Part One

Elasticity and the theory of strength

Chapter 2 Stresses and strains
or why you don't fall through the floor

'It had been his custom to engage Wan in philosophical discussion at the close of each day and on this occasion he was contrasting the system of Ka-ping, who maintained that the world was suspended from a powerful fibrous rope, with that of Tai-u who contended that it was supported upon a substantial bamboo pole. With the clear insight of an original and discerning mind Ah-shoo had already detected the fundamental weakness of both theories.'

Ernest Bramah, *Kai Lung unrolls his mat.*

We are so used to not falling through the floor that we never stop to think why we don't. However the problem of how any inanimate solid is able to resist a load at all worried both Galileo (1564–1642) and Hooke (1635–1702). The understanding of simple structures and how they resist loads is a good example of a problem which, except in its molecular aspects, requires no sophisticated apparatus and could, in theory, be solved almost entirely by pure reason. This is not to say that the subject is easy; it is intellectually very difficult. The genius of Galileo and Hooke lay as much in recognizing that an important problem existed as in their contributions towards solving it, significant as these were.

As a matter of fact the general problem was probably beyond the scientific potential of the seventeenth century and it was not until well into the nineteenth that any reasonably complete idea of what was happening in a structure existed; even then this knowledge was confined to a few rather despised theoreticians. For a long time 'practical' engineers went on as they always had done, by rule of thumb. It took a long history of controversy and a series of disasters like the Tay bridge to convince these people of the usefulness of proper strength calculations.* Also it was found that reliable calculations enabled structures to be made more cheaply because one could more safely economize in material. Nowadays the main difference between the qualified professional engineer on the one hand and the bench mechanic and the do-it-yourself amateur on the other lies not so much in mechanical ingenuity and skill as in an understanding of the problems of strength and energy.

Let us begin at the beginning with Newton (1642–1727) who said that action and reaction are equal and opposite. This means that every push must be matched and balanced by an equal and opposite push. It does not matter how the push arises. It may be a 'dead' load for instance: that is to say a stationary weight of some kind. If I weigh 200 pounds and stand on the floor, then the soles of my feet push downwards on the floor with a push or thrust of 200 pounds (or 900 Newtons, if you must); that is the business of feet. At the same time the floor must push upwards on my feet with a thrust of 200 pounds (or 900 Newtons); that is the business of floors. If the floor is rotten and cannot furnish a thrust of 200 pounds then I shall fall through the floor. If, however, by some miracle, the floor produced a larger thrust than my feet have called upon it to produce, say 201 pounds, then the result would be still more surprising because, of course, I should become airborne. Similarly, if a chair weighs 50 pounds, then the floor obliges by producing an upward force of exactly the 50 pounds which are needed to support the chair in its accustomed station in life. On the other hand, the force need not be a stationary weight. If I drive my car into a wall, the wall will respond by producing exactly enough force to stop the car at whatever speed it may be going, even if it kills me. Again, the wind, blowing where it listeth, pushes on my chimney pots but the chimney pots, bless them, push back at the wind just as hard, and that is why they don't fall off.

All this is merely a restatement of Newton's third law of motion which says, roughly speaking, that if the *status quo* is to be maintained then all the forces on an object must cancel each other out. This law does not say anything about how these various forces are generated. As far as the applied loads are concerned, the manner of their generation is usually straightforward: the weight of a 'dead' load arises from the action of the earth's gravitation upon the mass of the load and in the case of stopping a moving load (whether a solid, a liquid or a gas) the forces generated are those needed to decelerate the moving mass (Newton's second law of motion). The business of all structures is the conservative one of maintaining the *status quo* and in order to do this they must somehow generate adequate forces to oppose the loads which they have to carry. We can see how a weight presses down on the floor but how does the floor press up on the weight ?

The answer to this question is far from obvious and the problem was the more difficult for Galileo and Hooke, in the early days of scientific thought, because the biological analogy is confusing and the tendency is, or was, to begin thinking about a problem in an anthropomorphic way. An animal has really two mechanisms for resisting loads. Its inert parts – bones, teeth and hair – resist by just the same means as any other inert solid but the living animal as a whole behaves in a quite different manner. People and other animals resist mechanical forces by pushing back in an active way: they tense their muscles and push or pull as the situation may require. If I stretch out my hand and you put a weight on it such as a pint of beer, then I have to increase the tensions in certain muscles so as to sustain the load. I am enabled to do this because the tensions in our muscles can be continually adjusted by an elaborate biological mechanism. However, the maintenance of biological tensions requires the continual expenditure of actual work (like driving a car fitted with a fluid flywheel while it is hard up against a wall – the engine is working away and using petrol and the car is pushing against the wall but neither the car nor the wall are moving). For this reason my arm muscles will sooner or later get tired and so I shall have to drink the beer to relieve them. One remains standing, not like a tripod standing inertly on the ground, but by a series of deliberate, though perhaps unconscious, adjustments of the body muscles. One gets tired standing up, and, if the muscular processes are interrupted by fainting or death, there is a dramatic collapse.

In an inanimate solid these living processes are not available. Structural materials are passive and cannot push back deliberately, so that they do not, in the ordinary sense, get tired. They can only resist outside forces *when they are deflected;* that is, they must give way to the load to a greater or less extent in order to generate any resistance at all. By 'deflection', in this context, we do not mean that the solid moves bodily, as a whole and without changing its shape, but rather that the geometrical form of the solid is to some extent distorted so that some parts of it at least become shorter or longer by stretching or contracting within themselves. There is, and there can be, no such thing as a truly rigid material. Everything 'gives' to some extent and, as we have said, the realization that this is what structural engineering is about is what divides the professional from the amateur engineer.

When I climb a tree the deflections of the boughs under my weight will probably be very large, perhaps a matter of several inches, and are easily seen. However, when I walk across a bridge the deflections may be imperceptibly small. These are only questions of degree: there is always some deflection. Unless the deflections under loads are excessively large for the purpose of the structure they are not a fault but an inborn and unavoidable characteristic of structures with which it is the business of this chapter to come to terms. Most of us have sat in an aeroplane and watched the wing-tips going up and down. This is quite all right; the designer meant them to be like that.

It is probably obvious by this time that these deflections, be they large or small, generate the forces of resistance which make a solid hard and stiff and resistant to external loads. In other words, a solid deflects exactly far enough to build up forces which just counter the external load applied to it. This is the automatic process at the basis of all structures.

How are these forces generated? The atoms in a solid are held together by chemical forces or bonds (see Appendix 1) which may perhaps be thought of as electrical springs since there is nothing 'solid' in any crude sense to make any other kind of spring. It is these forces which bind solids together and also make the rules of chemistry. There is no distinction between the chemical bonds between atoms whose fracture yields the energy of gunpowder or petrol, and the chemical bonds which make steel and rubber strong and elastic.

When a solid is altogether free from mechanical loads (which, strictly speaking, is very seldom) these chemical bonds or springs are in their neutral or relaxed position (Figure 1). Any attempt to push them closer together (which we call compression) or to stretch them further apart (which we call tension) involves shortening or lengthening the interatomic springs, by however little, throughout the material. This is because the middle part of the atoms may be regarded as rigid and, furthermore, in a solid the atoms do not generally exchange places – at least at moderate or 'safe' loads. The only thing, therefore, which can 'give' is the interatomic bond. These bonds or springs vary a good deal in stiffness or springiness (or, as the layman might put it, in 'strength') but most of them are much stiffer than the metal springs to which we are accustomed in ordinary life. From this it follows, of course, that the forces between the atoms are often very large indeed. We should expect this if we think about the forces which can be released by chemical explosives and fuels.

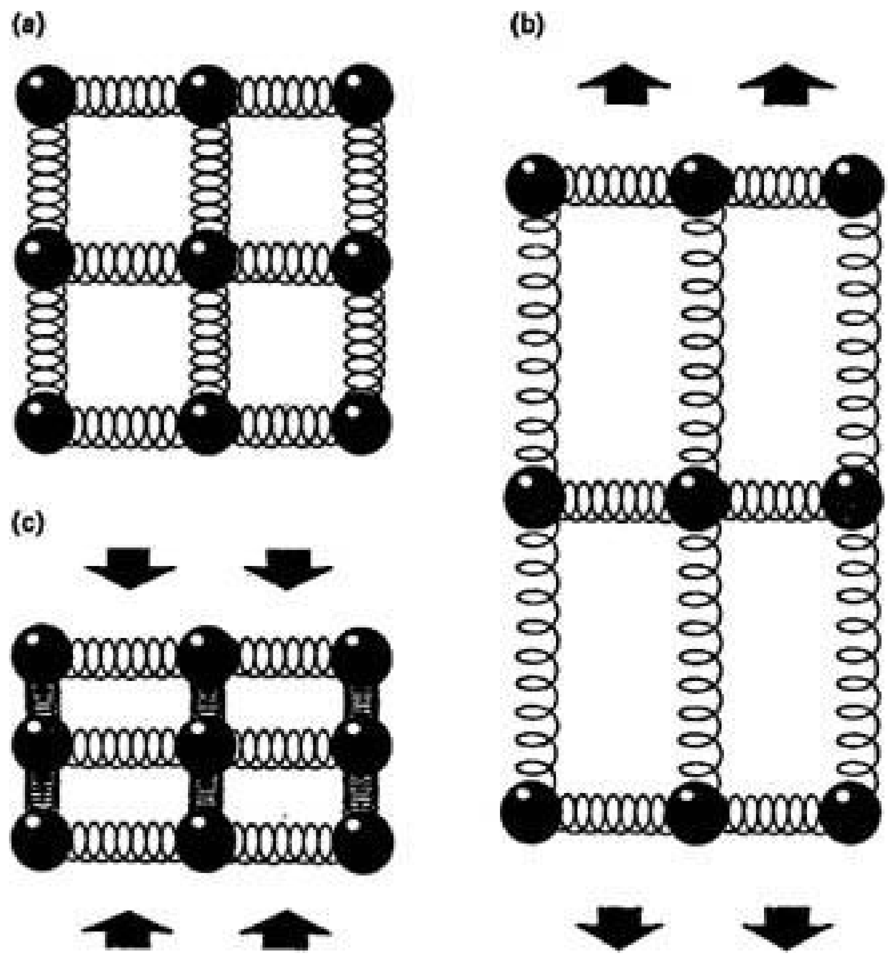


Figure 1. Simplified model of distortion of interatomic bonds under mechanical strain.
(a) Neutral, relaxed or strain-free position.
(b) Material strained in tension, atoms further apart, material gets longer.
(c) Material strained in compression, atoms closer together, material gets shorter.

