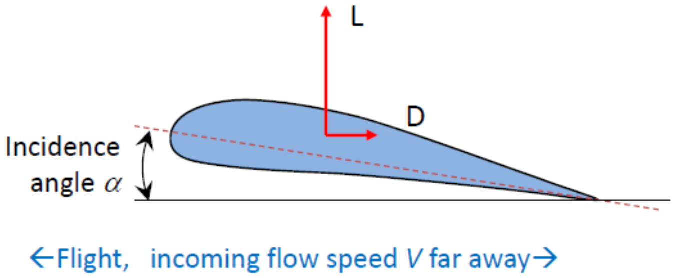
CH1 Aerodynamics

Lift (L)

Lift force is:

- A resultant force component
- Is perpendicular to the flight direction
- L >> D or high L/D ratio [Drag force (D) oppose flight velocity (V_{flight})]



Air foil shape:

- Blunt nose (finite local radius of curvature)
- Sharp tail (zero radius at the edges)

The origin of lift

Bernoulli's theorem between two points having the same elevation:

$$p_1 + \frac{1}{2}\rho V_1^2 = p_2 + \frac{1}{2}\rho V_2^2$$

Where:

- V = velocity
- p = pressure

Explanation for lift through Bernoulli's theorem:

1. Upper side of airfoil has higher flow speed
2. Lower pressure than lower side
3. Resultant air pressure force gives the lift (plus a small component of drag)

However, it is only the beginning since Bernoulli's theorem neglect the true origin, **fluid viscosity** and **vorticity shedding.**

Fluid Viscosity

- Originates from the random molecular interaction (collisions) and transportation of momentum
- Tending to eliminate velocity difference
- Viscosity is the resistance to shear deformation
- Very low in normal fluids (e.g. air, water)
- With fluid viscosity, no relative motion between a solid surface and the fluid particle on its surface → no-slip condition
- Increase with temperature

In the equation of shear stress: $\tau = \mu \frac{\partial u}{\partial y}$, μ is the dynamic viscosity and the kinematic viscosity v: $v = \frac{\mu}{\rho}$ where the unit of v is m^2/s.

For air at 15°C and sea-level pressure, ρ = 1.225 kg/m^3, v = 1.45 x 10^{-5} m^2/s

Kinematic viscosity:

- Momentum diffusivity (擴散)

Vorticity

- Twice the local rotational speed of fluid particle:

$$\overline{\Omega} = 2\omega$$

- Moving forms of fluid
  1. Uniform translation
  2. Straining motion (with volume expansion and without volume change)
  3. Effective local rotation (effective is required since a real fluid rarely undergoes rigid body rotation)

Vorticity is defined as curl of velocity:

$$\overline{\Omega} = \nabla \times \vec{V}$$

Kelvin's theorem (mathematical details are not required)

Circulation around a closed curve C moving with fluid remains unchanged in **inviscid** flow (無粘流).
[If the flow is not viscous, rate of vorticity flux is zero]
Therefore:

$$\frac{d\Gamma}{dt} = 0$$

Where Γ is defined as circulation and as the line integration:

$$\Gamma = \oint_c \vec{V} \cdot d\vec{l}$$

dl is the distance between two particles in the curve.

Kutta-Zhukowski's theorem

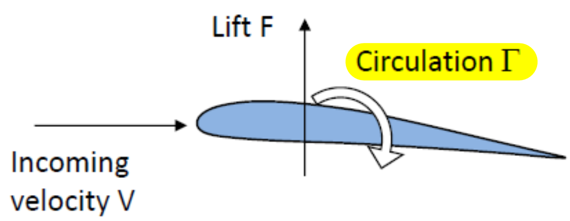
This theorem defines the lift on an arbitrary 2D body. **L_{2D}, lift force (L) per unit span (s)**, with the clockwise circulation **Γ** around that body has a relation below:

$$L_{2D} = \rho V \Gamma$$
$$L_{2D} = \frac{L}{s}$$

- The positive direction should be taken:
  1. Circulation is clockwise
  2. V is flow towards right
  3. Lift force towards up.

