Lecture 18

Simple Harmonic Motion

Damped Harmonic Motion

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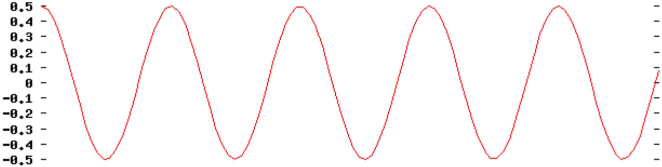
Damped Oscillations

Energy is usually dissipated in most systems: resistance



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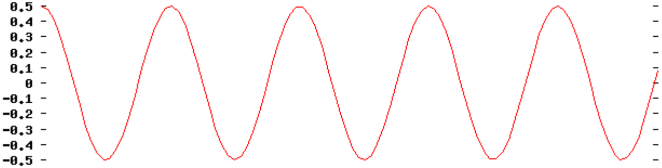
Amplitude of initial SHM



Damped Oscillations

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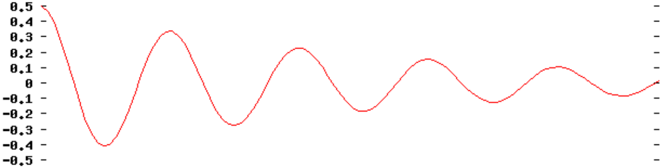

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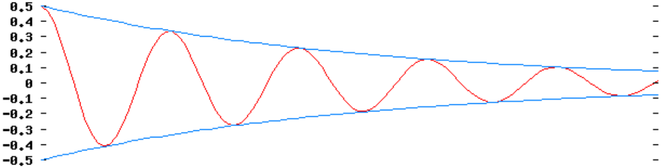

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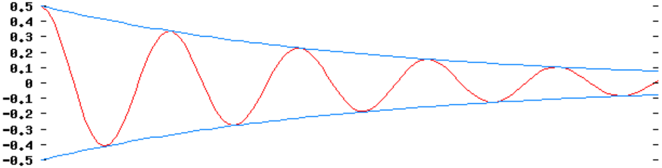

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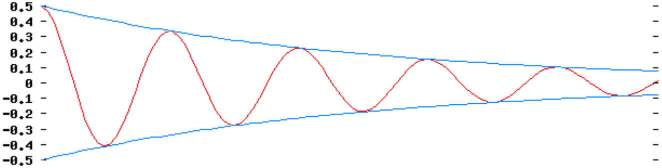

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- Viscous drag thru a fluid
- Resistance in an electrical oscillator

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Solution: Is it still oscillatory? Depends on the relative values of γ and ω_0!

Let's use complex exponentials.

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Let $\frac{1}{2}\sqrt{\gamma^2 - 4\omega_0^2} = \omega_0\sqrt{\frac{\gamma^2}{4\omega_0^2} - 1} = \beta.$

Two distinct solutions: $z(t) = \mathcal{A}e^{-\gamma t/2}e^{\beta t}$ or $\mathcal{B}e^{-\gamma t/2}e^{-\beta t}$.

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Three cases: β can be real or imaginary or zero.

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A & ϕ : arbitrary constants determined by initial conditions.

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Critical damping solution:

$$x(t) = (A + Bt)e^{-\frac{\gamma}{2}t}.$$

Analysis of Solution

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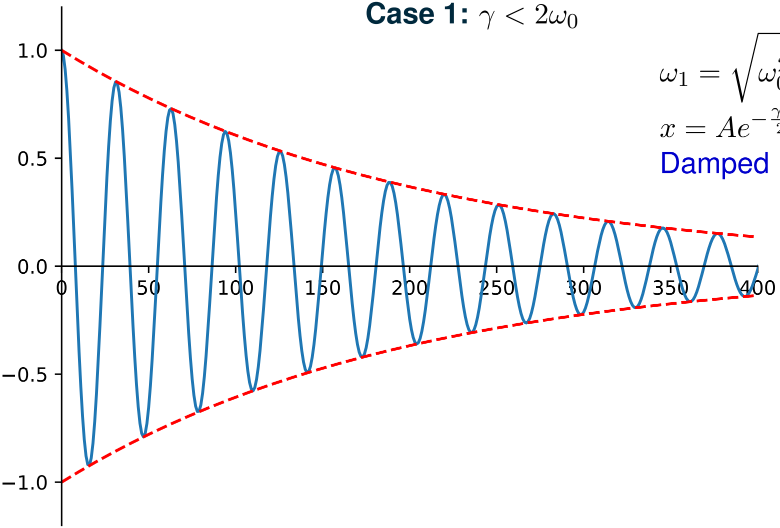

