Lecture 17

Simple Harmonic Motion

Superpositions

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Lecture 17

Simple Harmonic Motion

Superpositions

1. **One-dimensional Superposition**
   - Equal frequency
2. **Multiple vibrations of same frequency**
   - Beats
3. **Perpendicular Vibrations: Lissajous Figures**

Superposed Vibrations in One dimension

- Most physical situations involve combined vibrations

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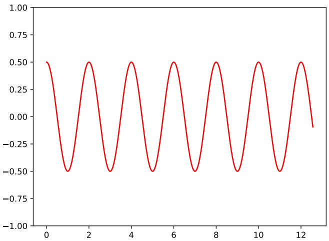
  SUPERPOSITION

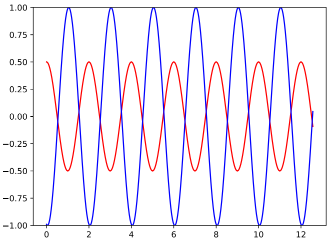
Two SHM's: same frequency, different phase



$$x_1 = A_1 \cos(\omega t + \phi_1);$$

$$x_2 = A_2 \cos(\omega t + \phi_2)$$

Two SHM's: same frequency, different phase



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Adding, we get the wave

$$x_1 + x_2 = R \cos(\omega t + \theta)$$

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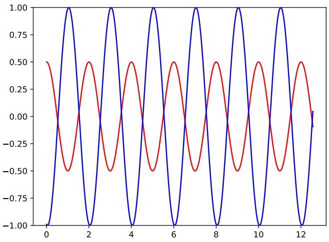

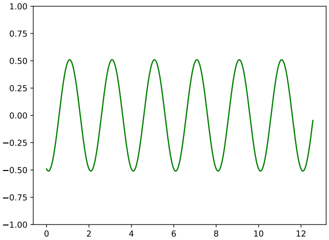

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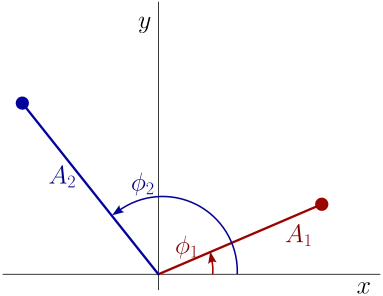
Resultant wave obtained by simply adding the component waves point by point!

Calculation of R and θ

Rotating vector picture



Adding the vectors $\vec{\boldsymbol{r}}_1$ and $\vec{\boldsymbol{r}}_2$,

Calculation of R and θ

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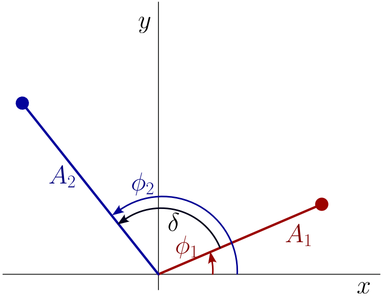

Adding the vectors $\vec{\boldsymbol{r}}_1$ and $\vec{\boldsymbol{r}}_2$,

$$R^2 = A_1^2 + A_2^2 + 2A_1A_2\cos\delta.$$

Calculation of R and θ

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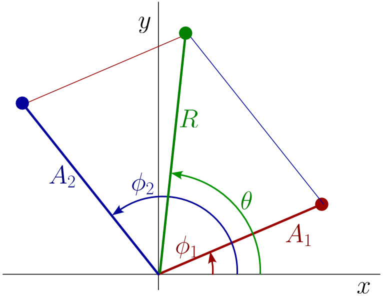

Adding the vectors $\vec{r}_1$ and $\vec{r}_2$,

$$R^2 = A_1^2 + A_2^2 + 2A_1A_2\cos\delta.$$

$$\tan\theta = \frac{A_1\sin\phi_1 + A_2\sin\phi_2}{A_1\cos\phi_1 + A_2\cos\phi_2},$$

