CH2: Engine
The Concept of Core Engine:

3 components:

- Compressor
  Gain mechanical energy from the turbine by a rotating shaft to compress air that passing through, the temperature of air also increases.

- Combustor
  Ignite the fuel to transfer chemical energy in the fuel to kinetic energy of the high-pressure gas.

- Turbine
  Driven by the high pressure, high temperature gas and gives out mechanical energy to compressor. However, only a part of the energy in the gas is transferred to the mechanical energy needed.

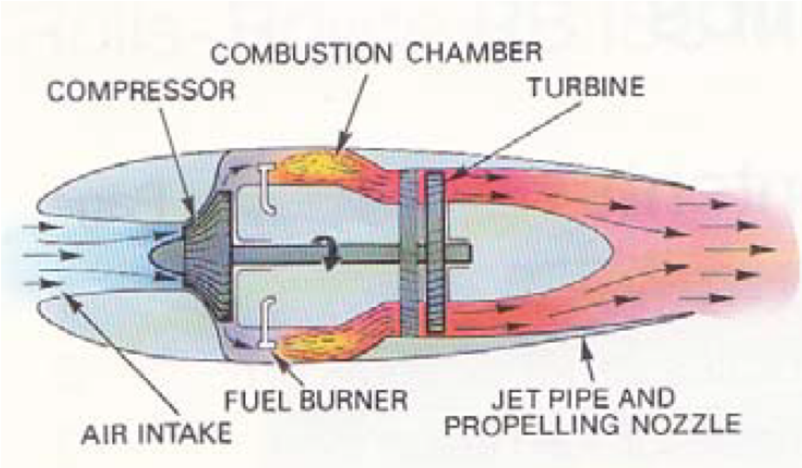
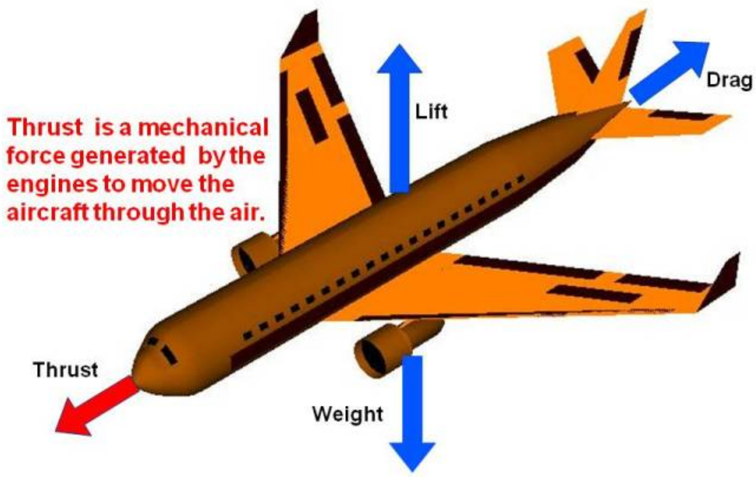


Fig. 1-2 A Whittle-type turbo-jet engine.

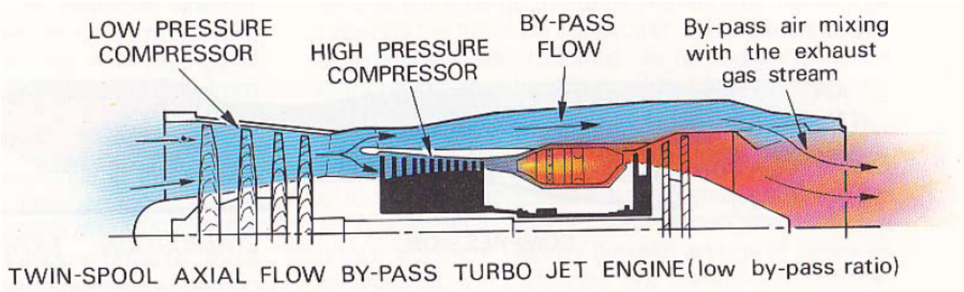
Turbojet engine uses the excess heat energy to expand and accelerate the gas through a nozzle. The reaction force presents itself as a thrust force:



The product of thrust with flight velocity is the useful propulsion power delivered. When a jet engine is tested on the ground, there is no flight velocity and hence no useful power output. Such that, all energy wasted in the jet in the form of turbulence and heat.

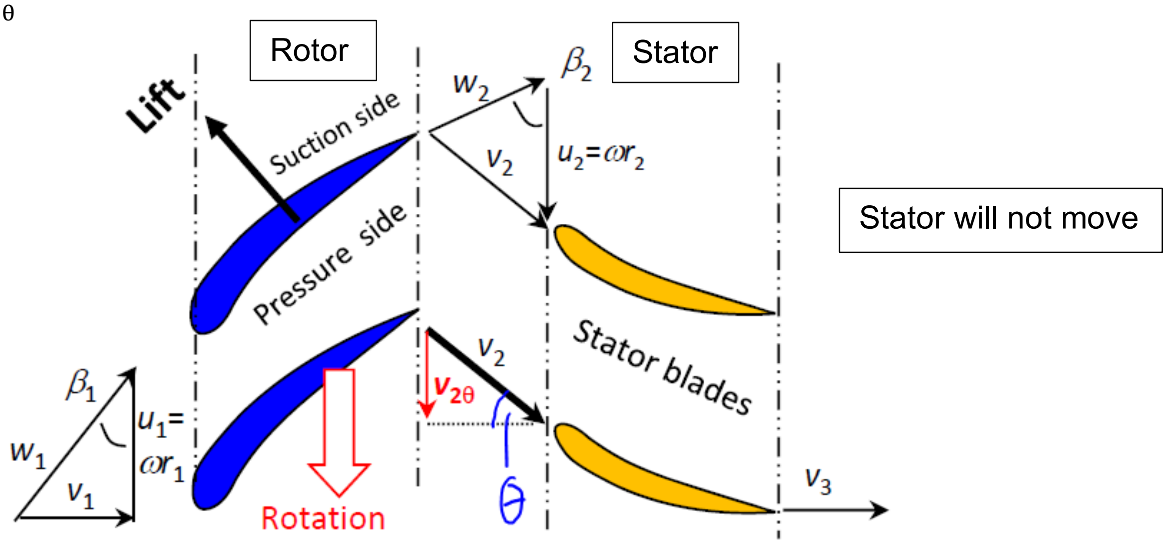
Later, a propeller is added, and the power output is used to overcome air resistance and so pulls the aircraft forward. But since the propeller have a large diameter, the shaft rotational speed of the compressor and turbine is too fast for the propeller. A better design would be to reduce propeller diameter and use many blades to compensate for the reduced power output from smaller blades. Then, the propeller blades become a fan, this is turbofan engine.

A turbofan requires at least two rotating shafts, one for the low pressure (LP) compressor and turbine, and another for the high pressure (HP) compressor and turbine.



Thermofluid principles of turbomachinery:

Air compression in a compressor is a mechanical process without heat addition. The energy adds to the flow by the reaction of the airfoil lift, it pushes air in the circumferential direction. This force, couples with the rotating velocity, add power to the flow. The flow then gains energy by gaining a circumferential velocity component $v_{2\theta}$



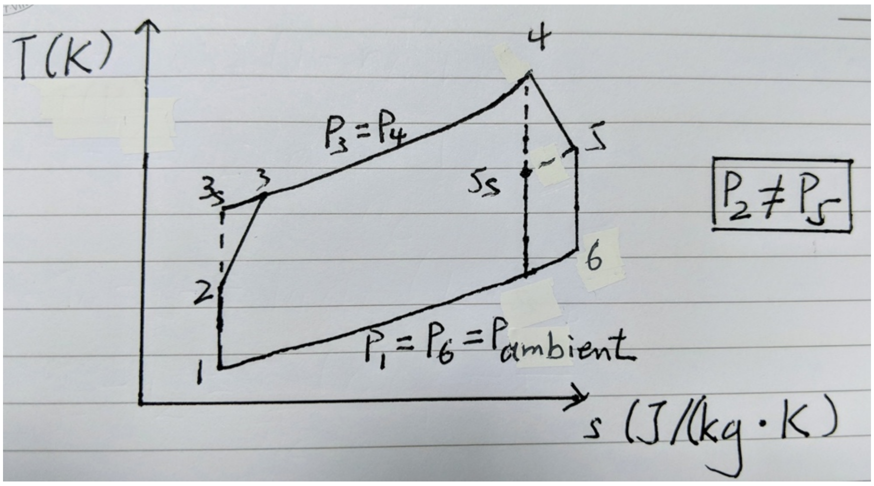
Therefore, the air flow enters the rotor at the axial direction v_1 and exists at v_2. The angle of turning is:

$$\theta = \tan^{-1}\left(\frac{v_{2\theta}}{v_1}\right)$$

If the inlet flow v_1 is too slow, the angle of turning θ required by the blade becomes too large which will cause flow separation (stall).

