Lecture 20

Simple Harmonic Motion

Forced Harmonic Motion with Damping

Radhika Vathsan, Physics@BITS-Goa, 2024

Lecture 20

Simple Harmonic Motion
Forced Harmonic Motion with Damping

Radhika Vathsan, Physics@BITS-Goa, 2024

Forced Oscillations with damping

$$\ddot{x} + \gamma \dot{x} + \omega_0^2 x = \frac{F_0}{m} \cos \Omega t$$

Forced Oscillations with damping

Mathematical analysis: $\ddot{x} + \gamma\dot{x} + \omega_0^2 x = \frac{F_0}{m}\cos\Omega t$

Forced Oscillations with damping

Mathematical analysis: $\ddot{x} + \gamma \dot{x} + \omega_0^2 x = \frac{F_0}{m} \cos \Omega t$

1. General solution: $x_2(t) = B e^{-\gamma t/2} \cos(\omega_0 t + \phi)$:

Forced Oscillations with damping

Mathematical analysis: $\ddot{x} + \gamma \dot{x} + \omega_0^2 x = \frac{F_0}{m} \cos \Omega t$

1. General solution: $x_2(t) = Be^{-\gamma t/2} \cos(\omega_0 t + \phi)$: Transient (damped oscillator).

Forced Oscillations with damping

Mathematical analysis: $\ddot{x} + \gamma\dot{x} + \omega_0^2 x = \dfrac{F_0}{m}\cos\Omega t$

1. General solution: $x_2(t) = Be^{-\gamma t/2}\cos(\omega_0 t + \phi)$: Transient (damped oscillator).
2. Particular solution:

   Solve $\ddot{z} + \gamma\dot{z} + \omega_0^2 z = \dfrac{F_0}{m}e^{i\Omega t}$ using $z(t) = Re^{i(\Omega t + \theta)}$

Forced Oscillations with damping

Mathematical analysis: $\ddot{x} + \gamma\dot{x} + \omega_0^2 x = \frac{F_0}{m}\cos\Omega t$

1. General solution: $x_2(t) = Be^{-\gamma t/2}\cos(\omega_0 t + \phi)$: Transient (damped oscillator).
2. Particular solution:

Solve $\ddot{z} + \gamma\dot{z} + \omega_0^2 z = \frac{F_0}{m}e^{i\Omega t}$ using $z(t) = Re^{i(\Omega t+\theta)}$

$$\implies \quad (-\Omega^2 + i\Omega\gamma + \omega_0^2)R = \frac{F_0}{m}e^{-i\theta},$$

Forced Oscillations with damping

Mathematical analysis: $\ddot{x} + \gamma \dot{x} + \omega_0^2 x = \frac{F_0}{m} \cos \Omega t$

1. General solution: $x_2(t) = Be^{-\gamma t/2} \cos(\omega_0 t + \phi)$: Transient (damped oscillator).
2. Particular solution:

Solve $\ddot{z} + \gamma \dot{z} + \omega_0^2 z = \frac{F_0}{m} e^{i\Omega t}$ using $z(t) = Re^{i(\Omega t + \theta)}$

$$\implies (-\Omega^2 + i\Omega\gamma + \omega_0^2)R = \frac{F_0}{m} e^{-i\theta},$$

real part: $$(\omega_0^2 - \Omega^2)R = \frac{F_0}{m} \cos \theta;$$

Forced Oscillations with damping

Mathematical analysis: $\ddot{x} + \gamma\dot{x} + \omega_0^2 x = \dfrac{F_0}{m}\cos\Omega t$

1. General solution: $x_2(t) = Be^{-\gamma t/2}\cos(\omega_0 t + \phi)$: Transient (damped oscillator).
2. Particular solution:

Solve $\ddot{z} + \gamma\dot{z} + \omega_0^2 z = \dfrac{F_0}{m}e^{i\Omega t}$ using $z(t) = Re^{i(\Omega t+\theta)}$

$$\implies \quad (-\Omega^2 + i\Omega\gamma + \omega_0^2)R = \frac{F_0}{m}e^{-i\theta},$$

real part: $$(\omega_0^2 - \Omega^2)R = \frac{F_0}{m}\cos\theta;$$

imaginary part: $$\Omega\gamma R = \frac{F_0}{m}\sin\theta.$$

Forced Oscillations with damping

Mathematical analysis: $\ddot{x} + \gamma\dot{x} + \omega_0^2 x = \dfrac{F_0}{m}\cos\Omega t$

1. General solution: $x_2(t) = Be^{-\gamma t/2}\cos(\omega_0 t + \phi)$: Transient (damped oscillator).
2. Particular solution:

Solve $\ddot{z} + \gamma\dot{z} + \omega_0^2 z = \dfrac{F_0}{m}e^{i\Omega t}$ using $z(t) = Re^{i(\Omega t+\theta)}$

$$\implies (-\Omega^2 + i\Omega\gamma + \omega_0^2)R = \frac{F_0}{m}e^{-i\theta},$$

real part: $$(\omega_0^2 - \Omega^2)R = \frac{F_0}{m}\cos\theta;$$

imaginary part: $$\Omega\gamma R = \frac{F_0}{m}\sin\theta.$$

$$\implies R = \frac{F_0}{m\sqrt{(\omega_0^2 - \Omega^2)^2 + (\Omega\gamma)^2}},$$

Forced Oscillations with damping

Mathematical analysis: $\ddot{x} + \gamma\dot{x} + \omega_0^2 x = \dfrac{F_0}{m}\cos\Omega t$

1. General solution: $x_2(t) = Be^{-\gamma t/2}\cos(\omega_0 t + \phi)$: Transient (damped oscillator).
2. Particular solution:

Solve $\ddot{z} + \gamma\dot{z} + \omega_0^2 z = \dfrac{F_0}{m}e^{i\Omega t}$ using $z(t) = Re^{i(\Omega t+\theta)}$

$$\Longrightarrow \quad (-\Omega^2 + i\Omega\gamma + \omega_0^2)R = \frac{F_0}{m}e^{-i\theta},$$

real part: $$(\omega_0^2 - \Omega^2)R = \frac{F_0}{m}\cos\theta;$$

imaginary part: $$\Omega\gamma R = \frac{F_0}{m}\sin\theta.$$

$$\Longrightarrow \quad R = \frac{F_0}{m\sqrt{(\omega_0^2 - \Omega^2)^2 + (\Omega\gamma)^2}}, \qquad \theta = \tan^{-1}\left(\frac{\Omega\gamma}{\omega_0^2 - \Omega^2}\right).$$