Although there is no such thing as a truly rigid solid – that is to say one which does not yield at all when a weight is put on it – in everyday life the deflections of common objects are often very small.

For instance, if I take an ordinary builder's ceramic brick, stand it upright on a firm surface and tread on it, then the brick will be compressed along its length by a total distance of about $\frac{1}{50,000}$ of an inch. Any two neighbouring atoms in the brick are pushed nearer together by about $\frac{1}{500,000}$ Ångström unit (2×10^{-14}cm., or about one hundredth of a millionth of a millionth of an inch). This is an inconceivably small distance but a perfectly real movement for all that. Actually, in large structures the deflections are not by any means always tiny. In order to support their load, that is to say the roadway and the cars, the suspension cables of the Forth road bridge are permanently stretched in tension by about 0.1 per cent, or something like ten feet (three metres) in their total length of nearly two miles, or three kilometres. In this case the atoms of iron which are normally about two Ångström units apart when at rest and unloaded are kept about $\frac{2}{1,000}$ Ångström units further apart than they would be in the unstressed state.

That atoms really do move further apart when a material is stretched has been checked experimentally many times and by different methods. The most obvious way is by X-ray diffraction of stretched and unstretched specimens. The standard way of measuring the distance between the atoms in a crystal is to study the way in which an X-ray beam is deflected when it passes through the crystal. This method has been used now for sixty years or more and it is nowadays capable of considerable accuracy. It is found that the atoms in a metal, for instance, move apart or together exactly in proportion to the amount by which the metal as a whole is stretched or compressed. Changes in interatomic spacing up to about 1·0 per cent have been observed. Some actual measurements up to about 0·3 per cent are shown in Figure 2.

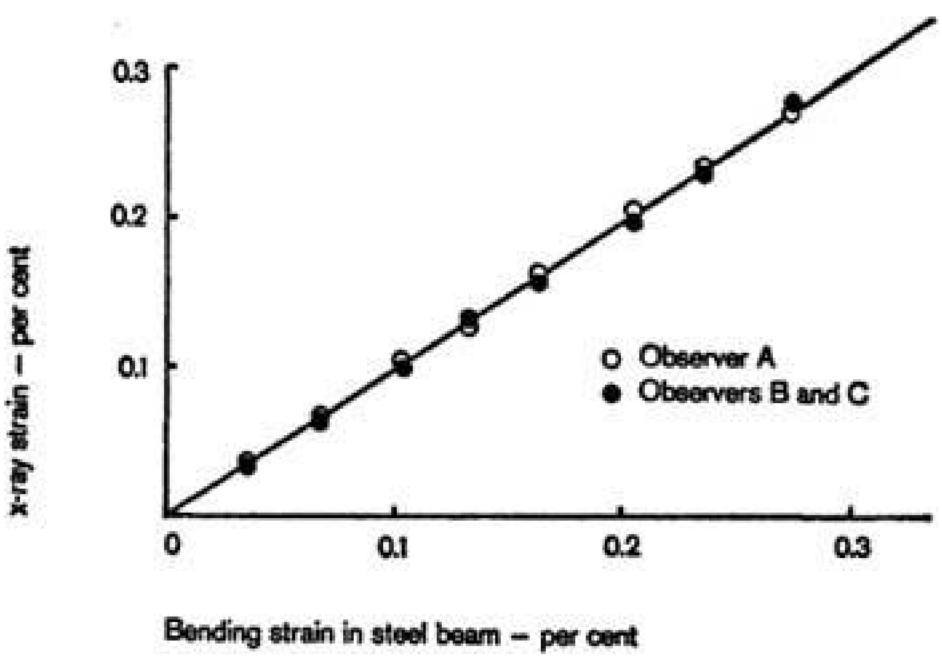


Figure 2. Comparison of strains determined by X-rays (two-exposure method) and strains computed from the curvature of a bent beam. Annealed mild steel.

What are stresses and strains, and why bother?

All this brings us to the question of stresses and strains, words which the layman is apt to regard as alarming, distressing and confusing. This is perhaps partly because the words may conjure up the idea of a wilderness of mathematics but probably more because the words have been borrowed or stolen by non-scientists to describe the mental condition of human beings. In this connotation the words have no very precise meaning and commonly stress and strain are used interchangeably as if they meant the same thing. All this is a pity because in science the two words have quite simple, clear and distinct meanings.

So far we have thought, as much as we have thought about it at all, of the force acting on a material as being the total load upon it. This might be any weight, and we have thought of the deflection under that load as being the total deflection, whatever the dimensions of the object, large or small. This is all very well but it gives us no proper standard of comparison between a big object under a big load and a small object under a little load. One might want to use the same kind of steel for a tiny part in a typewriter and also for the keel of an aircraft carrier: how can we compare its performance in the two jobs? Until we have some proper objective standards of comparison we cannot take the subject much further.

Stress is simply load per unit area. That is to say:

$$s = \frac{P}{A} \text{ (where } s = \text{stress, } P = \text{load, } A = \text{area.)}$$

This may possibly look frightening, but it is exactly analogous to such everyday remarks as ‘the cost of butter is 90p a pound’ or ‘my car does thirty miles to the gallon’.

Hence, to revert to the brick, if its cross section is 3 inches by 4 inches then its end has an area of 12 square inches and, if I tread on it with a weight of 200 pounds, the compressive stress which I cause in the brick is clearly:

$$s = \frac{P}{A} = \frac{200}{12} = 16\tfrac{2}{3} \text{ pounds on each square inch, or pounds per square inch, or lb./in.}^2\text{, or p.s.i.}$$

Similarly if the brickwork pier of a bridge has a cross-section measuring 20 feet by 5 feet and it is crossed by a railway engine weighing 100 tons (224,000 pounds) then the compressive stress in the brickwork will be roughly 16 p.s.i. We can say with confidence therefore that in both cases the stress in the bricks is similar and, if one structure is safe, so most probably will be the other. As far as the bricks are concerned, their molecules are being pushed together with an identical force although the engine is ponderous and I am relatively small. This is obviously the sort of things that engineers want to know.

In English-speaking countries stresses are traditionally expressed in pounds per square inch or tons per square inch. Continental engineers generally use kilogrammes per square centimetre. With S.I. the use of Newtons per square metre (N/m^2) usually produces embarrassingly large numbers and so we generally use Meganewtons per square metre (MN/m^2); (1 Meganewton = one million Newtons). In this book we shall use p.s.i. and MN/m^2 side by side.[*] Nevertheless we must be clear that we are applying the concept to the conditions at any cross-section or at a point and not especially to a square inch or to a square metre. Because the price of butter is 90p a pound this price does not apply especially to one pound; it is just as applicable to larger or smaller quantities.

Strain is just as simple: *Strain is the amount of stretch under load per unit length.*

Obviously, different lengths of material stretch different distances under the same load. So:

$$e = \frac{l}{L}$$

where e = strain, l = total amount of stretch, L = original total length.

So, if a rod 100 inches long stretches one inch under load, then it is subject to a strain of 1/100 or 0·01 or 1·0 per cent. So also is a rod SO inches long which stretches ½ inch, and so on. It does not matter how fat or thin the rod is or what is causing the extension. We are only concerned with how much the component atoms and molecules are stretched apart, and so strain is again, like stress, independent of the size of the specimen. Strain is a *fraction* of the original length and so it remains just a fraction or a ratio (in other words a number) and has no units, British, S.I. or anything else.

Hooke's law

The first man to grasp what was happening when an inert solid was loaded was Robert Hooke[*] who, besides being a physicist, was a notable architect and engineer and used to discuss the behaviour of springs and pendulums with the great clockmaker Thomas Tompion[†] (1639–1713). Hooke, or course, knew nothing about the chemical and electrical forces between atoms, but he realized that a 'spring' as a clockmaker might think of it is only a special case of the behaviour of any elastic solid and that there is no such thing as a truly rigid material, springiness being a property of every structure and of every solid.

Hooke, like Horace, did not suffer unduly from modesty and he staked his claim to priority in a number of fields by publishing in 1676 *A decimate of the centesme of the inventions I intend to publish* among which was 'The true theory of elasticity or springiness'. This heading was followed simply by the anagram 'ceiiinosssttuu'. The scientific public were left to make what they could of this until, in 1678, Hooke published *De potentia restitutiva, or of a spring* where the anagram was revealed as 'Ut tensio[‡] sic uis' – 'As the extension, so the force'.

In other words, stress is proportional to strain and vice versa. So, if an elastic body such as a wire is stretched one inch under a load of 100 pounds it will stretch two inches under 200 pounds and so on, *pro rata.* This is known as Hooke's law and is regarded as one of the pillars of engineering.

As a matter of fact Hooke's law is really an approximation which arises from the character of the forces between atoms. There are several kinds of chemical bonds between atoms (see Appendix 1) but they all result in interatomic force curves which are similar in general shape (Figure 3). At very large strains – 5 to 10 per cent or so – stress is anything but proportionate to strain. However, in practical engineering materials, strains nearly always lie in the range ± 1·0 per cent either side of the neutral or strain-free position and for this range the relation between stress and strain is pretty well a straight line. Furthermore for small strains the whole process of extension and recovery is reversible and can usually be repeated many thousands or millions of times with identical results; the hair-spring of a watch which is coiled and uncoiled 18,000 times each hour is a familiar example. This type of behaviour by solids under loading is called 'elastic' and is widespread. Elastic behaviour, which is shown by the majority of engineering materials, contrasts with 'plastic' behaviour, shown to the extreme by putty and Plasticine, where the material does not obey Hooke's law in the initial loading and does not recover properly when the load is removed. The word 'elastic' is not especially dedicated to indiarubber and sock-suspenders and the science of elasticity is the study of stresses and strains in solids.

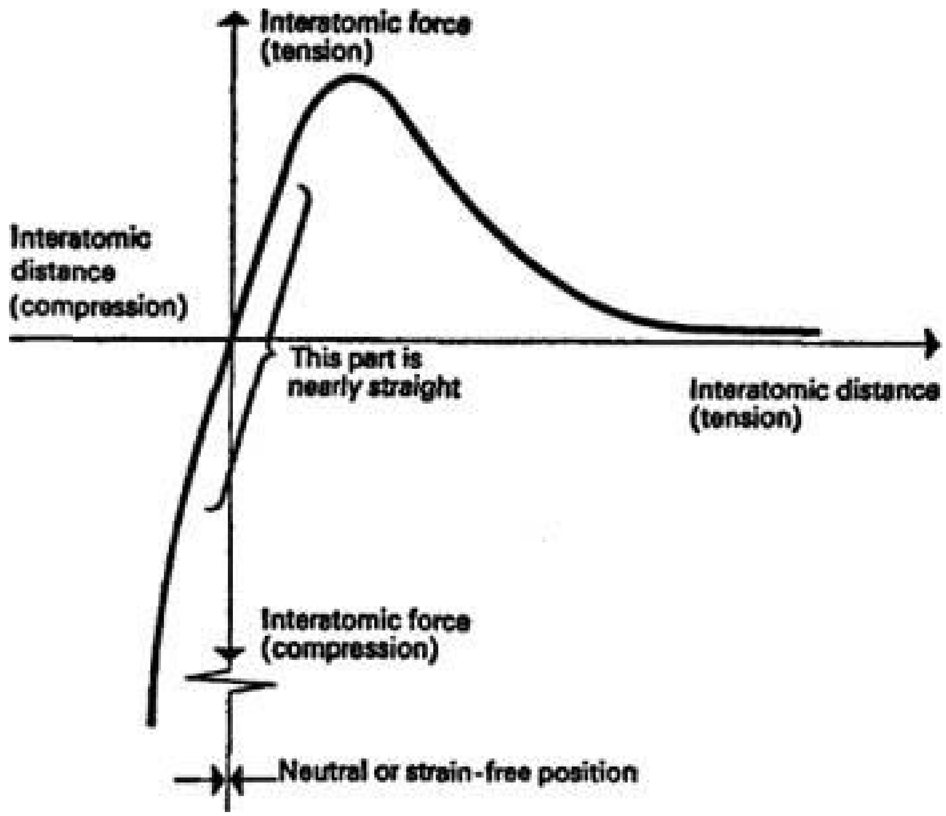


Figure 3. Relationship between the distance between two atoms and force between them.

In Hooke's day, and indeed down to the last few years, materials either broke or else flowed and ceased to be elastic when strains much over 1·0 per cent were applied to them. So the shape of the interatomic force curve at large deflections was only of the most academic interest because such stresses were never reached. Fairly recently, in the writer's laboratory and elsewhere, it has been possible to take very strong 'whisker' crystals up to strains between 3 and 6 per cent and the measurements confirm that Hooke's law is not literally true. The stress-strain curve bends over to follow the interatomic force curve which is derived from considerations, not of engineering, but of theoretical physics. Figure 4 shows such a curve for a silicon whisker strained to over 3 per cent.

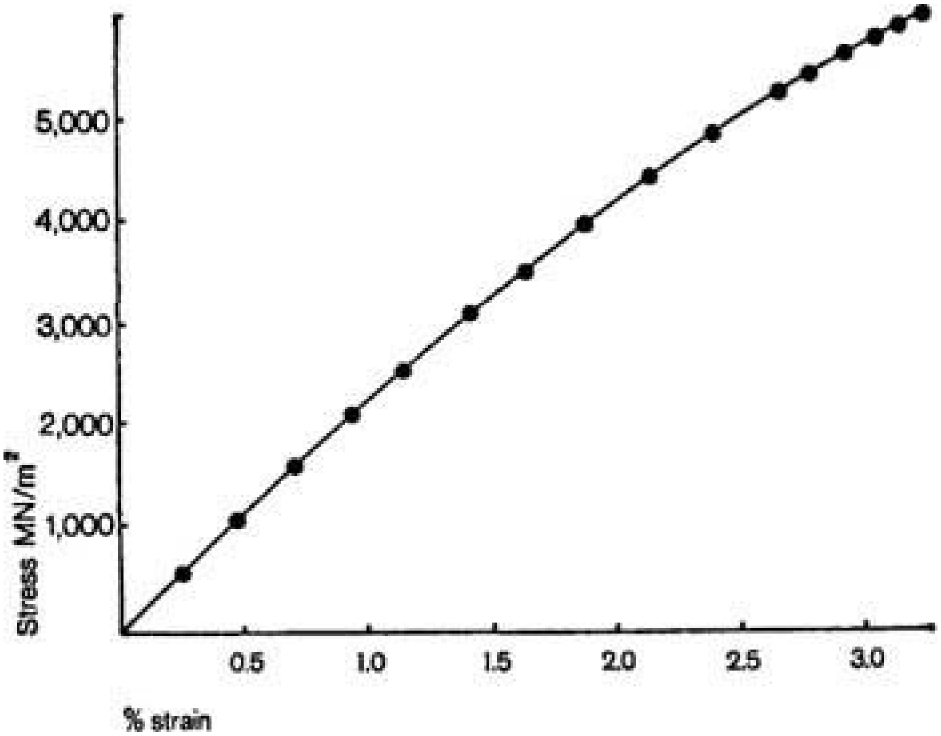


Figure 4. Stress-strain relationship for a very strong silicon whisker. This whisker or needle-like crystal was strained to 3·6 per cent in a testing machine and although the behaviour is ‘elastic’ it does not obey Hooke’s law at the higher strains, the top of the graph being distinctly curved. This is because the interatomic force relationship is also curved at the higher strains. Other strong filaments, such as iron whiskers, have similar non-linear stress-strain curves at high stresses.

Young’s modulus

Hooke stated that the deflections of springs and other elastic bodies were proportionate to the load which is applied to them but, of course, with different structures, the actual deflection under any given load will depend both upon the geometrical size and shape of the structure and also upon the material from which it is made. It is not clear how far Hooke distinguished elasticity as a property of a *material* from elasticity as a function of the *shape and dimensions* of the structure. We can get similar load-extension curves from a straight piece of rubber and from a helical piece of steel which we call a spring – this has always been a fruitful source of confusion. Certainly for something like a century after Hooke’s time a state of intellectual muddle seems to have invested the few people who thought about elasticity and no clear distinction seems to have been made between these ideas.

Around 1800 Thomas Young (1773–1829) realized that, if we consider the stresses and strains in the material rather than the gross deflections of the structure, then Hooke’s law can be written:

$$\frac{\text{stress}}{\text{strain}} = \frac{s}{e} = \text{constant}$$

Furthermore, Young realized that there was here a constant peculiarly characteristic of each chemical substance which, as he might have said, represents its ‘springiness’. We call this constant ‘Young’s modulus’ or *E*. There is no mystery about the word ‘modulus’, it just means a figure which describes a property of a material. Thus:

$$E = \frac{s}{e} = \frac{\text{stress}}{\text{strain}}$$

E therefore describes the elastic flexibility of a material as such; the flexibility of any given object will thus depend both upon the Young's modulus of the material from which it is made and also upon its geometrical shape.

It is said of Young that he was 'a man of great learning but unfortunately he never even began to realize the limitations of comprehension of ordinary minds'.[*] Young published the idea of his modulus in a rather incomprehensible paper in 1807 after he had been dismissed from his lectureship at the Royal Institution for not being sufficiently practical. Thus perhaps the most famous and the most useful of all concepts in engineering, which defines the stiffness or floppiness of a material, was not generally understood or absorbed into engineering practice until after Young's death. Young's modulus is often called 'stiffness' in casual engineering conversation and will sometimes be called stiffness and sometimes *E* in this book.

E is enormously important in engineering for two reasons. First, we need to know with accuracy the deflections in a structure, as a whole and in its various parts, when it is loaded. A moment's thought about bridges or aeroplanes or crankshafts will show that this is so (Figure 5). Things must still fit together, or have the proper clearances, when the load is on.[†] A knowledge of the *E* of the material being used is the first thing we need to know in making these calculations. Secondly, although the layman might suppose, as the early engineers seem to have done, that the stiffness of all common structural materials were very similar ('Well, it's stiff, isn't it, you can't *see* any deflections'), this is in fact very far from being the case and we not only need to know the *E*s of various materials such as wood and steel in order to calculate their deflections, but we must also arrange that the deflections of differing materials in a structure are compatible and that they share the load in the way we want them to.

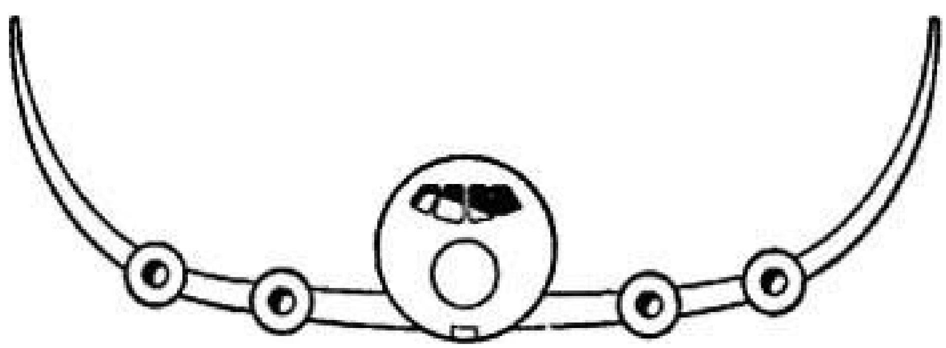


Figure 5. Aircraft with strain of 1·6 per cent in wing spar booms.

Since, if we divide stress by a ratio – that is by a number without dimensions[*] – we must still have a stress, Young's modulus is therefore a stress in pounds per square inch, or what you will. It is that stress which would in theory double the length of a specimen, if it did not break first. One can also regard it as the stress to produce 100 per cent strain. As it will easily be imagined, the actual figure is likely to be a high one, usually at least a hundred times larger than the breaking stress of the material, because, as we have said, materials are apt to fracture in the ordinary way at 1 per cent elastic strain or less. The Young's modulus of steel, for example, is about 30,000,000 pounds per square inch. As we have also said, *E* varies very much according to the kind of chemical substance we are dealing with. A few typical figures are shown on page 42.

