Robotic Systems Engineering
Coursework 1: Rigid Transformations and Forward Kinematics

Prof. Danail Stoyanov
Department of Computer Science
University College London
Gower Street, London, WC1E 6BT

October 23, 2020

To get full credit for an answer, you are *required* to provide a full working solution. For some questions, you will be asked to write code; for these you are expected to include a *full print out* of any requested results or graphs in the report, as well as a simple breakdown of the algorithms used in the coding solution. Furthermore, you will be required to *upload* your code to Moodle along with your submitted coursework manuscript in .zip extension. The necessary packages are available on https://github.com/surgical-vision/comp0127_lab

Rigid Transformations

1. a. Given an arbitrary 3D rotation matrix,

$$\mathbf{R} = \begin{bmatrix} r_1 & r_2 & r_3 \\ r_4 & r_5 & r_6 \\ r_7 & r_8 & r_9 \end{bmatrix}$$

   Prove that $||r_i|| \leq 1$ where $i = 1, 2, ..., 9$. [3 marks]

   b. For any rotation matrix $\mathbf{R}$, prove that $\mathbf{R}_{k,\theta} = \mathbf{R}_{-k,-\theta}$, where k is the unit vector defined axis of rotation and θ is the angle of rotation. [3 marks]

   c. Given two arbitrary Cartesian coordinate frames a and b, what does each row in a rotation matrix ${}^{a}\mathbf{R}_{b}$ represent? [3 marks]

   d. Identify the relationship between axis/angle of rotation and the eigenvector/eigenvalue of a rotation matrix. [3 marks]

2. a. Convert this rotation matrix into Z-Y-Z Euler angle representation, i.e. $\mathbf{R}_z\mathbf{R}_y\mathbf{R}_z$.

$$\mathbf{R} = \begin{bmatrix} -\frac{\sqrt{3}}{4} - \frac{\sqrt{6}}{8} & -\frac{\sqrt{2}}{4} & -\frac{3}{4} + \frac{\sqrt{2}}{8} \\ \frac{1}{4} - \frac{3\sqrt{2}}{8} & -\frac{\sqrt{6}}{4} & \frac{\sqrt{3}}{4} + \frac{\sqrt{6}}{8} \\ -\frac{\sqrt{6}}{4} & \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{4} \end{bmatrix}$$

[5 marks]

b. Why can a rotation matrix have more than one Z-Y-X Euler angle representation? [3 marks]

c. What is the limitation of using the Euler angle representation for a rotation matrix? How could one avoid this limitation in practice? [3 marks]

3. a. Convert the rotation matrix in Question 2.a into a quaternion. [5 marks]

   b. Prove that a rotation quaternion $\mathbf{q}$ and $-\mathbf{q}$ are equivalent. [3 marks]

   c. When do two arbitrary rotation matrices $\mathbf{R}_a$ and $\mathbf{R}_b$ become commutative? [5 marks]

4. Complete the following tasks by filling in the code templates in the packages ”cw1/cw1q4_srv” and ”cw1/cw1q4”. Please note that you can either use the cpp template ”cw1q4_node.cpp” or the py template ”cw1q4_node.py” to complete the tasks. **A simple code breakdown in the report is required for this subquestion.**

   a. Fill in the appropriate input and output for each service. [1 (report) + 2 (code) marks]

      - The quaternion message is ”geometry_msgs/Quaternion”, namely ”q”.
      - The rodrigues representation message is three ”std_msgs/Float64”, namely ”x” ,”y” and ”z”.
      - The Z-Y-X euler angle representation message is three ”std_msgs/Float64”, namely ”z” ,”y” and ”x”.
      - The rotation matrix is three ”std_msgs/Float64MultiArray” representing three rows vectors in the matrix, namely ”r1” ,”r2” and ”r3”.

   b. Write a ROS node for converting a quaternion representation to an euler angle representation $\mathbf{R}_z\mathbf{R}_y\mathbf{R}_x$. [1 (report) + 2 (code) marks]

   c. Write a ROS node for converting a quaternion representation to rodrigues representation. [1 (report) + 2 (code) marks]

   d. Write a ROS node for converting a rotation matrix to a quaternion representation. [1 (report) + 2 (code) marks]

Robot Kinematic

5. Apply forward kinematics on the KUKA YouBot manipulator to identify the end-effector pose with respect to the origin using the simplified dimensions of the Youbot shown in Figure 1.

   a. Use modified Denavit-Hartenberg convention. [7 marks]

b. What is the difference between the standard and the modified Denavit-Hartenberg convention? [3 mark]

c. Complete this task by filling in the code in the package ”cw1/cw1q5c”. **A simple code breakdown in the report is required for this subquestion.** Write a ROS node for computing the transformation using forward kinematics. Your code should be able to

- Take DH variables from the rostopic that publishes joint data.
- Publish a set of transformations relating the frame ”base_link” and each frame on the arm ”arm5c_link_i” where i is the frame, using tf messages.

