

MATHEMATICS-I (MATH F111)

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Tutorial Sheet-5

Topic: Infinite series

1. Determine if the following series is convergent or divergent. Give the reason of your answer.

(a) $\sum_{n=1}^{n=\infty} \frac{1}{n^2 + 2n}$

(b) $\sum_{n=1}^{n=\infty} \frac{\cos^2 n}{n^{3/2}}$

(c) $\sum_{n=1}^{n=\infty} \frac{5^n}{\sqrt{n}4^n}$

(d) $\sum_{n=1}^{n=\infty} \frac{1 + \cos n}{n^2}$

2. Suppose $\sum_{n=1}^{\infty} x_n$ be convergent where $x_n > 0$ for all n. Examine whether the following series are convergent or not.

(a) $\sum_{n=1}^{\infty} \frac{\sqrt{x_n}}{n}$

(b) $\sum_{n=1}^{\infty} \frac{x_n + 2^n}{x_n + 3^n}$

3. Determine whether following series are convergent or divergent.

(a) $\sum_{n=1}^{n=\infty} (1 - \cos(\frac{1}{n}))$

(b) $\sum_{n=1}^{n=\infty} (1 - \sin(\frac{1}{n}))$

4. Discuss whether the following series are convergent or not.

(a) $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^3 + 2}}$

(b) $\sum_{n=1}^{\infty} \frac{\sqrt{2n^2 + 3}}{\sqrt{5n^3 + 1}}$

$$(c) \sum_{n=1}^{\infty} \frac{(n+1)^n}{n^{n+3/2}}$$

$$(d) \sum_{n=1}^{\infty} \frac{1}{n^{1+\frac{1}{n}}}$$

$$(e) \sum_{n=1}^{\infty} \frac{2n-1}{2^n n!}$$

5. If $\sum_{n=1}^{\infty} a_n^4$ convergent. What can you say about $\sum_{n=1}^{\infty} a_n^5$? Justify your answer.

6. If $\sum_{k=1}^{\infty} a_k$ convergence absolutely and for each n we have $\left| \frac{b_{n+1}}{b_n} \right| \leq \left| \frac{a_{n+1}}{a_n} \right|$. Does $\sum_{k=1}^{\infty} b_k$ converges absolutely? Justify your answer.

7. Discuss the absolute convergence and conditional convergence of the following series.

$$(a) \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n \ln(n+1)}$$

$$(b) \sum_{n=1}^{\infty} \frac{(-1)^{n+1} (100)^n}{n!}$$

$$(c) \sum_{n=1}^{\infty} \frac{(-1)^n \ln n}{n - \ln(n)}$$

$$(d) \sum_{n=1}^{\infty} \frac{(-1)^n 3^n}{2^n + 3^n}$$