# Fórmulas de Gauss - [Hermite, Laguerre e Chebyshev]

José Douglas Gondim Soares[1] e Fernanda Costa de Sousa[2]

[1]douglasgondim@alu.ufc.br, 485347
[2]fernandacosta@alu.ufc.br, 485404

17 de maio de 2022

## 1 Polinômios de Gauss - [Hermite, Laguerre e Chebyshev] de grau 4:

$H_n(x) = (-1)^n e^{x^2} \frac{d^n}{dx^n} e^{-x^2} = (-1)^4 e^{x^2} \frac{d^4}{dx^4} e^{-x^2} = 16x^4 - 48x^2 + 12$

$L_n(x) = \frac{e^x}{n!} \frac{d^n}{dx^n}(e^{-x}x^n) = \frac{e^x}{4!} \frac{d^4}{dx^4}(e^{-x}x^4) = \frac{1}{24}(x^4 - 16x^3 + 72x^2 - 96x + 24)$

$T_n(x) = \frac{(-2)^n n!}{(2n)!} \sqrt{1-x^2} \frac{d^n}{dx^n}(1-x^2)^{n-\frac{1}{2}} = \frac{(-2)^4 4!}{(2*4)!} \sqrt{1-x^2} \frac{d^4}{dx^4}(1-x^2)^{4-\frac{1}{2}} = 8x^4 - 8x^2 + 1$

## 2 Calculando as raízes dos polinômios de Gauss - [Hermite, Laguerre e Chebyshev] de grau 4: $\alpha_1$, $\alpha_2$, $\alpha_3$ e $\alpha_4$.

### 2.1 Hermite:

$16x^4 - 48x^2 + 12 = 0 \implies$

$\alpha_1 = -\sqrt{\frac{3}{2} - \sqrt{\frac{3}{2}}},\ \alpha_2 = -\sqrt{\frac{3}{2} + \sqrt{\frac{3}{2}}},\ \alpha_3 = \sqrt{\frac{3}{2} - \sqrt{\frac{3}{2}}},\ \alpha_4 = \sqrt{\frac{3}{2} + \sqrt{\frac{3}{2}}}$

### 2.2 Laguerre:

$\frac{1}{24}(x^4 - 16x^3 + 72x^2 - 96x + 24) = 0 \implies$

$\alpha_1 \approx 0.32255,\ \alpha_2 \approx 1.7458,\ \alpha_3 \approx 4.5366,\ \alpha_4 \approx 9.3951$

### 2.3 Chebyshev:

$8x^4 - 8x^2 + 1 = 0 \implies$

$\alpha_1 = -\frac{1}{2}\sqrt{2-\sqrt{2}},\ \alpha_2 = -\frac{1}{2}\sqrt{2+\sqrt{2}},\ \alpha_3 = \frac{1}{2}\sqrt{2-\sqrt{2}},\ \alpha_4 = \frac{1}{2}\sqrt{2+\sqrt{2}}$

## 3 Calculando os pesos $w_1$, $w_2$, $w_3$ e $w_4$:

### 3.1 Hermite:

$w_k = \int_{-\infty}^{\infty} e^{-x^2} L_{g_k}(x)dx \implies$

$w_1 = \int_{-\infty}^{\infty} e^{-x^2} L_{g_1}(x)dx = \int_{-\infty}^{\infty} e^{-x^2} \prod_{j=1}^{4} \frac{\alpha-\alpha_j}{\alpha_1-\alpha_j} = \int_{-\infty}^{\infty} e^{-x^2} (\frac{\alpha-\alpha_2}{\alpha_1-\alpha_2})(\frac{\alpha-\alpha_3}{\alpha_1-\alpha_3})(\frac{\alpha-\alpha_4}{\alpha_1-\alpha_4}) \approx 0.804914$

