Deteriorating Market Conditions and Cartel Formation under

Manager Loss Aversion

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Abstract

Cartels often form following a deterioration in market conditions, such as a reduction in demand, an increase in costs or the entry of a new competitor. However, in conventional theoretical models of collusion, such deteriorations do not facilitate collusion and can even reduce incentives to collude. I show how deteriorations in market conditions can facilitate collusion and cause the formation of cartels when, contrary to standard models, colluding managers are averse to losses. Additionally, when managers are loss averse, a deterioration in market conditions can increase the maximum possible collusive payoff and

permit colluding managers to set higher prices than would otherwise be sustainable.

Keywords: collusion, loss aversion, prospect theory, managerial incentives

JEL Codes: D91, L41, L21

1 Introduction

Cartels often form after a deterioration in market conditions, such as a reduction in demand, an increase in marginal cost or the entry of a new competitor (see Aston and Pressey (2012), Herold and Paha (2018), and empirical findings presented in Section 2). However, under traditional theoretical models of collusion, the entry of a new competitor reduces incentives to collude. Additionally, permanent reductions in demand and increases in marginal cost do not impact the sustainability of collusion and, under certain conditions, can even reduce managers' incentives to collude (see Section 3 for additional discussion). Crucially, standard models of collusion assume that colluding managers are loss neutral (i.e., they treat gains and losses in utility symmetrically and do not perceive losses as more severe than an equivalent gain). In practice, managers may be averse to losses both due to a psychological aversion to losses in utility (Kahneman and Tversky, 1979) and because of negative career consequences (e.g., termination or the loss of a bonus) that may result from sub-par performance. I present a model of collusion between loss averse managers and find that, consistent with empirical evidence, deteriorations in market state can cause the formation of cartels¹ in previously competitive markets, increase the payoff managers earn from collusion, and result in higher prices. Intuitively, managers perceive continued competition as a loss after the market state deteriorates, which causes managers to collude (and refrain from cheating on collusion) in order to avoid painful losses in utility which would occur under competitive play.

Formally, I study a dynamic game wherein symmetric managers choose a strategic variable (e.g., prices, output levels or any other profit-relevant choice variable) in each period, and generate profits for their respective firms. Manager utility depends on the profits they generate and the manager's reference point. A reference point is a threshold utility level for which any utilities exceeding the reference point are perceived as gains and any utilities below the reference point are perceived as losses.² A manager's reference point is determined by past utility levels and evolves over time in response to experienced utilities. When colluding, managers choose, in each period, the strategic variable in order to maximize their utilities subject to the constraint that no manager wishes to defect from collusion (i.e., collusion must be incentive compatible). There are two regimes. The first regime (regime a) represents the market environment following a sudden deterioration in the profitability of the industry for all firms due to, for example, a reduction in demand. The second regime (regime b) represents the market environment absent any change in market conditions.

¹I refer to a group of colluding managers as a cartel throughout the analysis for ease of exposition. However, the model does not require collusion to be explicit/illegal. Collusion in the presence of an antitrust authority that can detect and penalize cartels is explored in Appendix D.

²Managerial loss aversion in the present study may take either of two forms. First, loss aversion may represent an inherent cognitive bias wherein losses loom larger than gains (termed psychological loss aversion). Second, if managers are evaluated relative to a target level of performance (e.g., a target profit level), then corporate managers may be averse to losses due to the likelihood of negative career consequences if performance targets are not met (termed target-based loss aversion).

To understand why a deterioration in market conditions may cause the formation of a cartel, consider a competitive industry consisting of loss averse managers. Over time, these managers become accustomed to competitive profit levels. Thus, when market conditions unexpectedly deteriorate in a way that reduces the amount of profit that can be earned from competitive play, managers perceive continued competition as a loss in utility. This effect causes managers to turn to collusion for two reasons. First, collusion, which raises profits above Nash equilibrium levels, avoids a painful loss in utility. Second, a deterioration in market conditions relaxes the incentive compatibility constraints necessary for successful collusion. To see this, suppose a cartel forms following a deterioration in market conditions and consider a manager who subsequently cheats on the collusive agreement. After the manager cheats, collusion breaks down and the market returns to competitive play. However, as competitive profit levels (after the deterioration in market state) are perceived a loss, the manager experiences a painful loss in utility when the cartel dissolves and competition resumes. As a result, managers have relatively weak incentives to cheat on the cartel as doing so would cause a breakdown in collusion and significant losses in utility.

As well as causing the formation of cartels, deteriorations in market conditions can, due to the considerations outlined above, also increase the collusive payoff (i.e., the discounted present value of utility from collusion) and the gain in utility managers experience from collusion. This is the case when managers are sufficiently loss averse, the deterioration in market state is not drastic and managers' discount factor is sufficiently small. Additionally, by enhancing the stability of collusion, a deterioration in market state can allow a cartel to set higher prices than would otherwise be incentive compatible/sustainable, particularly in early periods of collusion.

Shalev (1998) and Bernard (2011) both study repeated games with loss averse agents. However, neither study analyzes the impact of deteriorations in market conditions on incentives to collude when players are loss averse. Rotemberg and Saloner (1986) and Green and Porter (1984) study the impact of shocks to profit/demand, that last one period, on firms' ability to collude. Motivated by empirical evidence suggesting that many cartels form following a permanent (or long lasting) change in market conditions rather than a transitory shock,³ I analyze the impact of permanent changes in market state rather than temporary profit shocks.⁴

³Herold and Paha (2018) find that, among the cartels in their sample that formed in response to a reduction in demand, the decline was regarded as permanent in 80% of the cases with available information. Paha (2017) writes that "evidence suggests some cartels were established after a firm's profits had declined and were unlikely to recover in the near future." Paha (2017) also notes that "[e]vidence of cartels that were formed in times of volatile demand or following temporary shocks is much harder to find." For example, Grout and Sonderegger (2005) argue that the graphite electrodes cartel formed after technological innovation in the steelmaking industry (the primary purchaser of graphite electrodes) substantially reduced demand for graphite electrodes. Reductions in demand driven by improvements in technology of this kind are likely permanent. When the deterioration in market conditions is caused by the entry of a new firm, incumbent managers likely perceive this deterioration as permanent (unless there is reason to believe the entrant does not intend to remain in the market in the long term).

⁴Subsequent studies have also explored, among other topics, serially correlated demand shocks (Kandori, 1991), stochastic transitions between boom and bust periods (Bagwell and Staiger, 1997) and cyclic demand changes (Haltiwanger and Harrington Jr, 1991). The present study differs from these analyses for two primary reasons. First, unlike prior literature, the current

Most closely related to the current study is a series of insightful articles by Spagnolo (1999; 2000; 2005) who explores the impact of managerial incentives and compensation structures on managers' ability to collude. Spagnolo (2005) shows how collusion is easier to sustain when manager's have a preference for a smooth stream of income. Preferences for smooth income could be the result of a psychological aversion to losses or a tendency to evaluate managerial performance relative to past levels. However, unlike the present study, Spagnolo (2005)'s model is not tailored to the analysis of loss aversion in particular and is instead designed to capture the general impact of managers' aversion to profit variance. Additionally, Spagnolo (2005) does not analyze permanent deteriorations in market conditions.⁵

Section 2 presents empirical evidence indicating that cartels often form following a deterioration in market conditions. Section 3 introduces the model. Cartel formation is analyzed in Section 4. The value of/gain from collusion is studied in Section 5. Section 6 presents results related to cartel pricing, and Section 7 concludes. The appendix contains proofs, as well as additional derivations, discussion, and simulation results.

2 Empirical Evidence

European commission decisions in cartel cases typically contain detailed information regarding the relevant industry, the cartel's practices and structure, and the origins of the infringement. Within the decision's discussion of the cartel, the commission often indicates any changes in the market environment which may have instigated initial meetings between cartel members. For example, the EC decision in the calcium carbide cartel⁶ states: "[s]ince the beginning of the 21st century the price of calcium carbide powder for the steel industry has been under pressure, while costs increased and demand shrunk. These developments formed the basis for the meetings between the main European suppliers of calcium carbide powder." In other cases, the decisions explicitly state the producers' motivations for price fixing. For example, the EC decision in the professional videotapes cartel⁷ states: "the reasons for participation in the arrangements leading to the first two price increases (which were agreed on 1 September 1999 and 20 April 2000) comprised the following: i) the weakness of the Japanese Yen against the Deutsche Mark ... and ii) the fact that the prices ... were comparatively low."

To examine which specific changes in market conditions lead to the formation of a cartel, I review available European Commission decisions in cartel cases and record the purported causes each cartel's formation.⁸

study examines collusion between loss averse agents. Second, the current study focuses on a change in the market environment which is permanent (or long lasting) and not a part of a regular cycle or stochastic fluctuation.

⁵Spagnolo (2005) does analyze firms' ability collude under demand shocks/fluctuations in the spirit of Rotemberg and Saloner (1986).

 $^{^6}$ COMP/39.396 – Calcium carbide and magnesium based reagents for the steel and gas industries, 7/22/2009, Commission Decision (¶54).

⁷Case COMP/38.432 – Professional Videotape, 11/20/2007, Commission Decision (¶58).

⁸European Commission decisions are available at https://competition-policy.ec.europa.eu/index en. I specifically analyze

Specifically, I search for changes in market environment that are alleged to have caused or led producers to engage in price fixing. I restrict attention to cases with an available prohibition decision (in english) that was originally published between 1998 and 2024, which results in 90 cartel cases. Table 1 presents results. Panel A of Table 1 reports that, of the 92 decisions reviewed, 45 did not mention a clear cause of cartel formation. This is consistent with Herold and Paha (2018) who also review EC decisions and find that approximately half of EC decisions do not contain information about events that preceded the first cartel meetings. I record any causes of cartel formation indicated within the decisions. Thus, there are multiple causes for certain cartels. 35 cartels have a single cause while 12 have more than one cause.

Cartel members often argue that their industry was in crisis at the time of the cartel's formation in order to request a reduction in the fine imposed on them by the European Commission. In 30% of cases, either the infringing firms or the European commission contended that the industry was in a state of crisis around the time of the cartel's formation. Panel B of Table 1 presents results pertaining to improvements in market conditions.¹⁰ The most common improvement in market conditions causing a cartel's formation is a demand increase, which is cited as a cause in 3 cases (6% of all cases with at least one available cause). An improvement in market conditions of some kind (e.g., cost decrease, demand increase or the exit of a competitor/consolidation in the industry) is cited as a factor causing the cartel's formation in 5 cases, constituting 11% of all cases for which at least one cause is available. Panel C of Table 1 presents results pertaining to deteriorations in market conditions. Five different types of deteriorations in market conditions were cited within the decisions including an increase in cost, a decrease in demand, the entry of a new competitor, an increase in buyer power, and an increase in import competition. The most common deterioration in market conditions was a cost increase which was cited as a factor contributing to cartel formation in 12 cases (26% of all cases with at least one cause). For example, European stainless steel producers colluded to impose an alloy surcharge after the price of nickel (a key input in stainless steel production) increased in 1993. 11 Air cargo airlines formed a cartel in the early 2000s in the face of rising jet fuel prices.¹² The second most common deterioration in market conditions was the entry of a new competitor

prohibition decisions. While the US Department of Justice publishes brief press releases after a cartel member's plea or sentencing, these press releases are typically very brief and do not include detailed information about the cartel (Harrington, 2006). By contrast, European Commission decisions are highly detailed and can be hundreds of pages long.

⁹This information is typically contained in the section of the decision pertaining to the origin of the cartel, the cartel's history or a section titled "Description of the Events." I record only changes in market environment which the commission decision indicates were relevant for the cartel's formation (i.e., instigated its formation in some way). Often, when reviewing the relevant industry, the decisions discuss general trends in the market environment. For example, the EC decision in the industrial bags cartel (Case COMP/38354 – Industrial bags, 11/30/2005.) states that "[d]uring this period demand for industrial bags as a whole has stagnated or even declined." Unless a change in market conditions is in some way linked to the cartel's formation or the beginning of meetings between producers, this change in market conditions is not recorded.

¹⁰I do not attempt to distinguish between sudden and gradual changes in, for example, demand. The wording in the EC decisions typically did not clearly indicate whether changes in market conditions were abrupt or more gradual. Gradual deteriorations in market conditions are analyzed formally in Appendix E. Additionally, I do not attempt to determine if a change in market conditions was expected to be permanent or transitory.

 $^{^{11}\}mathrm{Case}\ \mathrm{IV}/35.814$ – Alloy surcharge, 1/21/1998, Commission Decision.

 $^{^{12}\}mathrm{COMP}/39258$ – Airfreight , 11/9/2010, Commission Decision.

which was cited as a factor contributing to cartel formation in 8 cases (17% of all cases with at least one cause). In total, at least one deterioration in market conditions was cited as a cause of cartel formation in 27 cases (57% of all cases with at least one cause). Thus, deteriorations in market conditions are significantly more likely to be cited as a cause of cartel formation than an improvement in market conditions. Panel D presents results pertaining to other causes of cartel formation including overcapacity (7 cases), ¹³ a regulatory policy change (3 cases), a price war (7 cases), ¹⁴ and low prices (9 cases). ¹⁵

In summary, results suggest that deteriorations in market state are significantly more likely to be cited as the cause of a cartel's formation than an improvement in market conditions. This finding is consistent with prior literature (Herold and Paha, 2018; Grout and Sonderegger, 2005, 2007; Aston and Pressey, 2012; Levenstein and Suslow, 2015) which has previously identified various deteriorations in market state as potential causes of cartel formation. Most closely related to the analysis in this section is Herold and Paha (2018) who also review EC cartel decisions, augmenting their review with information from external sources, and record events contributing to a cartel's formation. They find that events such as a reduction in demand or the entry of a new competitor often precede a cartel's formation. For example, Herold and Paha (2018) find that 16 of the 41 cartels included in their study were formed in response to declining demand while only 8 of 41 were formed after increases in demand. Aston and Pressey (2012) also review EU cartel cases and find similar results. Specifically, they find that market conditions declined or were declining prior to cartel formation in 24 cases while conditions were improving in only 12 cases. Reflecting this pattern, the European Commission frequently notes within its cartel decisions that "[a]s a general rule, cartels come into being when a sector encounters problems." 17

There are two important caveats to the preceding analysis. First, as mentioned previously, firms anticipating a fine for price fixing activity have an incentive to argue that their industry is/was in a state of turmoil or crisis in order to receive a fine reduction from the European Commission. The Commission has, in some cases, considered an industry's poor financial state as an extenuating circumstance when setting fines. Particularly, the Commission may reduce the fines imposed on a price fixer if it believes the firm is

¹³The emergence of overcapacity could also be classified as a deterioration in market conditions. However, without additional information regarding the cause of excess capacity in the industry, it is difficult to determine if overcapacity should be categorized as a deterioration or improvement in market conditions. For example, overcapacity may be the result of reduced output levels caused by relaxed competition between firms (which could be considered an improvement in market conditions).

¹⁴When low prices are discussed without reference to a price war, the cause is recorded as low prices. When a price war is explicitly discussed as a cause of the cartel's formation, I record the cause as a price war and not as low prices (recognizing that price wars, by definition, result in relatively low prices).

¹⁵Low prices may also represent a deterioration in market conditions in some cases. However, low prices could also be the result of an improvement in market conditions (a technological advance or reduction in input prices).

¹⁶I find, in Table 1, a higher likelihood of a cartel being caused by a deterioration in market conditions than prior literature. This difference is likely caused by my focus on changes in market conditions that the EC decision explicitly links to the formation of the cartel, rather than any changes in market environment preceding the cartel's formation. Additionally, Herold and Paha (2018) do not record changes in marginal cost which Table 1 reveals are a frequent deterioration in market conditions in my sample.

¹⁷See, for example, COMP/E-1/38.069 – Copper Plumbing Tubes, 9/3/2004, Commission Decision (¶742).

unable to pay due to economic or financial hardship. ¹⁸ As a result, firms may exaggerate problems in their industry, and de-emphasize prosperous market conditions, when arguing their case with the Commission. If this occurs, the above results may represent an overestimate of the frequency of deteriorations in market conditions as a cause of cartel formation. Second, as with any empirical analysis of illegal cartels, the above sample includes only detected cartels and is therefore not necessarily representative of the entire population of detected and undetected cartels. If cartels forming after a deterioration in market state are more likely to be detected and penalized, then the above results may overstate the prevalence of deteriorations in market state as causes of cartel formation.

 $^{^{18}\}mathrm{See},$ for example, the animal feed phosphates cartel (COMP/38866 – Animal Feed Phosphates, 7/20/2010, Commission Decision ($\P238$)).

Table 1: Causes of Cartel Formation from EC Decisions

	Num Cases	% of Cases	% of Cases w/ Causes
Panel A: Data Collection Statistics			
Total Num. Cases	92	100%	
No Cause Cited	45	49%	
At Least One Cause Cited	47	51%	
One Cause Cited	35	38%	
More than One Cause Cited	12	13%	
Crisis Mentioned	28	30%	
Panel B: Improvements in Market Cond.			
Cost Dec.	1	1%	2%
Demand Inc.	3	3%	6%
Exit	1	1%	2%
Improvement Cited	5	5%	11%
Panel C: Deteriorations in Market Cond.			
Cost Inc.	12	13%	26%
Demand Dec.	6	7%	13%
Entry	8	9%	17%
Inc. Buyer Power	5	5%	11%
Inc. Import Comp.	2	2%	4%
Deterioration Cited	27	29%	57%
Panel D: Other Causes			
Overcapacity	7	8%	15%
Regulatory Policy Change	3	3%	6%
Price War	7	8%	15%
Low Prices	9	10%	19%

Notes: This table presents results from a review of European Commissions cartel decisions since 1998.

3 Model

The empirical patterns identified in the preceding section suggest that deteriorations in market conditions, such as a reduction in demand, an increase in marginal cost or the entry of a new competitor, may cause the formation of a cartel. However, in conventional theoretical models of collusion, these changes typically hinder or have no impact on the sustainability of collusion. 19 In standard models of collusion, the entry of a new competitor reduces the sustainability of collusion (i.e., the critical discount factor is increasing in the number of firms (Ivaldi et al., 2003)). An increase in marginal cost or a reduction in demand typically does not impact the sustainability of collusion. As Klein and Schinkel (2019) note, "[i]n standard cartel theory, the overall level of marginal cost is immaterial to cartel stability, because common changes in marginal cost do not affect the classic critical discount factor(s)." Similarly, Gallice (2010) writes "[a]ccording to standard IO models, the parameters that characterize market demand (intercept, slope, and elasticity) and technology (the level of symmetric marginal costs) do not play any role in defining the sustainability of collusive behaviors." For example, the conventional critical discount factor in a Cournot market with N symmetric firms, constant marginal cost, linear demand, grim trigger strategies and monopoly pricing during collusion, is $\frac{(N+1)^2}{N^2+6N+1}$ which is increasing in the number of firms and does not depend on the demand intercept or the level of marginal cost. The corresponding discount factor for a homogenous product Bertrand market is $\frac{N-1}{N}$ which is also increasing in the number of firms and independent of the level of demand or marginal cost. When there are fixed costs of collusion, an increase in marginal cost or a reduction in demand can reduce the sustainability of collusion. This is the case as an increase in marginal cost or a decrease in demand reduces firms' variable profits and limits their ability to pay required fixed costs of collusion (see Appendix F.2). This result is consistent with Klein and Schinkel (2019) who find that the scope for collusion is generally declining in marginal cost when firms require a margin before colluding.

Thus, standard models of collusion cannot explain the empirical pattern identified in Section 2 (i.e., that cartels tend to form after increases in marginal cost, reductions in demand and the entry of a new competitor).²⁰ In this section, I present a model which illustrates how accounting for colluding managers' aversion to losses can explain this puzzle. Specifically, the following model demonstrates why deteriorations in market conditions (including those identified Section 2) can cause the formation of a cartel.

¹⁹Deteriorations in market conditions often reduce the output of each firm which generates idle, excess capacity. Thus, deteriorations in market conditions may facilitate collusion through the generation of excess capacity. However, prior literature has found that excess capacity does not necessarily have a pro-collusive effect (Brock and Scheinkman, 1985; Staiger and Wolak, 1992). This is the case as, while excess capacity enhances firms' ability to punish deviations, it also increases the potential gain from cheating on the cartel.

²⁰There are non-standard models in which the level of marginal cost or demand can impact the sustainability of collusion (e.g., Klein and Schinkel, 2019; Gallice, 2010; Lambertini and Sasaki, 2001). However, these models, at times, require specialized assumptions (e.g., discrete prices) and do not provide a general framework under which deteriorations in market conditions facilitate collusion.

Consider a market consisting of N symmetric firms interacting in each of infinitely many periods indexed by t=1,2,3... Each firm employs a manager responsible for overseeing its operations in the market. Specifically, in every period t, the manager chooses a strategic variable x_t (e.g., price, output level, or product quality) on behalf of the firm. Additionally, the manager's performance is evaluated on the basis of how much profit the firm earns in the market. For ease of exposition, I refer to the strategic variable x as the firm's price throughout the ensuing analysis, recognizing that x could represent any variable chosen by managers that can impact a firm's profit. Similarly, I refer to a sequence $\{x_t\}_{t=1}^{\infty}$ as a price path. $x_t \in \Omega$ for all $t \in \{1, 2...\}$ where $\Omega \subset \mathbb{R}$ is a compact set. Managers have a common discount factor $\delta \in (0, 1)$ and seek to maximize the discounted present value of their utility.

A manager's utility consists of two components: base utility and a separate term capturing managers' aversion to losses. A manager's base utility is denoted $u(\pi)$ where π is the profit a manager generates for the firm (through the manager's choice of the strategic variable x) in a given period. The following assumption governs $u(\pi)$.

Assumption 1. $u(\pi): \mathbb{R} \to \mathbb{R}$ satisfies the following assumptions:

- i) $u(\pi)$ is continuous for all $\pi \in \mathbb{R}$, and
- ii) $u(\pi)$ is strictly increasing for all $\pi \in \mathbb{R}$.

A manager's base utility depends on the level of profit that the manager generates for the firm because a manager's performance is evaluated, explicitly or implicitly, on the basis of profit. Managerial compensation structures may explicitly tie a manager's wage to the level of profit the manager generates. For example, a manager may receive a bonus proportional to the amount of profit their division generates over a certain period. Managers (who were tasked with setting prices) in the folding carton cartel received a base salary and a commission based on profits and volume (Sonnenfeld and Lawrence, 1978).²¹ An FW Cook report²² found that 91% of companies tie executive compensation to profits. Even if a manager's pay is not explicitly linked to profits, manager utility is likely increasing in profit due to other considerations. For example, managers that who generate large profits are more likely to be promoted and less likely to be terminated. Additionally, high performing managers may enjoy a higher status within the firm and profession. Reflecting these considerations, Assumption 1(ii) ensures $u(\pi)$ is strictly increasing in firm profit.

Managers are averse to losses. Loss aversion is when an individual is more sensitive to losses than to gains of an equivalent magnitude (Barberis, 2013). For example, a loss averse individual's disutility from losing \$1000 is greater than their utility from gaining \$1000. Managers may display a distaste for losses due to a

²¹Similarly, a manager's compensation may depend on the firm's overall performance (e.g., through stock options).

²²See 2019 Annual Incentive Plan Report. FW Cook. October 2019. https://www.fwcook.com/content/documents/publications/10-17-19 FWC 2019 Incentive Plan.pdf.

psychological aversion to losses (termed psychological loss aversion) or due to the negative career and wage consequences of a drop in performance below expected or target levels (termed target-based loss aversion). First, consider psychological loss aversion. Psychological loss aversion is an intrinsic cognitive aversion to utility levels below a pre-determined reference point. A vast experimental and empirical literature²³ has found evidence of loss aversion. One such experiment (Kahneman, Knetsch and Thaler, 1990) randomly assigned participants to act as sellers or buyers. Sellers were given a coffee mug and asked how much money they would accept in order to give up the mug. Buyers were not given a mug and asked how much they would pay for the mug. The minimum price sellers would accept to part with the mug exceeded the maximum price buyers would pay to receive the mug. Sellers, who received a mug, evaluated parting with the mug as a loss while buyers evaluated acquiring the mug as a gain. Participants, exhibiting loss aversion, required a greater amount of compensation to incur a loss (giving up the mug) than they would pay for an equivalent gain (receiving the mug).²⁴

While manager loss aversion may be the result of an inherent, psychological distaste for losses, as discussed in the previous paragraph, managers may also display an aversion to losses in profit or utility due to more practical considerations. Specifically, managers may be averse to losses if there are negative career or compensational consequences of performance below a predetermined target level (Sullivan and Kida, 1995; Crum, Laughhunn and Payne, 1981), termed target-based loss aversion.²⁵ As Sullivan and Kida (1995) write "[c]orporate managers typically operate in a decision environment that utilizes targets to both motivate and reward managerial performance." Exceeding or failing to meet a target performance level may determine whether a manager receives a bonus (Healy, 1985; Ely, 1991; Willman et al., 2002),²⁶ is awarded a promotion, or looses their job (Sullivan and Kida, 1995; Merchant and Manzoni, 1989). The inability to meet performance targets may also harm a manager's reputation and social status within the firm. "Managers who miss targets suffer credibility losses that harm promotion possibilities, chances for good salary increases, and ability to sell their ideas and to have resources allocated to their [profit centers]" (Merchant and Manzoni, 1989).²⁷ If managers incur negative consequences for failing to meet or exceed performance targets, then managers incur a pronounced loss in utility when performing below perceived target levels. Note that, unlike psychological

²³For reviews, see Novemsky and Kahneman (2005), Zank (2010), and Tversky and Kahneman (1991).

²⁴While loss aversion is often documented experimentally in laboratory studies involving college students, more experienced market participants, such as managers, are not immune to loss aversion. Prior literature has found that experienced corporate managers, in particular, exhibit loss aversion (e.g., executives in manufacturing companies (Sullivan and Kida, 1995), major league baseball managers (Pedace and Smith, 2013), professional futures and options pit traders (Haigh and List, 2005), managers in investment banks (Willman et al., 2002)). Additionally, loss aversion has been found to be important in competitive environments (Gill and Prowse, 2012) and strategic games (Feltovich, 2011).

²⁵Performance targets may be set by supervisors or shareholders. Additionally, performance targets may be based on past performance or the performance of comparable firms in other industries/markets.

²⁶Willman et al. (2002) interview traders at investment banks and conclude that "some traders described the bonus target as the reference point."

²⁷For example, division managers in the folding carton cartel evaluated the performance of lower-level employees (in charge of setting prices), not only for the purposes of bonuses but also for advancement within the firm, on the basis of profit (Sonnenfeld and Lawrence, 1978).

loss aversion, target-based loss aversion is not driven by an innate cognitive bias. Target-based loss aversion is driven entirely by the rational anticipation of losses in income, status or employment that result from performance below target levels. The utility functions employed in this study are consistent with both psychological and target-based loss aversion.²⁸

Whether a particular utility level is perceived as a loss or a gain depends on a manager's reference point. Utility levels above (below) the reference point are perceived as gains (losses) by managers. The formation and evolution of reference points over time will be discussed in greater detail later in this section. When a manager earns a profit of π , their utility is $u(\pi;r,l)$ where r denotes the reference point and l denotes the degree of loss aversion. Larger values of l are associated with a stronger distaste for losses. l=0 corresponds to a loss neutral manager who does not display an aversion to losses. Following Shalev (2000) and Tversky and Kahneman (1991), manager utility satisfies the following assumption.

Assumption 2. $u(\pi; r, l) \equiv u(\pi) - lL(r - u(\pi))$ where

- i) $l \geq 0$, and
- ii) $L(x): \mathbb{R} \to [0,\infty)$ is continuous, strictly increasing for x>0, and satisfies L(x)=0 for $x\leq 0$.

Assumption 2 permits a wide range of utility specifications including utility functions with a kink at the reference point²⁹ and S-shaped utility functions.³⁰ For example, $u(\pi) = \pi$ and L(x) = x result in linear loss aversion, also known as a kinked linear utility function (Shalev, 2000; Maggi, 2004). If $u(\pi) = \pi$ and L(x) is concave, then $u(\pi; r, l)$ is convex over losses.

Reference points divide utility levels into gains and losses. Reference points are defined differently under psychological and target-based loss aversion. First, consider psychological loss aversion. Reference points under psychological loss aversion are assumed to be backward-looking.³¹ Backward-looking reference points, or "classical" reference points, are based on past utility levels. 32 Intuitively, a manager who experiences a particular utility level over a significant length of time is likely to become accustomed to this utility level and perceive any greater (lower) utility levels as a gain (loss). Thus, past utility levels can determine an individual's reference point. Following this approach, Shalev (1998) assumes an individual's reference point

²⁸In Appendix C.6, I develop a two models capturing target-based loss aversion: a model wherein a bonus is awarded if a manager meets a pre-defined performance target and a model wherein a manager may be terminated if their performance fails to meet a target. These models illustrate how, under certain circumstances, assessing manager performance relative to a pre-defined target generates a loss averse utility function satisfying Assumption 2. Thus, the utility functions in the main text may represent either psychological or target-based loss aversion.

²⁹Let π^* satisfy $u(\pi^*) = r$. If $u(\pi; r, l)$ and $u(\pi)$ are differentiable in π and L is differentiable, then $\frac{\partial^+ u(\pi^*; r, l)}{\partial \pi} = u'(\pi^*)$ and $\frac{\partial^- u(\pi^*; r, l)}{\partial \pi} = (1 + lL'(0)) u'(\pi^*)$. Thus, $\frac{\partial^- u(\pi^*; r, l)}{\partial \pi} \neq \frac{\partial^+ u(\pi^*; r, l)}{\partial \pi}$ and $u(\pi; r, l)$ is kinked at π^* if L'(0) > 0.

 $^{^{30}}$ See Appendix C.2 for details.

 $^{^{31}}$ Forward-looking reference points are based on lagged expectations or beliefs regarding future utility (Kőszegi and Rabin, 2006; Abeler et al., 2011; Shaley, 2000). Intuitively, individuals experience disappointment or a sense of loss when experienced utility levels fail to match their recent expectations. If managers expect future utility to reflect past levels, forward and backward looking reference points may be similar.

³²Kahneman and Tversky (1979) consider the reference point to be "the status quo, or one's current assets." In the context of a manager's utility function, the status quo represents typical utility levels (or typical compensation) experienced in the past.

is their utility in the previous period. To capture the the possibility that reference points adjust gradually to changes in utility (i.e., reference point rigidity), Bowman, Minehart and Rabin (1999), Karlsson, Loewenstein and Seppi (2009), and Ryder Jr and Heal (1973) model reference points as a linear combination of lagged utility levels and lagged reference points. Similarly, DellaVigna et al. (2017) assume a worker's reference point is the average of their past income in a finite number of previous periods. To illustrate backward-looking reference points in the present setting, consider a manager who has engaged in Nash competition in each previous period. The manager's backward-looking reference point would be the utility level associated with Nash equilibrium profit. Profits in excess of Nash levels (e.g., due to collusion or an unanticipated increase in demand) would be perceived as gains while profits below Nash levels would represent losses.

Under target-based loss aversion, reference points are determined exogenously by performance targets (Sullivan and Kida, 1995; Crum, Laughhunn and Payne, 1981). These targets may be set explicitly by higher level managers (or shareholders) in the terms of a manager's compensation contract. For example, a manager's contract may award a bonus when the manager generates profits above a predetermined threshold.³³ Alternatively these targets may be the result of implicit performance expectations within the firm (e.g., a minimum performance level below which the manager anticipates a probability of losing their job or the performance levels of peers within the firm to which the manager is implicitly compared). When loss-aversion is driven by performance targets, l represents the likelihood and severity of the consequences a manager faces for failing to meet or exceed a performance target. For example, l may represent the likelihood of termination if a manager fails to meet a predetermined profit level.

Reference points, under both types of loss aversion, may evolve over time in response to experienced utility. A manager's reference point in period t is $r_t = m(r_{t-1}, u_{t-1})$ where r_{t-1} is the reference point in the previous period and u_{t-1} is utility in the previous period. m(r, u) satisfies the following assumption.

Assumption 3. m(r, u) satisfies the following assumptions:

```
i) r \le m(r, u) \le u if u > r, u \le m(r, u) \le r if u < r, m(r, u) = r if r = u,
```

- ii) m(r, u) is non-decreasing in u,
- iii) m(r, u) is non-decreasing in r, and
- iv) m(r, u) is continuous in u and r.

Assumption 3 permits a wide variety of reference point dynamics. For example, Assumption 3 is satisfied when an individual's reference point is constant (e.g., m(r, u) = r) or when an individual's reference point in period t is a convex combination of his/her reference point and utility level in the previous period as in (Bowman, Minehart and Rabin, 1999; Karlsson, Loewenstein and Seppi, 2009; Ryder Jr and Heal, 1973).

³³Heath, Larrick and Wu (1999) argue that reference points are determined by goals. A manager's goal may be to meet their target performance level.

Formally, $m(r,u) = \alpha r + (1-\alpha)u$ where $\alpha \in [0,1].^{34}$ Arkes et al. (2008) and Arkes et al. (2010) find that reference points respond differently to gains than losses. Specifically, the magnitude of the change in an individual's reference point following a gain is larger than the change in an individual's reference point following a corresponding loss. Assumption 3 permits asymmetric reference point adjustment of this kind. For example,

$$m(r, u) = \begin{cases} \alpha_I u + (1 - \alpha_I)r & \text{if } u \ge r \\ \alpha_D u + (1 - \alpha_D)r & \text{if } u < r \end{cases}$$

where $\alpha_I \in [0, 1]$, $\alpha_D \in [0, 1]$ and $\alpha_I > \alpha_D$. Assumption 3 also permits non-linear reference point dynamics (see Appendix C.3 for an example).

In some cases, managers' reference points may not adjust in response to experienced utility levels. For example, target-based reference points exogenously determined by a manager's employment contract may remain fixed irregardless of the manager's performance. Psychological reference points may not adjust in response to elevated utility levels if managers perceive those utility levels as abnormal. For example, colluding managers' may recognize that collusive profits are not the result of standard market competition and, as a result, may not become acclimatized to collusive utility levels as they would non-collusive increases in utility. m(r, u) = r represents a fixed reference point which is unaffected by utility. m(r, u) = r

Managers' incentives to collude depend not only on how reference points evolve over time but also managers' expectations regarding how their own reference point will adjust in the future. Do managers anticipate becoming acclimatized to higher or lower utility levels, or do managers fail to anticipate how their reference points will adjust to changes in circumstance?

I consider two approaches to modeling managers' expectations regarding the evolution of their reference points: naive and sophisticated managers. Naive managers evaluate future utilities relative to their current reference point. Ebert and Strack (2015) also consider naive, loss-averse agents who view their reference point as constant when a decision is made. Note that naive managers are not necessarily ignorant of the possibility of acclimatizing to future changes in utility, naive managers simply evaluate future utility levels relative to their current reference points when making a decision. Contrarily, sophisticated managers fully anticipate future changes in their reference point when making decisions.³⁷ The main results of this study hold, under appropriate assumptions, for both naive and sophisticated managers. In the main text, I restrict

³⁴When α is small, managers' reference points adjust rapidly in response to experienced utility levels. When α is large, managers' reference points adjust more gradually.

³⁵Karlsson, Loewenstein and Seppi (2009) show how an individuals attention can influence their reference point.

³⁶Additionally, if the length of a period in the model is sufficiently short, then constant reference points may represent an accurate approximation of reference point evolution if managers are slow to adjust their reference points in response to changes in utility.

³⁷DellaVigna et al. (2017) also assumes agents can anticipate future changes in their reference point. The case of sophisticated managers is also related to the concept of non-myopic equilibria presented by Shalev (2000).

attention to the case of sophisticated managers. Naive managers are analyzed in Appendix C.1.³⁸

As illustrated in Section 2, cartels often form immediately following a decline in the profitability of a market (i.e., a deterioration in market conditions). Formally, a parameter in firms' profit functions has changed in a way that negatively impacts profitability. The relevant parameter may be the market price of an input to production (e.g., the price of nickel in the stainless steel cartel³⁹), a parameter in the demand function (e.g., the graphite electrodes cartel (Grout and Sonderegger, 2005)), the number of firms in the market (e.g., the lysine cartel),⁴⁰ the bargaining power of buyers (e.g., the German coffee cartel (Holler and Rickert, 2022)), or the size of the competitive fringe (e.g., the Choline Chloride cartel (Herold and Paha, 2018)). In the ensuing analysis, this parameter is referred to as the market state, recognizing that any parameter appearing within firms' profit functions may represent the market state. The market environment before the deterioration in market state is referred to as regime b. The market environment after the deterioration in market state is referred to as regime a.

Let π_i^N denote Nash equilibrium profit under regime $i \in \{a, b\}$. The following assumption characterizes Nash equilibrium play under both regimes.

Assumption 4. For $i \in \{a, b\}$, there exists a unique symmetric Nash equilibrium wherein all managers play $x_i^N \in \Omega$ and earn a profit of π_i^N .

If a cartel forms, managers collude by setting a common price in each period t. Let $\pi_i(x)$ denote per-firm profit during collusion under regime i when all firms charge a common price x. $\pi_i(x)$ satisfies the following assumption.

Assumption 5. For $i \in \{a, b\}$, $\pi_i(x)$ satisfies the following assumptions:

- i) $\pi_i(x)$ is continuous in x for all $x \in \Omega$, and
- ii) $x_i^M \equiv \operatorname{argmax}_{x \in \Omega} \pi_i(x)$ exists and is unique.

Assumption 5(i) ensures $\pi_i(x)$ is continuous in x which is required for establishing the existence of a solution to the cartel's problem (see below). Assumption 5(ii) ensures that a unique monopoly price x_i^M exists under both regimes. Additionally, let $\pi_i^M \equiv \pi_i(x_i^M)$ denote monopoly profit. $\pi_i(x)$ includes any fixed costs of collusion. Fixed costs of collusion (Thomadsen and Rhee, 2007; Colombo, 2013; Klein and Schinkel,

³⁸When a manager's reference point is fixed across time, naive managers, by evaluating future utilities relative to their current reference point, correctly anticipate future reference point changes and, thus, the cases of naive and sophisticated managers are equivalent.

³⁹European Commission Decision 98/247/ECSC (Alloy Surcharge), 1998 O.J. (L 100) 55.

 $^{^{40}}$ The entry of a new firm represents a deterioration in market conditions regardless of whether the new firm participates in any subsequent collusion. As formalized in Assumption 8(ii), a deterioration in market conditions is defined as a change in the market environment that reduces Nash equilibrium profit. The entry of a new firm, regardless of whether the entrant decides to participate in collusion, reduces Nash equilibrium profit. Formally, let $N \equiv N_C + N_{NC}$ where N is the number of firms in the market, N_C is the number of firms which, if a cartel forms, participate in collusion, and N_{NC} is the number of firms that do not participate in collusion. An increase in either N_C or N_{NC} reduces Nash equilibrium profit. Therefore, both cases are consistent with Assumption 8(ii) and the model presented in this section.

2019) include any moral dis-utilities from participating in an illegal activity, fixed costs of monitoring rivals (e.g., payments made to a third party tasked with monitoring compliance with the collusive scheme), fixed costs involved in concealing collusive activities (including managerial effort) and communicating with other managers involved in the cartel, and costs of buying out potential entrants (Ganslandt, Persson and Vasconcelos, 2012).⁴¹

 $\pi_i^D(x)$ denotes the profit a manager earns if they choose to defect from collusion when the collusive level of the choice variable is x under regime i. The following assumption governs $\pi_i^D(x)$.

Assumption 6. For $i \in \{a, b\}$, $\pi_i^D(x)$ satisfies the following assumptions:

- i) $\pi_i^D(x)$ is continuous in x for all $x \in \Omega$, and
- ii) $\pi_i^D(x) \geq \pi_i(x)$ for all $x \in \Omega$.

Assumption 6(i) ensures $\pi_i^D(x)$ is continuous in x which is required for establishing the existence of a solution to the cartel's problem (see below). Assumption 6(ii) ensures that defection does not reduce a manager's profit. A broad variety of common oligopoly models satisfy Assumptions 4, 5, and 6, including homogenous product Cournot competition, differentiated product Bertrand competition and differentiated product Cournot competition.⁴²

The timing of the game is as follows. Prior to the initial period, the market environment reflects regime b. Under regime a, the market state unexpectedly⁴³ and permanently⁴⁴ In the beginning of the initial period (t=1), managers decide whether to form a cartel or not form a cartel. If a cartel does not form, managers engage in Nash competition in all future periods. If a cartel forms, then managers jointly set a common collusive price in each period. Specifically, the remainder of the initial period (t=1) and each subsequent period consists of three phases. In the first phase, managers determine x_t either collusively or competitively (depending on whether a cartel has formed or not). In the second phase, managers receive profits/utilities. In the third phase, managers' reference points are updated, in accordance with the utilities experienced in phase 2.

⁴¹Expected fines and penalties resulting from the detection of a cartel by an antitrust authority cannot be directly captured as fixed costs of collusion within the current model as detection by an antitrust authority also likely results in the breakdown of the cartel and a return to competition. Thus, the expected cost of detection depends on, among other factors, the discount rate, market state and degree of loss aversion. The impact of an antitrust authority that detects and penalizes colluding managers is analyzed formally in Appendix D.

⁴²Homogenous product Bertrand competition with constant marginal costs also satisfies Assumptions 4, 5, and 6. However, for many market state variables (e.g., number of firms, demand level, level of marginal cost), Assumption 8(ii) is violated under homogenous product Bertrand competition. This is the case as, for many important market state variables, a change in the market state does not impact Nash equilibrium profit due to the Bertrand paradox.

⁴³Thus, I do not consider changes in market state that can be easily predicted and anticipated by managers (e.g., regular increases in demand or an increase in costs due to a pre-planned tax increase). If managers fully anticipate future changes in the market state, then reference points may adjust to reflect the reduced utility levels expected in future periods. Thus, the change in market state, when it occurs, would not be perceived as a loss by managers.

⁴⁴See footnote 3. The deterioration in market state is assumed to be abrupt rather than gradual. The entry of a new firm, for example, is likely a abrupt change in market state. However, other deteriorations in market state, such as an decline in demand or increase in marginal cost may be gradual. In the main text, I restrict attention to abrupt changes in market conditions. Gradual deteriorations in market conditions are analyzed formally in Appendix E.

As all managers are symmetric and earn equal profits (along the equilibrium path), all managers have the same reference point in each period. Let r_t represent the reference point of all managers in period t. Initial reference points play an important role in the ensuing analysis as the initial reference point influences a manager's decision to form or not form a cartel in the initial period. The following assumption governs managers' initial reference point r_1 .

Assumption 7. $r_1 = u(\pi_b^N)$

Assumption 7 states that the initial reference point equals utility levels from Nash competition under regime b. Recall that regime b is the market state prior to the initial period. Thus, Assumption 7 is consistent with a setting where Nash competition has prevailed in the past and, as a result, managers have become accustomed to the Nash equilibrium utility level. This assumption is intended to reflect the focus of the current analysis on incentives to form cartels in industries where collusion did not otherwise prevail. Assumption 7 is made primarily for concreteness and ease of exposition in the main text. In Appendix B, I show that this assumption can be relaxed to permit reference points that exceed $u(\pi_b^N)$ and reference points below $u(\pi_b^N)$.

The following assumption characterizes the market state after the change (i.e., regime a).

Assumption 8. i)
$$r_1 < u(\pi_a^M)$$
, and ii) $\pi_a^N < \pi_b^N$.

In light of Assumption 7, Assumption 8(i) requires that the deterioration in market conditions is not drastic. Specifically, Assumption 8(i) requires that the change in market state is sufficiently moderate that monopoly profits after the change exceed Nash equilibrium profits prior to the change. In the results to follow, managers have an incentive to form a cartel after a deterioration in market state in order to avoid losses in utility. If the change in market state is sufficiently drastic that losses cannot be avoided by colluding (even if such collusion generates the maximum possible profits), then this effect will not occur. Assumption 8(ii) restricts attention to changes in market state that reduce the profitability of Nash competition, such as those discussed in Section 2.⁴⁶

If a cartel forms, collusion is sustained through grim trigger strategies. Specifically, collusion continues if all firms charge the agreed upon price. If any firm does not charge the agreed price (i.e., defects and earns profit $\pi_i^D(x)$), then collusion breaks down and all managers set prices competitively.

⁴⁵Note that any change in market state in the beginning of the initial period occurs after the initial reference point is determined at the end of the previous period. Thus, r_1 does not reflect the change in market state. If managers reference points instead adjusted downward (partially) after learning of the deterioration in market state, then the initial reference point may be less than $u(\pi_b^N)$. This possibility is explored in Appendix B.

⁴⁶In practice, most changes in market state will also reduce the profitability of collusion (e.g., a increase in marginal cost, a reduction in demand or the entry of a new competitor). However, no such restriction is necessary for the results of this study.

Assumption 9. Reference points do not update (i.e., m(r, u) = r) in response to utilities experienced during the defection period.

Assumption 9 reflects three considerations. First, the structure of grim trigger strategies implies that relatively large profit/utility levels earned during defection are transient as punishment phases immediately follow defection. Moreover, managers recognize that defection will result in elevated profits/utilities for a single period and therefore may not adjust or become accustomed to utility levels experienced during defection as they would to utility levels they believe may continue in future periods. As Chen and Rao (2002) write, "[i]f people are aware that a second event is going to undo the first, the reference point will likely not shift after the first event." Similarly, Joskow and Rose (1994) find that corporate boards discount extreme performance outcomes. Thus, expectations for future performance (i.e., a target-based reference point) may not adjust in response to extreme profits during defection. Second, the adjustment of reference points to utilities in the defection period can generate unusual manager behavior. Specifically, if reference points adjust to utilities in the defection period, managers may be hesitant to defect as they would anticipate becoming acclimatized to relatively high utilities during defection and, as a result, perceive the punishment phase as a particularly large loss in utility. This may result in managers distorting the prices they charge during defection in order to reduce the extent of the profits they earn when cheating on collusion or, alternatively, declining to defect whatsoever. Declining the possibility of obtaining large profits due to a fear of becoming acclimatized to the resulting increases in utility may be unrealistic.⁴⁷ Finally, permitting reference points to adjust during the defection phase creates a number of analytical difficulties as profits during defection will depend on managers' reference points. 48

Let $W(\{\pi_t\}_{t=T}^{\infty}; r_T)$ denote the discounted present value of the payoff, in period T and under regime i, from a profit sequence of $\{\pi_t\}_{t=T}^{\infty}$ when the reference point at time T is r_T . Thus,

$$W(\{\pi_t\}_{t=T}^{\infty}; r_T) = \sum_{t=T}^{\infty} \delta^{t-T} u(\pi_t; r_t)$$

where $r_t = m(r_{t-1}, u(\pi_t))$ for t > T.

Assumption 10. For $i \in \{a, b\}$, $W\left(\{\pi_i^N\}_{t=1}^{\infty}; r\right) \ge W\left(\{\pi_t\}_{t=1}^{\infty}; r\right)$ for any r and any $\{\pi_t\}_{t=1}^{\infty}$ where $\pi_t \le \pi_i^N$ for all t.

⁴⁷As discuss in Appendix C.1, research in behavioral economics and psychology suggests that humans are relatively poor predictors of future tastes and, in the context of the current study, their reference points. Thus, it seems unlikely that managers would engage in complex reasoning of this kind.

⁴⁸Strategically, managers have an incentive to not adjust their reference points to reflect utility levels during defection as doing so only enhances the sense of loss experienced when the punishment phase begins in the following period. Thus, managers may intentionally restrain themselves psychologically from adjusting to defection profits in order to limit the sense of loss in future periods.

Assumption 10 is sufficient to ensure that the infinite repetition of the static Nash equilibrium constitutes a sub-game perfect Nash equilibrium of the dynamic game. See Lemma 2 in Appendix A for a formal proof. Thus, if the cartel does not form, the infinite repetition of Nash equilibrium prices constitutes a Nash equilibrium of the dynamic game. Additionally, Assumption 10 ensures that no manager wishes to deviate from the punishment phase (repeated static Nash competition) under grim trigger strategies.⁴⁹

 $V_i^N(r)$ denotes the discounted present value of utility from repeated Nash equilibrium play in all future periods (hereafter, the competitive payoff) when the reference point is r under regime $i \in \{a, b\}$. Thus,

$$V_i^N(r) = u(\pi_i^N; r) + \delta V_i^N(m(r, u(\pi_i^N))).$$

If a cartel does not form, all managers earn payoff $V_i^N \equiv V_i^N(r_1)$ where r_1 is the initial reference point. For collusion to occur successfully, collusive prices must be such that no manager wishes to defect or cheat on collusion in any period. Suppose $\{x_t\}_{t=1}^{\infty}$ denotes a collusive price path and let $\{r_t\}_{t=1}^{\infty}$ denote the corresponding sequence of reference points (i.e., $r_t = m(r_{t-1}, u(\pi_i(x_t)))$ for t > 1). No manager wishes to defect in period T if (recall that managers collude using grim trigger strategies)⁵⁰

$$\sum_{t=T}^{\infty} \delta^{t-T} u(\pi_i(x_t); r_t) \ge u(\pi_i^D(x_T); r_T) + \delta V_i^N(r_T). \tag{1}$$

The inequality in Equation (1) is hereafter denoted the incentive compatibility constraint (ICC) in period T.

During collusion, managers set prices in order to maximize their discounted present value of utility subject to the constraint that no manager wishes to defect in the present period or any future period (the ICC in Equation (1) is satisfied for all T). V_i^C denotes the discounted present value of utility from collusion in the initial period under regime i. Thus, if a cartel forms, each manager earns a payoff V_i^C where

$$V_i^C = \max_{\{x_t\}_{t=1}^\infty \in \Psi} \sum_{t=1}^\infty \delta^{t-1} u(\pi_i(x_t); r_t)$$
 (2)

 $^{^{49}}$ Assumption 10 is not an implication of the fact that u is strictly increasing in π as profit/utility levels in period t impact reference points in future periods. Conceivably, managers may wish to intentionally reduce their utility in the current period in order to reduce their reference point and limit the extent of perceived losses in later periods. This assumption rules out such behavior.

 $^{^{50}}$ For expositional clarity, the degree of loss aversion l is dropped in Equation (1). Throughout the manuscript, I drop functional arguments (e.g. x or l) when the dependence of the relevant functions on the argument is apparent.

and

$$\Psi_{i} = \left\{ \left\{ x_{t} \right\}_{t=1}^{\infty} : x_{t} \in \Omega \text{ and } \sum_{t=T}^{\infty} \delta^{t-T} u(\pi_{i}(x_{t}); r_{t}) \geq u(\pi_{i}^{D}(x_{T}); r_{T}) + \delta V_{i}^{N}(r_{T}) \quad \text{ for all } T \in \left\{ 1, 2 \dots \right\} \right\}$$
(3)

where, in each of the inequalities in Ψ , r_t denotes the reference point in period t consistent with the price path $\{x_t\}_{t=1}^{\infty}$. Ψ is the set of price paths $\{x_t\}_{t=1}^{\infty}$ that are incentive compatible in all periods. Let $V_i^C = -\infty$ when $\Psi = \emptyset$ (i.e., no price path satisfies the ICCs).⁵¹

A cartel forms if the discounted present value of utility from collusion (hereafter, the "collusive payoff") is greater than the discounted present value of utility from Nash competition (i.e., $V_i^C > V_i^N$). This condition involves two requirements. First, there must exist a price path that satisfies the inequalities in Equation (3) (i.e., $\Psi \neq \emptyset$). If no such price path exists, then $V_i^C = -\infty < V_i^N$ and a cartel never forms. Second, given an incentive compatible price path exists, the optimal price path (i.e., the price path that solves the maximization problem in Equation (2)) must result in a collusive payoff which is greater than the competitive payoff.⁵² If $V_i^C < V_i^N$, then a cartel does not form and managers set Nash equilibrium prices in all periods and earn a payoff of V_i^N . A solution to (2) is referred to as an optimal price path. Additionally, let V_i^M denote the discounted present value of utility, under regime i, if the cartel charges the monopoly price in all periods (hereafter, the monopoly payoff).

The following lemma establishes that an optimal price path exists under the above assumptions.⁵³

Lemma 1. If $\Psi_i \neq \emptyset$, then a solution to (2) exists under regime $i \in \{a, b\}$.

⁵¹ Note that $\{x_i^N\}_{t=1}^{\infty} \in \Psi$ does not necessarily hold as $\pi_i(x_i^N)$ (collusive profits from charging the Nash equilibrium price) may not equal π_i^N due to the presence of, for example, fixed costs of collusion.

⁵²In Appendix C.4, I show that the existence of an incentive compatible price path (i.e., $\Psi \neq \emptyset$) does not necessarily imply that the collusive payoff exceeds the payoff from Nash competition.

 $^{^{53}}$ If $\Psi_i = \emptyset$, then a cartel does not form and managers set Nash equilibrium prices in all periods. A solution to (2) does not imply a cartel forms as the collusive payoff must exceed the Nash payoff (see Appendix C.4).

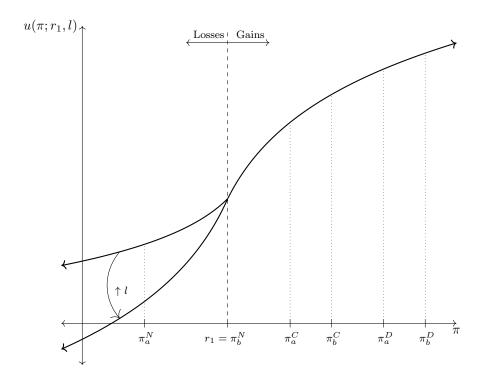


Figure 1: A loss averse utility function where π_i^C denotes collusive profits, π_i^D denotes defection profits, and π_i^N denotes Nash profit under regime $i \in \{a, b\}$.

4 Cartel Formation

In this section, I analyze the impact of a deterioration in market conditions on loss averse managers' incentives to form a cartel. The following condition is assumed to hold throughout this section.

Condition 1.
$$V_b^C(l) \leq V_b^N$$

Condition 1 ensures that a cartel does not form under regime b (i.e., a cartel does not form absent a change in market conditions). This condition holds when, for example, the discount factor is sufficiently low (i.e., firms are too impatient for successful collusion) or fixed costs of collusion are sufficiently large.⁵⁴

Proposition 1. Suppose Condition 1 holds. There exists a \bar{l} such that $V_a^C(l) > V_a^N(l)$ when $l \geq \bar{l}$.

Proposition 1 implies that a cartel forms under regime a when managers are sufficiently loss averse. Note that Proposition 1 holds irregardless of the discount factor or the size of fixed costs of collusion. In light of Condition 1, Proposition 1 states that, when managers are sufficiently loss averse, a deterioration in market conditions will cause a cartel to form in an otherwise competitive market. Thus, the entry of a new firm, a reduction in demand, or an increase in the cost of an input can actually cause the formation of a cartel.

 $^{^{54}}$ Note that when m(r, u) = r, $V_b^C(l)$ does not depend on l. Thus, Condition 1 is independent of l. See Figure 2.

Proposition 1 suggests that the empirical patterns identified in Section 2 may, to some extent, reflect the behavior of loss averse managers.

To understand this result, recall that, in the initial period when managers decide whether to form or not form a cartel, managers are accustomed to competitive utility levels (i.e., $r_1 = u(\pi_b^N)$ by Assumption 7). When the market state unexpectedly deteriorates in a way that reduces the profit that can be earned through competitive play (Assumption 8(ii)), managers' perception of continued competition changes. Specifically, continued competition is now perceived as a loss because the utilities that can be earned through competitive play are less than managers' reference points. When managers are particularly averse to losses, the incentive to avoid this painful loss in utility causes the managers to form a cartel which, by Assumption 8(i), can generate utility levels which exceed the reference point and are therefore perceived as a gain, not a loss. In summary, a deterioration in market state causes managers to perceive continued competition as a loss. To avoid a loss in utility, managers form a cartel which elevates their profits/utilities above competitive levels. Thus, the threat of a loss in utility drives loss averse managers to form a cartel after the deterioration in market state. Note that such an effect does not occur unless the market state deteriorates because continued competition is not be perceived as a loss absent a deterioration in market conditions.

Recall that loss aversion may reflect the negative consequences a manager faces for failing to meet performance targets set by supervisors (i.e., target-based loss aversion). When the market state deteriorates, continued competition is perceived as a drop in performance below pre-specified target levels, which may lead to termination, demotion or the loss of a bonus. In this context, Proposition 1 captures the incentive of managers to form a cartel, after a deterioration in market state, in order to protect their employment, status or compensation.

Cartel activity becomes increasing attractive to managers after a deterioration in market state for another, more subtle, reason. Under grim trigger strategies, defection from the cartel is punished through a reversion to competitive play in all subsequent periods. However, as discussed previously, managers perceive competitive utility levels, after the deterioration in market conditions, as a loss. As a result, managers have relatively weak incentives to cheat on the cartel and endure relatively large losses in utility during the subsequent punishment phase. This effect relaxes the incentive compatibility constraints, stabilizes collusion and enables managers to successfully collude after the deterioration in market state.

Figure 1 illustrates managers incentives to collude in the initial period. Specifically, Figure 1 depicts a classical S-shaped utility function indicating competitive, collusive and defection profits under both regimes.⁵⁵ Figure 1 illustrates how a deterioration in market conditions reduces competitive profit levels and, as a

 $^{^{55}}$ While Figure 1 depicts a fixed level of collusive profits to facilitate the visualization, the profit managers earn during collusion may change over time as reference points adjust.

result, causes continued competition to be perceived as a loss. Crucially, collusive profits represent a gain (under both regimes). Thus, managers can avoid painful losses in utility from continued competition by turning to collusion. Figure 1 also illustrates that an increase in the degree of loss aversion reduces the utility managers earn from competition under regime a, which enhances incentives for cartel formation and relaxes the incentive compatibility constraints necessary for successful collusion.

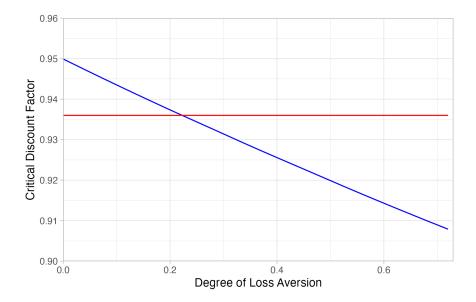


Figure 2: Critical Discount Factor by Degree of Loss Aversion Before (Red) and After (Blue) the Entry of a New Firm.

Notes: This figure depicts the critical discount factor as a function of the degree of loss aversion. See Appendix F.1 for additional details regarding the simulations employed to generate this figure. Parameters: $a = 100, b = 2, e = 1, c = 0, \alpha = 1$ and F = 125. The blue curve denotes the critical discount factor after entry (i.e., N = 6) and the red curve denotes the critical discount factor absent entry (i.e., N = 5).

Figure 2 depicts the critical discount factor, the smallest discount factor for which a cartel forms, before (red) and after (blue) the entry of a new firm. See Appendix F.1 for additional details regarding the simulations conducted to generate this figure and other figures presented in the main text.⁵⁶ First, observe that the entry of a new firm, as expected, increases the critical discount factor (i.e., reduces incentives to collude) when managers are loss neutral (i.e., l = 0).⁵⁷ This finding reflects the standard theoretical result

$$U(q_1, \dots q_N) = a \sum_{i=1}^{N} q_i - \left(\frac{1}{2}\right) \left(b \sum_{i=1}^{N} q_i^2 + e \sum_{i=1}^{N} \sum_{j \neq i} q_i q_j\right)$$

where a > 0, b > e > 0 and q_i is the quantity of firm i's product consumed. Managers have a loss function of L(x) = x, linear base utility u(x) = x, and $m(r, u) = \alpha r + (1 - \alpha) u$ where $\alpha \in [0, 1]$. Additionally, there is a fixed cost of collusion $F \ge 0$.

 $^{^{56}}$ All figures in the main text reflect outcomes from a setting involving N firms choosing prices and selling symmetrically differentiated products where the representative consumer has a utility function of (Singh and Vives, 1984)

⁵⁷The new firm is assumed to participate in the cartel (if it forms). This assumption is consistent with, for example, the formation of the lysine cartel. The lysine cartel formed following the entry of Archer Daniels Midlands, which participated in the cartel, into the lysine market (Connor, 2001).

that the sustainability of collusion in a repeated game is declining in the number of firms. However, when managers are sufficiently loss averse, the entry of a new firm can, counterintuitively, reduce the critical discount factor and enhance incentives to collude. Thus, when managers are loss averse, there exists a range of discount factors under which the entry of a new firm causes the formation of a cartel in a previously competitive market. Note that Condition 1 holds when $\delta < .936$ in the simulation in Figure 2.

In summary, a deterioration in market conditions can cause the formation of a cartel in a previously competitive market when two conditions are satisfied. First, as established in managers must be sufficiently loss averse (i.e., $l \ge \bar{l}$ in Proposition 1). Second, the discount factor must be sufficiently small that a cartel does not form absent the change (i.e., Condition 1 must be satisfied).

5 Collusive Payoff and Gain from Collusion

Section 4 establishes that a deterioration on market state may cause a cartel to form in an otherwise competitive market. However, Proposition 1 does not speak to the magnitude of the collusive payoff managers obtain after forming a cartel. In the model analyzed in this study, I assume a cartel forms whenever the collusive payoff exceeds the competitive payoff (regardless of the size of the collusive payoff or the magnitude of the difference between collusive and competitive payoffs). In practice, cartely seem more likely to form when the potential gain from collusive activity is large. This may reflect two distinct considerations. First, there may be a fixed cost (not explicitly incorporated in the current model) of initiating a collusive agreement that is paid once in the beginning of the initial period. This cost may reflect a moral disutility from conspiring to commit an illegal activity (Klein and Schinkel, 2019; Boulu-Reshef and Monnier-Schlumberger, 2023), initial costs of communication/coordination necessary to establish the cartel, or other start-up costs involved in successfully forming and organizing a cartel.⁵⁸ The potential payoff from collusion may need to exceed the competitive payoff by a sufficiently large margin⁵⁹ for managers' to choose to incur these costs and collude. Second, managers' may be more likely to coordinate on a collusive equilibrium than a competitive equilibrium when the collusive payoff is relatively large. Recall that the infinite repetition of the Nash outcome always constitutes an equilibrium of the dynamic game. In Section 3, I assume that managers' coordinate on a collusive equilibrium whenever such an equilibrium results in a payoff that exceeds the competitive payoff. However, this equilibrium selection assumption is more likely to hold when the collusive payoff exceeds the competitive payoff by a significant margin.⁶⁰

⁵⁸To illustrate, suppose the collusive payoff equals the competitive payoff by an infinitesimal margin. In this case, it seems unlikely that managers' will expend the effort and time necessary to form a cartel for a negligible gain. I refrain from explicitly including startup costs of this kind in the model for simplicity.

 $^{^{59}}$ See Klein and Schinkel (2019) for an analysis of collusion when firms require a margin before colluding.

⁶⁰This argument mirrors the Pareto criterion for equilibrium selection.

Motivated by the above considerations, I explore in this section how a deterioration in market conditions impacts both the collusive payoff $(V_i^C(l))$ and the gain from collusion $(V_i^C(l) - V_i^N(l))$.

Condition 2. i)
$$V_b^C(l) > V_b^N(l)$$
 , and
 ii) $V_a^M > V_b^C(l)$.

Condition 2(i), the reverse of Condition 1, restricts attention to markets wherein cartel forms absent a change in market state. This condition is not strictly necessary for the following proposition, but is made primarily for expositional purposes and to focus the analysis of this section on the impact of a change in market state on the gain/value from collusion rather than incentives to form cartels (which was analyzed in the previous section).

Condition 2(ii) ensures that the collusive payoff under regime b is less than the monopoly payoff under regime a. If managers can obtain a collusive payoff, absent a change in market state, which exceeds the maximum possible collusive payoff (i.e., the monopoly payoff) after a deterioration in market state, then a deterioration in market state will never increase the value of collusion. In Appendix C.5, I provide a lower-level sufficient condition that ensures Condition 2(ii) holds. Specifically, I show that Condition 2(ii) is satisfied when the 1) discount factor is sufficiently low and 2) the magnitude of the change in market state is sufficiently moderate.⁶¹ Intuitively, when managers are relatively impatient, only relatively low collusive prices and moderate collusive payoffs are obtainable. If the cartel instead set high prices during collusion, then impatient managers would have an incentive to cheat on collusion, undermining the cartel's stability. Thus, $V_b^C(l)$ is relatively low when managers are impatient which helps ensure Condition 2(ii) holds. A drastic deterioration market state (e.g., an extreme reduction in demand) would substantially reduce monopoly profits (i.e., would result in a low value of V_a^M) which would violate Condition 2(ii).

Proposition 2. Suppose Condition 2 holds. There exists an \bar{l} such that the following hold when $l \geq \bar{l}$:

i)
$$V_a^C(l) > V_b^C(l)$$
, and

$$ii) \ V_a^C(l) - V_a^N(l) > V_b^C(l) - V_b^N.$$

Proposition 2 states that a deterioration in market conditions enhances the collusive payoff (part i) and increases the potential gain from collusion (part ii) when managers are sufficiently loss averse. Note that this result occurs despite the fact that a deterioration in market state, such as a reduction in demand or an increase in input cost, typically reduces collusive payoffs when managers are loss neutral.

Proposition 2(i) is driven by the fact, discussed in the previous section, that a deterioration in market state stabilizes collusion and reduces incentives to defect when managers are loss averse. In short, managers

 $[\]overline{}^{61}$ If the change in market state increases the monopoly payoff (i.e., $V_b^M < V_a^M$), then Condition 2(ii) holds trivially.

have weaker incentives to defect from collusion after the change in market state as such a defection would lead to competitive play in the subsequent punishment phase which, after the change in market state, is perceived by managers as a painful loss in utility. Weaker incentives to defect after the deterioration in market state stabilizes collusion and relaxes the incentive compatibility constraints in Equation (1). Relaxed ICCs after the deterioration in market state permit the cartel to set higher prices than would otherwise be incentive compatible. This enables the cartel to earn higher collusive profits and utilities after the deterioration.

Proposition 2(ii) is driven by the effect outlined in the previous paragraph, but also by a distinct, albeit closely related, consideration. At the beginning of the initial period, when managers decide whether to form or not form a cartel, the competitive payoff is smaller after the deterioration in market state (i.e., $V_a^N(l) < V_b^N$). This is the case not only due to the fact that the deterioration in market state reduces Nash profits (Assumption 8(ii)), but also due to the fact that managers perceive the competitive utility level as a loss.

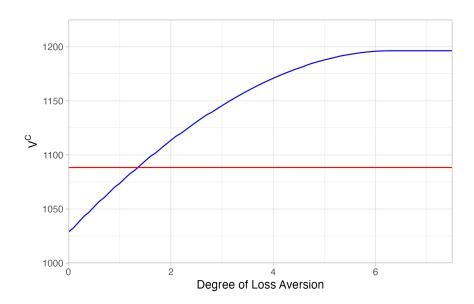


Figure 3: Collusive Payoff by Degree of Loss Aversion Before (Red) and After (Blue) a 25% Increase in Marginal Cost.

Notes: This figure depicts the collusive payoff as a function of the degree of loss aversion. Parameters: $a=100, b=2, e=1, N=7, F=0, \alpha=1$ and $\delta=.8$. Absent the deterioration in market conditions, all firms have a marginal cost of c=10. The blue curve denotes the collusive payoff V_a^C after a 25% increase in marginal cost (i.e., c=12.5) and the red curve denotes the collusive payoff V_b^C absent an increase in marginal cost (i.e., c=10).

Figure 3 depicts the collusive payoff after an increase in marginal cost (in blue) and absent an increase in marginal cost (in red) as a function of the degree of loss aversion.⁶² As expected, an increase in marginal

⁶²There are no fixed costs of collusion in the simulations presented in this section. Excluding fixed costs of collusion ensures that a cartel always forms for any discount rate which concentrates the analysis on the collusive payoff rather than decisions regarding cartel formation. Formally, Condition 2(i) is always satisfied in the simulations in this section as there is no fixed

cost reduces the collusive payoff when managers are relatively loss neutral. When managers are sufficiently loss averse, the increase in marginal cost enhances the collusive payoff. Note that when l is particularly large, managers earn the monopoly payoff after the deterioration in market conditions (i.e., $V_a^C(l) = V_a^M$).

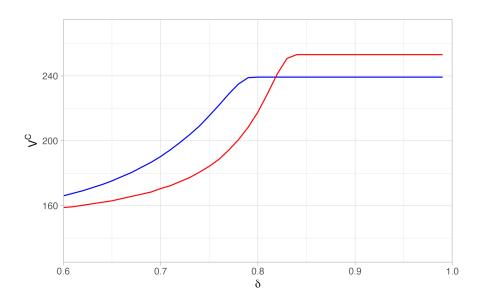


Figure 4: Collusive Payoff by Discount Factor Before (Red) and After (Blue) a 25% Increase in Marginal Cost.

Notes: This figure depicts $1-\delta$ times the collusive payoff as a function of the discount factor. Parameters: $a=100, b=2, e=1, N=7, F=0, \alpha=1$ and l=7.5. Absent the deterioration in market conditions, all firms have a marginal cost of c=10. The blue curve depicts $(1-\delta) V_a^C$ after a 25% increase in marginal cost (i.e., c=12.5) and the red curve depicts $(1-\delta) V_b^C$ absent an increase in marginal cost (i.e., c=10).

Figure 3 depicts the (normalized) collusive payoff after an increase in marginal cost (in blue) and absent an increase in marginal cost (in red) as a function of the discount rate when managers are loss averse. An increase in marginal cost enhances the collusive payoff for moderate discount factors. If the discount factor is instead relatively large, then the cartel can, prior to the increase in marginal cost, set monopoly prices in all periods (i.e., $V_b^C(l) = V_b^M$). If this is the case, then a deterioration in market state will never enhance the collusive payoff because no price path, after the marginal cost increase, can generate a collusive payoff exceeding the monopoly payoff. Formally, Condition 2(ii) is violated when the discount factor is relatively large.

cost of collusion. The simulations in this section depict the collusive payoff. Corresponding figures depicting the cartel's gain from collusion are presented in Appendix F.4.

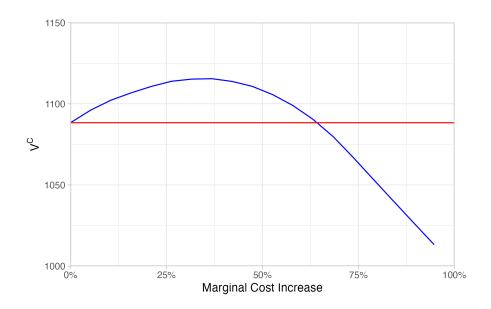


Figure 5: Collusive Payoff Before (Red) and After (Blue) Deterioration by Size of Marginal Cost Increase. Notes: This figure depicts the collusive payoff as a function of the size of the marginal cost increase. Parameters: $a = 100, b = 2, e = 1, N = 7, \alpha = 1, F = 0, l = 2$ and $\delta = .8$. Prior to the deterioration in market conditions, all firms have a marginal cost of c = 10. Thus, a 50% increase in marginal cost results in a marginal cost of c = 15. The blue curve denotes V_a^C and the red curve denotes V_b^C .

Figure 5 depicts the collusive payoff after an increase in marginal cost (in blue) and absent an increase in marginal cost (in red), as a function of the size of the marginal cost increase when managers are loss averse. Moderate increases in marginal cost increase the collusive payoff (which is consistent with Proposition 2). However, drastic increases in marginal cost reduce the collusive payoff. To see this, note that substantial increases in marginal cost reduce the maximal/monopoly payoff (i.e., V_a^M) below the collusive payoff absent the marginal cost increase (i.e., V_b^C). If this is the case, managers cannot, for any degree of loss aversion, obtain a collusive payoff after the marginal cost increase which exceeds the collusive payoff prior to the increase. Formally, Condition 2(ii) is violated when the deterioration in market conditions is sufficiently severe.

In summary, Proposition 2 suggests that events such as the entry of a new firm, an increase in the price of an input or a reduction in demand, which are typically associated with a reduction in profitability, can actually enhance the utility a manager can obtain from collusion and, similarly, the gain in utility from collusion. This is the case when managers are sufficiently loss averse, the discount factor is relatively low and the change in market state is not drastic.

6 Pricing Results

In this section, I show that a deterioration in market state can result in higher prices, particularly during early periods of collusion. To characterize optimal cartel prices, it is necessary to place a number of additional restrictions on the model. These restrictions are contained in the following assumption which is assumed to hold for the remainder of this section.

Assumption 11. i) $\Omega = [c, d] \subset \mathbb{R}$ where c < d,

- ii) for all $i \in \{a, b\}$, $\pi_i(x)$ is strictly increasing in x for $x < x_i^M$ and strictly decreasing in x for $x > x_i^M$,
- iii) for all $i \in \{a,b\}$, $x_i^N < x_i^M$, and
- iv) for all $i \in \{a,b\}$, $u(\pi_i^D(x)) u(\pi_i(x))$ is strictly increasing in x for all $x \in \Omega$ such that $x \ge x_i^N$.

Assumption 11(i) implies that the set of feasible prices is a closed interval. Assumption 11(ii) ensures that collusive profits are strictly quasi-concave. This assumption prevents the existence of multiple optimal price paths, simplifying the analysis. Assumption 11(iii) ensures that the monopoly value of the choice variable x exceeds the Nash value of the choice variable. This assumption is made without loss of generality as if $x^N > x^M$, then the choice variable can instead be taken as $\tilde{x} = -x$. Recall that the choice variable x is referred to as a price, for expositional ease, throughout the analysis. However, the model does not require x to represent prices. For example, x could instead represent output levels x (for which the Nash output level is likely to exceed the monopoly output level), in which case Assumption 11(iii) is satisfied by a choice variable of x = -q. Assumption 11(iv) mirrors Assumption C3 in Harrington (2004) and ensures that collusion involving higher prices is more difficult to sustain than collusion involving lower prices.

The main result of this section applies when the following condition holds.

Condition 3. i)
$$\delta < \frac{u(\pi_b^D(x_a^M)) - u(\pi_b(x_a^M))}{u(\pi_b^M) - u(\pi_b^N) + u(\pi_b^D(x_a^M)) - u(\pi_b(x_a^M))}$$
, and ii) $x_b^N < x_a^M$.

Condition 3(i) holds if managers' discount factors are sufficiently low.⁶³ When managers are sufficiently patient that incentive compatibility constraints do not bind prior to the deterioration in market state, managers can set the monopoly price in all periods. In this case, a deterioration in market state, unless the deterioration increases the monopoly price, cannot result in increased collusion prices. This is the case as, even if firms can successfully charge the monopoly price x_a^M after the deterioration in market state, these monopoly prices will not exceed collusive/monopoly prices absent the deterioration.⁶⁴

 $^{^{63}}$ See the proof of Proposition 3 for a proof that the numerator and denominator of this expression are positive.

 $^{^{64}}$ A reduction in demand or the entry of a new competitor, for example, generally do not increase monopoly prices (i.e., $x_a^M \leq x_b^M$). However, an increase in marginal cost, for example, increases monopoly prices. In this case, Condition 3(i) may be unnecessary for the following results. As the majority of deteriorations in market state tend to reduce monopoly prices, Condition 3(i) is maintained (even when not strictly necessary) throughout this section.

Condition 3(ii) ensures the monopoly price after a deterioration in market state exceeds the Nash equilibrium price absent a deterioration in market state. This condition holds if the magnitude of the deterioration is moderate. If the change in market state is more drastic (e.g., the entry of multiple new competitors simultaneously or an extreme reduction in demand), then monopoly prices after the change in market state may be less than competitive prices absent the change in market state. In this case, a deterioration in market state would cause lower prices even if a cartel formed and was capable of charging monopoly prices in all periods. Condition 3(ii) rules out this possibility.

Let $\{x_{b,t}\}_{t=1}^{\infty}$ and $\{x_{a,t}\}_{t=1}^{\infty}$ denote optimal price paths under regime b and a, respectively.

Proposition 3. Suppose Condition 3 holds. There exists an \bar{l} such that if $l \geq \bar{l}$, then there exists a $T \in \{1, 2...\} \cup \{\infty\}$ such that $x_{b,t} < x_{a,t}$ for all $t \leq T$.

Proposition 3 establishes that a deterioration in market state can increase collusive prices in early time periods when managers are sufficiently loss averse.⁶⁵ To understand this result, suppose a cartel would form both with and without the deterioration in market state. The pricing dynamics in Proposition 3 reflect two countervailing effects. First, due to conventional considerations, deteriorations in market state tend to reduce collusive prices. This is the case, for example, when the deterioration in market conditions is a reduction in demand or the entry of a new firm. This effect is termed the *standard effect* and, all else equal, causes collusive prices to decline following a deterioration in market state.⁶⁶

Second, when managers are loss averse, a reversion to Nash equilibrium play is perceived as a loss. The size of this loss depends on the difference between the current reference point r_t and the Nash equilibrium utility level. When this difference is large, managers perceive the punishment phase to be a significant loss in utility and, as a result, are hesitant to defect from collusion. This effect enhances the stability of collusion (i.e., relaxes the incentive compatibility constraints) and permits the cartel to set higher prices. This effect is termed the *stability effect* and is largest when $r_t - u(\pi^N)$ is large (i.e., the difference between the reference point and the competitive utility level is large). Which of these two effects dominates depends crucially on the degree of loss aversion, how reference points adjust over time, and the current reference point.

When managers are loss neutral, the stability effect does not occur and, as a result, a deterioration in market conditions reduces collusive prices due to the standard effect. However, when managers are loss averse, the stability effect occurs and can overpower the standard effect. As a result, a deterioration in market conditions can enhance collusive prices in early periods of collusion (as Proposition 3 states). In

 $^{^{65}}$ Proposition 3 is not the result of a cartel forming under regime a and a cartel not forming under regime b. While this may be the case, Proposition 3 also applies when a cartel forms under both regimes (see Figure 6 and Figure 7, as well as supplementary figures in Appendix F).

⁶⁶For certain deteriorations in market state, such as an increase in marginal cost, the standard effect may lead to higher prices after the deterioration in market state. In this case, the deterioration in market state could result in higher prices even absent the presence of loss aversion.

early periods of collusion, the reference point is $r_1 = u(\pi_b^N)$ or, depending on how reference points adjust over time, slightly above $r_1 = u(\pi_b^N)$. Absent a deterioration in market state, a reversion to Nash competition results in a utility level of $u(\pi_b^N)$ which is not perceived as a loss (or, at a minimum, not as a significant loss). However, after a deterioration in market state, a reversion to Nash competition results in a utility of $u(\pi_a^N) < r_1$ which is perceived as a pronounced loss. Thus, in early periods of collusion, the stability effect is approximately zero absent a deterioration in market state, but is positive following a deterioration. This results in higher prices in early periods of collusion under regime a than under regime b.

Cartel prices in later periods of collusion are more difficult to characterize. As collusion continues, reference points adjust upwards and, in later periods of collusion, reference points exceed Nash equilibrium utility levels under both regimes. Thus, the stability effect is large in both cases and, as a result, prices absent the deterioration in market state can exceed prices after the deterioration in market state.

Figure 6 plots the optimal price path before (red) and after (blue) a reduction in the demand for a variety of degrees of loss aversion. Note that the standard effect of a decrease in demand (specifically, the demand intercept) is to reduces prices. When managers are loss neutral (Panel A of Figure 6), the stability effect does not occur and, as a result, a reduction in demand reduces cartel prices in all periods. However, when managers are loss averse (Panel B of Figure 6), the reduction in demand increases the collusive price in early periods of collusion due to the stability effect. In later periods of collusion, the stability effect is present under both regimes and prices are instead driven by the standard effect, resulting in higher prices absent a reduction in demand.

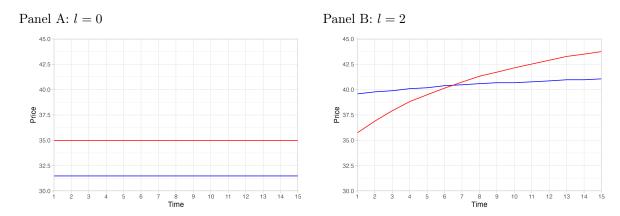


Figure 6: Optimal Price Paths Before (Red) and After (Blue) a 10% Reduction in Demand by Degree of Loss Aversion.

Notes: This figure depicts optimal price paths before (red) and after (blue) a 10% reduction in a for loss neutral managers (Panel A) and a loss degree of loss aversion of 2 (Panel B). Parameters: $b=2, e=1, c=0, N=3, \delta=.25, F=0$ and $\alpha=.9$. Prior to the deterioration in market conditions, a=100. The blue curve depicts the optimal price path after the deterioration in market conditions (i.e., a=90) and the red curve depicts the optimal price path prior to the deterioration in market conditions (i.e., a=100). The Nash equilibrium price before (after) the change is 25 (22.5). The monopoly price before (after) the change is 50 (45).

While the exact length of time for which $x_{b,t} < x_{a,t}$ depends on a variety of factors (especially, the function m characterizing reference point dynamics), the following proposition demonstrates that this length of time may be substantial.

Proposition 4. Suppose Condition 3 holds and m(r,u) = r. There exists an \bar{l} such that if $l \geq \bar{l}$, then $x_{b,t} < x_{a,t}$ for all t.

Proposition 4 implies that, when reference points are constant, collusive prices after the deterioration in market state exceed prices absent a deterioration in market state in all periods. Recall that constant reference points can occur in a variety of situations including, for example, when reference points are the result of (fixed) expectations set by higher level executives or when reference points correspond to a profit level below which managers face the risk of job loss. To illustrate Proposition 4, first consider regime b. If reference points remain constant at $r_1 = u(\pi_b^N)$, the punishment for defection (Nash competition) is never perceived as a loss in utility. Thus, the stability effect is zero in all periods absent a deterioration in market state. However, after the market state deteriorates, the punishment for defection is perceived as a loss in all future periods. Thus, the stability effect occurs under regime a and, when managers are sufficiently loss averse, overpowers the standard effect and results in higher prices in all periods.

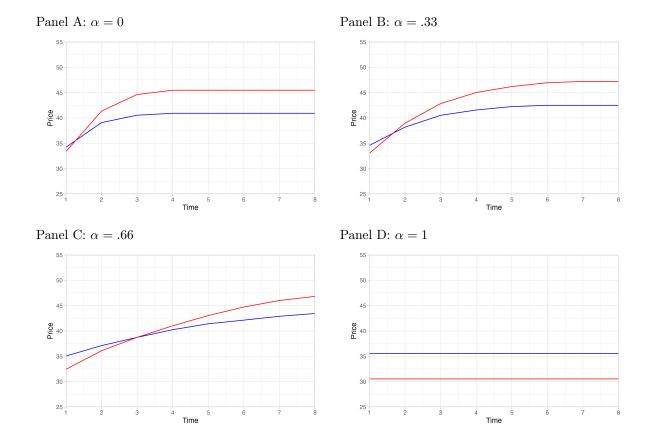


Figure 7: Optimal Price Paths Before (Red) and After (Blue) a 10% Reduction in Demand by α .

Notes: This figure depicts optimal price paths before (red) and after (blue) a 10% reduction in the demand parameter a for a variety of α values. Parameters: $b=2, e=1, c=0, N=4, \delta=.25, F=0$ and l=10. Prior to the deterioration in market conditions, a=100. The blue curve depicts the optimal price path after the deterioration in market conditions (i.e., a=90) and the red curve depicts the optimal price path prior to the deterioration in market conditions (i.e., a=100).

Figure 7 presents the optimal price path before (red) and after (blue) a reduction in demand for a range of α values. In these simulations, $m(r,u) = \alpha r + (1-\alpha)u$ where $\alpha \in [0,1]$ determines the speed of reference point adjustment. $\alpha = 1$ corresponds to constant reference points that do not adjust in response to experienced utility levels (i.e., m(r,u) = r as in Proposition 4). $\alpha = 0$ corresponds to reference points that immediately and fully adjust to experienced utility levels (i.e., m(r,u) = u). Figure 7 illustrates that the length of time that collusive prices under regime a exceed collusive prices under regime b increases with a. Consistent with Proposition 4, the reduction in demand reduces collusive prices in all periods when reference points are constant ($\alpha = 1$).

The results of this section establish that a deterioration in market conditions can cause cartels to set higher prices, particularly in the periods immediately following a cartel's formation. Whether consumer surplus when a cartel is active under under regime a is less than consumer surplus when a cartel is active under regime b depends crucially on consumers' discount rate and how rapidly reference points adjust to changes in utility (i.e., the function m).⁶⁷ When consumers' are impatient (i.e., a low discount factor), they heavily discount future periods wherein prices are higher under regime b and place a relatively large weight on earlier periods wherein prices are higher under regime a. As Proposition 4 illustrates, if reference points adjust sufficiently slowly to changes in utility, then cartel prices under regime a may exceed cartel prices under regime b for a long period of time, resulting in diminished consumer surplus.

7 Conclusion

Empirical evidence suggests that deteriorations in market conditions, such as a decrease in demand, an increase in marginal cost or the entry of a competitor, often precede the formation of a cartel. However, conventional theoretical models, which assume managers are loss neutral, do not imply that deteriorations of this kind enhance the sustainability of collusion. The preceding analysis establishes that, when colluding managers are instead averse to losses, deteriorations in market conditions can cause the formation of a cartel. Loss averse managers perceive continued competition, after a deterioration in market conditions, as a painful loss in utility. To avoid a loss, managers turn to collusion. Due to similar considerations, I find that deteriorations in market conditions can also enhance the payoff managers receive from collusion and the gain in utility from collusion. Deteriorations in market state can also result in higher collusion prices, particularly in periods immediately following a cartel's formation.

While the preceding model is general in many respects (e.g., it does not assume a particular demand or cost specification), there remain a number of important modeling restrictions and conditions. First, the preceding analysis considers collusion by means of grim trigger strategies rather than more sophisticated stick and carrot punishments (Abreu, 1986). If a deterioration in market state permits firms to enforce stronger punishments (i.e., lower profit levels in the "stick" phase of collusion), then the deterioration may facilitate collusion between loss averse managers, as under grim trigger strategies. Second, I assume managers/firms are symmetric throughout the analysis. The results of this study seem like to hold when managers/firms are asymmetric as long as the deterioration in market conditions causes continued competition to be perceived as a loss by all managers. Third, this study focuses on permanent changes in the market environment. Fourth, the preceding results apply only when managers are sufficiently loss averse and when the deterioration in market conditions is moderate. If managers are relatively loss neutral, deteriorations in market state are likely to instead hinder or have no impact on incentives to collude. Additionally, the impact of extreme changes in market state, which result in losses regardless of whether managers collude, on incentives to

⁶⁷Note that the change in market state may influence the consumer surplus function (e.g., a change in a demand parameter) which must be taken into account when assessing the impact of a change in market state on consumer surplus.

collude is unclear and likely depends on a number of subtle considerations such as the curvature of the function $L(\cdot)$, and the speed that reference points update over time. For expositional purposes, the model presented in the main text does not include an antitrust authority that detects and penalizes cartels.⁶⁸ The model is extended to incorporate the presence of such an antitrust authority in Appendix D.

Results of this study, together with prior empirical evidence, suggest that difficult conditions in a market may indicate an increased risk of cartelization. Additionally, managerial compensation structures that severely penalize managers for performing below a pre-defined threshold may create an incentive to collude during an industry crisis. Antitrust authorities may wish to examine or preventatively screen industries where compensation structures of this kind are prevalent and/or the market environment has deteriorated.

⁶⁸Recall that collusion in the present study need not be illegal. I refer to a group of colluding managers as a cartel for ease of exposition.

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A Proofs

Proof of Lemma 1. First, I show that the payoff function $F\left(\{x_t\}_{t=1}^{\infty}\right) = \sum_{t=1}^{\infty} \delta^{t-1} u(\pi(x_t); r_t)$ is a continuous function $F: \Omega^{\infty} \to \mathbb{R}$ where \mathbb{R} is endowed with the standard topology. Ω^{∞} is endowed with the product topology which implies the projection $p_T\left(\{x_t\}_{t=1}^{\infty}\right) = x_T$ is continuous for all T.

First, I show that $u(\pi(x_T); r_T)$ is continuous in $\{x_t\}_{t=1}^{\infty}$ on the product space Ω^{∞} for all T. Let $u_T = u(\pi(p_T(\{x_t\}_{t=1}^{\infty})))$ denote base utility in period T as a function of $\{x_t\}_{t=1}^{\infty}$. u_T is continuous in $\{x_t\}_{t=1}^{\infty}$ by the continuity of p_T , the continuity of π (Assumption 5(i)), and the continuity of u (Assumption 1(i)). Next, I show that r_T is continuous in $\{x_t\}_{t=1}^{\infty}$ for all T. The proof follows by induction. $r_2 = m(r_1, u_1)$ is continuous in $\{x_t\}_{t=1}^{\infty}$ by the continuity of m (Assumption 3(iv)) and the continuity of u_1 in $\{x_t\}_{t=1}^{\infty}$. Suppose r_{T-1} is continuous in $\{x_t\}_{t=1}^{\infty}$. Then $r_T = m(r_{T-1}, u_T)$ is continuous in $\{x_t\}_{t=1}^{\infty}$ by the continuity of m (Assumption 3(iv)), the continuity of r_{T-1} , the continuity of u_T , and the fact that compositions of continuous functions are continuous. Finally, $u(\pi(x_T); r_T) = u_T - lL(r_T - u_T)$ is continuous in $\{x_t\}_{t=1}^{\infty}$ by the continuity of t_T in t_T

As sums of continuous functions are continuous,

$$F_n(\{x_t\}_{t=1}^{\infty}) = \sum_{t=1}^{n} \delta^{t-1} u(\pi(x_t); r_t)$$

is continuous for all n. $\{F_n(\{x_t\}_{t=1}^{\infty})\}_{n=1}^{\infty}$ is a sequence of continuous functions that converges uniformly to $F(\{x_t\}_{t=1}^{\infty})$. To see that $\{F_n(\{x_t\}_{t=1}^{\infty})\}_{n=1}^{\infty}$ converges uniformly, note that $\pi(x)$ is a continuous function (Assumption 5(i)) on a compact set. Thus, $\pi(x)$ is bounded above and below. Let $\bar{\pi}$ denote the upper bound and let $\underline{\pi}$ denote the lower bound. $u(\pi(x); r_t)$ is therefore bounded above and below as

$$\underline{u} \equiv u(\pi; \bar{r}) \le u(\pi(x); r_t) \le u(\bar{\pi}) \equiv \bar{u}$$

where $r_t \leq \bar{r} = \max\{\bar{u}, r_1\}$ by Assumption 3(i). Let $\epsilon > 0$, then

$$|F_n(\{x_t\}_{t=1}^{\infty}) - F(\{x_t\}_{t=1}^{\infty})| = \left|\sum_{t=n+1}^{\infty} \delta^{t-1} u(\pi(x_t); r_t)\right| \le \sum_{t=n+1}^{\infty} \delta^{t-1} \max\{|\bar{u}|, |\underline{u}|\} = \delta^n \frac{\max\{|\bar{u}|, |\underline{u}|\}}{1 - \delta} < \epsilon$$

for sufficiently large n, for all $\{x_t\}_{t=1}^{\infty} \in \Omega^{\infty}$. Thus, $\{F_n\left(\{x_t\}_{t=1}^{\infty}\right)\}_{n=1}^{\infty}$ converges uniformly to $F\left(\{x_t\}_{t=1}^{\infty}\right)$.

Thus, the uniform limit theorem implies that $F(\{x_t\}_{t=1}^{\infty})$ is continuous on Ω^{∞} . Ω is compact which implies, by Tychonoff's Product Theorem, Ω^{∞} is compact. The continuity of $F(\{x_t\}_{t=1}^{\infty})$ in $\{x_t\}_{t=1}^{\infty}$, the

 $\text{continuity of } u(\pi^D(x_t); r_t) = u(\pi^D(p_t\left(\{x_t\}_{t=1}^\infty\right)); r_t) \text{ in } \{x_t\}_{t=1}^\infty, ^{69} \text{ and the continuity of } V^N(r_T) \text{ in } \{x_t\}_{t=1}^\infty, ^{70} \text{ in } \{x_t\}_{t=1}^\infty, ^{69} \text{ and the continuity of } V^N(r_T) \text{ in } \{x_t\}_{t=1}^\infty, ^{70} \text{ in } \{x_t\}_{t$ implies that both sides of the inequality constraints in (3) are continuous. Thus, Ψ is a closed set. Ψ is a closed subset of a compact set Ω^{∞} which implies Ψ is compact. $\Psi \neq \emptyset$ by supposition. A solution to (2) exists as the cartel's problem involves maximizing a continuous function $F(\lbrace x_t \rbrace_{t=1}^{\infty})$ over a compact set Ψ.

The following lemma establishes that the infinite repetition of the Nash equilibrium constitutes a subgame perfect equilibrium of the dynamic game. This result ensures that Nash competition in the punishment phase is self-enforcing (i.e., no firm wishes to deviate during the punishment phase). Let x^N denote the Nash equilibrium price and let π^N denote Nash equilibrium profits. Let $\tilde{\pi}(x;x^N)$ denote a managers payoff when charging price x when all rivals charge price x^N .

Lemma 2. A strategy profile wherein all managers charge price x^N in all periods (regardless of prior play) constitutes a sub-game perfect equilibrium of the dynamic game.

Proof. First, I establish that the proposed equilibrium is a Nash equilibrium of the dynamic game. Strategy profiles in the dynamic game are characterized by a function $s:\mathcal{H}\to\Omega$ where \mathcal{H} is the set of possible pricing histories of all managers. The proposed equilibrium strategy profile is s^N where $s^N(H) = x^N$ for all H. Suppose manager 1 deviates to an alternative strategy function \tilde{s} . Let \tilde{x}_1, \tilde{x}_2 ... denote the sequence of prices consistent with manager 1 following strategy function \tilde{s} and all rival managers following the strategy function s^N . As x^N is the Nash equilibrium price, manager 1 earns profit $\tilde{\pi}(\tilde{x}_t; x^N) \leq \pi^N$ in each period t. By Assumption 10, the discounted present value of manager 1's payoff is less than or equal to its payoff from repeated Nash competition. Thus, the manager does not wish to defect.

Next, I establish that the proposed equilibrium is a sub-game perfect equilibrium. Note that the history of the game (i.e., prices set by all managers in previous periods) impacts a manager's future payoff only through the manager's reference point. As sub-games in the dynamic game correspond to distinct pricing histories, it suffices to establish that no manager wishes to defect for any (current) reference point r. The result follows by the above argument, which holds for any r, and the fact that Assumption 10 applies for any possible reference point.

The following Lemma establishes that $\left\{x_a^M\right\}_{t=1}^{\infty}$ is incentive compatible when l is sufficiently large. Let $\Psi_i(l)$ denote the set of incentive compatible price paths under regime i when the degree of loss aversion is l. Additionally, let $\pi_i^{DM} \equiv \pi_i^D(x_i^M)$ denote the profit a manager earns when defecting from collusion when the collusive price is x_i^M under regime i.

The continuity of $u(\pi^D(x_t); r_t)$ in $\{x_t\}_{t=1}^{\infty}$ follows from an analogous argument as above. Note that $\pi^D(x; a)$ is continuous by Assumption 6(i).

The continuity of $V^N(r_T)$ in $\{x_t\}_{t=1}^{\infty}$ follows from the continuity of r_T in $\{x_t\}_{t=1}^{\infty}$, shown earlier in the proof.

Lemma 3. $\left\{x_a^M\right\}_{t=1}^{\infty} \in \Psi_a(l)$ if

$$l \geq \bar{l} = \frac{u(\pi_a^{DM}) + \delta \frac{u(\pi_a^N)}{1-\delta} - \frac{u(\pi_a^M)}{1-\delta}}{\delta L \left(r_1 - u(\pi_a^N)\right)}.$$

Proof. It suffices to show that

$$V_a^M(r_T; l) \equiv \sum_{t=T}^{\infty} \delta^{t-T} u(\pi_a^M; r_t, l) \ge u(\pi_a^{DM}; r_T, l) + \delta V_a^N(r_T; l)$$
(4)

for all $T \in \{1, 2, 3...\}$ where $r_t = m(r_{t-1}, u(\pi_a^M))$ for t > 1. Assumption 3(i) and Assumption 8(i) imply that $r_t \le r_{t+1}$ for all t and $r_t \le u(\pi_a^M)$ for all t. Thus,

$$u(\pi_a^M; r_t, l) = u(\pi_a^M) \tag{5}$$

for all t. Additionally, $u(\pi_a^{DM}; r_T, l) = u(\pi_a^{DM})$ by Assumption 6(ii) and $r_t \leq u(\pi_a^M)$ for all t. Therefore, the inequalities in (4) become

$$\sum_{t=T}^{\infty} \delta^{t-T} u(\pi_a^M) \ge u(\pi_a^{DM}) + \delta V_a^N(r_T; l)$$

for all $T \in \{1, 2, 3...\}$, or

$$\frac{u(\pi_a^M)}{1-\delta} - u(\pi_a^{DM}) \ge \delta V_a^N(r_T; l).$$

for all $T \in \{1, 2, 3...\}$. Fix $T \in \{1, 2, 3...\}$. Let $\tilde{r}_T = r_T$ and let $\tilde{r}_t = m(\tilde{r}_{t-1}, u(\pi_a^N))$ for t > T. Note that

$$V_{a}^{N}(r_{T};l) = \sum_{t=T}^{\infty} \delta^{t-T} u(\pi_{a}^{N}; \tilde{r}_{t}, l) \leq u(\pi_{a}^{N}; r_{1}, l) + \sum_{t=T+1}^{\infty} \delta^{t-T} u(\pi_{a}^{N}; \tilde{r}_{t}, l)$$

$$\leq u(\pi_{a}^{N}; r_{1}, l) + \sum_{t=T+1}^{\infty} \delta^{t-T} u(\pi_{a}^{N})$$

$$= u(\pi_{a}^{N}; r_{1}, l) + \sum_{t=1}^{\infty} \delta^{t} u(\pi_{a}^{N})$$

$$= u(\pi_{a}^{N}; r_{1}, l) + \delta \frac{u(\pi_{a}^{N})}{1 - \delta}$$
(6)

where the first inequality follows from $\tilde{r}_T = r_T \ge r_1$ for all T and Assumption 2. The second inequality in Equation (6) follows from $u(\pi; r, l) \le u(\pi)$ (Assumption 2).

 $r_1 > u(\pi_a^N)$ by $r_1 = u(\pi_b^N)$ (Assumption 7) and $\pi_a^N < \pi_b^N$ (Assumption 8(ii)). Thus, $L(r_1 - u(\pi_a^N)) > 0$ by Assumption 2(ii).

Suppose $l \geq \bar{l}$. Then,

$$u(\pi_{a}^{DM}) + \delta V_{a}^{N}(r_{T}; l) \leq u(\pi_{a}^{DM}) + \delta u(\pi_{a}^{N}; r_{1}, l) + \delta^{2} \frac{u(\pi_{a}^{N})}{1 - \delta}$$

$$= u(\pi_{a}^{DM}) + \delta u(\pi_{a}^{N}) - \delta lL(r_{1} - u(\pi_{a}^{N})) + \delta^{2} \frac{u(\pi_{a}^{N})}{1 - \delta}$$

$$\leq u(\pi_{a}^{DM}) + \delta u(\pi_{a}^{N}) - \delta \bar{l}L(r_{1} - u(\pi_{a}^{N})) + \delta^{2} \frac{u(\pi_{a}^{N})}{1 - \delta}$$

$$= u(\pi_{a}^{DM}) + \delta u(\pi_{a}^{N})$$

$$- \left[u(\pi_{a}^{DM}) + \delta \frac{u(\pi_{a}^{N})}{1 - \delta} - \frac{u(\pi_{a}^{M})}{1 - \delta} \right] + \delta^{2} \frac{u(\pi_{a}^{N})}{1 - \delta}$$

$$= \delta u(\pi_{a}^{N}) - \delta \frac{u(\pi_{a}^{N})}{1 - \delta} + \frac{u(\pi_{a}^{M})}{1 - \delta} + \delta^{2} \frac{u(\pi_{a}^{N})}{1 - \delta}$$

$$= \delta u(\pi_{a}^{N}) - (1 - \delta) \delta \frac{u(\pi_{a}^{N})}{1 - \delta} + \frac{u(\pi_{a}^{M})}{1 - \delta}$$

$$= \frac{u(\pi_{a}^{M})}{1 - \delta}$$

$$= \frac{u(\pi_{a}^{M})}{1 - \delta}$$
(8)

where the first inequality follows from (6). The first equality follows from the definition of $u(\pi_a^N; r_1, l)$. The second inequality follows from $l \geq \bar{l}$. The second equality follows from the definition of \bar{l} and $L(r_1 - u(\pi_a^N)) > 0$ (shown earlier in the proof). Thus, Equation 8 implies $\{x_a^M\}_{t=1}^{\infty} \in \Psi_a(l) \text{ when } l \geq \bar{l}$.

The following Lemma establishes that the monopoly price path is the unique optimal price path when the monopoly price path is ICC.

Lemma 4. $\left\{x_a^M\right\}_{t=1}^{\infty}$ is the unique optimal price path when $\left\{x_a^M\right\}_{t=1}^{\infty} \in \Psi_a(l)$.

Proof. It suffices to show that $\{x_a^M\}_{t=1}^{\infty}$ generates a strictly larger collusive payoff than any other path when $\{x_a^M\}_{t=1}^{\infty} \in \Psi_a(l)$. Let $\{x_t\}_{t=1}^{\infty} \in \Omega^{\infty}$ be an alternative path where $\{x_t\}_{t=1}^{\infty} \neq \{x_a^M\}_{t=1}^{\infty}$. Let $W(\{\pi_a(x_t)\}_{t=1}^{\infty}; r_1)$ denote the payoff from collusion when the path is $\{x_t\}_{t=1}^{\infty}$ and the initial reference point is r_1 .

$$V_a^M(r_1; l) = \frac{u(\pi_a^M)}{1 - \delta} > \sum_{t=1}^{\infty} \delta^{t-1} u(\pi_a(x_t)) \ge W(\{\pi_a(x_t)\}_{t=1}^{\infty}; r_1)$$

where the equality follows from $r_t \leq u(\pi_a^M)$ for all t (which follows from Assumption 3(i) and Assumption 8(i)) and $\pi_a^M \geq \pi_a(x_t)$ for all t (which follows from Assumption 5(ii)). The first inequality follows from $\pi_a^M > \pi_a(x_t)$ for some t as $\{x_t\}_{t=1}^{\infty} \neq \{x_a^M\}_{t=1}^{\infty}$ and by Assumption 5(ii). The second inequality follows from from $u(\pi; r, l) \leq u(\pi)$ (Assumption 2). Therefore, $\{x_a^M\}_{t=1}^{\infty}$ is the unique optimal price path when $\{x_a^M\}_{t=1}^{\infty} \in \Psi_a(l)$.

Lemma 5. $V_a^C(l) = V_a^M(l)$ when $\{x_a^M\}_{t=1}^{\infty} \in \Psi_a(l)$.

Proof. The result follows immediately from Lemma 4.

The following Lemma establishes that a cartel forms when managers are sufficiently loss averse.

