MATH F111- Mathematics I

Saranya G. Nair
Department of Mathematics

BITS Pilani

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Properties

Example (Example 1)

Graph the curve $r = 1 - \cos\theta$ in the Cartesian xy-plane.

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Graph the curve $r = 1 - \cos\theta$ in the Cartesian xy-plane.

Check if the curve is symmetric about x-axis?

p Suppose the point (r_1, θ_1) lies in the curve.

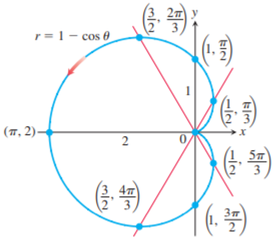
$\Leftrightarrow r_1 = 1 - \cos\theta_1$

$\Leftrightarrow r_1 = 1 - \cos -\theta_1$, $\Leftrightarrow$ the point $(r_1, -\theta_1)$ lies in the curve and thus the curve is symmetric about the x-axis. Due to symmetry about x axis, we need to trace the curve in $0 \leq \theta \leq \pi$ and then reflect it wrt x axis.

As θ increases from 0 to π, $\cos\theta$ decreases from 1 to -1. Therefore $1 - \cos\theta$ increases from 0 to 2. Now as we increase θ from π to 2π, $\cos\theta$ increases from -1 to 1. Thus $1 - \cos\theta$ decreases from 2 to 0. The curve leaves the origin with slope $\tan(0) = 0$ and returns to the origin with slope $\tan(2\pi) = 0$.

We make a table of values from $\theta = 0$ to $\theta = \pi$, plot the points, draw a smooth curve through them with a horizontal tangent at the origin, and reflect the curve across the X-axis to complete the graph. The curve is called a cardioid because of its heart shape.

θ	$r = 1 - \cos\theta$
0	0
$\frac{\pi}{3}$	$\frac{1}{2}$
$\frac{\pi}{2}$	1
$\frac{2\pi}{3}$	$\frac{3}{2}$
π	2



General Method

Another method of graphing that is usually quicker and more reliable is to

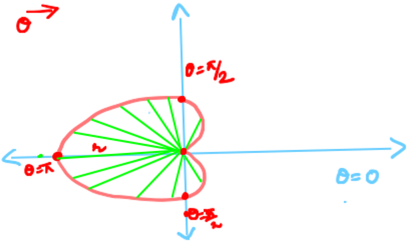
1. first graph the function $r = f(\theta)$ in the Cartesian $r\theta$-plane,
2. then use that Cartesian graph as a TABLE and guide to sketch the polar coordinate graph in the xy-plane.

This method is sometimes better than simple point plotting because the first Cartesian graph, shows at a glance where r is positive, negative, and nonexistent as well as where r is increasing and decreasing.

Example

Plot the graph $r = 1 - \cos\theta$ in General method.

First we plot the graph in the Cartesian $r\theta$- plane.



Example

Graph the curve $r^2 = 4\cos\theta$ *in the Cartesian xy-plane.*

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Graph the curve $r^2 = 4\cos\theta$ in the Cartesian xy-plane.

- $\cos\theta \geq 0$ when θ from $-\frac{\pi}{2}$ to $\frac{\pi}{2}$.
- Is the curve Symmetric about X axis?

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- Can you use the above 2 symmetries to conclude if the curve is symmetric wrt Y axis?

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- What are the values of θ for which the curve passes through the origin? What are values of tan at those θ's?

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Thus the curve has vertical tangents at $\theta = -\frac{\pi}{2}$ and $\theta = \frac{\pi}{2}$ because $\tan\theta$ is infinite.

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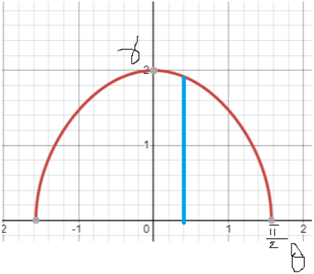
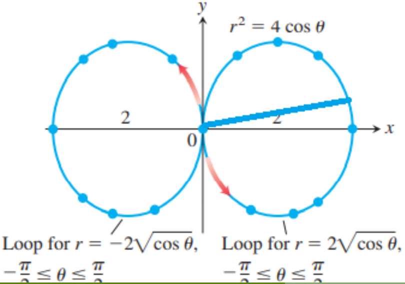
Remark

In general, if we know the curve is Symmetric wrt X axis and Y axis, can you derive it is symmetric wrt origin? If we know the curve is Symmetric wrt Y axis and origin can you derive it is symmetric wrt X axis?