$\omega_1 = \sqrt{\omega_0^2 - \frac{\gamma^2}{4}}$, real:

$x = Ae^{-\frac{\gamma t}{2}} \cos(\omega_1 t + \phi)$

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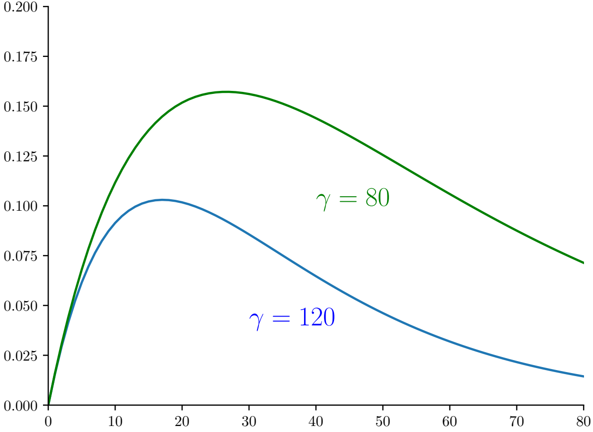

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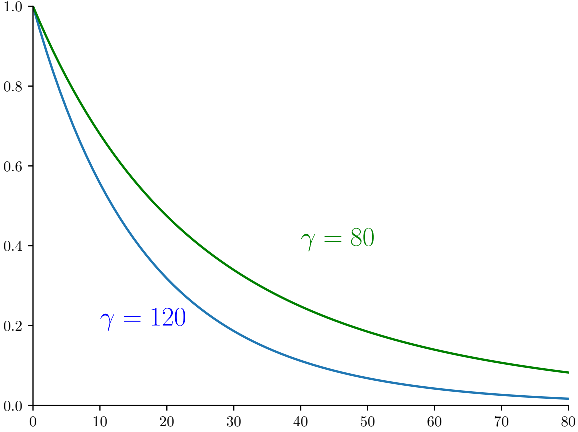

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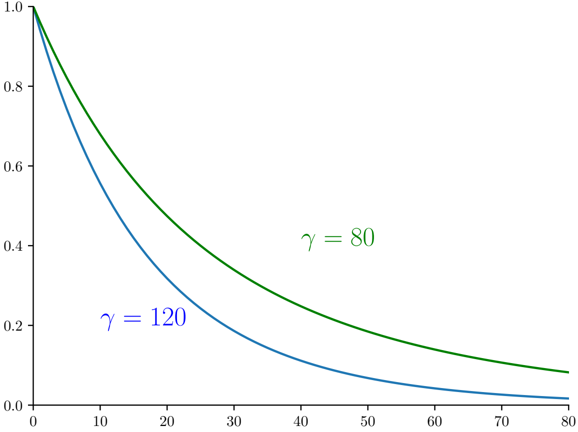

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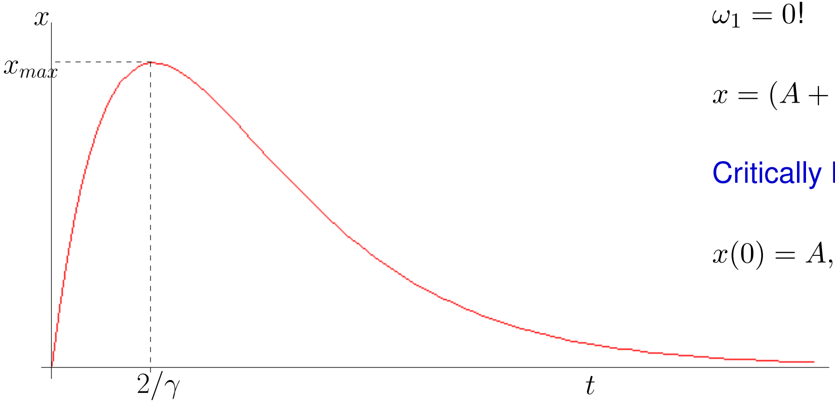

