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| **Analysis of Uncertainties** | **Fall 2014** |

Lecture 5: 12 November 2014

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## 5.1 Synopsis of error matrix equations

Recall from the previous lecture the error matrix (for correlated variables $x_1, \ldots, x_n$):

$$\mathbf{M} = \begin{pmatrix} \sigma_{x_1}^2 & \mathrm{cov}(x_1, x_2) & \ldots & \mathrm{cov}(x_1, x_n) \\ \mathrm{cov}(x_1, x_2) & \sigma_{x_2}^2 & \ldots & \mathrm{cov}(x_2, x_n) \\ \vdots & \vdots & \ddots & \vdots \\ \mathrm{cov}(x_1, x_n) & \mathrm{cov}(x_2, x_n) & \ldots & \sigma_{x_n}^2 \end{pmatrix} \tag{5.1}$$

And this came from:

$$P(x_1, x_2, \ldots, x_n) = \frac{1}{(2\pi)^{n/2}} \frac{1}{|\mathbf{M}|^{1/2}} \exp\left[-\frac{1}{2}(\mathbf{x} - \boldsymbol{\mu})^{\mathrm{T}} \mathbf{M}^{-1} (\mathbf{x} - \boldsymbol{\mu})\right] \tag{5.2}$$

We can use the error matrix for three scenarios:

1. Error propagation of a variable as a function of independent variables $x_1, \ldots, x_n$.

$$r = f(x_1, \ldots, x_n) \quad \rightarrow \quad \sigma_r^2 = \mathbf{D}^{\mathrm{T}}\mathbf{M}\mathbf{D} \quad \text{where} \quad \mathbf{D} = \begin{pmatrix} \partial r/\partial x_1 \\ \vdots \\ \partial r/\partial x_n \end{pmatrix} \tag{5.3}$$

2. Changing variables. If $\{y_m\}$ are all functions of $\{x_n\}$.

$$\mathbf{A} = \begin{pmatrix} \partial y_1/\partial x_1 & \ldots & \partial y_m/\partial x_1 \\ \vdots & \ddots & \vdots \\ \partial y_1/\partial x_n & \ldots & \partial y_m/\partial x_n \end{pmatrix} \quad \rightarrow \quad \mathbf{M}_{\{y_m\}} = \mathbf{A}^{\mathrm{T}}\mathbf{M}_{\{x_n\}}\mathbf{A} \tag{5.4}$$

3. And uncertainties in values dependent on changed variables. If $z$ is a function of all $\{y_m\}$:

$$z = f(y_1, \ldots, y_m) \quad \rightarrow \quad \sigma_z^2 = \mathbf{D}'^{\mathrm{T}}\mathbf{A}^{\mathrm{T}}\mathbf{M}_{\{x_n\}}\mathbf{A}\mathbf{D}' = \mathbf{D}'^{\mathrm{T}}\mathbf{M}_{\{y_m\}}\mathbf{D}' \quad \text{where} \quad \mathbf{D}' = \begin{pmatrix} \partial z/\partial y_1 \\ \vdots \\ \partial z/\partial y_m \end{pmatrix} \tag{5.5}$$

## 5.2 Error matrices example uses