Thus the whole range of solids vary in E by about 200,000 to 1. Even substances which we normally think of as 'rigid' vary by about 1000 to 1, which is still an enormous range. E is very low in rubber because rubber is made of long molecular chains which are flexible and in the resting material they are generally much bent, kinked and convoluted, like a heap of bits of string such as one finds in a drawer in the hall at home. When rubber is stretched, the bent chains are straightened and, as one can easily see, the force needed to do so is very much less than that which is needed to stretch an arrangement of strings which were initially straight. Nothing of this kind happens in a normal crystal where one is pulling directly on the interatomic bonds and the only reason for the large variations in Young's modulus is that the chemical bonds themselves vary a great deal in stiffness. So with crystals, although the general shape of the interatomic force curves is similar, the slope of the straight part of the curves varies greatly according to the bond energy and other chemical conditions.

Approximate Young's moduli of various substances

	E Pounds per square inch	E MN/m^2
Rubber	$0{\cdot}001 \times 10^6$(i.e. 1,000)	7
Unreinforced plastics	$0{\cdot}2 \times 10^6$	1,400
Organic molecular crystal, phthalocyanine, a blue pigment	$0{\cdot}2 \times 10^6$	1,400
Wood (about)	$2{\cdot}0 \times 10^6$	14,000
Concrete	$2{\cdot}5 \times 10^6$	17,000
Bone	$3{\cdot}0 \times 10^6$	21,000
Magnesium metal	$6{\cdot}0 \times 10^6$	42,000
Ordinary glasses	$10{\cdot}0 \times 10^6$	70,000
Aluminium	$10{\cdot}5 \times 10^6$	73,000
Steel	$30{\cdot}0 \times 10^6$	210,000
Aluminium oxide (sapphire)	$60{\cdot}0 \times 10^6$	420,000
Diamond	$170{\cdot}0 \times 10^6$	1,200,000

Note. Because the interatomic force curve (Figure 3) passes smoothly through the point of zero stress and strain the true E of a material is always the same in compression as it is in tension at all normal strains, if this were not so then the mathematics of elasticity would be even more complicated than they are. In practice, however, materials such as cast iron and cement, which contain quite gross internal cracks, may sometimes show an E which is lower in tension than it is in compression. This is simply because the cracks gape under tension and' come up solid' under compression.

The figure for the E of phthalocyanine tells us at once why a great many solid chemical compounds are not candidates for the status of structural materials. Generally speaking we want a structure to be as rigid as possible: bridges and buildings sway quite enough as it is and there are excellent reasons for making other things rigid as well. Any structure made from a material with a stiffness as low as phthalocyanine would be far too floppy. Steel is about the stiffest reasonably cheap material, which is one of the reasons why it is used so widely. As much as anything it is the relatively low stiffness of plastics, even when 'reinforced', which restricts their use for large objects.

Strength

Next to 'heat-proof' I suppose that 'unbreakable' is one of the most useful words in advertising. Although most of us know that advertising is not an entirely objective profession, somehow or other the message sinks in so that one still meets people who really believe that there are unbreakable objects or, if there aren't, then there ought to be. Since there is always some force which will tear the atoms apart in a solid (since the chemical bonds have a finite energy or, in other words, they are only so strong) nothing is unbreakable. You have only to get hold of the thing firmly and pull hard enough and it will break. The only question is 'how soon?' There is however a very large variation between the strengths of various materials.

Lest there be any possible, probable, shadow of doubt, strength is not, repeat not, the same thing as stiffness. Stiffness, Young's modulus or E, *is concerned with how stiff, flexible, springy or floppy a material is. Strength is the force or stress needed to break a thing. A biscuit is stiff but weak, steel is stiff and strong, nylon is flexible (low* E) *and strong, raspberry jelly is flexible (low* E) *and weak. The two properties together describe a solid about as well as you can reasonably expect two figures to do.*

It is easiest to think about strength in terms of tensile strength. This is the stress needed to pull a material asunder by breaking all the bonds between the atoms along the line of fracture. One can perhaps most conveniently think of it as the stress required to break a bar by pulling it along its axis like a rope. A very strong steel may withstand a tensile stress of 450,000 pounds (200 tons) per square inch (3,000 MN/m^2), while ordinary brick or cement may perhaps withstand 600 or 800 p.s.i. or only 4 or 5 MN/m^2.[*] The strength of commonly used engineering materials thus varies over a range of about a thousand to one[†]. The tensile strengths of some common materials are given in the table.

Some typical tensile strengths in round figures

	p.s.i.	MN/m^2
METALS		
Steels		
Steel piano wire (very brittle)	450,000	3,000
High tensile engineering steel	225,000	1,500
Commercial mild steel	60,000	400
Wrought iron		
Traditional	20,000–40,000	140–280
Cast iron		
Traditional	10,000–20,000	70–140

Modern	20,000–40,000	140–280
Other metals		
Aluminium		
cast, pure	10,000	70
alloys	20,000–80,000	140–550
Copper	20,000	140
Brasses	18,000–60,000	120–400
Magnesium alloys	30,000–40,000	200–280
Titanium alloys	100,000–200,000	700–1,400
NON-METALS		
Wood, spruce		
along grain	15,000	100
across grain	500	3
Glass (window or beer-mug)	5,000–25,000	30–170
Good ceramics	5,000–50,000	30–340
Ordinary brick	800	5
Cement and concrete	600	4
Flax	100,000	700
Cotton	50,000	350
Catgut	50,000	350
Silk	50,000	350
Spider’s thread	35,000	240
Tendon	15,000	100
Hemp rope	12,000	80
Leather	6,000	40
Bone	20,000	140

When we talk about ‘strength’ we usually mean tensile strength although materials are more often used in compression than they are in tension. At first sight it is not very easy to see why a material should ever want to break at all in compression. After all, if one is pressing the atoms closer together, why should they come apart? Compressive failure is more complicated than tensile failure, especially as there are several different ways in which a material can run away from a compressive load.

If the material is in the form of a fairly short, squat column, or chock or wedge or something of the sort, then, if the material is at all soft or ductile like mild steel or copper, it will simply squish out sideways, like Plasticine. If the material is brittle, like stone or glass, it will explode sideways (and very dangerous it can be) into dust and splinters. However, if the specimen is long and slender, like a thin rod or panel, then it may fail by 'buckling' such as happens when you lean too hard on a walking stick which bends and ultimately snaps in two. If you put too much weight on a tin can, as by driving a car over it, it will crumple in the same sort of way. This is the kind of failure which is apt to happen to shell structures such as steel ships and metal aeroplanes when they hit things, not to mention the wings of motor cars. For these reasons it is not easy to quote figures in tables for '*the* compressive strength of so and so'. Broadly speaking there isn't one or at least it must be estimated with knowledge and experience. This is one of the reasons why structural engineering isn't particularly easy.

There is no general relationship between the tensile and compressive strengths of various materials and structures, partly because the distinction between a material and a structure is never very clear. For instance, a pile of bricks is strong in compression but has no tensile strength at all. A pile of bricks is undoubtedly a structure and not a material, but then cast iron, cement, plaster and masonry are much stronger in compression than they are in tension and for much the same reason as a pile of bricks: they are full of cracks. Chains and ropes are strong in tension and have no compressive strength because they fold up in compression. They are probably structures not materials. Wood is three or four times as strong in tension as it is in compression because the cell walls fold up in compression, yet wood is thought of as a material not a structure.

Tension and compression structures

For a great many centuries engineers and architects avoided using materials in tension as much as they could. This was not so much because they had no materials strong in tension – wood, for example, is excellent – but rather because of the difficulty of making reliable strong joints to withstand tension. Most of us intuitively feel that a compression structure is safer than a tension one; that a brick tower is safer than an aerial cableway, for instance. In the old days, when tension joints were unavoidable, as they were in ships, they were a perpetual source of trouble. Now that we can make good joints with bolts or rivets, glues or welding, there is no special justification for distrusting a tension structure.

However, in a primitive technology, the problem of a compression joint is very much easier and in the simplest case resolves itself into merely heaping one stone or brick upon another in such a way that the house does not fall down. Dry-walling is a skilled job but not much more so than doing a jigsaw puzzle. As architects became more ambitious and walls higher, it was necessary to arrange firmer, better fitting joints lest the wall slide down with a rumble into a heap of stones. If the stones do not fit each other reasonably well they will roll over each other like a pile of balls and be pushed outward under the superincumbent weight, just as, on a finer scale, Plasticine is pushed outwards. In this way we get magnificently fitted joints between large blocks of stone in ancient buildings. How much of this laborious accuracy was born of engineering necessity and how much of a morbid desire for prestige on the part of men or gods is arguable. Many of these buildings, like a famous car, strike one as being 'a triumph of workmanship over design'.

However high and impressive a wall may be, it remains technologically a very unsophisticated structure because the designer is really only thinking about stresses in one dimension; that is to say vertically. He is always in a difficulty about bridging roofs, doors and other openings. Once he starts thinking imaginatively of stress systems in two and three dimensions, all kinds of possibilities open up, even if he is still restricted to compressive systems. This is why the arch is important. The ordinary simple arch utilizes compression in two directions simultaneously to bridge a gap (Figure 6).

This is an apparent impossibility which works extremely well. A masonry arch can span 200 feet (60 metres) or so (though 100–200 feet is more common) without much difficulty. This is a very much greater span than any primitive beam or architrave or lintel could bridge. An arch is also durable and there are innumerable Roman arches, such as the aqueducts, in excellent condition today.

