Lecture 1

VECTORS

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1. Physical Quantities
2. Operations with Vectors
3. Cartesian Coordinates
4. Time rate of change of a vector

VECTORS

Physical Quantities

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- **Scalars**:

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Physical Quantities

- **Scalars**: described by a single number (magnitude)
- **Vectors**: direction & magnitude

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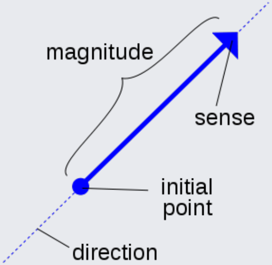

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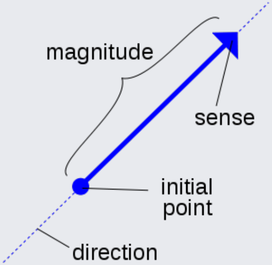

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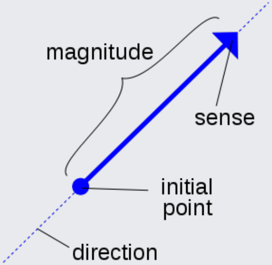

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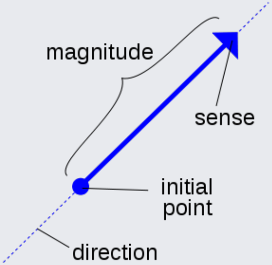

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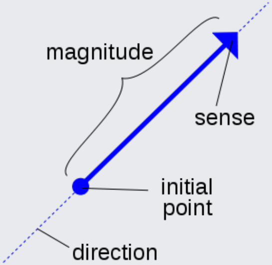

- **Tensors**: Generalization of vectors:

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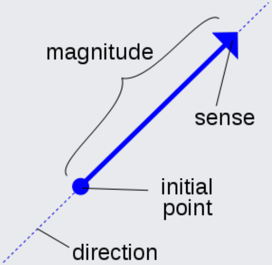

- **Tensors**: Generalization of vectors: products of vectors

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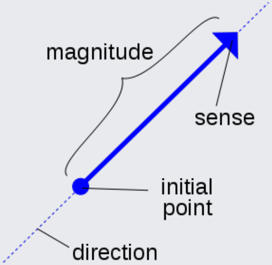

- **Tensors**: Generalization of vectors: products of vectors Moment of Inertia, Stress tensor, permeability, energy-momentum

Operations with Vectors

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- Addition:

$$\overrightarrow{A} + \overrightarrow{B} = \overrightarrow{C}$$

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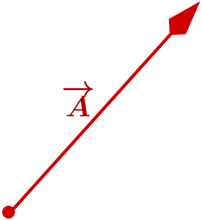
- Cross Product

$$\overrightarrow{\boldsymbol{A}} \times \overrightarrow{\boldsymbol{B}} = \overrightarrow{\boldsymbol{C}}$$

Vectors: Component form

Description relative to a set of coordinates in 3d Space.



Vectors: Component form

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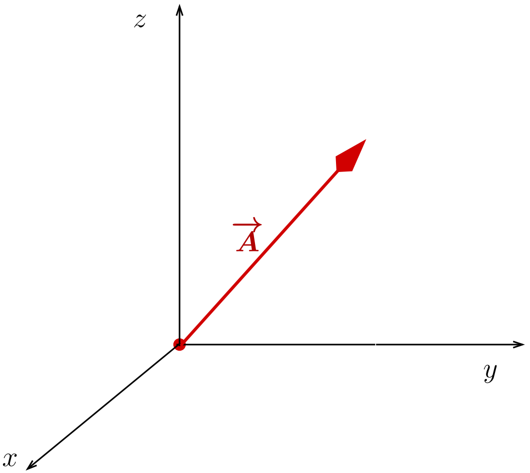
Cartesian coordinates



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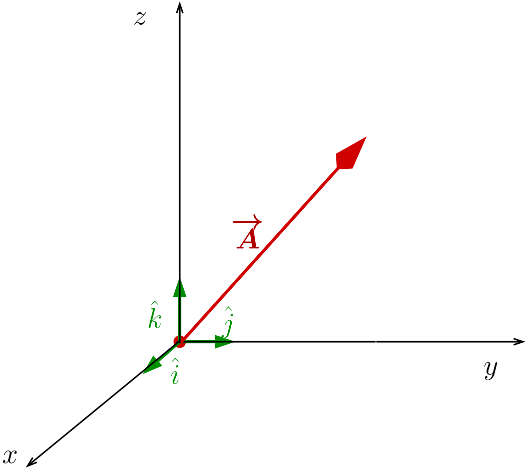