- Another way to remember the positive direction:
  1. Treat them as vector
  2. $L_{2D} = \rho V \Gamma$
  3. Applying right hand rule

Apply right-hand rule to Γ



Simplistic (approximate) proof of Kutta-Zhukowski's theorem

Assumptions:
1. Irrotational flow (vorticity = 0) everywhere outside the body
2. Incompressible (no change of volume and density when a change of pressure is applied)

Proof:
1. By taking the position on the two sides of the airfoil, we have

$$L_{2D} \approx \int_0^{\text{Chord}} (p_{\text{lower}} - p_{\text{upper}})\, dx$$

   Where "chord" means the distance between the leading and trailing edges of airfoil (blunt nose to sharp tail)

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2. Applying the Bernoulli's theorem:

$$\begin{aligned} p_{\text{lower}} - p_{\text{upper}} &= \frac{1}{2}\rho\left(u_{\text{upper}}^2 - u_{\text{lower}}^2\right) \\ &= \frac{1}{2}\rho(u_{\text{upper}} + u_{\text{lower}})(u_{\text{upper}} - u_{\text{lower}}) \end{aligned}$$

Where $\frac{1}{2}(u_{\text{upper}} + u_{\text{lower}}) \approx V$ (average velocity / incoming velocity), and thus

$$\begin{aligned} L_{2D} &\approx \rho V \int_0^{\text{Chord}} (u_{\text{upper}} - u_{\text{lower}})\, dx \\ L_{2D} &= \rho V \Gamma \end{aligned}$$

Vortex generation and its shedding

Vorticity Ω is a measurement of cross-stream velocity gradient. It is generated whenever flow passes over a solid surface, on which velocity = 0. The free stream velocity is found to be the total vorticity generated by unit distance in flow direction

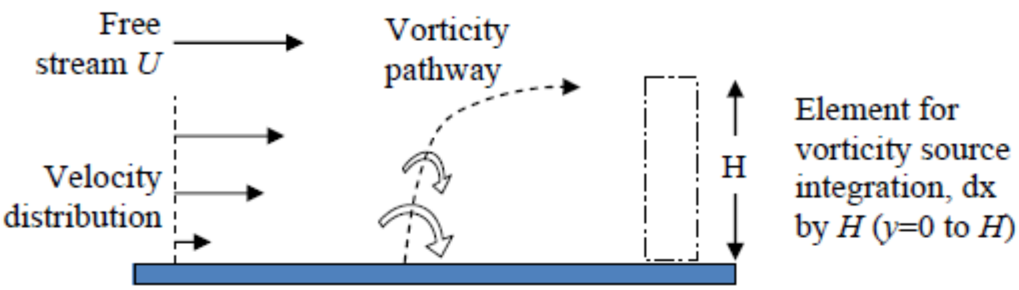
$$\Omega = \Omega_z e_z, \qquad \Omega_z = \frac{\partial V_x}{\partial y} - \frac{\partial V_y}{\partial x} \approx \frac{\partial V_x}{\partial y}$$

A few more points to be noted:

1. For convenience, we may write $\Omega = \frac{du}{dy}$ for steady flow where velocity V_x is u(y).
2. Vorticity cannot be generated in the fluid interior under normal circumstances.
3. It can only be diffused in fluid by viscosity (like heat conduction).

Vortex on a moving flat plate:

1. Flat plate suddenly moves (to the left at speed **U**)
2. Pulls fluid particles on its surface with it
3. In a reference frame that moves with the plate, all fluids tend to move right at speed U while those on the surface stays.
4. The total vorticity generated in a small element of width **dx** and height **dy**, where 0 < y < H, presented below:



$$\begin{aligned}\Omega_{Total} &= \int_{A=ydx} \Omega dA \qquad \boxed{dA = dydx}\\ &= \int_{y=0}^{y=H} \frac{du}{dy} dydx\\ &= \left(u_{y=H} - u_{y=0}\right)dx\\ &= Udx\end{aligned}$$

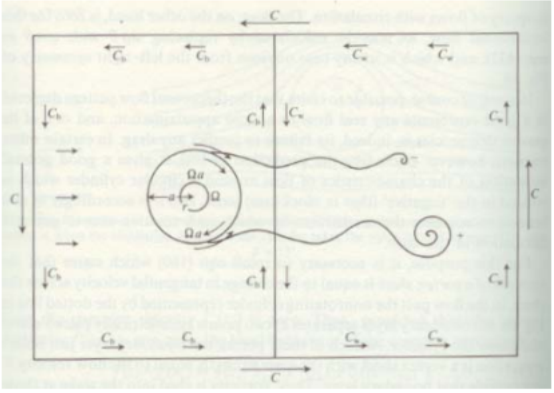
The vorticity generated per unit width (dx = 1) is the free stream flow velocity.

Convection (對流):

- Vorticity which is generated on the surface is diffused into the irrotational or uniform flow field.
- It is carried downstream by the fluid particle when diffusing away from surface.
- Therefore, it cannot be diffused too far away from the surface.

