Lecture 16

Simple Harmonic Motion

Introduction

Radhika Vathsan, Physics@BITS-Goa, 2024

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Importance of SHM:

- Fourier theorem: *"Any periodic motion can be decomposed into harmonic (sine-form) components."*
- **Small oscillations** about the mean of ANY system: nearly harmonic $\Longrightarrow$ SHM is important!

Prototype example: Frictionless Spring Mass system

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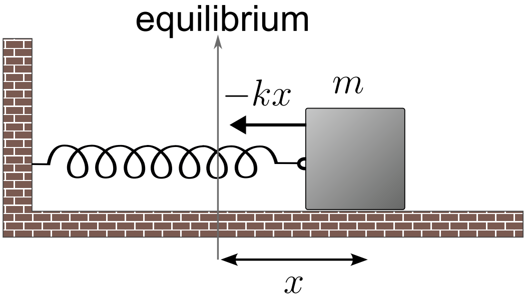
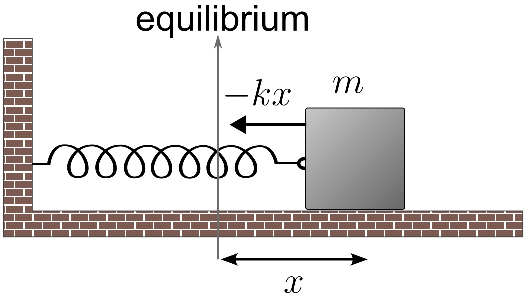


Prototype example: Frictionless Spring Mass system

Restoring force = $-kx$



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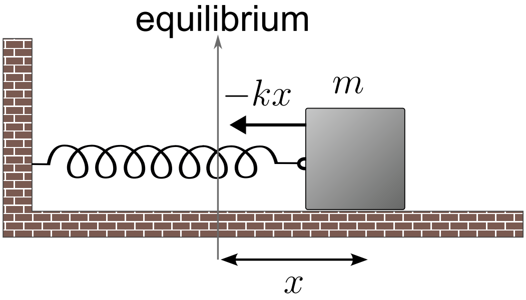
$$F(x) = -\frac{dU}{dx} \implies$$



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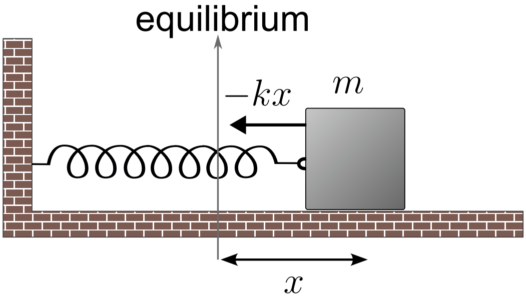

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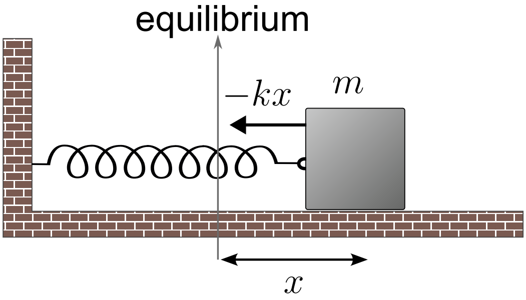

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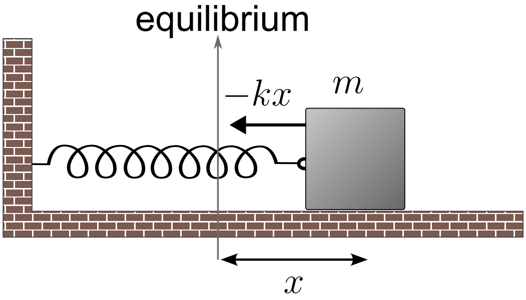
Solution:

$$x(t) = A\cos(\omega_0 t + \phi)$$



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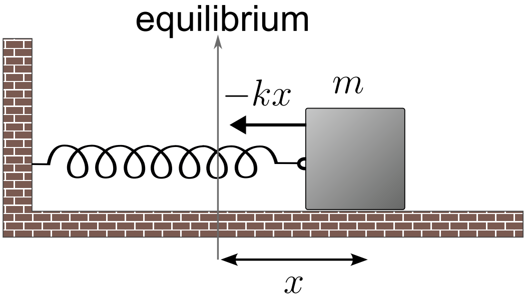

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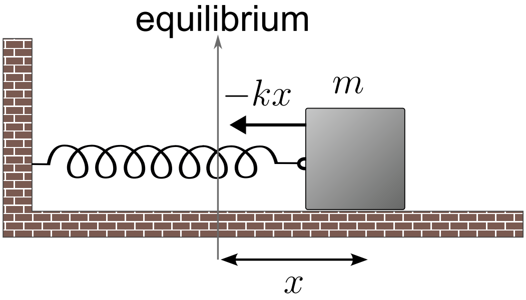
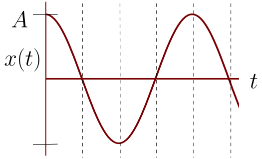


A: amplitude
ϕ: initial phase.

