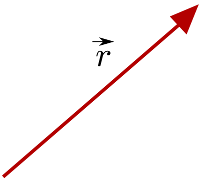
Polar Coordinates

1. Polar Coordinates
2. Unit vectors $\hat{r}$ and $\hat{\theta}$
3. Polar & Cartesian components of a vector
4. Velocity and Acceleration in Polar Coordinates
5. Examples

POLAR COORDINATES

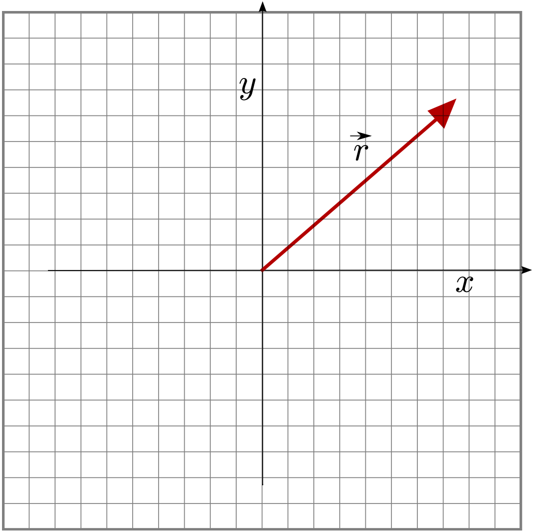
Polar Coordinates

Two Dimensional Plane

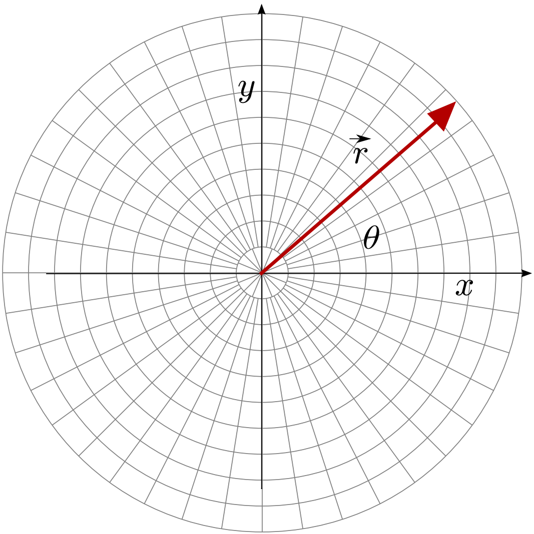


Polar Coordinates

Two Dimensional Plane



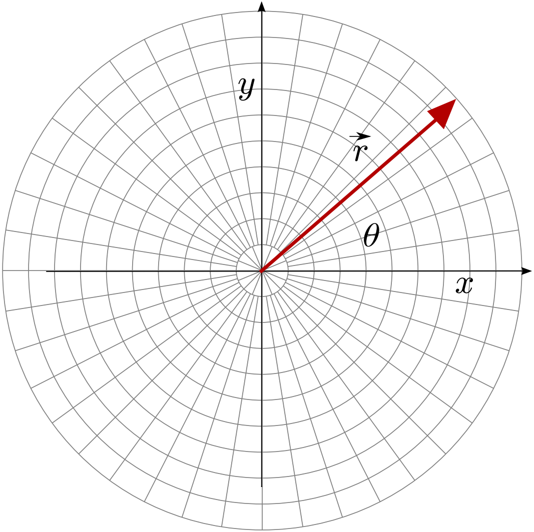
- Cartesian Coordinates: (x, y)



- Cartesian Coordinates: (x, y)
- Polar Coords: $(r,\ \theta)$

Polar Coordinates

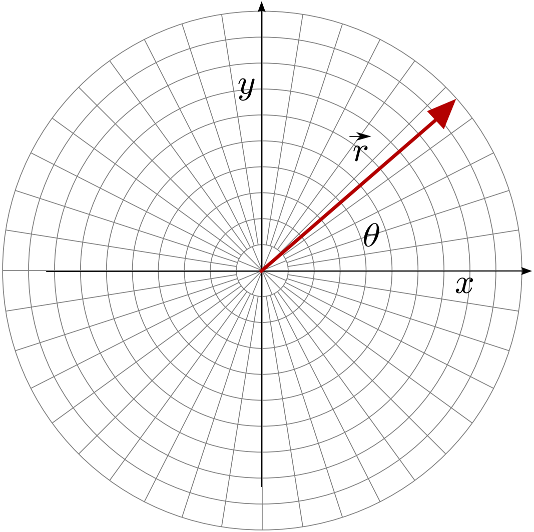
Two Dimensional Plane



- Cartesian Coordinates: (x, y)
- Polar Coords: $(r,\ \theta)$
- $r \in [0, \infty)$, $\theta \in [0, 2\pi]$

Polar Coordinates

Two Dimensional Plane



- Cartesian Coordinates: (x, y)
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- $r \in [0, \infty)$,
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- $r = \sqrt{x^2 + y^2}$,
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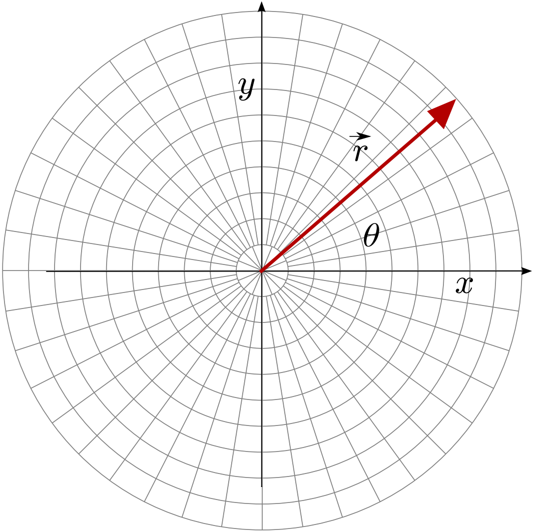
Polar Coordinates

Two Dimensional Plane



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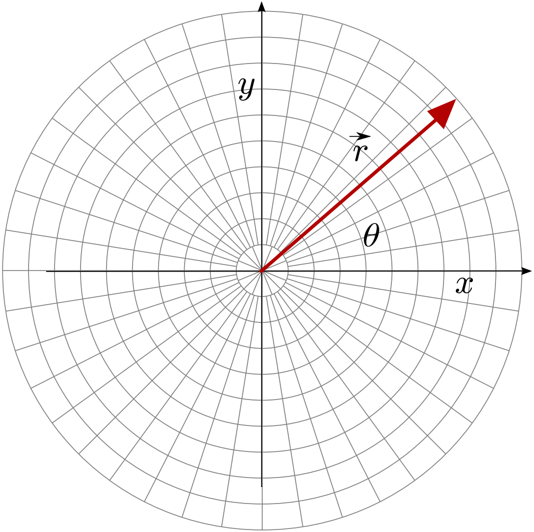