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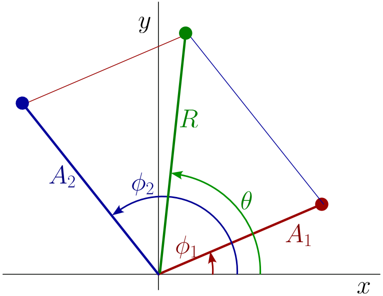

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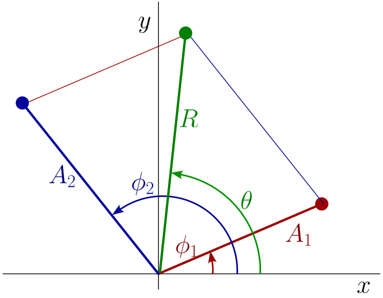
When $A_2 = A_1 = A$,

$$\theta = \frac{\delta}{2},$$
$$R = 2A\cos(\delta/2).$$

Calculation of R and θ

Rotating vector picture



Adding the vectors $\vec{r}_1$ and $\vec{r}_2$,

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When $A_2 = A_1 = A$,

$$\begin{aligned} \theta &= \frac{\delta}{2}, \\ R &= 2A\cos(\delta/2). \end{aligned}$$

Eg: Interference

Calculation of R and θ

Complex Exponential Method

Calculation of R and θ

Complex Exponential Method

$$z_1 = A_1 e^{i(\omega t + \phi_1)}$$

Calculation of R and θ

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or, $$R e^{i\omega t + \theta} = e^{i(\omega t + \phi_1)} \left(A_1 + A_2 e^{i\delta} \right)$$

Calculation of R and θ

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or, $$Re^{i\omega t + \theta} = e^{i(\omega t + \phi_1)} \left(A_1 + A_2 e^{i\delta}\right)$$

where $R = \sqrt{(Re(z))^2 + (Im(z))^2}$,

Calculation of R and θ

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where $$R = \sqrt{(Re(z))^2 + (Im(z))^2},$$

$$\tan\theta = \frac{Im(z)}{Re(z)}$$

Ex.: Show that you get the same formulae as from the geometric picture.

Multiple vibrations of same frequency

and same relative phase

$$x_1 = a\cos(\omega t),$$

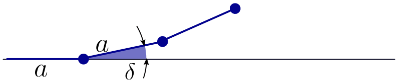
Multiple vibrations of same frequency

and same relative phase

$$
\begin{aligned}
x_1 &= a\cos(\omega t),\\
x_2 &= a\cos(\omega t + \delta),
\end{aligned}
$$

Multiple vibrations of same frequency

and same relative phase



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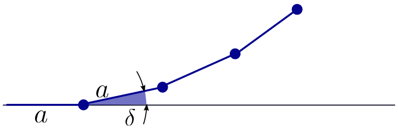

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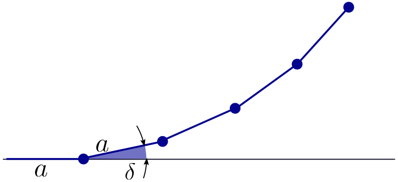
&\vdots \\
x_n &= a\cos(\omega t + (n-1)\delta).
\end{aligned}
$$

and same relative phase



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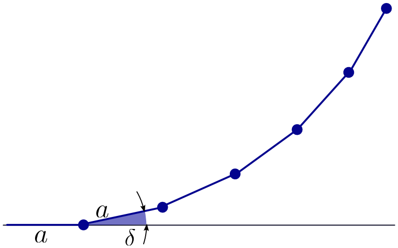

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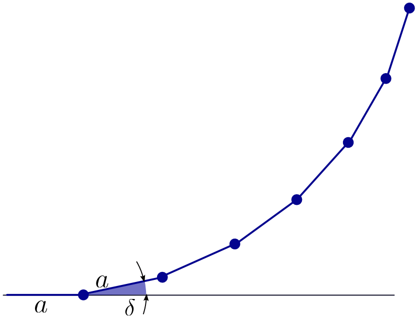

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$$X = A\cos(\omega t + \alpha)$$