Characteristics of SHM

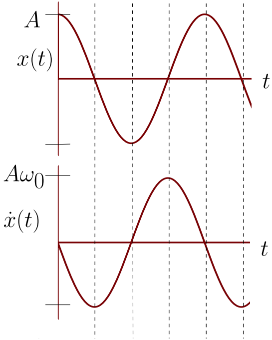
Displacement: $x(t) = A\cos(\omega_0 t + \phi)$,



Characteristics of SHM

Displacement: $x(t) = A\cos(\omega_0 t + \phi)$,

Velocity: $\dot{x}(t) = -A\omega_0 \sin(\omega_0 t + \phi)$,



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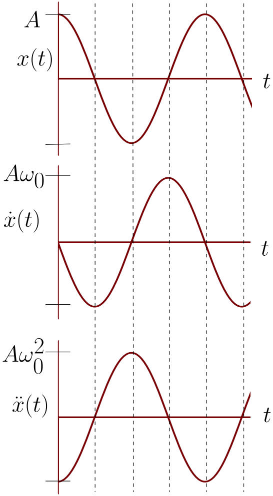

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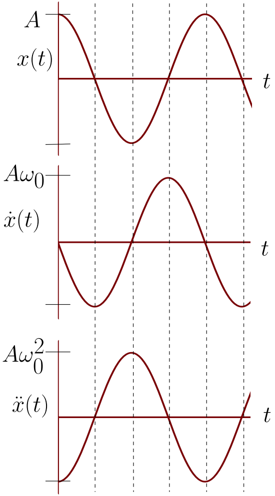
Converse:

$m\ddot{x} = -m\omega_0^2 x$, restoring force.



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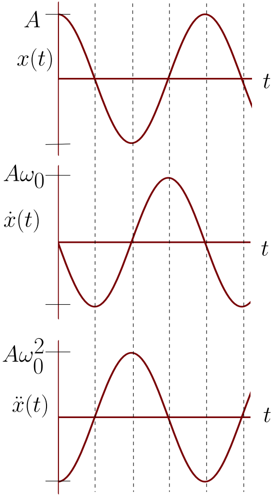
Converse:
$m\ddot{x} = -m\omega_0^2 x$, restoring force. $m\omega_0^2 = k$: stiffness or spring constant.



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$\implies$ Amplitude $A = \sqrt{x_0^2 + \left(\frac{v_0}{\omega_0}\right)^2};$

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$$\implies \text{Amplitude } A = \sqrt{x_0^2 + \left(\frac{v_0}{\omega_0}\right)^2};$$

$$\text{Phase } \tan(\phi) = \frac{v_0}{\omega_0 x_0}.$$

Energy in SHM

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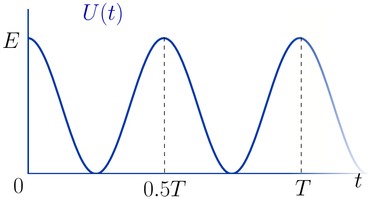
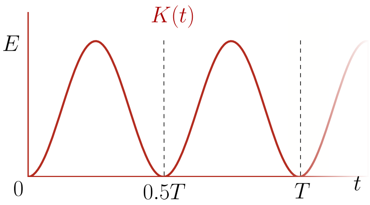


Potential Energy

$$U(t) = \frac{1}{2}kx^2$$

Energy in SHM



Kinetic Energy

$$K(t) = \frac{1}{2}m\dot{x}^2$$

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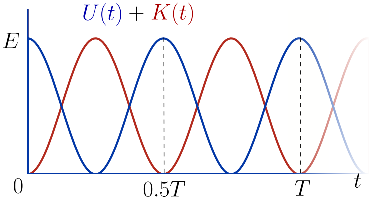
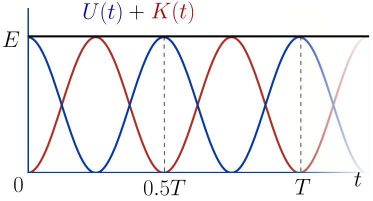
Energy in SHM