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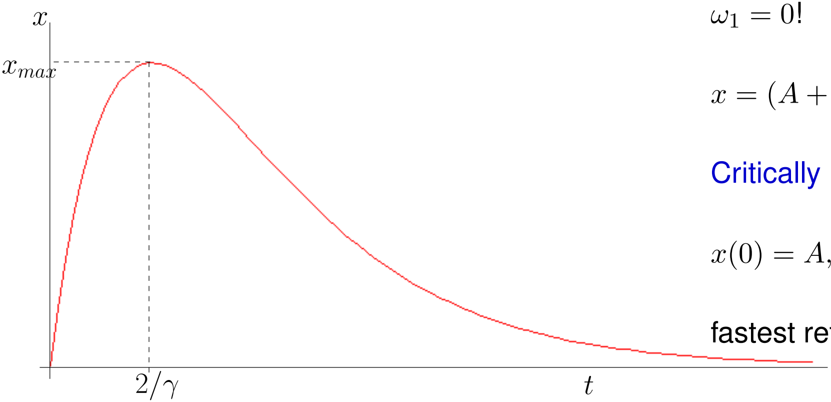

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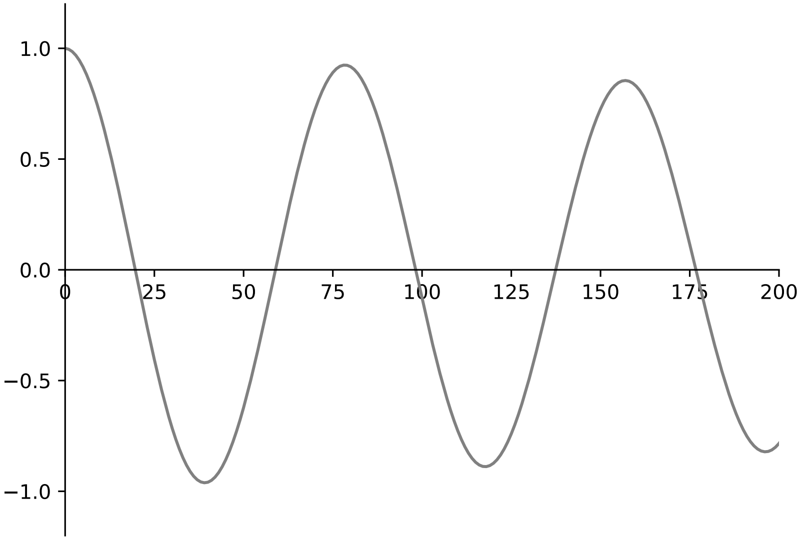
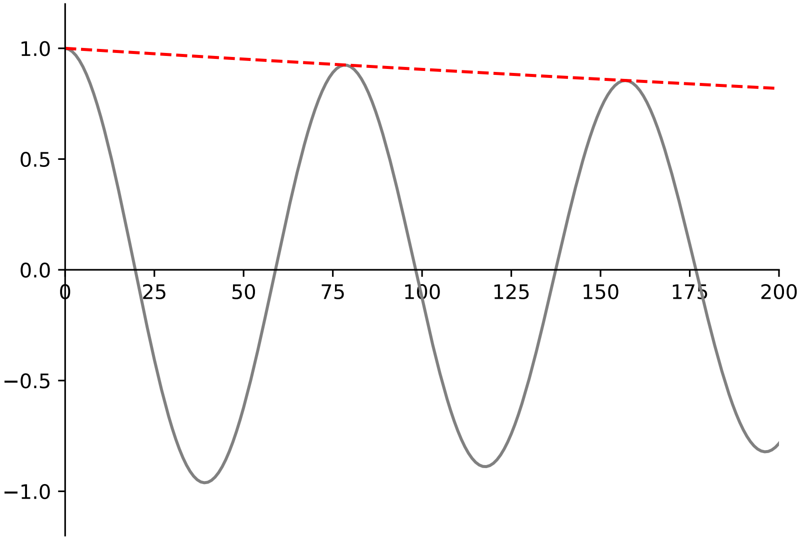
fastest return to equilibrium