Multiple vibrations of same frequency

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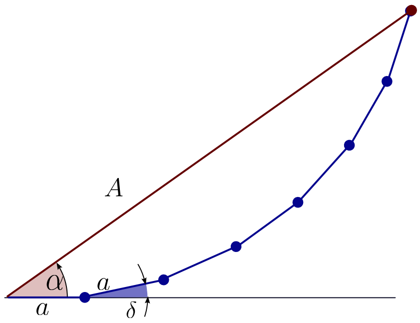

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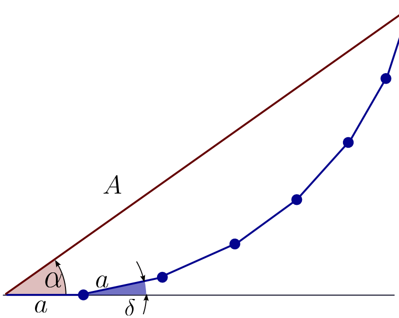
$$X = A\cos(\omega t + \alpha)$$

What are A and α?



$$X = A\cos(\omega t + \alpha).$$

Geometric method:

Construct a circle superscribing the polygon, of radius R centered at O.

Multiple vibrations of same frequency

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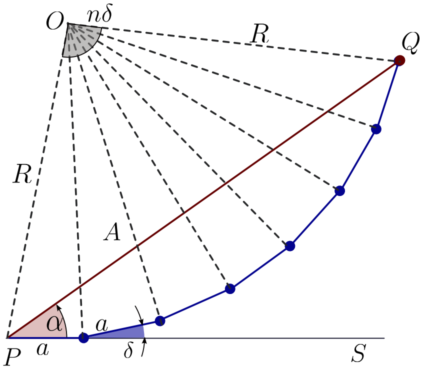

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Construct a circle superscribing the polygon, of radius R centered at O.

Observe that $\sin\left(\frac{n\delta}{2}\right) = \frac{A}{2R}, \quad \sin\left(\frac{\delta}{2}\right) = \frac{a}{2R}.$

Multiple vibrations of same frequency

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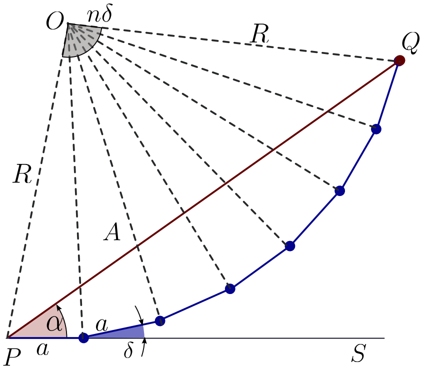

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$$A = 2R\sin\left(\frac{n\delta}{2}\right),$$

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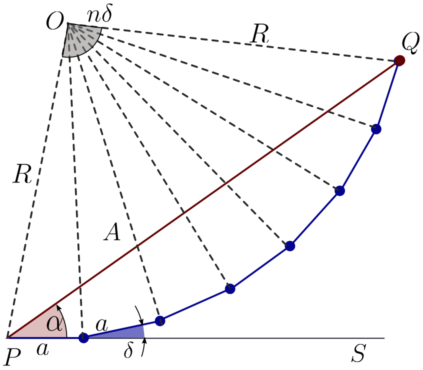

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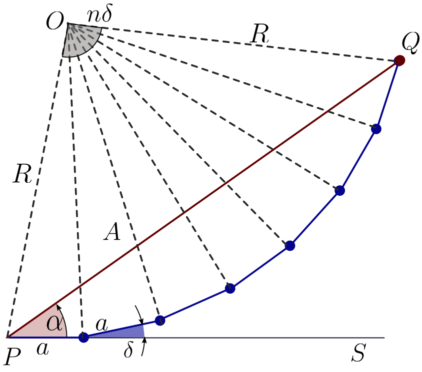