Total Energy

$$U(t) + K(t) = \frac{1}{2}kx^2 + \frac{1}{2}m\dot{x}^2 = E$$

Energy in SHM



Total Energy decided by the initial conditions:

$$\begin{aligned} U(t) + K(t) &= \frac{1}{2}kx^2 + \frac{1}{2}m\dot{x}^2 = E \\ &= \frac{1}{2}m{v_0}^2 + \frac{1}{2}kx_0^2 = \text{ const.} \end{aligned}$$

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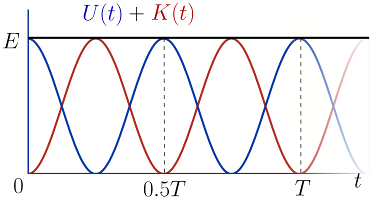

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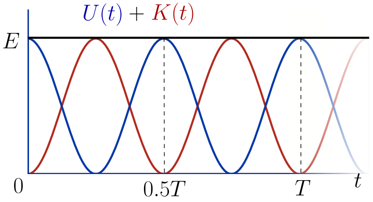
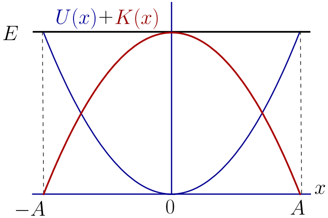
$$\frac{dE}{dt} = 0 \implies m\ddot{x} + kx = 0 \text{ (EoM).}$$

Energy in SHM





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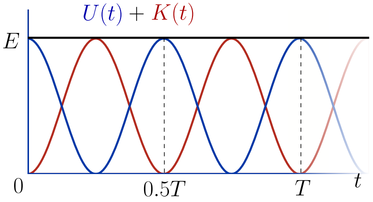

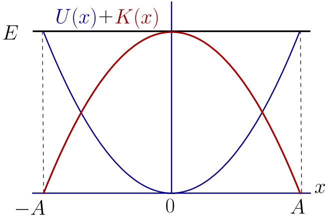

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1. $U_{\text{max}} = \frac{1}{2}kA^2 = K_{\text{max}}$.
2. $U_{\text{min}} = 0$ at $x = 0$ (equilibrium)

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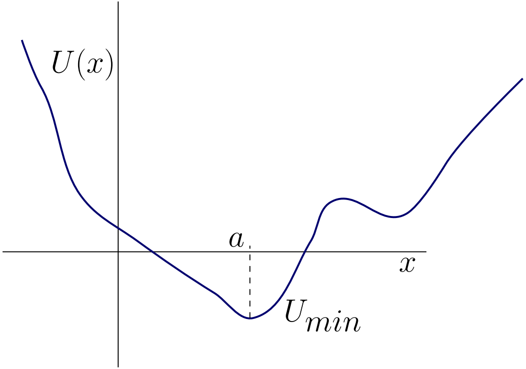

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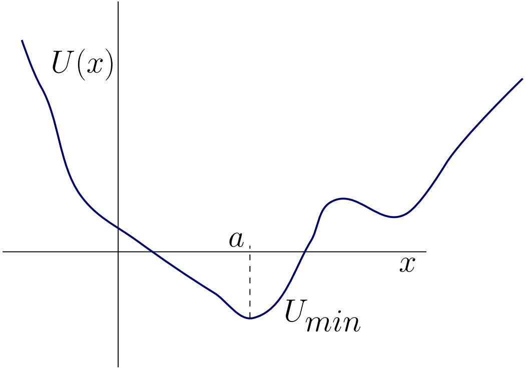
Near $x = a$, use Taylor expansion

$$U(x) \approx U(a) + xU'(a) + \frac{x^2}{2}U''(a) + \ldots$$



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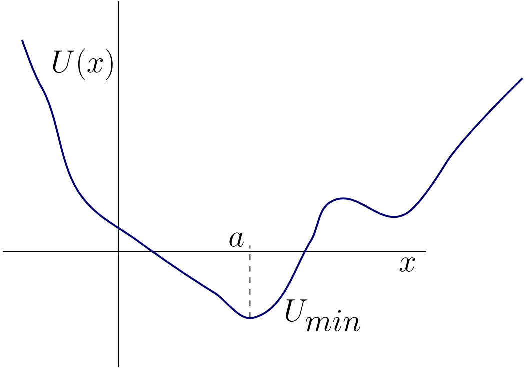