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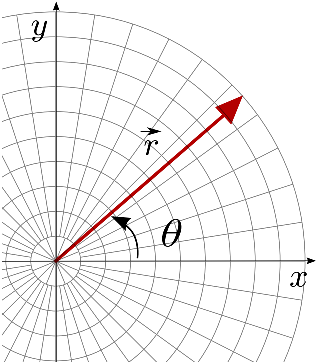
- $x = r\cos\theta$, $y = r\sin\theta$
- Use of Polar Coords nothing to do with rotational motion

Unit vectors $\hat{r}$ and $\hat{\theta}$

Unit vectors $\hat{r}$ and $\hat{\theta}$



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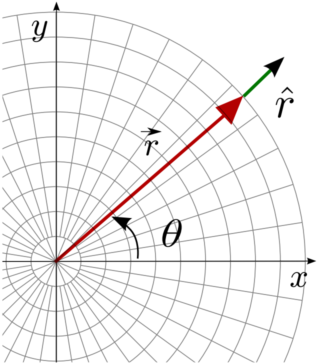
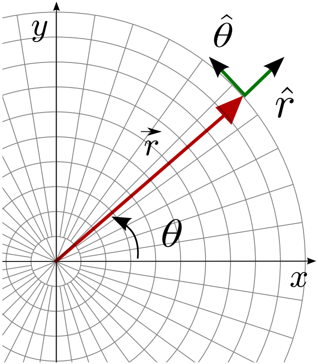


$\hat{r}$ points along increasing r

Unit vectors $\hat{r}$ and $\hat{\theta}$



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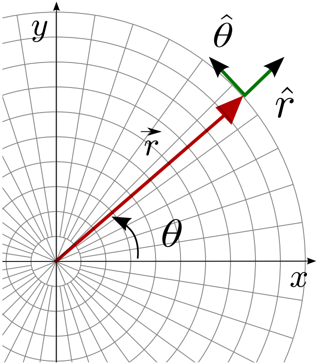

$\hat{\boldsymbol{r}}$ points along increasing r

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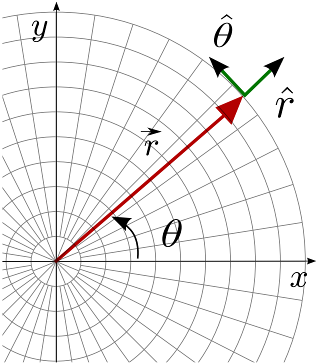
$\hat{\boldsymbol{r}}$ and $\hat{\boldsymbol{\theta}}$ not constant unit vectors!

$$\hat{\boldsymbol{r}}(\theta), \hat{\boldsymbol{\theta}}(\theta)$$

Unit vectors $\hat{r}$ and $\hat{\theta}$



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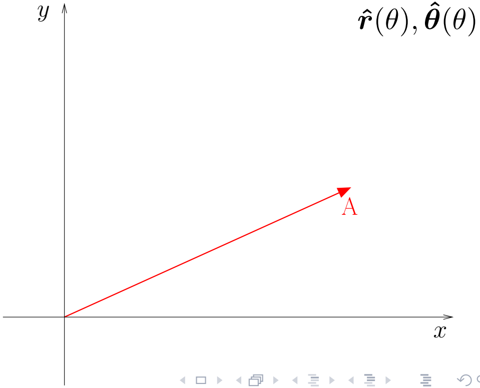

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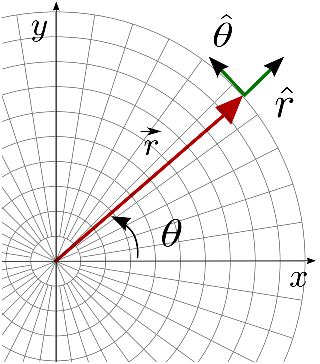

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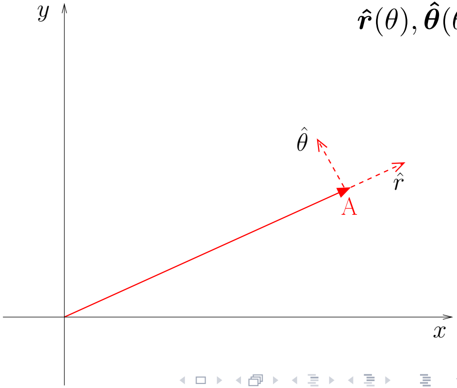

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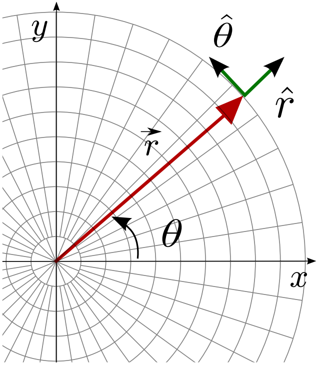

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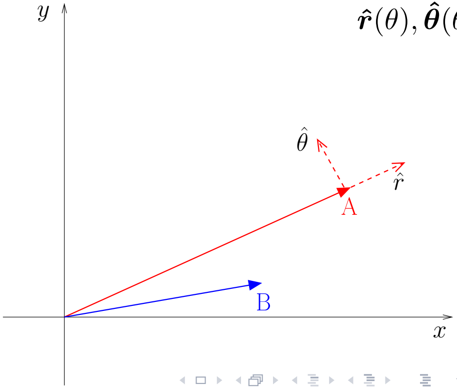

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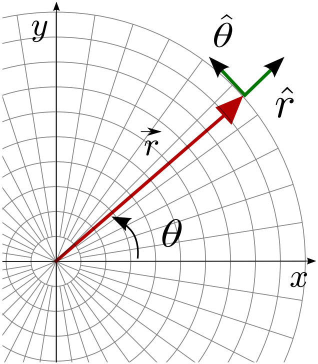

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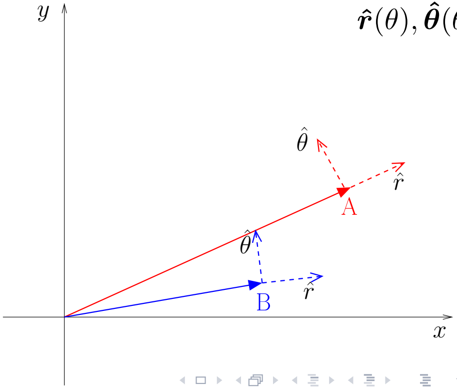
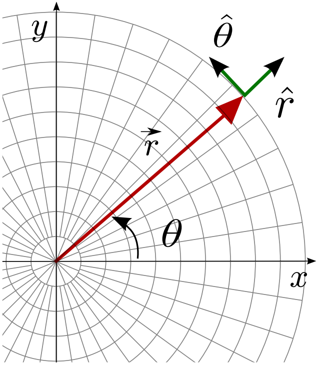