Summary



Under-damped



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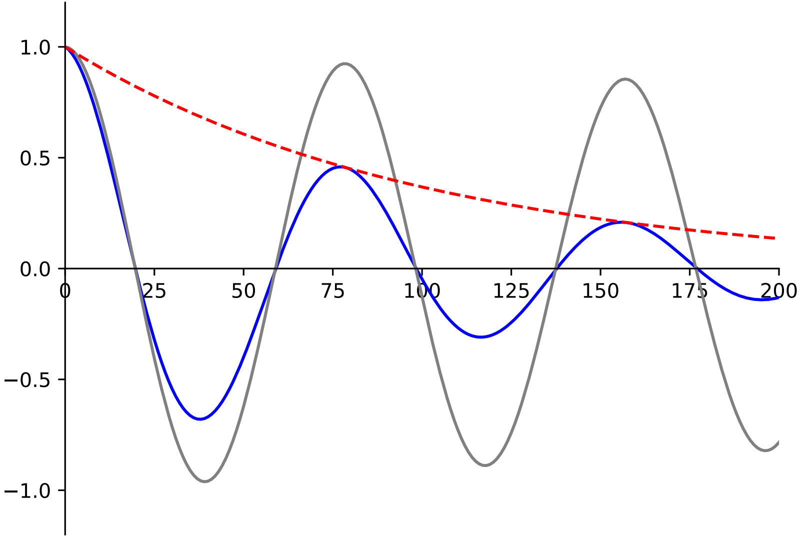

Under-damped
Damped

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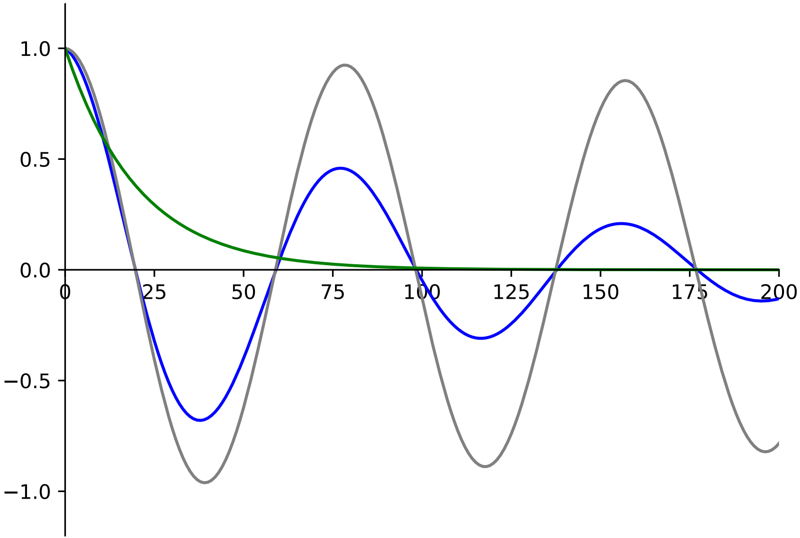
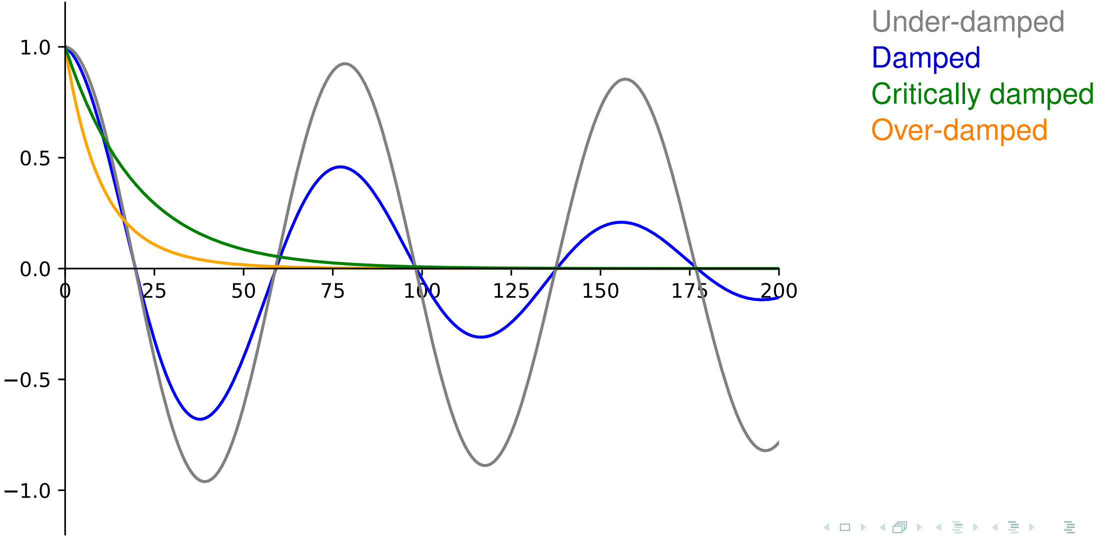