Turbine blades do exactly the opposite as flow pushes the rotor blades to transfer energy to the rotating shaft. In the turbine, the stator act as a nozzle, so the heat energy of the flow is partially converted to KE of the flow because the velocity is increased. Turbine do not need to consider the inlet flow because it depends on the compressor.

Compression



If the compression is done very slowly, the process is reversible and thus isentropic. Then the entropy remains constant.

$$\mathrm{dq} = \mathrm{Tds} = \mathrm{du} + \mathrm{pdv} = \mathrm{dh} - \mathrm{vdp} = 0$$

This means that the mechanical energy given to the flow remains mechanical instead of being degrading to heat. This is the ideal process we want to have, and this does not occur in reality. However, we use this to calculate the efficiency of the compressor.

Accompanying the above equation with pv = mRT, will give:

$$\frac{T_{3s}}{T_2} = \left(\frac{p_{3s}}{p_2}\right)^{\frac{k-1}{k}}$$

Where:

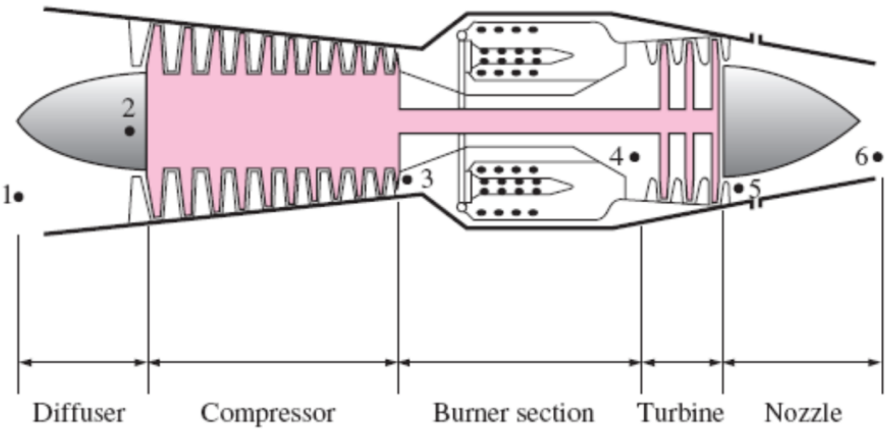
$$r_{comp} = \frac{p_{3s}}{p_2}$$

is the compression pressure ratio, and

$$k = \frac{c_p}{c_v}$$

Which typically have a value of k = 1.4 for air and 1.3 for burned gas going through the turbine.
Also:

- For burnt gas, c_p = 1244 J/kg
- For pure air, c_p = 1005 J/kg



*For now, assume the pressures at state 1 and 6 are the same as the ambient pressure.

The compression efficiency is defined by:

$$\begin{aligned}\eta_{comp} &= \frac{\text{minimum energy required by isentropic compression}}{\text{actual energy used to achieve the targeted pressure from stage 2 to 3}} \\ &= \frac{h_{3s} - h_2}{h_3 - h_2} \\ &= \frac{c_p(T_{3s} - T_2)}{c_p(T_3 - T_2)} = \frac{r_{comp}^{\frac{k-1}{k}} - 1}{\frac{T_3}{T_2} - 1}\end{aligned}$$

In reality, $T_3 > T_{3s}$, more heat is added to compress the gas.
We also use the above equation to calculate T_3 if the efficiency is given.

The turbine efficiency is defined by:

$$\begin{aligned}\eta_{turb} &= \frac{\text{the actual energy output}}{\text{the isentropic amont of energy output where pressure falls from stage 4 to 5}} \\ &= \frac{h_4 - h_5}{h_4 - h_{5s}} \\ &= \frac{c_p(T_4 - T_5)}{c_p(T_4 - T_{5s})} = \frac{1 - \frac{T_5}{T_4}}{1 - r_{turb}^{\frac{k-1}{k}}}\end{aligned}$$

$$\boxed{r_{turb}^{\frac{k-1}{k}} = \frac{T_{5s}}{T_4}}$$

As $T_5 > T_{5s}$, the energy output is less. So there is less energy converted to mechanical energy than expected.

If we **assume the air at the compressor inlet (state 2) and the turbine exit (state 5) are also under ambient pressure**, we have:

$$p_2 = p_5$$
$$\frac{p_3}{p_2} = \frac{p_4}{p_5}$$
$$r_T = \left(\frac{p_{3s}}{p_2}\right)^{\frac{k-1}{k}} = \left(\frac{p_4}{p_{5s}}\right)^{\frac{k-1}{k}}$$

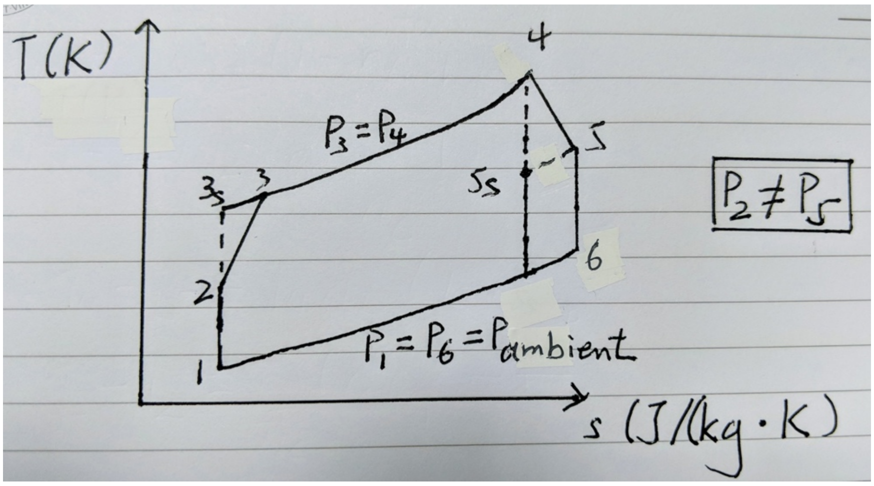
where r_T is the isentropic temperature ratio for the common pressure ratio.

Then we have the thermodynamic efficiency of the gas turbine generator η_{th}^{gen}:

$$\eta_{th}^{gen} = \frac{w_{turb} - w_{comp}}{c_p(T_4 - T_3)}$$
$$= \frac{T_4\eta_{turb}\left(1 - \frac{1}{r_T}\right) - \frac{T_2}{\eta_{comp}}(r_T - 1)}{T_4 - \frac{T_2}{\eta_{comp}}(r_T - 1)}$$

The above equation is NOT FOR JET ENGINE.

Initial cycle analysis of a turbojet:



Assuming isentropic process for compressor, turbine and nozzle, Flow through the diffuser (1 – 2) and nozzle (5 – 6) is a process of exchanging heat energy to KE, such that:

$$h_1 + \frac{V_1^2}{2} = h_2 + \frac{V_2^2}{2}$$
$$h_5 + \frac{V_5^2}{2} = h_6 + \frac{V_6^2}{2}$$

Where velocities V_2 and V_5 are negligible since they are stagnation point, collaborating with:

$$\frac{T_{n+1}}{T_n} = \left(\frac{p_{n+1}}{p_n}\right)^{\frac{k-1}{k}} \text{ or } \frac{T_n}{T_{n+1}} = \left(\frac{p_n}{p_{n+1}}\right)^{\frac{k-1}{k}}$$

then the above energy equations becomes:

Isentropic diffuser:

$$V_1^2 = 2c_P(T_2 - T_1) = 2c_pT_2\left[1 - \left(\frac{p_1}{p_2}\right)^{\frac{k-1}{k}}\right]$$

Isentropic nozzle expansion:

$$V_6^2 = 2c_P(T_5 - T_6) = 2c_pT_5\left[1 - \left(\frac{p_6}{p_5}\right)^{\frac{k-1}{k}}\right]$$

Then as the turbine drives the compressor, we have:

$$\mathrm{W_{turb}} = \mathrm{W_{comp}}$$
$$\mathrm{h_3} - \mathrm{h_2} = \mathrm{h_4} - \mathrm{h_5}$$

Now, consider the power and energy prospect of the engine, we know that the combustion of the engine in the combustion chamber provides the propulsion power, the heat lost and KE in the gas:

$$\mathrm{Q_{in}} = \mathrm{Q_{out}} + \mathrm{W_p} + \Delta\mathrm{KE}$$

First, we have the thrust equals:

$$\mathrm{F} = \dot{\mathrm{m}}_{\mathrm{air}}(\mathrm{V_6} - \mathrm{V_1})$$

Where $V_1 = V_{flight}$, and therefore the propulsion power is:

$$\begin{aligned} \mathrm{W_P} &= \mathrm{FV_{flight}} \\ &= \dot{\mathrm{m}}_{\mathrm{air}}\mathrm{V_{flight}}(\mathrm{V_6} - \mathrm{V_{flight}}) \end{aligned}$$

