Question 6

Modified DH parameters:

```
as      = [0.0,              0.0,    0.155,  0.135,  0.0,    0.0];
alphas  = [0.0,              -pi/2,  0.0,    0.0,    pi/2,   0.0];
thetas  = [0.0,              -pi/2,  0.0,    pi/2,   0.0,    0.0];
ds      = [0.072 + 0.075,    0.0,    0.0,    0.0,    0.113,  0.105];
```

Declare Modified DH Transformation matrix:

```
syms a alpha theta d
T_MDH = [cos(theta)             -sin(theta)             0               a;
         sin(theta)*cos(alpha)  cos(theta)*cos(alpha)   -sin(alpha)     -d*sin(alpha);
         sin(theta)*sin(alpha)  cos(theta)*sin(alpha)   cos(alpha)      d*cos(alpha);
         0                      0                       0               1]
```

T_MDH =

$$\begin{pmatrix} \cos(\theta) & -\sin(\theta) & 0 & a \\ \cos(\alpha)\sin(\theta) & \cos(\alpha)\cos(\theta) & -\sin(\alpha) & -d\sin(\alpha) \\ \sin(\alpha)\sin(\theta) & \sin(\alpha)\cos(\theta) & \cos(\alpha) & d\cos(\alpha) \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Ground Truths:

```
T_gt1 = subs(T_MDH, {a alpha theta d}, {as(1) alphas(1) thetas(1) ds(1)});
T_gt2 = subs(T_MDH, {a alpha theta d}, {as(2) alphas(2) thetas(2) ds(2)});
T_gt3 = subs(T_MDH, {a alpha theta d}, {as(3) alphas(3) thetas(3) ds(3)});
T_gt4 = subs(T_MDH, {a alpha theta d}, {as(4) alphas(4) thetas(4) ds(4)});
T_gt5 = subs(T_MDH, {a alpha theta d}, {as(5) alphas(5) thetas(5) ds(5)});
T_gt6 = subs(T_MDH, {a alpha theta d}, {as(6) alphas(6) thetas(6) ds(6)});
T_gt  = T_gt1*T_gt2*T_gt3*T_gt4*T_gt5*T_gt6
```

T_gt =

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \frac{131}{200} \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Estimations with 0.5 degree offset in θ_i:

```
T_est1 = subs(T_MDH, {a alpha theta d}, {as(1) alphas(1) thetas(1) + 0.5*pi/180 ds(1)})
T_est2 = subs(T_MDH, {a alpha theta d}, {as(2) alphas(2) thetas(2) + 0.5*pi/180 ds(2)})
T_est3 = subs(T_MDH, {a alpha theta d}, {as(3) alphas(3) thetas(3) + 0.5*pi/180 ds(3)})
T_est4 = subs(T_MDH, {a alpha theta d}, {as(4) alphas(4) thetas(4) + 0.5*pi/180 ds(4)})
T_est5 = subs(T_MDH, {a alpha theta d}, {as(5) alphas(5) thetas(5) + 0.5*pi/180 ds(5)})
T_est6 = subs(T_MDH, {a alpha theta d}, {as(6) alphas(6) thetas(6) + 0.5*pi/180 ds(6)})

T_est_1 = T_est1*T_gt2*T_gt3*T_gt4*T_gt5*T_gt6;
T_est_2 = T_gt1*T_est2*T_gt3*T_gt4*T_gt5*T_gt6;
T_est_3 = T_gt1*T_gt2*T_est3*T_gt4*T_gt5*T_gt6;
```

```
T_est_4 = T_gt1*T_gt2*T_gt3*T_est4*T_gt5*T_gt6;
T_est_5 = T_gt1*T_gt2*T_gt3*T_gt4*T_est5*T_gt6;
T_est_6 = T_gt1*T_gt2*T_gt3*T_gt4*T_gt5*T_est6;

T_ERR_1 = inv(T_gt)*T_est_1;
T_ERR_1_a = T_gt*inv(T_est_1);
T_ERR_1_b = inv(T_est_1)*T_gt;

T_ERR_2 = inv(T_gt)*T_est_2
```

