound on F.

(b) If the point of contact moves as well (rolling and slipping) then the acceleration is greater than $R\alpha$, so that $F > 3\mu Mg$. What is its value?

$$F - \mu Mg = Ma \implies a = F/M - \mu g.$$
$$\tau_C = fR \implies \alpha = \frac{\tau}{I} = \frac{\mu MgR}{MR^2/2} = \frac{2\mu g}{R}$$

P2. Here the only force on the disc is the frictional force. Let's assume it acts in the forward direction. The maximum acceleration and torque can be calculated by taking $f = \mu Mg$.

$$f = \mu Mg = Ma \implies v_C(t) = v(0) + \mu gt.$$
$$\tau_C = \mu MgR \implies \omega(t) = \omega_0 - \frac{2\mu g}{R}t.$$

If rolling without slipping occurs at some time t_1,

$$v(t_1) = R\omega(t_1) \implies \mu g t_1 = \omega_0 R - 2\mu g t_1$$
$$t_1 = \frac{\omega_0 R}{3\mu g}.$$

rolling and slipping for $t < t_1$.

P3. (a) $\vec{\boldsymbol{L}}_{O_1} = 0$

(b) $\vec{\boldsymbol{L}}_{O_2} = mvb$

P4. $\omega = \sqrt{\frac{k}{I}}$; $\quad T = 2\pi\sqrt{\frac{MR^2}{2k}}$.