Unit vectors $\hat{r}$ and $\hat{\theta}$



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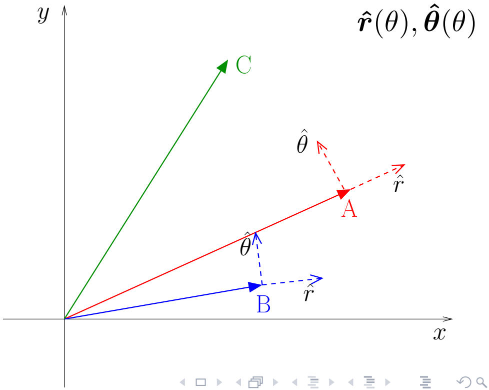

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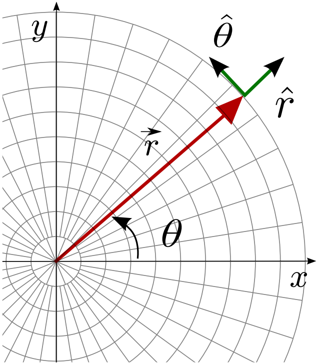

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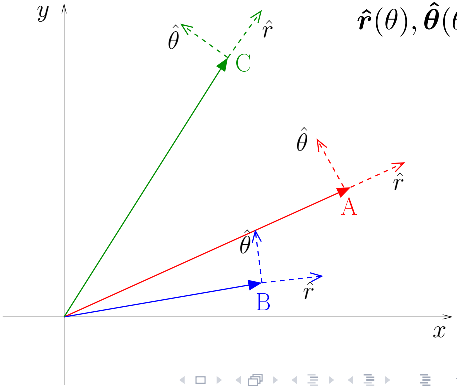

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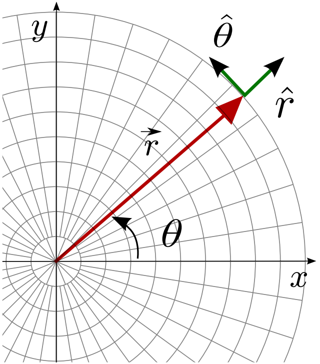

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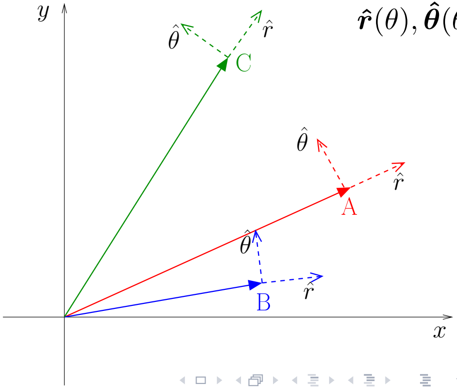
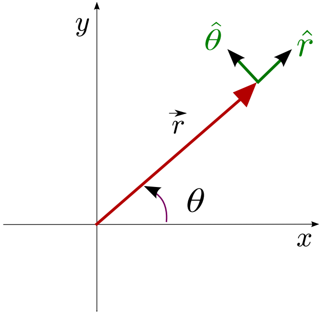


Ex: What are $\hat{\boldsymbol{r}}$, $\hat{\boldsymbol{\theta}}$ at the points

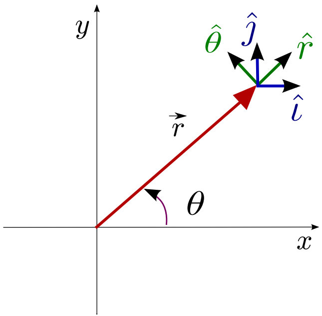
$$\begin{aligned}(x, y) \quad = \quad & (1,0); (-1,0); \\ & (0,1); (0,-1)?\end{aligned}$$

$\hat{r}$&$\hat{\theta}$ in terms of $\hat{i}$&$\hat{j}$

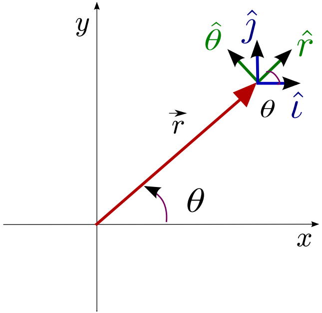
$\hat{r}$&$\hat{\theta}$ in terms of $\hat{i}$&$\hat{j}$



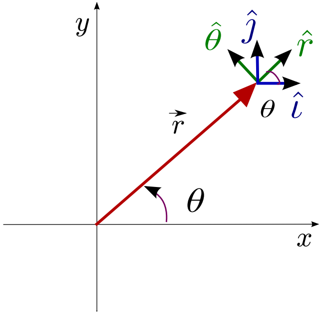
$\hat{r}\&\hat{\theta}$ in terms of $\hat{i}\&\hat{j}$



$\hat{r}$&$\hat{\theta}$ in terms of $\hat{i}$&$\hat{j}$

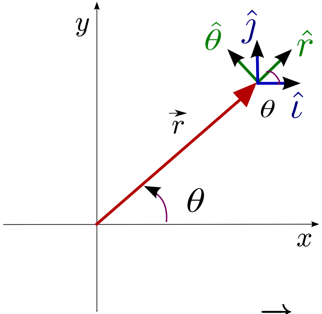


$\hat{r}$&$\hat{\theta}$ in terms of $\hat{i}$&$\hat{j}$



$$
\begin{aligned}
\hat{\boldsymbol{r}} &= \cos\theta\,\hat{\boldsymbol{i}} + \sin\theta\,\hat{\boldsymbol{j}} \\
\hat{\boldsymbol{\theta}} &= -\sin\theta\,\hat{\boldsymbol{i}} + \cos\theta\,\hat{\boldsymbol{j}}
\end{aligned}
$$

$\hat{r}\&\hat{\theta}$ in terms of $\hat{i}\&\hat{j}$



$$
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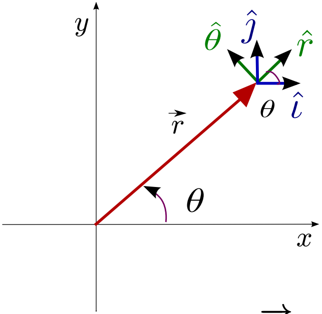
\end{aligned}
$$

$$
\overrightarrow{\boldsymbol{A}} = A_x\,\hat{\boldsymbol{i}} + A_y\,\hat{\boldsymbol{j}}
$$

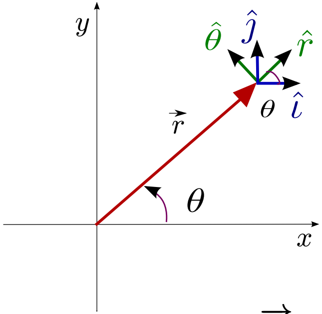
$\hat{r}\&\hat{\theta}$ in terms of $\hat{i}\&\hat{j}$



$$\begin{aligned} \hat{\boldsymbol{r}} &= \cos\theta\,\hat{\boldsymbol{i}} + \sin\theta\,\hat{\boldsymbol{j}} \\ \hat{\boldsymbol{\theta}} &= -\sin\theta\,\hat{\boldsymbol{i}} + \cos\theta\,\hat{\boldsymbol{j}} \end{aligned}$$

$$\begin{aligned} \overrightarrow{\boldsymbol{A}} &= A_x\,\hat{\boldsymbol{i}} + A_y\,\hat{\boldsymbol{j}} \\ &= A_r\,\hat{\boldsymbol{r}} + A_\theta\,\hat{\boldsymbol{\theta}} \end{aligned}$$