Second, the energy input Q_{in} and the energy output Q_{out} are:

$$\mathrm{Q_{in}} = \dot{\mathrm{m}}_{\mathrm{air}}\mathrm{c_P}(\mathrm{T_4} - \mathrm{T_3})$$
$$\mathrm{Q_{out}} = \dot{\mathrm{m}}_{\mathrm{air}}\mathrm{c_p}(\mathrm{T_6} - \mathrm{T_1})$$

So the (overall) propulsion efficiency η_P is:

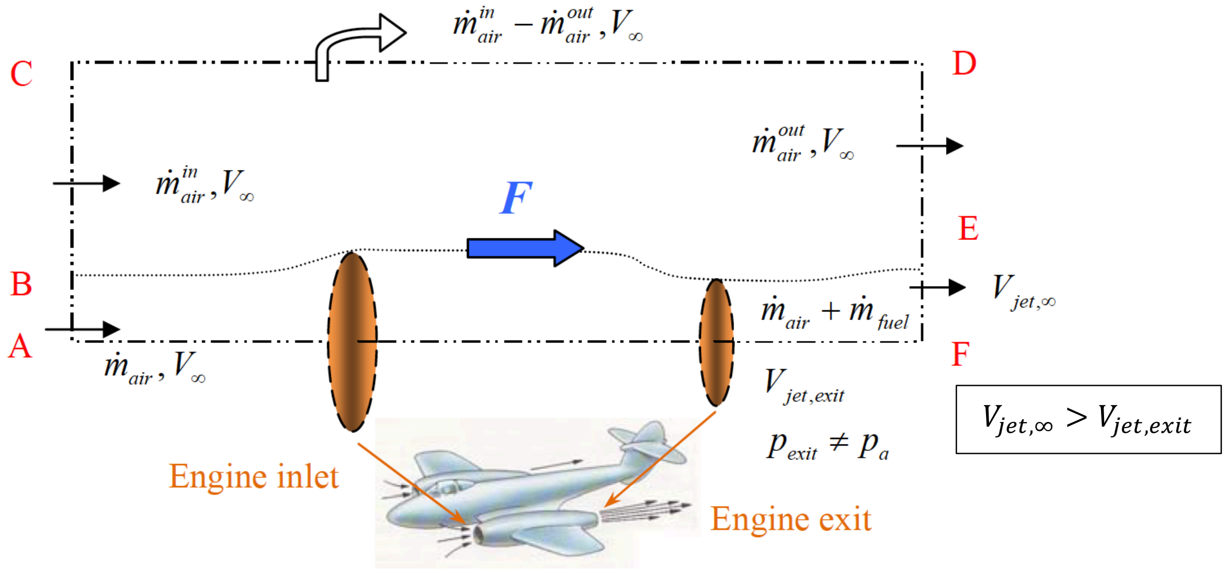
$$\begin{aligned} \eta_\mathrm{P} &= \frac{\mathrm{W_P}}{\mathrm{Q_{in}}} \\ &= \frac{\mathrm{FV_{flight}}}{\dot{\mathrm{m}}_{\mathrm{air}}(\mathrm{h_4} - \mathrm{h_3})} \end{aligned}$$

Lastly, the KE gained by the exhaust gas when viewed from the ground is:

$$\Delta\mathrm{KE} = \frac{1}{2}\dot{\mathrm{m}}_{\mathrm{air}}(\mathrm{V_6} - \mathrm{V_1})^2$$

Derivation of thrust:

Consider:



Where:

- AC and DF is the radius of the control volume and is sufficiently far from the engine such that the pressure along the line is ambient pressure
- Pressure only acts on surface AC and DF
- The flow external to the engine goes along the curve BE
- V_∞ is the velocity of air, which is equal to the velocity of the plane: $V_\infty = V_{flight}$
- F is the thrust force ON THE ENTIRE AIR FLOW, NOT THE PLANE

We can obtain the force equilibrium in this control volume:

$$\dot{m}_{air}V_\infty + \dot{m}_{air}^{in}V_\infty + F = \left(\dot{m}_{air}^{in} - \dot{m}_{air}^{out}\right)V_\infty + \dot{m}_{air}^{out}V_\infty + (\dot{m}_{air} + \dot{m}_{fuel})V_{jet,\infty}$$
$$F = (\dot{m}_{air} + \dot{m}_{fuel})V_{jet,\infty} - \dot{m}_{air}V_\infty$$

The fuel mass flow can often be ignored as very little fuel is required when compares to the air. Therefore, finally:

$$\begin{aligned} F &= \dot{m}_{air}\left(V_{jet} - V_\infty\right) \\ &= \dot{m}_{air}(V_{jet} - V_{flight}) \end{aligned}$$

Assuming an isentropic process, one can relate the parameter at the nozzle exit to the far field point where the ambient pressure p_a is reached using energy conservation:

$$\frac{1}{2}V_{jet,exit}^2 + c_P T_{jet,exit} = \frac{1}{2}V_{jet,\infty}^2 + c_P T_{jet\infty}$$

and:

$$\frac{T_{jet,\infty}}{T_{jet,exit}} = \left(\frac{p_a}{p_{jet,exit}}\right)^{\frac{k-1}{k}}$$

gives:

$$V_{jet,\infty}^2 = V_{jet,exit}^2 + 2c_P T_{jet,exit}\left[1 - \left(\frac{p_a}{p_{jet,exit}}\right)^{\frac{k-1}{k}}\right]$$

Further analysis of turbojet efficiency:
We now view the aircraft from ground. From this perspective, we see engine being pushed at force F at the flight velocity V_∞, the propulsive power W_P is then:

$$\mathrm{W_P} = \mathrm{FV_\infty}$$

The energy released from fuel is decomposed into 3 components:

$$\dot{\mathrm{Q}}_{\mathrm{in}} = \dot{\mathrm{Q}}_{\mathrm{out}} + \mathrm{W_p} + \Delta\mathrm{KE}$$

The sum of $W_p + \Delta KE$ is total enthalpy:

$$\begin{aligned}\mathrm{W_p} + \Delta\mathrm{KE} &= \dot{\mathrm{m}}_{\mathrm{air}}\left(\mathrm{V_{jet}} - \mathrm{V_\infty}\right)\mathrm{V_\infty} + \frac{1}{2}\dot{\mathrm{m}}_{\mathrm{air}}\left(\mathrm{V_{jet}} - \mathrm{V_\infty}\right)^2 \\ &= \frac{1}{2}\dot{\mathrm{m}}_{\mathrm{air}}(\mathrm{V_{jet}^2} - \mathrm{V_\infty^2})\end{aligned}$$

Also:

$$\dot{\mathrm{Q}}_{\mathrm{in}} - \dot{\mathrm{Q}}_{\mathrm{out}} = \dot{\mathrm{m}}_{\mathrm{air}}(\mathrm{h_4} - \mathrm{h_3}) - \dot{\mathrm{m}}_{\mathrm{air}}\left(\mathrm{h_{jet}} - \mathrm{h_\infty}\right)$$

In which, since:

$$\begin{aligned}\mathrm{h_3} - \mathrm{h_2} &= \mathrm{h_4} - \mathrm{h_5} \\ \mathrm{h_4} - \mathrm{h_3} &= \mathrm{h_5} - \mathrm{h_2} \\ &= \left(\mathrm{h_{jet}} + \frac{1}{2}\mathrm{V_{jet}^2}\right) - \left(\mathrm{h_\infty} + \frac{1}{2}\mathrm{V_\infty^2}\right)\end{aligned}$$

The above 2 equations proved that in the engine:

$$\dot{\mathrm{Q}}_{\mathrm{in}} - \dot{\mathrm{Q}}_{\mathrm{out}} = \mathrm{W_p} + \Delta\mathrm{KE}$$

Where the energy loss is the KE in the jet.

For the (overall) propulsion efficiency η_p:

$$\begin{aligned}\eta_{\mathrm{P}} &= \frac{\mathrm{W_P}}{\dot{\mathrm{Q}}_{\mathrm{in}}} \\ &= \frac{\mathrm{W_P}}{\mathrm{W_P} + \Delta\mathrm{KE}} \times \frac{\mathrm{Q_{in}} - \mathrm{Q_{out}}}{\dot{\mathrm{Q}}_{\mathrm{in}}} \\ &= \eta_{\mathrm{KE}}\eta_{\mathrm{th}}\end{aligned}$$

Where:
- η_{KE} is the kinematic propulsion efficiency
- η_{th} is the thermodynamic cycle efficiency

η_{KE} is entirely depends on velocities, shown as below:

$$\eta_{\mathrm{KE}} = \frac{\mathrm{W_P}}{\mathrm{W_P} + \Delta\mathrm{KE}} = \frac{\dot{\mathrm{m}}_{\mathrm{air}}\left(\mathrm{V_{jet}} - \mathrm{V_\infty}\right)\mathrm{V_\infty}}{\frac{1}{2}\dot{\mathrm{m}}_{\mathrm{air}}(\mathrm{V_{jet}^2} - \mathrm{V_\infty^2})} = \frac{2}{1 + \frac{\mathrm{V_{jet}}}{\mathrm{V_\infty}}}$$

When the thrust required is small, the jet speed can be small and the efficiency is high, vice versa.