$w_2 = \int_{-\infty}^{\infty} e^{-x^2} L_{g_2}(x)dx = \int_{-\infty}^{\infty} e^{-x^2} \prod_{j=1}^{4} \frac{\alpha-\alpha_j}{\alpha_2-\alpha_j} = \int_{-\infty}^{\infty} e^{-x^2} (\frac{\alpha-\alpha_1}{\alpha_2-\alpha_1})(\frac{\alpha-\alpha_3}{\alpha_2-\alpha_3})(\frac{\alpha-\alpha_4}{\alpha_2-\alpha_4}) \approx 0.0813128$

$w_3 = w_1 = 0.804914$

$w_4 = w_2 = 0.0813128$

### 3.2 Laguerre:

$w_k = \int_0^\infty e^{-x} L_{g_k}(x)dx \implies$

$w_1 = \int_0^\infty e^{-x} L_{g_1}(x)dx = \int_0^\infty e^{-x} \prod_{j=1}^4 \frac{\alpha-\alpha_j}{\alpha_1-\alpha_j} = \int_0^\infty e^{-x} (\frac{\alpha-\alpha_2}{\alpha_1-\alpha_2})(\frac{\alpha-\alpha_3}{\alpha_1-\alpha_3})(\frac{\alpha-\alpha_4}{\alpha_1-\alpha_4}) \approx 0.603154$

$w_2 = \int_0^\infty e^{-x} L_{g_2}(x)dx = \int_0^\infty e^{-x} \prod_{j=1}^4 \frac{\alpha-\alpha_j}{\alpha_2-\alpha_j} = \int_0^\infty e^{-x} (\frac{\alpha-\alpha_1}{\alpha_2-\alpha_1})(\frac{\alpha-\alpha_3}{\alpha_2-\alpha_3})(\frac{\alpha-\alpha_4}{\alpha_2-\alpha_4}) \approx 0.357419$

$w_3 = \int_0^\infty e^{-x} L_{g_3}(x)dx = \int_0^\infty e^{-x} \prod_{j=1}^4 \frac{\alpha-\alpha_j}{\alpha_3-\alpha_j} = \int_0^\infty e^{-x} (\frac{\alpha-\alpha_1}{\alpha_3-\alpha_1})(\frac{\alpha-\alpha_2}{\alpha_3-\alpha_2})(\frac{\alpha-\alpha_4}{\alpha_3-\alpha_4}) \approx 0.0388879$

$w_4 = \int_0^\infty e^{-x} L_{g_4}(x)dx = \int_0^\infty e^{-x} \prod_{j=1}^4 \frac{\alpha-\alpha_j}{\alpha_4-\alpha_j} = \int_0^\infty e^{-x} (\frac{\alpha-\alpha_1}{\alpha_4-\alpha_1})(\frac{\alpha-\alpha_2}{\alpha_4-\alpha_2})(\frac{\alpha-\alpha_3}{\alpha_4-\alpha_3}) \approx 0.000539295$

### 3.3 Chebyshev:

$w_k = \int_{-1}^1 \frac{1}{\sqrt{1-x^2}} L_{g_k}(x)dx \implies$

$w_1 = \int_{-1}^1 \frac{1}{\sqrt{1-x^2}} L_{g_1}(x)dx = \int_{-1}^1 \frac{1}{\sqrt{1-x^2}} \prod_{j=1}^4 \frac{\alpha-\alpha_j}{\alpha_1-\alpha_j} = \frac{1}{\sqrt{1-x^2}} (\frac{\alpha-\alpha_2}{\alpha_1-\alpha_2})(\frac{\alpha-\alpha_3}{\alpha_1-\alpha_3})(\frac{\alpha-\alpha_4}{\alpha_1-\alpha_4}) = \frac{\pi}{4}$

$w_2 = w_3 = w_4 = \frac{\pi}{4}$

**Agora, já temos todos os valores das raízes $\alpha$ e de seus respectivos pesos $w$ para as fórmulas de Gauss - [Hermite, Laguerre e Chebyshev] de grau 4**.