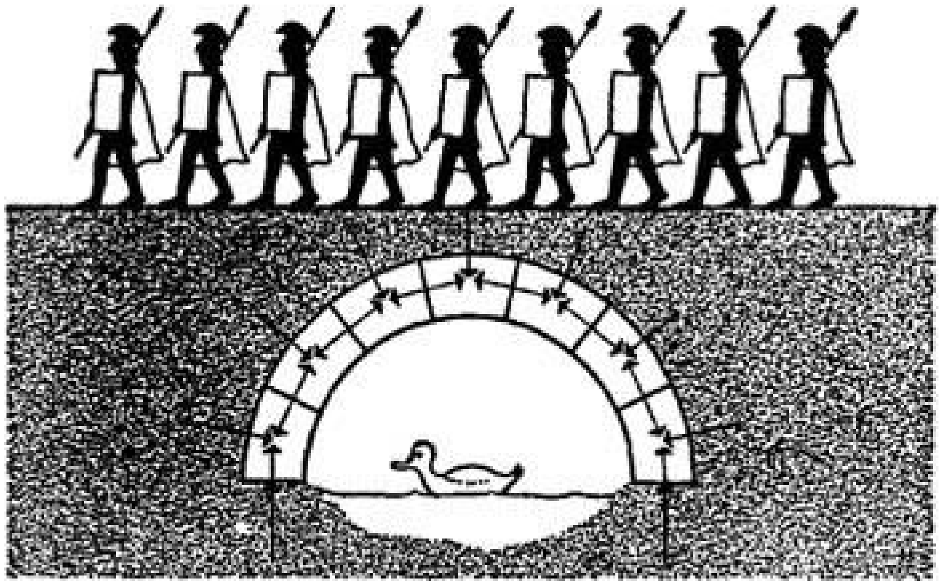


Figure 6. The arch – a two-dimensional compression structure – enables vertical forces to be transmitted laterally around the arch-ring into the abutments. (The wedge-shaped pieces which make up the arch-ring are called 'voussoirs'.)

The trick of thinking in terms of stresses acting in more than one direction simultaneously is really the key to most advanced architecture and engineering. Once one accepts the two-dimensional concept of the arch or the three-dimensional concept of the dome which is the next logical step, then one can start playing elaborate architectural games. St Sophia, built by Justinian at Constantinople about 530 A.D., is a great dome 107 feet (32 metres) in diameter and made of pumice bricks for lightness, poised upon four great arches which are propped in turn by auxiliary half domes (Figure 7). The result was a nave, completely clear of pillars, measuring more than 200 feet by 100 feet (60 metres x 30 metres) and about 240 feet (72 metres) high, a clear roofed area greater, probably, than any achieved until the advent of the modern railway station which is roofed with steel trusses. The shapes of Byzantine architecture are usually simple but the Gothic architects ran riot in aisles, fan-vaulting and clerestories. All this, though rather expensive, is great technical and artistic fun as long as you know what you are doing. The essential thing about a masonry structure is that it must be a compression structure everywhere, because masonry is incapable of resisting tensions since the stones will come apart at the joints.

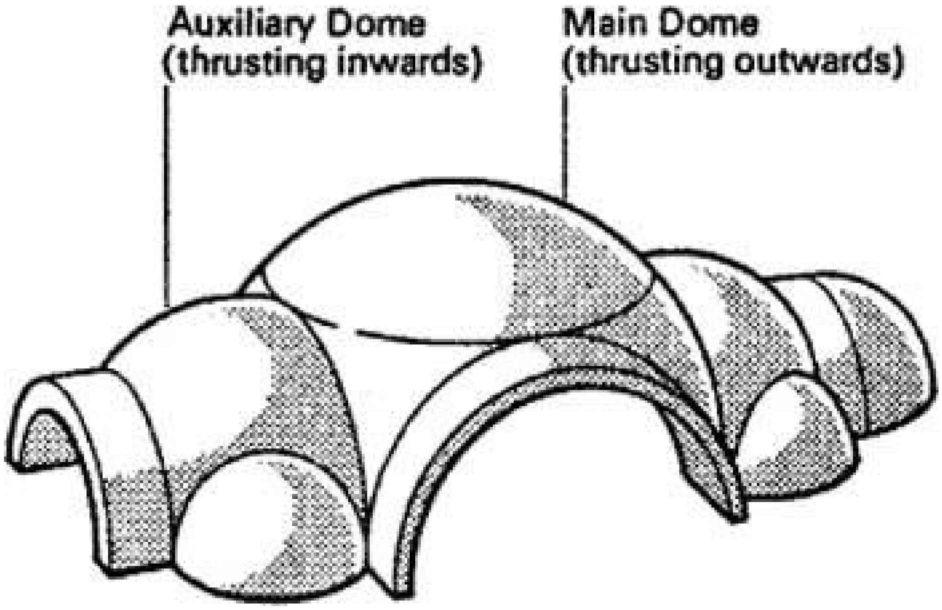


Figure 7. How the outward thrusts of the main dome of St Sophia at Constantinople are supported by means of subsidiary domes and vaults.

In the three-dimensional labyrinth of a cathedral roof, where thrust chases thrust in Gothic disregard of mathematics, strange things were apt to happen. Tensions crept in, like devils among the gargoyles. In one of the greatest of the Gothic cathedrals, Beauvais (1247), the tower fell once and the roof fell twice. Contemporary architects knew what was wrong in a qualitative sort of way and they propped their structures up with a maze of flying buttresses (Figure 8), just as St Sophia is set about, in a more rational and successful way, with auxiliary domes which thrust inwards and maintain a state of compression in the critical regions. Sometimes the Gothic architects overdid the business of inward thrusting and had to strut their naves internally to prevent the building collapsing inwards. This strutting was sometimes done by inserting inverted arches, as at Wells Cathedral (Plate 1) which, whatever may be thought of it aesthetically, is a mess structurally. It is not surprising that the roofs of churches continued to fall upon the heads of their congregations with fair regularity throughout the ages of faith.

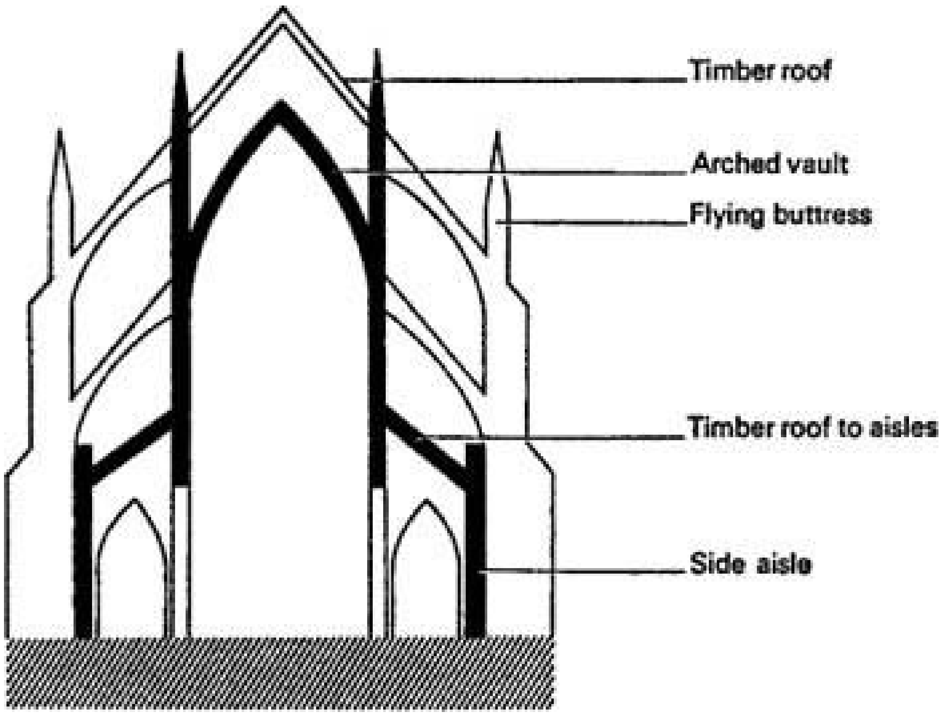


Figure 8. In a 'Gothic' structure the outward thrust of the roof is taken mainly by buttresses.

A masonry structure is kept together by gravity; that is, if it is properly designed, the weight of the stones keeps everything safely in compression. If necessary one can pile on pinnacles and towers to get more weight in the right place. Once we start dealing with tension, however, or mixed tension and compression structures, we have to accept that the tensions and compressions must balance out, taking into account the weights of the various parts, of course. In a suspension bridge the cables are maintained in tension by a corresponding horizontal compression in the ground beneath the bridge (Figure 9). In a tent the tension in the canvas and in the guys is reacted vertically by the tent pole and horizontally by the ground on which the tent is pitched. In a sailing ship the tension in the sails and in the standing and running rigging is reacted by compressions in the masts and spars. In an animal the bones, and especially the backbone, are chiefly compressive members reacting, not only the weight of the animal, but also the tensions in the muscles and tendons. I raise my arm by shortening a muscle, that is to say by pulling on it, and this puts the bone into compression – which is what bones are usually designed to take. Getting one's leg into bending, which involves tensions, is the easiest way to break it.

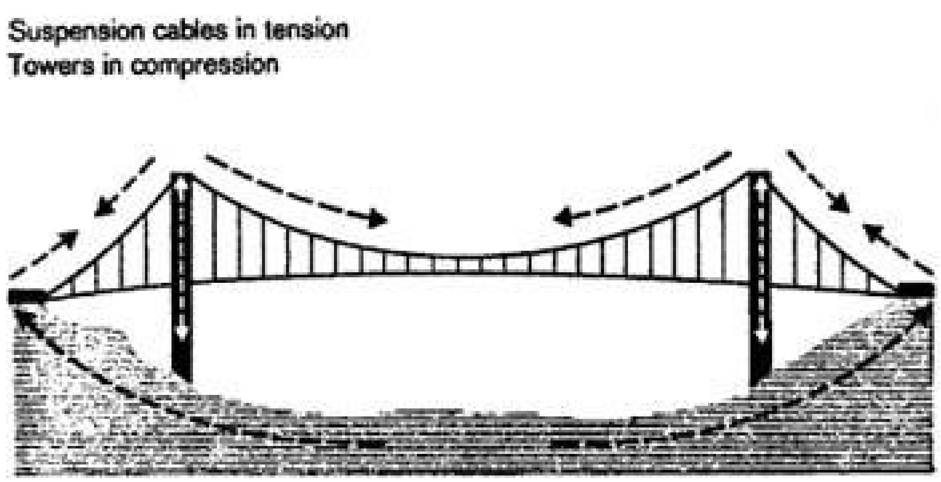


Figure 9. The tension in the cables is reacted by a corresponding compression in the ground beneath.

It is a great convenience and a great source of safety to be able to pass, as it were, from compression into tension and back again, either deliberately or by accident. In architecture this is one of the arguments for reinforced concrete and for steel-framed buildings, both materials being strong in tension and compression. It is also one of the reasons why iron and steel are such a godsend to engineers, putting a cloak over their ignorances and uncertainties. Boilers, for instance, are tension structures which may occasionally get into compression (if you let the fire out you can drive an engine under the negative pressure, that is the vacuum in the boiler) without anything very dangerous happening.

The compressive stresses in a submarine can lead to some rather interesting and unexpected effects of strains, which have to be thought about and guarded against. When a submarine is on the surface she floats, like any other ship, because the weight of the submarine is less than the total weight of water which would be displaced if the vessel were totally submerged. If, for any reason, the hull sinks a little in the water, a greater volume is immersed and the extra buoyancy pushes the vessel up again. When a submarine dives, she fills her ballast tanks with water until her total weight just about equals her submerged displacement and so she has no reserve of buoyancy. In this condition she can dive and manoeuvre under water in much the same way as an airship does in the air. However, as the submarine dives deeper, the water pressure increases and the hull is put under more and more compression. Because the air inside the hull is not under pressure the steel in the hull can only resist this compression by contracting. So the volume of the hull, and thus its displacement, is reduced, although the weight of the submarine and her ballast water is, naturally, not changed. There is therefore a tendency for the submarine to sink further, or to become apparently heavier, the deeper she goes – and in certain circumstances this can be dangerous.

At the safe limit of diving depth the compressive strain in the hull plates might be about 0·7 per cent. Since this strain occurs in all three directions the hull may shrink by about 2 per cent in volume. As water is only very slightly compressed, this may represent a loss of about 20 tons or so of buoyancy for a 1,000-ton submarine.[*] If this weight cannot be counteracted by blowing the ballast tanks or working the hydrofoils, the submarine will sink deeper and deeper until she is crushed by the water pressure in the depths of the ocean. This is one of the difficulties about making submarines out of reinforced plastics, such as fibreglass, which are otherwise rather attractive, but which have low Young's moduli. It is nonsense to think, as used sometimes to be said, that sinking submarines and wrecks will float somewhere short of the bottom of the sea. Even if the pressure hulls and compartments which contain air do not actually burst inwards, which must usually happen, they will progressively contract and lose buoyancy and so the wreck sinks faster and faster.

Balloons, pneumatic tyres and the like are a special case of a tension structure where the tension in the skin is reacted by the pressure of the gas or liquid inside. In this way Dracones (large bag-like barges for conveying liquids) and pneumatic boats are usually very light and efficient structures. The air-supported roof – a building held up entirely by internal pressure – reverses architectural tradition in that everything except the air inside is in tension. Since only a very small air pressure is needed, a modest electric blower keeps everything taut and even supports any reasonable snow load for less expense than the capital charges of a conventional building. Plants and animals use the osmotic pressure of their internal fluids in a similar way.

Beams and bending

It is quite easy to see how tension and compression structures work but it is not at all self-evident how the tensions and compressions which we have been discussing really support a load on a beam. This is a pity, since beams of one kind or another (Figure 10) make up a high proportion of everyday structures. The ordinary floorboard is as good an example as any other of a simple beam. As we said earlier it is the function of a floorboard to press upwards on the soles of our feet with a thrust equal to our own weight. Yet it must perform this function even when we are standing in the middle of the floor and the walls which eventually support the plank are remote. Exactly how does this thrust get from the wall to our feet and vice versa?

The answer to this question is known as 'beam theory' to engineers and is more or less the backbone of engineering. Unfortunately it is less of a backbone than a *pons asinorum* to engineering students. Most students merely learn the formulae of beam theory off by heart and regurgitate them at examination times; understanding only comes much later when they have to struggle with designing something. We shall therefore leave out all that stuff about integrating the shearing force diagram and try to tackle the problem by the light of nature. Once beam theory is understood the world becomes a more lucid and an altogether better place, so take courage.

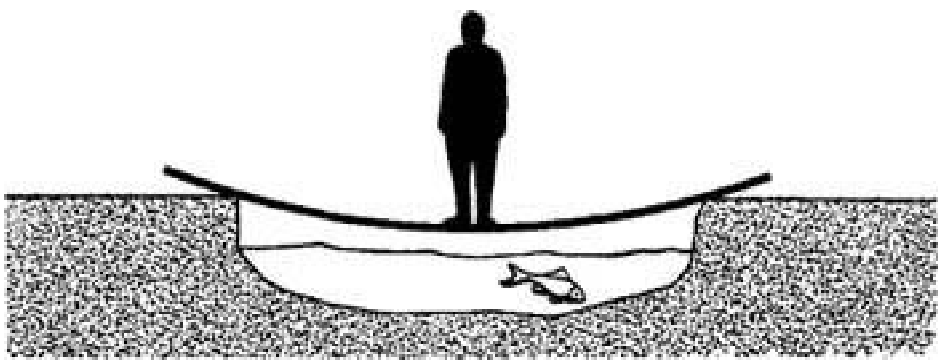


Figure 10. Simply supported beam.

To understand about beams it is perhaps easiest to return to the idea that there is no very clear distinction between a material and a structure. Large beams are often fabricated, Meccano fashion, from many small tension and compression rods, as anyone can see who looks at a railway bridge. Yet the means by which the load is transmitted in such a lattice beam or girder is not different in kind from the means by which it is transmitted in a solid beam, even so humble a one as a plank or floorboard. In the lattice structure we can generally see the individual members which resist all the pushes and pulls – all of them, since of course no load can get across the empty spaces between the lattice-work. In a solid beam we have to consider the lattice members as diffused throughout the beam, but the stresses are working in the same way.

We might as well start with a cantilever, a beam one end of which is built into a wall or otherwise fixed to a firm base (what engineers call *encastré)* while it is loaded in some way on the projecting part. Galileo's picture (Plate 2) of a cantilever will serve as well as any other although Galileo, rather excusably, got his cantilever sums wrong. Let us, however, build up a cantilever entirely by tension in wires and rods.

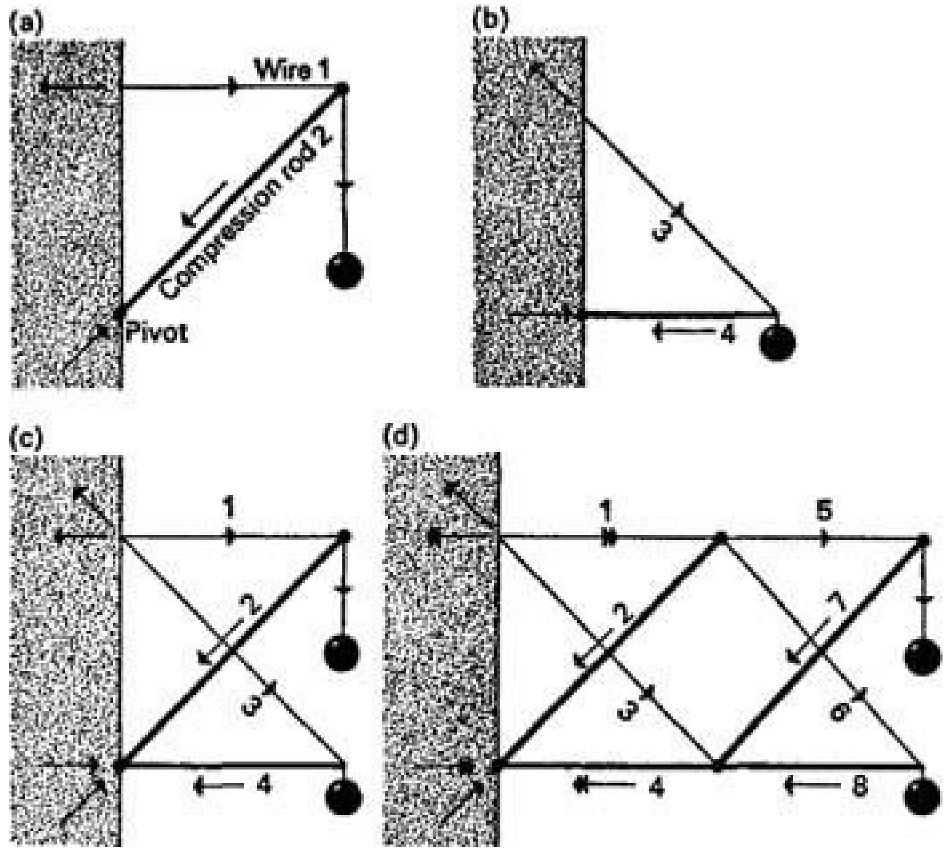
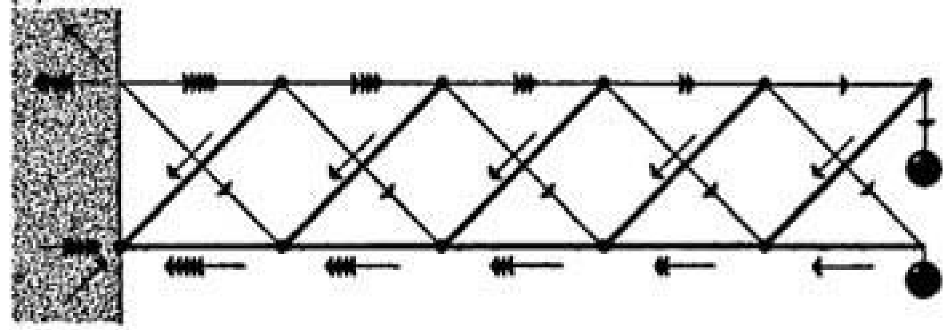


Figure 11. Beam theory – a beam may be considered as made up from a number of separate panels.