Propellers have large diameters and can process a huge amount of air mass flow but without accelerating it too much. It gives the best efficiency. However, a propeller driven aircraft cannot run too fast because the propeller tip velocity will exceed the speed of sound, giving a huge drag and loss of thrust power.

Turbojet is exactly the opposite, it's most inefficient due to its fast jet flow but is the most powerful in producing thrust F. Therefore, turbofan and turboprop are the compromise between propeller and jet engines.

Specific fuel consumption (sfc) is defined as the fuel weight per unit thrust generated by the jet engine:

$$\begin{aligned}\mathrm{sfc} &= \frac{\dot{m}_{fuel} g}{F} \\ &= \frac{3600g}{F} \times \dot{m}_{fuel} \\ &= \frac{3600g}{F} \times \frac{\dot{m}_{air} c_P (T_4 - T_3)}{LCV} \\ &= \frac{3600g}{LCV} \times \frac{\dot{Q}_{in}}{F} \\ &= \frac{3600g}{LCV} \times \frac{V_\infty}{\eta_P}\end{aligned}$$

$$\begin{aligned}\dot{Q}_{in} &= \dot{m}_{air} c_p (T_4 - T_3) \\ &= \dot{m}_{fuel} \times LCV\end{aligned}$$

Gives:

$$\dot{m}_{fuel} = \frac{\dot{m}_{air} c_P (T_4 - T_3)}{LCV}$$

$$\eta_P = \frac{FV_\infty}{\dot{Q}_{in}}$$

With unit: per hour

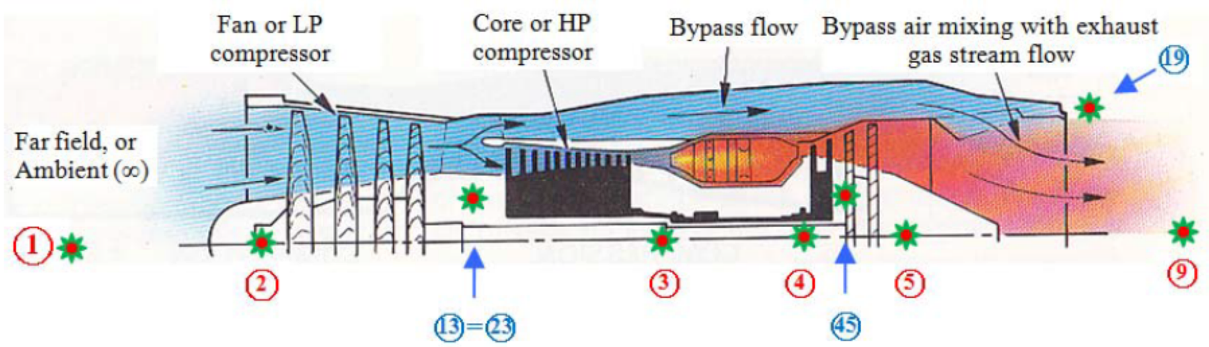
Where:
LCV is the lower calorific value (熱值) of kerosene: $LCV = 43 \times 10^6$ J/kg
(with the assumption that water in the product remains in vapour state)

**Formulas that related to Mach number (M):

$$\frac{T_2}{T_1} = 1 + \frac{1}{2}(k-1)M^2$$

Where:
T_1 is the temperature of the ambient.

$$V_\infty = V_1 = M\sqrt{kRT_1}$$



**When the compressor is divided into two parts, a fan and the core compressor (cc), and the inlet flow is split into two streams, the engine becomes a turbofan engine.

Pressure ratio across the core turbine is always less than the pressure ratio across the corresponding compressor because the pressure ratio is determined by the temperature ratio. However:

$$\mathrm{T}_3 - \mathrm{T}_{23} = \mathrm{T}_4 - \mathrm{T}_{45}$$
$$\frac{\mathrm{T}_3}{\mathrm{T}_{23}} - 1 = \left(\frac{\mathrm{T}_{45}}{\mathrm{T}_{23}}\right)\left(\frac{\mathrm{T}_4}{\mathrm{T}_{45}} - 1\right)$$

Since $\mathrm{T}_{45} > \mathrm{T}_{23}$:

$$\frac{\mathrm{T}_3}{\mathrm{T}_{23}} - 1 > \frac{\mathrm{T}_4}{\mathrm{T}_{45}} - 1$$
$$\frac{\mathrm{T}_3}{\mathrm{T}_{23}} > \frac{\mathrm{T}_4}{\mathrm{T}_{45}}$$

So, the pressure ratio across the core turbine is always less than the pressure ratio across the corresponding compressor.

Turbofan engine analysis:

Turbofan engine now dominates the commercial aircraft due to its superior fuel efficiency. The most important parameter for a turbofan engine is the bypass ratio (bpr) defined as:

$$\begin{aligned}\mathrm{bpr} &= \frac{\text{amount of airflow through the bypass}}{\text{amount of airflow through the core engine}} \\ &= \frac{\dot{\mathrm{m}}_{\mathrm{air,bypass}}}{\dot{\mathrm{m}}_{\mathrm{air,core}}}\end{aligned}$$

So, the total mass of air is:

$$\dot{\mathrm{m}}_{\mathrm{air,\ total}} = \dot{\mathrm{m}}_{\mathrm{air,\ bypass}} + \dot{\mathrm{m}}_{\mathrm{air,\ core}}$$

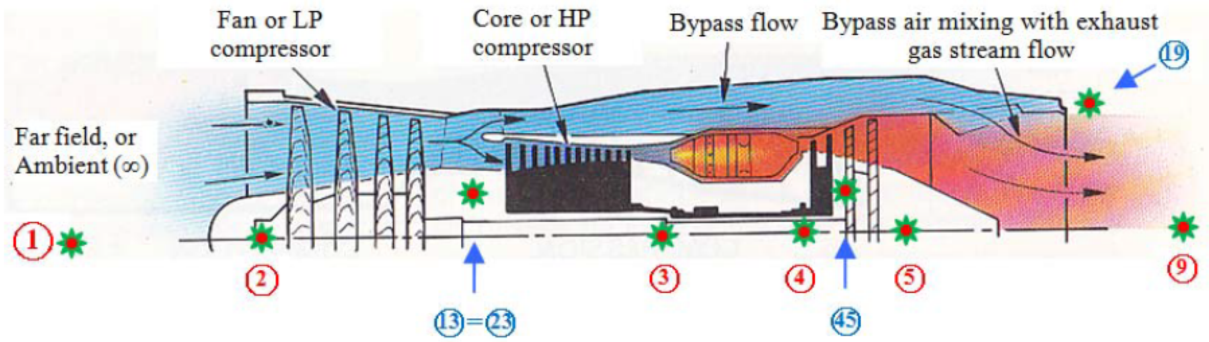
Adding a bypass means a set of LP fan and LP turbine is also added, this makes the engine more efficient and produce more thrust because additional kinetic energy is transferred to the LP turbine to drive the LP fan, so there will be less KE loss and there will be additional thrust created by the LP fan. Therefore, the jet velocity will be reduced.

The reduced jet velocity in turns reduce the noise because the power of noise:

$$\mathrm{P}_{\text{jet noise}} \propto \mathrm{V}_{\text{jet}}^{8}$$

The cool bypass air flowing around the engine core can also be used for general engine cooling.

Theoretically, the efficiency of the engine increases monotonically with bpr, but the benefit for the whole aircraft will be capped by the increasing drag caused by the increasing frontal area of the engine.



In the above turbofan engine, for a given height and flight velocity:

- Parameters for stations 1, 2 are completely fixed.

- Peak temperature occurs in the combustor, limited by the melting point of metal and the concern of producing NO_x. This fixes T_4.

- High overall compression ratio is good for efficiency but reduces the room for heat addition, because T3 will be too high and the difference of $T_4 - T_3$ will be low, Therefore, the total compression ratio:

$$\mathrm{r_{total}} = \frac{\mathrm{p_3}}{\mathrm{p_2}}$$

  is also fixed.
- Therefore, the main variable is the compression ratio through the fan stage (LP):

$$\mathrm{r_{fan}} = \frac{\mathrm{p_{13}}}{\mathrm{p_2}}$$

  with this, the core engine compression ratio r_{cc} is determined by:

$$\begin{aligned}\mathrm{r_{total}} &= \mathrm{r_{fan}} \times \mathrm{r_{cc}} \\ &= \frac{\mathrm{p_{13}}}{\mathrm{p_2}} \times \frac{\mathrm{p_3}}{\mathrm{p_{13}}}\end{aligned}$$

  which means that we can know stage 13/23, 3.

