Tutorial 9 **Central Force motion** 19 September 2024

P1. Find the forces for the following potential energies.

(a) $U = Ax^2 + By^2 + Cz^2$

(b) $U = A\ \ln(x^2 + y^2 + z^2)$

(c) $U = A\cos\theta/r^2$ (plane polar coordinates)

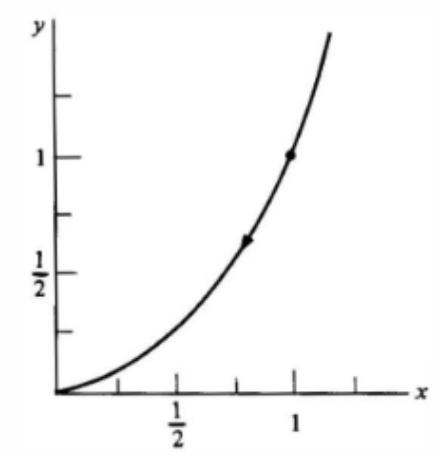
P2. Determine whether each of the following forces is conservative. Find the potential energy function if it exists. A, α, β are constants.

(a) $\mathbf{F} = A(3\hat{\mathbf{i}} + z\hat{\mathbf{j}} + y\hat{\mathbf{k}})$

(b) $\mathbf{F} = Axyz(\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}})$

(c) $F_x = A\ \sin(\alpha y)\cos(\beta z)$, $F_y = -Ax\alpha\ \cos(\alpha y)\cos(\beta z)$, $F_z = Ax\ \sin(\alpha y)\sin(\beta z)$

P3.



A particle of mass m moves in a horizontal plane along the parabola $y = x^2$. At $t = 0$ it is at the point $(1, 1)$, moving in the direction shown with speed v_0. Aside from the force of constraint holding it to the path, it is acted upon by the following external forces:

A radial force $\mathbf{F}_a = -Ar^3\hat{\mathbf{r}}$

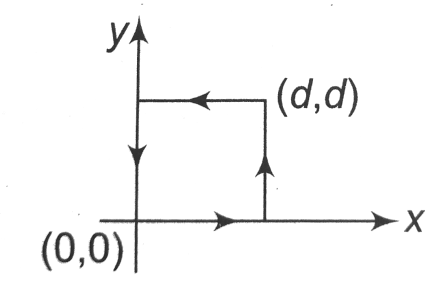
A force given by $\mathbf{F}_b = B(y^2\hat{\mathbf{i}} - x^2\hat{\mathbf{j}})$

where A and B are positive constants.

(a) Are these forces conservative?

(b) What is the speed v_f of the particle when it arrives at the origin?

P4.



How much work is done around the path that is shown by the force $\mathbf{F} = A(y^2\hat{\mathbf{i}} + 2x^2\hat{\mathbf{j}})$, where A is a constant and x and y are in meters? Find the answer by evaluating the line integral, and also by using Stokes' theorem.