Under-damped
Damped
Critically damped

Summary


1.0
0.5
0.0
−0.5
−1.0
0
25
50
75
100
125
150
175
200
Under-damped
Damped
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Rate of Energy loss in Damped SHM

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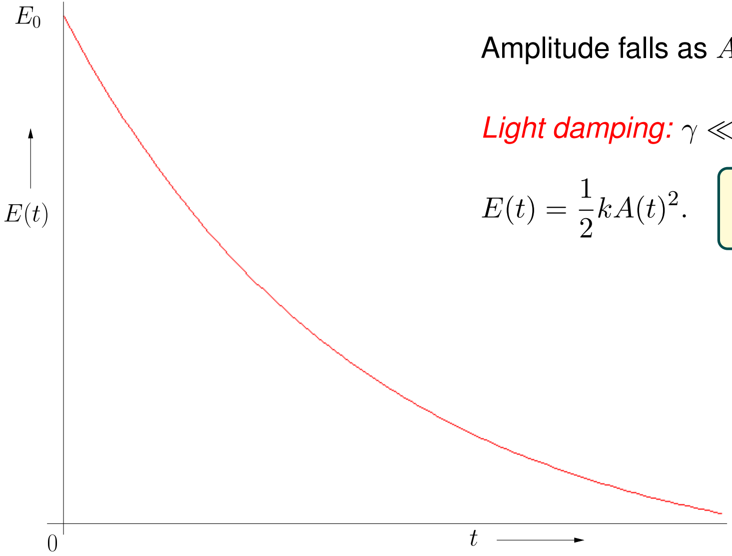

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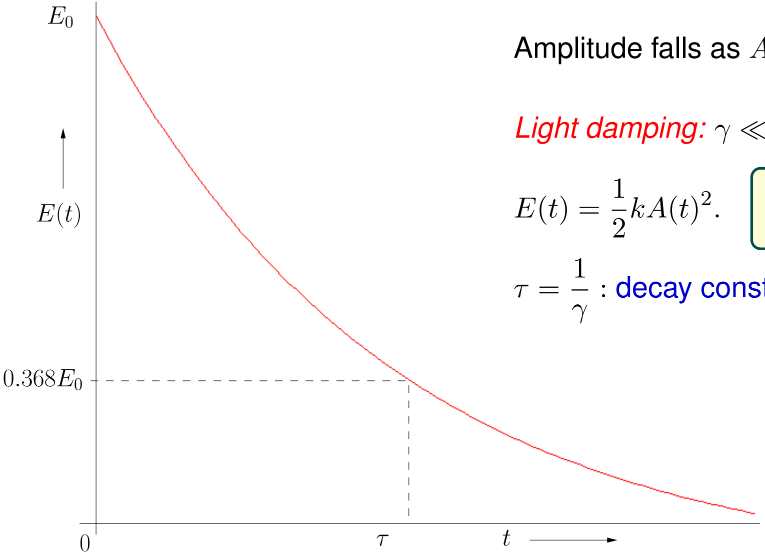