  In general, component efficiency is known, with the best quality fan and compressor giving around 85-90% and best turbine 90-95%.

- The partition of the turbine into high and low pressure stages, HP and LP, is governed by the power matching requirement. The work output of HP turbine drives the core compressor:

$$\mathrm{h_3} - \mathrm{h_{23}} = \mathrm{h_{45}} - \mathrm{h_4}$$

- The pressure at state 5 is a matter of design choice. The requirement is that the LP turbine power output can fully supplies the work needed for the fan driving both streams of the entry flow.

  With higher bpr, more work from LP turbine is needed, or lower p_5 is needed, the lowest being the value required to obtain an exhaust speed of:

  $$V_9 \geq V_\infty$$

  when the gas expands from pressure p_5 at state 5 to the ambient pressure $p_9 = p_\infty$ at state 9, which may be achieved at some distance downstream of the physical nozzle for supersonic case. Otherwise, there would be negative thrust generation by the core jet ($V_9 \leq V_\infty$). When bpr and turbine efficiency η_{turb} are fixed, state 5 is fixed, so we can find V_9.

- The expansion from p_{13} to p_∞ determines the jet speed in the bypass ($p_\infty > p_{13}$).
  The kinetic energy in the jet derives solely from the fan compression. Note that the jets in the bypass and the core engine may have different velocity.

All in all, if:

- Given height, we can find ρ_1, p_1, T_1
- Given Mach number M, we can find $V_1 = M\sqrt{kRT_1}$
- Given the basic parameters:
  r_{total}
  η_{fan}
  η_{comp}
  $\eta_{turb} = \eta_{LPturb} = \eta_{HPturb}$ (assumption)
  $T_4 = T_{max}$

- Choose bpr and r_{fan}

→We can then determine all variables at all states and this can determine the jet velocities V_{19}, V_9, the thrust per unit core engine air mass flow F_{1c}:

$$\begin{aligned} F_{1c} &= \frac{F}{\dot{m}_{air,core}} \\ &= (V_9 - V_\infty) + bpr(V_{19} - V_\infty) \end{aligned}$$

where the thrust force F for turbofan engine is:

$$F = \dot{m}_{air,core}[(V_9 - V_\infty) + bpr(V_{19} - V_\infty)]$$

and the final sfc [unit: per hour] using F_{1c}:

$$sfc = \frac{3600gc_p(T_4 - T_3)}{F_{1c} \times LCV}$$

Then to compare results for different bpr designs, we use the overall specific thrust F_1 (thrust per unit overall air mass flow):

$$\begin{aligned} F_1 &= \frac{F_{1c}}{1 + bpr} \\ &= \frac{(V_9 - V_\infty) + bpr(V_{19} - V_\infty)}{1 + bpr} \end{aligned}$$

This means, in the big picture, for any given bpr, there would be one particular value of the fan compression ratio r_{fan} that can optimize the thrust per unit core engine air mass flow F_{1c} and minimize sfc.

The bypass ratio will always increase engine efficiency but large diameter would impose drag and weight penalties on the aircraft. Also, when the fan diameter is too large, its rotational speed has to be limited and there would be a need to install a gear box between the LP turbine and fan to rotate at different speeds. The **three shafts** Trent has would certainly improve its off-design performance, but it also means **structural complications**. Typical tip speed of the fan is supersonic (up to 450m/s at take-off).

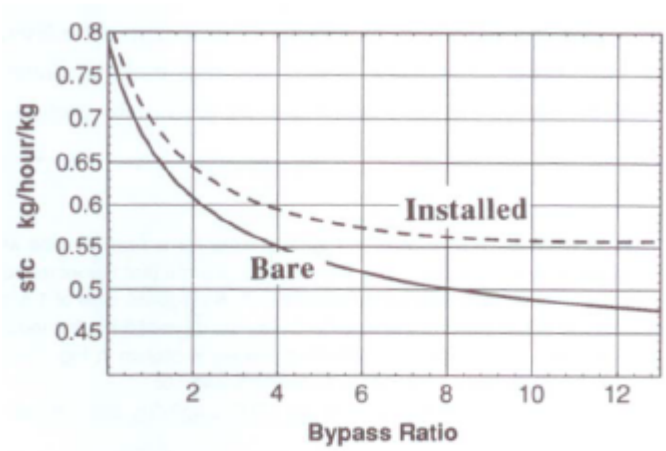
It is discovered that, when the best ratio r_{fan} occur, the core jet velocity V_9 is higher than the bypass jet velocity V_{19}. It is also found that the best ratio of the two velocity is close to the product of the efficiencies of the LP fan η_{fan} and turbine $\eta_{LP\ Turbine}$:

$$\frac{V_{bypass}}{V_{nozzle}} = \eta_{LP\ Turbine} \times \eta_{fan}$$

When the engine in installed in the aircraft, the variation of sfc with bpr is important for bpr optimization:

$$sfc = sfc_{bare\ engine}[1 + bpr \times \delta_{nacelle}]$$

where $\delta_{nacelle}$ is a coefficient, and nacelle is the engine casing.

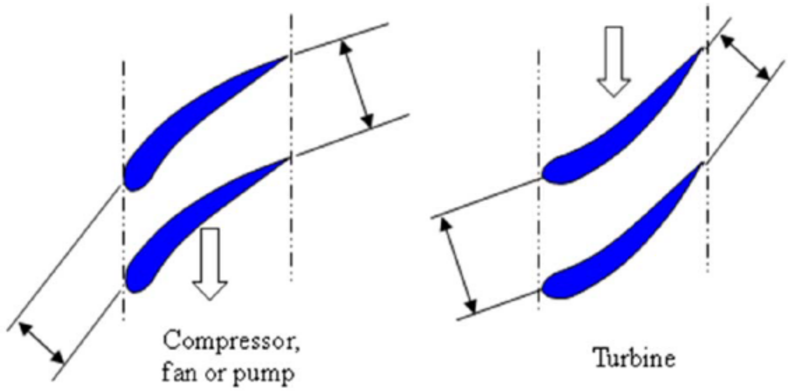


Compressor instability and turbine cooling

Compressor and turbine perform exactly the opposite roles in the core engine. We need these two parts operate together to improve the cycle efficiency when heat is added at higher temperature.

Compressor instability:

the effective passage expands for compressor and narrows in turbine, corresponding to pressure increase in compressor and decrease in turbine:



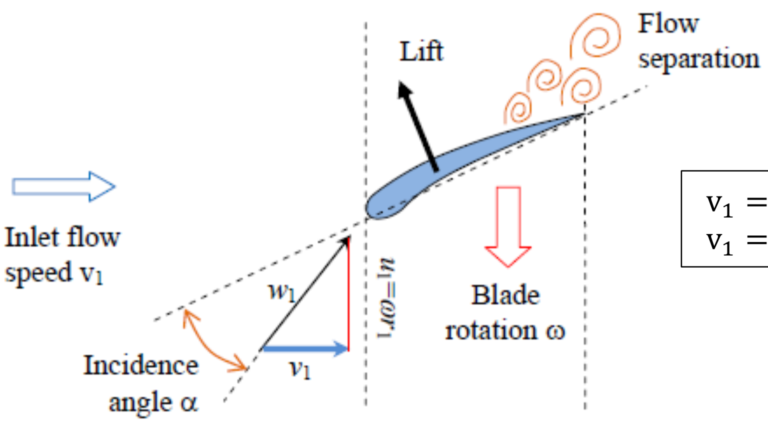
Compressor blades rotate against the lift (acting on blades) and turbine is driven by the lift force acting on the blades.

During compression, velocity decreases and pressure increases in the direction of flow, this is adverse pressure gradient. Ideally, flow can convert all its energy to potential energy by slowing down to zero speed for a stream of uniform velocity. The trouble arises in compressor as the flow velocity is far from uniform from one curved blade to another.

On the blade surface, no-slip condition requires zero velocity so a boundary layer is formed and thickens. The flow in the boundary layer has little kinetic energy to transfer to the flow at the middle of the two blades because it moves slow. So, when the pressure gradient is too high, the boundary layer separates from the blades (move backward). And the lift force suddenly drops. This is called stall, which also happens in airfoil.

Compressor blades also experience a dangerous, unstable working condition called “flutter”, defined as the growing blade vibration by unsteady flow coupled with the blade vibration velocity. This causes energy transfer from the flow back to the blades at a frequency close to the structural resonance frequency. This can lead to the blade and entire engine failure.

