

MATHEMATICS-I (MATH F111)

Department of Mathematics
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Tutorial Sheet-6

Topic: Power Series

1. Find the radius of convergence and interval of convergence of the following power series.

(a) $\sum_{n=1}^{\infty} \frac{(x-3)^n}{n \cdot 4^n}$

(b) $\sum_{n=1}^{\infty} \frac{(x+2)^n}{n^2}$

(c) $\sum_{n=0}^{\infty} \frac{2^n (x-1)^n}{n!}$

(d) $\sum_{n=1}^{\infty} \frac{n! (x-1)^n}{n^n}$

2. For what values of x , the series $\sum_{n=1}^{\infty} (-1)^n \frac{x^{3n}}{n^{\frac{2}{n}}}$ is convergent?

3. For what values of x , the series $\sum_{n=2}^{\infty} \frac{x^n}{n(\log n)^2}$ is convergent?

4. Consider the function $g(x) = \frac{1}{1-x^2}$. Find the power series expansion of $g(x)$ centered at $x = 2$. Determine the radius of convergence.

5. Consider the function $f(x) = \frac{1}{(1-x)^2}$. Find the power series expansion of $f(x)$ centered at $x = 0$. Determine the radius of convergence.

6. Find the power series expansion around $x = 0$ and the interval of convergence of the following functions

(a) $f(x) = \frac{x}{1+x^2}$

(b) $f(x) = \frac{\sin x}{x}$

(c) $f(x) = \frac{2+x}{1-x}$

(d) $f(x) = \frac{x}{x^2+2x+5}$