Lemma 6. $V_a^C(l) > V_a^N(l)$ (thus, a cartel forms) if

$$l \ge \bar{l} = \frac{u(\pi_a^{DM}) + \delta \frac{u(\pi_a^N)}{1-\delta} - \frac{u(\pi_a^M)}{1-\delta}}{\delta L \left(r_1 - u(\pi_a^N)\right)}.$$

Proof. When $l \geq \bar{l}$, $\left\{x_a^M\right\}_{t=1}^{\infty} \in \Psi_a(l)$ (by Lemma 3) and $V_a^C(l) = V_a^M(l)$ (by Lemma 5). It remains to establish that $V_a^M(l) > V_a^N(l)$. Note that

$$u(\pi_a^N) < u(\pi_b^N) = r_1 < u(\pi_a^M)$$
 (9)

where the first inequality follows from Assumption 8(ii), the equality follows from Assumption 7, and the second inequality follows from Assumption 8(i). Thus,

$$\begin{aligned} V_a^N(l) &\leq \sum_{t=1}^\infty \delta^{t-1} u(\pi_a^N) \\ &< \sum_{t=1}^\infty \delta^{t-1} u(\pi_a^M) \\ &= \frac{u(\pi_a^M)}{1-\delta} = V_a^C(l) \end{aligned}$$

where the first inequality follows from the fact that $u(\pi; r, l) \leq u(\pi)$ (Assumption 2). The second inequality follows from Equation 9. As a result, the cartel forms when $l \geq \bar{l}$.

Proof of Proposition 1. Suppose $l \geq \bar{l} = \frac{u(\pi_a^{DM}) + \delta \frac{u(\pi_a^N)}{1-\delta} - \frac{u(\pi_a^M)}{1-\delta}}{\delta L(r_1 - u(\pi_a^N))}$. Lemmas 3-6 imply that $V_a^C(l) = V_a^M(l) > V_a^N(l)$ (thus, a cartel forms) when $l \geq \bar{l}$.

Proof of Proposition 2. Suppose $l \geq \bar{l} = \frac{u(\pi_a^{DM}) + \delta \frac{u(\pi_a^N)}{1-\delta} - \frac{u(\pi_a^M)}{1-\delta}}{\delta L(r_1 - u(\pi_a^N))}$. Part (i) follows from

$$V_b^C(l) < V_a^M = V_a^C(l)$$

when $l \geq \bar{l}$. The first inequality follows from Condition 2(ii). The equality follows from Lemma 5.

Next, consider Part (ii). Assumption 8(ii) and Assumption 10 imply

$$V_b^N(l) = W\left(\{\pi_b^N\}_{t=1}^{\infty}; r_1\right) \ge W\left(\{\pi_a^N\}_{t=1}^{\infty}; r_1\right) = V_a^N(l).$$

 $V_a^N(l) \leq V_b^N(l)$ and $V_b^C(l) < V_a^C(l)$ (from part (i)) imply

$$V_a^C(l) - V_a^N(l) > V_b^C(l) - V_b^N(l)$$

Proof of Proposition 3. The proof follows from Proposition 5.⁷¹

Proof of Proposition 4. Suppose $l \geq \bar{l} = \frac{u(\pi_a^{DM}) + \delta \frac{u(\pi_a^N)}{1-\delta} - \frac{u(\pi_a^M)}{1-\delta}}{\delta L(r_1 - u(\pi_a^N))}$. By Lemmas 3-6, $\left\{x_a^M\right\}_{t=1}^{\infty} \in \Psi_a(l)$ and a cartel forms when $l \geq \bar{l}$ after the change. $\left\{x_a^M\right\}_{t=1}^{\infty}$ is the unique optimal price path by Lemma 4. Thus, it suffices to show that $x_a^M > x_{b,1}$, when $l \geq \bar{l}$. First, note that if a cartel does not form before the change, then $x_{b,t}=x_b^N$ for all t and, by Condition 3(ii), $x_{a,t}=x_a^M>x_b^N=x_{b,t}$ for all t. For the remainder of the proof, assume a cartel forms before the change in market state.

First, note that $u(\pi_b^D(x_a^M)) - u(\pi_b(x_a^M)) \ge 0$ by Assumption 6(ii). Suppose $u(\pi_b^D(x_a^M)) - u(\pi_b(x_a^M)) = 0$. By Assumption 11(iv), $u(\pi_b^D(x)) - u(\pi_b(x))$ is strictly increasing in x for all $x \geq x_b^N$. Thus, $u(\pi_b^D(x_a^M))$ $u(\pi_b(x_a^M)) = 0$ implies $u(\pi_b^D(x)) < u(\pi_b(x))$ for $x < x_a^M$ which contradicts Assumption 6(ii). Thus, $u(\pi_b^D(x_a^M)) - u(\pi_b(x_a^M)) > 0$. As a cartel forms before the change, $\pi_b^M \ge \pi_b^N$ must hold.⁷² As m(r, u) = r, $r_t = r_1$ for all t.

Suppose there exists a T_1 such that $x_{b,T_1} \geq x_{a,T_1} = x_a^M$. It follows directly from the derivations in the proof of Proposition 3 that $G(x_a^M) < 0$ (where G is defined in Proposition 3) under Condition 3. G(x) is strictly decreasing in x for $x \ge x_b^N$ by Assumption 11(iv). Thus, G(x) < 0 for all $x \ge x_a^M \ge x_b^N$ (where the last inequality follows from Condition 3(ii))

If
$$x_{b,T_1} \geq x_a^M$$
, then

⁷¹Proposition 5 establishes the conclusion under the more general Assumption 12 and Condition 4. ⁷²If $\pi_b^M < \pi_b^N$, then the maximum payoff from collusion is less than the payoff from Nash competition. Therefore, the cartel

$$u(\pi_{b}(x_{b,T_{1}}); r_{1}) + \sum_{t=T_{1}+1}^{\infty} \delta^{t-T_{1}} u(\pi_{b}(x_{b,t}); r_{1})$$

$$\leq u(\pi_{b}(x_{b,T_{1}})) - lL(r_{1} - u(\pi_{b}(x_{b,T_{1}}))) + \sum_{t=T_{1}+1}^{\infty} \delta^{t-T_{1}} u(\pi_{b}(x_{b,t}))$$

$$\leq u(\pi_{b}(x_{b,T_{1}})) - lL(r_{1} - u(\pi_{b}(x_{b,T_{1}}))) + \delta \frac{u(\pi_{b}(x_{b}^{M}))}{1 - \delta}$$

$$< u(\pi_{b}^{D}(x_{b,T_{1}})) - lL(r_{1} - u(\pi_{b}(x_{b,T_{1}}))) + \delta V_{b}^{N}(r_{1})$$

$$\leq u(\pi_{b}^{D}(x_{b,T_{1}})) - lL(r_{1} - u(\pi_{b}^{D}(x_{b,T_{1}}))) + \delta V_{b}^{N}(r_{1})$$

$$= u(\pi_{b}^{D}(x_{b,T_{1}}); r_{1}) + \delta V_{b}^{N}(r_{1})$$

$$(10)$$

where the first inequality follows from the fact that $u(\pi;r) \leq u(\pi)$ (Assumption 2). The second inequality follows from Assumption 5(ii). The third inequality follows from $G(x_{b,T_1}) < 0$ for $x_{b,T_1} \geq x_a^M$. The fourth inequality follows from Assumption 6(ii) and Assumption 2(ii). (10) implies that $x_{b,T_1} < x_a^M$ as the ICC in period T_1 is not satisfied if $x_{b,T_1} \geq x_a^M$. Thus, $x_{b,T_1} < x_a^M$, a contradiction.

B Robustness of Assumption 7

In this section, I examine the robustness of results to Assumption 7. Specifically, I consider alternative values for the initial reference point r_1 . Throughout this section, I place the following restriction on r_1 , in place of Assumption 7.

Assumption 12. $r_1 > u(\pi_a^N)$

Assumption 12 ensures the Nash equilibrium utility level (after the change) is perceived as a loss. If $r_1 \leq u(\pi_a^N)$, then the change in market conditions is not perceived as a loss by managers and the effects highlighted in this study are not applicable. Recall that $r_1 < u(\pi_a^M)$ by Assumption 8(i). Thus, Assumption 12 and Assumption 8(i) together imply $r_1 \in (u(\pi_a^N), u(\pi_a^M))$.

Lemmas 3-6 continue to hold under Assumption 12 with only slight modification to the proof of Lemma 6 in Appendix A. Specifically, Equation (9) becomes

$$u(\pi_a^N) < r_1 < u(\pi_a^M) \tag{11}$$

where the first inequality follows from Assumption 12 and the second inequality follows from Assumption 8(i). Thus, a cartel forms and $V_a^C(l) = V_a^M(l)$ when $l \geq \bar{l} = \frac{u(\pi_a^{DM}) + \delta \frac{u(\pi_a^N)}{1-\delta} - \frac{u(\pi_a^M)}{1-\delta}}{\delta L(r_1 - u(\pi_a^N))}$ under regime a. Proposition 1,

Proposition 2 and their corresponding proofs from Appendix A hold without modification under Assumption 12.

Next, I establish that pricing results from Section 6 continue to hold when Assumption 12 holds (in place of Assumption 7). The following condition generalizes Condition 3 from the main text to reflect Assumption 12, and is equivalent to Condition 3 when $r_1 = u(\pi_b^N)$ (i.e., Assumption 7 holds).

Condition 4. i)
$$\delta < \frac{u(\pi_b^D(x_a^M)) - u(\pi_b(x_a^M))}{u(\pi_b(x_b^M)) - \left(u(\pi_b^N) - lL(r_1 - u(\pi_b^N))\right) + u(\pi_b^D(x_a^M)) - u(\pi_b(x_a^M))}$$
, and ii) $x_b^N < x_a^M$.

Let $\{x_{b,t}\}_{t=1}^{\infty}$ and $\{x_{a,t}\}_{t=1}^{\infty}$ denote optimal price paths under regime b and regime a, respectively. The following proposition establishes that a deterioration in market state can result in higher collusive prices when managers are sufficiently loss averse.

Proposition 5. Suppose Condition 4 holds. Then, there exists an \bar{l} such that if $l \geq \bar{l}$, then there exists a $T \in \{1, 2...\} \cup \{\infty\}$ such that $x_{b,t} < x_{a,t}$ for all $t \leq T$.

Proof. Suppose $l \geq \bar{l} = \frac{u(\pi_a^{DM}) + \delta \frac{u(\pi_a^N)}{1-\delta} - \frac{u(\pi_a^M)}{1-\delta}}{\delta L(r_1 - u(\pi_a^N))}$. By Lemma 6, $\{x_a^M\}_{t=1}^{\infty} \in \Psi_a(l)$ and a cartel forms under regime a and $l \geq \bar{l}$. $\{x_a^M\}_{t=1}^{\infty}$ is the unique optimal price path by Lemma 4. Thus, it suffices to show that $x_{a,1} = x_a^M > x_{b,1}$ when $l \geq \bar{l}$. First, note that if a cartel does not form before the change, then $x_{b,t} = x_b^N$ for all t and, by Condition 4(ii), $x_{a,1} = x_a^M > x_b^N = x_{b,1}$. For the remainder of the proof, assume a cartel forms before the change in market state.

Note that $u(\pi_b^D(x_a^M)) - u(\pi_b(x_a^M)) \ge 0$ by Assumption 6(ii). First, suppose $u(\pi_b^D(x_a^M)) - u(\pi_b(x_a^M)) = 0$. By Assumption 11(iv), $u(\pi_b^D(x)) - u(\pi_b(x))$ is strictly increasing in x for all $x \ge x_b^N$. Thus, $u(\pi_b^D(x_a^M)) - u(\pi_b(x_a^M)) = 0$ implies $u(\pi_b^D(x)) < u(\pi_b(x))$ for $x < x_a^M$ which contradicts Assumption 6(ii). Thus, $u(\pi_b^D(x_a^M)) - u(\pi_b(x_a^M)) > 0$. As a cartel forms before the change, $u(\pi(x_b^M; a_0)) \ge u(\pi_b^N) - lL(r_1 - u(\pi_b^N))$ must hold.⁷³ $u(\pi_b^D(x_a^M)) > u(\pi_b(x_a^M))$ and $u(\pi(x_b^M; a_0)) \ge u(\pi_b^N) - lL(r_1 - \pi_b^N)$ imply that the numerator and denominator of the expression in Condition 4(i) are positive.

Let $\hat{r}_1 = r_1$ and let $\hat{r}_{t+1} = m(\hat{r}_t, u(\pi_b^N))$. If $r_1 > u(\pi_b^N)$, then

$$\frac{u(\pi_b^N) - lL(r_1 - u(\pi_b^N))}{1 - \delta} \le V_b^N = \sum_{t=1}^{\infty} \delta^{t-1} \left[u(\pi_b^N) - lL(\hat{r}_t - u(\pi_b^N)) \right].$$

as $\hat{r}_t \leq r_1$ for all t by Assumption 3(i). If $r_1 \leq u(\pi_b^N)$, then

$$\frac{u(\pi_b^N) - lL(r_1 - u(\pi_b^N))}{1 - \delta} = \frac{u(\pi_b^N)}{1 - \delta} = V_b^N = \sum_{t=1}^{\infty} \delta^{t-1} u(\pi_b^N)$$

 $^{73 \}text{Otherwise, } V_b^C \leq \frac{u(\pi_b^M)}{1-\delta} < \frac{u(\pi_b^N) - lL(r_1 - u(\pi_b^N))}{1-\delta} \leq V_b^N \text{ and a cartel does not form.}$

as $\hat{r}_t \leq u(\pi_b^N)$ for all t by Assumption 3(i). Thus,

$$\frac{u(\pi_b^N) - lL(r_1 - u(\pi_b^N))}{1 - \delta} \le V_b^N.$$
(12)

Let

$$G(x) = u(\pi_b(x)) + \delta \frac{u(\pi_b^M)}{1 - \delta} - \left(u(\pi_b^D(x)) + \delta V_b^N\right).$$

Condition 4(i) states

$$\frac{u(\pi_b^D(x_a^M)) - u(\pi_b(x_a^M))}{u(\pi_b^M) - (u(\pi_b^N) - lL(r_1 - u(\pi_b^N))) + u(\pi_b^D(x_a^M)) - u(\pi_b(x_a^M))} > \delta$$

$$\iff \delta \left[u(\pi_b^M) - \left(u(\pi_b^N) - lL(r_1 - u(\pi_b^N)) \right) + u(\pi_b^D(x_a^M)) - u(\pi_b(x_a^M)) \right]$$

$$< u(\pi_b^D(x_a^M)) - u(\pi_b(x_a^M))$$

$$\Leftrightarrow \delta \left[u(\pi_b^M) - \left(u(\pi_b^N) - lL(r_1 - u(\pi_b^N)) \right) + u(\pi_b^D(x_a^M)) - u(\pi_b(x_a^M)) \right]$$

$$+ u(\pi_b(x_a^M)) - u(\pi_b^D(x_a^M)) - \delta \left(u(\pi_b(x_a^M)) - u(\pi_b^D(x_a^M)) \right)$$

$$\Leftrightarrow \delta \left[u(\pi_b^M) - \left(u(\pi_b^N) - lL(r_1 - u(\pi_b^N)) \right) + \delta \left(u(\pi_b^M) - (u(\pi_b^N) - lL(r_1 - u(\pi_b^N))) \right) \right] < 0$$

$$\Leftrightarrow \delta \left[u(\pi_b(x_a^M)) - u(\pi_b^D(x_a^M)) + \delta \left(u(\pi_b^M) - u(\pi_b^N) - u(\pi_b^N) \right) \right] < 0$$

$$\Leftrightarrow \delta \left[u(\pi_b(x_a^M)) - u(\pi_b^D(x_a^M)) + \delta \left(u(\pi_b^M) - u(\pi_b^N) - u(\pi_b^N) \right) \right] < 0$$

$$\Leftrightarrow \delta \left[u(\pi_b(x_a^M)) - u(\pi_b^D(x_a^M)) + \delta \left(u(\pi_b^M) - u(\pi_b^N) - u(\pi_b^N) - u(\pi_b^N) \right) \right] < 0$$

$$\Leftrightarrow \delta \left[u(\pi_b(x_a^M)) - u(\pi_b^D(x_a^M)) + \delta \left(u(\pi_b^M) - u(\pi_b^N) - u(\pi_b^N) - u(\pi_b^N) \right) \right] < 0$$

$$\Leftrightarrow \delta \left[u(\pi_b(x_a^M)) - u(\pi_b^D(x_a^M)) + \delta \left(u(\pi_b^M) - u(\pi_b^N) - u(\pi_b^M) - u(\pi_b^N) \right) \right] < 0$$

$$\Leftrightarrow \delta \left[u(\pi_b(x_a^M)) - u(\pi_b^D(x_a^M)) + \delta \left(u(\pi_b^M) - u(\pi_b^M) - u(\pi_b^M) - u(\pi_b^M) \right) \right] < 0$$

$$\Leftrightarrow \delta \left[u(\pi_b(x_a^M)) - u(\pi_b^D(x_a^M)) + \delta \left(u(\pi_b^M) - u(\pi_b^M) - u(\pi_b^M) - u(\pi_b^M) \right) \right] < 0$$

$$\Leftrightarrow \delta \left[u(\pi_b(x_a^M)) - u(\pi_b^D(x_a^M)) + \delta \left(u(\pi_b^M) - u(\pi_b^M) - u(\pi_b^M) - u(\pi_b^M) \right) \right] < 0$$

$$\Leftrightarrow \delta \left[u(\pi_b(x_a^M)) - u(\pi_b^D(x_a^M)) + \delta \left(u(\pi_b^M) - u(\pi_b^M) - u(\pi_b^M) - u(\pi_b^M) \right) \right] < 0$$

$$\Leftrightarrow \delta \left[u(\pi_b(x_a^M)) - u(\pi_b^D(x_a^M)) + \delta \left(u(\pi_b^M) - u(\pi_b^M) - u(\pi_b^M) - u(\pi_b^M) \right) \right] < 0$$

$$\Leftrightarrow \delta \left[u(\pi_b(x_a^M)) - u(\pi_b^D(x_a^M)) + \delta \left(u(\pi_b^M) - u(\pi_b^M) - u(\pi_b^M) - u(\pi_b^M) \right) \right] < 0$$

$$\Leftrightarrow \delta \left[u(\pi_b(x_a^M)) - u(\pi_b^D(x_a^M)) + \delta \left(u(\pi_b^M) - u(\pi_b^M) - u(\pi_b^M) - u(\pi_b^M) \right) \right] < 0$$

$$\Leftrightarrow \delta \left[u(\pi_b(x_a^M)) - u(\pi_b^D(x_a^M)) + \delta \left(u(\pi_b^M) - u(\pi_b^M) - u(\pi_b^M) - u(\pi_b^M) \right) \right] < 0$$

$$\Leftrightarrow \delta \left[u(\pi_b(x_a^M)) - u(\pi_b^M) - u(\pi_b^M) - u(\pi_b^M) - u(\pi_b^M) \right] < 0$$

$$\Leftrightarrow \delta \left[u(\pi_b(x_a^M)) - u(\pi_b(x_a^M)) + u(\pi_b(x_a^M) - u(\pi_b^M) - u(\pi_b^M) - u(\pi_b^M) \right) \right]$$

$$\Leftrightarrow \delta \left[u(\pi_b(x_a^M)) - u(\pi_b(x_a^M)) + u(\pi_b(x_a^M) - u(\pi_b^M)$$

where the second to last line follows from Equation (12). G(x) is strictly decreasing in x for $x \geq x_b^N$ by Assumption 11(iv). Thus, G(x) < 0 for all $x \geq x_a^M \geq x_b^N$ (where the last inequality follows from Condition

4(ii)). If $x_{b,1} \geq x_a^M$, then

$$u(\pi_{b}(x_{b,1}); r_{1}) + \sum_{t=2}^{\infty} \delta^{t-1} u(\pi_{b}(x_{b,t}); r_{t})$$

$$\leq u(\pi_{b}(x_{b,1})) - lL (r_{1} - u(\pi_{b}(x_{b,1}))) + \sum_{t=2}^{\infty} \delta^{t-1} u(\pi_{b}(x_{b,t}))$$

$$\leq u(\pi_{b}(x_{b,1})) - lL (r_{1} - u(\pi_{b}(x_{b,1}))) + \delta \frac{u(\pi_{b}^{M})}{1 - \delta}$$

$$< u(\pi_{b}^{D}(x_{b,1})) - lL (r_{1} - u(\pi_{b}(x_{b,1}))) + \delta V_{b}^{N}$$

$$\leq u(\pi_{b}^{D}(x_{b,1})) - lL (r_{1} - u(\pi_{b}^{D}(x_{b,1}))) + \delta V_{b}^{N}$$

$$= u(\pi_{b}^{D}(x_{b,1}); r_{1}) + \delta V_{b}^{N}$$

$$(13)$$

where the first inequality follows from the fact that $u(\pi;r) \leq u(\pi)$ (Assumption 2). The second inequality follows from Assumption 5(ii). The third inequality follows from $G(x_{b,1}) < 0$ for $x_{b,1} \geq x_a^M$. The fourth inequality follows from Assumption 6(ii) and Assumption 2(ii). (13) implies that $x_{b,1} < x_a^M$ as the ICC in period 1 is not satisfied if $x_{b,1} \geq x_a^M$.

C Additional Results and Examples

C.1 Naive Managers

Research in behavioral economics and psychology suggests that people are relatively poor predictors of their future tastes. Particularly, people tend to underestimate the extent that their future tastes will differ from their current preferences. Loewenstein, O'Donoghue and Rabin (2003) refers to this cognitive bias as "projection bias." In general, humans adjust to changes in their circumstances, but tend to underestimate their own ability to become acclimatized to gains or losses. A number of experiments⁷⁴ have compared individuals' predictions of how a major life event or change in fortune will impact their subjective well-being to actual reports of individuals who have experienced the event. Subjects tend to overestimate the impact of changes in circumstance on their well-being. Kahneman and Snell (1992) find almost no connection between individual's predictions of their changes in preference and their actual changes in preference. For example, Jepson, Loewenstein and Ubel (2001) asked patients waiting for a kidney transplant to predict their (subjective) quality of life one year later if they receive a transplant and if they do not receive a transplant. They then surveyed the same individuals a year later and found that "[p]atients who received transplants predicted a higher quality of life than they ended up reporting, and those who did not predicted

⁷⁴See Helson (1964); Frederick and Loewenstein (1999); Loewenstein, O'Donoghue and Rabin (2003); Wilson and Gilbert (2005) and Wilson and Gilbert (2005) for examples and reviews.

a lower quality of life than they ended up reporting" (Jepson, Loewenstein and Ubel, 2001). These results are consistent with individuals failing to entirely anticipate how they would psychologically adapt to receiving or not receiving a transplant. Barkan and Busemeyer (1999) specifically analyze individuals' ability to predict changes in their reference points and, consistent with projection bias, find that individuals do not correctly anticipate changes in their reference point over time.

In the current setting, projection bias suggests that managers may not fully anticipate or account for the extent that their reference points will adjust to changes in utility.⁷⁵ Naive managers suffer from projection bias. When considering joining a cartel, a naive manager views the relatively high utility levels during collusion as a large gain and may underestimate the extent that they will become accustomed to those utility levels. Conversely, a naive manager considering defecting from a cartel may underestimate the extent that they will become accustomed to lower utility levels after cartel breakdown.

To explore the robustness of results to the possibility of naive managers, suppose that managers do not anticipate any changes in their reference points. Formally, if a manager's reference point in period t is r_t , the manager believes its reference point will remain fixed at r_t in all future periods. Thus, colluding managers set prices expecting their current reference points to prevail in all future periods. In the subsequent period, reference points adjust according to $r_{t+1} = m(r_t, u_t)$. Managers then set prices expecting the updated reference point r_{t+1} to prevail in all future periods. Note that the pricing decisions of naive managers may be dynamically inconsistent. Specifically, a manager may regret a pricing decision made in period t (under the expectation that the reference point in all future periods would remain fixed at r_t) in period t+1 because the manager's reference point unexpectedly updates between period t and period t+1. While the possibility of dynamic inconsistency complicates the analysis of pricing decisions, the choice whether to form or not form a cartel can be analyzed more tractably.⁷⁶ As in the main text, a cartel forms if $V_i^C(r_1) > V_i^N(r_1)$ where

$$V_i^C(r_1) = \max_{\{x_t\}_{t=1}^{\infty} \in \Psi} \sum_{t=1}^{\infty} \delta^{t-1} u(\pi_i(x_t); r_1)$$
(14)

⁷⁵While people are likely capable of anticipating simple changes in their preferences, reference points are a subtle and non-obvious psychological phenomenon. Some individuals may be entirely unaware of innate cognitive biases such as loss aversion. ⁷⁶Note that the price path decided upon by managers in period 1 (when forming the cartel) may be revised in subsequent periods when the managers' reference points unexpectedly adjust. Thus, the solution to (14) in the initial period is a sequence of prices which may in fact not occur.

and

$$\Psi_{i}(r_{1}) = \left\{ \left\{ x_{t} \right\}_{t=1}^{\infty} : x_{t} \in \Omega \text{ and } \sum_{t=T}^{\infty} \delta^{t-T} u(\pi_{i}(x_{t}); r_{1}) \geq u(\pi_{i}^{D}(x_{T}); r_{1}) + \delta V_{i}^{N}(r_{1}) \text{ for all } T \in \left\{ 1, 2 \dots \right\} \right\}. \tag{15}$$

Note that, unlike in the main text, $V_i^C(r_1)$ represents the manager's perceived discounted present value of collusion in the initial period. Crucially, the manager evaluates all utility levels relative to their current reference point r_1 , failing to anticipate future adjustments to their reference points. Thus, whether the manager wishes to form a cartel (in period 1) depends only on the manager's initial reference point. In period 1, the optimization problem in Equation (14) is equivalent to the problem in the main text with m(r,u) = r. Thus, the results of the main text regarding cartel formation (in Section 4 and 5) continue to hold for naive managers.

