Tutorial 9 (Answer key) **Central Force motion** 19 September 2024

P1. (a) $\vec{F} = -\vec{\nabla}U = -2Ax\hat{i} - 2By\hat{j} - 2Cz\hat{k}$

(b) $\vec{F} = -2A\left(\frac{x}{x^2+y^2+z^2}\hat{i} + \frac{y}{x^2+y^2+z^2}\hat{j} + \frac{z}{x^2+y^2+z^2}\hat{k}\right)$

(c) $\vec{F} = -\frac{\partial U}{\partial r}\hat{r} - \frac{1}{r}\frac{\partial U}{\partial \theta}\hat{\theta} = \frac{2A\cos\theta}{r^3}\hat{r} - \frac{A\sin\theta}{r^3}\hat{\theta}$

P2. (a) $\vec{\nabla}\times\vec{F} = 0$, So $\mathbf{F} = A(3\hat{\mathbf{i}} + z\hat{\mathbf{j}} + y\hat{\mathbf{k}})$ is conservative force,

Potential energy is defined with respect to a reference. Our U expression should satisfy $\vec{F} = -\vec{\nabla}U$.

Let $U = -3Ax + U_1$, then $-\frac{\partial U_1}{\partial y}$ should be Az and $-\frac{\partial U_1}{\partial z}$ should be Ay. This is possible if $U_1 = -Ayz$.

So, $U = -A(3x + yz)$.

(b) $\vec{\nabla}\times\vec{F} \neq 0$, So $\mathbf{F}$ is non-conservative force,

(c) $\vec{\nabla}\times\vec{F} = 0$, So $\mathbf{F}$ is a conservative force,

$U = -Ax^3y^5e^{\alpha z}$

P3. $\mathbf{F}_a = -Ar^3\hat{\mathbf{r}}$ is conservative, $\vec{\nabla}\times\vec{F} = 0$,

$\mathbf{F}_b = B(y^2\hat{\mathbf{i}} - x^2\hat{\mathbf{j}})$ is a non-conservative force, $\vec{\nabla}\times\vec{F} \neq 0$,

Let the initial coordinate be $P(1,1)$ and the final coordinate be $Q(0,0)$. Now, Work-energy theorem

$W^{Total} = \int_P^Q \vec{F}^{Conservative}\cdot d\vec{r} + \int_P^Q \vec{F}^{Non-conservative}\cdot d\vec{r}$

$KE_Q - KE_P = -U_Q + U_P + \int_P^Q \vec{F}^{Non-conservative}\cdot d\vec{r}$

Now, $KE_Q = \frac{1}{2}mv_Q^2$; $KE_P = \frac{1}{2}mv_0^2$; $U_Q = 0$; and $U_P = A$

$\int_P^Q \vec{F}^{Non-conservative}\cdot d\vec{r} = -\frac{B}{5} + \frac{B}{2}$. So

$v_Q = (v_0^2 + \frac{2A}{m} + \frac{3B}{5m})^{1/2}$.

P4. $W = \int_{(0,0)}^{(d,0)} \vec{F}\cdot d\vec{r} + \int_{(d,0)}^{(d,d)} ... + \int_{(d,d)}^{(0,d)} ... + \int_{(0,d)}^{(0,0)}$

$W = 0 + 2Ad^3 - Ad^3 + 0 = Ad^3$

Stoke's theorem

$\iint_S (\vec{\nabla}\times\vec{F}).d\vec{a} = \oint \vec{F}.d\vec{l}$. (You can check this yourself).