Forced Oscillations with dampeding

$$x_1(t) = R\cos(\Omega t + \theta),$$

- Steady state solution

Forced Oscillations with dampeding

$$x_1(t) = R\cos(\Omega t + \theta), \quad R = \frac{F_0}{m\sqrt{(\omega_0^2 - \Omega^2)^2 + (\Omega\gamma)^2}},$$

- Steady state solution

Forced Oscillations with dampeding

$$x_1(t) = R\cos(\Omega t + \theta), \quad R = \frac{F_0}{m\sqrt{(\omega_0^2 - \Omega^2)^2 + (\Omega\gamma)^2}}, \qquad \theta = \tan^{-1}\left(\frac{\Omega\gamma}{\omega_0^2 - \Omega^2}\right)$$

- Steady state solution

Forced Oscillations with dampeding

$$x_1(t) = R\cos(\Omega t + \theta), \quad R = \frac{F_0}{m\sqrt{(\omega_0^2 - \Omega^2)^2 + (\Omega\gamma)^2}}, \qquad \theta = \tan^{-1}\left(\frac{\Omega\gamma}{\omega_0^2 - \Omega^2}\right)$$

- Steady state solution
- R is maximum when $\Omega \sim \omega_0$: Resonance

Forced Oscillations with dampeding

$$x_1(t) = R\cos(\Omega t + \theta), \quad R = \frac{F_0}{m\sqrt{(\omega_0^2 - \Omega^2)^2 + (\Omega\gamma)^2}}, \qquad \theta = \tan^{-1}\left(\frac{\Omega\gamma}{\omega_0^2 - \Omega^2}\right)$$

- Steady state solution
- R is maximum when $\Omega \sim \omega_0$: Resonance
- Phase changes sign thru resonance.

Forced Oscillations with dampeding

$$x_1(t) = R\cos(\Omega t + \theta), \quad R = \frac{F_0}{m\sqrt{(\omega_0^2 - \Omega^2)^2 + (\Omega\gamma)^2}}, \qquad \theta = \tan^{-1}\left(\frac{\Omega\gamma}{\omega_0^2 - \Omega^2}\right)$$

- Steady state solution
- R is maximum when $\Omega \sim \omega_0$: Resonance
- Phase changes sign thru resonance.
- Full solution:

$$x(t) = A(\text{Transient state}) + B(\text{Steady state}).$$

Forced Oscillations with dampeding

$$x_1(t) = R\cos(\Omega t + \theta), \quad R = \frac{F_0}{m\sqrt{(\omega_0^2 - \Omega^2)^2 + (\Omega\gamma)^2}}, \qquad \theta = \tan^{-1}\left(\frac{\Omega\gamma}{\omega_0^2 - \Omega^2}\right)$$

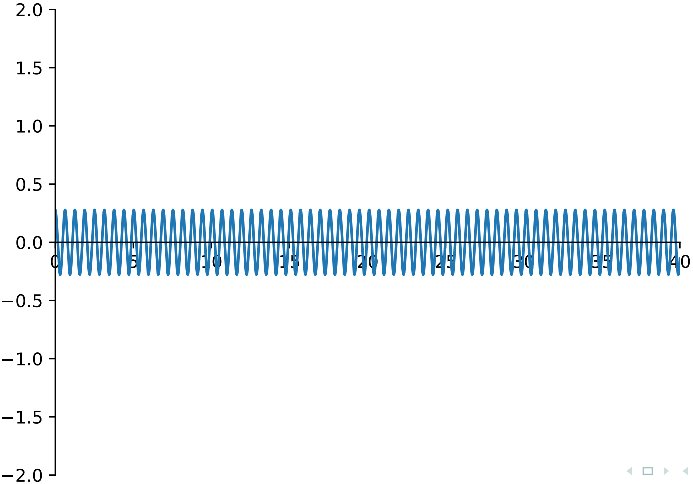
- Steady state solution
- R is maximum when $\Omega \sim \omega_0$: Resonance
- Phase changes sign thru resonance.
- Full solution:

$$x(t) = A(\text{Transient state}) + B(\text{Steady state}).$$

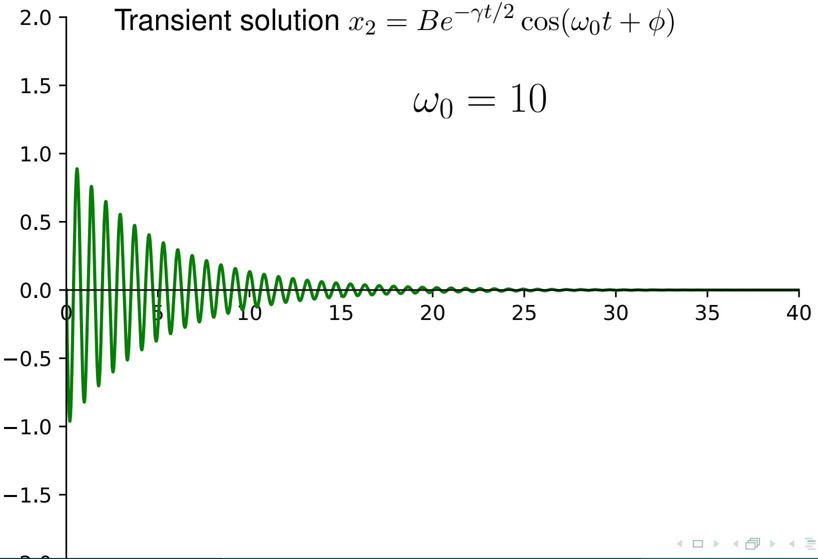
Transient vibrations with freq ω_0: satisfy IC's, die out at rate $e^{-\gamma t/2}$.

