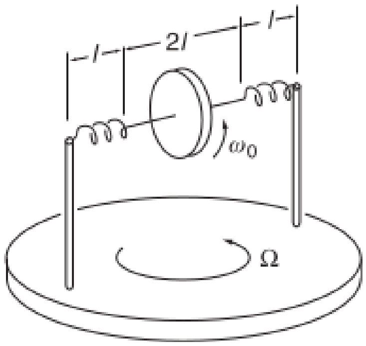
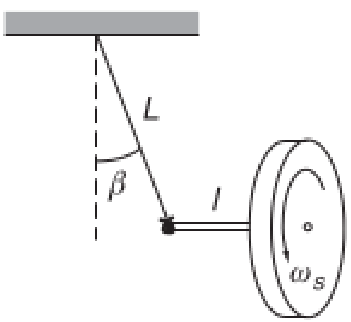
Tutorial 10 **Rigid Body Motion (Gyroscope)** 24 September 2024

P1.



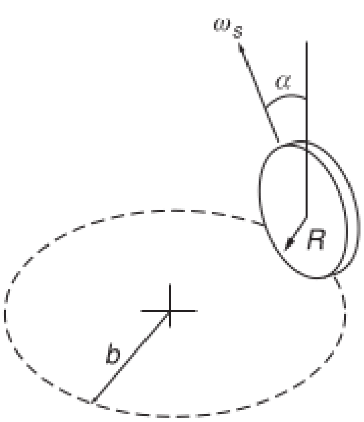
Flywheel on rotating table
A flywheel of moment of inertia I_0 rotates with angular velocity ω_0 at the middle of an axle of length $2l$. Each end of the axle is attached to a support by a spring which is stretched to length l and provides tension T. You may assume that T remains constant for small displacements of the axle. The supports are fixed to a table that rotates at constant angular velocity Ω, where $\Omega << \omega_0$. The center of mass of the flywheel is directly over the center of rotation of the table. Neglect gravity and assume that the motion is completely uniform so that nutational effects are absent. The problem is to find the direction of the axle with respect to a straight line between the supports.

P2.



Suspended gyroscope
A gyroscope wheel is at one end of an axle of length l_a. The other end of the axle is suspended from a string of length l_b. The wheel is set into motion so that it executes uniform precession in the horizontal plane. The wheel has mass M and moment of inertia about its center of mass I_0 . Its spin angular velocity is ω_s . Neglect the masses of the shaft and string.
Find the angle β that the string makes with the vertical. Assume that β is so small that approximations like $\sin\beta \approx \beta$ are justified.

P3.



Rolling coin
A coin of radius b and mass M rolls on a horizontal surface at speed V. If the plane of the coin is vertical the coin rolls in a straight line. If the plane is tilted, the path of the coin is a circle of radius R. Find an expression for the tilt angle of the coin α in terms of the given quantities. (Because of the tilt of the coin the circle traced by its center of mass is slightly smaller than R but you can ignore the difference.)

P4. *Measuring latitude with a gyro*

Latitude can be measured with a gyro composed of a spinning disk mounted to pivot freely on an axis through the plane of the disk, with its axle horizontal and lying along the east–west axis.

(a) Show that the gyro can remain stationary when its spin axis is parallel to the polar axis and is at the latitude angle λ with the horizontal.

(b) If the gyro is released with the spin axis at a small angle to the polar axis show that the gyro spin axis will oscillate about the polar axis with a frequency $\omega_{osc} = \sqrt{I_1 \omega_s \Omega_e / I_\perp}$, where I_1 is the moment of inertia of the gyro about its spin axis, $I_\perp$ is its moment of inertia about the fixed horizontal axis, and Ω_e is the Earth's rotational angular velocity.

What value of ω_{osc} is expected for a gyro rotating at 40000 rpm, assuming that it is a thin disk and that the mounting frame makes no contribution to the moment of inertia?