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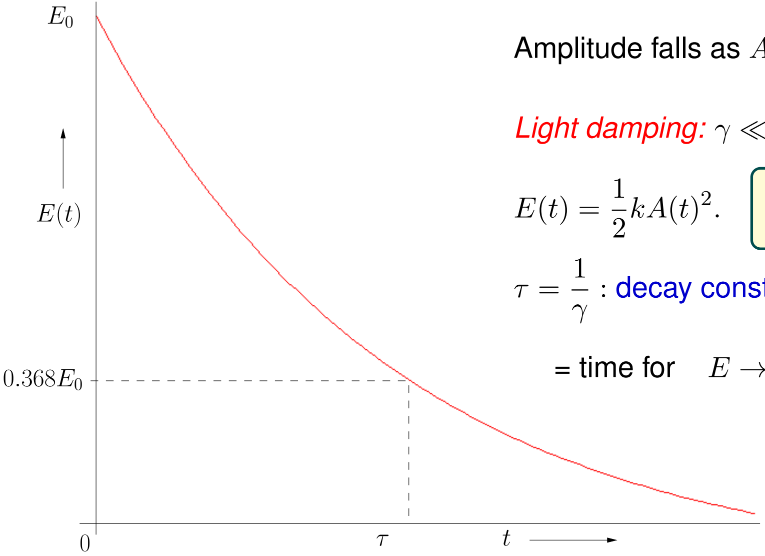

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= time for $E \to \frac{E_0}{e} = 0.368 E_0$

$(e \simeq 2.718)$

Exercise:

- Find the total energy of the lightly damped oscillator system and show that it can be written as $\frac{1}{2}kA^2e^{-\gamma t}$.
- Show that the energy loss rate $\frac{dE}{dt}$ is the rate of work done by the viscous force.

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Q is large for less energy loss: Quality of the oscillation.

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$$\omega_1 = \sqrt{\omega_0^2 - \gamma^2/4} \sim \omega_0 \text{ (light damping)}$$

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$$\omega_0 = 2\pi \times 440 = 2765 \text{ rad/sec}; \quad E_0 e^{-4\gamma} = \frac{E_0}{5}$$

$$\implies \gamma = \frac{\ln 5}{4} = 0.4 s^{-1}$$

$$\omega_1 = \sqrt{\omega_0^2 - \gamma^2/4} \sim \omega_0 \text{ (light damping)}$$

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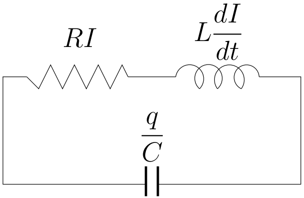
Example: LCR Circuit



Voltage drop across each element:

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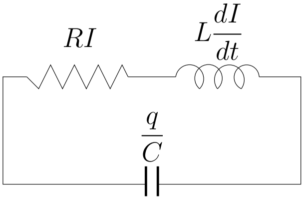

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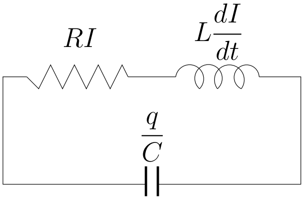

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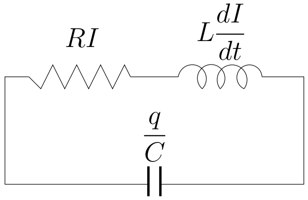

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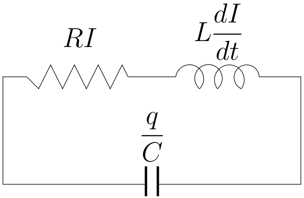

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  mass $\sim L$—inertia
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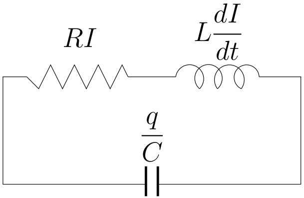

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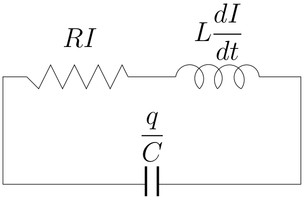

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- Damping constant $\gamma = \frac{R}{L}$.

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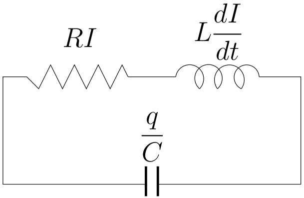

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- Quality $Q = \frac{1}{R}\sqrt{\frac{L}{C}}$.