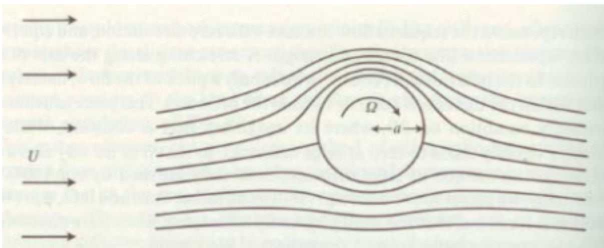
Vorticity is carried by fluid and is detached from surface when fluid separates from the surface.

E.g.1



- A circular cylinder rotates clockwise in a mean flow U
- Upper surface total velocity > lower.
- When flow separates from the cylinder, the clockwise circulation shed (upper) < anti-clockwise circulation shed(lower) in magnitude.
- In total, the wake has anti-clockwise circulation.

E.g.2



- A circulation around a large rectangle containing the cylinder and the entire wake
- → Total circulation = 0 (Kelvin's theorem)
- If separates this domain into two parts:
  1. The cylinder and
  2. The another wake
- The zero circulation for the large domain is the sum of the circulation for the two sub-domains since the common interface does not contribute to the circulation (upward circulation is cancelled by the downward).
- Knowing that the airfoil domain has clockwise circulation which is indicated by the cylinder rotation.
- The circulation magnitude = that circulation shed into the wake.
- The cylinder has a clockwise circulation and it experiences an upward lift force.

d’Alembert’s paradox

For flow past a flat plate with an incidence angle, if viscosity is ignored, the complete symmetry between the upper and lower surfaces would guarantee zero lift, and the lack of skin friction also means zero drag. Two stagnation points are formed, one near the leading edge at the lower surface and another near the trailing edge on the upper surface. This conclusion (zero drag when ignoring viscosity) contradicts the common sense of drag, hence “paradox”.

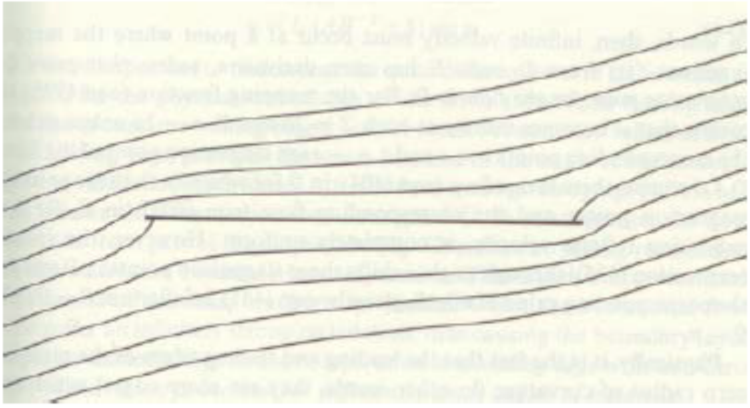


Figure: d’Alembert’s analysis

<u>Zhukowski's hypothesis</u>

If inviscid flow is assumed:

Trailing edges Leading edges	**Blunt**	**Sharp**
Blunt	Not mentioned	**Zhukowski's hypothesis, described the most:** When a 2D aerofoil moves steadily through fluid in a direction only slightly inclined to its chord, the effect of viscosity on the flow at high Reynolds number (ratio of inertia to friction forces. An increasing Reynolds number indicates an increasing turbulence of flow) is to cause: • The generation of circulation round the aerofoil, the magnitude of the circulation being just that required to move the <u>rear stagnation point</u> to the sharp trailing edge • To eliminate boundary layer separation from both surfaces of the aerofoil. At trailing edge: • Start from the very beginning, the rapid flow around the edge is slowed down to the stagnation point • Anti-clockwise vorticity is shed more than the clockwise from the upstream part of the upper surface → The wake region would have more anti-clockwise vorticity. • According to Kelvin's law. A clockwise circulation (also called the attached (or bound) vorticity) is generated by the airfoil itself to match the shedding. • This vorticity would induce its own velocity field and drive the stagnation point towards the trailing edge. • Both anti-clockwise and clockwise circulation generation stop when 1. the stagnation point is finally pushed to the trailing edge 2. there is no more flow climbing around the trailing edge • Then the sharp trailing edge has a finite velocity. • The amount of circulation that enables such a stable flow is totally determined by the airfoil geometry and the incidence angle.
Sharp	Infinite speed is avoided.	The flow at the two points has a theoretical speed of infinity. Since it takes infinitely negative pressure to suck flow around a corner of zero radius. Such low pressure can only be generated if a great velocity is achieved.