$\hat{r}\&\hat{\theta}$ in terms of $\hat{\imath}\&\hat{\jmath}$



$$\begin{aligned} \hat{\boldsymbol{r}} &= \cos\theta\ \hat{\boldsymbol{i}} + \sin\theta\ \hat{\boldsymbol{j}} \\ \hat{\boldsymbol{\theta}} &= -\sin\theta\ \hat{\boldsymbol{i}} + \cos\theta\ \hat{\boldsymbol{j}} \end{aligned}$$

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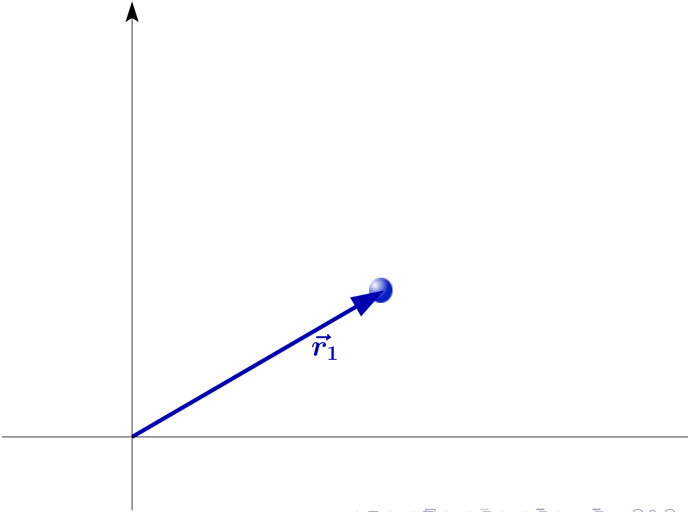
$$\begin{aligned} A_r &= A_x\ \cos\theta + A_y\ \sin\theta \\ A_\theta &= -A_x\ \sin\theta + A_y\ \cos\theta \end{aligned}$$

Velocity & Acceleration in Polar Coordinates

Particle moving from

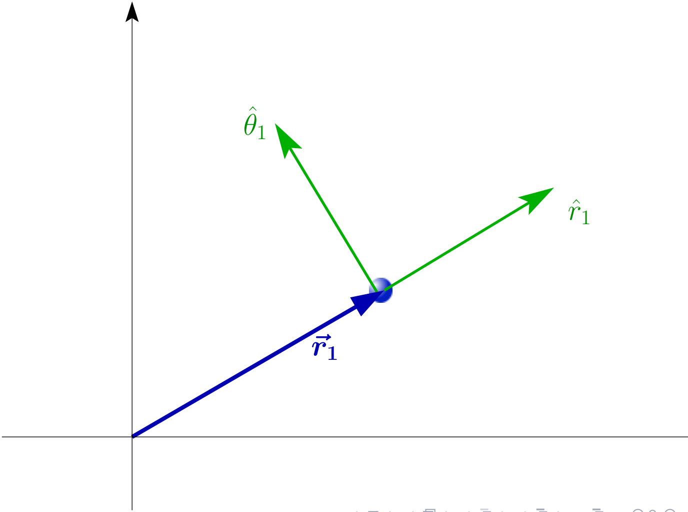
Velocity & Acceleration in Polar Coordinates

Particle moving from $\vec{r}_1$ at time t_1



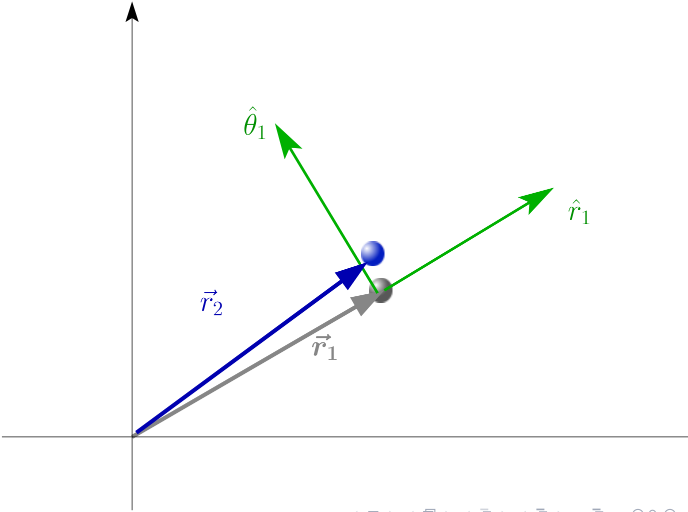
Velocity & Acceleration in Polar Coordinates

Particle moving from $\vec{\boldsymbol{r}}_1$ at time t_1



Velocity & Acceleration in Polar Coordinates

Particle moving from $\vec{r}_1$ at time t_1 to $\vec{r}_2$ at time t_2



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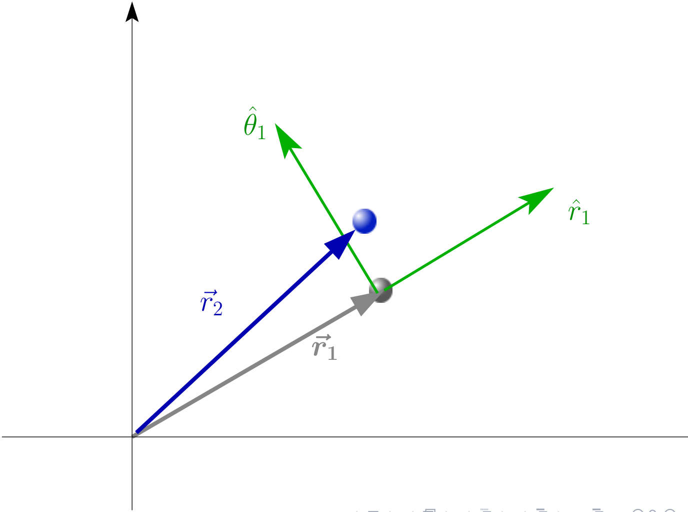

