Tutorial 2 **Vectors and Polar coordinates** 9 August 2024

Problem 1. $\dot{r} = 4$ m/s, $\dot{\theta} = 2$ rad/s.

(1.1) (a) $r(t=0) = 0m$ (b) $\dot{r}(t=0) = 4m/s$ (c) $\theta(t=0) = 0$ (d) $\dot{\theta}(t=0) = 2rad/s$

(1.2) At time t = t_1 the particle is at $r = 3$ m. $r(t) = 4t$

(a) $t_1 = 3/4\ s$.

(b) $r(t_1) = 3\text{m}$, $\theta(t_1) = 3/2\text{rads}$

(c) $x(t_1) = 3\cos\dfrac{3}{2} = 0.2122$ m, $\quad y(t_1) = 3\sin\dfrac{3}{2} = 2.9925$ m.

(d) $v_x(t_1) = -5.7022$ m/s, $\quad v_y(t_1) = 4.4142$ m/s.

(e) $v_r(t_1) = 4$ m/s and $v_\theta(t_1) = 6$ m/s.

(f) $a_r = \ddot{r} - r\dot{\theta}^2 = -r\dot{\theta}^2 \implies a_r(t_1) = -12\ \text{m/s}^2$.
$a_\theta = r\ddot{\theta} + 2\dot{r} = 2\dot{r}\dot{\theta} \implies a_\theta(t_1) = 16\ \text{m/s}^2$.

(g) The Cartesian components of acceleration:
$a_x = a_r\cos\theta - a_\theta\sin\theta \implies a_x(t_1) = -16.8084\ \text{m/s}^2$,
$a_y = a_r\sin\theta + a_\theta\cos\theta \implies a_y(t_1) = -10.8388\ \text{m/s}^2$.

(1.3) If the mass of the particle is 100 gm, determine (at time t_1)

(a) The Cartesian components of the Force and Torque:
$F_x = ma_x = -1.69$ N, $F_y = ma_y = -1.08$ N,
$\vec{\boldsymbol{\tau}} = (xF_y - yF_x)\hat{\boldsymbol{k}} = 4.83\hat{\boldsymbol{k}}$ Nm.

(b) The polar components of the Force and the Torque:
$F_r = ma_r = -1.2$ N, $F_\theta = ma_\theta = 1.6$ N, $\tau_z = rF_\theta = 4.8$ Nm.

Problem 2. $\vec{\boldsymbol{v}} = v_0\hat{\boldsymbol{i}}$, $y = a$. $\vec{\boldsymbol{r}}(0) = a\hat{\boldsymbol{j}}$

(2.1) $r(t) = \sqrt{v_0^2t^2 + a^2}$, and $\theta(t) = \tan^{-1}\left(\dfrac{a}{v_0 t}\right)$.

(2.2) Velocity vector:
$v_x = v_0, v_y = 0$;
$$v_r = \frac{v_0^2 t}{\sqrt{v_0^2t^2 + a^2}}, \quad v_\theta = -\frac{v_0 a}{\sqrt{(v_0^2t^2 + a^2)}}.$$

(2.3) The acceleration vector:
$a_x = 0 = a_y$
$a_r = a_\theta = 0$.