MATH F111 - MATHEMATICS 1
Tutorial Sheet 12

October 19, 2024

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1. Evaluate the iterated integral.

   (i) $\displaystyle\int_{-3}^{2}\int_{3}^{5}\left(2-3x^2+y^2\right)\,dxdy$

   (ii) $\displaystyle\int_{0}^{1}\int_{0}^{\pi/2}e^{y}\cos x\,dxdy$

   (ii) $\displaystyle\int_{0}^{1}\int_{0}^{1}xy\sqrt{x^2+y^2}\,dydx$

   (iv) $\displaystyle\int_{1}^{4}\int_{1}^{2}\left(\frac{x}{y}+\frac{y}{x}\right)\,dydx$

   (v) $\displaystyle\int_{1}^{6}\int_{3}^{8}(x+\ln y)\,dydx$

2. Evaluate the double integral over the given region R.

   (i) $\displaystyle\iint_R\frac{1}{(2x+3y)^2}\,dA,\ R=[0,1]\times[1,2]$

   (ii) $\displaystyle\iint_R\left(x^2y^2+\cos(\pi x)+\sin(\pi y)\right)\,dA,\ R=[-2,-1]\times[0,1]$

   (iii) $\displaystyle\iint_R xe^{xy}\,dA,\ R=[-1,2]\times[0,1]$

   (iv) $\displaystyle\iint_R xy\cos(yx^2)\,dA,\ R=[-2,3]\times[-1,1]$

3. Find the volume of the region bounded above by the surface $z=3\sin x\sin y$ and below by the rectangle $R:0\le x\le\pi/3,\ 0\le y\le\pi/2$.

4. Find the volume of the solid lying under the elliptic paraboloid $\dfrac{x^2}{4}+\dfrac{y^2}{9}+z=1$ and above the rectangle $R=[-1,1]\times[-2,2]$.

5. Find the volume of the solid bounded by the graphs of the equations $z=\dfrac{1}{1+y^2}$, $x=0$, $x=2$, and $y\ge 0$.

6. Write an iterated integral $\displaystyle\iint_R dA$ over the described region R using (a) vertical cross-sections and (b) horizontal cross-sections. Then evaluate the double integral in two ways.

   (i) Bounded by $y=2x$ and $y=x^2$

   (ii) Bounded by $y=\sqrt{x}$ and $y=x^3$

7. Sketch the region of integration, reverse the order of integration, and evaluate the integral.

   (*i*) $\displaystyle\int_0^1 \int_0^{\sqrt{1-x^2}} \sqrt{1-y^2}\, dydx$

   (*ii*) $\displaystyle\int_0^1 \int_y^1 x^2 e^{xy}\, dxdy$

8. Use double integral to evaluate $\displaystyle\int_0^2 \left(\tan^{-1}(\pi x) - \tan^{-1} x\right)\, dx$.

9. Find $\displaystyle\iint_D f(x,y)dA$, where $f(x,y) = e^{x^2}$ and D is the region bounded by the lines $y = 0$, $x = 1$ and $y = 2x$.

10. Find the volume of the solid which is common to the cylinder $x^2 + y^2 = 1$ and $x^2 + z^2 = 1$.

11. Evaluate the improper integral $\displaystyle\iint_R xye^{-x^2-y^2}\, dA$, where R is the first quadrant of $\mathbb{R}^2$.

12. Sketch the region bounded by the curves $y^2 = x$ and $y^2 = 4 - 3x$ and find the area enclosed between them.

13. Find the first quadrant area bounded by the following curves:

   (*i*) $y = x^2 + 2$, $y = 4$ and $x = 0$

   (*ii*) $y = \arctan x$, $y = \dfrac{\pi}{4}$, and $x = 0$

14. Find the area of the region enclosed by the curves:

   (*i*) $y = e^x$, $y = x^2 - 1$, $x = -1$ and $x = 1$

   (*ii*) $x = y^2 - 2$ and $x = y$

15. Find the volume under the surface $z = \sqrt{1 - x^2}$ and above the triangle formed by $y = x$, $x = 1$ and the x-axis.

16. Find the average value of the funtion over the region R:

   (*i*) $f(x,y) = \sinh x + \sinh y$, $R = [0,1] \times [0,2]$

   (*ii*) $f(x,y) = xy$, R is the triangle with vertices $(0,0)$, $(1,0)$ and $(1,3)$.