T_ERR_2 =

$$\begin{pmatrix} \sin\left(\frac{179\,\pi}{360}\right) & 0 & \cos\left(\frac{179\,\pi}{360}\right) & \frac{127\cos\left(\frac{179\,\pi}{360}\right)}{250} \\ 0 & 1 & 0 & 0 \\ -\cos\left(\frac{179\,\pi}{360}\right) & 0 & \sin\left(\frac{179\,\pi}{360}\right) & \frac{127\sin\left(\frac{179\,\pi}{360}\right)}{250} - \frac{127}{250} \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```
T_ERR_2_a = T_gt*inv(T_est_2)
```

T_ERR_2_a =

$$\begin{pmatrix} \frac{\sigma_2}{\sigma_1} & 0 & -\frac{\sigma_3}{\sigma_1} & \frac{147\,\sigma_3}{1000\,\sigma_1} \\ 0 & 1 & 0 & 0 \\ \frac{\sigma_3}{\sigma_1} & 0 & \frac{\sigma_2}{\sigma_1} & \frac{131}{200} - \frac{508\,{\sigma_3}^2 + 508\,{\sigma_2}^2 + 147\,\sigma_2}{1000\,\sigma_1} \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

where

$$\sigma_1 = {\sigma_3}^2 + {\sigma_2}^2$$

$$\sigma_2 = \sin\left(\frac{179\,\pi}{360}\right)$$

$$\sigma_3 = \cos\left(\frac{179\,\pi}{360}\right)$$

```
T_ERR_2_b = inv(T_est_2)*T_gt;

T_ERR_3 = inv(T_gt)*T_est_3
```

T_ERR_3 =

$$\begin{pmatrix} \cos\left(\frac{\pi}{360}\right) & 0 & \sin\left(\frac{\pi}{360}\right) & \frac{353\sin\left(\frac{\pi}{360}\right)}{1000} \\ 0 & 1 & 0 & 0 \\ -\sin\left(\frac{\pi}{360}\right) & 0 & \cos\left(\frac{\pi}{360}\right) & \frac{353\cos\left(\frac{\pi}{360}\right)}{1000} - \frac{353}{1000} \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```
T_ERR_3_a = T_gt*inv(T_est_3)
```

```
T_ERR_3_a =
```

$$\begin{pmatrix} \sigma_2 & 0 & -\sigma_1 & \frac{151\sin\left(\frac{\pi}{360}\right)}{500\ (\sigma_4+\sigma_3)} \\ 0 & 1 & 0 & 0 \\ \sigma_1 & 0 & \sigma_2 & \frac{131}{200} - \frac{353\,\sigma_4 + 302\cos\left(\frac{\pi}{360}\right) + 353\,\sigma_3}{1000\ (\sigma_4+\sigma_3)} \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

where

$$\sigma_1 = \frac{\sin\left(\frac{\pi}{360}\right)}{\sigma_4+\sigma_3}$$

$$\sigma_2 = \frac{\cos\left(\frac{\pi}{360}\right)}{\sigma_4+\sigma_3}$$

$$\sigma_3 = \sin\left(\frac{\pi}{360}\right)^2$$

$$\sigma_4 = \cos\left(\frac{\pi}{360}\right)^2$$

```
T_ERR_3_b = inv(T_est_3)*T_gt;

T_ERR_4 = inv(T_gt)*T_est_4
```

```
T_ERR_4 =
```

$$\begin{pmatrix} \sin\left(\frac{179\,\pi}{360}\right) & 0 & \cos\left(\frac{179\,\pi}{360}\right) & \frac{109\cos\left(\frac{179\,\pi}{360}\right)}{500} \\ 0 & 1 & 0 & 0 \\ -\cos\left(\frac{179\,\pi}{360}\right) & 0 & \sin\left(\frac{179\,\pi}{360}\right) & \frac{109\sin\left(\frac{179\,\pi}{360}\right)}{500} - \frac{109}{500} \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```
T_ERR_4_a = T_gt*inv(T_est_4)
```

T_ERR_4_a =

$$\begin{pmatrix} \frac{\sigma_2}{\sigma_1} & 0 & -\frac{\sigma_3}{\sigma_1} & \frac{437\,\sigma_3}{1000\,\sigma_1} \\ 0 & 1 & 0 & 0 \\ \frac{\sigma_3}{\sigma_1} & 0 & \frac{\sigma_2}{\sigma_1} & \frac{131}{200} - \frac{218\,{\sigma_3}^2 + 218\,{\sigma_2}^2 + 437\,\sigma_2}{1000\,\sigma_1} \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

where

$$\sigma_1 = {\sigma_3}^2 + {\sigma_2}^2$$

$$\sigma_2 = \sin\left(\frac{179\,\pi}{360}\right)$$

$$\sigma_3 = \cos\left(\frac{179\,\pi}{360}\right)$$

