Tutorial 7 **Rotation** 29 August 2024

P1. The string is being pulled slowly

$$\Rightarrow \dot{r} \sim \ddot{r} \sim 0$$

$$\Rightarrow F = m\omega^2 r$$

Angular momentum conservation implies

$$ml_1^2\omega_1 = mr^2\omega = ml_2^2\omega_2$$

Work done by F

$$W_F = \frac{ml_1^4\omega_1^2}{2}\left(\frac{1}{l_2^2} - \frac{1}{l_1^2}\right)$$

Change in kinetic energy

$$\Delta KE = \frac{1}{2}m\omega_2^2 l_2^2 - \frac{1}{2}m\omega_1^2 l_1^2 = \frac{ml_1^4\omega_1^2}{2}\left(\frac{1}{l_2^2} - \frac{1}{l_1^2}\right)$$

P2. Torque about the centre of the shaft

$$\tau = Fr$$

where r is the radius of the shaft

$$\Rightarrow \omega(t) = \left(\frac{Fr}{I}\right)t$$

where $\omega(t)$ is the angular velocity at time t

$$\Rightarrow l = \frac{Fr^2t^2}{2I}$$

where l is the length of the string unwound in time t. Therefore,

$$I = \frac{2FL}{\omega^2}$$

P3.

$$r(t) = r_\circ e^{\omega t}$$

$$\dot{\theta} = \omega = \text{constant}$$

$$\Rightarrow a_r = 0 \text{ and } a_\theta = 2\omega^2 r$$

The contact force which causes this acceleration must be

$$\mathbf{N} = 2m\omega^2 r\hat{\theta}$$

Power put in by the external agent

$$P_{ext} = 2m\omega^3 r_\circ^2 e^{2\omega t}$$

Kinetic energy of the bead

$$KE_{bead} = m\omega^2 r^2$$

$$\Rightarrow \frac{dKE_{bead}}{dt} = 2m\omega^3 r_\circ^2 e^{2\omega t}$$

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P4. The cm of the combined system lies at a distance

$$r_{cm} = \frac{Ml}{2(M+m)}$$

from the point of contact.

Conservation of linear momentum implies that the velocity of the cm is

$$v_{cm} = \frac{mv_\circ}{M+m}$$

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Conservation of angular momentum about the point of contact gives the angular velocity of the system just after the collision to be

$$\omega = \frac{6mv_\circ}{l(M+4m)}$$

The point at rest just after the collision is at a distance $2l/3$ from the point of contact.