Tutorial 8

MATH F111 Mathematics I

September 24, 2024

1. Find $\vec{\mathbb{T}}, \vec{\mathbb{N}}$ and κ for the curve
   (a) $\vec{r}(t) = (\ln \sec t)\vec{i} + t\vec{j}, -\frac{\pi}{2} < t < \frac{\pi}{2}$.
   (b) $\vec{r}(t) = t\vec{i} + (\ln \cos t)\vec{j}, -\frac{\pi}{2} < t < \frac{\pi}{2}$.
2. (a) The graph $y = f(x)$ in the $xy-$ plane automatically has the parametrization $x = x, y = f(x)$, and the vector formula $\vec{r}(x) = x\vec{i} + f(x)\vec{j}$. Use this formula to show that if f is a twice differentiable function of x, then
   $$\kappa(x) - \frac{|f''(x)|}{(1 + (f'(x))^2)^{\frac{3}{2}}}.$$
   (b) Use the formula for κ in part (a) to find the curvature of $y = \ln(\cos x), -\frac{\pi}{2} < x < \frac{pi}{2}$. Compare your answer with the answer in Exercise $11(b)$.
   (c) Show that the curvature is zero at a point of inflection. (The points where the first derivative changes sign, from positive to negative or negative to positive)
3. Determine maximum curvature for the graph $f(x) = x/(x + 1)$ for $x > -1$.
4. Show that $\vec{n}(t) = -g'(t)\vec{i} + f'(t)\vec{j}$, and $-\vec{n}(t) = -g'(t)\vec{i}0f'(t)\vec{j}$ are both normal to the curve $\vec{r}(t) = f(t)\vec{i} + g(t)\vec{j}$ at the point $(f(t), g(t))$.
5. Find the equation for the circle of curvature of the curve $\vec{r}(t) = t\vec{i} + \sin(t)\vec{j}$ at the point $(\pi/2, 1)$.
6. Show that the center of the osculating circle for the parabola $y = x^2$ at the point (a, a^2) is located at $(-4a^3, 3a^2 + (1/2))$.
7. An object of mass m travels along the parabola $y = x^2$ with a constant speed of 10 units/sec. What is the force on the object due to its acceleration at $(0, 0)$? At $(2^{1/2}, 2)$?
8. Find the tangential and normal components of the acceleration for the following curves:
   (a) $\vec{r}(t) = (t^2)\vec{i} + (t + (1/3)t^3)\vec{j} + (t - (1/3)t^3)\vec{k}, -\frac{\pi}{2}$ at $t = 0$
   (b) $\vec{r}(t) = e^t \cos(t)\vec{i} + e^t \sin(t)\vec{j} + \sqrt{2}e^t\vec{k}, -\frac{\pi}{2}$ at $t = 0$
9. Find κ, τ and osculating, normal and rectifying planes for
   $$\mathbf{r}(t) = \left(\frac{2t^3}{3}\right)\mathbf{i} + \left(\frac{3t^2}{2}\right)\mathbf{j} \qquad t > 0.$$
10. Consider the helix $\vec{r}(t) = a\cos t\vec{i} + a\sin t\vec{j} + bt\vec{k}$, with $a, b \geq 0$. Show that the curvature and torsion of helix is $\frac{a}{a^2+b^2}$ and $\frac{b}{a^2+b^2}$ respectively.