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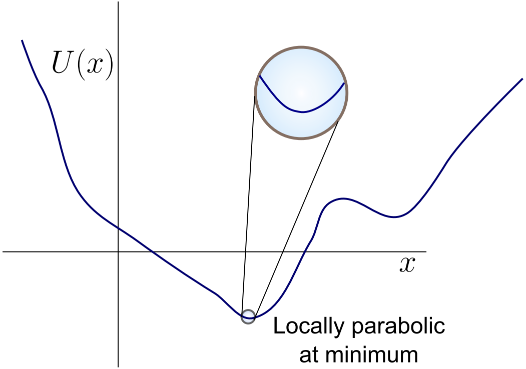

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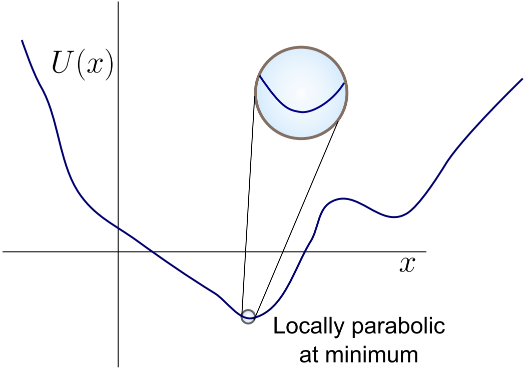
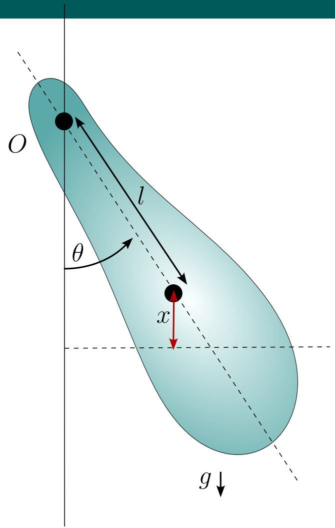
where $k \equiv U''(a)$.



Physical Pendulum

Angle θ : displacement from equilibrium ($\theta = 0$).



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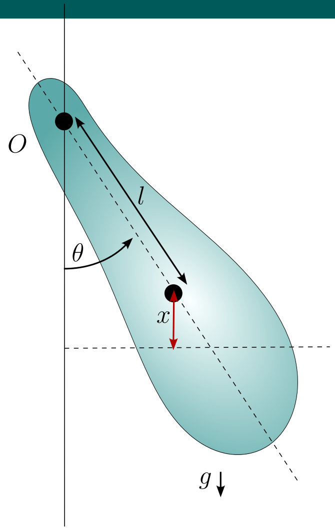
$U(x) = mgx;$



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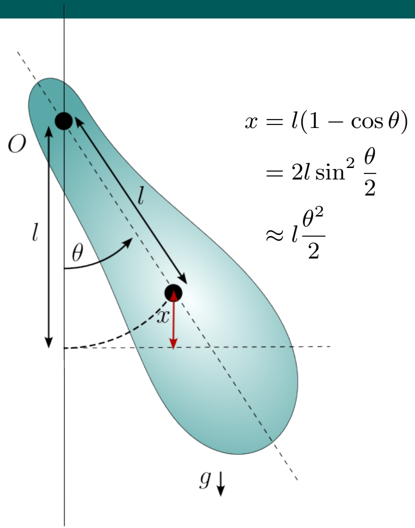
$U(x) = mgx; \approx mgl\dfrac{\theta^2}{2}$ (for small θ).



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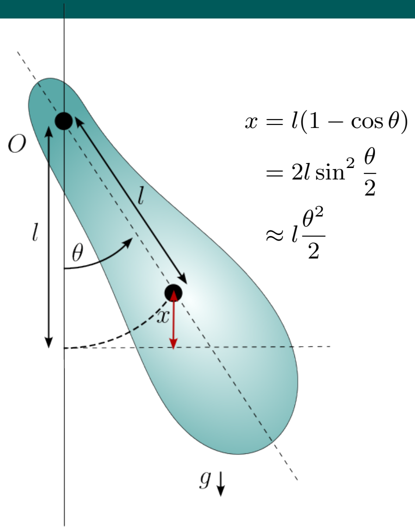
Energy conservation:

$$\frac{1}{2}I\dot{\theta}^2 + \frac{1}{2}mgl\theta^2 = E.$$



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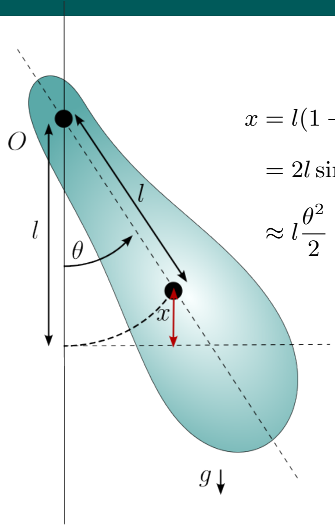

$$x = l(1-\cos\theta) = 2l\sin^2\frac{\theta}{2} \approx l\frac{\theta^2}{2}$$

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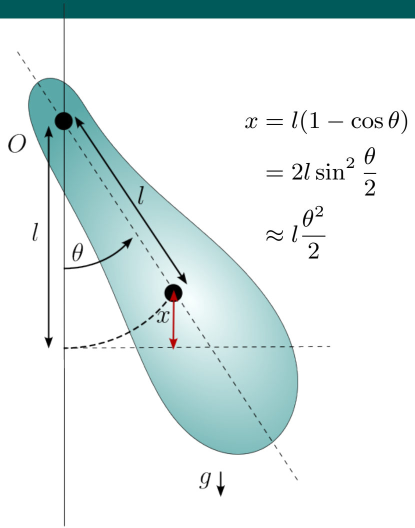
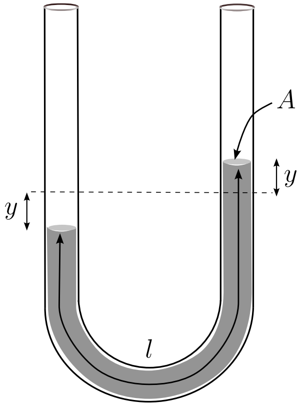
$$\therefore \boxed{\omega_0^2 = \frac{mgl}{I}}.$$



Non-viscous liquid in a U-tube

Displacement of level: y
Level oscillates about mean $y = 0$.



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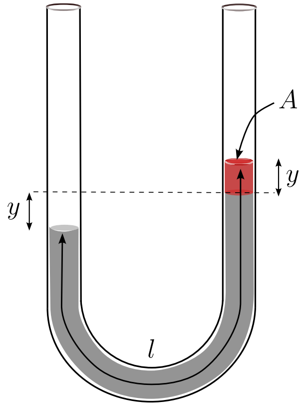

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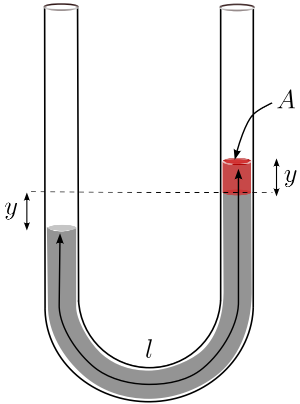

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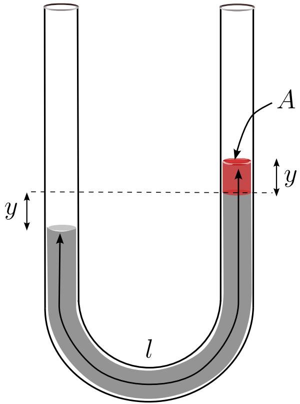
Kinetic energy : $\frac{1}{2}(\rho A l)\dot{y}^2$.

$$\frac{1}{2}\rho A l \dot{y}^2 + \rho g A y^2 = E$$



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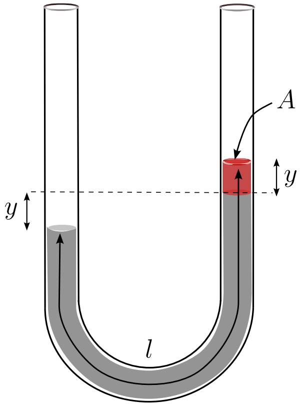
$$\frac{1}{2}\rho A l \dot{y}^2 + \rho g A y^2 = E$$

$$\implies \ddot{y} + \frac{2g}{l} y = 0.$$



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Level oscillates about mean $y = 0$.

Mass of liquid displaced: $m = \rho A y$.

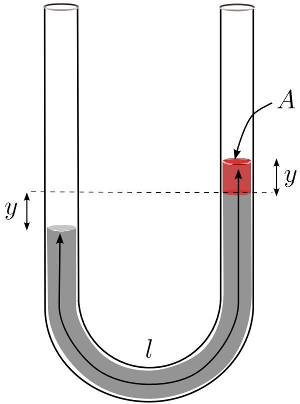
Potential energy : $U = \rho g A y^2$.