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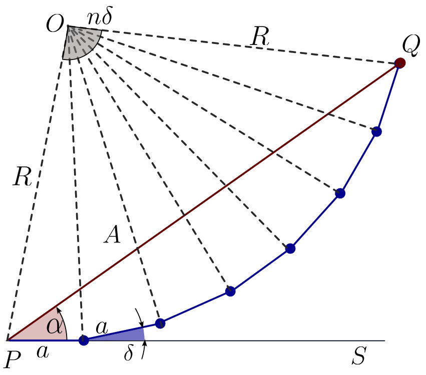
$$A = 2R\sin\left(\frac{n\delta}{2}\right), \quad a = 2R\sin\left(\frac{\delta}{2}\right),$$

$$\alpha = \angle OPS - \angle OPQ.$$



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$$A = 2R\sin\left(\frac{n\delta}{2}\right), \quad a = 2R\sin\left(\frac{\delta}{2}\right),$$

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$$A = a\frac{\sin(n\delta/2)}{\sin(\delta/2)}, \quad \alpha = (n-1)\frac{\delta}{2}.$$

Multiple vibrations of same frequency

and same relative phase

Using Complex Exponentials: $Z(t) = X(t) + iY(t) = Ae^{i(\omega t+\alpha)}$

Multiple vibrations of same frequency

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Using Complex Exponentials: $Z(t) = X(t) + iY(t) = Ae^{i(\omega t+\alpha)}$

$$Z = ae^{i\omega t} + ae^{i\omega t+\delta} + ae^{i\omega t+2\delta} \ldots + ae^{i\omega t+(n-1)\delta}.$$

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$$= ae^{i\omega t}(1 + f + f^2 + ... + f^{n-1}) \quad \text{where } f = e^{i\delta}.$$

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Sum of geometric progression:

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Subtracting, $S(f) = \dfrac{1 - f^n}{1 - f}$

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Subtracting, $S(f) = \dfrac{1-f^n}{1-f} = \dfrac{1-e^{in\delta}}{1-e^{i\delta}} = \dfrac{e^{in\delta/2}(e^{-in\delta/2} - e^{in\delta/2})}{e^{i\delta/2}(e^{-i\delta/2} - e^{i\delta/2})}$

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Subtracting, $S(f) = \dfrac{1-f^n}{1-f} = \dfrac{1-e^{in\delta}}{1-e^{i\delta}} = \dfrac{e^{in\delta/2}(e^{-in\delta/2} - e^{in\delta/2})}{e^{i\delta/2}(e^{-i\delta/2} - e^{i\delta/2})} = e^{i(n-1)\delta/2} \; \dfrac{\sin(n\delta/2)}{\sin(\delta/2).}$

Multiple vibrations of same frequency

and same relative phase

When n is very large and δ very small, $\alpha \sim n\delta/2$,

$$\boxed{X = na\frac{\sin\alpha}{\alpha}\cos(\omega t + n\delta/2)}$$

Amplitude depends

on δ as a

“sinc” function.

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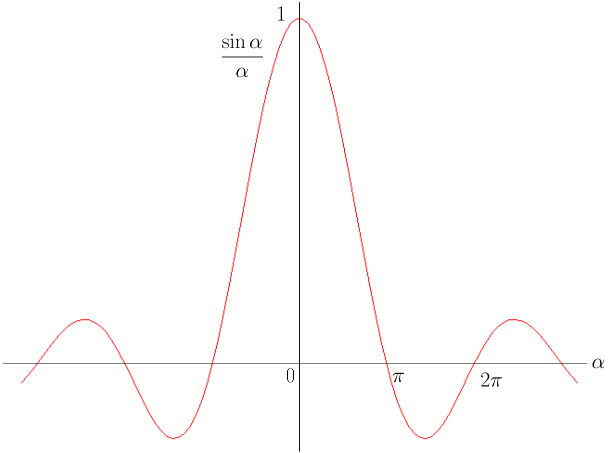
Amplitude depends

on δ as a

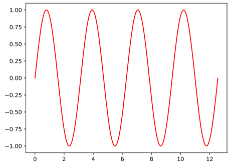
“sinc” function.

This is the pattern for diffraction from a thin slit!

