Tutorial 11. **Simple Harmonic Motion** Date: 11 October 2024

P1. (APF 1-1) Consider a vector z defined by the equation $z = z_1 z_2$, where $z_1 = a + jb$, $z_2 = c + jd$.

(a) Show that the length of z is the product of the lengths of z_1 and z_2.

(b) Show that the angle between z and the x-axis is the sum of the angles made by z_1 and z_2 separately.

P2. (APF 1-3) Show that the multiplication of any complex number z by $e^{j\theta}$ is describable, in geometrical terms, as a positive rotation through the angle θ & of the vector by which z is represented without any alteration of its length.

P3. (APF 1-4) (a) If $z = Ae^{j\theta}$, deduce that $dz = jzd\theta$, and explain the meaning of this relation in a vector diagram.
(b) Find the magnitudes and directions of the vectors $(2 + j\sqrt{3})$ and $(2 - j\sqrt{3})^2$.

P4. (APF, 1-12) A small particle moves in a circle at a constant speed of 50cm/s. The time period of completing a circle is 6s. at $t = 0$, the radial line from the center of the circle to the particle makes an angle of 30° with the x-axis.

(a) Find the equation of the x-coordinate of the mass as a sinusoidal function of time. Identify the amplitude, frequency and phase of the motion.

(b) Find the values of x, $\dot{x}$ and $\ddot{x}$ at $t = 2$s.

(c) Plot the x and y coordinates of the particle as a function of time from $t = 0$ to $t = 6$s.

P5. (APF 1-11) A mass of 1g is attached to the end of a spring of constant 10 dynes/cm. At $t = 0$, the mass is pulled away from its equilibrium position $x = 0$ by 5cm and released.

(a) Find an equation describing its motion in the form $x = A\cos(\omega t + \phi)$, giving the values of A, ω and ϕ.

(b) Find the values of x, $\dot{x}$ and $\ddot{x}$ at $t = 8/3$s

P6. (APF 1-10) Verify that the differential equation $d^2y/dx^2 = -ky$ has as its solution
$y = A\ \cos(kx) + B\ \sin(kx)$
where A and B are arbitrary constants. Show also that this solution can be written in the form
$y = C\ \cos(kx + \alpha) = C\ \mathrm{Re}[e^{j(kx+\alpha)}] = \mathrm{Re}[(Ce^{j\alpha})e^{jkx}]$.
and express C and α as functions of A and B.

P7. (APF 2-1) Express the following in the form $z = \mathrm{Re}[A\exp(j(\omega t + \alpha))]$:

(a) $z = \sin\omega t + \cos\omega t$.

(b) $z = cos(\omega t - \pi/3) - \cos\omega t$.

(c) $z = 2\sin\omega t + 3\cos\omega t$.

(d) $z = \sin\omega t - 2\cos(\omega t - \pi/4) + \cos\omega t$.

P8. (APF 2-2) A particle is simultaneously subjected to three simple harmonic motions, all of the same frequency and in the x direction. If the amplitudes are 0.25, 0.20, and 0.15 mm, respectively, and the phase difference between the first and second is 45°, and between the second and third is 30°, find the amplitude of the resultant displacement and its phase relative to the first (0.25-mm amplitude) component.

P9. (APF 2-3) Two vibrations along the same line are described by the equations
$y_1 = A \cos 10\pi t$
$y_2 = A \cos 12\pi t.$
Find the beat period, and draw a careful sketch of the resultant disturbance over one beat period.

P10. (APF 2-4) Find the frequency of the combined motion of each of the following:

(a) $\sin(2\pi t - \sqrt{2}) + \cos(2\pi t)$.

(b) $\sin(12\pi t) + \cos(13\pi t - \pi/4)$.

(c) $\sin(3t) - \cos(\pi t)$.

P11. (APF 2-5) Two vibrations at right angles to one another are described by the equations
$x = 10 \cos(5\pi t)$
$y = 10 \cos(10\pi t + \pi/3)$
Construct the Lissajous figure of the combined motion.