- Orthogonal unit vectors, right-handed system $\hat{\boldsymbol{i}}, \hat{\boldsymbol{j}}, \hat{\boldsymbol{k}}$

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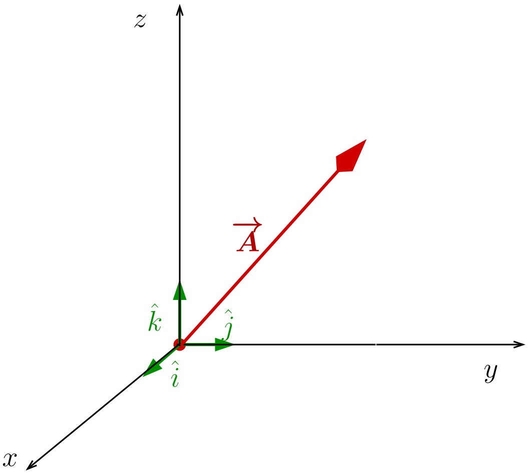

- Orthogonal unit vectors, right-handed system $\hat{\boldsymbol{i}}, \hat{\boldsymbol{j}}, \hat{\boldsymbol{k}}$

$$\hat{\boldsymbol{i}} \cdot \hat{\boldsymbol{i}} = \hat{\boldsymbol{j}} \cdot \hat{\boldsymbol{j}} = \hat{\boldsymbol{k}} \cdot \hat{\boldsymbol{k}} = 1$$

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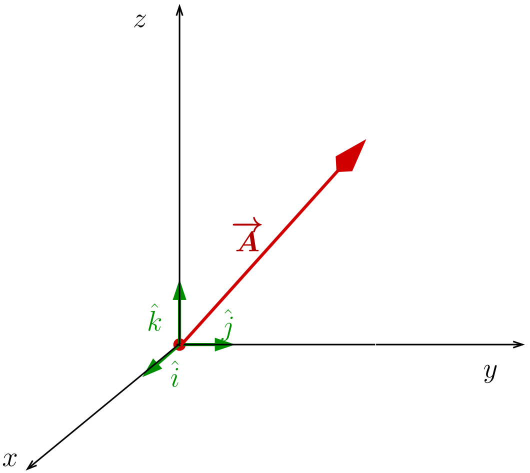

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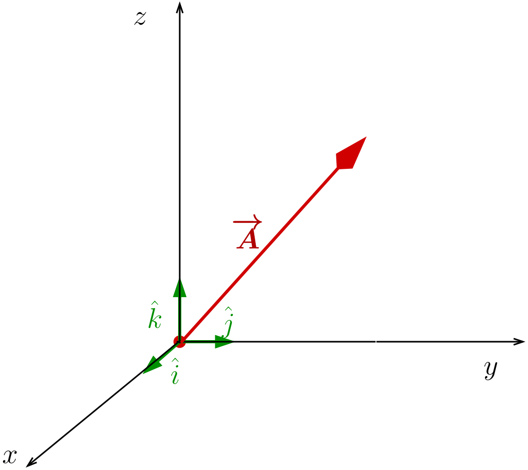

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$$\hat{\boldsymbol{i}} \times \hat{\boldsymbol{j}} = \hat{\boldsymbol{k}};$$
$$\hat{\boldsymbol{j}} \times \hat{\boldsymbol{k}} = \hat{\boldsymbol{i}};$$
$$\hat{\boldsymbol{k}} \times \hat{\boldsymbol{i}} = \hat{\boldsymbol{j}}.$$

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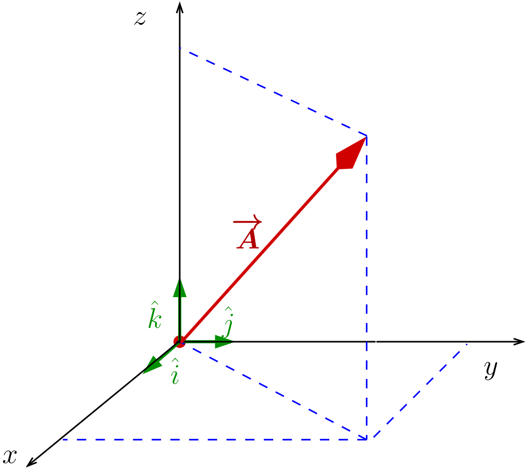

- Decomposition of Vector into components:

Vectors: Component form

Description relative to a set of coordinates in 3d Space.

