Sol^n. 3 (A) In polar co-ordinates centered at O,
The position vector of the car at any time $0 \leq t < t_0$

$$\vec{r}(t) = v_0 t \, \hat{r} \quad \text{— (1)}$$

such that $\hat{r}$ makes an angle $\omega_0 t$ with the x-axis

$$\Rightarrow \dot{\vec{r}}(t) = v_0 \hat{r} + v_0 t \, \omega_0 \hat{\theta} \quad \text{— (2)}$$

$$\Rightarrow \ddot{\vec{r}}(t) = 2 v_0 \omega_0 \hat{\theta} - v_0 t \, \omega_0^2 \hat{r} \quad \text{— (3)}$$

If $\mu g > 2 v_0 \omega_0$ then the frictional forces acting on the wheels of the car can support the acceleration in eq^n (3) for $0 < t < t_0$

$$\vec{F} = M (2 v_0 \omega_0 \hat{\theta} - v_0 t \omega_0^2 \hat{r})$$

$\Rightarrow$ The force exerted by the wheels of the car on the platform

$$\vec{F}' = -\vec{F} = M (v_0 \omega_0^2 t \, \hat{r} - 2 v_0 \omega_0 \hat{\theta}) \quad \text{— (4)}$$

clearly at $t = t_0$ $\quad |\vec{F}| = \mu M g$

$$\Rightarrow 4 v_0^2 \omega_0^2 + v_0^2 t_0^2 \omega_0^4 = \mu^2 g^2$$

$$\Rightarrow t_0^2 = (\mu^2 g^2 - 4 v_0^2 \omega_0^2) / v_0^2 \omega_0^4 \quad \text{— (5)}$$

$$\Rightarrow R = v_0 t_0 = (\mu^2 g^2 - 4 v_0^2 \omega_0^2)^{1/2} / \omega_0^2 \quad \text{— (6)}$$

(B) From eq^n (3) and eq^n (5), radial acceleration at $t = t_0$

$$a_r(t_0) = v_0 t_0 \omega_0^2 = (\mu^2 g^2 - 4 v_0^2 \omega_0^2)^{1/2} \quad \text{— (7)}$$

$$a_\theta(t_0) = 2 v_0 \omega_0 \quad \text{— (8)}$$

(c) In an infinitesimal time interval dt after time t such that $0<t<t_0$ & $0<t+dt<t_0$, the work done by the car

$$dw = \vec{F}'(t) \cdot \dot{\vec{r}}(t)\, dt$$

$$= M(v_0 \omega_0^2 t\, \hat{r} - 2 v_0 \omega_0\, \hat{\theta}) \cdot (v_0\, \hat{r} + v_0 \omega_0 t\, \hat{\theta})\, dt$$

$$= M(v_0^2 \omega_0^2 t\, dt - 2 v_0^2 \omega_0^2 t\, dt) = -M v_0^2 \omega_0^2 t\, dt \quad \text{--- (5)}$$

$\therefore$ Total work done by the car in the time interval $0<t<t_0$

$$W = \int dw = -M v_0^2 \omega_0^2 \int_0^{t_0} t\, dt = -M v_0^2 \omega_0^2 \frac{t_0^2}{2} = -\frac{M}{2}\left(\frac{\mu^2 g^2}{\omega_0^2} - 4 v_0^2\right)$$

$$= \frac{M}{2}\left(4 v_0^2 - \frac{\mu^2 g^2}{\omega_0^2}\right) \quad \text{--- (6)}$$

Consistency with work energy theorem.

Newton's 3rd law implies

work done by the car = $-$(work done on the car by friction)

= $-$ (change in kinetic energy of the car)

Eq^n (2) implies

Initial kinetic energy of car = $\frac{1}{2} M v_0^2$

Kinetic energy at t_0 = $\frac{1}{2} M v_0^2 + \frac{1}{2} M v_0^2 t_0^2 \omega_0^2$

$\Rightarrow$ Work done by the car $= -\frac{1}{2} M v_0^2 t_0^2 \omega_0^2$

$$= \frac{M}{2}\left(4 v_0^2 - \frac{\mu^2 g^2}{\omega_0^2}\right)$$