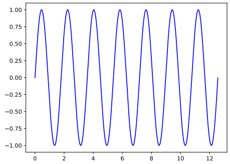


Addition of Different Frequencies

$$x_1 = A\cos(\omega_1 t), \quad x_2 = A\cos(\omega_2 t);$$

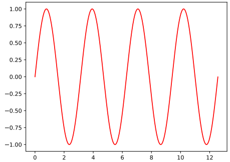


$$\omega_1 = 2$$

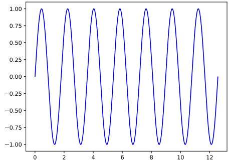


$$\omega_2 = 3.5$$

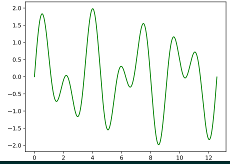
Addition of Different Frequencies



$$\omega_1 = 2$$



$$\omega_2 = 3.5$$

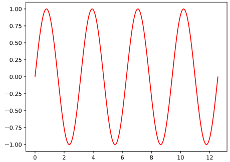


$$x_1 = A\cos(\omega_1 t), \quad x_2 = A\cos(\omega_2 t);$$

Resultant wave is a complicated function of time:

$$x = 2A\cos\left(\frac{\omega_1 - \omega_2}{2}t\right)\cos\left(\frac{\omega_1 + \omega_2}{2}t\right).$$

Addition of Different Frequencies



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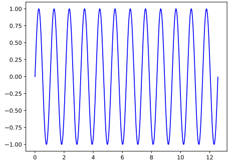
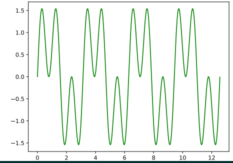


$\omega_2 = 6$



$x_1 + x_2$

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Resultant wave is a complicated function of time:

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Periodic with period T only if T_1 and T_2 are commensurate:

if $n_1 T_1 = n_2 T_2 = T$

for integer n_1 and n_2.

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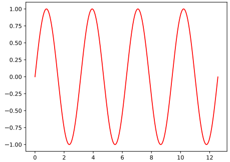

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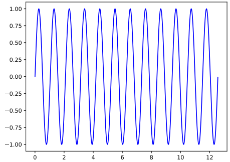

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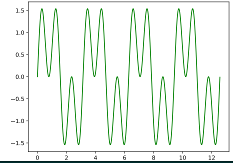

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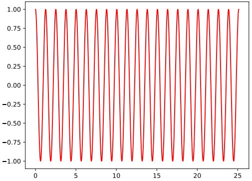
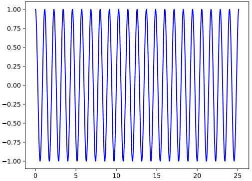
for integer n_1 and n_2.

Here, $T = T_1 = 3T_2$

When the frequencies are nearly equal...Beats!



$$\omega_1 = 5$$



$$\omega_2 = 5.5$$

$$x = x_1 + x_2 = A(\cos\omega_1 t + \cos\omega_2 t) = 2A \cos\left(\frac{\omega_1 - \omega_2}{2} t\right) \cos\left(\frac{\omega_1 + \omega_2}{2} t\right)$$

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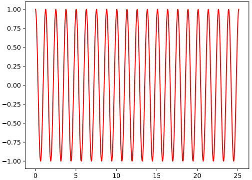

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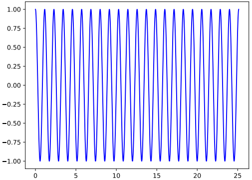

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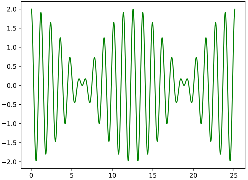

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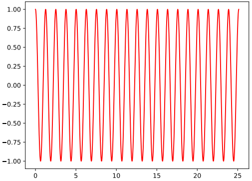

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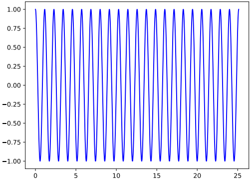

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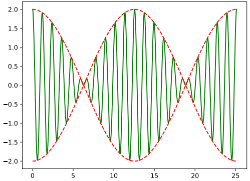

