BIRLA INSTITUTE OF TECHNOLOGY AND SCIENCE PILANI -
K. K. BIRLA GOA CAMPUS
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Course title: Mathematics-I | Tutorial Sheet 11
Course No. MATH F111 | Date: Oct 22, 2024

Textbook (14th Edition) Problems of 14.5

1, 6, 9, 17, 23, 27, 29, 31, 34, 35, 36, 37, 38, 39, 40, 42, 44

Textbook (14th Edition) Problems of 14.6

1, 9, 13, 15, 19, 21, 25, 27, 31, 37, 39, 40, 45, 49, 51, 53, 54, 57, 59, 60, 61

Exercise : Directional derivative

1. Find the directional derivative of $f(x, y) = y\cos(xy)$ at the point (0, 1) in the direction indicated by the angle $\theta = \frac{\pi}{4}$.
2. (a) Find the gradient of f.
   (b) Evaluate the gradient at the point P.
   (c) Find the rate of change of f at P in the direction of the vector u.
   (i) $f(x, y) = \frac{x}{y}$, $P(2, 1)$, $\mathbf{u} = \frac{3}{5}\mathbf{i} + \frac{4}{5}\mathbf{j}$.
   (ii) $f(x, y, z) = x^2yz - xyz^3$, $P(2, -1, 1)$, $\mathbf{u} = \frac{4}{5}\mathbf{j} - \frac{3}{5}\mathbf{k}$.
3. Find the directions in which the directional derivative of $f(x, y) = x^2 + xy^3$ at the point $(2, 1)$ has the value 2.
4. Find all points at which the direction of the fastest change of the function $f(x, y) = x^2 + y^2 - 2x - 4y$ is $\mathbf{i} + \mathbf{j}$.
5. The second directional derivative of $f(x, y)$ is $D_u^2 f(x, y) = D_u(D_u f(x, y))$. If $f(x, y) = x^3 + y^3 + 5x^2y$ and $\mathbf{u} = \frac{3}{5}\mathbf{i} + \frac{4}{5}\mathbf{j}$. Calculate $D_u^2 f(2, 1)$.

Exercise : Tangent line, normal line, tangent plane

1. Find equations of (a) the tangent plane and (b) the normal line to the given surface $x = y^2 + z^2 + 1$ at the specified point $(3, 1, -1)$.
2. Find an equation of the tangent plane $z = 2x^2 + y^2 - 5y$ to the given surface at the specified point $(1, 2, -4)$.
3. At what point on the ellipsoid $x^2 + y^2 + 2z^2 = 1$ is the tangent plane parallel to the plane $x + 2y + z = 1$?
4. Show that every plane that is tangent to the cone $x^2 + y^2 = z^2$ passes through the origin.

Exercise: Linearization and standard linear approximation

1. Find the linearization of $f(x, y) = \sqrt{xy}$ of the function at the point (1,4). What is its standard linear approximation?

2. Given that f is a differentiable function with $f(2, 5) = 6, f_x(2, 5) = 1, f_y(2, 5) = -1$. Use standard linear approximation to estimate $f(2.2, 4.9)$.

3. Find the differential of the function $R = \alpha\beta^2 \cos(\gamma)$.