Tarefa 01 - Diferenciação Numérica

Dupla:
José Douglas Gondim Soares, 485347
Fernanda Costa de Sousa, 485404

1. a)

$$f(x_k + \Delta x) = f(x_k) + \frac{1}{1!}f'(x_k)(\Delta x) + \frac{1}{2!}f''(x_k)(\Delta x)^2 + \frac{1}{3!}f'''(x_k)(\Delta x)^3 + \frac{1}{4!}f^{(iv)}(x_k)(\Delta x)^4 + \frac{1}{5!}f^{(v)}(x_k)(\Delta x)^5 + \frac{1}{6!}f^{(vi)}(x_k)(\Delta x)^6$$

$$f(x_k - \Delta x) = f(x_k) - \frac{1}{1!}f'(x_k)(\Delta x) + \frac{1}{2!}f''(x_k)(\Delta x)^2 - \frac{1}{3!}f'''(x_k)(\Delta x)^3 + \frac{1}{4!}f^{(iv)}(x_k)(\Delta x)^4 - \frac{1}{5!}f^{(v)}(x_k)(\Delta x)^5 + \frac{1}{6!}f^{(vi)}(x_k)(\Delta x)^6$$

$$f(x_k + 2\Delta x) = f(x_k) + \frac{1}{1!}f'(x_k)(2\Delta x) + \frac{1}{2!}f''(x_k)(2\Delta x)^2 + \frac{1}{3!}f'''(x_k)(2\Delta x)^3 + \frac{1}{4!}f^{(iv)}(x_k)(2\Delta x)^4 + \frac{1}{5!}f^{(v)}(x_k)(2\Delta x)^5 + \frac{1}{6!}f^{(vi)}(x_k)(2\Delta x)^6$$

$$f(x_k - 2\Delta x) = f(x_k) - \frac{1}{1!}f'(x_k)(2\Delta x) + \frac{1}{2!}f''(x_k)(2\Delta x)^2 - \frac{1}{3!}f'''(x_k)(2\Delta x)^3 + \frac{1}{4!}f^{(iv)}(x_k)(2\Delta x)^4 - \frac{1}{5!}f^{(v)}(x_k)(2\Delta x)^5 + \frac{1}{6!}f^{(vi)}(x_k)(2\Delta x)^6$$

$$f' * \Delta\,(\alpha - \alpha + 2\beta - 2\beta)$$
$$\frac{1}{3!}f'''(x_k)(\Delta x)^3(\alpha - \alpha + 8\beta - 8\beta)$$
$$\frac{1}{5!}f^{(v)}(x_k)(\Delta x)^5(\alpha - \alpha + 32\beta - 32\beta)$$
$$\frac{1}{4!}f^{(iv)}(x_k)(\Delta x)^4(\alpha + \alpha + 16\beta + 16\beta) = 0$$

$$\alpha + 16\beta = 0,\ \beta = 1\ e\ \alpha = -16$$

$$-16f(x_k + \Delta x) = -16\,f(x_k) - 16\frac{1}{2!}f''(x_k)(\Delta x)^2 + -16\frac{1}{6!}f^{(vi)}(x_k)(\Delta x)^6$$

$$-16f(x_k - \Delta x) = -16\,f(x_k) - 16\frac{1}{2!}f''(x_k)(\Delta x)^2$$

$$- 16\frac{1}{6!}f^{(vi)}(x_k)(\Delta x)^6$$

$$f(x_k + 2\Delta x) = f(x_k) + \frac{1}{2!}f''(x_k)(2\Delta x)^2 + \frac{1}{6!}f^{(vi)}(x_k)(2\Delta x)^6$$

$$f(x_k - 2\Delta x) = f(x_k) + \frac{1}{2!}f''(x_k)(2\Delta x)^2 + \frac{1}{6!}f^{(vi)}(x_k)(2\Delta x)^6$$

$$- 16f(x_k + \Delta x) - 16f(x_k - \Delta x) + f(x_k + 2\Delta x) + f(x_k - 2\Delta x) = - 30\ f(x_k) + f''(x_k)(\Delta x)^2(-12) + \frac{1}{6!}f^{(vi)}(x_k)(2\Delta x)^6 - (32 + 128)$$

$$f''(x_k) = \frac{1}{12(\Delta x)^2}[-f(x_k - 2\Delta x) + 16f(x_k - \Delta x) - 30f(x_k) + 16f(x_k + \Delta x) - f(x_k + 2\Delta x)] + 0.71f^{(vi)}(x_k)(\Delta x)^4$$

