MATHEMATICS-I

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Department of Mathematics

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August 2, 2024

Text Book:

- Thomas' Calculus by M.D. Weir, J. Hass and F.R. Giordano, Pearson Education 12th Edition, 2015/16.

References:

1. Essential Calculus Early Transcendentals by J. Stewart, Thomson Learning, 2014.

2. A First Course in Calculus by Searge Lang, Springer-Verlag 5th Edition, 2009.

3. Advanced Engineering Mathematics by Erwin Kreyszig, Wiley 10h edition, 2015.

4. Calculus Vol 1 & 2, by T M Apostol, 2nd edition, 2007.

5. Basic Multivariable Calculus by Jerrold E. Marsden, Anthony Tromba, Alan Weinstein, 3rd edition, 1993.

Teachers:

1. **Prof. Saranya Nair** (IC)

2. **Prof. Anushaya Mohapatra** (A-404)

3. **Prof. Gunja Sachdev**

4. **Prof. Amit Setia**

5. **Prof. Prasanna Kumar**

6. **Prof. Shilpa Gondhali**

7. **Prof. Mukesh Kumar Nagar**

Evaluation Scheme:

	Components	Duration	Marks	Nature
1.	Mid-term	1Hr. 30 Min.	105	CB
2.	Compre.	3 Hrs.	120	CB
3.	Quizzes: AQ1 and AQ2	1 Hrs.	75	OB

- Chamber consultation hour: TBA.
  email ID: anushayam@goa.bits-pilani.ac.in
- Make-up Policy: Make-up will be given only for very genuine cases and prior permission has to be obtained from Instruction Division and I/C.
- Notices: All notices regarding the course MATH F111 will be displayed on online course platform; QUANTA.
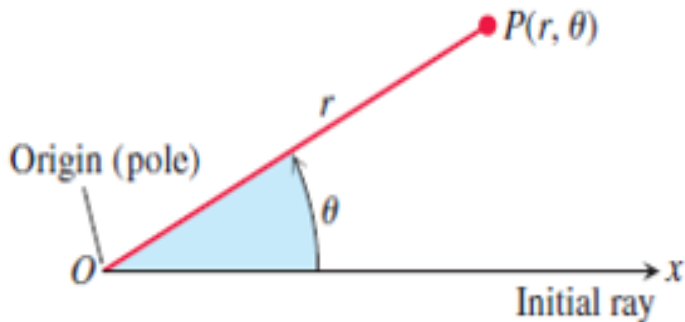
Lecture 1

Polar Coordinates

Text book chapter: 11.3

Polar coordinates

To define polar coordinates for a point, fix an origin O, called the pole, and an initial ray from O. (initial ray is called polar axis).(r, θ) represent the polar coordinate of the point P, where r is the directed distance from O to P and θ is the directed angle from the initial ray to OP.

Polar Coordinates

Some of the common fields that have the application of polar coordinates are:

- Navigation
- Quantum Dynamics
- Biomedical Imaging
- Sonar and Radar Systems

Conventions

- **Positive** θ: It is measured in the counterclockwise direction from the polar axis.

- **Negative** θ: It is measured in the clockwise direction from the polar axis.

- **Positive r**: It is measured in the same direction of the initial ray OP form the origin O.

- **Negative r**: It is measured in the reverse direction of the initial ray OP from the origin O.
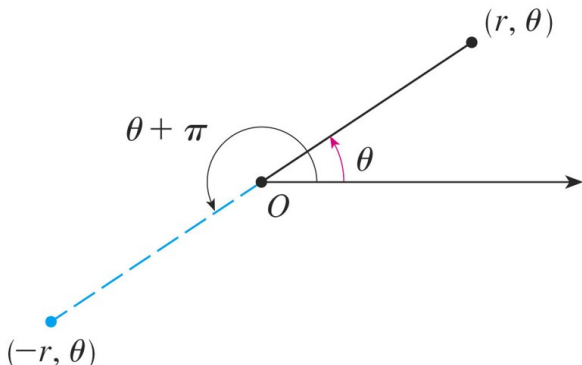
Polar coordinates

- If $P = 0$, then $r = 0$, the point (r, θ) represents the the pole for any value of θ.
- The points (r, θ) and $(-r, \theta)$ lie on the same line through O and at the same distance from O but on opposite sides of O.
- Note that $(-r, \theta)$ represent the same point as $(r, \theta + \pi)$.
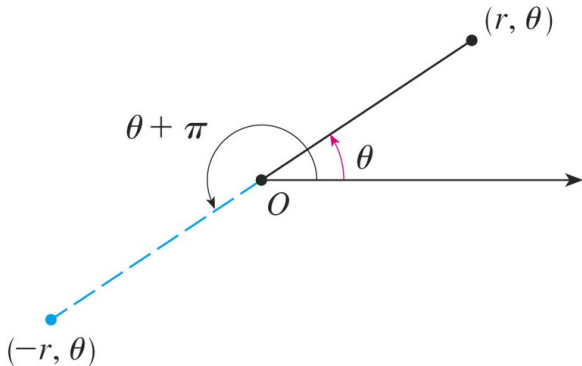
Polar coordinates

- If $r > 0$, the point (r, θ) lies in the same quadrant as θ.
- If $r < 0$, the point (r, θ) lies on the opposite side of the pole.