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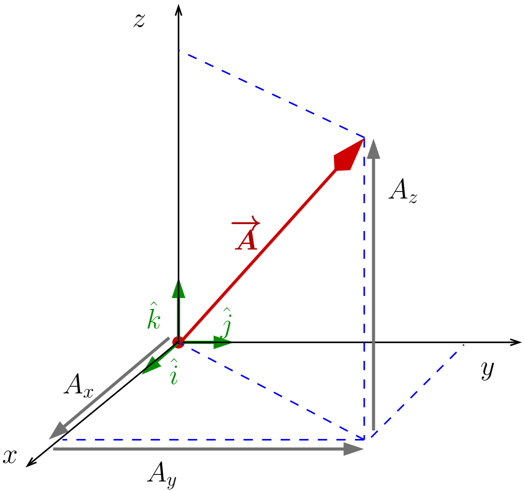

- Decomposition of Vector into components:

  Projections along the axes

Vectors: Component form

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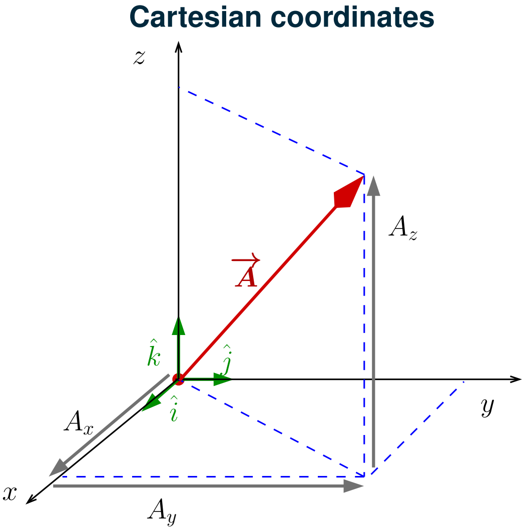

- Decomposition of Vector into components:

  Projections along the axes

$$\vec{A} = A_x \hat{\boldsymbol{i}} + A_y \hat{\boldsymbol{j}} + A_z \hat{\boldsymbol{k}}$$

Vectors: Component form

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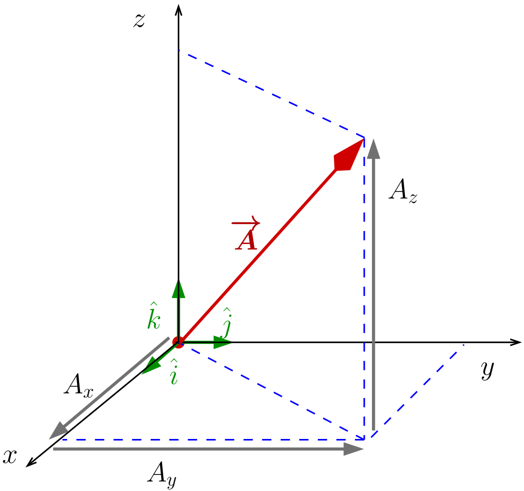

- Decomposition of Vector into components:

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  $$\overrightarrow{A} = A_x\hat{\boldsymbol{i}} + A_y\hat{\boldsymbol{j}} + A_z\hat{\boldsymbol{k}}$$

- $\hat{\boldsymbol{i}}, \hat{\boldsymbol{j}}, \hat{\boldsymbol{k}}$ are constant vectors.

Operations with Vectors: Component form

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- Magnitude of $\overrightarrow{\boldsymbol{A}}$: $A = \sqrt{A_x^2 + A_y^2 + A_z^2}$

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- Vector Addition:

$$\begin{aligned}
\overrightarrow{\boldsymbol{C}} &= \overrightarrow{\boldsymbol{A}} + \overrightarrow{\boldsymbol{B}} \\
&= (A_x + B_x)\hat{\boldsymbol{i}} + (A_y + B_y)\hat{\boldsymbol{j}} + (A_z + B_z)\hat{\boldsymbol{k}} \\
&= C_x\hat{\boldsymbol{i}} + C_y\hat{\boldsymbol{j}} + C_z\hat{\boldsymbol{k}}
\end{aligned}$$

Operations with Vectors: Component form

- Magnitude of $\overrightarrow{\boldsymbol{A}}$: $A = \sqrt{A_x^2 + A_y^2 + A_z^2}$
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- Scalar Product:

$$\overrightarrow{\boldsymbol{A}} \cdot \overrightarrow{\boldsymbol{B}} = A_xB_x + A_yB_y + A_zB_z$$

