Tutorial 5 **Moment of Inertia** 22 August 2024

Preliminaries:

You should know that the **Moments of Inertia** of a distribution of masses $\{m_i(\vec{\boldsymbol{r}}_i)\}$, with respect to a specific coordinate system, are defined by

$$I_{xx} = \sum_i m_i(y_i^2 + z_i^2); \qquad I_{yy} = \sum_i m_i(z_i^2 + x_i^2); \qquad I_{zz} = \sum_i m_i(x_i^2 + y_i^2).$$
$$I_{xy} = -\sum_i m_i x_i y_i = I_{yx}, \quad I_{yz} = -\sum_i m_i y_i z_i = I_{zy}, \quad I_{zx} = -\sum_i m_i z_i x_i = I_{zx}.$$

These are required to calculate the angular momentum of the distribution with respect to the origin:

$$\begin{pmatrix} L_x \\ L_y \\ L_z \end{pmatrix} = \begin{pmatrix} I_{xx} & I_{xy} & I_{xz} \\ I_{yx} & I_{yy} & I_{yz} \\ I_{zx} & I_{zy} & I_{zz} \end{pmatrix} \begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix}.$$

P1. Two equal masses m are attached to each other by a rigid, massless rod of length $2l$. This rigid system is undergoing rotation with uniform angular speed ω about an axis passing through the center of mass, at an angle α to the vertical.

(a) Draw a neat diagram indicating this configuration, the angular velocity vector and the angular momentum vector. Understand that the angular momentum is NOT parallel to the angular velocity.

(b) Calculate the moment of inertia tensor.

(c) Calculate the angular momentum vector and show that it changes with time. Find this rate of change.

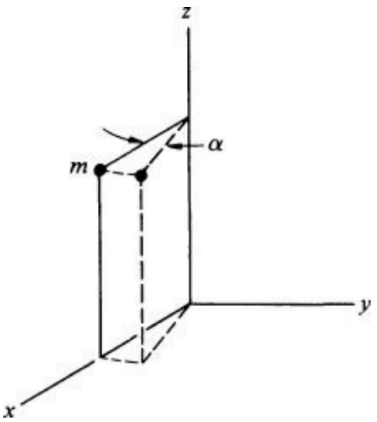
(d) What is the torque acting on this system? Can you attribute and origin to this torque?

P2. (K.K 8.11)

A particle of mass m is located at $x = 2, y = 0, z = 3$.

(a) Find the moment of inertia tensor with respect to the origin.

(b) The particle is rotated about the z axis by a very small angle α.
What are the changes in the moment of inertia tensor, to *first order* in α?



P3. A rigid body consists of three point masses $m_1 = 2\text{ kg}, m_2 = 1\text{ kg}, m_3 = 4\text{ kg}$, located at $\vec{\boldsymbol{r}}_1 = (1, -1, 1)\text{ m}, \vec{\boldsymbol{r}}_2 = (2, 0, 2)\text{ m}, \vec{\boldsymbol{r}}_3 = (-1, 1, 0)\text{ m}$. Find the moment of inertia tensor of this distribution. Suppose these masses are held in place with respect to each other by rigid massless rods, and rotated with an angular velocity $\vec{\boldsymbol{\omega}} = 3\hat{\boldsymbol{i}} - 2\hat{\boldsymbol{j}} + 4\hat{\boldsymbol{k}}$ rads/s. Find its angular momentum vector.