Birla Institute of Technology and Science Pilani, K K Birla Goa Campus
First Semester 2024-2025 Quiz-1(Open Book)

Course Title: Mathematics-I (MATH F111) Max. Marks: 75
Date: Sept 21, 2024 Duration: 60 min.

INSTRUCTIONS: A (1) Each question is worth 5 marks. **(2)** Each question has 4 alternative answers. Write the correct choice (A, B, C, or D) in the table provided in this question paper using a pen only, and submit it to the invigilator. **(3) Do not encircle (◯)** or tick ✓ your answers on the question itself; this will be considered invalid. **(4)** Answers written in pencil or any overwriting will be treated as invalid. **(5)** No rough work should be done on the question paper.

Student's Name	**ID No.**	**Roll No.**

Write answers in this table														
Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Q11	Q12	Q13	Q14	Q15
B	C	B	A	C		C	C	C	C	D	A	B	D	D

For Evaluator's use only		
No. of Correct Ans	No. of Incorrect Ans	Marks awarded

1. The distance between the points with polar coordinates $(2, \frac{\pi}{3})$ and $(4, \frac{2\pi}{3})$ is
   (A) 5 (B) $\sqrt{12}$ (C) $\sqrt{2}$ (D) None of these.

2. Which of the following represents the area (bounded) inside both the polar curves $r = 3+\cos(\theta)$ and $r = 5 - 3\cos(\theta)$?
   (A) $\int_0^{\pi/3} (3+\cos(\theta))^2 d\theta + \int_{\pi/3}^{\pi} (5-3\cos(\theta))^2 d\theta$ (B) $\int_{-\pi/3}^{\pi/3} \frac{1}{2}(3+\cos(\theta))^2 - (5-3\cos(\theta))^2) d\theta$
   (C) $\int_0^{\pi/3} (5-3\cos(\theta))^2 d\theta + \int_{\pi/3}^{\pi} (3+\cos(\theta))^2 d\theta$ (D) None of these.

3. Let $r = 4(1+\cos(\theta))$, $0 \le \theta \le \pi/2$ be a curve in polar form. Determine the point (r, θ) at which the slope of the tangent line to the curve is -1.
   (A) $(4(1+\sqrt{1/2}), \pi/4)$ (B) $(4(1+\sqrt{3}/2), \pi/6)$ (C) $(0, \pi)$ (D) None of these.

4. The number of points of intersection of the curves $r = 5 - 3\cos(\theta)$ and $r = 1 + \theta$ when $0 \le \theta \le 2\pi$ is
   (A) 1 (B) 2 (C) 3 (D) None of these.

5. Suppose $x_n = \frac{n}{4} - [\frac{n}{4}]$, where $[n]$ is the greatest integer less than or equal to n. Suppose $y_{n+1} = \frac{5y_n}{2+y_n}$ for all $n \in \mathbb{N}$ with $y_1 = 1$. Which of the following is TRUE?
   (A) x_n is convergent and y_n is not convergent (B) Both x_n and y_n are divergent
   (C) x_n is not convergent and y_n is convergent (D) Both x_n and y_n are convergent.

6. For a positive integer n, let a_n, b_n, c_n, d_n be the real numbers such that

$$\begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}^n = \begin{pmatrix} a_n & b_n \\ c_n & d_n \end{pmatrix},$$

   where A^n denotes multiplying the matrix A by itself n times. Then $\lim_{n\to\infty} \frac{a_n}{b_n}$ is
   (A) 1 (B) e (C) $\frac{3}{2}$ (D) Limit doesn't exist.

7. Let (x_n) and (y_n) be sequences of positive real numbers such that $\left(\frac{x_n}{n}\right)$ is monotonically increasing and $(2^n y_n)$ is monotonically decreasing. Then
(A) x_n converges to 0, but y_n is divergent (B) Both x_n and y_n converge to 0
(C) x_n doesn't converge to 0 and y_n converges to 0 (D) None of these.

8. Suppose $A := \sum_{n=2}^{\infty} \frac{1}{n\ln(n)\ln(\ln(n))}$ and $B := \sum_{n=100}^{\infty} \frac{1}{(\ln(n))\sqrt{\ln(n)}}$. Which of the following is true?
(A) Both A and B are convergent (B) A is convergent and B is divergent
(C) Both A and B are divergent (D) A is divergent and B is convergent.

9. Consider the series $\sum_{n=1}^{\infty} \frac{n^d}{c^n}$, and consider the following combinations of c and d values:
(I) $c = 1,\ d = -1$. (II) $c = 2,\ d = 1$. (III) $c = 1,\ d = -2$. Then the series converges for:

(A) I and II (B) I and III (C) II and III (D) None of these.

10. Let $A := \sum_{n=2}^{\infty} \frac{\sin\left((2n+1)\frac{\pi}{2}\right)}{n\ln n}$ and $B := \sum_{n=2}^{\infty} (-1)^n \frac{n!}{(n+2)!}$. Which of the following is true?
(A) Both A and B are absolutely convergent
(B) A is absolutely convergent and B is conditionally convergent
(C) A is conditionally convergent and B is absolutely convergent
(D) Both A and B are conditionally convergent.

11. Find all possible values of x for which the series $\sum_{n=1}^{\infty} \frac{9+x^n}{5^n}$ converges.
(A) It is not possible to find such x because the series diverges. (B) $x > 0$
(C) $|x| < 1$ (D) $-5 < x < 5$

12. Let R be the radius of convergence and IOC be the interval of convergence. Which of the following is true for the power series expansion of the function $\frac{1}{x^2-4x+5}$ around $x = 2$?
(A) R is 1 and IOC is $(1, 3)$ (B) R is 2 and IOC is $[0, 4)$
(C) R is 4 and IOC is $(-2, 6)$ (D) None of these

13. The $\lim_{t\to\infty} \left(\frac{t^2\sin(\frac{1}{t})}{\sqrt{t^2+1}}\hat{\imath} + \sin\left(\frac{\pi}{2}+\frac{1}{t}\right)\hat{\jmath}\right)$ is
(A) $\hat{\jmath}$ (B) $\hat{\imath}+\hat{\jmath}$ (C) Limit doesn't exist (D) None of these.

14. Consider the vector-valued function $r(t) = (2\cos(t) - \cos(2t))\hat{\imath} + (2\sin(t) - \sin(2t)\hat{\jmath} + \hat{k}$ for $0 \le t \le 2\pi$. Then $r(t)$ fails to be smooth at how many points of the domain?

(A) 4 (B) 5 (C) 6 (D) None of these.

15. Let s be the arc length parameter of a curve $r(t)$ and $r(s) = \frac{1}{\sqrt{2}}\sin(s/4)\hat{\imath} + \frac{1}{\sqrt{2}}\cos(s/4)\hat{\jmath} + \left(\frac{s}{a}\right)\hat{k}$ be the arc length parametrization of the curve. Then the value of a can be
(A) $a = \pm 1$ (B) $a = \pm\sqrt{2}$ (C) $a = \pm\sqrt{3}$ (D) None of these.

**********ALL THE BEST **********

Recheck request after exam: