Lista 10

1- a) $T(n) = 4 \cdot T(n/2) + n =$

$4 \cdot (4 \cdot T(n/2^2) + \frac{n}{2}) + n$

$4^2 \cdot T(n/2^2) + \frac{4n}{2} + n =$

$4^2 \cdot (4 \cdot T(n/2^3) + \frac{n}{2^2}) + \frac{4n}{2} + n =$

$4^3 \cdot T(n/2^3) + \frac{4^2 \cdot n}{2^2} + \frac{4n}{2} + n$

Temos

$4^k \cdot T(n/2^k) + \frac{4^{k-1} \cdot n}{2^{k-1}} + \frac{4^{k-2} \cdot n}{2^{k-2}} + \dots + \frac{4^{k-k} \cdot n}{2^{k-k}}$

assumindo $k = \log_2 n$

$4^{\log_2 n} \cdot T(1) + \dots + \underbrace{4n + 2n + n}_{\log_2 n \text{ vezes}} =$

$n^2 + 5n\left(n \cdot \frac{(1 - 2^{\log_2 n})}{(1-2)}\right) = n^2 + n \cdot \frac{(1-n)}{-1} =$

$n^2 + \frac{n - n^2}{-1} = n^2 + n^2 - n = 2n^2 - n = O(n^2)$.

T.M. $4 \cdot T(n/2) + n$

$\frac{4}{2^1} = 2 \to O(n^{\log_b a}) = O(n^{\log_2 4}) = O(n^2)$.

b) $T(n) = 2.T(n/3) + n^2 =$

$2.(2.T(n/3^2) + \frac{n^2}{3}) + n^2 =$

$2^2.T(n/3) + \frac{2n^2}{3} + n^2 =$

$2^2.(2.T(n/3^3) + \frac{n^2}{3^2}) + \frac{2n^2}{3} + n^2 =$

$2^3.T(n/3^3) + \frac{2^2.n^2}{3^2} + \frac{2n^2}{3} + n^2$

Temos

$2^k.T(n/3^k) + \frac{2^{k-1}.n^2}{3^{k-1}} + \frac{2^{k-2}.n^2}{3^{k-2}} + \dots + \frac{2^{k-k}.n^2}{3^{k-k}}$

assumindo $K = \log_3 n$

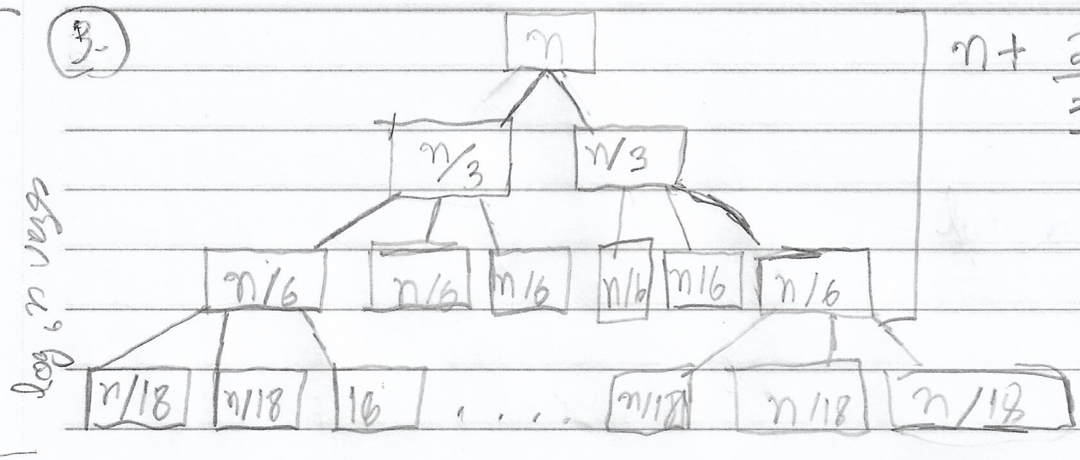
$$2^{\log_3 n}.T(1) + \underbrace{\frac{4}{9}.n^2 + \frac{2}{3}.n^2 + n^2}_{\log_3 n \text{ vezes}} =$$

$$n + S_n\left(\frac{n^2.(1-\frac{2}{3}^{\log_3 n})}{(1-\frac{2}{3})}\right) = n + \frac{n^2 - n^2.\frac{2}{3}^{\log_3 n}}{\frac{1}{3}}$$

$\rightarrow O(n^2)$

T.M. $\frac{a}{b^c} = \frac{2}{3^2} = \frac{2}{9} < 1 \rightarrow O(n^c) = O(n^2)$

3-




$$n + \frac{2n}{3} = \frac{5n}{3}$$

| | | |
|---|---|---|
| 1 | $n$ | $= n$ |
| 2 | $2n/3$ | $= 2n/3$ |
| 6 | $6n/6$ | $= n$ |
| 12 | $12n/18$ | $= 2n/3$ |
| 36 | $36n/36$ | $= n$ |

$$T(n) = \frac{5n}{3} \cdot \log_6 n = O(n \cdot \log_6 n)$$