The dynamic inconsistency of naive managers creates an additional complication: managers may, in theory, wish to disband the cartel after their reference points (unexpectedly) change in future periods. Alternatively, no incentive compatible price path may exist in a future period, in light of updated reference points, when a cartel has previously formed. However, dynamic inconsistency of this kind is unlikely to arise in the present setting. To see this, note that reference points typically increase over time as managers become accosted to higher profits during collusion. As the reference point increases, the punishment for cheating on the cartel is perceived as an increasingly large loss and, as a result, incentives to defect decline over time. Additionally, disbanding the cartel and returning to Nash competition is perceived as an increasingly large loss as reference points adjust upwards. Thus, as managers become accustomed to collusive profits, the cartel becomes more stable, expanding the set of price paths which satisfy the incentive compatibility constraints and reducing the perceived utility of disbanding the cartel and returning to Nash competition.

C.2 S-Shaped Utility and Assumption 2

In this subsection, I explore when a utility function that satisfies Assumption 2 will display risk aversion over gains and risk seeking preferences over losses (i.e., an "S-shape").

For the purposes of this subsection, it facilitates the analysis to define a minimum possible per-period profit level $\underline{\pi}$ and restrict attention to $\pi \geq \underline{\pi}$.⁷⁷ For example, $\underline{\pi} < 0$ may represent the maximum possible perperiod loss that a manager can incur. For consistency, the reference point r is assumed to satisfy $r > u(\underline{\pi})$. Under Assumption 2, a utility function $u(\pi; r, l)$ is risk averse over gains if base utility $u(\pi)$ is strictly concave for all π such that $u(\pi) > r$. $u(\pi; r, l)$ is risk seeking over losses if $u(\pi; r, l)$ is strictly convex for all π such

⁷⁷For example, power utility functions such as $u(\pi) = \pi^b$ for $b \in (0,1)$ may be undefined for $\pi < 0$.

that $u(\pi) < r$. Assuming $u(\pi; r, l)$ is differentiable in π and L is differentiable, $u(\pi; r, l)$ is strictly convex if

$$u''(\pi; r, l) = u''(\pi) - lL''(r - u(\pi)) \left[u'(\pi) \right]^2 + lL'(r - u(\pi))u''(\pi) > 0$$

for all $u(\pi) < r$ and $\pi \ge \underline{\pi}$. The above inequality holds and $u(\pi; r, l)$ is convex over losses if base utility is linear (i.e., $u''(\pi) = 0$) and L is strictly concave (i.e., L'' < 0).

To further explore when utility functions satisfying Assumption 2 are S-shaped, I consider a power utility function (Maggi, 2004) for the remainder of this section. Specifically, suppose $u(\pi) = \pi^b$ where $b \in (0,1)$ and $L(x) = x^a$ where $a \in (0,1)$. To ensure $u(\pi)$ is real, suppose $\underline{\pi} \geq 0$.

Lemma 7. $u(\pi;r,l)$ with $u(\pi)=\pi^b$ where $b\in(0,1)$ and $L(x)=x^a$ where $a\in(0,1)$ is S-shaped if

- i) l is sufficiently large and $[(1-a)b+(1-b)]\underline{\pi}^b>(1-b)r,$ or
- ii) b is sufficiently close to 1 and l > 0.

Proof. Clearly $u(\pi; r, l)$ exhibits risk aversion over gains as $u(\pi; r, l) = u(\pi) = x^b$ when $u(\pi) > r$ and $b \in (0, 1)$. The remainder of the proof establishes that $u(\pi; r, l)$ is convex when $u(\pi) < r$. It suffices to show that the second derivative of $u(\pi; r, l)$ when $u(\pi) < r$ is positive:

$$u''(\pi; r, l) = b (b - 1) \pi^{b-2} - la (a - 1) (r - u(\pi))^{a-2} \left[b \pi^{b-1} \right]^{2}$$

$$+ la (r - u(\pi))^{a-1} b (b - 1) \pi^{b-2}$$

$$= b (b - 1) \pi^{b-2} - la \pi^{b-2} b \left[(a - 1) (r - u(\pi))^{a-2} b \pi^{b} - (r - u(\pi))^{a-1} (b - 1) \right] > 0.$$
(16)

i) $[(1-a)b + (1-b)] \underline{\pi}^b > (1-b)r$ implies that

$$[(1-a)b + (1-b)] \underline{\pi}^b - (1-b)r > 0.$$

As $b \in (0,1)$ and $a \in (0,1)$, it follows that

$$[(1-a)b + (1-b)] \pi^b - (1-b)r > 0$$

for all $\pi \geq \underline{\pi}$.

$$[(1-a)b + (1-b)] \pi^b - (1-b)r > 0$$

$$\implies (1-a)b\pi^b - (1-b)(r-\pi^b) > 0$$

$$\implies (1-a)b\pi^b (r-\pi^b)^{a-2} - (1-b)(r-\pi^b)^{a-1} > 0$$

$$\implies (a-1)b\pi^b (r-\pi^b)^{a-2} - (b-1) (r-\pi^b)^{a-1} < 0.$$

The above inequality implies that the term in brackets on the left hand side of (16) is positive. Thus, the inequality in (16) holds if l is sufficiently large.

ii) As $b \to 1$, the expression on the left hand side of (16) approaches

$$-la\pi^{-1}(a-1)(r-u(\pi))^{a-2}\pi = la(1-a)(r-u(\pi))^{a-2} \ge la(1-a)(r-u(\underline{\pi}))^{a-2} > 0.$$

Lemma 7 establishes that the power utility function is S-shaped when one of two conditions is met. The first condition holds when the manager is sufficiently loss averse. When managers are highly loss averse, the curvature of the utility function is determined primarily by L(x) rather than base utility. As $L(x) = x^a$ where $a \in (0,1)$, utility is risk seeking over losses and the manager is risking over losses. The concavity of base utility ensures that managers are risk averse over gains. The second condition holds when base utility is sufficiently linear (i.e., b is close to 1). When base utility is highly linear, the curvature of the utility function (for losses) is driven primarily by the curvature of L(x). As L(x) is convex, the manager is risk seeking over losses.

C.3 Non-linear m(r, u)

In this subsection, I present an example of a non-linear function m(r, u) which satisfies Assumption 3. Let \bar{u} denote an upper bound on base utility and let \underline{u} denote a lower bound on base utility. Additionally, suppose $r \in [\bar{u}, \underline{u}]$. Let

$$m(r,u) = \begin{cases} r + \beta(u-r)^2 & \text{if } u > r \\ r & \text{if } u = r \\ r - \beta(u-r)^2 & \text{if } u < r \end{cases}$$

$$(17)$$

where $\beta \in \left[0, \frac{1}{2(\bar{u}-\underline{u})}\right]$. First, I show that Assumption 3(i) holds. If u > r, then

$$\beta \le \frac{1}{2(\bar{u} - \underline{u})}$$

$$\implies 2\beta(\bar{u} - \underline{u}) \le 1$$

$$\implies \beta(u - r) \le 1$$

$$\implies \beta (u-r)^2 \le u-r$$

$$\implies m(r,u) = r + \beta (u-r)^2 \le u.$$

 $m(r, u) \ge r$ when u > r follows immediately from $\beta \ge 0$. If u < r, then

$$\beta \le \frac{1}{2(\bar{u} - \underline{u})}$$

$$\implies 2\beta (\bar{u} - \underline{u}) \le 1$$

$$\implies -\beta (u - r) \le 1$$

$$\implies -\beta (u - r)^2 \ge u - r$$

$$\implies m(r, u) = r - \beta (u - r)^2 \ge u.$$

 $m(r, u) \le r$ when u < r follows immediately from $\beta \ge 0$.

Next, consider Assumption 3(ii). It suffices to show that $\frac{\partial}{\partial u}m(r,u) \geq 0$. $\frac{\partial}{\partial u}m(r,u) = 2\beta(u-r) \geq 0$ if u > r and $\frac{\partial}{\partial u}m(r,u) = -2\beta(u-r) \geq 0$ if u < r.

Next, consider Assumption 3(iii). It suffices to show that $\frac{\partial}{\partial r}m(r,u) \geq 0$. If u > r, then

$$\beta \le \frac{1}{2(\bar{u} - \underline{u})}$$

$$\implies 2\beta(\bar{u} - \underline{u}) \le 1$$

$$\implies 2\beta(u - r) \le 1$$

$$\implies 0 \le 1 - 2\beta(u - r) = \frac{\partial}{\partial r}m(r, u).$$

If u < r, then

$$\beta \leq \frac{1}{2(\bar{u} - \underline{u})}$$

$$\implies 2\beta(\bar{u} - \underline{u}) \leq 1$$

$$\implies -2\beta(u - r) \leq 1$$

$$\implies 0 \leq 1 + 2\beta(u - r) = \frac{\partial}{\partial r}m(r, u).$$

Assumption 3(iv) holds as m from Equation (17) is continuous in u and r.

Cartel Formation Conditions

Note that $\{x_t\}_{t=1}^{\infty} \in \Psi$ does not imply that the payoff from collusion with a price path $\{x_t\}_{t=1}^{\infty}$ exceeds the payoff from Nash competition. Thus, both conditions are required. To see this, suppose l = 0 (i.e., managers are loss neutral), $u(\pi) = \pi$ (i.e., linear utility) and the cartel sets a price x_1 in the first period and a price x in all other periods. $\{x_1, x, x \dots\} \in \Psi$ if

$$\pi(x_1) + \delta \frac{\pi(x)}{1 - \delta} \ge \pi^D(x_1) + \frac{\delta}{1 - \delta} \pi^N$$

and

$$\frac{\pi(x)}{1-\delta} \ge \pi^D(x) + \frac{\delta}{1-\delta} \pi^N.$$

Both inequalities hold, for example, if $\pi(x) \ge \pi^N$, $\pi^D(x) = \pi(x)$ and $\pi^D(x_1) = \pi(x_1)^{.78}$ $V^C < V^N$ if

$$V^{C} = \pi(x_1) + \delta \frac{\pi(x)}{1 - \delta} < \frac{\pi^{N}}{1 - \delta} = V^{N}$$

or

$$\pi(x_1) < \frac{1}{1-\delta} \left(\pi^N - \delta \pi(x) \right)$$

which holds, for example, when δ is sufficiently small and $\pi(x_1) < \pi^N$.

Sufficient Condition for $V_b^C(l) < V_a^M(l)$ (Condition 2(ii)) C.5

In this subsection, let the market state parameter in firms' profit functions be $a \in \Gamma \subset \mathbb{R}$. Thus, $\pi^M(a)$ and $\pi^{N}(a)$ denote monopoly and Nash equilibrium firm profits, respectively, when the market state is a. Additionally, let a_0 denote the value of the market state parameter prior to the deterioration in market conditions. Thus, $\pi_b^M = \pi^M(a_0)$ and $\pi_b^N = \pi^N(a_0)$. Analogously, let a_1 denote the value of the market state parameter after the deterioration in market conditions. Thus, $\pi_a^M = \pi^M(a_1)$ and $\pi_a^N = \pi^N(a_1)$. Let $\delta_b^M = \frac{u(\pi_b^{DM}) - u(\pi_b^M)}{u(\pi_b^{DM}) - u(\pi_b^N)}. \quad \delta_b^M \text{ represents the smallest discount factor such that } \{x_b^M\}_{t=1}^\infty \in \Psi_b(0) \text{ (i.e., is ICC)}$ when l=0)

Proposition 6. Suppose

- i) $\pi^M(a)$ is continuous in a for all $a \in [c,d] \subset \Gamma$ where $a_0 \in (c,d)$,
- ii) $\delta < \delta_h^M$, and
- $iii) \ \pi_b^M > \pi_b^N.$

Then, there exists an $\epsilon > 0$ such that $V_b^C(l) < V_a^M(l)$ for a_1 such that $|a_0 - a_1| < \epsilon$. Assumption 6(ii) ensures $\pi^D(x) \ge \pi(x)$ and $\pi^D(x_1) \ge \pi(x_1)$.

Proof. Note that $V_b^M = \frac{u(\pi_b^M)}{1-\delta}$ by $\pi_b^M > \pi_b^N$ (Assumption (iii)), Assumption 7 and Assumption 3(i). First, I show that $V_b^C(l) < V_b^M$ when $\delta < \delta_b^M$. $V_b^C(l) = V_b^M$ if and only if $x_t = x_b^M$ for all t (Assumption 5(ii)). The IC in the first period associated with this path (when $a = a_0$) is

$$\frac{u(\pi_b^M)}{1-\delta} \ge u(\pi_b^{DM}) + \frac{\delta}{1-\delta}u(\pi_b^N)$$

which is violated as $\delta < \delta_b^M = \frac{u(\pi_b^{DM}) - u(\pi_b^M)}{u(\pi_b^{DM}) - u(\pi_b^M)}$. Thus, $V_b^C(l) < V_b^M = \frac{u(\pi_b^M)}{1 - \delta}$.

As $\pi^M(a)$ is continuous in a for all $a \in [c,d] \subset \Gamma$, $a_0 \in (c,d)$, $V_b^C(l) < \frac{u(\pi_b^M)}{1-\delta} = \frac{u(\pi^M(a_0))}{1-\delta}$ and $\pi_b^M > \pi_b^N$, there exists an ϵ such that for all a_1 such that $|a_0 - a_1| < \epsilon$, i) $\frac{u(\pi_a^M)}{1-\delta} = \frac{u(\pi^M(a_1))}{1-\delta} > V_b^C(l)$, ii) $\pi_a^M > \pi_b^N$, and iii) $a_1 \in [c,d]$. Thus, for a_1 such that $|a_0 - a_1| < \epsilon$, $V_b^C(l) < \frac{u(\pi_a^M)}{1-\delta} = V_a^M$ where the equality follows from $\pi_a^M > \pi_b^N$, Assumption 7 and Assumption 3(i).

If $V_a^M > V_b^M$, then Condition 2(ii) holds trivially as $V_a^M > V_b^M \ge V_b^C(l)$.

C.6 Target-based Loss Aversion

C.6.1 Bonuses

In this subsection, I present a model wherein managers receive a bonus if they meet or exceed a performance target. The model illustrates how the loss averse utility function from the main text can also reflect the incentives of a manager subject to a performance-based bonus structure.

Suppose manager pay includes a fixed wage w, a constant fraction $\alpha \in [0,1]$ of the profit that the manager generates and a possible bonus B. Thus, a managers compensation is $w + \alpha \pi + B$ if they receive the bonus and $w + \alpha \pi$ if they do not receive the bonus. The manager receives the performance based bonus if they meet or exceed a pre-defined performance target π^T . If the manager fails to meet the performance target, then the manager may or may not receive a bonus.⁷⁹ A manager generating profit π receives the bonus with probability $\gamma(\pi^T - \pi)$ where $\gamma(x)$ satisfies the following assumption.

Assumption 13. $\gamma(x): \mathbb{R} \to [0,1]$ satisfies the following conditions:

- i) $\gamma(x) = 1$ if $x \le 0$, and
- ii) $\gamma(x)$ is continuous and strictly decreasing for x > 0.

Assumption 13(i) implies that the manager receives the bonus if they reach their performance target. Assumption 13(ii) reflects the fact that managers are less likely to receive their bonus if they perform significantly below their performance target than if the manager performs only slightly below their target.

⁷⁹Even if a manager fails to meet a pre-defined performance target, the manager may be awarded a bonus due to, for example, leniency on the part of higher-level executives awarding bonuses, past service to the firm, or strong overall performance by the firm

A manager's expected wage when generating profit π is therefore

$$w + \alpha \pi + \gamma (\pi^T - \pi)B = w + \alpha \pi + B - \left[1 - \gamma (\pi^T - \pi)\right]B$$
$$= w + \alpha \pi + B - \left[1 - \gamma \left(\frac{w + \alpha \pi^T + B - (w + \alpha \pi + B)}{\alpha}\right)\right]B.$$

The above expression is equivalent to the utility function $u(\pi; r, l)$ from the main text with base utility $u(\pi) = w + \alpha \pi + B$, reference point $r = u(\pi^T)$, $L(x) = 1 - \gamma\left(\frac{x}{\alpha}\right)$ and l = B. Note that $L(x) = 1 - \gamma\left(\frac{x}{\alpha}\right)$ satisfies Assumption 2(ii) by Assumption 13.

In summary, the loss averse utility function $u(\pi; r, l)$ from the main text can also reflect the incentives of a manager that earns a bonus when profits exceed a pre-defined target level and, with a certain probability, does not receive the bonus when performing below the target level.

C.6.2 Managerial Job Loss

In this subsection, I present a model wherein managers face the risk of job loss if their performance drops below a pre-specified target level (i.e., target-based loss aversion). I demonstrate how the threat of termination can result in payoffs which closely resemble payoffs from the loss averse utility function $u(\pi; r, l)$ employed in the main text.

Suppose managers are evaluated relative to a target profit level π^T . If the manager fails to generate a level of profit that matches or exceeds the target profit level, then the manager may be fired, effective immediately. Formally, if π represents a manager's profit, then the manager faces a risk of termination if $\pi < \pi^T$ or, in terms of base utility, $u(\pi) < u(\pi^T)$. Let $\phi(u(\pi^T) - u(\pi))$ denote the likelihood of termination when the manager generates a profit level of $u(\pi)$.⁸⁰ $\phi(x)$ satisfies the following assumption.

Assumption 14. $\phi(x): \mathbb{R} \to [0,1]$ satisfies the following assumptions:

- i) $\phi(x) = 0$ if $x \le 0$, and
- ii) $\phi(x)$ is continuous and strictly increasing for x > 0.

Additionally, suppose that performance targets are set equal to the previous period's profit/utility level. Formally, performance targets (which will act as reference points in the subsequent analysis) update according to $r_{t+1} = m(r_t, u_t) = u_t$.

Each period, the manager's performance is evaluated relative to the performance target. If the manager meets or exceeds their performance target, then the manager retains their employment with probability 1 (see Assumption 14(i)). If the manager performs below their target by a margin $u(\pi^T) - u(\pi) > 0$, the

⁸⁰If base utility is $u(\pi) = \pi$, then the likelihood of termination depends only on the difference between the target profit level and the actual profit level.

manager is fired with probability $\phi\left(u(\pi^T) - u(\pi)\right)$. If the manager maintains their employment, they receive utility $u(\pi)$ and remain employed for the subsequent period. Additionally, the managers performance target updates according to $r_{t+1} = m(r_t, u_t) = u_t$. If the manager is terminated, then the manager receives a constant, expected utility u_F in all future periods.⁸¹ By Assumption 9 and 7, loss aversion typically impacts incentives to collude strictly through managers' payoffs during the punishment phase (i.e., the discounted present value of utility from repeated Nash competition). The managers discounted present value of utility from repeated Nash competition (i.e., a constant stream of profits π^N) when the performance target is $u(\pi^T)$ is $u(\pi^T)$

$$V^{N}(u(\pi^{T})) = \left(1 - \phi\left(u(\pi^{T}) - u(\pi^{N})\right)\right) \frac{u(\pi^{N})}{1 - \delta} + \phi\left(u(\pi^{T}) - u(\pi^{N})\right) \frac{u_{F}}{1 - \delta}$$

$$= \frac{u(\pi^{N})}{1 - \delta} - \phi\left(u(\pi^{T}) - u(\pi^{N})\right) \frac{u(\pi^{N})}{1 - \delta} + \phi\left(u(\pi^{T}) - u(\pi^{N})\right) \frac{u_{F}}{1 - \delta}$$

$$= \frac{u(\pi^{N})}{1 - \delta} - \phi\left(u(\pi^{T}) - u(\pi^{N})\right) \left[\frac{u(\pi^{N})}{1 - \delta} - \frac{u_{F}}{1 - \delta}\right]$$

$$= \frac{1}{1 - \delta} \left[u(\pi^{N}) - \phi\left(u(\pi^{T}) - u(\pi^{N})\right) \left(u(\pi^{N}) - u_{F}\right)\right]. \tag{18}$$

For comparison, recall that the payoff from repeated Nash competition in the model of the main text when the reference point is r and $r_{t+1} = m(r_t, u_t) = r_t$ is

$$V^{N}(r) = \frac{1}{1 - \delta} \left[u(\pi^{N}) - lL(r - u(\pi^{N})) \right]. \tag{19}$$

The payoff in (18) is equivalent to the payoff in (19) with $r = u(\pi^T)$, $L(x) = \phi(x)$ and $l = u(\pi^N) - u_F$. Thus, the loss averse utility function characterized in Assumption 2 can capture target-based loss aversion driven by the threat of managerial job loss.

The preceding analysis involves a number of assumptions, particularly those related to the updating of reference points, which may not hold in practice. Broadly, the preceding discussion is intended to illustrate how the utility functions in the main text can, under certain circumstances, reflect loss aversion driven by the threat of job loss.

C.7 Explanation of $r_1 < u(\pi_a^M)$ (Assumption 8(i))

The below proposition provides a sufficient condition for $r_1 < u(\pi_a^M)$ (Assumption 8(i)). For the remainder of this subsection, let the market state parameter in firms' profit functions be $a \in \Gamma \subset \mathbb{R}$. Thus, $\pi^M(a)$

 $^{^{81}}$ Note that u_F represents the manager's expectations regarding his/her utility levels following termination.

⁸²Recall that performance targets/reference points update immediately following a change in performance/profit (i.e., $r_{t+1} = m(r_t, u_t) = u_t$). Thus, the risk of termination occurs only during the period of a drop in performance and not during subsequent periods.

denotes monopoly firm profits when the market state is a. Additionally, let a_0 denote the value of the market state parameter prior to the deterioration in market conditions. Thus, $\pi_b^M = \pi^M(a_0)$. Analogously, let a_1 denote the value of the market state parameter after the deterioration in market conditions. Thus, $\pi_a^M = \pi^M(a_1)$.

Proposition 7. Suppose i) $\pi^M(a)$ is continuous in a for all $a \in [c, d] \subset \Gamma$ where $a_0 \in (c, d)$, and ii) $u(\pi_b^M) > r_1 = u(\pi_b^N)$.

Then, $r_1 < u(\pi_a^M)$ if $|a_0 - a_1| < \epsilon$ for some $\epsilon > 0$.

Proof. As $u(\pi)$ is strictly increasing in π (Assumption 1(ii)), it suffices to show that $\pi_b^N < \pi_a^M$.

There are two cases:

Case 1: Suppose $\pi_b^M > \pi_a^M$. By the continuity of $\pi^M(a)$ in a (Assumption 5(i)), there exists an ϵ such that

$$|\pi_b^M - \pi_a^M| = |\pi^M(a_0) - \pi^M(a_1)| < \pi_b^M - \pi_b^N$$

if $|a_0 - a_1| < \epsilon$. Then,

$$\begin{aligned} \left| \pi_b^M - \pi_a^M \right| &< \pi_b^M - \pi_b^N \\ \iff \pi_b^M - \pi_a^M &< \pi_b^M - \pi_b^N \\ \iff \pi_a^M > \pi_b^N. \end{aligned}$$

Case 2: If $\pi_b^M \leq \pi_a^M$, then $\pi_b^N < \pi_b^M \leq \pi_a^M$ where the first inequality follows from Assumption (ii) in the statement of the proposition.

D Presence of an Antitrust Authority

In this section, I introduce an antitrust authority, which may detect and penalize managers engaged in cartel activity, into the model of the main text. The timing of the game proceeds as follows when a cartel is active. First, managers set prices. Second, firms earn profits and managers receive utilities. Third, reference points are updated for the following period according to $r_{t+1} = m(r_t, u_t)$ where r_t is the reference point in period t (the current period) and u_t is utility experienced in period t. Fourth, an antitrust authority detects the cartel with probability $\beta \in (0,1)$. If the cartel is detected, then collusion ceases and managers experience a stream of utilities u_1^P , u_2^P ... where u_t^P denotes manager utility t periods after detection. The discounted present value of manager utility immediately after detection is $V^P(\bar{r}_1) = \sum_{t=1}^{\infty} \delta^{t-1} \left(u_t^P - lL(\bar{r}_t - u_t^P)\right)$

where \bar{r}_1 is the reference point in the first period following detection (recall that reference points update prior to the cartel's detection) and $\bar{r}_{t+1} = m(\bar{r}_t, u_t^P)$. Manager utility after detection reflects, among other considerations, criminal and civil punishments such as prison sentences, job loss, reputational damage, and lost income due to diminished career prospects. If a cartel is not detected, collusion continues into the next period. Cartels are only detected when collusion is active (i.e., cartels cannot be detected during defection or punishment phases).

A manager's ex-ante collusive payoff in the presence of an antitrust authority is

$$V^{C}(r_{T}) = u(\pi_{T}; r_{T}) + \delta (1 - \beta) V^{C}(r_{T+1}) + \beta \delta V^{P}(r_{T+1})$$

$$= u(\pi_{T}; r_{T}) + \delta (1 - \beta) \left[u(\pi_{T+1}; r_{T+1}) + \delta (1 - \beta) V^{C}(r_{T+2}) + \beta \delta V^{P}(r_{T+2}) \right] + \beta \delta V^{P}(r_{T+1})$$

$$\vdots$$

$$= \sum_{t=T}^{\infty} \left[\delta (1 - \beta) \right]^{t-T} u(\pi_{t}; r_{t}) + \beta \delta \sum_{t=T+1}^{\infty} \left[\delta (1 - \beta) \right]^{t-(T+1)} V^{P}(r_{t})$$

where reference points update according to m as in the main text. The monopoly payoff is 83

$$V^{M}(r_{T}) = \sum_{t=T}^{\infty} \left[\delta\left(1-\beta\right)\right]^{t-T} u(\pi^{M}; r_{t}) + \beta \delta \sum_{t=T+1}^{\infty} \left[\delta\left(1-\beta\right)\right]^{t-(T+1)} V^{P}(r_{t}).$$

Nash payoff is unchanged. The following assumption is assumed to hold for the remainder of this section.

Assumption 15. $i) \beta \leq \frac{1}{\delta l}$,

ii)
$$u_t^P \geq u^P$$
 for all t for some $u^P \in \mathbb{R}$, and

iii)
$$u_t^P < u(\pi_a^M)$$
 for all t .

Assumption 15(i) requires that the probability of detection is sufficiently small relative to the degree of loss aversion. If detection by an antitrust authority is particularly likely, then the following results will not hold as a cartel will never form due to the risk of penalization. Assumption 15(ii) ensures a lower bound on manager utilities following detection by an antitrust authority. Assumption 15(iii) ensures that managers earn a higher level of utility when earning monopoly collusive profits than after detection by an antitrust authority. Put differently, Assumption 15(iii) holds if managers prefer successful collusion to detection.