[4 (report) + 8 (code) marks]

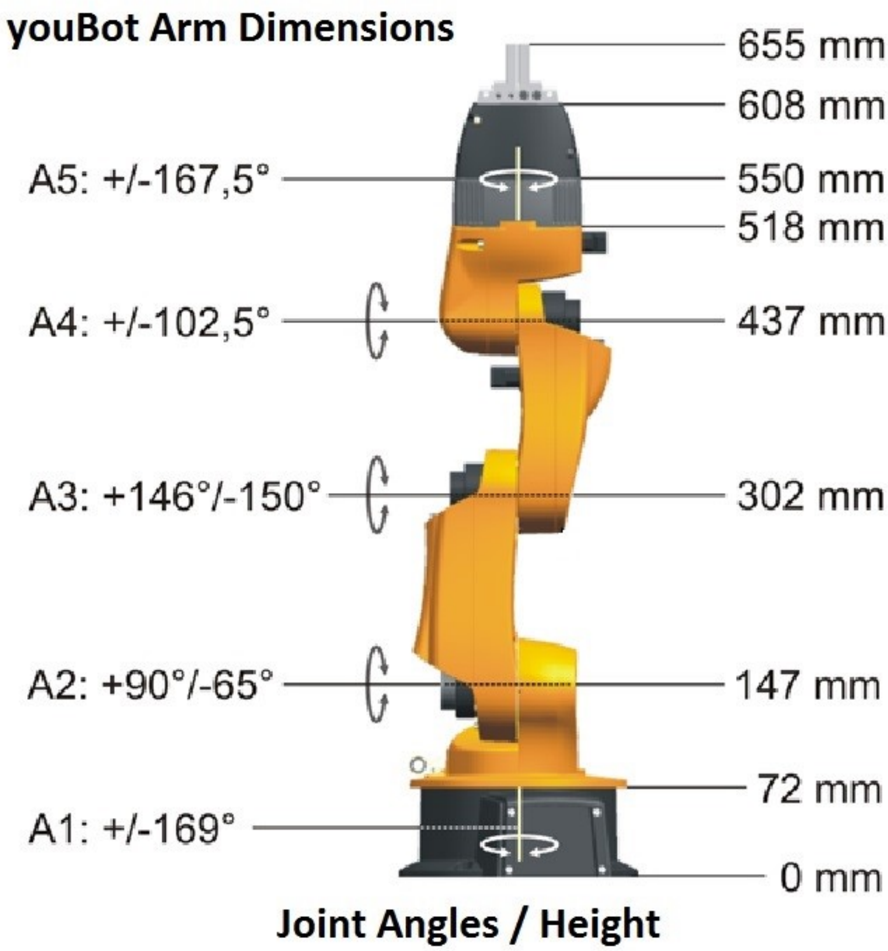

Figure 1: Kuka youBot Manipulator’s simplified dimensions

6. In robotic applications, the error in rigid transformation is represented as $\mathbf{T}_{\text{error}} = \mathbf{T}_{\text{gt}}^{-1}\mathbf{T}_{\text{est}}$, where $\mathbf{T}_{\text{gt}}$ is the ground truth of the transformation and $\mathbf{T}_{\text{est}}$ is the estimation of the transformation. The residue in rotation is represented by the rotation part of $\mathbf{T}_{\text{error}}$ (also depending on the interested representation) whereas the error in translation is represented

by the norm of the translation component of $\mathbf{T}_{\text{error}}$. Use **THIS FORMULA** and **SIMPLIFIED MODEL** of Youbot (from question 5) to answer the following questions.

NOTE: For the subquestions 6c-6e, The full calculation of $\mathbf{T}_{\text{gt}}$ and $\mathbf{T}_{\text{est}}$ is not required.

a. If we use $\mathbf{T}_{\text{gt}}\mathbf{T}_{\text{est}}^{-1}$ to calculate the error instead of using $\mathbf{T}_{\text{gt}}^{-1}\mathbf{T}_{\text{est}}$, will the error in the rotation (in rodrigues representation) and the translation be the same? [3 marks]

b. If we use $\mathbf{T}_{\text{est}}^{-1}\mathbf{T}_{\text{gt}}$ to calculate the error instead of using $\mathbf{T}_{\text{gt}}^{-1}\mathbf{T}_{\text{est}}$, will the error in the translation be the same? [1 marks]

c. Suppose the Youbot is in the zero position, i.e. $\theta_i = 0$ for $i = 1, 2, ..., 5$, but there is an error in joint position reading which produces $\theta_1 = 0.5°$. Compute the positioning error in the translation component. [1 mark]

d. Suppose the Youbot is in the zero position, i.e. $\theta_i = 0$ for $i = 1, 2, ..., 5$, but there is an error in joint position reading which produces $\theta_4 = 0.5°$. Compute the positioning error in the translation component. [1 mark]

e. Suppose the Youbot is in the zero position, i.e. $\theta_i = 0$ for $i = 1, 2, ..., 5$, but there is an error in joint position reading which produces $\theta_2 = 0.5°$. Compute the positioning error in the translation component. [1 mark]

f. Explain why the error in position reading in each joint produce different errors. What is the implication of this situation in a bigger robot? [6 marks]

7. Apply forward kinematics on the KUKA YouBot manipulator to identify the end-effector pose with respect to the origin using the complete dimensions of the Youbot described in the xacro file "robot_description/youbot_description/urdf/youbot_arm/arm.urdf.xacro".

   a. Use **STANDARD** Denavit-Hartenberg convention to determine the location of each frame. [7 marks]

   b. Complete this task by filling in the code in the package "cw1/cw1q7b". **A simple code breakdown in the report is required for this subquestion.** Write a ROS node for computing the transformation using forward kinematics. Your code should be able to

      - Take DH variables from the rostopic that publishes joint data.
      - Publish a set of transformations relating the frame "base_link" and each frame on the arm "arm7b_link_i" where i is the frame, using tf messages.

[4 (report) + 6 (code) marks]

END OF COURSEWORK