Operations with Vectors: Component form

- Vector Product:

$$\begin{aligned}\overrightarrow{\boldsymbol{C}} &= \overrightarrow{\boldsymbol{A}} \times \overrightarrow{\boldsymbol{B}} = \begin{vmatrix} \hat{\boldsymbol{i}} & \hat{\boldsymbol{j}} & \hat{\boldsymbol{k}} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} \\ &= (A_y B_z - A_z B_y)\hat{\boldsymbol{i}} + (A_z B_x - A_x B_z)\hat{\boldsymbol{j}} + (A_x B_y - A_y B_x)\hat{\boldsymbol{k}}\end{aligned}$$

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Direction of vector product: right hand screw rule

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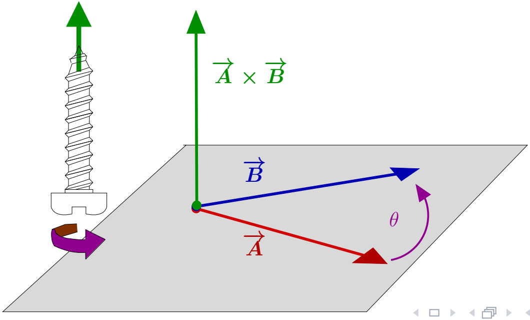
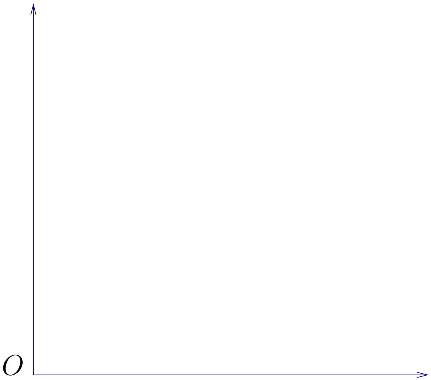


Time Rate of Change of a Vector



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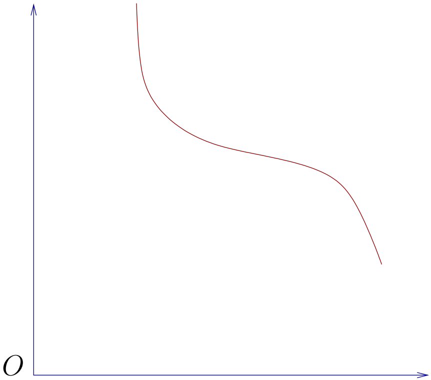

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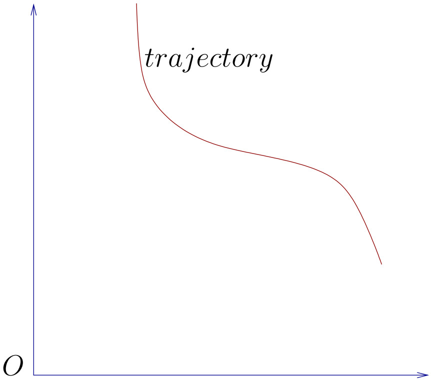

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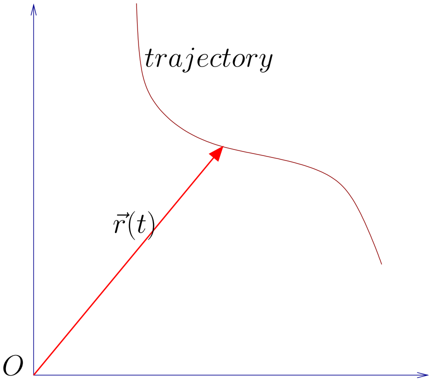

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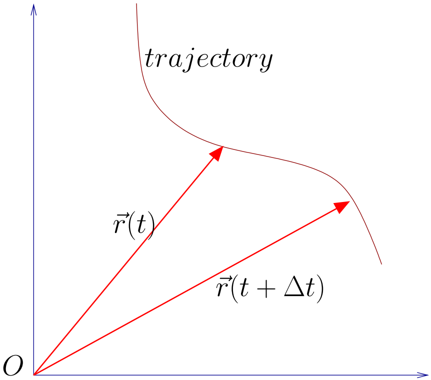

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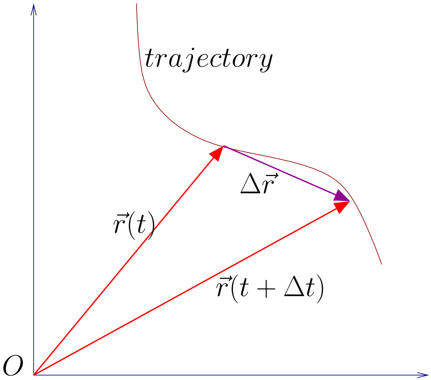