θ	$\cos\theta$	$r = \pm 2\sqrt{\cos\theta}$ (approx)
0	1	± 2
$\pm\frac{\pi}{6}$	$\frac{\sqrt{3}}{2}$	± 1.9
$\pm\frac{\pi}{4}$	$\frac{1}{\sqrt{2}}$	± 1.7
$\pm\frac{\pi}{3}$	$\frac{1}{2}$	± 1.4
$\pm\frac{\pi}{2}$	0	0



2
1
-1
0
1
2



y
$r^2 = 4\cos\theta$
2
0
2
x
Loop for $r = -2\sqrt{\cos\theta}$, $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$
Loop for $r = 2\sqrt{\cos\theta}$, $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$

Example 3

Example

Graph the lemniscate curve $r^2 = sin 2\theta$ *in the Cartesian* xy*-plane.*

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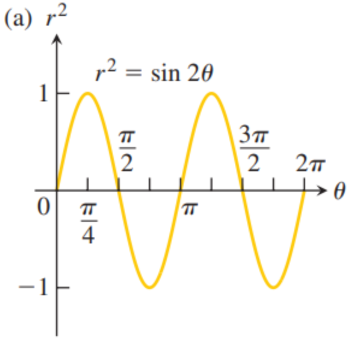
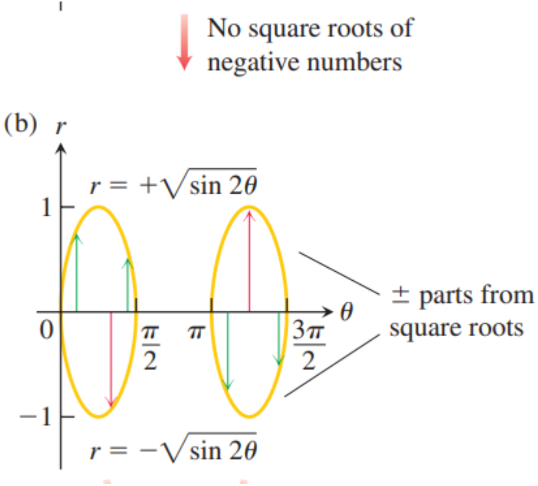
Solution: We begin by plotting r^2 as a function of θ in the Cartesian $r^2 - \theta$ plane.



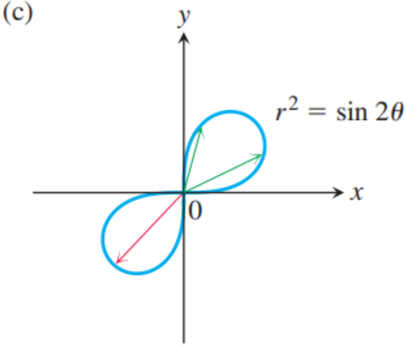
Example 3

We pass from there to the graph of $r = \pm\sqrt{\sin 2\theta}$ in the $r\theta$ plane.



Example 3

Now we plot the polar graph.



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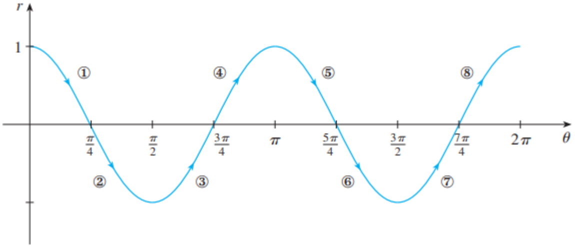

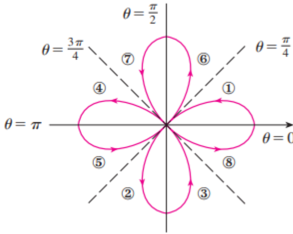

4 leaved rose

Graphing Calculators:
Desmos, Geogebra
https://www.intmath.com/plane-analytic-geometry/8-curves-polar-coordinates.php

Graphing Limacons

Limaçon ("lee-ma-sahn") is Old French word for "snail." Equations for limaçons have the form $r = a \pm b\cos\theta$ or $r = a \pm b\sin\theta$. There are four basic shapes.

1. Limaçons with an inner loop: (a).$r = \frac{1}{2} + \cos\theta$, (b). $r = \frac{1}{2} + \sin\theta$
2. Cardioids: (a). $r = 1 - \cos\theta$ (b). $r = -1 + \sin\theta$
3. Dimpled limaçons (a). $r = \frac{3}{2} + \cos\theta$ (b). $r = \frac{3}{2} - \sin\theta$
4. Oval limaçons (a). $r = 2 + \cos\theta$ (b). $r = -2 + \sin\theta$.