Tutorial 12 **Systems exhibiting SHM** Date

There are many physical systems in nature where the different constituents that comprise the system are coupled due to forces exerted on each other and the motion of each of the constituent in bounded. For some of these systems, there may be a state of stable equilibrium. Any perturbation that displaces the constituents from the equilibrium state results in a motion which can be understood in terms of the abstract simple harmonic oscillator. The following problems involve such systems.

P1. A particle of mass m is subject to a central force $-\frac{k}{r^2}\hat{\boldsymbol{r}}$. Consider that it is in the stable state corresponding to a circular orbit of radius r_0.

(a) Find the energy E_0 and angular momentum L_0 of the particle in terms of r_0.

(b) The orbit is perturbed by adding a small amount of energy ΔE to the particle. Show that the subsequent radial motion can be approximated to simple harmonic oscillation.

(c) Find the time period and amplitude of these oscillations.

(d) How does the time period of radial oscillations compare with the time period for the particle to complete its orbit?

(e) What sort of orbit corresponds to the new energy $E + \Delta E$?

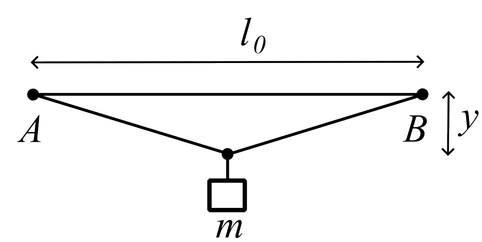
P2. (APF 3-3) A platform is executing simple harmonic motion in a vertical direction with an amplitude of 5 cm and a frequency of $10/\pi$ vibrations per second. A block is placed on the platform at the lowest point of its path.

(a) At what point will the block leave the platform?

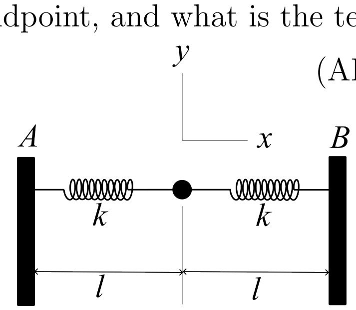
(b) How far will the block rise above the highest point reached by the platform?

P3. (APF 3-5) A uniform rod of length L is nailed to a post so that two-thirds of its length is below the nail. What is the period of small oscillations of the rod?

P4. (APF 3-7) A wire of unstretched length l_0 is extended by a distance $10^{-3}l_0$ when a certain mass is hung from its bottom end. If this same wire is connected between two points, A and B, that are a distance l_0 apart on the same horizontal level, and the same mass is hung from the mid-point of the wire as shown, what is the depression y of the midpoint, and what is the tension in the wire?



P5. (APF 3-19) A mass m rests on a frictionless horizontal table and is connected to rigid supports via two identical springs each of relaxed length l_0 and spring constant k (see figure). Each spring is stretched to a length l considerably greater than l_0. Horizontal displacements of m from its equilibrium position are labeled x (along AB) and y (perpendicular to AB).



(a) Write down the differential equation of motion (i.e., Newton's law) governing small oscillations in the x direction.

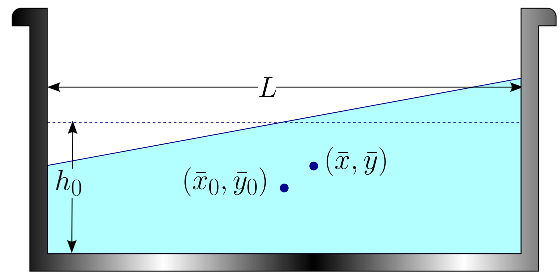
(b) Write down the differential equation of motion governing small oscillations in the y direction (assume $y \ll l$).

(c) In terms of l and l_0, calculate the ratio of the periods of oscillation along x and y.

(d) If at $t = 0$ the mass m is released from the point $x = y = A_0$ with zero velocity, what are its x and y coordinates at any later time t?

(e) Draw a picture of the resulting path of m under the conditions of part (d) if $l = 9l_0/5$.

P6. Sloshing modes in a pan of water: A rectangular pan of width L is partly filled with water. At equilibrium the level is flat and horizontal, with a height h_0 from the bottom. When the side of the pan is nudged a bit, the water starts sloshing. Assume that this motion keeps the surface of the water flat.
Suppose $\{\bar{x}, \bar{y}\}$ are the coordinates of the center of gravity of the body of water, with equilibrium values $\{\bar{x}_0, \bar{y}_0\}$.



(a) Work out the relationship between $\bar{y} - \bar{y}_0$ and $\bar{x} - \bar{x}_0$. You can use the potential and kinetic energies of the center of mass, and apply Newton's laws to the system as if its entire mass were concentrated at the CM.

(b) Show that the motion is SHM and find its frequency.