Tutorial 7

MATH F111 Mathematics I

September 13, 2024

1. Find the domain of the vector valued functions
   - $\vec{r} = (t^2,\ \sqrt{t-1},\ \sqrt{5-t})$.
   - $\vec{r} = (\sqrt{2-t},\ \ln(t+1),\ e^t)$.
2. Find a vector equation and parametric equations for the line segment that joins the point $P(1, 3, -2)$ to the point $Q(2, -1, 3)$.
3. Find a vector function that represents the curve of intersection of the cylinder $x^2 + y^2 = 1$ and the plane $y + z = 2$. Is the curve a smooth curve?
4. Draw the curve with vector equation $\vec{r} = (t, t^2, t^3)$ This curve is called a twisted cubic.
5. Two particles travel along the space curves $\vec{r_1}(t) = (t, t^2, t^3)$ and $\vec{r_2}(t) = (1 + 2t, 1 + 6t, 1 + 14t)$. Do the particles collide? Do their paths intersect?
6. - If $r(t) = \frac{1}{(t^2+1)}\hat{\mathrm{i}} + \ln(t+1)\hat{\mathrm{j}} + \frac{1}{t}\hat{\mathrm{k}}$, then find $\lim_{t\to 0} r(t)$.
   - If $r(t) = \frac{2t-4}{(t+1)}\hat{\mathrm{i}} + \frac{t}{t^2+1}\hat{\mathrm{j}} + (4t-3)\hat{\mathrm{k}}$, then find $\lim_{t\to 3} r(t)$.
7. What the value of the t in which the vector function $r(t) = \tan(t)\hat{\mathrm{i}} + (\ln t)\hat{\mathrm{j}} + (\sqrt{1-t})\hat{\mathrm{k}}$ is continuous.
8. Sketch the position vector $\vec{r}(t)$ and the tangent vector $\vec{r}\,'(t)$ for the given value of t .
   - $\vec{r}(t) = (t-2, t^2+1), t = -1$
   - $\vec{r}(t) = (1 + \cos t, 2 + \sin t), t = \frac{\pi}{6}$
9. Find parametric equations for the tangent line to the helix with parametric equations $x = 2\cos t, y = \sin t, z = t$ at the point $(0, 1, \frac{\pi}{2})$.
10. Determine whether the curves are smooth:
    1. $\vec{r}(t) = (1 + t^3, t^2)$
    2. $\vec{r}(t) = (t^3, t^4, t^5)$
    3. $\vec{r}(t) = (t^3 + t, t^4, t^5)$
11. If a curve has the property that the position vector is always perpendicular to the tangent vector, show that the curve lies on a sphere with center the origin.
12. Calculate the following integrals
    - $$\int (t, t^2, t^3) \times (t^3, t^2, t)\ dt.$$
    - $$\int_0^{\frac{\pi}{3}} (\sin(2t), \tan(t), e^{-2t})\ dt.$$

13. During an Independence Day celebration, a cannonball is fired from a cannon on a cliff toward the water. The cannon is aimed at an angle of 30° above horizontal and the initial speed of the cannonball is 600 feet/sec. The cliff is 100 feet.

    (a) Find the maximum height of the cannonball.

    (b) How long will it take for the cannonball to splash into the sea?

    (c) How far out to sea will the cannonball hit the water?

14. Find the arc-length parameterization for each of the following curves:

    - $r(t) = 4\cos t\hat{\imath} + 4\sin t\hat{\jmath}, \quad t \geq 0$
    - $r(t) = (t+3, 2t-4, 2t), \quad t \geq 3.$

15. Determine where on the curve given by $r(t) = (t^2, 2t^3, 1-t^3)$ we are after traveling a distance of 20.