MATHEMATICS-I (MATH F111)

Tutorial Sheet-4

Topic: Sequences, Infinite series: nth term test, Integral test

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1. Determine if the following series is convergent or divergent. If a series converges find the sum.

   (a) $\displaystyle\sum_{n=1}^{n=\infty}\left(\frac{1}{n^2}-\frac{1}{(n+1)^2}\right)$

   (b) $\displaystyle\sum_{n=1}^{n=\infty}(\ln\sqrt{(}n+1)-\ln\sqrt{(}n))$

   (c) $\displaystyle\sum_{n=1}^{n=\infty}(\tan^{-1}(n)-\tan^{-1}(n+1))$

   (d) $\displaystyle\sum_{n=1}^{n=\infty}\left(\frac{2^n-1}{3^n}\right)$

   (e) $\displaystyle\sum_{n=1}^{n=\infty}\left(\frac{n^n}{n!}\right)$

2. Determine if the following series is convergent or divergent. Give the reason of your answer.

   (a) $\displaystyle\sum_{n=1}^{n=\infty}\ln\left(\frac{1}{3^n}\right)$

   (b) $\displaystyle\sum_{n=1}^{n=\infty}\frac{\ln n}{n}$

   (c) $\displaystyle\sum_{n=1}^{n=\infty}\frac{1}{\sqrt{n}(\ln n)}$

   (d) $\displaystyle\sum_{n=1}^{n=\infty}\frac{\tan^{-1}n}{1+n^2}$

   (e) $\displaystyle\sum_{n=1}^{n=\infty}\frac{\sqrt{n}}{\ln n}$

3. Show that the improper integral $\displaystyle\int_2^{\infty}\frac{dx}{x(\ln x)^p}$ where p a positive constant converges if and only if $p>1$. Discuss the convergence of the series $\displaystyle\sum_{n=2}^{n=\infty}\frac{1}{n(\ln n)^p}$.

4. Determine if the following series is convergent or divergent.

(a) $\displaystyle\sum_{n=2}^{n=\infty} \frac{1}{n(\ln n)}$

(b) $\displaystyle\sum_{n=1}^{n=\infty} \frac{1}{n(\ln n)^{1.000001}}$

5. Find the values of x for which the series converges.

(a) $\displaystyle\sum_{n=1}^{n=\infty} (-1)^n x^n$

(b) $\displaystyle\sum_{n=1}^{n=\infty} \frac{(-1)^n}{2} \left(\frac{1}{3+\sin x}\right)^n$

6. Construct an infinite series of nonzero terms whose sum is

(a) 1

(b) -3

7. If $\displaystyle\sum_{n=1}^{n=\infty} a_n$ converges and $a_n > 0$ for all n, can anything be said about $\displaystyle\sum_{n=1}^{n=\infty} \frac{1}{a_n}$? Give reasons for your answer.

8. For a sequence a_n the terms of even index are denoted by a_{2k} and the terms of odd index by a_{2k+1}. Prove that if $a_{2k} \to L$ and $a_{2k+1} \to L$, then $a_n \to L$.