Solution

Steady-state solution $x_1 = R(\Omega)\cos(\Omega t + \theta(\Omega))$



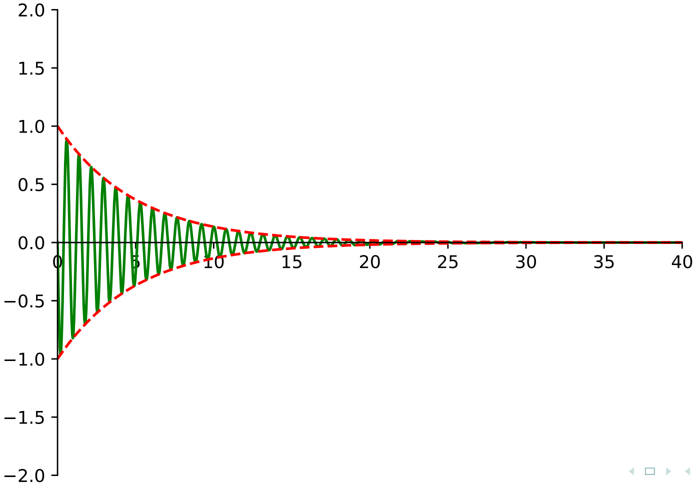
Solution



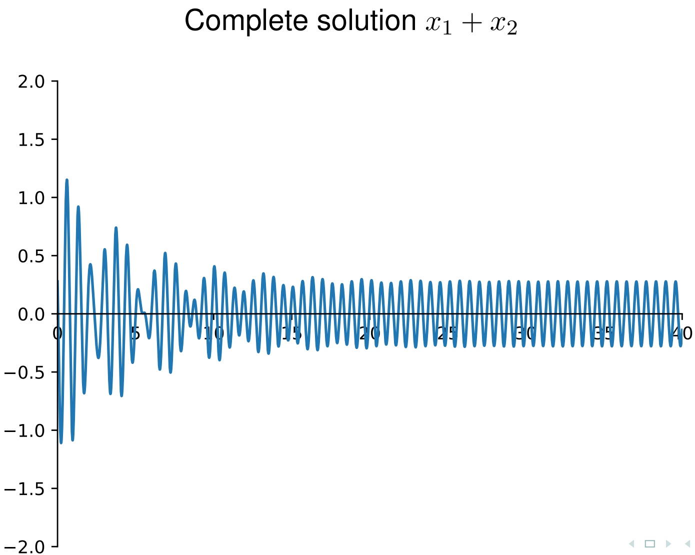
Transient solution $x_2 = Be^{-\gamma t/2}\cos(\omega_0 t + \phi)$
$\omega_0 = 10$
2.0
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
0
5
10
15
20
25
30
35
40

Solution

Transient solution $x_2 = Be^{-\gamma t/2}\cos(\omega_0 t + \phi)$

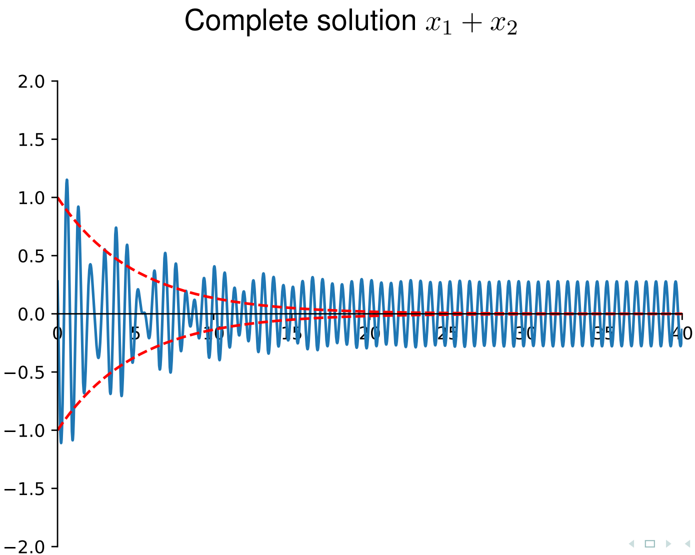


Solution



Complete solution $x_1 + x_2$
2.0
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−2.0
0
5
10
15
20
25
30
35
40

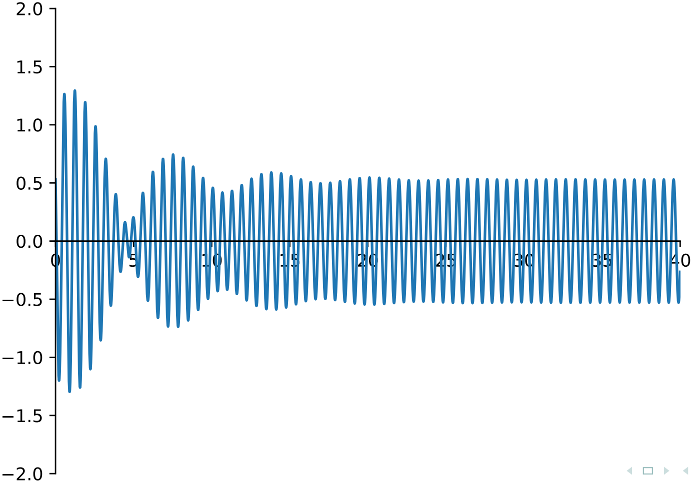
Solution



Complete solution $x_1 + x_2$
2.0
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−2.0
0
5
10
15
20
25
30
35
40