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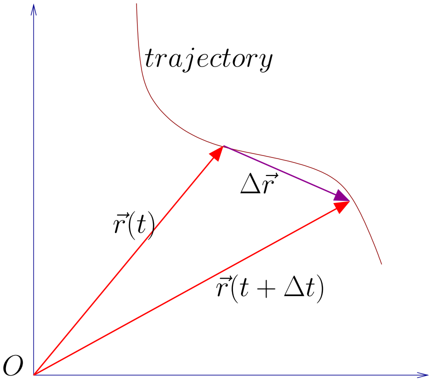

$$\lim_{\Delta t \to 0} \frac{\overrightarrow{\boldsymbol{\Delta r}}}{\Delta t} =$$

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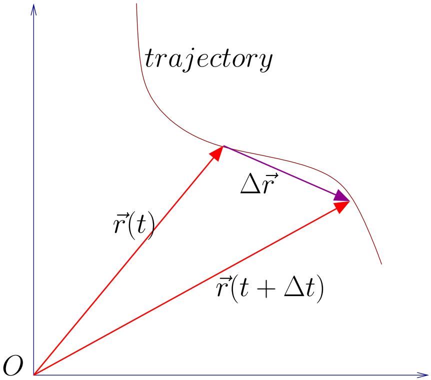

$$\lim_{\Delta t \to 0} \frac{\overrightarrow{\boldsymbol{\Delta r}}}{\Delta t} = \frac{d\,\overrightarrow{\boldsymbol{r}}}{dt} =$$

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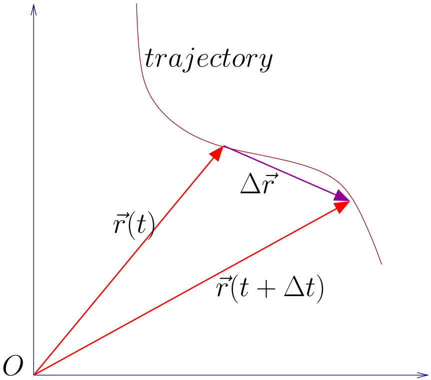

$$\lim_{\Delta t \to 0} \frac{\overrightarrow{\boldsymbol{\Delta r}}}{\Delta t} = \frac{d\,\overrightarrow{\boldsymbol{r}}}{dt} = \frac{dx}{dt}\hat{\boldsymbol{i}} + \frac{dy}{dt}\hat{\boldsymbol{j}} + \frac{dz}{dt}\hat{\boldsymbol{k}}$$

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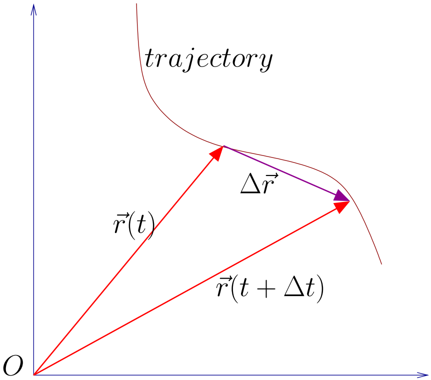

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$$\frac{d\,\overrightarrow{\boldsymbol{A}}}{dt} =$$

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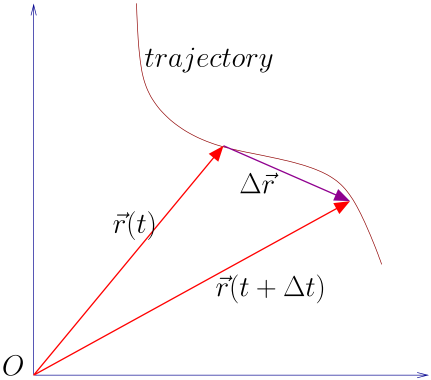

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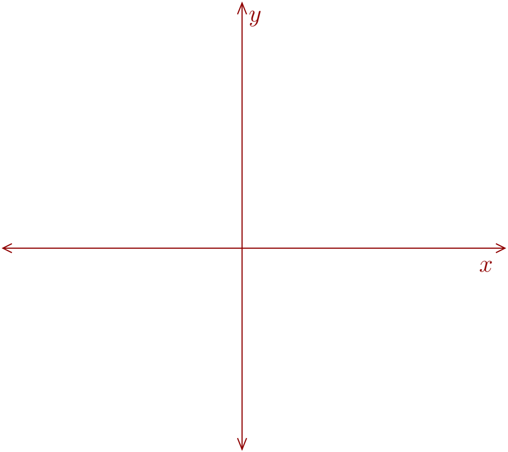
$$\frac{d\,\overrightarrow{\boldsymbol{A}}}{dt} = \frac{dA_x}{dt}\hat{\boldsymbol{i}} + \frac{dA_y}{dt}\hat{\boldsymbol{j}} + \frac{dA_z}{dt}\hat{\boldsymbol{k}}$$