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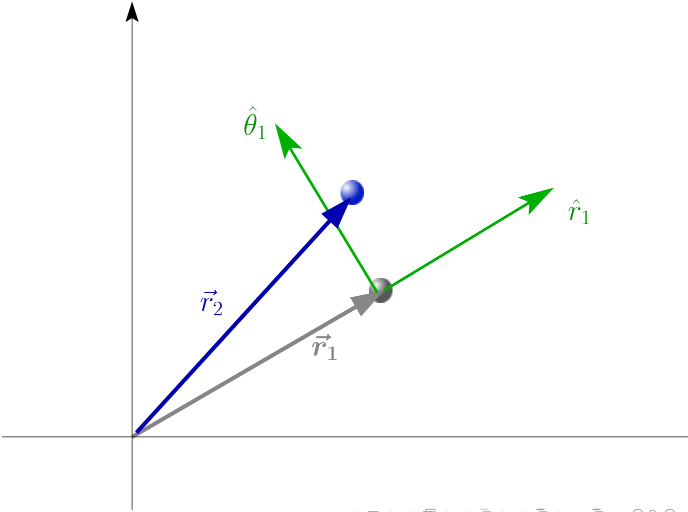

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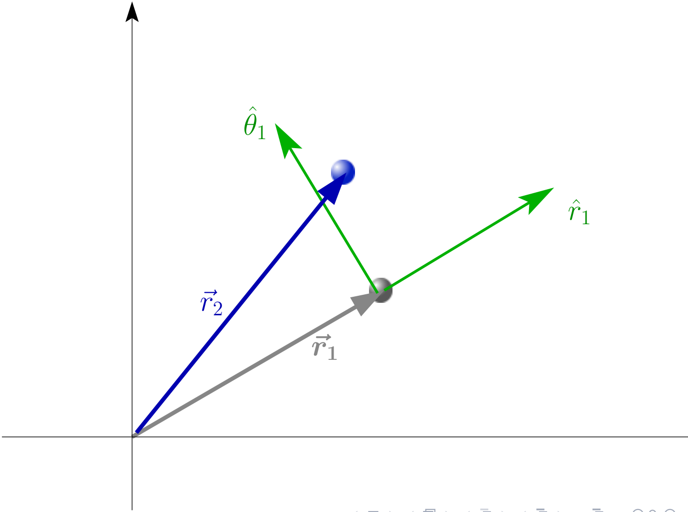

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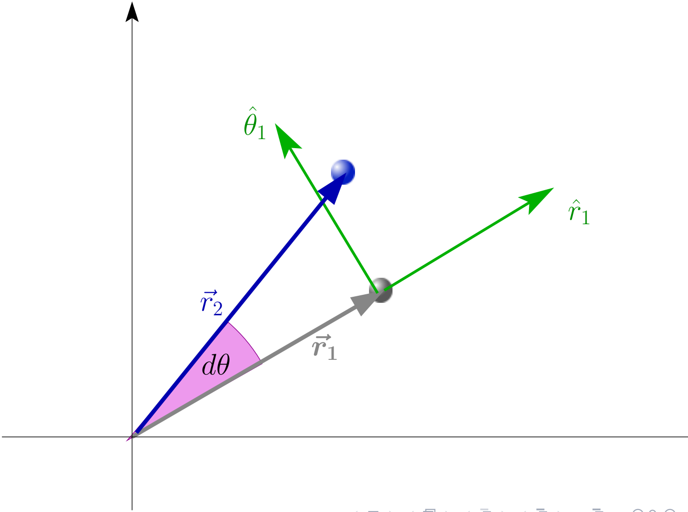

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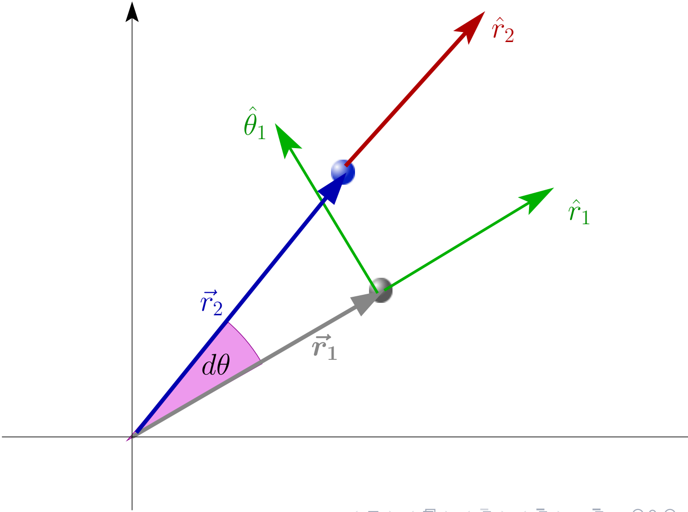

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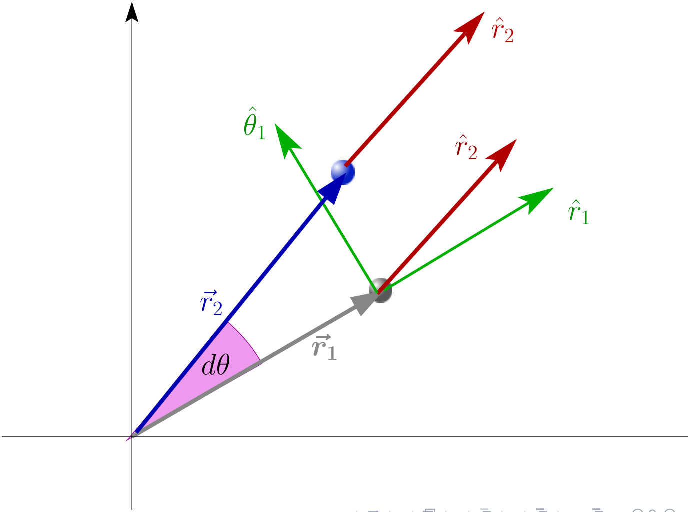

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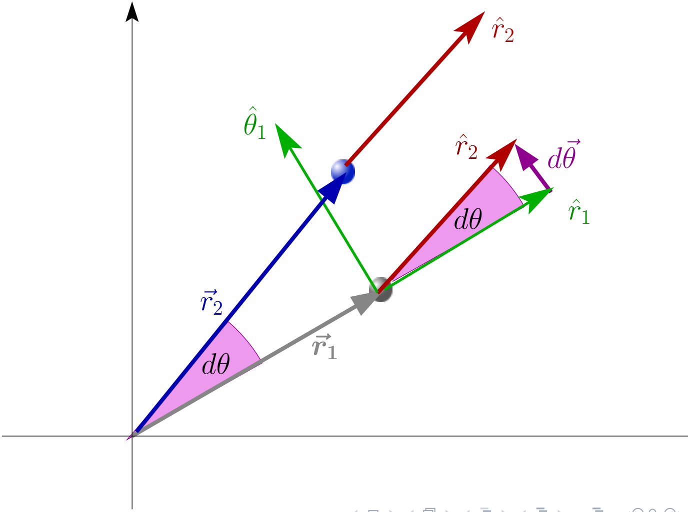

