Individual Coursework

The submission of this coursework includes:

- written report with individual answers to each question (docx, pdf);
- Simulink file (.slx) for question 2 c)
- Matlab file(s) (.m) for question 2 e)

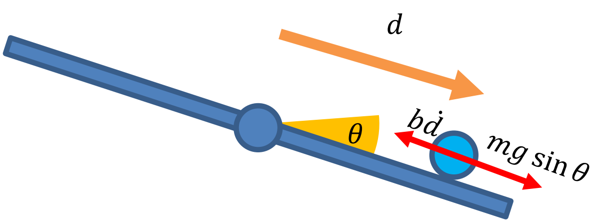
IMPORTANT: Only in questions that are marked with [MATLAB] you can use Matlab results/plots/code to justify your answers. For all other questions, justify your answers based on mathematical derivations and/or written argument as necessary.

This coursework aims at controlling the position $d(t)$ of a sphere on a rotating platform. The only forces actuating on the sphere are the gravity and a surface friction.

Assume the following

- the distance d is measured and can be used for control feedback
- the initial conditions are: $d = \dot{d} = 0$
- the mass, gravitational constant, and friction coefficient are respectively

$$m = 10\text{Kg}; \quad g = 9.8\text{m/s}; \quad b = 1.5$$



Questions

1) **[35 marks]** Assume a simplified scenario where the platform angle θ (in radians) is directly and instantly controlled by a servo-motor.

   a) **[4 marks]** *"We can assume, without any loss of generality, that the control input is $u(t) = sin\,\theta(t)$"*. Justify this statement and explain why this is useful.

   b) **[4 marks]** Determine the continuous open-loop transfer function of this system, using the input defined in a).

   c) **[7 marks]** Suppose that we are designing a continuous P controller with a gain K. What is the range of values K that make the system both stable and without any oscillations for a step response?

   d) **[4 marks] [MATLAB]** Confirm the answer to c) by reference to the root locus and the closed-loop step response plots.

   e) **[14 marks] [MATLAB]** Consider as a starting point a P controller with $K = 0.05$.
      - Determine its crossover frequency and phase margin.
      - From its closed-loop step response, determine the maximum overshoot and the approximate time it takes to converge to 1.
      - Using this information, design a lead compensator with the same crossover frequency as above, but reducing both the maximum overshoot and the convergence time of the closed-loop step response.

2) **[65 marks]** Assume now a slightly more realistic scenario where the platform angle θ does not change instantly according to the servo-motor input anymore. For a control input $u(t)$ to the motor, the platform angle changes continuously according to

$$\dot{\theta}(t) = K_m(u(t) - \theta(t))$$

Where $K_m = 25$ is a constant parameter.

a) **[5 marks]** For a small angle approximation ($\theta \approx 0 \rightarrow \sin\theta \approx \theta$), determine the continuous open-loop transfer function of this new system.

b) **[5 marks] [MATLAB]** Between the following continuous controllers
   - P controller
   - PD controller with a single gain $K\big(e(t) + \dot{e}(t)\big)$
   - A PI controller with a single gain $K\left(e(t) + \int_0^t e\,dt\right)$

   Show that the PD controller is the only one that can stabilise the sphere.

c) **[25 marks] [MATLAB]** For the continuous PD controller defined in b), find a value K with phase margin between 76 and 78 degrees. Discretise the transfer function of this controller with a sampling time of 0.1 seconds. Develop a Simulink model containing a simulation of the linearised system and both the continuous PD controller and the discretised PD controller. Move the sphere according to the following dynamic reference:

$$r_d(t) = \begin{cases} 0.3, & 0 \le t < 4 \\ 1.1 - 0.2t, & 4 \le t < 8 \\ -0.5, & 8 \le t < \infty \end{cases}$$

   Compare the performance of both controllers:
   - With/without input disturbances (Uniform noise with [-0.05 0.05] rad)
   - With/without sensor noise (Gaussian noise with variance 0.005m)

   In addition to the written answer, submit the Simulink .slx file.

d) **[5 marks]** Convert the discrete PD controller developed in c) to a discrete time domain function (that could be implemented as code in a programming loop).

e) **[25 marks] [MATLAB]** In a Matlab .m script, implement a simulation of the non-linear system (now assuming $\sin\theta \neq \theta$), together with the controller developed in d). Follow the trajectory:

$$r_d(t) = \begin{cases} 0.15, & 0 \le t < 8 \\ 0.95 - 0.1t, & 8 \le t < 16 \\ -0.65, & 16 \le t < \infty \end{cases}$$

   Assume
   - Input disturbances (Uniform noise with [-0.05 0.05] rad)
   - Sensor noise (Gaussian noise with variance 0.005m)

   In addition to the written answer, submit the Matlab .m file(s) and instructions to run the code as necessary.