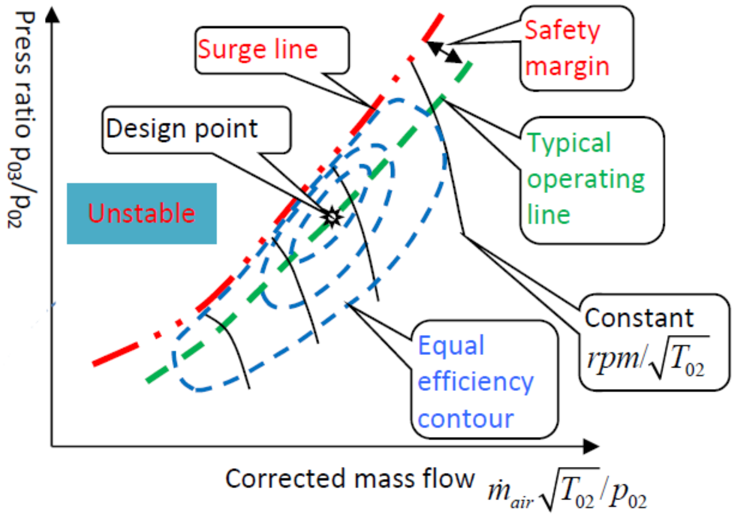
For a given pressure ratio, very low mass flow is unstable:



$$v_1 = w_1 + u_1$$
$$v_1 = w_1 + \omega r$$

When the magnitude of the inlet velocity v_1 is reduced, the relative flow velocity w_1 is more inclined towards the circumferential direction. The incidence angle α is increased.
When α exceeds a certain limit, such as 15 - 20°, the flow will separate at the upper surface, causing drastic loss of lift. This is called rotating stall.

The compressor performance characteristics below are illustrated using the compression ratio $\frac{p_3}{p_2}$, against the normalized mass flow rate $\dot{m}_{air} \times \frac{\sqrt{T_2}}{p_2}$ at the first stagnation point (stage 2), where T_2 and p_2 are the stagnation temperature and pressure at stage 2:



In the figure, the surge line is the critical condition that separates stable and unstable operating conditions. The ideal operating line is away from the surge line. The constant normalized rotational speed lines $\frac{\text{rpm}}{\sqrt{T_2}}$ and the isentropic efficiency (equal efficiency) contours are given, which reach the peak at the design point.

The reason why the mass flow rate and the rotational speed (rpm) are normalized is because:

Mass flow rate:
the mass flow is influenced by the entry flow condition. Therefore, we can normalize the mass flow rate by the theoretical mass flow rate:

$$\dot{m} = \rho V A$$

of the entry flow (stage 2). The density is taken as the stagnation density (using ideal condition: p = ρRT) and the velocity taken as the speed of sound ($V_{sound} = \sqrt{kRT_2}$), so the normalized mass flow rate becomes:

$$\frac{\dot{m}_{air}}{\rho VA} = \dot{m}_{air} \times \left(\frac{1}{\frac{p_2}{RT_2} \times \sqrt{kRT_2} \times A} \right)$$

$$= \dot{m}_{air} \times \left(\frac{\sqrt{T_2}}{p_2} \times \frac{1}{A}\sqrt{\frac{R}{k}} \right)$$

In the above expression, the term $\frac{1}{A}\sqrt{\frac{R}{k}}$ contains all the terms that will not change with working condition of the compressor. Therefore, this term is dropped out. Then the final form of the normalized (corrected) mass flow becomes:

$$\text{Corrected } \dot{m}_{air} = \dot{m}_{air} \times \frac{\sqrt{T_2}}{p_2}$$

Rotational speed (rpm):
Same reason as in the mass flow rate, we reference to the speed of sound at stage 2. Therefore, the corrected rotation speed becomes:

$$\frac{\mathrm{rpm}}{\mathrm{V_{sound}}} = \mathrm{rpm} \times \frac{1}{\sqrt{\mathrm{kRT_2}}}$$

Again, the term $\frac{1}{\sqrt{\mathrm{kR}}}$ contains constants which will not change with working condition of the compressor. Therefore, this term is dropped out. Then the final form of the normalized (corrected) rotational speed becomes:

$$\text{Corrected rotational speed} = \frac{\mathrm{rpm}}{\sqrt{\mathrm{T_2}}}$$

$$\boxed{\begin{gathered}1\mathrm{Hz} = 60\mathrm{rpm} \\ \mathrm{f} = \frac{\omega}{2\pi}\end{gathered}}$$

Turbine cooling:
The main issue for turbine is the high temperature of the gas exhaust from the combustor, without this, the turbine is much easier to design than compressor.

The metal melting temperature of turbine blades is around 1550 K and the turbine entry temperature at take off is roughly 1700 K. Although the inlet temperature varies a lot from take off to cruise, the temperature ratio T4/T2 is about the same with the top of climb (near cruise state) being the highest.

When T4 is high, the temperature ratio (hence the isentropic pressure ratio) will be low for HP, leaving more energy from LP to do useful propulsion work. The cycle efficiency will be higher.

The use of new materials has contributed to the tolerance of high T_4, but **the crucial technology has been the cooling of HP blades by the air released from the compressor (about 900 K)**. Air is taken inside the turbine blades and comes out from small holes on the blade surface forming a thin film shielding the blades from hot gas.

The amount of cool air from the core engine used in the cooling system of HP blades is about 15 - 25%, reduce thrust significantly. So, cooling is a balance act between engine performance and engine life. Hot engine has shorter life, therefore, the time that the engine spends on hot environment during take off should be strictly limited.

Selection of engine

There are 4 types of aero engines today:

1. Reciprocating engine with propeller
2. Turbojet
3. Turbofan
4. Turboprop and propfan

The appropriate engine for an aircraft mainly depends on its cruise flight speed.
The order of low to high speed is:

1. Piston engine
2. Turboprop
3. Turbofan
4. Turbojet

We still use propeller engine nowadays because it is the most efficient engine at low speed. Propellers are simply twisted wings, and they are normally driven by reciprocating engines running on Otto Cycles (isentropic thermodynamic cycles). Its power depends on:

- Number of cylinder and each cylinder volume
- Mean pressure
- Rotational speed

It is relatively independent of the properties of surrounding air. So, normally the piston engine power is fixed. And its thrust is therefore approximated by:

$$F = \frac{P}{V_{flight}}$$

where P here is constant power.

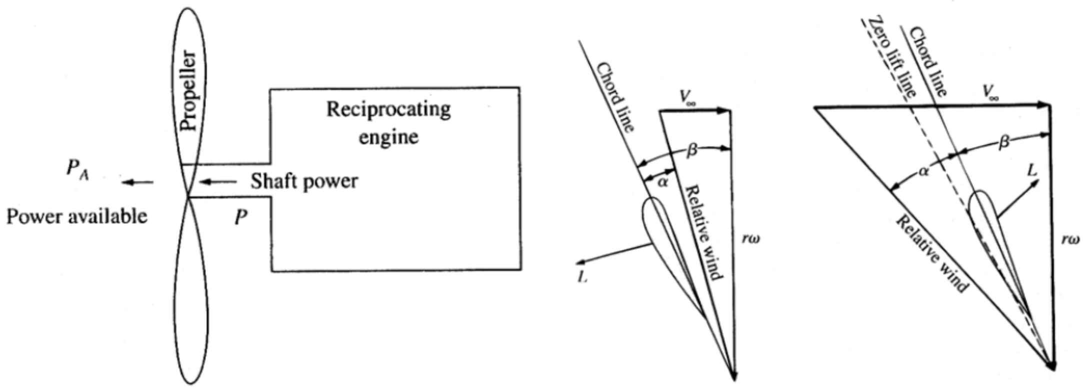


Illustration of propeller engine, top view of low flight speed, and high speed.