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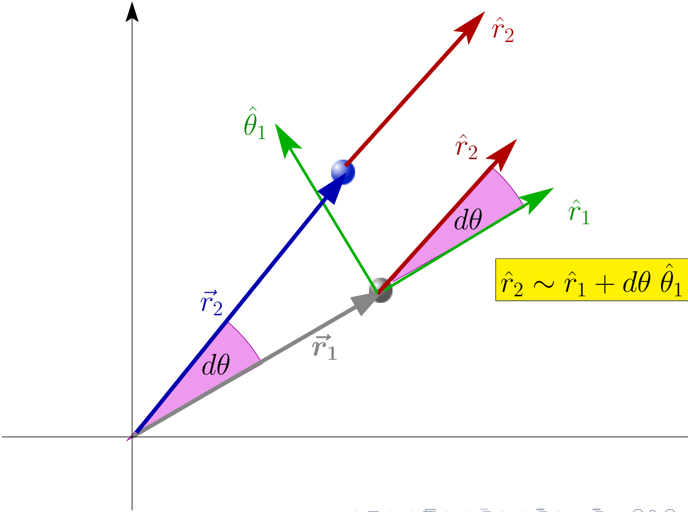

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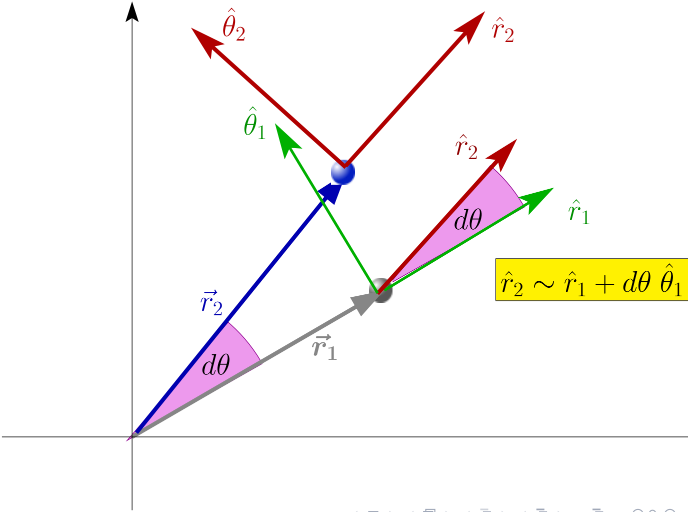

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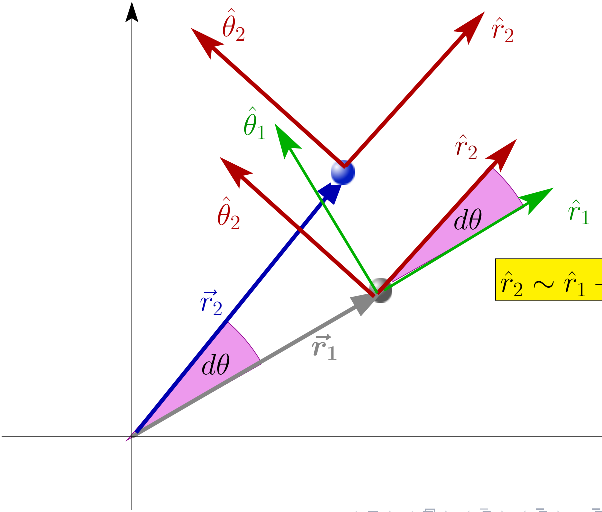
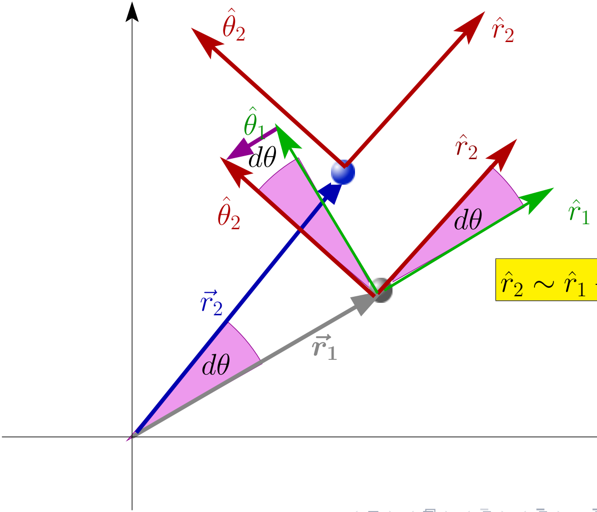


$\hat{r}_2 \sim \hat{r}_1 + d\theta\ \hat{\theta}_1$

Velocity & Acceleration in Polar Coordinates

Particle moving from $\vec{r}_1$ at time t_1 to $\vec{r}_2$ at time t_2



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Velocity & Acceleration in Polar Coordinates

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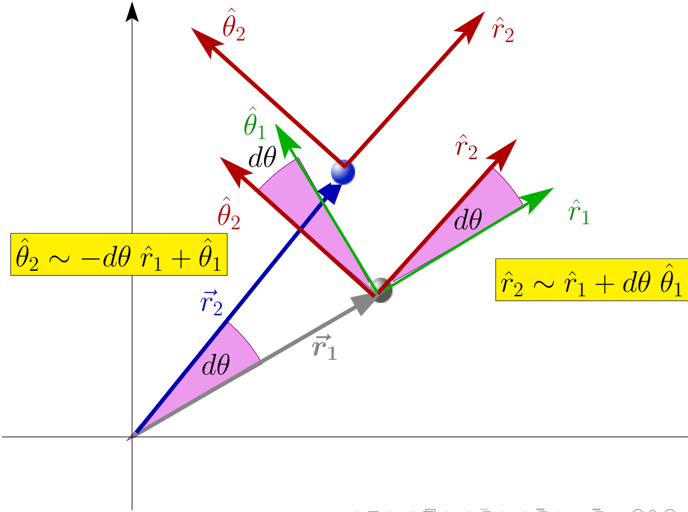
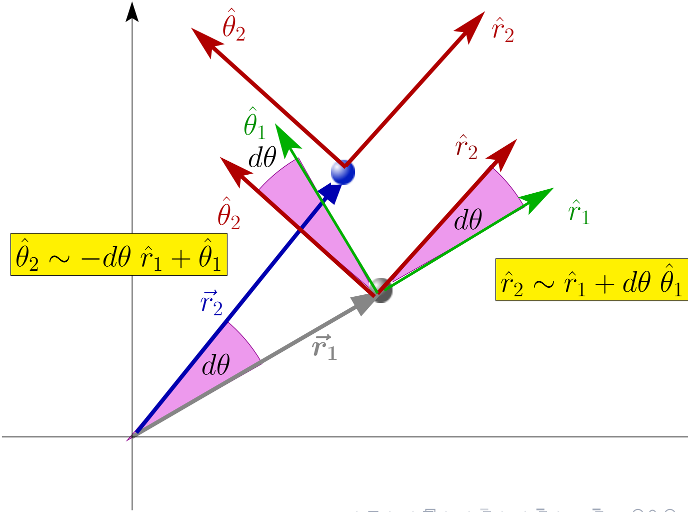