The following lemma establishes a result analogous to Lemma 3 under the presence of an antitrust authority.

⁸³Note that $V^M(r_T)$ does not necessarily correspond to the maximum payoff under collusion when an antitrust authority is present. This is the case as a price path $\left\{x_a^M\right\}_{t=1}^\infty$ does not necessarily yield the maximum payoff under the presence of an antitrust authority. For example, cartels may wish to reduce their price below the monopoly level in order to reduce their reference points and limit the losses incurred if detected by an antitrust authority. Formally, the term $V^P(r_{T+1})$ in $V^C(r_T)$ may impact the cartel's pricing decisions.

Lemma 8. $\left\{x_a^M\right\}_{t=1}^\infty \in \Psi_a(l)$ if

$$l \geq \bar{l} = \frac{u(\pi_a^{DM}) + \delta \frac{u(\pi_a^N)}{1 - \delta} - \frac{u(\pi_a^M)}{1 - \delta(1 - \beta)} - \beta \delta \frac{V^P(\underline{u}^P) - \frac{1}{\beta \delta} \frac{L(u(\pi_a^M) - \underline{u}^P)}{1 - \delta(1 - \beta)}}{\delta L \left(r_1 - u(\pi_a^N) \right)}.$$

Proof. It suffices to show that

$$\sum_{t=T}^{\infty} \left[\delta \left(1 - \beta \right) \right]^{t-T} u(\pi_a^M; r_t, l) + \beta \delta \sum_{t=T+1}^{\infty} \left[\delta \left(1 - \beta \right) \right]^{t-(T+1)} V^P(r_t) \ge u(\pi_a^{DM}; r_T, l) + \delta V_a^N(r_T; l) \tag{20}$$

for all $T \in \{1, 2, 3...\}$ where $r_t = m(r_{t-1}, u(\pi_a^M))$ for t > 1. Assumption 3(i) and Assumption 8(i) imply that $r_t \le r_{t+1}$ for all t and $r_t \le u(\pi_a^M)$ for all t. Thus,

$$u(\pi_a^M; r_t, l) = u(\pi_a^M) \tag{21}$$

for all t. Additionally, $u(\pi_a^{DM}; r_T, l) = u(\pi_a^{DM})$ by Assumption 6(ii) and $r_t \leq u(\pi_a^M)$ for all t. Thus, the inequalities in 4 simplify to

$$\frac{u(\pi_a^M)}{1 - \delta(1 - \beta)} + \beta \delta \sum_{t=T+1}^{\infty} \left[\delta(1 - \beta) \right]^{t - (T+1)} V^P(r_t) \ge u(\pi_a^{DM}) + \delta V_a^N(r_T; l)$$
 (22)

for all $T \in \{1, 2, 3 \dots \}$.

Note that reference points after detection never exceed $u(\pi_a^M)$ as $r_t \leq u(\pi_a^M)$ for all $t, u_t^P < u(\pi_a^M)$ for all t (Assumption 15(iii)) and by Assumption 3(i). Thus, for all $T \in \{1, 2, 3 \dots\}$,

$$V^{P}(r_{T}) \geq \sum_{t=1}^{\infty} \delta^{t-1} \left[u_{t}^{P} - lL(u(\pi_{a}^{M}) - u_{t}^{P}) \right]$$

$$\geq \sum_{t=1}^{\infty} \delta^{t-1} \left[u_{t}^{P} - lL(u(\pi_{a}^{M}) - \underline{u}^{P}) \right]$$

$$= V^{P}(\underline{u}^{P}) - l\frac{L(u(\pi_{a}^{M}) - \underline{u}^{P})}{1 - \delta}$$

$$\geq V^{P}(\underline{u}^{P}) - \frac{1}{\beta \delta} \frac{L(u(\pi_{a}^{M}) - \underline{u}^{P})}{1 - \delta} \equiv \underline{V}^{P}$$

$$(23)$$

where the first inequality follows from the fact that $r_t \leq u(\pi_a^M)$ for all t, the fact that post-detection reference points are bounded above by $u(\pi_a^M)$ (shown earlier in the proof) and Assumption 2. The second inequality follows from Assumption 2(ii) and Assumption 15(ii). The equality follows from Assumption 15(ii).

Thus, if

$$\frac{u(\pi_a^M)}{1 - \delta(1 - \beta)} + \beta \delta \sum_{t=T+1}^{\infty} \left[\delta(1 - \beta) \right]^{t - (T+1)} \underline{V}^P \ge u(\pi_a^{DM}) + \delta V_a^N(r_T, l)$$
 (24)

holds for all $T \in \{1, 2, 3...\}$, or equivalently,

$$\frac{u(\pi_a^M)}{1 - \delta(1 - \beta)} + \beta \delta \frac{\underline{V}^P}{1 - \delta(1 - \beta)} \ge u(\pi_a^{DM}) + \delta V_a^N(r_T; l).$$

for all $T \in \{1, 2, 3...\}$, then the inequalities in 22 holds for all $T \in \{1, 2, 3...\}$. Fix $T \in \{1, 2, 3...\}$, Let $\tilde{r}_T = r_T$ and let $\tilde{r}_t = m(\tilde{r}_{t-1}, u(\pi_a^N))$ for t > T. Note that

$$V_{a}^{N}(r_{T};l) = \sum_{t=T}^{\infty} \delta^{t-T} u(\pi_{a}^{N}; \tilde{r}_{t}, l) \leq u(\pi_{a}^{N}; r_{1}, l) + \sum_{t=T+1}^{\infty} \delta^{t-T} u(\pi_{a}^{N}; \tilde{r}_{t}, l)$$

$$\leq u(\pi_{a}^{N}; r_{1}, l) + \sum_{t=T+1}^{\infty} \delta^{t-T} u(\pi_{a}^{N})$$

$$= u(\pi_{a}^{N}; r_{1}, l) + \sum_{t=1}^{\infty} \delta^{t} u(\pi_{a}^{N})$$

$$= u(\pi_{a}^{N}; r_{1}, l) + \delta \frac{u(\pi_{a}^{N})}{1 - \delta}$$
(25)

where the first inequality follows from $\tilde{r}_T = r_T \ge r_1$ for all T and Assumption 2. The second inequality follows from $u(\pi; r, l) \le u(\pi)$ for all π , r, and l.

 $r_1 > u(\pi_a^N)$ by $r_1 = u(\pi_b^N)$ (Assumption 7) and $\pi_a^N < \pi_b^N$ (Assumption 8(ii)). Thus, $L(r_1 - u(\pi_a^N)) > 0$ by Assumption 2(ii).

Suppose $l \geq \bar{l}$. Then,

$$u(\pi_{a}^{DM}) + \delta V_{a}^{N}(r_{T}; l)$$

$$\leq u(\pi_{a}^{DM}) + \delta u(\pi_{a}^{N}; r_{1}, l) + \delta^{2} \frac{u(\pi_{a}^{N})}{1 - \delta}$$

$$= u(\pi_{a}^{DM}) + \delta u(\pi_{a}^{N}) - \delta lL(r_{1} - u(\pi_{a}^{N})) + \delta^{2} \frac{u(\pi_{a}^{N})}{1 - \delta}$$

$$\leq u(\pi_{a}^{DM}) + \delta u(\pi_{a}^{N}) - \delta \bar{l}L(r_{1} - u(\pi_{a}^{N})) + \delta^{2} \frac{u(\pi_{a}^{N})}{1 - \delta}$$

$$= u(\pi_{a}^{DM}) + \delta u(\pi_{a}^{N}) + \delta^{2} \frac{u(\pi_{a}^{N})}{1 - \delta}$$

$$- \left[u(\pi_{a}^{DM}) + \delta \frac{u(\pi_{a}^{N})}{1 - \delta} - \frac{u(\pi_{a}^{M})}{1 - \delta(1 - \beta)} - \beta \delta \frac{\underline{V}^{P}}{1 - \delta(1 - \beta)} \right]$$

$$= \delta u(\pi_{a}^{N}) - \delta \frac{u(\pi_{a}^{N})}{1 - \delta} + \frac{u(\pi_{a}^{M})}{1 - \delta(1 - \beta)} + \delta^{2} \frac{u(\pi_{a}^{N})}{1 - \delta}$$

$$+ \beta \delta \frac{\underline{V}^{P}}{1 - \delta(1 - \beta)}$$

$$= \delta u(\pi_{a}^{N}) - (1 - \delta) \delta \frac{u(\pi_{a}^{N})}{1 - \delta} + \frac{u(\pi_{a}^{M})}{1 - \delta(1 - \beta)} + \beta \delta \frac{\underline{V}^{P}}{1 - \delta(1 - \beta)}$$

$$= \frac{u(\pi_{a}^{M})}{1 - \delta(1 - \beta)} + \beta \delta \frac{\underline{V}^{P}}{1 - \delta(1 - \beta)}$$

$$\leq \frac{u(\pi_{a}^{M})}{1 - \delta(1 - \beta)} + \beta \delta \sum_{t=T+1}^{\infty} [\delta(1 - \beta)]^{t-(T+1)} V^{P}(r_{t})$$
(26)

where the first inequality follows from Equation (25). The first equality follows from the definition of $u(\pi_a^N; r_1, l)$. The second inequality follows from $l \geq \bar{l}$. The second inequality follows from the definition of \bar{l} and $L\left(r_1 - u(\pi_a^N)\right) > 0$. The last inequality follows from Equation (23). Thus, Equation (26) implies the inequalities in 22 are satisfied and $\left\{x_a^M\right\}_{t=1}^{\infty} \in \Psi_a(l)$ when $l \geq \bar{l}$.

The following lemma establishes a result analogous to Lemma 6 under the presence of an antitrust authority.

Lemma 9. $V_a^C(l) > V_a^N(l)$ if

$$l \geq \bar{l} = \frac{u(\pi_a^{DM}) + \delta \frac{u(\pi_a^N)}{1 - \delta} - \frac{u(\pi_a^M)}{1 - \delta(1 - \beta)} - \beta \delta \frac{V^P(\underline{u}^P) - \frac{1}{\beta \delta} \frac{L(u(\pi_a^M) - \underline{u}^P)}{1 - \delta(1 - \beta)}}{\delta L \left(r_1 - u(\pi_a^N) \right)}.$$

Proof. Lemma 8 implies that $V_a^C(l) \ge V_a^M(l)$ when $l \ge \bar{l}$. Thus, it suffices to show $V_a^M(l) > V_a^N(l)$ when $l \ge \bar{l}$.

Next, note that

$$u(\pi_a^N) < u(\pi_b^N) = r_1 < u(\pi_a^M) \le u(\pi_a^{DM})$$
 (27)

where the first inequality follows from Assumption 8(ii), the equality follows from Assumption 7, and the third inequality follows from Assumption 8(i). The fourth inequality follows from Assumption 6(ii).

Let $r_1 = \tilde{r}_1$ and let $\tilde{r}_t = m(\tilde{r}_{t-1}, u(\pi_a^N))$ for t > 1. When $l \ge \bar{l}$,

$$\begin{split} V_a^N(l) &= \sum_{t=1}^{\infty} \delta^{t-1} u(\pi_a^N) - \sum_{t=1}^{\infty} \delta^{t-1} lL(\tilde{r}_t - u(\pi_a^N)) \\ &< u(\pi_a^N) + \sum_{t=2}^{\infty} \delta^{t-1} u(\pi_a^N) - \delta \sum_{t=1}^{\infty} \delta^{t-1} lL(\tilde{r}_t - u(\pi_a^N)) \\ &\leq u(\pi_a^{DM}) + \delta \left[\sum_{t=1}^{\infty} \delta^{t-1} u(\pi_a^N) - \sum_{t=1}^{\infty} \delta^{t-1} lL(\tilde{r}_t - u(\pi_a^N)) \right] \\ &= u(\pi_a^{DM}) + \delta V_a^N(l) \\ &\leq \frac{u(\pi_a^M)}{1 - \delta \left(1 - \beta \right)} + \beta \delta \sum_{t=2}^{\infty} \left[\delta \left(1 - \beta \right) \right]^{t-2} V^P(r_t) = V_a^M(r_1; l) \end{split}$$

where the first inequality follows from $\delta < 1$, Assumption 2(ii) and $u(\pi_a^N) < u(\pi_b^N) = r_1$ (Assumption 7 and Assumption 8(ii)). The second inequality follows from Equation (27). The third inequality follows from Lemma 8. Thus, $V_a^N(l) < V_a^M(l)$.

Note that there always exists an l that satisfies $l \geq \bar{l}$ and Assumption 15(i) for sufficiently small β . To see this, note that Assumption 15(i) holds if $l \leq \frac{1}{\delta\beta}$. As $\beta \to^+ 0$, $\bar{l} \to \frac{u(\pi_a^{DM}) + \delta \frac{u(\pi_a^N)}{1-\delta} - \frac{u(\pi_a^M)}{1-\delta} + \frac{L(\pi_a^M - \underline{u}^P)}{(1-\delta)^2}}{\delta L(r_1 - u(\pi_a^N))} < \infty$ and $\frac{1}{\delta\beta} \to \infty$. Therefore, the lower bound on l is finite as $\beta \to^+ 0$ while the upper bound on l approaches infinity.

Proofs of Proposition 1 and 2 follow directly from Lemma 9. Pricing results are more difficult to establish when an antitrust authority is present. This is the case as the threat of detection (and corresponding losses in utility due to penalization) impacts the cartel's pricing decisions. Thus, properly characterizing the optimal price path and comparing price paths before and after the change in market state would require additional assumptions regarding the nature of antitrust penalties and their impact on manager utility (i.e., the stream of utilities $\{u_t^P\}_{t=1}^{\infty}$). However, numerical solutions suggest that pricing dynamics under the presence of an antitrust authority are qualitatively similar to the pricing dynamics outlined in the main text. To illustrate, Figure 8 plots optimal price paths before (red) and after (blue) a reduction in demand. As in the main text, a deterioration in market conditions increases cartel prices in early periods of collusion when managers are sufficiently loss averse.

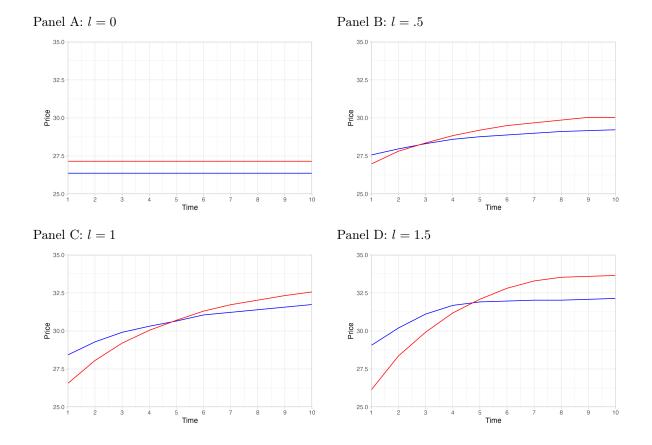


Figure 8: Optimal Price Paths Before (Red) and After (Blue) a 5% Reduction in Demand by Degree of Loss Aversion (with an Antitrust Authority).

Notes: This figure depicts optimal price paths before and after a 5% reduction in the demand parameter a for a variety of degrees of loss aversion with an antitrust authority. Parameters: $b=2, e=1, c=0, N=4, \delta=.25, \beta=.125, u_t^P=.7\times\pi_b^N$ for all t, and $\alpha=.75$. Prior to the deterioration in market conditions, a=100. The blue curve depicts the optimal price path after the deterioration in market conditions (i.e., a=95) and the red curve depicts the optimal price path prior to the deterioration in market conditions (i.e., a=100). The Nash equilibrium price before (after) the change is 20 (19). The monopoly price before (after) the change is 50 (47.5).

E Gradual Deteriorations in Market Conditions

In the main text, I restrict attention to abrupt and instantaneous deteriorations in market state. This approach is primarily for analytical tractability and simplicity. Considering gradual changes in market state introduces an additional dynamic variable (i.e., the market state) which complicates the analysis. In this section, I demonstrate that gradual deteriorations in market state can also cause the formation of cartels when managers are loss averse.

Certain types of deteriorations in market state, such as the entry of a new firm or a technological advance that rapidly eliminates a portion of demand for a product, are likely abrupt and are, therefore, best captured by the model presented in the main text. However, other changes in market state may be more gradual (e.g., steadily increasing marginal costs due to rising input prices or a gradual reduction in demand due to changing preferences over time).

Note that a gradual deterioration in market conditions typically hinders the sustainability of collusion between loss neutral agents. For example, gradually declining demand reduces the sustainability of collusion because managers anticipate lower collusive profits in subsequent periods and, thus, have weaker incentives to refrain from cheating on the agreement in the current period (Ivaldi et al., 2007). Put differently, the temptation to cheat in the current period is strong as current demand is high relative to future levels. Thus, when agents are loss neutral, gradual reductions in demand hinder the sustainability of collusion. The subsequent analysis will establish that this is not necessarily the case when managers are loss averse.

Throughout this section, regime b is unchanged from the main text. Under regime a, market conditions begin deteriorating prior to the initial period and, unlike the model in the main text, continue deteriorating gradually in subsequent periods. Managers recognize and anticipate subsequent deteriorations in market state when deciding whether to form a cartel in the initial period.

Formally, let $\pi_{a,t}(x)$ denote profits in period t under regime a when the cartel sets a common price $x \in \Omega$. Assumption 5 holds for each $\pi_{a,t}(x)$. Collusive profits decline over time. Formally, suppose $\pi_{a,t}(x) \ge \pi_{a,t+1}(x)$ for all $t \in \{1, 2...\}$ and $x \in \Omega$. Let $\pi_{a,t}^M$ denote the monopoly profit in period t under regime a.

Let $\pi^D_{a,t}(x)$ denote the profit a manager earns when cheating on collusion in period t when the collusive price is $x \in \Omega$. Assumption 6 is assumed to hold for each $\pi^D_{a,t}(x)$. Assumption 4 holds in each period t. Let $\pi^N_{a,t}$ denote the Nash equilibrium price in period t. Let $\pi^N_{a,t}$ denote Nash equilibrium profit in period t and suppose $\pi^N_{a,t} \geq \pi^N_{a,t+1}$ for all $t \in \{1, 2...\}$. Thus, the profitability of both collusion and continued competition is gradually declining over time under regime a.

The following assumption replaces Assumption 8(ii).

Assumption 16. $\pi_{a,1}^N < \pi_b^N$

Assumption 16 implies that market conditions have deteriorated prior to the initial period. As Nash equilibrium profits decline over time, Assumption 16 implies that $\pi_{a,t}^N < \pi_b^N$ for all $t \in \{1, 2...\}$. The following assumption ensures that there exists a collusive price path generating a constant stream of utilities that exceeds the initial reference point.

Assumption 17. There exists a price path $\{\bar{x}_{a,t}\}_{t=1}^{\infty}$ such that $\pi_{a,t}(\bar{x}_{a,t}) = \bar{\pi}_a$ for all t where $u(\bar{\pi}_a) > r_1$.

 $\{\bar{x}_{a,t}\}_{t=1}^{\infty}$ represents a price path wherein managers adjust the collusive price over time, as the market state deteriorates, to ensure a profit of $\bar{\pi}_a$ is earned in each period. Such a price path is not necessarily an optimal price path or incentive compatible, but the existence of such a path is employed in the following

proofs. Note that $\bar{\pi}_a \leq \pi_{a,t}^M$ holds, by definition, for all t. Thus, Assumption 17 implies that $\pi_{a,t}^M > r_1$ for all t. Assumption 17 holds if, for example, $\Omega = [c, d]$ for c < d and the size of the deterioration in market state is sufficiently moderate.84

Let $\bar{\pi}_{a,t}^D = \pi_{a,t}^D(\bar{x}_{a,t})$ and let $\bar{\pi}_a^D = \sup_t \bar{\pi}_{a,t}^D$. Let $V_{a,T}^N(r_T; l)$ denote the discounted present value of manager utility in period T from competitive play when the current reference point is r_T . Note that $V_{a,T}^{N}(r_T;l)$ depends on time T as Nash equilibrium payoffs are declining over time.

Lemma 10. $\{\bar{x}_{a,t}\}_{t=1}^{\infty} \in \Psi_a(l)$ if

$$l \ge \bar{l} = \frac{u(\bar{\pi}_a^D) + \delta \frac{u(\pi_{a,1}^N)}{1-\delta} - \frac{u(\bar{\pi}_a)}{1-\delta}}{\delta L \left(r_1 - u(\pi_{a,1}^N)\right)}.$$

Proof. It suffices to show that

$$\sum_{t=T}^{\infty} \delta^{t-T} u(\bar{\pi}_a; r_t, l) \ge u(\bar{\pi}_{a,t}^D; r_T, l) + \delta V_{a,T}^N(r_T; l)$$
(28)

for all $T \in \{1, 2, 3...\}$ where $r_t = m(r_{t-1}, u(\bar{\pi}_a))$ for t > 1. Assumption 3(i) and Assumption 17 imply that $r_t \leq r_{t+1}$ for all t and $r_t \leq u(\bar{\pi}_a)$ for all t. Thus,

$$u(\bar{\pi}_a; r_t, l) = u(\bar{\pi}_a) \tag{29}$$

for all t. Additionally, $u(\bar{\pi}_{a,t}^D; r_T, l) = u(\bar{\pi}_{a,t}^D)$ by Assumption 6(ii) and $r_t \leq u(\bar{\pi}_a)$ for all t. Therefore, the inequalities in (28) become

$$\sum_{t=T}^{\infty} \delta^{t-T} u(\bar{\pi}_a) \ge u(\bar{\pi}_{a,T}^D) + \delta V_{a,T}^N(r_T; l)$$

for all $T \in \{1, 2, 3...\}$. As $\bar{\pi}_{a,t}^D \leq \bar{\pi}_a^D$ for all T, it suffices to show that

$$\sum_{t=T}^{\infty} \delta^{t-T} u(\bar{\pi}_a) \ge u(\bar{\pi}_a^D) + \delta V_{a,T}^N(r_T; l)$$

for all $T \in \{1, 2, 3 \dots\}$, or, equivalently,

$$\frac{u(\bar{\pi}_a)}{1-\delta} - u(\bar{\pi}_a^D) \ge \delta V_{a,T}^N(r_T; l).$$

⁸⁴To see this, suppose $\pi_{a,t}(x_{a,t}^N) = \pi_{a,t}^N$ for all t, and $r_1 = u(\pi_b^N) < u(\pi_{a,t}^M)$ for all t (i.e., the size of the deterioration is moderate). Let $\pi^M_{a,inf} \equiv \inf_t \pi^M_{a,t}$ and let $\bar{\pi}_a \in \left(\pi^N_b, \pi^M_{a,inf}\right)$. Thus, $\bar{\pi}_a > \pi^N_b > \pi^N_{a,1} \geq \pi^N_{a,t} = \pi_{a,t}(x^N_{a,t})$ and $\bar{\pi}_a < \pi^M_{a,inf} \leq \pi^M_{a,t} = \pi_{a,t}(x^M_{a,t})$ for all t. Therefore, by the intermediate value theorem, there exists an $\bar{x}_{a,t}$, for each t, such that $\bar{\pi}_a \neq (\bar{\tau}_{a,t}) = \bar{\pi}_a$. $a_{i,n,l} = a_{i,t}$ $\pi_{a,t}(\bar{x}_{a,t}) = \bar{\pi}_a$.

85The supremem exists if, for example, $\max_x \pi_{a,t}^D(x) \ge \max_x \pi_{a,t+1}^D(x)$ for all $t \in \{1, 2 \dots\}$.

for all $T \in \{1, 2, 3...\}$. Fix $T \in \{1, 2, 3...\}$. Let $\tilde{r}_T = r_T$ and let $\tilde{r}_t = m(\tilde{r}_{t-1}, u(\pi_{a,t-1}^N))$ for t > T. Note that

$$V_{a,T}^{N}(r_{T};l) = \sum_{t=T}^{\infty} \delta^{t-T} u(\pi_{a,t}^{N}; \tilde{r}_{t}, l) \leq u(\pi_{a,1}^{N}; r_{1}, l) + \sum_{t=T+1}^{\infty} \delta^{t-T} u(\pi_{a,t}^{N}; \tilde{r}_{t}, l)$$

$$\leq u(\pi_{a,1}^{N}; r_{1}, l) + \sum_{t=T+1}^{\infty} \delta^{t-T} u(\pi_{a,t}^{N})$$

$$\leq u(\pi_{a,1}^{N}; r_{1}, l) + \sum_{t=T+1}^{\infty} \delta^{t-T} u(\pi_{a,1}^{N})$$

$$= u(\pi_{a,1}^{N}; r_{1}, l) + \sum_{t=1}^{\infty} \delta^{t} u(\pi_{a,1}^{N})$$

$$= u(\pi_{a,1}^{N}; r_{1}, l) + \delta \frac{u(\pi_{a,1}^{N})}{1 - \delta}$$
(31)

where the first inequality follows from $\tilde{r}_T = r_T \ge r_1$ for all T and Assumption 2. The second inequality in Equation (31) follows from $u(\pi; r, l) \le u(\pi)$ (Assumption 2). The third inequality follows from $\pi_{a,t}^N \le \pi_{a,t+1}^N$. $r_1 > u(\pi_{a,1}^N)$ by $r_1 = u(\pi_b^N)$ (Assumption 7) and $\pi_{a,1}^N < \pi_b^N$ (Assumption 16). Thus, $L(r_1 - u(\pi_{a,1}^N)) > 0$ by Assumption 2(ii).