$x_1 + x_2, \, \omega_b = 0.25$

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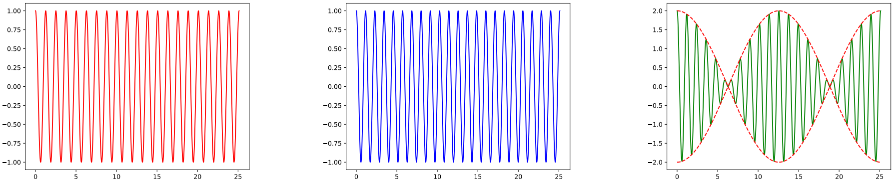

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Amplitude oscillates with frequency $\Delta\omega = |\omega_1 - \omega_2|$: beat frequency.

Perpendicular Vibrations: Lissajous Figures

1. In phase:

$$x = A_1 \sin(\omega t),$$

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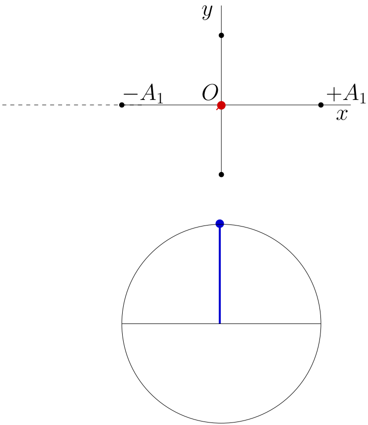

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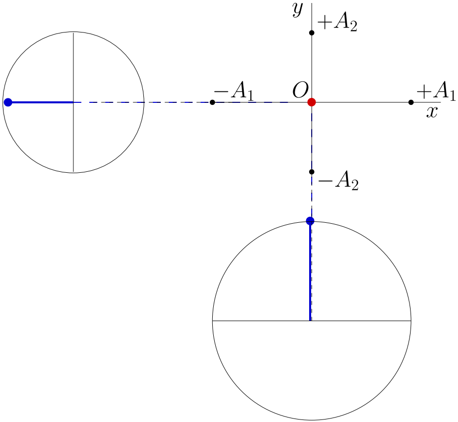

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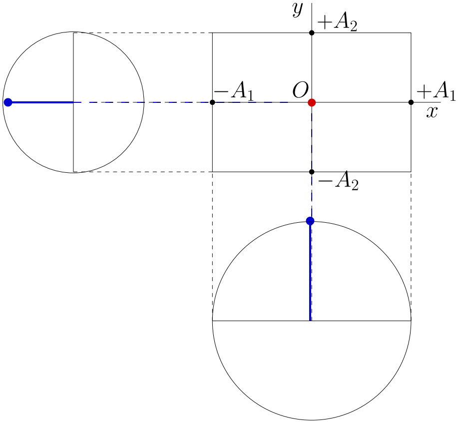

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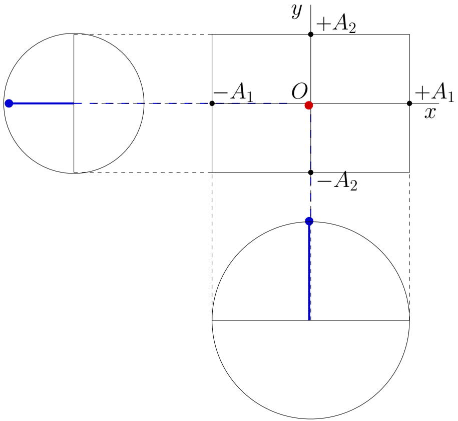

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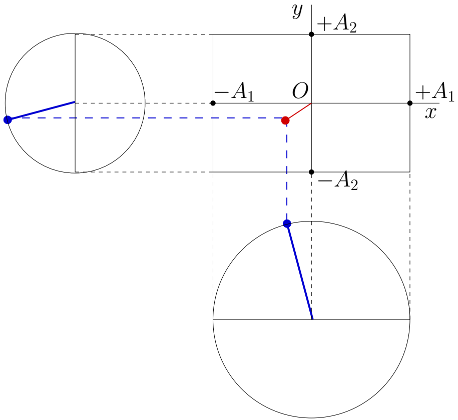