Kinetic energy : $\frac{1}{2}(\rho A l)\dot{y}^2$.

$$\frac{1}{2}\rho A l \dot{y}^2 + \rho g A y^2 = E$$

$$\implies \ddot{y} + \frac{2g}{l} y = 0.$$

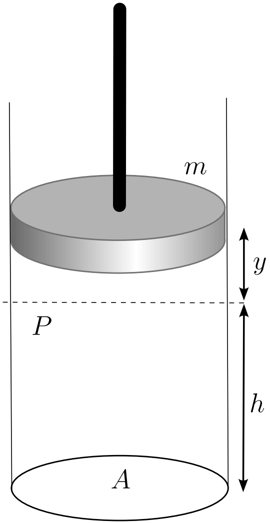
$$\therefore \; \omega_0^2 = \frac{2g}{l}.$$



"Air spring"

Insulated container of gas at pressure P.

Piston of mass m oscillates about $y = 0$.



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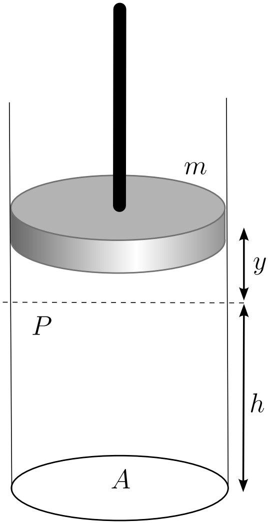

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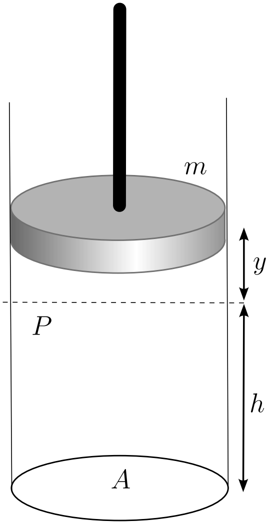

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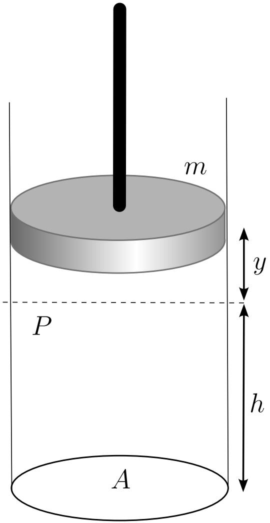

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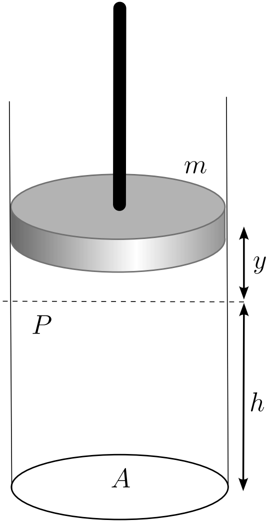

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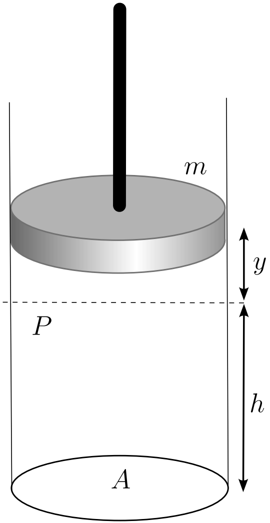

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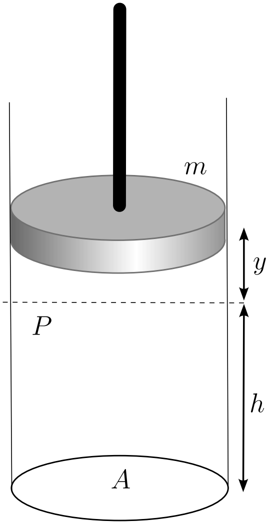