```
T_ERR_4_b = inv(T_est_4)*T_gt;

T_ERR_5 = inv(T_gt)*T_est_5
```

T_ERR_5 =

$$\begin{pmatrix} \cos\left(\frac{\pi}{360}\right) & -\sin\left(\frac{\pi}{360}\right) & 0 & 0 \\ \sin\left(\frac{\pi}{360}\right) & \cos\left(\frac{\pi}{360}\right) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```
T_ERR_5_a = T_gt*inv(T_est_5)
```

T_ERR_5_a =

$$\begin{pmatrix} \sigma_2 & \sigma_1 & 0 & 0 \\ -\sigma_1 & \sigma_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

where

$$\sigma_1 = \frac{\sin\left(\frac{\pi}{360}\right)}{\cos\left(\frac{\pi}{360}\right)^2 + \sin\left(\frac{\pi}{360}\right)^2}$$

$$\sigma_2 = \frac{\cos\left(\frac{\pi}{360}\right)}{\cos\left(\frac{\pi}{360}\right)^2 + \sin\left(\frac{\pi}{360}\right)^2}$$

```
T_ERR_5_b = inv(T_est_5)*T_gt;

T_ERR_6 = inv(T_gt)*T_est_6;
T_ERR_6_a = T_gt*inv(T_est_6)
```

```
T_ERR_6_a =
```

$$\begin{pmatrix} \sigma_2 & \sigma_1 & 0 & 0 \\ -\sigma_1 & \sigma_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

where

$$\sigma_1 = \frac{\sin\left(\frac{\pi}{360}\right)}{\cos\left(\frac{\pi}{360}\right)^2 + \sin\left(\frac{\pi}{360}\right)^2}$$

$$\sigma_2 = \frac{\cos\left(\frac{\pi}{360}\right)}{\cos\left(\frac{\pi}{360}\right)^2 + \sin\left(\frac{\pi}{360}\right)^2}$$

```
T_ERR_6_b = inv(T_est_6)*T_gt;

N1 = norm(T_ERR_1(1:3,4)) % = 0 --> 6c)
```

```
N1 = 0
```

```
N1_a = norm(T_ERR_1_a(1:3,4))
```

N1_a = 0

```
N1_b = norm(T_ERR_1_b(1:3,4)) % = 0
```

N1_b = 0

```
N2 = norm(T_ERR_2(1:3,4)) % = 0.0044 --> 6e)
```

N2 =

$$\sqrt{\left(\frac{127\sin\left(\frac{179\,\pi}{360}\right)}{250}-\frac{127}{250}\right)^2+\frac{16129\cos\left(\frac{179\,\pi}{360}\right)^2}{62500}}$$

```
N2_a = norm(T_ERR_2_a(1:3,4)) % = 0.0013
```

N2_a =

$$\sqrt{\left(\frac{508\,\sigma_2+508\sin\left(\frac{179\,\pi}{360}\right)^2+147\sin\left(\frac{179\,\pi}{360}\right)}{1000\,\sigma_1}-\frac{131}{200}\right)^2+\frac{21609\,\sigma_2}{1000000\,{\sigma_1}^2}}$$

where

$$\sigma_1=\sigma_2+\sin\left(\frac{179\,\pi}{360}\right)^2$$

$$\sigma_2=\cos\left(\frac{179\,\pi}{360}\right)^2$$

```
N2_b = norm(T_ERR_2_b(1:3,4)) % = 0.0044
```

N2_b =

$$\sqrt{\frac{16129\cos\left(\frac{179\,\pi}{360}\right)^2}{62500\,{\sigma_1}^2}+\left(\frac{131\sin\left(\frac{179\,\pi}{360}\right)}{200\,\sigma_1}-\frac{508\cos\left(\frac{179\,\pi}{360}\right)^2+508\sin\left(\frac{179\,\pi}{360}\right)^2+147\sin\left(\frac{179\,\pi}{360}\right)}{1000\,\sigma_1}\right)^2}$$

where

$$\sigma_1=\cos\left(\frac{179\,\pi}{360}\right)^2+\sin\left(\frac{179\,\pi}{360}\right)^2$$

```
N3 = norm(T_ERR_3(1:3,4)) % = 0.0031
```

