Tutorial 2 **Vectors and Polar coordinates** 9 August 2024

Problem 1. A particle moves in the x - y plane with a constant radial velocity of 4 m/s starting at $t = 0$ from the origin. It is also undergoing rotation with a constant angular velocity of 2 rad/s. Assume that its initial angular position is zero. Note the radial coordinate of the particle is labelled by r and its angular coordinate is labelled by θ, which is the angle that its position vector makes with the positive direction of the x- axis.

(1.1) Write down the values of the following quantities from the description in the problem statement.
(a)) $r(t = 0)$, $(b)\, \dot{r}(t = 0)$, $(c)\, \theta(t = 0)$, $(d)\, \dot{\theta}(t = 0)$

(1.2) At time t $= t_1$ the particle is at $r = 3$ m. Determine

(a) The value of the time t_1.
(b) The polar coordinates r and θ at time t_1.
(c) The x and y coordinates of the particle at $t = t_1$.
(d) The Cartesian components of the velocity vector, viz v_x and v_y at time t_1.
(e) The radial and tangential velocities v_r and v_θ at time t_1.
(f) The Cartesian components of the particle's acceleration viz. a_x and a_y at time t_1.
(g) The radial and angular components of the acceleration a_r and a_θ at time t_1.

(1.3) If the mass of the particle is 100 gm, determine (at time t_1)

(a) The Cartesian components of the Force $\overrightarrow{\boldsymbol{F}}$ and the Torque $\vec{\boldsymbol{\tau}}$.
(b) The polar components of the Force $\overrightarrow{\boldsymbol{F}}$ and the Torque $\vec{\boldsymbol{\tau}}$.

Problem 2. Consider a particle moving in the x-y plane with a constant velocity $v_0\hat{\boldsymbol{i}}$ along the line $y = a$. Let its radial and angular coordinates be r and θ.The particle is at $x = 0$, $y = a$ at time $t = 0$. Determine as a function of time

(2.1) $r = r(t)$ and $\theta = \theta(t)$

(2.2) The Cartesian and Polar components of the velocity vector $\vec{\boldsymbol{v}}$ of the particle.

(2.3) The Cartesian and the Polar components of the acceleration vector $\vec{\boldsymbol{a}}$

(2.4) Substitute for $\hat{\boldsymbol{r}}$ and for $\hat{\boldsymbol{\theta}}$ in terms of $\hat{\boldsymbol{i}}$ and $\hat{\boldsymbol{j}}$, and show explicitly that $\vec{\boldsymbol{v}}$ is indeed a constant in both coordinate systems. Thus a constant vector in any one coordinate system is also constant in any other coordinate system whose origin and whose axes are stationary (i.e. not moving).