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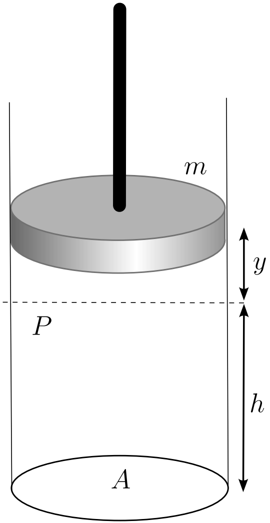
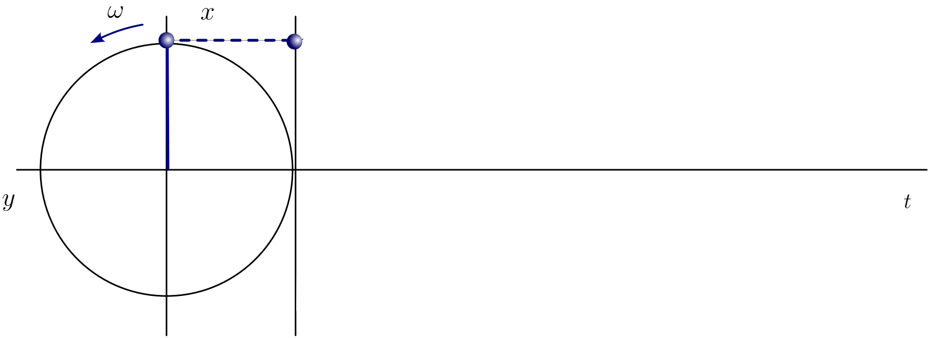


SHM and Circular Motion

Particle moves with uniform angular speed ω along circle of radius A.



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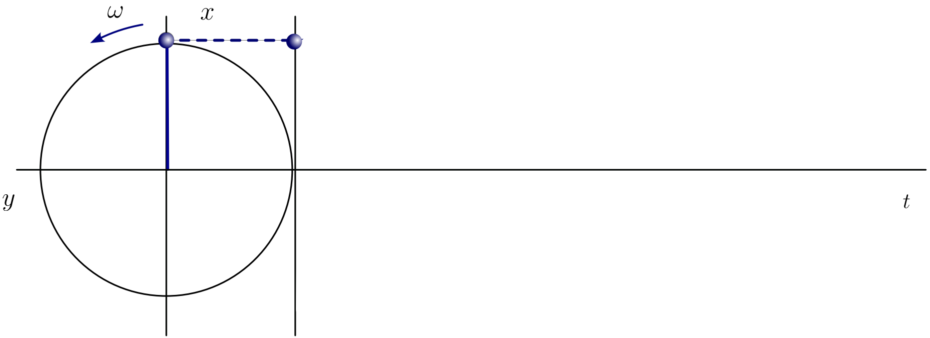

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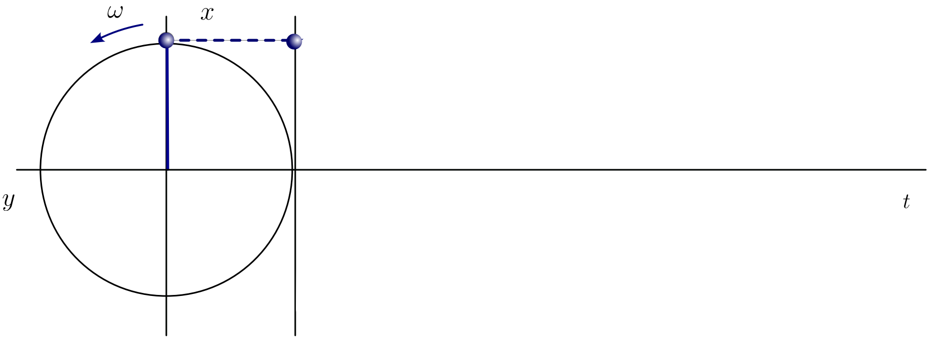

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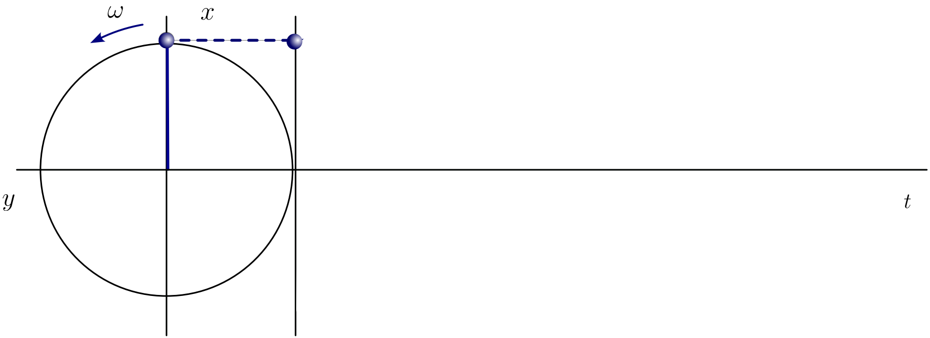