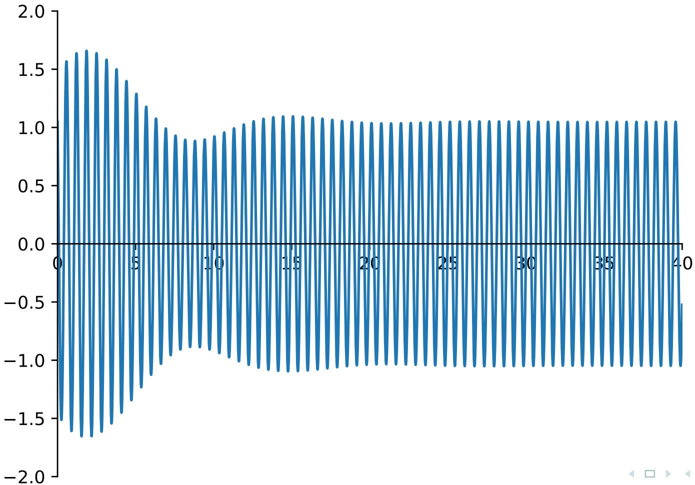
Solution

Complete solution $x_1 + x_2$: closer to resonance

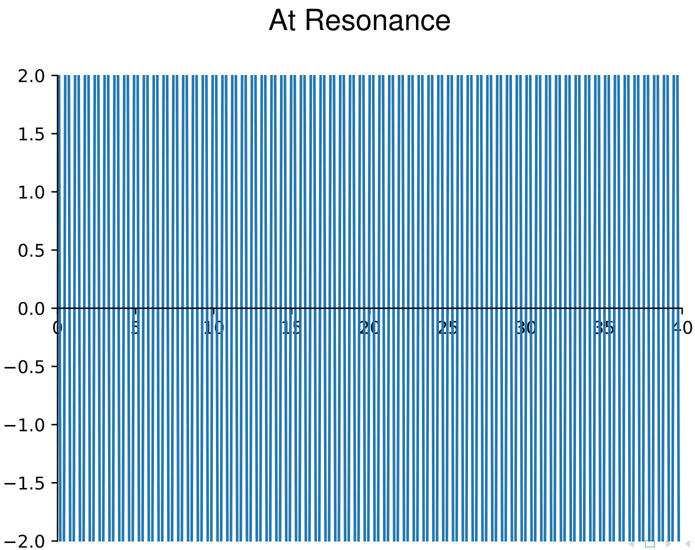


Solution

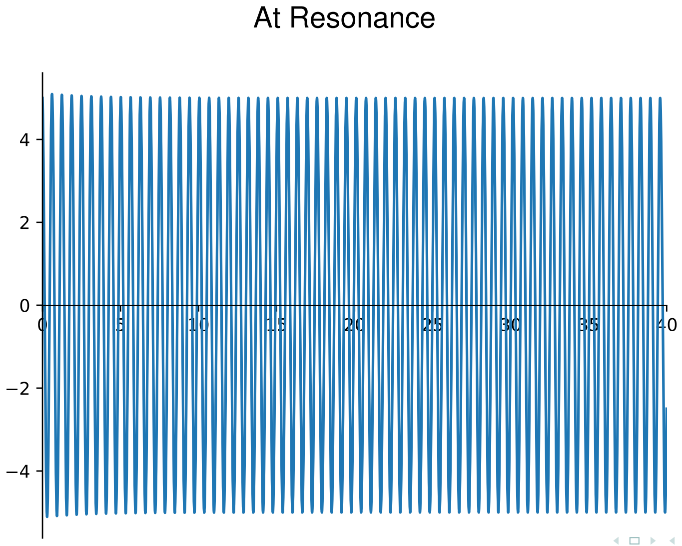
Complete solution $x_1 + x_2$: closer to resonance



Solution



Solution

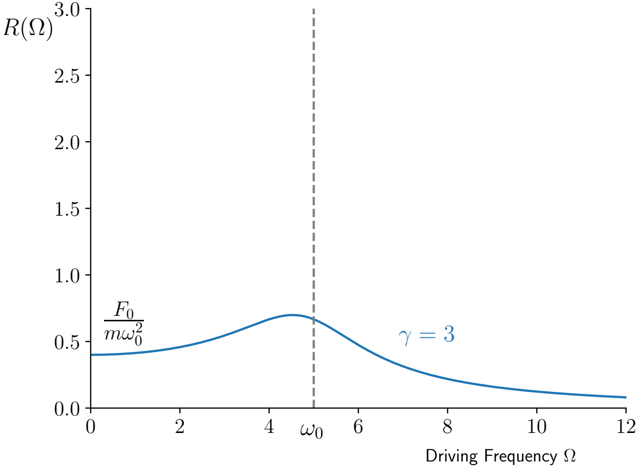


Resonance: displacement vs driving frequency

$$R(\Omega) = \frac{F_0/m}{\sqrt{(\omega_0^2 - \Omega^2)^2 + (\Omega\gamma)^2}}$$

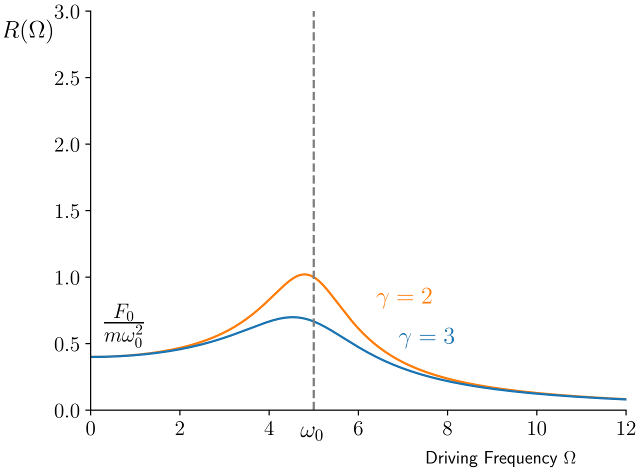
Resonance: displacement vs driving frequency

$$R(\Omega) = \frac{F_0/m}{\sqrt{(\omega_0^2 - \Omega^2)^2 + (\Omega\gamma)^2}}$$



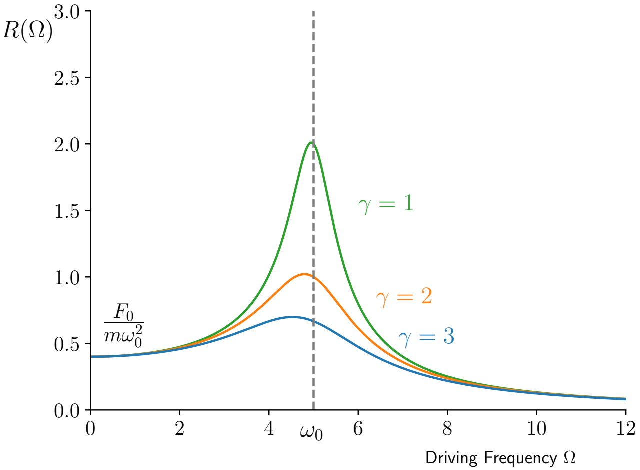
Resonance: displacement vs driving frequency

$$R(\Omega) = \frac{F_0/m}{\sqrt{(\omega_0^2 - \Omega^2)^2 + (\Omega\gamma)^2}}$$



