Fórmulas de Gauss-Legendre

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1 Ingredientes da fórmula de Gauss-Legendre com 4 pontos de interpolação.

Por definição temos:

$I = \int_{x_i}^{x_f} f(x)\,dx \ \approx \ \frac{x_f - x_i}{2}[\sum_{k=1}^{4} f(x(\alpha_k))w_k] =$

$\frac{x_f - x_i}{2}[f(x(\alpha_1))w_1 + f(x(\alpha_2))w_2 + f(x(\alpha_3))w_3 + f(x(\alpha_4))w_4]$

2 Calculando as raízes do polinômio de Legendre de grau 4 α_1, α_2, α_3, α_4 e α_5:

$P_n(\alpha) = \frac{1}{2^n n!}\frac{d^n}{d\alpha^n}[(\alpha^2 - 1)^n] \implies$

Grau 4: $P_4(\alpha) = \frac{1}{2^4 4!}\frac{d^4}{d\alpha^4}((\alpha^2 - 1)^4) = \frac{1}{8}(35x^4 - 30x^2 + 3)$

2.1 Cálculo das raízes do polinômio de grau 4:

$P_4(\alpha) = \frac{1}{8}(63x^5 - 70x^3 + 15x) = 0 \implies$

$\overline{\alpha_1} = -\sqrt{\frac{3}{7} - \frac{2\sqrt{\frac{6}{5}}}{7}}, \ \ \overline{\alpha_2} = -\sqrt{\frac{3}{7} + \frac{2\sqrt{\frac{6}{5}}}{7}}, \ \ \overline{\alpha_3} = \sqrt{\frac{3}{7} - \frac{2\sqrt{\frac{6}{5}}}{7}}, \ \ \overline{\alpha_4} = \sqrt{\frac{3}{7} + \frac{2\sqrt{\frac{6}{5}}}{7}}$

3 Calculando $x(\alpha_1)$, $x(\alpha_2)$, $x(\alpha_3)$ e $x(\alpha_4)$:

$x(\alpha_k) = \frac{x_i + x_f}{2} + \frac{x_f - x_i}{2}\alpha_k \implies$

$x(\alpha_1) = x(-\sqrt{\frac{3}{7} - \frac{2\sqrt{\frac{6}{5}}}{7}}) = \frac{x_i + x_f}{2} + \frac{x_f - x_i}{2}(-\sqrt{\frac{3}{7} - \frac{2\sqrt{\frac{6}{5}}}{7}}).$

$x(\alpha_2) = x(-\sqrt{\frac{3}{7} + \frac{2\sqrt{\frac{6}{5}}}{7}}) = \frac{x_i + x_f}{2} + \frac{x_f - x_i}{2}(-\sqrt{\frac{3}{7} + \frac{2\sqrt{\frac{6}{5}}}{7}}).$

$x(\alpha_3) = x(\sqrt{\frac{3}{7} - \frac{2\sqrt{\frac{6}{5}}}{7}}) = \frac{x_i + x_f}{2} + \frac{x_f - x_i}{2}(\sqrt{\frac{3}{7} - \frac{2\sqrt{\frac{6}{5}}}{7}}).$

$x(\alpha_4) = x(\sqrt{\frac{3}{7} + \frac{2\sqrt{\frac{6}{5}}}{7}}) = \frac{x_i + x_f}{2} + \frac{x_f - x_i}{2}(+\sqrt{\frac{3}{7} + \frac{2\sqrt{\frac{6}{5}}}{7}}).$

4 Calculando os pesos w_1, w_2, w_3 e w_4:

$w_k = \int_{-1}^{1} L_k(\alpha)d\alpha$

$w_1 = \int_{-1}^{1} L_1(\alpha) = \int_{-1}^{1} \prod_{j=1}^{4} \frac{\alpha-\alpha_j}{\alpha_1-\alpha_j} = \int_{-1}^{1} (\frac{\alpha-\alpha_2}{\alpha_1-\alpha_2})(\frac{\alpha-\alpha_3}{\alpha_1-\alpha_3})(\frac{\alpha-\alpha_4}{\alpha_1-\alpha_4}) = 0.652145$

$w_2 = \int_{-1}^{1} L_2(\alpha) = \int_{-1}^{1} \prod_{j=1}^{4} \frac{\alpha-\alpha_j}{\alpha_2-\alpha_j} = \int_{-1}^{1} (\frac{\alpha-\alpha_1}{\alpha_2-\alpha_1})(\frac{\alpha-\alpha_3}{\alpha_2-\alpha_3})(\frac{\alpha-\alpha_4}{\alpha_2-\alpha_4}) = 0.347855$

$w_3 = w_1 = 0.652145$

$w_4 = w_2 = 0.347855$

Agora, já temos todos os ingreditente para a fórmula de Legendre de grau 4.