Advanced Topic 1

Transformation of Vectors under Rotation of Coordinate Axes



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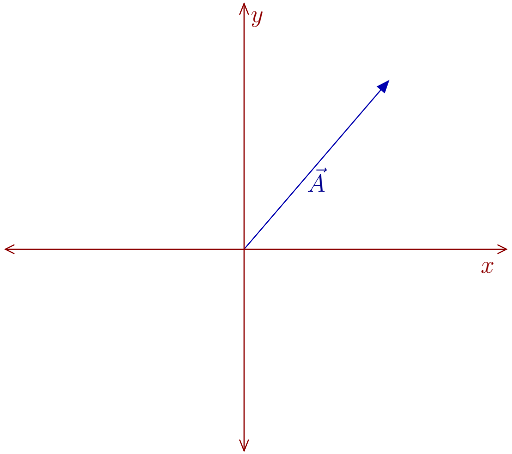

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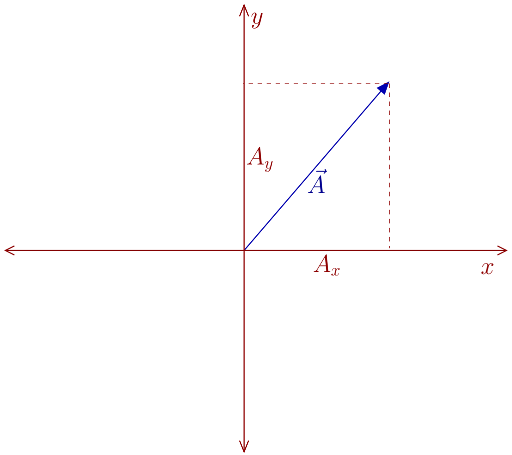

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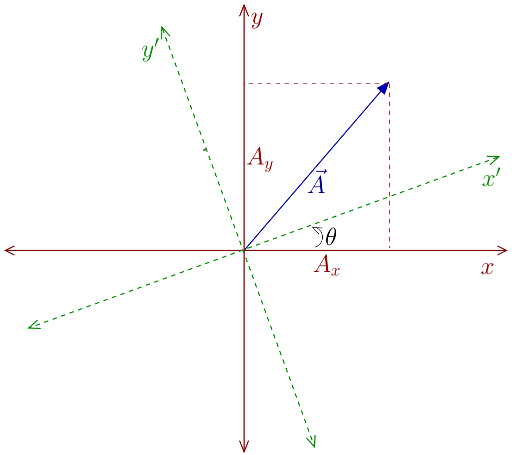

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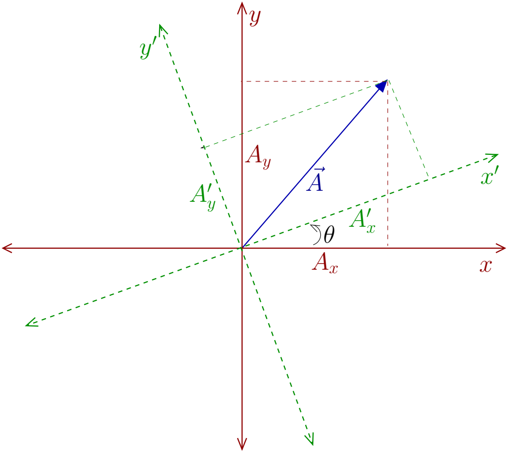

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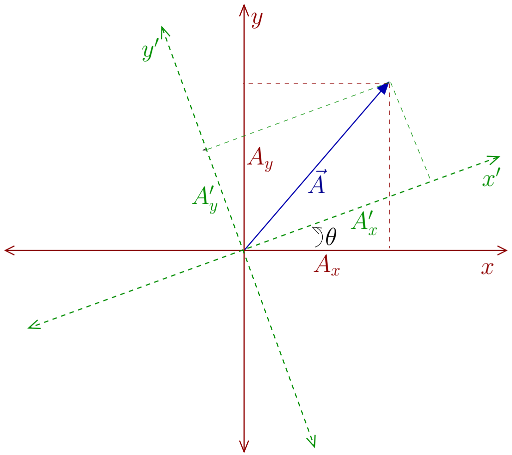

$$
\begin{aligned}
\vec{\boldsymbol{A}} &= A_x \ \hat{\boldsymbol{i}} + A_y \ \hat{\boldsymbol{j}} \\
&= A'_x \ \hat{\boldsymbol{i}}' + A'_y \ \hat{\boldsymbol{j}}'
\end{aligned}
$$