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Resultant motion is a straight line with slope

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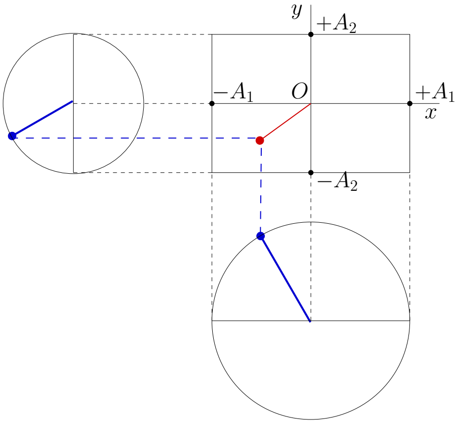

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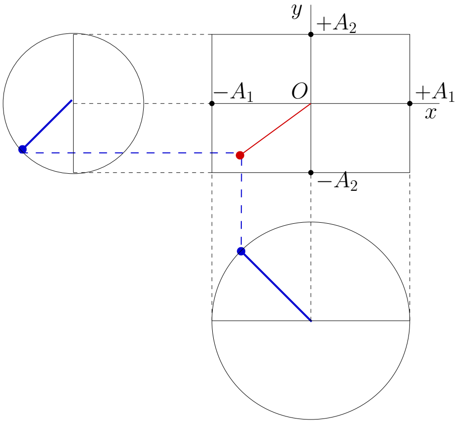


Perpendicular Vibrations: same frequency

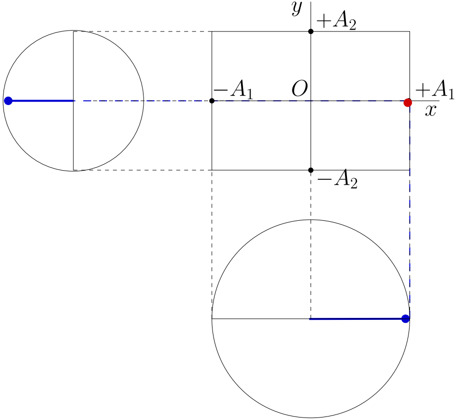
2. Out of phase by $\pi/2$:

$$x = A_1 \cos(\omega t)$$

$$y = A_2 \sin(\omega t)$$

Resultant motion traces the ellipse

$$\frac{x^2}{A_1^2} + \frac{y^2}{A_2^2} = 1.$$



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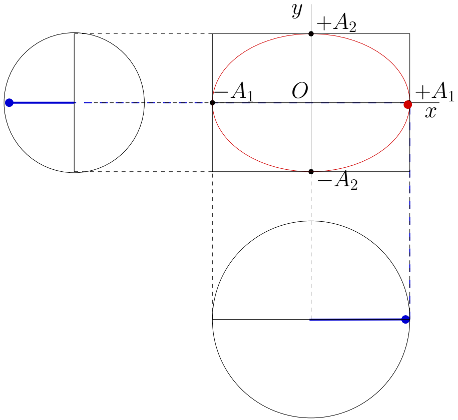

Perpendicular Vibrations: same frequency

In general,

$$x = A_1 \cos(\omega t), \quad y = A_2 \cos(\omega t + \delta).$$

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Let $\dfrac{x}{A_1} = \tilde{x}$ and $\dfrac{y}{A_2} = \tilde{y}$,

we get $\tilde{x}^2 + \tilde{y}^2 - 2\tilde{x}\tilde{y}\cos\delta = \sin^2\delta$,

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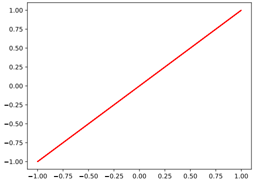
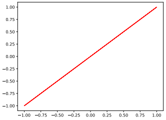
If $A_1 = A_2$, you get a circle for $\delta = \pi/2$.

