Tarefa 2 - Métodos Numéricos II

Jose Douglas Gondim Soares, 485347
Fernanda Costa de Sousa, 485404

Polinomio grau 4 (n=4):

$$\sum_{0}^{4}\binom{s}{k}\Delta^k r_0 = \sum_{0}^{4}\frac{s!}{k!(s-k)!}\Delta^k r_0 = \frac{s!}{0!(s-0)!}\Delta^0 r_0 + \frac{s!}{1!(s-1)!}\Delta^1 r_0 + \frac{s!}{2!(s-2)!}\Delta^2 r_0 + \frac{s!}{3!(s-3)!}\Delta^3 r_0 +$$

$$\frac{s!}{4!(s-4)!}\Delta^4 r_0 = g(s)$$

$$g(s) = \Delta^0 r_0 + s\,\Delta^1 r_0 + \frac{s(s-1)}{2}\Delta^2 r_0 + \frac{s(s-1)(s-2)}{6}\Delta^3 r_0 + \frac{s(s-1)(s-2)(s-3)}{24}\Delta^4 r_0$$

$$\Delta^0 r_0 = r(0)$$

$$\Delta^1 r_0 = r(1) - r(0)$$

$$\Delta^2 r_0 = \Delta^1 r_1 - \Delta^1 r_0 = (r(2) - r(1)) - r(1) + r(0) = r(2) - 2r(1) + r(0)$$

$$\Delta^3 r_0 = (\Delta^2 r_1 - \Delta^2 r_0) = (\Delta^1 r_2 - \Delta^1 r_1) - [r(2) - 2r(1) + r(0)] =$$
$$\{[r(3) - r(2)] - [r(2) - r(1)]\} - r(2) + 2r(1) - r(0) =$$
$$r(3) - r(2) - r(2) + r(1) - r(2) + 2r(1) - r(0) = r(3) - 3r(2) + 3r(1) - r(0)$$

$$\Delta^4 r_0 = (\Delta^3 r_1 - \Delta^3 r_0) = (\Delta^2 r_2 - \Delta^2 r_1) - (r(3) - 3r(2) + 3r(1) - r(0)) =$$
$$[(\Delta^1 r_3 - \Delta^1 r_2) - (\Delta^1 r_2 - \Delta^1 r_1)] - r(3) + 3r(2) - 3r(1) + r(0) =$$
$$\{[(r(4) - r(3)) - (r(3) - r(2))] - [(r(3) - r(2)) - (r(2) - r(1))]\} - r(3) + 3r(2) - 3r(1) + r(0) =$$
$$r(4) - r(3) - r(3) + r(2) - r(3) + r(2) + r(2) - r(1) - r(3) + 3r(2) - 3r(1) + r(0) =$$
$$r(4) - 4r(3) + 6r(2) - 4r(1) + r(0)$$

$$g(s) = r(0) + [\, s(r(1) - r(0)) + (\frac{s(s-1)}{2}(r(2) - 2r(1) + r(0)))\,] +$$
$$[(\frac{s(s-1)(s-2)}{6})(r(3) - 3r(2) + 3r(1) - r(0))] + [(\frac{s(s-1)(s-2)(s-3)}{24})(r(4) - 4r(3) + 6r(2) - 4r(1) + r(0))]$$

Abordagem fechada: polinômio de substituição de grau 4:

$$h = \frac{\Delta x}{4}$$

$$h\int_0^4 \Big(r(0) + \big[s(r(1) - r(0)) + \big(\frac{s(s-1)}{2} \big(r(2) - 2r(1) + r(0) \big) \big) \big] +$$

$$[(\frac{s(s-1)(s-2)}{6})(r(3) - 3r(2) + 3r(1) - r(0))] + [(\frac{s(s-1)(s-2)(s-3)}{24}) (r(4) - 4r(3) + 6r(2) - 4r(1) + r(0))]\Big)$$

$$= \int_{x1}^{x5} f(x)dx = \frac{2}{45}h(7r(1) + 32r(2) + 12r(3) + 32r(4) + 7r(5)) - \frac{8}{945}h^7 f^{(6)}(\xi)$$

Abordagem aberta: polinômio de substituição de grau 4:

$$h = \frac{\Delta x}{6}$$

$$h\int_0^4 \Big(r(0) + \big[s(r(1) - r(0)) + \big(\frac{s(s-1)}{2} \big(r(2) - 2r(1) + r(0) \big) \big) \big] +$$

$$[(\frac{s(s-1)(s-2)}{6})(r(3) - 3r(2) + 3r(1) - r(0))] + [(\frac{s(s-1)(s-2)(s-3)}{24}) (r(4) - 4r(3) + 6r(2) - 4r(1) + r(0))]\Big)$$

$$= \int_{x0}^{x6} f(x)dx = \frac{6}{20}h(11r(1) - 14r(2) + 26r(3) - 14r(4) + 11r(5)) - \frac{41}{140}h^7 f^{(6)}(\xi)$$