Advanced Topic 1

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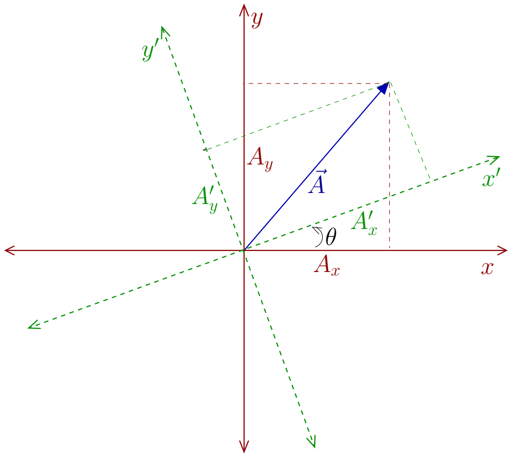
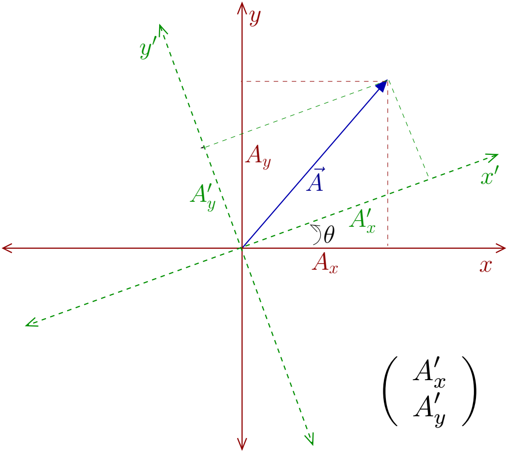


$$\vec{\boldsymbol{A}} = A_x\ \hat{\boldsymbol{i}} + A_y\ \hat{\boldsymbol{j}}$$
$$= A'_x\ \hat{\boldsymbol{i}}' + A'_y\ \hat{\boldsymbol{j}}'$$

$$A'_x = A_x \cos\theta + A_y \sin\theta$$
$$A'_y = -A_x \sin\theta + A_y \cos\theta$$

Advanced Topic 1

Transformation of Vectors under Rotation of Coordinate Axes



$$\vec{A} = A_x\ \hat{\boldsymbol{i}} + A_y\ \hat{\boldsymbol{j}}$$
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$$A'_x = A_x \cos\theta + A_y \sin\theta$$
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$$\begin{pmatrix} A'_x \\ A'_y \end{pmatrix}$$

Advanced Topic 1

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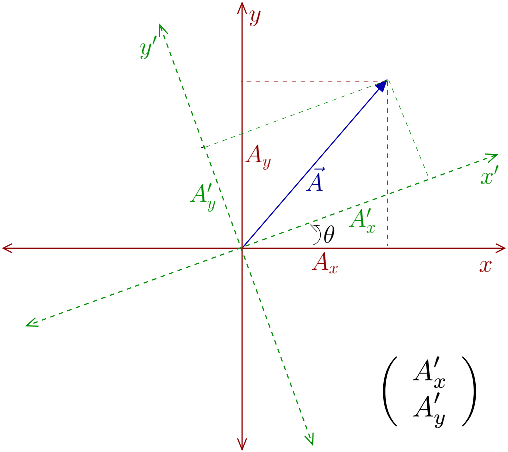


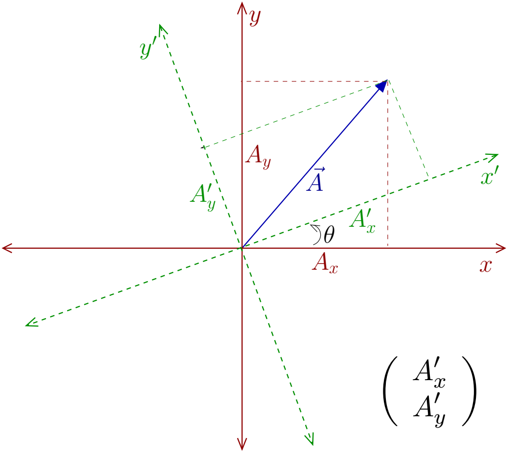
$$\begin{aligned} \vec{\boldsymbol{A}} &= A_x \ \hat{\boldsymbol{i}} + A_y \ \hat{\boldsymbol{j}} \\ &= A'_x \ \hat{\boldsymbol{i}}' + A'_y \ \hat{\boldsymbol{j}}' \end{aligned}$$

$$\begin{aligned} A'_x &= A_x \cos\theta + A_y \sin\theta \\ A'_y &= -A_x \sin\theta + A_y \cos\theta \end{aligned}$$

$$\begin{pmatrix} A'_x \\ A'_y \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