Lift coefficient (C_L)

- α: incidence angle between the flight direction and the chord,
- β: a parameter that controls the camber (β = 0 for symmetrical airfoil) but is not equal to any obvious, specific physical angle in the geometry.

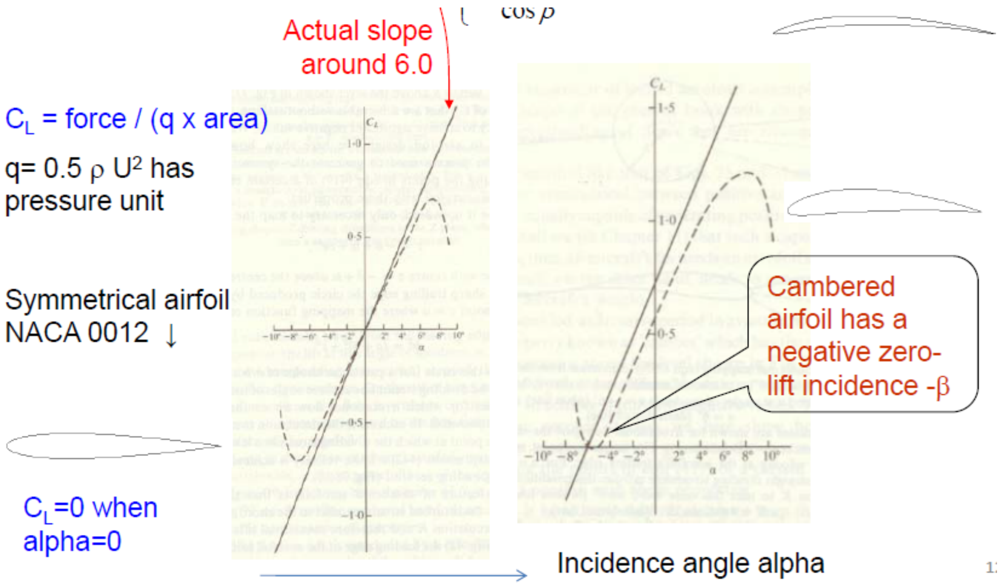
2π is the theoretical slope of C_L against α for thin and symmetrical airfoil, but the actual value is around 6.0. Theoretically:

$$C_L = \frac{L}{\frac{1}{2}\rho V_{\text{flight}}^2 A_w} = \begin{cases} 2\pi\sin\alpha\left(1 + 0.77\dfrac{\text{thickness}}{\text{chord}}\right), & \text{symmetrical} \\ 2\pi\dfrac{\sin(\alpha+\beta)}{\cos\beta}, & \text{circular arc of 0 thickness} \end{cases}$$

What we do in exercise 1.8

$$m = \frac{C_{\text{Lmax}} - C_{\text{Lzero}}}{\sin\alpha_{\text{max}} - \sin\alpha_{\text{zero}}}$$

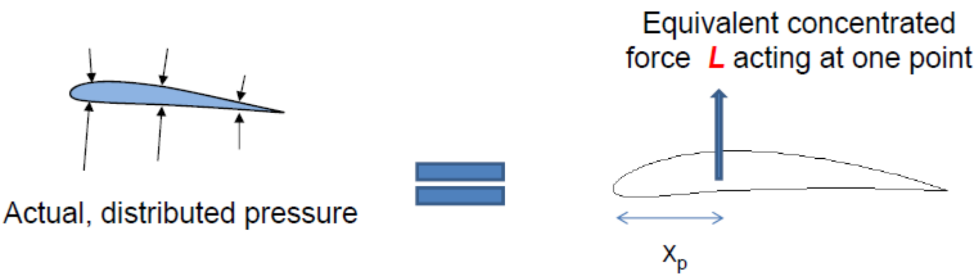
$$C_{\text{Li}} = \frac{L}{\frac{1}{2}\rho V_{\text{flight}}^2 A_w} = m(\alpha_i - \alpha_{\text{zero}})$$



* cambered airfoil has a negative zero-lift incidence

Pressure centre

The concentrated 2D lift force (lift per unit span) generated by the resultant force on the airfoil:



$$L_{2D} = \int_0^{chord} (p_{lower} - p_{upper})\, dx$$

The moment of the distributed force:

$$L_{2D} x_p = \int_0^{chord} x(p_{lower} - p_{upper})\, dx$$

x_p is the pressure centre at a certain distance from the leading edge such that the resultant force on the airfoil can be represented by a concentrated lift force placed at this position:

$$\frac{x_p}{chord} = \frac{1}{chord} \times \left[\frac{1}{L_{2D}} \int_0^{chord} x(p_{lower} - p_{upper})\, dx\right]$$