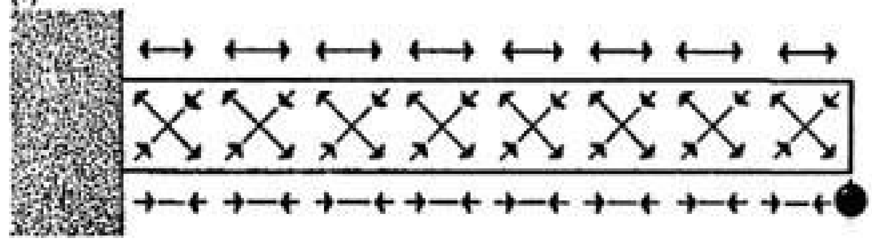
Consider the simple crane-like structure in Figure 11(*a*). A compression rod (2) is pivoted against a firm wall and is supported by a wire or tension member (1), so that it can carry at its outer end a load, *W,* say. Now it is clear that the push against gravity which is actually supporting the load *W* is generated from the compression in the sloping rod (2). The tension in the wire (1) acts horizontally and only prevents the compression rod (2) from rotating and falling down.

Now we might equally support the weight *W* from another triangular structure such as *(b)* in which the compression rod (4) was horizontal and was kept from falling down by the diagonal tension wire (3). In this case the upthrust to support the weight *W* comes from the slanting wire (3) and all the horizontal compression rod does is to prevent the wire from collapsing inwards onto the wall.

(e)



(f)



But the stress system in a solid cantilever is not very different from that in a lattice truss.

(g)

And, of course, in order to support the weight, the material must be strained and so the cantilever will droop.

These two structures are each as good as the other and we might combine the two to support *2W,* as in (c). Clearly the weight 2*W* is directly supported by the two slanting members (2) and (3), one in tension and the other in compression. The horizontal members (1) and (4) pull and push on the wall to prevent the whole structure swinging downwards but their thrusts do not directly sustain the weight.

We can now repeat this structure by duplicating (*c*) so as to get (*d*). Here we have a lattice girder with two panels. In this case the same load 2*W* is again actually supported by the tensions and compressions in the slanting members (2), (3), (6), and (7) while (1), (5), (4), and (8) pull and push horizontally, and though they do not directly support the load, they keep the whole girder from collapsing; indeed every member performs an essential, though different, function and the failure of any of the eight members would be catastrophic.

Notice the way the loads are building up in our simple girder. The outer panel in *(d)* is in all respects similar to the single panel in (c). Consider, however, the inner panel in *(d);* that is the one next the wall. A little consideration will show that the tension in the wire (1) is greater that in (5) and in the same way the compression in (4) is greater that in (8). This is because the diagonal or 'shear' members are feeding load in progressively towards the root of the cantilever. In the shear or slanting members, however, the loads are the same in each panel, however long the girder may be.

We can go on and build up a long girder of many panels like *(e)* and here again, if we look at the lattice intelligently, it is obvious that the load in all the diagonal shear members is constant along the length of the girder, however many panels there may be. On the other hand, the tensions and compressions in the top and bottom horizontal members (called the booms or flanges of the girder) are building up and increasing as we move from the loaded tip to the built-in root of the girder, in fact in proportion to its length. For this reason a cantilever will usually break in its most highly stressed members – the horizontal members which are up against the wall – unless we have gone to the trouble of making the thickness of each part proportional to the load which it has to carry. In such a case the lattice may break anywhere, which is an ideal state of affairs and the aim of most stress calculations. When this occurs all the material is equally stressed (like the one-horse shay) and the material is used in the most efficient way. Hence the least quantity of material can be used and the lightest structure will result.

If we now convert our lattice or Meccano girder to a simple continuous beam we shall get a stress system like (*f*). The middle of the beam is mostly occupied with resisting shear, which turns out to be another name for tensions and compressions at forty-five degrees to the axis. These shear stresses are of constant magnitude for the whole length of the beam. The material near the top and bottom surfaces is concerned with resisting the tensions and compressions which the shear stresses have generated. These horizontal stresses near the surfaces build up rapidly and in the worst place are usually far greater than the shears. They are the stresses which are most liable to break structures and kill people. Stressing is not a dry academic exercise of interest only to experts but an affair affecting the safety and pockets of most of us.

If all this stress-chasing seems confusing the best thing to do is to make a model out of Meccano or drinking straws joined with ordinary pins. If one makes a lattice model in this way it is quite easy to understand what it is in a cantilever which actually keeps the load from falling down. Of course, in all this rather complicated pattern of stresses, each stress is only gained at the expense of a proportionate strain and so the cantilever does not stick out rigidly but inevitably droops *(g)* to a greater or less extent.

Cantilevers are common enough in engineering but ordinary beams, particularly those that engineers call 'simply supported', are commoner still (Figure 10). This is the sort of thing you get when you put a plank across any simple gap such as a stream. How is this related to the cantilever? The answer is really quite obvious from Figure 12. The simply supported beam is really two cantilevers turned back to back and upside down. While the biggest stresses in the cantilever are near the root, those in a simply supported beam are in the middle and so such a beam will generally break in the middle.

We can now see that the reason why we don't fall through the floor is that the floor boards and joists produce tensions and compressions at forty-five degrees to the surface of the floor and these stresses, elaborately zig-zagging all the way from my shoes to the skirting-board, provide the upward sustenance which I need. As well as these shears, and much larger in magnitude, there are tensions and compressions horizontally near the top and bottom faces of the floor boards. If these horizontal stresses become too large, either because I am too heavy or the floor boards are too thin we shall first get alarming deflection in the floor and finally it will break.

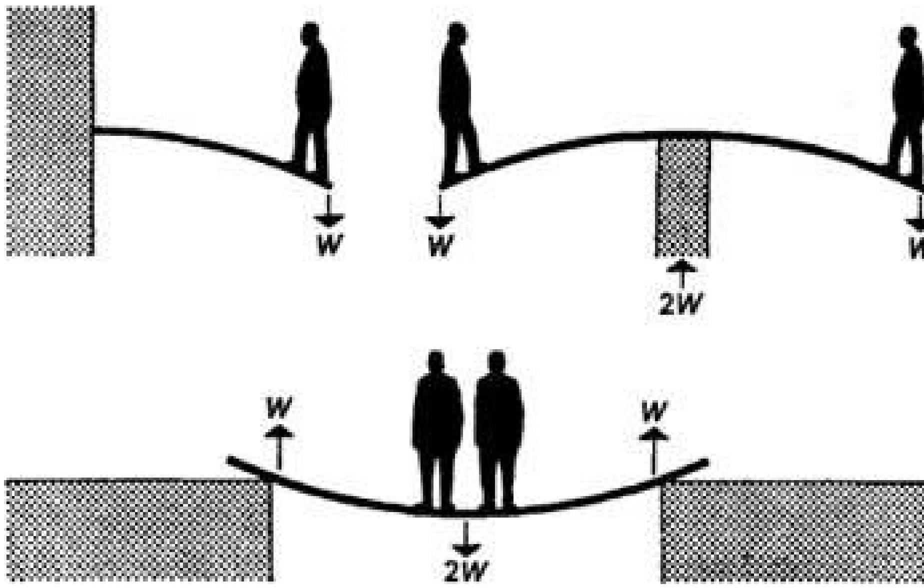


Figure 12. A simply supported beam may be considered as two cantilevers back to back and upside down.

The simplest experiment will show that the stresses and deflections induced by bending are, other things being equal, much more severe than those caused by direct tension and compression. If we take a piece of fairly thin wooden plank or rod in our hands, it is quite impossible to break it in tension by pulling on it by hand and the deflections we can cause by hand tension or compression are far too small to see by eye. If we bend the rod, however, we can nearly always produce quite a big and obvious deflection and in many cases it is quite easy to break it. For these sorts of reasons, although beams are extremely convenient, we nearly always have to be careful that they are strong enough and don't produce excessive deflections. The strengths and deflections of a given beam can be calculated by anyone with a knowledge of elementary algebra from the standard formulae which are given in the appendix at the end of this book.

As we have said, all this is not particularly easy to understand but it is really no more difficult than, say, French verbs, and it can be comprehended by quite a moderate intellectual effort. Once this is done, a great deal of engineering becomes much clearer. The truth is that many professional engineers use very little more than elementary beam theory when they design quite ambitious structures. As we shall see this is apt to be dangerous because beam theory in itself, though extremely useful, does not really tell us all we need to know about the strength of a sophisticated structure. However, it is very widely used as a guide to the strength of all kinds of things from crankshafts to ships.

The deliberate and confident use of large beams in engineering is not much more than a century old. Telford (1757–1834) – the 'Colossus of roads', or 'Pontifex Maximus', as Southey called him – probably built more bridges than anyone else in history. He used masonry or cast-iron arches in compression and for the longer spans he pioneered the suspension bridge, using wrought-iron tension chains, notably in the Menai road bridge (1819). Telford hardly ever used large beams. This was partly because a suitable material, such as wrought-iron plates, was not easily available and partly because of the lack of a trustworthy beam theory. An interesting sidelight on the status of strength calculations in Telford's time is that the shape of the chain catenaries for the Menai bridge was determined not by calculation but by setting up a large model across a dry valley.

Working thirty years later, Robert Stephenson (1803–59) had large wrought-iron boiler plates available and he also had the courage of his calculations. He had the brilliant idea* of making a hollow box-like beam of iron plates and running the trains inside it. The idea found its best-known expression in the Menai railway bridge which was opened in 1850, almost alongside Telford's bridge. Stephenson's beams, which weighed 1,500 tons each, were built beside the Straits and were floated into position between the towers on rafts across a swirling tide. They were raised rather over a hundred feet up the towers by successive lifts with primitive hydraulic jacks. All this was not done without both apprehension and adventure; they were giants on the earth in those days.

At one time, when Stephenson's faith weakened, it was proposed to add suspension chains to help sustain the tubes, but this proved quite unnecessary. Both bridges, stood side by side until recently as elegant demonstrations of tension and bending upon the grand scale.[*] Telford's suspension bridge was lacking in stiffness at first and the bridge swayed alarmingly in the gales which blew down the Straits. There is an account of how one winter night the oscillations were such that the horses of the mail-coach could not keep their feet on the bridge and were thrown down in a dangerous tangle of hooves and harness so that the traces had to be cut by lantern light before the mess could be sorted out. After this the bridge was stiffened and now carries modern traffic.