Velocity & Acceleration in Polar Coordinates

Particle moving from $\vec{\boldsymbol{r}}_1$ at time t_1 to $\vec{\boldsymbol{r}}_2$ at time t_2

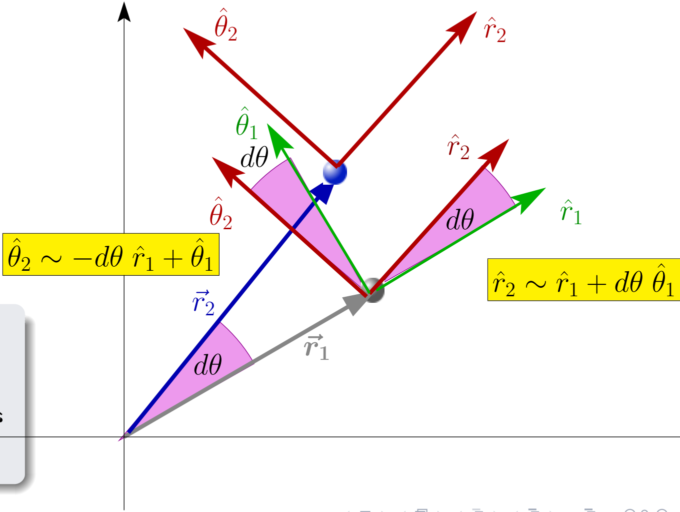
$$d\hat{\boldsymbol{r}} \sim d\theta\hat{\boldsymbol{\theta}}$$
$$d\hat{\boldsymbol{\theta}} \sim -d\theta\hat{\boldsymbol{r}}$$



Velocity & Acceleration in Polar Coordinates

Particle moving from $\vec{\boldsymbol{r}}_1$ at time t_1 to $\vec{\boldsymbol{r}}_2$ at time t_2

$$\begin{aligned} d\hat{\boldsymbol{r}} &\sim d\theta\hat{\boldsymbol{\theta}} \\ d\hat{\boldsymbol{\theta}} &\sim -d\theta\hat{\boldsymbol{r}} \end{aligned}$$



$$\begin{aligned} \frac{d\hat{\boldsymbol{r}}}{dt} &= \frac{d\theta}{dt}\hat{\boldsymbol{\theta}} = \dot{\theta}\hat{\boldsymbol{\theta}} \\ \frac{d\hat{\boldsymbol{\theta}}}{dt} &= -\frac{d\theta}{dt}\hat{\boldsymbol{r}} = -\dot{\theta}\hat{\boldsymbol{r}} \end{aligned}$$

Velocity & Acceleration in Polar Coordinates

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Velocity & Acceleration in Polar Coordinates

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- But we are in a inertial frame with no fictitious forces
- Polar coords used for general motion in plane,
  NOT for rotational motion alone

Eg. 1: Straight Line motion

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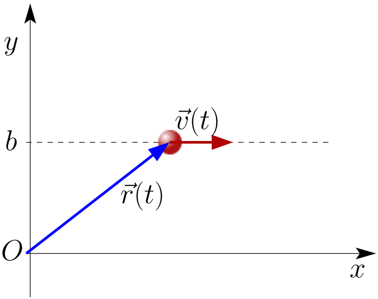

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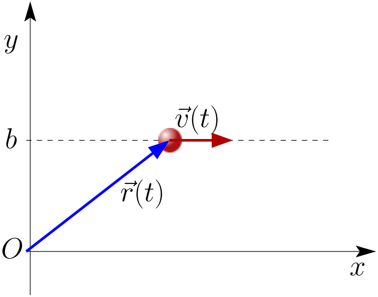

Cartesian:

$$
\begin{aligned}
\vec{\boldsymbol{r}}(t) &= (vt)\hat{\boldsymbol{i}} + b\hat{\boldsymbol{j}} \\
\vec{\boldsymbol{v}}(t) &= v\hat{\boldsymbol{i}}
\end{aligned}
$$

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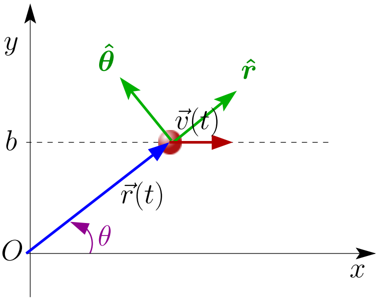

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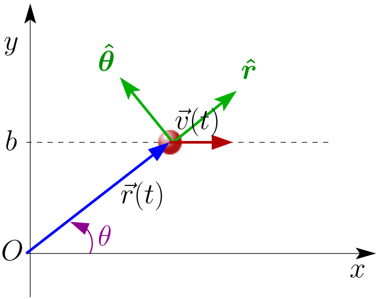

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Polar:

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$$\begin{aligned}\vec{\boldsymbol{r}}(t) &= r\hat{\boldsymbol{r}} \\ \vec{\boldsymbol{v}}(t) &= v\left(\frac{vt}{\sqrt{(vt)^2+b^2}}\hat{\boldsymbol{r}} - \frac{b}{\sqrt{(vt)^2+b^2}}\hat{\boldsymbol{\theta}}\right)\end{aligned}$$

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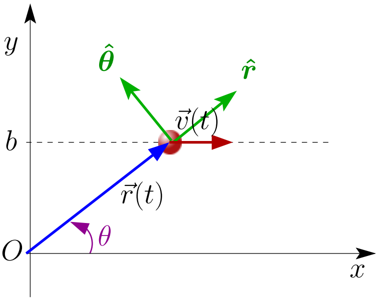

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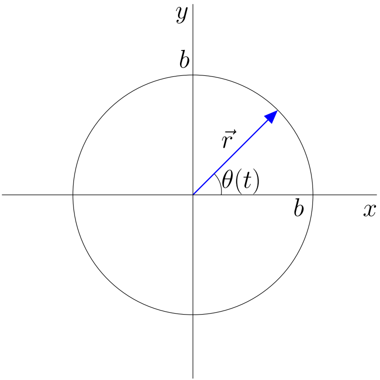
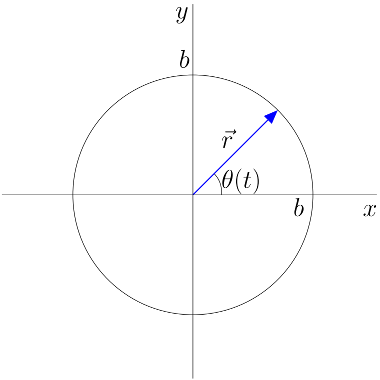
Question: What is $\vec{a}$?