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Use: geometric calculation of superpositions of SHMs, solutions of eqns involving SHMs

SHM and Complex Phasors

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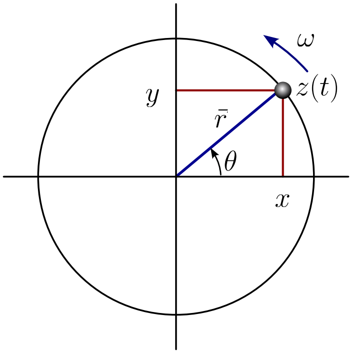

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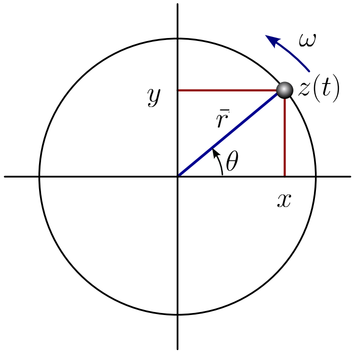

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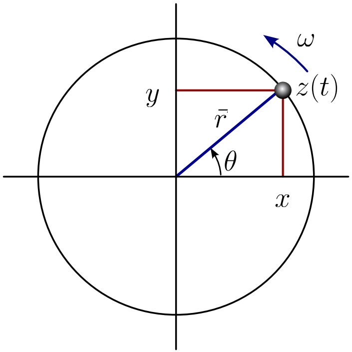

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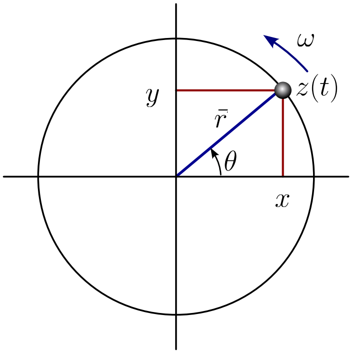

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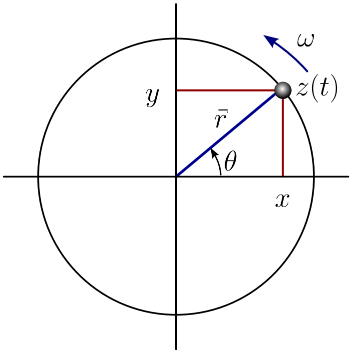
Examples: $-1 = e^{i\pi}$, $i = e^{i\pi/2}$, $i^i = e^{-\pi/2}$.



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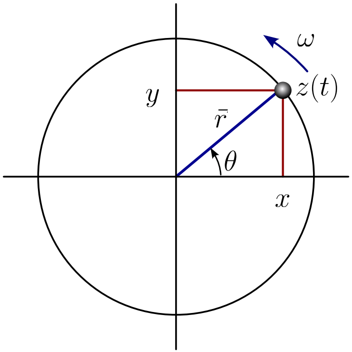

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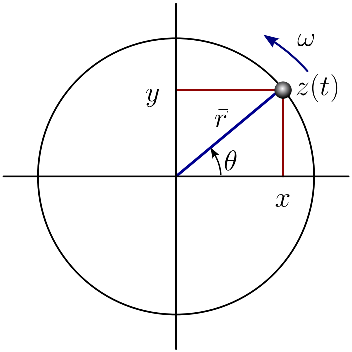

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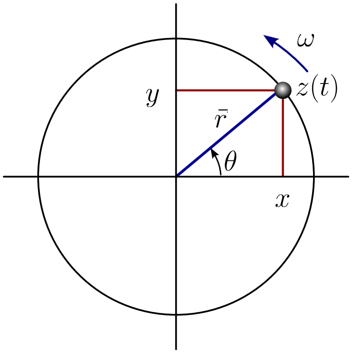

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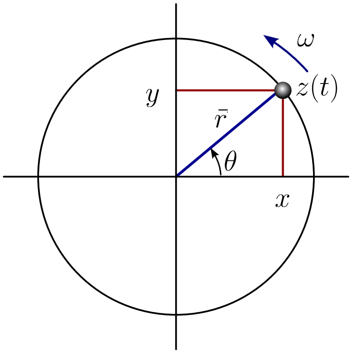

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Use: Superposition of SHMs, solution to SHM differential eqs.

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Exercise: Find C and ϕ in terms of A and B.