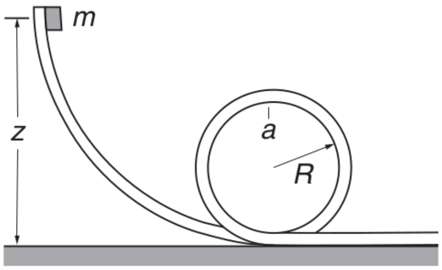
Tutorial 8 **Energy** 6 September 2024

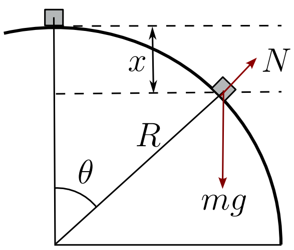
P1.



When the block is in the circular track, $a_r = \ddot{r} - r\dot{\theta}^2$ where $r = R$ and $\dot{r} = 0$.
At the top point 'a', radial forces on the block are $\vec{\boldsymbol{F}}_r = (mg + N)(-\hat{\boldsymbol{k}})$.
Since $N = mg$, we have $R\dot{\theta}^2 = 2g$.
Using conservation of energy,

$$mgz = 2Rmg + \frac{1}{2}mv_a^2,\ \ v_a^2 = \dot{r}_a^2 + r_a^2\dot{\theta}^2 = 2Rg \implies z = 3R.$$

P2.



Radial force $= N - mg\cos\theta$. Radial acceleration $= m(\ddot{r} - r\dot{\theta}^2) = -mv^2/R$
When the block is at the point where it loses contact, $N = 0 \implies v^2 = Rg\cos\theta = Rg\left(1 - \dfrac{x}{R}\right)$.

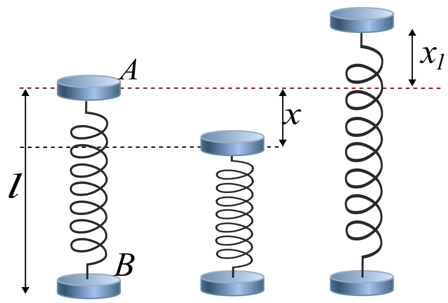
Conservation of energy $\implies mgx = \dfrac{1}{2}mv^2 \implies v^2 = 2gx$.

$2gx = (R - x)g \implies x = \dfrac{R}{3}$.

P3. The kinetic energy of the sphere is $\dfrac{1}{2}mv^2 + \dfrac{r^2}{5R^2}mv^2$.

So conservation of energy gives $v^2 = 5gx\dfrac{R^2}{r^2} \implies x = \dfrac{R}{5\left(\frac{R}{r}\right)^2 + 1}$.

P4.



Suppose the zero of potential energy is when the system is in equilibrium.
(a) When the top disc is depressed by x, the potential energy of the system is
$E_1 = \dfrac{1}{2}kx^2 - M_A gx$.
Now A accelerates upwards under the influence of the spring force, until it stops at x_1 above the equilibrium position. Total energy in this configuration is
$E_2 = \dfrac{1}{2}kx_1^2 + M_A gx_1$.
Now the total force on B is $kx_1 - M_B g$. For B to rise this force must be positive. So minimum condition is when $kx_1 = M_B g$.

Conservation of energy demands that

$$\frac{1}{2}kx^2 - M_A gx = \frac{1}{2}kx_1^2 + M_A gx_1 \implies \frac{1}{2}(x - x_1) = M_A g \implies x = (M_B + 2M_A)\frac{g}{k} = 3\frac{Mg}{k}.$$

(b) If the disc A is depressed by $2x$, then the total energy imparted to the system is $2(kx - M_A g)x$.

Disc A rises upward upto a point x_2 above equilibrium such that energy is now

$$\frac{1}{2}kx_2^2 + M_A g x_2 = \frac{1}{2}k(2x)^2 - M_A g(2x) \implies x_2 = 4\frac{Mg}{k}.$$

Meanwhile B has risen, and the two masses execute SHM about the CM....until B hits the ground again...