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Examples

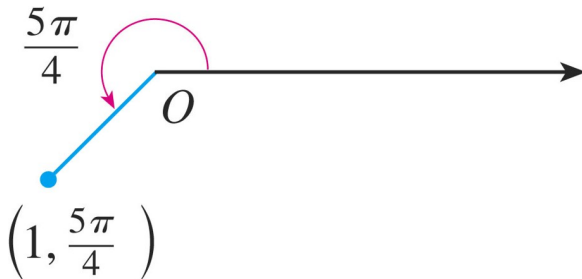**Plot the points whose polar coordinates are given**.
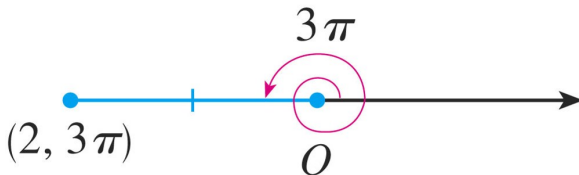
  a. $(1, 5\pi/4)$

  b. $(2, 3\pi)$

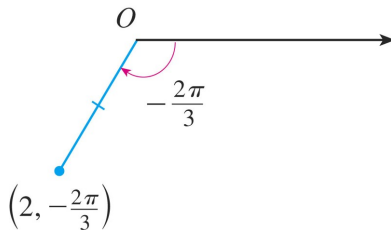 *c.* $(2, -2\pi/3)$

  d. $(-3, 3\pi/4)$

Examples

$$\frac{5\pi}{4}$$

O

$$\left(1, \frac{5\pi}{4}\right)$$

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$$3\pi$$

$(2, 3\pi)$

O

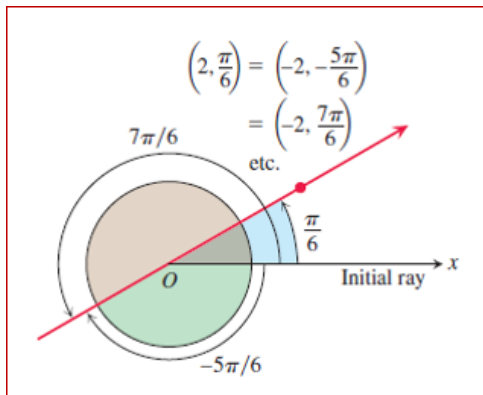© Thomson Higher Education

Examples
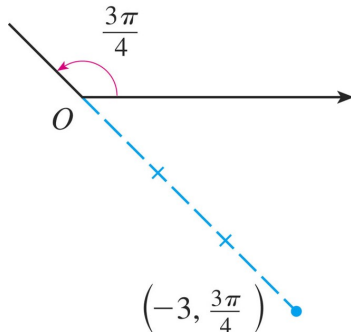
- In cartesian system, each point has only one representation, however in polar system each point has many representation.

$$\left(2, \frac{\pi}{6}\right) = \left(-2, -\frac{5\pi}{6}\right)$$
$$= \left(-2, \frac{7\pi}{6}\right)$$
etc.

$7\pi/6$

$\frac{\pi}{6}$
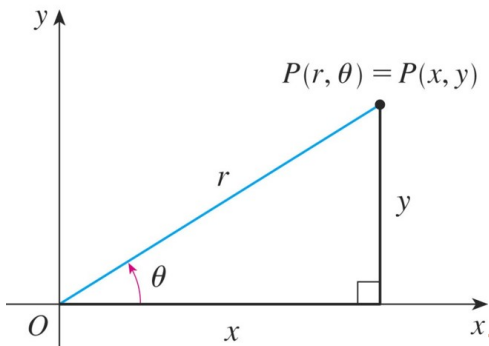
Initial ray

x

O

$-5\pi/6$

Cartesian and Polar

If the point P has Cartesian coordinates (x, y) and polar coordinates (r, θ), then we have

$$x = r\cos(\theta)$$
$$y = r\sin(\theta)$$

$$r^2 = x^2 + y^2, \tan\theta = \frac{y}{x}$$

Example-1

Convert the point $(2, \pi/3)$ from polar to Cartesian coordinates:

$$x = r\cos\theta = 2\cos\frac{\pi}{3} = 2 \cdot \frac{1}{2} = 1$$

$$y = r\sin\theta = 2\sin\frac{\pi}{3} = 2 \cdot \frac{\sqrt{3}}{2} = \sqrt{3}$$

So $(1, \sqrt{3})$ is the cartesian coordinate.

Example-2

1. Represent the point with Cartesian coordinates $(-1, -1)$ in terms of polar coordinates.

$$r = \sqrt{x^2 + y^2} = \sqrt{2}$$

$tan(\theta) = \frac{y}{x} = 1$, So one possibility is $\theta = \frac{\pi}{4}$.
Thus, one possible answer is: $(\sqrt{2}, \pi/4)$. Is the answer correct ?