Circular Motion

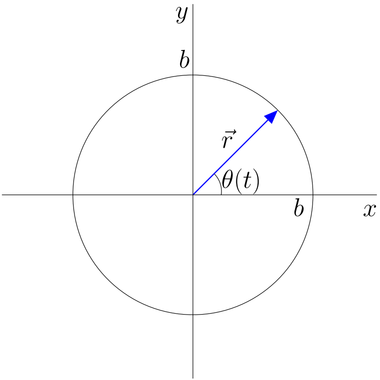


Circular Motion



Cartesian:

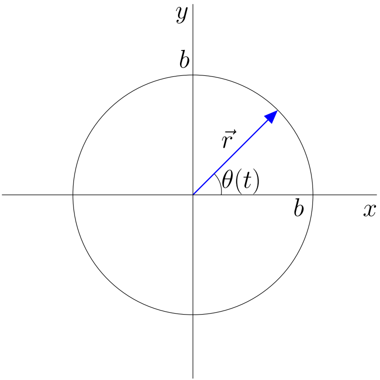
Circular Motion



Cartesian:

$$\vec{\boldsymbol{r}}(t) \quad = \quad b\cos\theta(t)\hat{\boldsymbol{\imath}} + b\sin\theta(t)\hat{\boldsymbol{\jmath}}$$

Circular Motion



Cartesian:

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\begin{aligned}
\vec{\boldsymbol{r}}(t) &= b\cos\theta(t)\hat{\boldsymbol{\imath}} + b\sin\theta(t)\hat{\boldsymbol{\jmath}} \\
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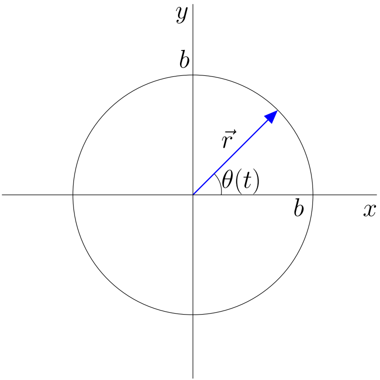
\end{aligned}
$$

Circular Motion



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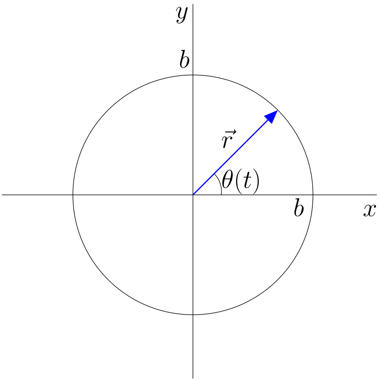
Polar:

Circular Motion



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\end{aligned}
$$

Polar:

$$
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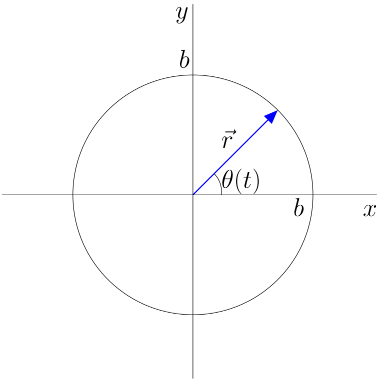
$$

Circular Motion



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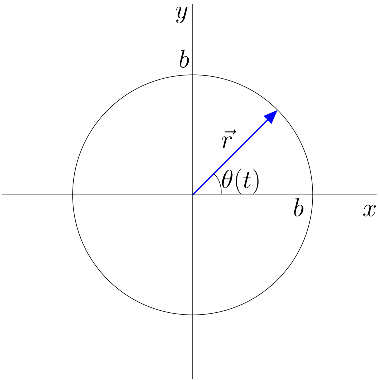

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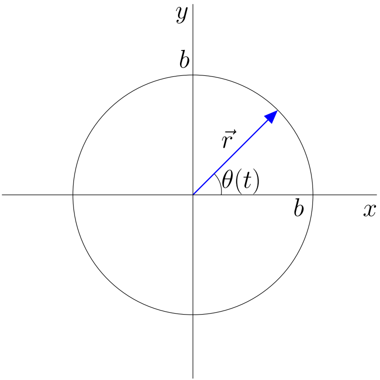
Circular Motion



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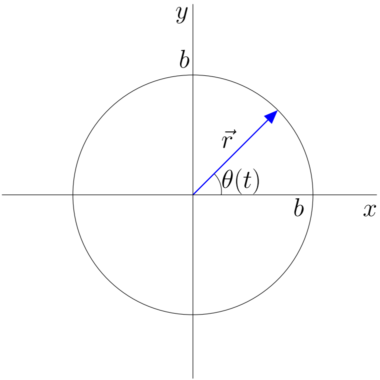

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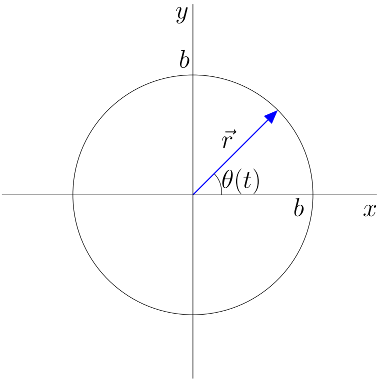
Centripetal +

Circular Motion



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\vec{\boldsymbol{v}}(t) &= -b\sin\theta(t)\hat{\boldsymbol{i}} + b\cos\theta(t)\hat{\boldsymbol{j}}
\end{aligned}
$$

Polar:

$$
\begin{aligned}
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&= b\dot{\theta}\hat{\boldsymbol{\theta}} \\
\vec{\boldsymbol{a}} &= (\ddot{r} - r\dot{\theta}^2)\hat{r} + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\hat{\theta} \\
&= -b\dot{\theta}^2\,\hat{\boldsymbol{r}} + b\ddot{\theta}\,\hat{\boldsymbol{\theta}}
\end{aligned}
$$

Centripetal + Tangential acceleration

Spiral motion

Archimedian spiral

$r = a + b\theta$

Spiral motion

Archimedian spiral

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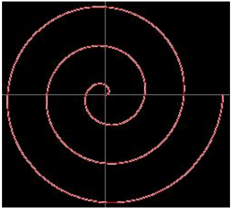
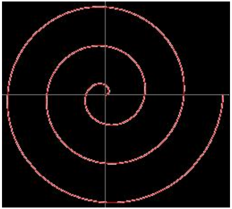


Spiral motion

Archimedian spiral

$r = a + b\theta$



Logarithmic spiral

$r = ab^{\theta}$

Spiral motion

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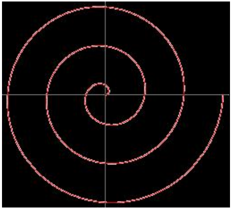

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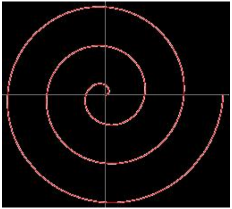

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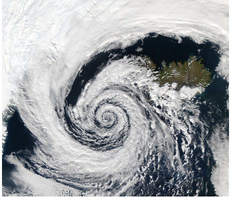

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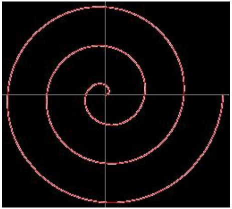

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