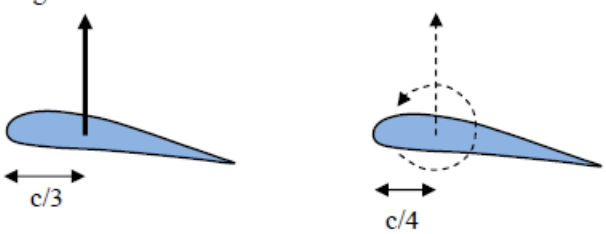
$$x_p \approx \frac{\alpha + 2\beta}{4(\alpha + \beta)} \times chord$$

$$x_p = \begin{cases} \frac{1}{4} \times chord, & \beta = 0 \text{ (symmetrical)} \\ \frac{1}{2} \times chord, & \alpha = 0 \quad \text{(pure arc)} \end{cases}$$

Symmetrical, x_p=chord/4

Arc with α=0, x_p=chord/2

Example: If the centre of pressure is at 0.3 chord (c) from leading edge and the lift is L, shown below on the left, what is the moment around the $c/4$ as shown on the right?



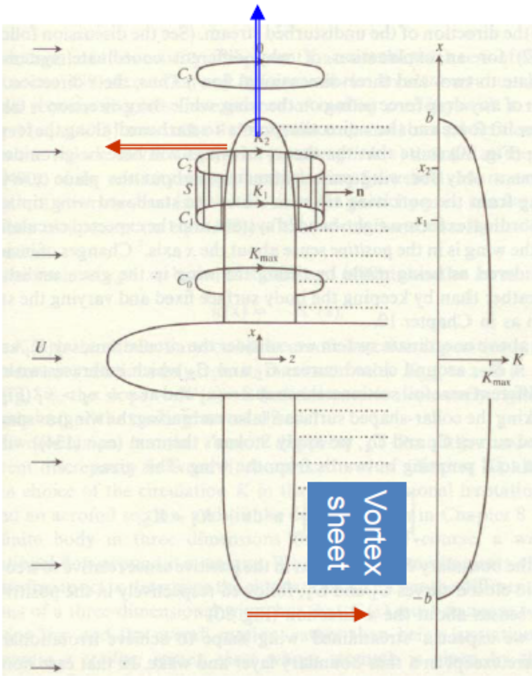
Ans: moment $M_{c/4}$=L(0.3-0.25)c=L 0.05 c in anti-clockwise direction

Discussions: This direction is taken to be negative if we choose positive to be the one that would increase incidence angle. Note that the moment can be further transferred to the leading edge by $M_{LE}=M_{c/4}+L\,(c/4)$.

Flow over a finite wing: spanwise and streamwise vorticity

Applying the theory over a particular cross section of the wing as an example.

Spanwise vorticity (for lift)

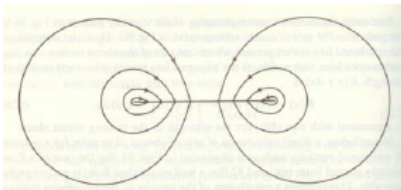


Streamwise vorticity (from tip)

K is circulation (Γ) here

- The distribution of Γ along the span is shown on the right. This circulation vanishes at the two tips.
- There is two different direction for vorticity: Spanwise (for lift, arrow in blue), Streamwise (from tip).
- The higher pressure on the lower wing surface drives the air to flow around the wing tip to the upper surface forming a circulation around the tips (shown in the following graph).

Front view of aircraft tip vortex



Tip vortex drag (Induced Drag)

Mechanism:

1. The streamwise vorticity induce a downward velocity on the whole airflow (downwash)
2. Two related effects due to downwash
   i. The angle between the chord and incoming flow is changed in such a way that the effective incidence angle is reduced, hence the lift is reduced.
   ii. The resultant force is very closely perpendicular to the inlet flow.
3. The inlet flow is deflected downward for a region of considerable size around the wing, the direction of the resulting lift force is also tilted backward.
4. The backward tilting gives rise to a force component against the flight.

$$\text{Induced Drag} = \frac{L^2}{\frac{\pi}{2}\rho V_{\text{flight}}^2 A_w^2}$$

Where L is the lift over the wing.

Vortex drag is:

- Inevitable (finite wing)
- Proportional to L^2