N3 =

$$\sqrt{\left(\frac{353\cos\left(\frac{\pi}{360}\right)}{1000}-\frac{353}{1000}\right)^2+\frac{124609\sin\left(\frac{\pi}{360}\right)^2}{1000000}}$$

```
N3_a = norm(T_ERR_3_a(1:3,4)) % = 0.0026
```

```
N3_a =
```

$$\sqrt{\left(\frac{353\cos\left(\frac{\pi}{360}\right)^2+302\cos\left(\frac{\pi}{360}\right)+353\,\sigma_2}{1000\,\sigma_1}-\frac{131}{200}\right)^2+\frac{22801\,\sigma_2}{250000\,{\sigma_1}^2}}$$

where

$$\sigma_1=\cos\left(\frac{\pi}{360}\right)^2+\sigma_2$$

$$\sigma_2=\sin\left(\frac{\pi}{360}\right)^2$$

```
N3_b = norm(T_ERR_3_b(1:3,4)) % = 0.0031
```

```
N3_b =
```

$$\sqrt{\left(\frac{131\cos\left(\frac{\pi}{360}\right)}{200\,\sigma_1}-\frac{353\cos\left(\frac{\pi}{360}\right)^2+302\cos\left(\frac{\pi}{360}\right)+353\sin\left(\frac{\pi}{360}\right)^2}{1000\,\sigma_1}\right)^2+\frac{124609\sin\left(\frac{\pi}{360}\right)^2}{1000000\,{\sigma_1}^2}}$$

where

$$\sigma_1=\cos\left(\frac{\pi}{360}\right)^2+\sin\left(\frac{\pi}{360}\right)^2$$

```
N4 = norm(T_ERR_4(1:3,4)) % = 0.0019 --> 6d)
```

```
N4 =
```

$$\sqrt{\left(\frac{109\sin\left(\frac{179\,\pi}{360}\right)}{500}-\frac{109}{500}\right)^2+\frac{11881\cos\left(\frac{179\,\pi}{360}\right)^2}{250000}}$$

```
N4_a = norm(T_ERR_4_a(1:3,4)) % = 0.0038
```

```
N4_a =
```

$$\sqrt{\left(\frac{218\,\sigma_2 + 218\sin\left(\frac{179\,\pi}{360}\right)^2 + 437\sin\left(\frac{179\,\pi}{360}\right)}{1000\,\sigma_1} - \frac{131}{200}\right)^2 + \frac{190969\,\sigma_2}{1000000\,{\sigma_1}^2}}$$

where

$$\sigma_1 = \sigma_2 + \sin\left(\frac{179\,\pi}{360}\right)^2$$

$$\sigma_2 = \cos\left(\frac{179\,\pi}{360}\right)^2$$

```
N4_b = norm(T_ERR_4_b(1:3,4)) % = 0.0019
```

```
N4_b =
```

$$\sqrt{\frac{11881\cos\left(\frac{179\,\pi}{360}\right)^2}{250000\,{\sigma_1}^2} + \left(\frac{131\sin\left(\frac{179\,\pi}{360}\right)}{200\,\sigma_1} - \frac{218\cos\left(\frac{179\,\pi}{360}\right)^2 + 218\sin\left(\frac{179\,\pi}{360}\right)^2 + 437\sin\left(\frac{179\,\pi}{360}\right)}{1000\,\sigma_1}\right)^2}$$

where

$$\sigma_1 = \cos\left(\frac{179\,\pi}{360}\right)^2 + \sin\left(\frac{179\,\pi}{360}\right)^2$$

```
N5 = norm(T_ERR_5(1:3,4)) % = 0
```

```
N5 = 0
```

```
N5_a = norm(T_ERR_5_a(1:3,4))
```

```
N5_a = 0
```

```
N5_b = norm(T_ERR_5_b(1:3,4)) % = 0
```

```
N5_b = 0
```

```
N6 = norm(T_ERR_6(1:3,4)) % = 0
```

```
N6 = 0
```

```
N6_a = norm(T_ERR_6_a(1:3,4))
```

```
N6_a = 0
```

```
N6_b = norm(T_ERR_6_b(1:3,4)) % = 0
```

```
N6_b = 0
```

```
plot([0,1,2,3,4,5,6], [0,N1,N2,N3,N4,N5,N6]) % --> 6f)
xlabel("Number of Joints (i)")
ylabel("Translation positioning error (m)")
```