Propeller blade is installed at a pitch angle β with respect to the chord line. From the above figure, we can calculate the relative wind velocity angle (β – α) by the velocity triangle:

$$\frac{V_{flight}}{\omega r} = \tan(\beta - \alpha)$$

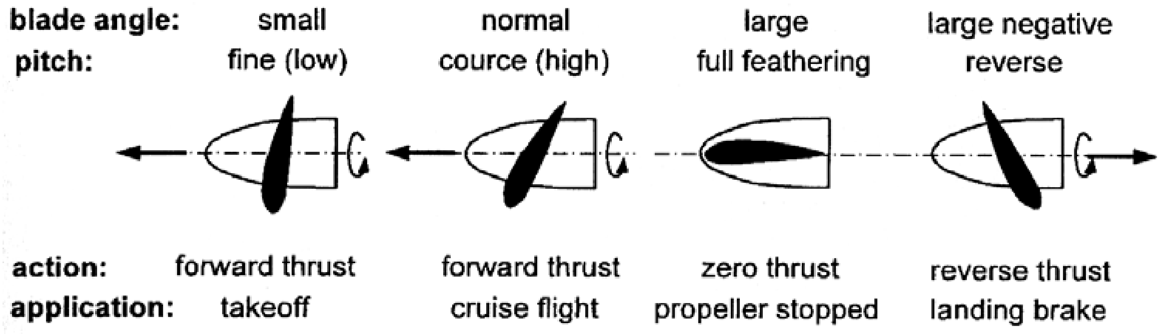
From this, we can obtain an expression for the incidence angle α:

$$\alpha = \beta - \tan^{-1}\left(\frac{V_{flight}}{\omega r}\right)$$

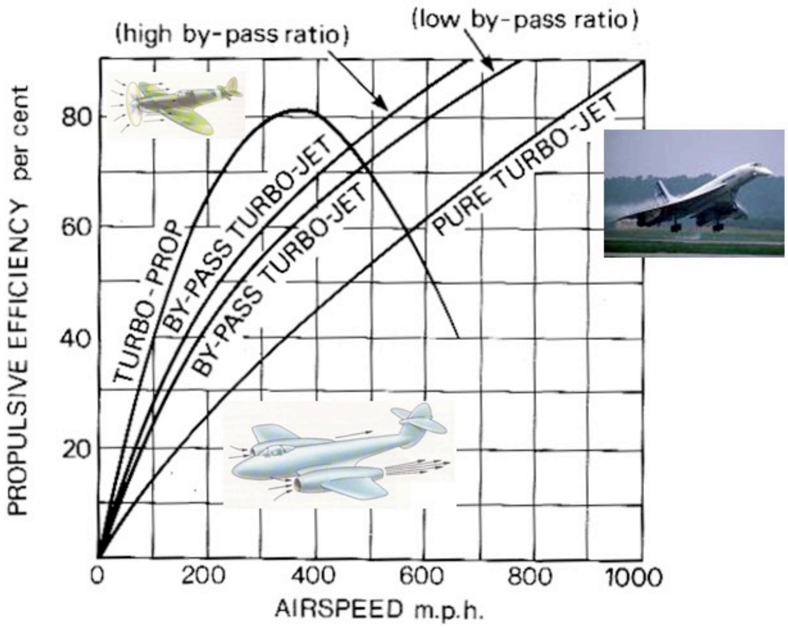
which is a function of radius. This explains why the blade is twisted.

The efficiency of propeller increases from 0 at $V_{flight} = 0$ to a peak of 85%. Then, it falls back to 0 when fast flight speed gives negative incidence angle α for the relative wind velocity to the chord line.

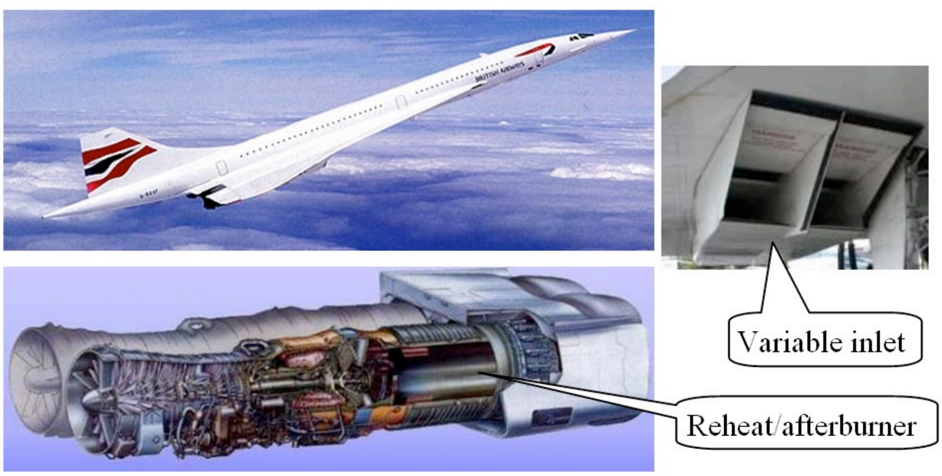
In order to maintain good efficiency, the pitch angle and rotational speed can be adjusted as below:



The variation of efficiency with flight speed is illustrated below:



An after burner is used in some aircrafts for take off when extra thrust is required.



Combustion

In the combustion chamber, heat is added to the flow through the chemical reaction of kerosene $C_{12}H_{23}$:

$$\mathrm{C_{12}H_{23} + 17.75O_2 + 66.7755N_2 \rightarrow 12CO_2 + 11.5H_2O + 66.7766N_2 + Heat(Q)}$$

Kerosene is generated by fractional distillation of petroleum at the temperature between 150°C to 275°C. The molar mass of kerosene is 167kg/kmol. This is also the stoichiometric fuel needed.

From the above chemical equation, we can calculate the required mass of oxygen for the combustion of 167kg of kerosene as:

$$\mathrm{m_{O_2}} = \frac{167}{167} \times 1 \times 17.75 \times (16 \times 2) = 568\mathrm{kg}$$

The above 2 number gives a meaningful ratio called stoichiometric ratio:

$$\begin{aligned}\text{stoichiometric ratio} &= \frac{\mathrm{m_{air}}}{\mathrm{m_{fuel}}} \\ &= \frac{\frac{568}{0.232}}{167} = 14.66\end{aligned}$$

However, the actual amount of air is more than this ratio requires. But oxygen can be locally deficient because the combustion only occurs in the flame region.

Equivalence fuel mass ratio Φ:

The equivalence fuel mass ratio is defined as:

$$\Phi = \frac{\text{actual fuel}}{\text{stoichiometric fuel}}$$

$$\Phi \begin{cases} < 1, & \text{fuel lean} \\ > 1, & \text{fuel rich} \end{cases}$$

CO formation

CO formation in the combustion chamber is mainly due to incomplete combustion by low combustor inlet temperature or fuel lean environment.

NO_x formation

When local temperature is high and held high for sufficient time, nitrogen in air reacts with oxygen to form harmful nitrogen oxide:

$$\mathrm{N_2 + O + Heat \leftrightarrow NO + N}$$

The rate of NO formation is governed by:

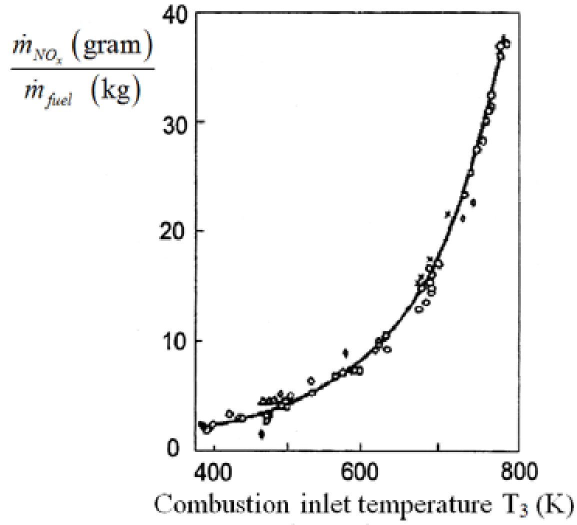
$$\mathrm{R_{NO}} = \mathrm{A}e^{\frac{\mathrm{T_{st}}}{\mathrm{B}}}$$

Where T_{st} is the stoichiometric flame temperature.

For the formation rate of NO_x, it depends on the stoichiometric flame temperature T_{st},
local O_2 concentration and residence time (停留時間).
There is ~10% of NO reacts with O_2 to form NO_2 further downstream

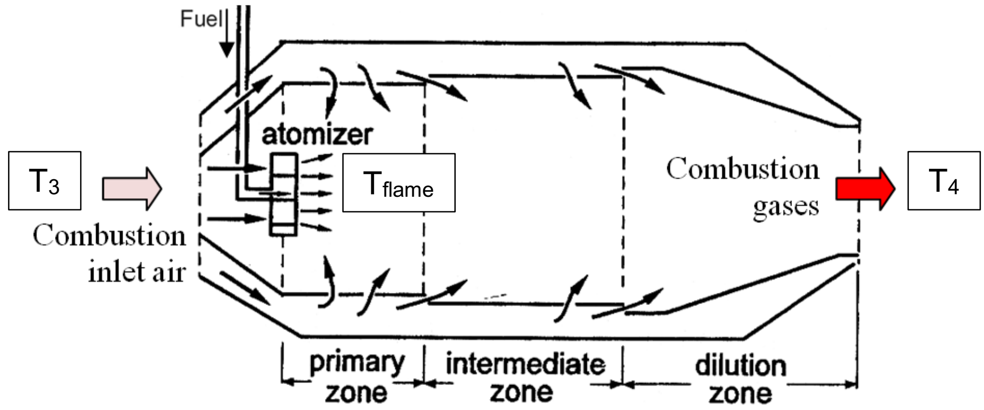
It is found that the combustion inlet temperature T_3 have a correlation with NO_x emission:

$$\frac{\dot{m}_{NO_x}}{\dot{m}_{fuel}} = 10^{1+0.0032(T_3-581.25)}\sqrt{\frac{p_a}{p_{sea\ level}}}\left(\frac{g}{kg}\right)$$



(Lipert plot, 1972)

T_3 is used instead because the flame temperature is not available, so T_3 is used as a tool of correlation study since it influences the peak flame temperature in the primary combustion zone.