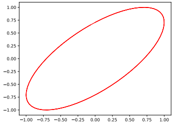
Lissajous figures: same frequency

Changing δ from 0 through 2π.

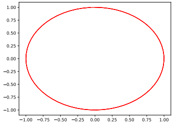




$\delta = 0$



$\delta = \pi/4$



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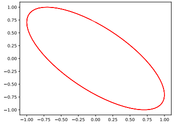
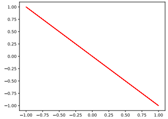


$\delta = 3\pi/4$



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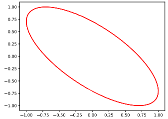
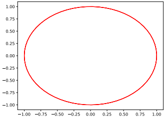


$\delta = 4\pi/4$



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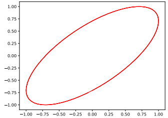
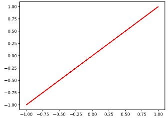


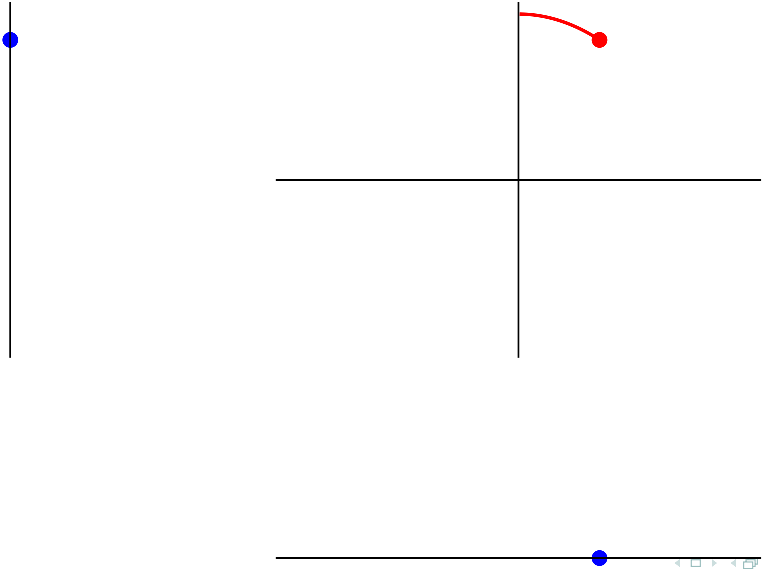
$\delta = 7\pi/4$



$\delta = 2\pi$

Perpendicular vibrations with different frequencies

$\omega_1 = 2, \omega_2 = 3$



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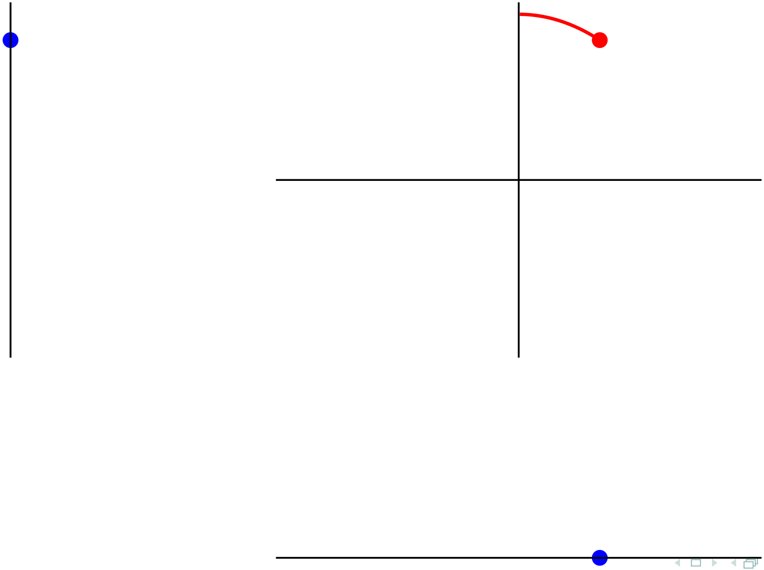

Perpendicular vibrations with different frequencies

For different frequency ratios