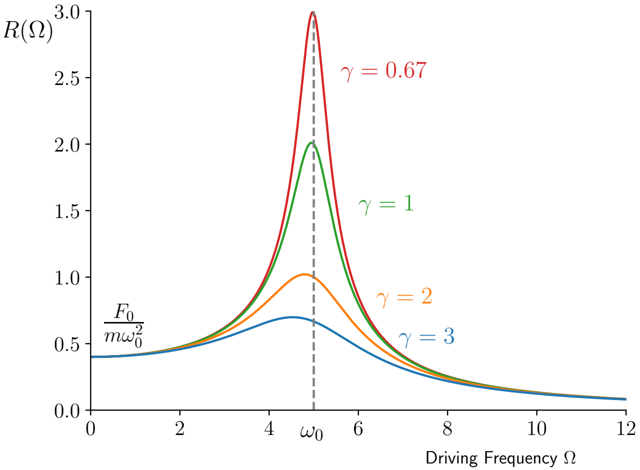
Resonance: displacement vs driving frequency

$$R(\Omega) = \frac{F_0/m}{\sqrt{(\omega_0^2 - \Omega^2)^2 + (\Omega\gamma)^2}}$$



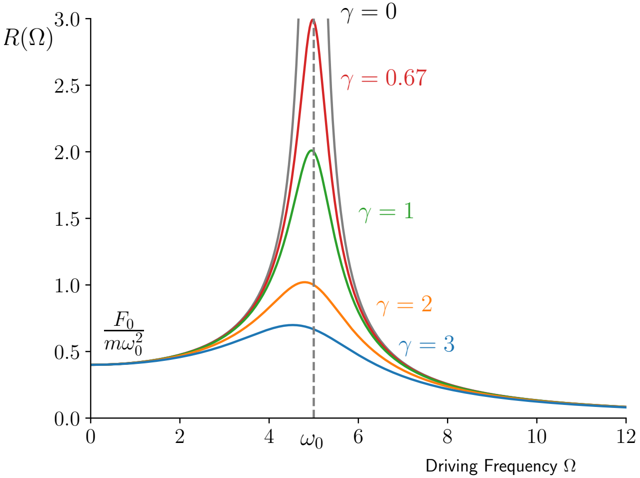
Resonance: displacement vs driving frequency

$$R(\Omega) = \frac{F_0/m}{\sqrt{(\omega_0^2 - \Omega^2)^2 + (\Omega\gamma)^2}}$$



Resonance: displacement vs driving frequency

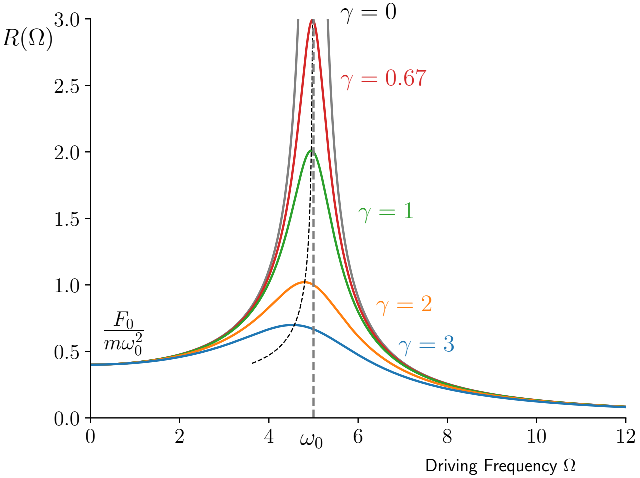
$$R(\Omega) = \frac{F_0/m}{\sqrt{(\omega_0^2 - \Omega^2)^2 + (\Omega\gamma)^2}}$$



Resonance: displacement vs driving frequency

$$R(\Omega) = \frac{F_0/m}{\sqrt{(\omega_0^2 - \Omega^2)^2 + (\Omega\gamma)^2}}$$

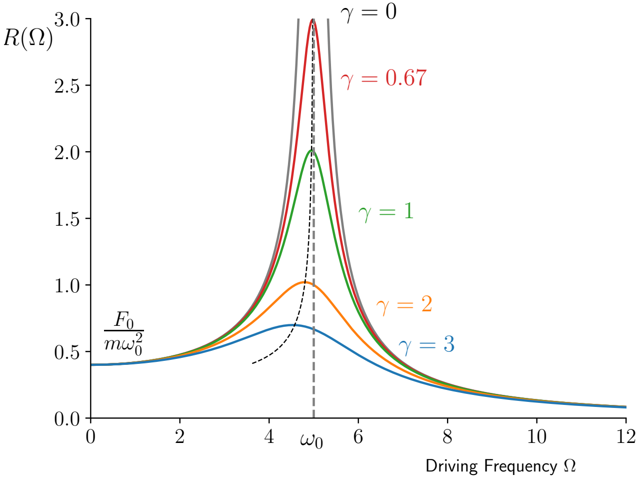
- Max amplitude resonance: when $\Omega_r^2 = \omega_0^2 - \dfrac{\gamma^2}{2}$
- $R_{\text{max}} = \dfrac{F_0}{m\omega_0\gamma}$



Resonance: displacement vs driving frequency

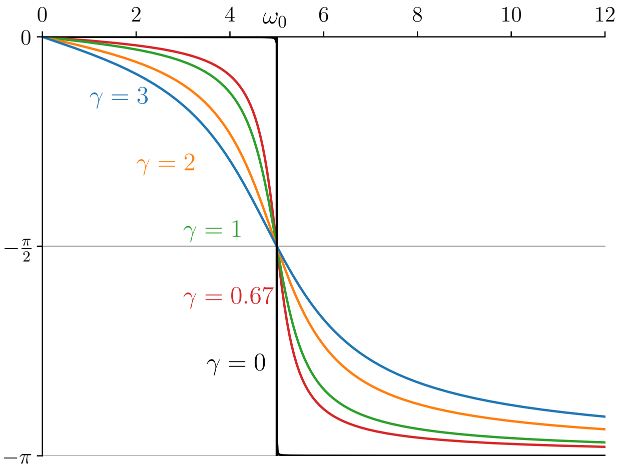
$$R(\Omega) = \frac{F_0/m}{\sqrt{(\omega_0^2 - \Omega^2)^2 + (\Omega\gamma)^2}}$$

- Max amplitude resonance: when $\Omega_r^2 = \omega_0^2 - \dfrac{\gamma^2}{2}$
- $R_{\text{max}} = \dfrac{F_0}{m\omega_0\gamma}$
- For *light damping*, resonance at $\Omega_r = \omega_0$.



Resonance: phase vs driving frequency

$$\theta(\Omega) = \tan^{-1}\left(\frac{\Omega\gamma}{\omega_0^2 - \Omega^2}\right)$$



Energy in a forced damped oscillator

- Energy of forced oscillator NOT constant in time.