Suppose $l \geq \bar{l}$. Then,

$$u(\bar{\pi}_{a}^{D}) + \delta V_{a,T}^{N}(r_{T}; l) \leq u(\bar{\pi}_{a}^{D}) + \delta u(\pi_{a,1}^{N}; r_{1}, l) + \delta^{2} \frac{u(\pi_{a,1}^{N})}{1 - \delta}$$

$$= u(\bar{\pi}_{a}^{D}) + \delta u(\pi_{a,1}^{N}) - \delta l L(r_{1} - u(\pi_{a,1}^{N})) + \delta^{2} \frac{u(\pi_{a,1}^{N})}{1 - \delta}$$

$$\leq u(\bar{\pi}_{a}^{D}) + \delta u(\pi_{a,1}^{N}) - \delta \bar{l} L(r_{1} - u(\pi_{a,1}^{N})) + \delta^{2} \frac{u(\pi_{a,1}^{N})}{1 - \delta}$$

$$= u(\bar{\pi}_{a}^{D}) + \delta u(\pi_{a,1}^{N})$$

$$- \left[u(\bar{\pi}_{a}^{D}) + \delta \frac{u(\pi_{a,1}^{N})}{1 - \delta} - \frac{u(\bar{\pi}_{a})}{1 - \delta} \right] + \delta^{2} \frac{u(\pi_{a,1}^{N})}{1 - \delta}$$

$$= \delta u(\pi_{a,1}^{N}) - \delta \frac{u(\pi_{a,1}^{N})}{1 - \delta} + \frac{u(\bar{\pi}_{a})}{1 - \delta} + \delta^{2} \frac{u(\pi_{a,1}^{N})}{1 - \delta}$$

$$= \delta u(\pi_{a}^{N}) - (1 - \delta) \delta \frac{u(\pi_{a,1}^{N})}{1 - \delta} + \frac{u(\bar{\pi}_{a})}{1 - \delta}$$

$$= \frac{u(\bar{\pi}_{a})}{1 - \delta}$$

$$(33)$$

where the first inequality follows from (31). The first equality follows from the definition of $u(\pi_{a,1}^N; r_1, l)$. The second inequality follows from $l \geq \bar{l}$. The second equality follows from the definition of \bar{l} and $L(r_1 - u(\pi_{a,1}^N)) > 0$ (shown earlier in the proof). Thus, Equation 33 implies $\{\bar{x}_{a,t}\}_{t=1}^{\infty} \in \Psi_a(l)$ when $l \geq \bar{l}$.

Lemma 11. $V_a^C(l) > V_a^N(l)$ (thus, a cartel forms) if

$$l \geq \bar{l} = \frac{u(\bar{\pi}_a^D) + \delta \frac{u(\pi_{a,1}^N)}{1-\delta} - \frac{u(\bar{\pi}_a)}{1-\delta}}{\delta L\left(r_1 - u(\pi_{a,1}^N)\right)}.$$

Proof. When $l \geq \bar{l}$, $\{\bar{x}_{a,t}\}_{t=1}^{\infty} \in \Psi_a(l)$ (by Lemma 10). It remains to establish that $V_a^C(l) > V_a^N(l)$. Note that

$$u(\pi_{a,1}^N) < u(\pi_b^N) = r_1 < u(\bar{\pi}_a) \tag{34}$$

where the first inequality follows from Assumption 16, the equality follows from Assumption 7, and the second inequality follows from Assumption 17. Thus,

$$\begin{aligned} V_a^N(l) &\leq \sum_{t=1}^{\infty} \delta^{t-1} u(\pi_{a,1}^N) \\ &< \sum_{t=1}^{\infty} \delta^{t-1} u(\bar{\pi}_a) \\ &= \frac{u(\bar{\pi}_a)}{1-\delta} \leq V_a^C(l) \end{aligned}$$

where the first inequality follows from the fact that $u(\pi; r, l) \leq u(\pi)$ (Assumption 2) and $\pi^N_{a,t} \geq \pi^N_{a,t+1}$. The second inequality follows from Equation 34. The third inequality follows from Lemma 10. As a result, the cartel forms when $l \geq \bar{l}$.

Lemma 11 establishes a result analogous to Proposition 1 in the main text. Formally, Lemma 11 demonstrates that a cartel forms during a gradual deterioration in market state when managers are sufficiently loss averse. Recall that the market state begins deteriorating immediately prior to the initial period (see Assumption 16). Managers decide whether to form a cartel in the initial period and, following the initial period, market conditions continue to deteriorate. The effects outlined in the main text also arise when the deterioration in market state is gradual because a reversion to Nash competition is perceived as a loss in both cases. Loss averse managers wish to avoid the painful losses in utility that competitive play would cause. To avoid this outcome, managers turn to collusion.

Note that the analysis in this section has assumed that a cartel forms (or does not form) in the beginning of a gradual deterioration in market state. However, similar effects can arise if the cartel instead decides whether to form after multiple periods of gradual deterioration in the market state. To illustrate, suppose reference points are constant (i.e., m(r,u) = r) and suppose managers choose whether to form a cartel in period T. T periods following the beginning of a gradual deterioration in market state, Nash equilibrium profits satisfy $u(\pi_{a,T}^N) \leq u(\pi_{a,1}^N) < r_1$ (where the second inequality follows from Assumption 16). Thus,

the cartel's problem is identical to the setting outlined above with period T representing the initial period, period T+1 representing the second period etc. The above conclusions will therefore apply without additional modification.

Lemma 11 implies that a deterioration in market conditions can also enhance the value of collusion. Note that the proof of Lemma 11 establishes that $V_a^C(l) \geq \frac{u(\bar{\pi}_a)}{1-\delta}$ when managers are sufficiently loss averse. Thus, if $V_b^C(l) < \frac{u(\bar{\pi}_a)}{1-\delta}$ due to, for example, high fixed costs of collusion or a relatively low discount factor, then a gradual deterioration in market state can also enhance the collusive payoff.

Determining the impact of a gradual change in market state on collusive prices is more challenging. The effects outlined in the main text will, all else equal, result in higher prices (at least, in early periods of collusion). However, other considerations arise when the change in market state is gradual. For example, managers may have a stronger incentive to defect during early periods of collusion because they anticipate relatively low collusive profits in future periods due to continued deterioration of the market state (Ivaldi et al., 2007). This may cause the cartel to reduce prices in early periods of collusion to ensure no manager wishes to defect. Which effect dominates likely depends on, among other factors, the degree of loss aversion and the dynamics of market state changes over time. Generally, if market state changes during collusion are relatively moderate, then it seems likely that the effects outlined in Proposition 2 and Proposition 3 will continue to hold. When changes in the market state during collusion are more extreme or volatile, other considerations may overpower the effects captured in the main text.

F Simulation Results

F.1 Simulation Framework

In this subsection, I provide additional details regarding numerical simulations conducted to generate the figures in the main text (as well as additional figures presented later in this section). All figures reflect outcomes from a setting involving N firms selling symmetrically differentiated products and engaging in price competition. The representative consumer has a utility function of (Singh and Vives, 1984; Harrington, 2004)

$$U(q_1, \dots q_N) = a \sum_{i=1}^{N} q_i - \left(\frac{1}{2}\right) \left(b \sum_{i=1}^{N} q_i^2 + e \sum_{i=1}^{N} \sum_{j \neq i} q_i q_j\right)$$

where a > 0, b > e > 0 and q_i is the quantity of firm i's product consumed. If consumers demand a positive quantity of all firms' products, then firm i's demand is

$$D(p_i, p_{-i}) = \left(\frac{a}{b + (N-1)e}\right) - \left(\frac{b + (N-2)e}{(b + (N-1)e)(b-e)}\right)p_i + \left(\frac{e(N-1)}{(b + (N-1)e)(b-e)}\right)p_{-i}$$
(35)

where p_i is the price set by firm i and p_{-i} is the common price set by all of firm i's rivals. All firms have constant marginal cost c where $0 \le c < a$. In all figures, the base utility function is u(x) = x (i.e., base utility is risk neutral) and L(x) = x. Thus, mangers have a utility function of

$$u(\pi; r, l) = \pi - l (r - \pi).$$

Reference points evolve over time according to $m(r, u) = \alpha r + (1 - \alpha) u$ where $\alpha \in [0, 1]$. There is a fixed cost of collusion of $F \ge 0$.⁸⁶ Thus, collusive profit when all managers charge a common price of p is

$$\pi(p) = (p - c) D(p, p) - F.$$

 $\pi^D(p)$ denotes profits when defecting from collusion when the collusive price is p.⁸⁷ Nash equilibrium profits are

$$\pi^{N} = \frac{(a-c)^{2} (b+e (N-2)) (b-e)}{(b+(N-1)e) (2b+e (N-3))^{2}}.$$

In all figures, the deterioration in market state is either a reduction in the demand intercept (a decrease in a), an increase in marginal cost (an increase in c), or the entry of a new competitor (an increase from N to N+1). Note that, as expected, π^N is increasing in a and decreasing in c. Additionally, routine calculations imply that π^N is decreasing in N. Thus, each of the three possible deteriorations in market state are consistent with Assumption 8(ii). Under the differentiated product demand system in Equation (35), a cartel forms for any discount factor unless there is a fixed cost of collusion.⁸⁸ The optimal price path and the value of collusion are determined through value function iteration techniques.

Throughout the main text, parameter values in numerical simulations are selected in order to ensure the relevant effects can be clearly distinguished visually in the figures and, to the greatest extent possible, be

⁸⁶Recall that fixed costs of collusion include any moral dis-utilities from participating in an illegal activity, fixed costs of monitoring rivals (e.g., payments made to a third party tasked with monitoring compliance with the collusive scheme), fixed costs involved in concealing collusive activities (including managerial effort) and communicating with other managers involved in the cartel (Klein and Schinkel, 2019), and costs of buying out potential entrants (Ganslandt, Persson and Vasconcelos, 2012). Fixed costs of collusion are paid only when the cartel is active.

⁸⁷In a the demand system employed in this section, defection can occur in two distinct ways. The defecting manager can choose a defection price sufficiently low that demand for rival products is 0. Alternatively, the defecting manager can set a higher defection price for which rival demands are positive.

⁸⁸See, for example, the proof of Proposition 2 in Bos et al. (2018).

representative of patterns observed more generally in alternative parameter settings. Parameter values are not intended to reflect a particular industry/cartel nor empirical estimates from prior literature. Determining realistic values for all parameters (including the rate of reference point adjustment (α) , degree of loss aversion (l), and fixed cost of collusion (F)) as well as accurate functional forms for utility $u(\cdot)$ and the loss function $L(\cdot)$ is challenging and beyond the scope of this study. A variety of additional simulations, including variations in the parameter values employed in figures in the main text, are presented in the following subsections.

F.2 Cartel Formation

Figure 9 plots the critical discount factor both before and after the entry of a new firm for alternative α values. Note that when $\alpha=1$ (i.e., reference points are constant), an increase in the degree of loss aversion does not impact the critical discount factor under regime b. This is the case as, when reference points are constant at the Nash equilibrium profit level, a reversion to Nash competition is not, in any period, perceived as a loss by managers. Thus, the degree of loss aversion does not impact manager utility. However, when $\alpha < 1$, reference points adjust upward in response to elevated utilities during collusion. After reference points adjust upwards, a return to Nash competition is perceived as a loss. Thus, an increase in the degree of loss aversion reduces the utility managers experience in the punishment phase, enhancing incentives to collude.

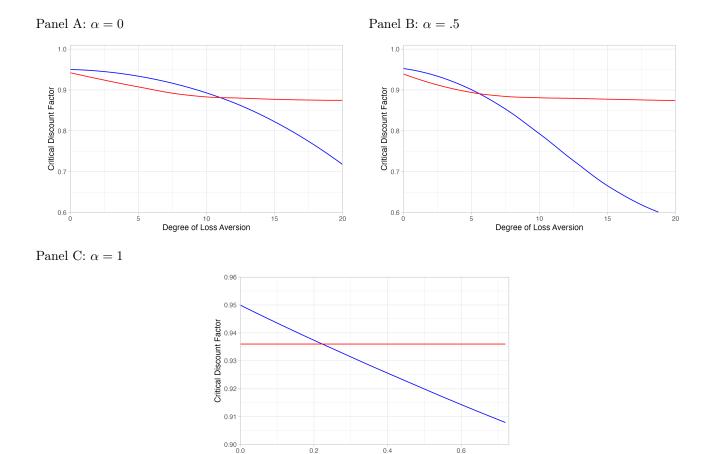


Figure 9: Critical Discount Factor by Degree of Loss Aversion Before (Red) and After (Blue) the Entry of a New Firm.

Degree of Loss Aversion

0.6

Notes: These figures depict the critical discount factor when $\alpha = 0$ (Panel A), $\alpha = .5$ (Panel B) and $\alpha = 1$ (Panel C) as a function of the degree of loss aversion. The critical discount factor is the smallest discount factor for which a cartel forms. Parameters: a = 100, b = 2, e = 1, c = 0, and F = 125. The blue curve denotes the critical discount factor after entry (i.e., N=6) and the red curve denotes the critical discount factor absent entry (i.e., N=5).

Figure 10 depicts the critical discount factor before (red) and after (blue) a 50% increase in marginal cost for a parameter configuration involving a fixed cost of collusion. When managers are loss neutral, an increase in marginal cost increases the critical discount factor and reduces incentives to collude. This is the case as an increase in marginal cost reduces the variable profits each firm earns during collusion which makes it more difficult to cover fixed costs of collusion. However, when managers are sufficiently loss averse, an increase in marginal cost can reduce the critical discount factor and increase the range of discount factors for which a cartel forms.

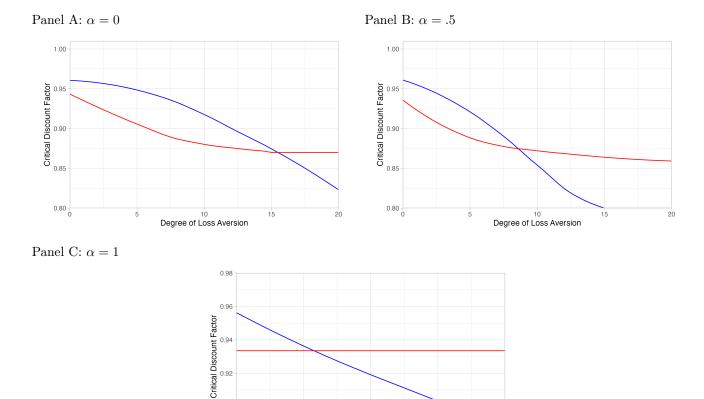


Figure 10: Critical Discount Factor by Degree of Loss Aversion Before (Red) and After (Blue) a 50% Increase in Marginal Cost.

Degree of Loss Aversion

1.5

0.90

0.88

Notes: These figures depict the critical discount factor when $\alpha=0$ (Panel A), $\alpha=.5$ (Panel B) and $\alpha=1$ (Panel C) as a function of the degree of loss aversion. The critical discount factor is the smallest discount factor for which a cartel forms. Parameters: a=100, b=2, e=1, N=5 and F=100. The blue curve denotes the critical discount factor after a 50% increase in marginal cost (i.e., c=15) and the red curve denotes the critical discount factor absent an increase in marginal cost (i.e., c=10).

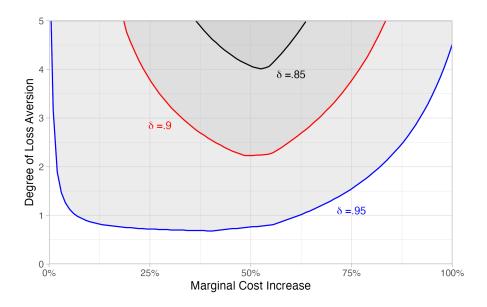


Figure 11: Threshold Degree of Loss Aversion \bar{l} by Size of Marginal Cost Increase.

Notes: This figure depicts the threshold degree of loss aversion \bar{l} by the size of a marginal cost increase for various discount factors. The grey shaded region depicts marginal cost increases and degrees of loss aversion for which a cartel forms. Parameters: $a=100, b=2, e=1, N=5, \alpha=1$ and F=105. Absent a deterioration in market conditions, all firms have a marginal cost of c=10. Thus, a 50% increase in marginal cost corresponds to a marginal cost of 15.

Figure 11 depicts the threshold degree of loss aversion \bar{l} (from Proposition 1) when the deterioration in market conditions is an increase in marginal cost, for various discount factors. When the degree of loss aversion exceeds \bar{l} , a cartel forms after the marginal cost increase. Thus, the shaded grey regions in Figure 11 denote combinations of the degree of loss aversion (l) and the size of the marginal cost increase that result in the formation of a cartel. Note that, for the discount factors depicted in Figure 11, a cartel does not form prior to the marginal cost increase (i.e., regime b) for any degree of loss aversion (thus, Condition 1 holds for all l).

Figure 11 indicates that a moderate increase in marginal cost causes a cartel to form for the widest range of l values (i.e., \bar{l} is smallest for moderate increases in marginal cost). This finding reflects two considerations. First, small increases in marginal cost do not significantly reduce Nash equilibrium profit and, therefore, are not perceived as significant losses, which weakens the effects outlined in the preceding discussion. Second, pronounced increases in marginal cost substantially reduce each firm's variable profits and limit the firm's ability to cover fixed costs of collusion, weakening incentives to collude.⁸⁹

⁸⁹If the increase in marginal cost is exceptionally large, collusive profits may be less than Nash equilibrium profits prior to the deterioration in market conditions (i.e., a violation of Assumption 8(i)). In this case, collusion would be perceived as a loss and the effects outlined in Section 4 do not occur.

F.3 Collusive Payoff

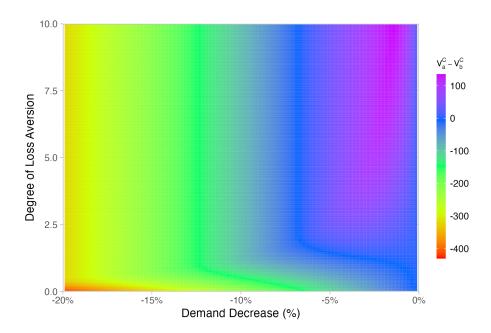


Figure 12: Difference in Collusive Payoff by Size of Demand Decrease and Degree of Loss Aversion.

Notes: This figure depicts $V_a^C - V_b^C$ as a function of the size of the decrease in the demand parameter a and the degree of loss aversion. Parameters: $b = 2, e = 1, N = 7, \alpha = 1, F = 0$ and $\delta = .8$. Prior to the deterioration in market conditions, a = 100. Thus, a 10% decrease in a implies a = 90.

Figure 13 depicts the difference between the collusive payoff after a decrease in the demand parameter a and the collusive payoff absent a change in demand (i.e., $(V_a^C - V_b^C)$) as a function of the size of the demand reduction and the degree of loss aversion l. A reduction in demand enhances the collusive payoff when 1) the reduction in demand is moderate and 2) managers are loss averse. Recall that extreme reductions in demand violate Condition 2(ii).

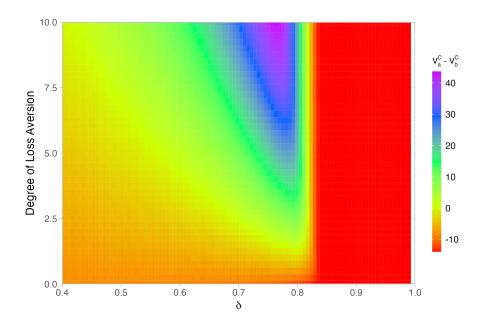


Figure 13: Change in Collusive Payoff from a 25% Increase in Marginal Cost by Discount Factor and Degree of Loss Aversion.

Notes: This figure depicts $(1-\delta) \left(V_a^C - V_b^C\right)$ as a function of the discount factor and the degree of loss aversion. Parameters: a=100, b=2; e=1, N=7, F=0 and $\alpha=1$. The deterioration in market conditions is an increase in marginal cost of 25%. Prior to the deterioration in market conditions, all firms have a marginal cost of c=10. Thus, a 25% increase in marginal cost results in a marginal cost of c=12.5.

Figure 13 depicts the normalized difference between the collusive payoff after a 25% increase in marginal cost and the collusive payoff absent a change in marginal cost (i.e., $(1 - \delta) (V_a^C - V_b^C)$) as a function of the discount rate δ and the degree of loss aversion l. Consistent with Figure 3, an increase in marginal cost reduces the collusive payoff for relatively high discount factors. However, an increase in marginal cost can enhance the collusive payoff for moderate discount factors when managers are sufficiently loss averse.

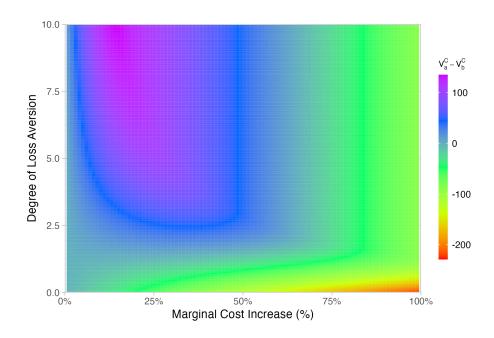


Figure 14: Difference in Collusive Payoff by Size of Marginal Cost Increase and Degree of Loss Aversion. Notes: This figure depicts $V_a^C - V_b^C$ as a function of the size of the marginal cost increase and the degree of loss aversion. Parameters: $a = 100, b = 2, e = 1, N = 7, \alpha = 1, F = 0$ and $\delta = .8$. Prior to the deterioration in market conditions, all firms have a marginal cost of c = 10. Thus, a 50% increase in marginal cost results in a marginal cost of c = 15.

Figure 14 depicts the difference between the collusive payoff after an increase in marginal cost and absent a change in marginal cost (i.e., $(V_a^C - V_b^C)$) as a function of the size of the marginal cost increase and the degree of loss aversion l. Consistent with Figure 5, moderate increases in marginal cost increase the collusive payoff when managers are loss averse. Alternatively, large increases in marginal cost reduce the collusive payoff, regardless of the degree of loss aversion. The simulations in this subsection illustrate that a moderate deterioration in market state (e.g., a moderate increase in marginal cost) can enhance the collusive payoff when the discount factor is relatively low and managers are loss averse.

F.4 Gain From Collusion

Figure 15 depicts the gain from collusion after an increase in marginal cost by the size of the marginal cost increase and the degree of loss aversion. The gain from collusion is increasing in the degree of loss aversion and, when managers are sufficiently loss averse, increasing in the size of the marginal cost increase. Recall that increases in loss aversion impact the gain from collusion in two distinct ways. First, increases in loss aversion stabilize collusion and enhance the collusion payoff. Second, increases in loss aversion reduce V_a^N , the payoff from Nash competition, which also increases the gain from collusion in this simulation.

Figure 16 depicts the normalized gain from collusion after a 25% increase in marginal cost for various

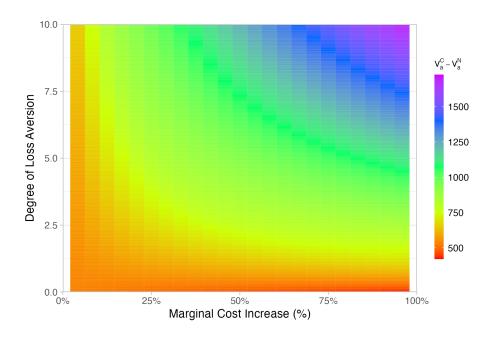


Figure 15: Gain from Collusion by Size of Marginal Cost Increase and Degree of Loss Aversion.

Notes: This figure depicts $V_a^C - V_a^N$ as a function of the increase in marginal cost and the degree of loss aversion. Parameters: $a=100, b=2, e=1, N=7, \alpha=1, F=0$ and $\delta=.8$. Prior to the deterioration in market conditions, all firms have a marginal cost of c=10. Thus, a 25% increase in marginal cost results in a marginal cost of c=12.5.

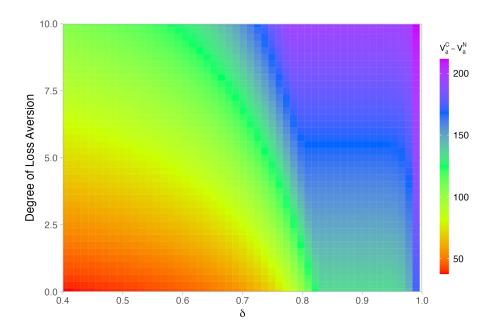


Figure 16: Gain from Collusion by Discount Factor and Degree of Loss Aversion After a 25% Increase in Marginal Cost.

Notes: This figure depicts $(1-\delta) \left(V_a^C - V_a^N\right)$ as a function of the discount factor and the degree of loss aversion. Parameters: a=100, b=2, e=1, c=10, N=7, F=0 and $\alpha=1$. The deterioration in market conditions is a 25% increase in marginal cost. Prior to the deterioration in market conditions, all firms have a marginal cost of c=10. Thus, a 25% increase in marginal cost results in a marginal cost of c=12.5.

discount factors and degrees of loss aversion. Increases in the degree of loss aversion enhance the gain from collusion for all discount factors. Recall that a cartel forms for any discount factor as there are no fixed costs of collusion.

F.5 Pricing Results

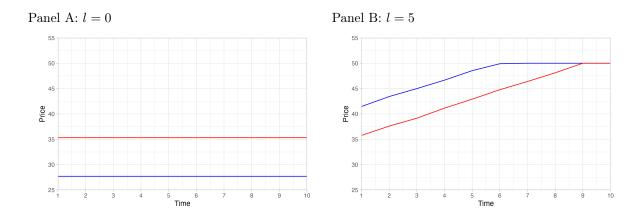


Figure 17: Optimal Price Paths Before (Red) and After (Blue) the Entry of a New Firm by Degree of Loss Aversion.

Notes: This figure depicts optimal price paths before and after the entry of a new firm for a variety of degrees of loss aversion. Parameters: $a=100, b=2, e=1, c=0, , \delta=.5$ and $\alpha=.9$. Prior to the entry of a new firm, N=4. The blue curve depicts the optimal price path after the deterioration in market conditions (i.e., N=5) and the red curve depicts the optimal price path prior to the deterioration in market conditions (i.e., N=4). The Nash equilibrium price before (after) the change is 20 (16.67). The monopoly price before and after the change is 50.

Figure 17 presents the optimal price paths before and after the entry of a new firm. When managers are loss neutral, prices under regime b always exceed prices under regime a. However, when managers are loss averse, prices under regime a exceed prices under regime b in early periods of collusion. In later periods of collusion, prices under both regimes equal the monopoly price (50).