Agora vamos calcular a derivada segunda da função: $\sqrt{e^{3x} + 4x^2}$ no ponto 2.

$$f = \sqrt{e^{3x} + 4x^2}$$

$$\Delta x = 0.5$$

$$f''(2) = \frac{1}{12(0.5)^2}[-f(1) + 16f(1.5) - 30f(2) + 16f(2.5) - f(3)] + 0.71f^{(vi)}(2)(0.5)^4$$

$$f''(2) = \frac{1}{3}[-\sqrt{e^3 + 4} + 16 * \sqrt{e^{4.5} + 9} - 30 * \sqrt{e^6 + 16} + 16 * \sqrt{e^{7.5} + 25} - \sqrt{e^9 + 36}] + 0.71f^{(vi)}(2)(0.5)^4 = 44.9044$$

$$\Delta x = 0.25$$

$$f''(2) = \frac{1}{12(0.25)^2}[-f(1.5) + 16f(1.75) - 30f(2) + 16f(2.25) - f(2.5)] + 0.71f^{(vi)}(2)(0.25)^4$$

$$f''(2) = \frac{1}{0.75}[-\sqrt{e^{4.5} + 9} + 16 * \sqrt{e^{5.25} + 12.25} - 30 * \sqrt{e^6 + 16} + 16 * \sqrt{e^{6.75} + 20.25} -$$

$\sqrt{e^{7.5} + 25}] + 0.71f^{(vi)}(2)(0.5)^4 = 45.0634$

$\Delta x = 0.125$

$f(2) = 20.48$

$f''(2) = \frac{1}{12(0.125)^2}[-f(1.75) + 16f(1.875) - 30f(2) + 16f(2.125) - f(2.25)] + 0.71f^{(vi)}(2)(0.25)^4$

$f''(2) = \frac{1}{0.1875}[-\sqrt{e^{5.25} + 12.25} + 16 * \sqrt{e^{5.625} + 14.0625} - 30 * \sqrt{e^{6} + 16} + 16 * \sqrt{e^{6.375} + 18.0625} -$

$\sqrt{e^{6.75} + 20.25}] + 0.71f^{(vi)}(2)(0.5)^4 = 45.0729$

$\Delta x = 0.0625$

$f''(2) = \frac{1}{12(0.0625)^2}[-f(1.875) + 16f(1.9375) - 30f(2) + 16f(2.0625) - f(2.125)] + 0.71f^{(vi)}(2)(0.25)^4$

$f''(2) = \frac{1}{0.046875}[-\sqrt{e^{5.625} + 14.0625} + 16 * \sqrt{e^{5.8125} + 15.015625}$

$- 30 * \sqrt{e^{6} + 16} + 16 * \sqrt{e^{6.1875} + 17.015625} -$

$\sqrt{e^{6.375} + 18.0625}] + 0.71f^{(vi)}(2)(0.5)^4 = 45.07346$

$\Delta x = 0.03125$

$f''(2) = \frac{1}{12(0.03125)^2}[-f(1.9375) + 16f(1.96875) - 30f(2) + 16f(2.03125) - f(2.0625)]$
$+ 0.71f^{(vi)}(2)(0.25)^4$

$f''(2) = \frac{1}{0.01171875}[-\sqrt{e^{5.8125} + 15.015625} + 16 * \sqrt{e^{5.90625} + 15.50390625}$

$- 30 * \sqrt{e^{6} + 16} + 16 * \sqrt{e^{6.09375} + 16.50390625} -$

$\sqrt{e^{6.1875} + 17.015625}] + 0.71f^{(vi)}(2)(0.5)^4 = 45.073469$

$(\Delta)^k$	$f(x)$	$f''(x)$	$e(x) = \left\lvert \frac{f''(\Delta)^{(k)} - f''(\Delta)^{(k-1)}}{f''(\Delta)^{(k)}} \right\rvert$
0.5	20.48	44.9044	-
0.25	20.48	45.0634	0.159
0.125	20.48	45.0729	0.0095
0.0625	20.48	45.07346	0.00056
0.03125	20.48	45.073469	0.000009