Energy in a forced damped oscillator

- Energy of forced oscillator NOT constant in time.

$$x(t) = R(\Omega)\cos(\Omega t + \theta); \quad \text{Steady State}$$

Energy in a forced damped oscillator

- Energy of forced oscillator NOT constant in time.

$$
\begin{aligned}
x(t) &= R(\Omega)\cos(\Omega t + \theta); \quad \text{Steady State} \\
\dot{x}(t) &= -R\Omega \sin(\Omega t + \theta)
\end{aligned}
$$

Energy in a forced damped oscillator

- Energy of forced oscillator NOT constant in time.

$$
\begin{aligned}
x(t) &= R(\Omega)\cos(\Omega t + \theta); \quad \text{Steady State} \\
\dot{x}(t) &= -R\Omega \sin(\Omega t + \theta) \\
E &= \frac{1}{2}m\dot{x}^2 + \frac{1}{2}kx^2
\end{aligned}
$$

Energy in a forced damped oscillator

- Energy of forced oscillator NOT constant in time.

$$
\begin{aligned}
x(t) &= R(\Omega)\cos(\Omega t+\theta); \quad \textcolor{blue}{\text{Steady State}} \\
\dot{x}(t) &= -R\Omega\sin(\Omega t+\theta) \\
E &= \frac{1}{2}m\dot{x}^2+\frac{1}{2}kx^2 \\
&= \frac{1}{2}R^2\left(m\Omega^2\cos^2(\Omega t+\theta)+k\sin^2(\Omega t+\theta)\right)
\end{aligned}
$$

Energy in a forced damped oscillator

- Energy of forced oscillator NOT constant in time.

$$\begin{aligned} x(t) &= R(\Omega)\cos(\Omega t+\theta); \quad \textcolor{blue}{\text{Steady State}} \\ \dot{x}(t) &= -R\Omega\sin(\Omega t+\theta) \\ E &= \frac{1}{2}m\dot{x}^2+\frac{1}{2}kx^2 \\ &= \frac{1}{2}R^2\left(m\Omega^2\cos^2(\Omega t+\theta)+k\sin^2(\Omega t+\theta)\right) \end{aligned}$$

- Energy taken out by viscous damping: fed in by driving force

Energy in a forced damped oscillator

- Energy of forced oscillator NOT constant in time.

$$\begin{aligned} x(t) &= R(\Omega)\cos(\Omega t + \theta); \quad \text{Steady State} \\ \dot{x}(t) &= -R\Omega\sin(\Omega t + \theta) \\ E &= \frac{1}{2}m\dot{x}^2 + \frac{1}{2}kx^2 \\ &= \frac{1}{2}R^2\left(m\Omega^2\cos^2(\Omega t + \theta) + k\sin^2(\Omega t + \theta)\right) \end{aligned}$$

- Energy taken out by viscous damping: fed in by driving force
- Phase difference between motion and force $\Longrightarrow$ energy changes with time.

Energy in a forced damped oscillator

- Energy of forced oscillator NOT constant in time.

$$\begin{aligned} x(t) &= R(\Omega)\cos(\Omega t+\theta); \quad \textcolor{blue}{\text{Steady State}} \\ \dot{x}(t) &= -R\Omega\sin(\Omega t+\theta) \\ E &= \frac{1}{2}m\dot{x}^2+\frac{1}{2}kx^2 \\ &= \frac{1}{2}R^2\left(m\Omega^2\cos^2(\Omega t+\theta)+k\sin^2(\Omega t+\theta)\right) \end{aligned}$$

- Energy taken out by viscous damping: fed in by driving force
- Phase difference between motion and force $\implies$ energy changes with time.
- Average power dissipated by friction = Average power supplied by external force

Power Absorbed by a driven Oscillator

Instantaneous Power supplied by external force: $P(t) = F(t)\dot{x}(t)$

Power Absorbed by a driven Oscillator

Instantaneous Power supplied by external force: $P(t) = F(t)\dot{x}(t)$

$$P(t) \quad = -\Omega F_0 R \cos \Omega t \sin(\Omega t + \theta).$$

Power Absorbed by a driven Oscillator

Instantaneous Power supplied by external force: $P(t) = F(t)\dot{x}(t)$

$$P(t) \quad = -\Omega F_0 R \cos \Omega t \sin(\Omega t + \theta).$$

Average Power per cycle:

Power Absorbed by a driven Oscillator

Instantaneous Power supplied by external force: $P(t) = F(t)\dot{x}(t)$

$$P(t) \quad = -\Omega F_0 R \cos\Omega t \sin(\Omega t + \theta).$$

Average Power per cycle:

$$\langle P \rangle \qquad = \frac{\text{work per oscillation}}{\text{time period}}$$

Power Absorbed by a driven Oscillator

Instantaneous Power supplied by external force: $P(t) = F(t)\dot{x}(t)$

$$P(t) \quad = -\Omega F_0 R \cos \Omega t \sin(\Omega t + \theta).$$

Average Power per cycle:

$$\langle P \rangle \qquad = \frac{\text{work per oscillation}}{\text{time period}} \qquad = \frac{1}{T}\int_0^T P dt$$

Power Absorbed by a driven Oscillator

Instantaneous Power supplied by external force: $P(t) = F(t)\dot{x}(t)$

$$P(t) \quad = -\Omega F_0 R \cos\Omega t \sin(\Omega t + \theta).$$

Average Power per cycle:

$$\langle P \rangle \qquad = \frac{\text{work per oscillation}}{\text{time period}} \qquad = \frac{1}{T}\int_0^T P dt$$

$$= -\Omega F_0 R \frac{\sin\theta}{2}$$

Power Absorbed by a driven Oscillator

Instantaneous Power supplied by external force: $P(t) = F(t)\dot{x}(t)$

$$P(t) \quad = -\Omega F_0 R \cos\Omega t \sin(\Omega t + \theta).$$

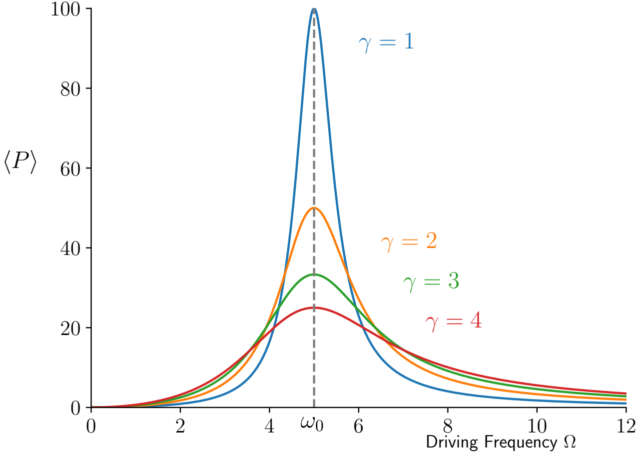
Average Power per cycle:

$$\langle P \rangle \quad = \frac{\text{work per oscillation}}{\text{time period}} \quad = \frac{1}{T}\int_0^T P dt$$

$$= -\Omega F_0 R \frac{\sin\theta}{2}$$

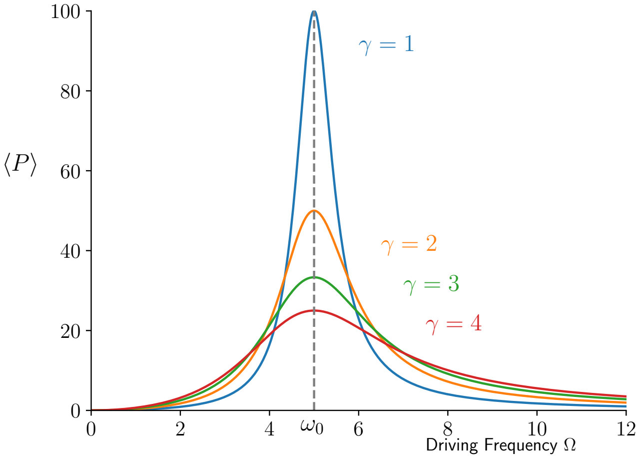
Exercise: Show that this equals the power dissipated by viscous force.

Power Resonance



$$\langle P \rangle = -\Omega F_0 R \frac{\sin\theta}{2}$$

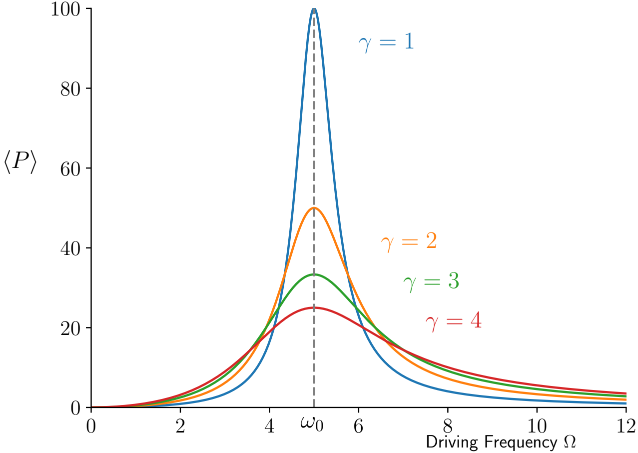
Power Resonance



$$\langle P \rangle = -\Omega F_0 R \frac{\sin\theta}{2}$$

$$= \frac{F_0^2}{2m}\left[\frac{\gamma\Omega^2}{\Omega^2\gamma^2 + (\Omega^2 - \omega_0^2)^2}\right]$$

Power Resonance

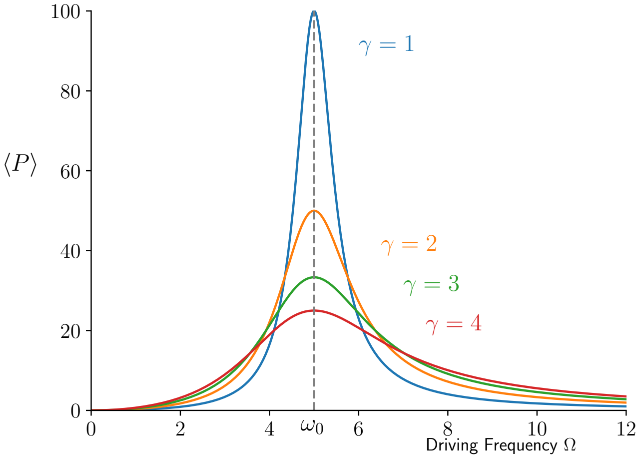


$$\langle P \rangle = -\Omega F_0 R \frac{\sin\theta}{2}$$

$$= \frac{F_0^2}{2m}\left[\frac{\gamma\Omega^2}{\Omega^2\gamma^2 + (\Omega^2 - \omega_0^2)^2}\right]$$

$$= \frac{F_0^2}{2k}\frac{\omega_0}{Q}\left[\frac{1}{\left(\frac{\omega_0}{\Omega} - \frac{\Omega}{\omega_0}\right)^2 + \frac{1}{Q^2}}\right]$$

Power Resonance



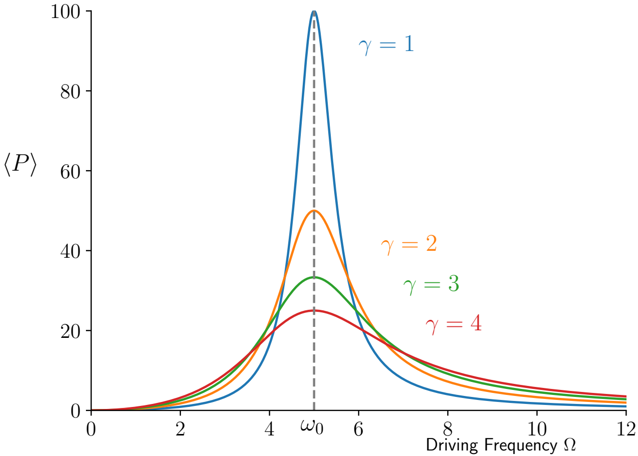
$$\langle P \rangle = -\Omega F_0 R \frac{\sin\theta}{2}$$

$$= \frac{F_0^2}{2m}\left[\frac{\gamma\Omega^2}{\Omega^2\gamma^2 + (\Omega^2 - \omega_0^2)^2}\right]$$

$$= \frac{F_0^2}{2k}\frac{\omega_0}{Q}\left[\frac{1}{\left(\frac{\omega_0}{\Omega} - \frac{\Omega}{\omega_0}\right)^2 + \frac{1}{Q^2}}\right]$$

Max when $\Omega = \omega_0$.