Advanced Topic 1

Transformation of Vectors under Rotation of Coordinate Axes



$$\vec{A} = A_x\ \hat{\boldsymbol{i}} + A_y\ \hat{\boldsymbol{j}}$$
$$= A'_x\ \hat{\boldsymbol{i}}' + A'_y\ \hat{\boldsymbol{j}}'$$

$$A'_x = A_x \cos\theta + A_y \sin\theta$$
$$A'_y = -A_x \sin\theta + A_y \cos\theta$$

$$\begin{pmatrix} A'_x \\ A'_y \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} A_x \\ A_y \end{pmatrix}$$

Advanced Topic 1

Transformation of Vectors under Rotation of Coordinate Axes

$$\begin{pmatrix} A'_x \\ A'_y \end{pmatrix}$$

Advanced Topic 1

Transformation of Vectors under Rotation of Coordinate Axes

$$\begin{pmatrix} A'_x \\ A'_y \end{pmatrix} = \begin{pmatrix} R_{xx} & R_{xy} \\ R_{yx} & R_{yy} \end{pmatrix}$$

$$\begin{pmatrix} A'_x \\ A'_y \end{pmatrix} = \begin{pmatrix} R_{xx} & R_{xy} \\ R_{yx} & R_{yy} \end{pmatrix} \begin{pmatrix} A_x \\ A_y \end{pmatrix}$$

Advanced Topic 1

Transformation of Vectors under Rotation of Coordinate Axes

$$\begin{pmatrix} A'_x \\ A'_y \end{pmatrix} = \begin{pmatrix} R_{xx} & R_{xy} \\ R_{yx} & R_{yy} \end{pmatrix} \begin{pmatrix} A_x \\ A_y \end{pmatrix}$$

In 3D

Advanced Topic 1

Transformation of Vectors under Rotation of Coordinate Axes

$$\begin{pmatrix} A'_x \\ A'_y \end{pmatrix} = \begin{pmatrix} R_{xx} & R_{xy} \\ R_{yx} & R_{yy} \end{pmatrix} \begin{pmatrix} A_x \\ A_y \end{pmatrix}$$

In 3D

$$\begin{pmatrix} A'_x \\ A'_y \\ A'_z \end{pmatrix} = \begin{pmatrix} R_{xx} & R_{xy} & R_{xz} \\ R_{yx} & R_{yy} & R_{yz} \\ R_{zx} & R_{zy} & R_{zz} \end{pmatrix} \begin{pmatrix} A_x \\ A_y \\ A_z \end{pmatrix}$$

$$\begin{pmatrix} A'_x \\ A'_y \end{pmatrix} = \begin{pmatrix} R_{xx} & R_{xy} \\ R_{yx} & R_{yy} \end{pmatrix} \begin{pmatrix} A_x \\ A_y \end{pmatrix}$$

In 3D

$$\begin{pmatrix} A'_x \\ A'_y \\ A'_z \end{pmatrix} = \begin{pmatrix} R_{xx} & R_{xy} & R_{xz} \\ R_{yx} & R_{yy} & R_{yz} \\ R_{zx} & R_{zy} & R_{zz} \end{pmatrix} \begin{pmatrix} A_x \\ A_y \\ A_z \end{pmatrix}$$

Transformation of Vector under Rotation of Axes

$$[A'] = [R] \times [A]$$

Advanced Topic 1

Transformation of Vectors under Rotation of Coordinate Axes

$$\begin{pmatrix} A'_x \\ A'_y \end{pmatrix} = \begin{pmatrix} R_{xx} & R_{xy} \\ R_{yx} & R_{yy} \end{pmatrix} \begin{pmatrix} A_x \\ A_y \end{pmatrix}$$

In 3D

$$\begin{pmatrix} A'_x \\ A'_y \\ A'_z \end{pmatrix} = \begin{pmatrix} R_{xx} & R_{xy} & R_{xz} \\ R_{yx} & R_{yy} & R_{yz} \\ R_{zx} & R_{zy} & R_{zz} \end{pmatrix} \begin{pmatrix} A_x \\ A_y \\ A_z \end{pmatrix}$$

Transformation of Vector under Rotation of Axes

$$[A'] = [R] \times [A]$$

Definition: A Physical Quantity which transforms under rotation as above is a Vector