In the chamber, compressed air is brought to contact with fuel and burned products in three zones: primary, intermediate and dilution zones.

The primary zone provides a stable environment for flame to hold and sufficient turbulence to allow good mixing and heat transfer. Typically, fuel is sprayed into this zone without pre-mixing. Combustion initially occurs only at the interface between air and droplets, leading to high local temperature.

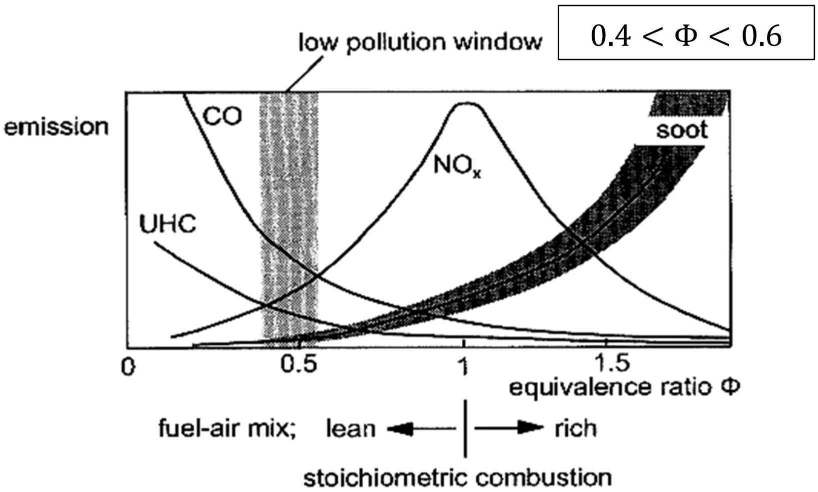
In **the intermediate zone**, a small amount of air is introduced and the temperature is dropped. Reaction is generally completed in this zone. Incomplete combustion products and some original hydrocarbon are burned here, and the heat lost in dissociation is recovered. CO is allowed to oxidise finally. When airplane flies at high altitudes, the intermediate zone serves as an extension of the primary combustion zone as the lower temperature and pressure give much slower pace of chemical reaction.

In **the dilution zone**, more compressed air enters through holes in the annular walls of the chamber. This further mix the gas mixture to distribute the temperature more evenly before the gas enter the HP turbine. This reduce the suffer of the turbine root because it works under maximum centrifugal stress. Therefore, the temperature should be lower.

Details of pollutants formation

There are 3 different types of pollutant mechanisms:

1. CO and unburned hydrocarbon (UHC) are products of incomplete combustion (low temperature, insufficient O_2, poor fuel-air mixing, etc) and they increase or decrease simultaneously.
2. NO_x is a result of high temperature complete combustion, peaks when equivalence fuel mass ratio $\Phi = 1$.
3. Soot (~100% C) is primarily a result of complex physical interaction, not a product of equilibrium chemical reaction.

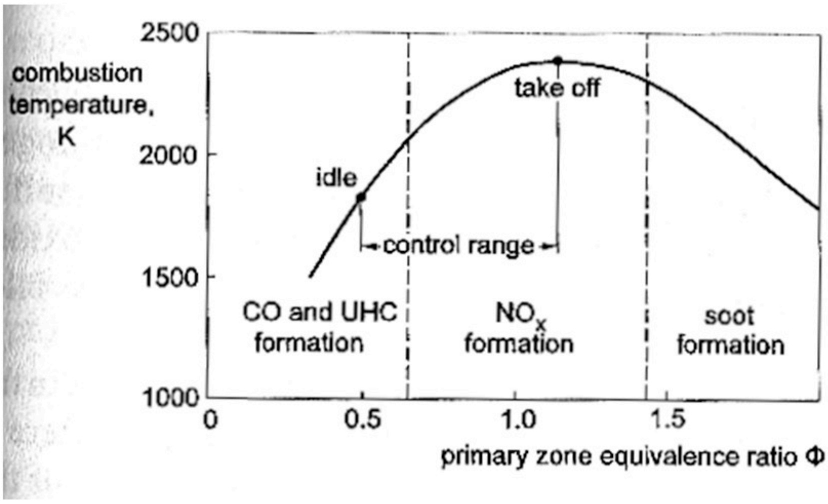


Pollutants except NO_x have been reduced to very low level by design. Therefore, NO_x remains the dominant concern because it contradicts the efficiency for high T_4.

The above pollutants do have impacts on environment and humans:

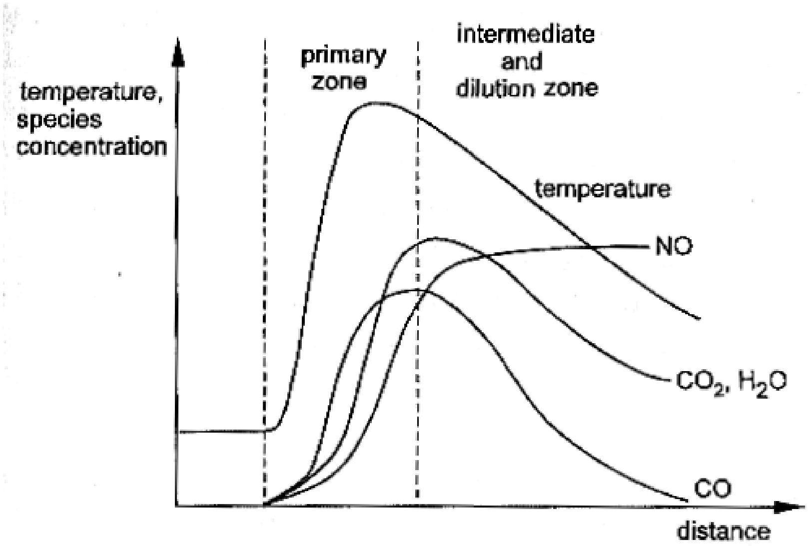
- **Tropospheric ozone production and toxic**: UHC, CO
- **Respiratory disease**: UHC, NO_x, soot
- **Global warming**: CO_2, H_2O, UHC, NO_x, soot
- **Acid rain**: NO_x
- **Smog**: UHC, NO_x

High temperature is good for cycle efficiency and this is ultimately responsible for the production of NOx. The temperature in the primary zone is the crucial parameter controlling NOx production:



In **fuel-lean condition**, small fuel flow in the injection nozzle leads to poor atomization, and the extra air lowers the peak temperature reached in combustion. The residence time of fuel in the combustion zone is also short. All together there is a lot of incomplete combustion with high production rate of CO and UHC.

In **fuel-rich condition**, insufficient O_2 simply means much less rate of combustion leading to low peak temperature in combustion.



UHC basically goes with CO and they decrease in the intermediate and dilution zone, while NO formation takes time and continues to growth downstream.

All in all:

- Combustion is normally incomplete in the primary zone, and NOx takes time to form, well into downstream zones.
- CO would be burned out in the downstream zones, mostly.
- Fuel-rich is a primary zone condition (overall ratio being lean). It favors soot formation.
- Landing and take-off are the main ground level environmental concerns, with lots of CO and UHC in take-off, but NO_x is still the most challenging.
- CO_2 and H_2O can only be reduced by reducing the specific fuel consumption (sfc).

Clean combustion

All main NOx reduction starts from combustion chamber, there are 3 methods to reduce the emission of NOx:

1. Give the primary combustion zone more air to create fuel-lean condition and use of pre-mixed, pre-vaporized combustor. The danger of this is:
    a. The combustion instability in the lean zone (called blowout).
    b. Possible flashback (flame entering the pre-mixing zone), and the lack of further cooling air for the combustor walls.
2. Use of staged combustion. A small amount of fuel is burned in fuel-rich environment and a large amount is burned in fuel-lean environment, avoiding the stoichiometric condition $\Phi = 1$.
3. Use rich burn quick quench lean (RQL) combustor. This combustor has the primary zone operates in the rich region, followed by a secondary burning zone of lean environment through quick introduction of cooling air.

Jet noise control

Noise is defined as unwanted sound that propagates through air at the speed of sound $\mathrm{V_{sound}} = \sqrt{\mathrm{kRT_1}}$.

It is discovered that the power of jet noise is proportional to the 8th power of jet velocity:

$$\mathrm{P_{jet\ noise}} \propto \mathrm{V_{jet}^8}$$

Therefore, turbofan is much quieter than turbojet (pg. 11).