Power Resonance



$$\langle P \rangle = -\Omega F_0 R \frac{\sin\theta}{2}$$

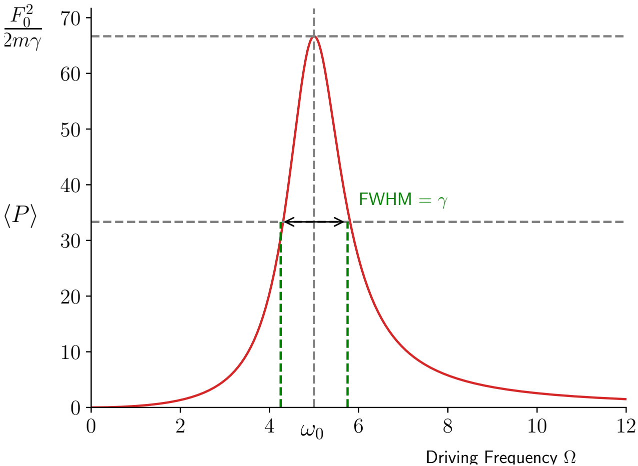
$$= \frac{F_0^2}{2m}\left[\frac{\gamma\Omega^2}{\Omega^2\gamma^2 + (\Omega^2 - \omega_0^2)^2}\right]$$

$$= \frac{F_0^2}{2k}\frac{\omega_0}{Q}\left[\frac{1}{\left(\frac{\omega_0}{\Omega} - \frac{\Omega}{\omega_0}\right)^2 + \frac{1}{Q^2}}\right]$$

Max when $\Omega = \omega_0$.

$$\langle P(\Omega) \rangle_{max} = \frac{F_0^2}{2m\gamma}$$

Power Resonance



$$\langle P \rangle = -\Omega F_0 R \frac{\sin\theta}{2}$$

$$= \frac{F_0^2}{2m}\left[\frac{\gamma\Omega^2}{\Omega^2\gamma^2 + (\Omega^2 - \omega_0^2)^2}\right]$$

$$= \frac{F_0^2}{2k}\frac{\omega_0}{Q}\left[\frac{1}{\left(\frac{\omega_0}{\Omega} - \frac{\Omega}{\omega_0}\right)^2 + \frac{1}{Q^2}}\right]$$

Max when $\Omega = \omega_0$.

$$\langle P(\Omega) \rangle_{max} = \frac{F_0^2}{2m\gamma}$$

Width of resonance at half maximum (FWHM): $\Delta\Omega = \gamma$

Q-factor

$$
\begin{aligned}
Q &= \frac{\omega_0}{\gamma} \\
&= \frac{\text{resonant frequency}}{\text{FWHM}}
\end{aligned}
$$

Q-factor

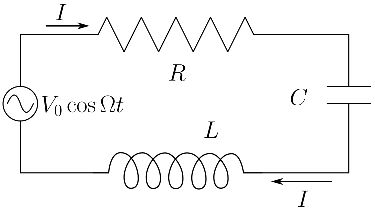
$$
\begin{aligned}
Q &= \frac{\omega_0}{\gamma} \\
&= \frac{\text{resonant frequency}}{\text{FWHM}}
\end{aligned}
$$

Q-factor and amplification:

As $\omega \to 0$, amplitude $R \to R_0 = \dfrac{F_0}{k}$

$$
\frac{R_{max}}{R_0} = \frac{1}{\gamma} = \frac{Q}{\omega_0}
$$

The electrical analog: Forced LCR circuit



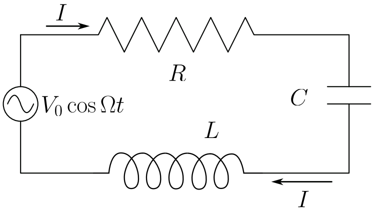
Voltage drops across various elements:

$$IR = R\dot{q}, \quad L\dot{I} = L\ddot{q}, \quad \frac{q}{C}.$$

Equation for charge:

$$\ddot{q} + \frac{R}{L}\dot{q} + \frac{1}{LC}q = \frac{V_0}{L}\cos\Omega t.$$

The electrical analog: Forced LCR circuit



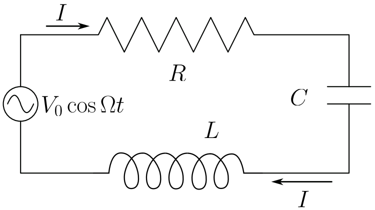
Voltage drops across various elements:

$$IR = R\dot{q}, \quad L\dot{I} = L\ddot{q}, \quad \frac{q}{C}.$$

Equation for charge:

$$\ddot{q} + \frac{R}{L}\dot{q} + \frac{1}{LC}q = \frac{V_0}{L}\cos \Omega t.$$

The electrical analog: Forced LCR circuit



Voltage drops across various elements:

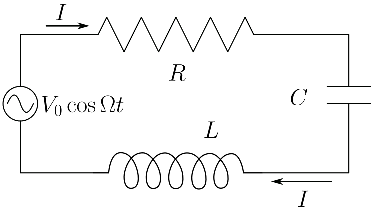
$$IR = R\dot{q}, \quad L\dot{I} = L\ddot{q}, \quad \frac{q}{C}.$$

Equation for charge:

$$\ddot{q} + \frac{R}{L}\dot{q} + \frac{1}{LC}q = \frac{V_0}{L}\cos \Omega t.$$

Mechanical Analogy:

The electrical analog: Forced LCR circuit



Voltage drops across various elements:

$$IR = R\dot{q}, \quad L\dot{I} = L\ddot{q}, \quad \frac{q}{C}.$$

Equation for charge:

$$\ddot{q} + \frac{R}{L}\dot{q} + \frac{1}{LC}q = \frac{V_0}{L}\cos \Omega t.$$

Mechanical Analogy:

displacement	x	$\sim$	q
inertia	m	$\sim$	L
spring constant	k	$\sim$	$1/C$
visc. force const.	b	$\sim$	R
driving force	F	$\sim$	V
resonant freq. ω_0	$\sqrt{k/m}$	$\sim$	$1/\sqrt{LC}$
resonance width γ	b/m	$\sim$	R/L
KE	$mv^2/2$	$\sim$	$LI^2/2$
PE	$kx^2/2$	$\sim$	$q^2/2C$
power abs at res	$F_0^2/(2m\gamma)$	$\sim$	$V_0^2/(2R)$