The general lack of stiffness of suspension bridges made them unsuitable for railways, since the trains might have been rolled off the rails. This was why Stephenson and I. K. Brunei (1806–59) developed beam-like bridges for long spans. Though the Menai tubular bridge is splendidly stiff and has never given any trouble, equivalent modern beam bridges are generally lattice structures because lattices are easier to paint and 'To keep the Menai bridge from rust by boiling it in wine' was impracticable.

A ship is a long tube closed at both ends which happens to be afloat but is not otherwise structurally very different from Stephenson's Menai bridge. The support which the water gives to the hull does not necessarily coincide with the weights of engines, cargo and fuel which are put into the ship and so there is a tendency for the hull to bend. It ought to be impossible to break a ship, floating alongside a quay, by careless and uneven loading of the holds and tanks, but this has happened often enough and will probably happen again. In dry-dock ships are supported with care upon keel-blocks arranged to give even support but there is not much even support at sea where a ship may be picked up by rude waves at each end, leaving her heavy middle unsustained, or else exposing a naked forefoot and propeller at the same moment.

As ships tended to get longer and more lightly built, the Admiralty decided to make some practical experiments on the strength of ships. In 1903 a destroyer, H.M.S. *Wolf,* was specially prepared for the purpose. The ship was put into dry-dock and the water was pumped out while she was supported, in succession, amidships and at the ends. The stresses in various parts of the hull were measured with strain-gauges, which are sensitive means of measuring changes of length, and therefore of strain, in a material. The ship was then taken to sea to look for bad weather. It does not require very much imagination to visualize the observers, struggling with seasickness and with the old-fashioned temperamental strain-gauges, wedged into Plutonic compartments in the bottom of the ship, which was put through a sea which was described in the official report as 'rough and especially steep with much force and vigour'. Her captain seems to have given the *Wolf as* bad a time as he could manage but, whatever they did, no stress greater than about 12,000 p.s.i. or 80 MN/m^2 could be found in the ship's hull.

As the tensile strength of the steel used in ships was about 60,000 p.s.i. or 400 MN/m^2, and no stress anywhere near this figure could be measured, either at sea or during the bending trials in dry-dock, not only the Admiralty Constructors but Naval Architects in general concluded that the methods of calculating the strength of ships by simple beam theory, which had become standardized, were satisfactory and ensured an ample margin of safety. Sometimes nobody is quite as blind as the expert.

Ships continued to break from time to time. A 300-foot (90 metres) ore-carrying steamer, for instance, broke in two and sank in a storm on one of the Great Lakes of America. The maximum calculated stress under the probable conditions was not more than a third of the breaking stress of the ship's material. Even when major disasters did not actually happen, cracks appeared around hatchways and other openings in the hull and decks.* These openings are of course the key to the problem. Stephenson's tubular bridge was eminently satisfactory because it is a continuous shell with no holes in it except the rivet holes. Ships have hatchways and all sorts of other openings. Naval Architects are not especially stupid and they made due allowance for the material which was cut away at the openings, increasing the calculated stresses around the holes *pro rata.* Professor Inglis, in a famous paper in 1913, showed however that *'pro rata'* was not good enough and he introduced the concept of 'stress-concentration' which, as we shall see (Chapter 4), is of vital importance both in calculating the strength of structures and in understanding materials.

What Inglis was saying was that if we remove, say, a third of the cross-section of a member by cutting a hole in it then the stress at the edge of the hole is not $\frac{3}{2}$ (or 1·5) of the average but it may, locally, be many times as high. The amount by which the stress is raised above the average by the hole – the stress-concentration factor – depends both upon the shape of the hole and upon the material, being worst for sharp re-entrants and for brittle materials. This conclusion, which Inglis arrived at by mathematical analysis, was regarded with the usual lack of respect by that curiously impractical tribe who call themselves 'practical men'. This was largely because mild steel is, of all materials, perhaps the least susceptible to the effects of stress concentrations though it is by no means impervious (Plate 3). It is significant that, in the *Wolf* experiments, none of the strain gauges seems to have been put close to the edge of any important opening in the hull.

Note: problems of walls, arches, beams and so on are dealt with more fully in the author's *Structures,* Penguin Books, 1978.

Chapter 3 Cohesion
or how strong ought materials to be?

'Again, the things that we see to be hard and dense must needs consist of particles more mutually hooked and must be deeply held compact by branch-like elements. In this class, for example, stands adamantine rock, accustomed to laugh blows to scorn, and stalwart flint, and the hard strength of iron, and the copper bolts that scream as they resist their rooves.'[*]

Lucretius, *De rerum natura*[†]

Before one can start arguing about how strong materials ought to be one should be able to measure how strong they actually are. Although nowadays a certain amount of mechanical testing is done for what might be called academic reasons, by far the most of it is done for strictly practical ends and in fact a thorough knowledge of the actual strength of its materials is, like drains and income tax, one of the things which no advanced civilization can do without.

There are generally two pragmatic reasons for knowing the strength of a material. The first and the most obvious is to have a figure to put into one's calculations on the strength of structures. However since proper scientific strength analysis is a recent affair, much the older and the commoner is that of maintaining the quality of materials. In other words, is this batch as good as the last? A variant of this is, can I use this as a substitute for that?

Of course anything as scientific as a mechanical test has not always found favour with traditional craftsmen or indeed with business men.[*] The procedure described in Weston Martyr's (1885–1966) delightful book *The Southseaman*[†] (which is about wooden shipbuilding in Nova Scotia in the nineteen twenties) was probably much commoner.

> Before any plank was put into place, MacAlpine and Tom and anyone else who happened to be about held a consultation over it. First they examined it very carefully, and then they bent it, tapped it, listened to it, and, as I live by bread, I swear that once, at least, I saw MacAlpine tasting it. At any rate he applied his tongue to the wood, and then went through all the motions of an expert tea-taster – even to that final feat of expectorating through the clenched teeth with precision and gusto.

The first published tensile tests seem to have been done by the French philosopher and musician Marin Mersenne (1588–1648) who was interested in the strength of the wires used in musical instruments. In 1636 Mersenne made a series of tests on wires of different materials but it is doubtful if any use was made of the information.

As far as I know, the first actual record of an objective mechanical test, which had results of practical consequence, occurs in Pepys' diary for 4 June 1662.

> Povey and Sir W. Batten and I by water to Woolwich; and there saw an experiment made of Sir R. Ford's Holland's yarn (about which we have lately made so much stir; and I have much concerned myself of our rope-maker, Mr Hughes who represented it so bad) and we found it to be very bad, and broke sooner than, upon a fair triall, five threads of that against four of Riga yarne; also that some of it had old stuffe that had been tarred, covered over with new hempe, which is such a cheat as has not been heard of.

The Woolwich people may have broken these ropes in direct tension by hanging weights on them, having tied some sort of scale pan to one end and the other end to an overhead beam. On the whole, however, it is more likely that they used a comparative test, tying the two competing ropes end to end, in series, and breaking them by means of a capstan. The number of strands in each rope would then be adjusted until there was an equal chance of failure.

Ropes and wires are fairly simple to test since it is easy to grip the ends by winding them round the barrel of a winch or capstan. Rigid solids are much harder to get hold of in tension and so for a long time such testing as was done was confined to compression and bending. Testing machines now exist which have vice-like grips, called 'friction grips', so that one can take an ordinary bar of metal, cut off a short length, and break it in tension. In practice however, this is generally an unsatisfactory arrangement since the grips damage the metal and cause premature failure at the ends so that the result is unreliable. It is usually better to cause an hour-glass or wasp-waisted specimen to be made as this can be arranged to break in the middle where it is thinnest. Even so, the design and making of satisfactory test-pieces calls for a modest degree of skill and experience as the best shape will be different for each kind of material.

With regard to the actual mechanics of testing, it is of course possible to apply the load to each specimen directly, by means of weights. However, since the breaking loads for convenient sized test pieces (say ¼ inch thick) are typically between about one and ten tons (a motor car weighs about a ton) and since most testing is done by girls, it is usual to apply the load mechanically or hydraulically and there are a large number of more or less automatic testing machines on the market. All that the operator has to do is to insert the specimen, watch the machine break it, then divide the recorded breaking load by the area of the cross-section at fracture, which is easily measured. The result is the breaking stress.

Of course this figure tells one nothing at all about why the material has the strength which it has and whether it ought to be stronger. On the other hand, in practice, the strength of any one individual engineering material tends to be constant. There therefore grew up a tendency to ignore the whys and wherefores and to regard the tensile and other strengths as innate properties with which the material happened to have been endowed by Providence in a rather arbitrary way. Metallurgists knew that this or that ingredient or heat treatment would strengthen or weaken an alloy but this knowledge was empirical and the effects were not susceptible to an obvious rational explanation.

Engineers like their materials to be consistent and are not too deeply interested in reasons, so they encouraged the idea that each material has a characteristic strength which could be determined accurately, once for all, if only one did enough tests. Materials laboratories of a generation ago centred upon magnificent collections of large testing machines. We filled a great many notebooks with testing data but learnt very little about the strength of materials.

Indeed it is difficult to exaggerate the impenetrability of the mystery which for centuries hung over the problem of the strength and fracture of solids. Lucretius (95–55 B.C.) set forth at great length the theory of the atomic nature of matter which had been propounded earlier by Democritus (460–370 B.C.). Though the theory was many years ahead of its time it was almost wholly guesswork and rested on no satisfactory contemporary experimental evidence. However Lucretius recognized the problem of cohesion and suggested that the atoms of strong materials were provided with hooks with which to grip each other. In the middle of the nineteenth century Faraday (1791–1867), one of the wisest of men, could do no better than to say that the strength of solids was due to the cohesion between their fine particles and that the subject was a very interesting one. Though both these statements were true they were not a great advance on Lucretius.

Chapter 2 contains a list of the practical strengths of various materials. Like the values for Young's modulus or stiffness, the figures vary a great deal between different substances, but then, so do the strengths of the chemical bonds within them and one might expect the engineering strengths to be proportional to the strengths of the chemical bonds. This is one of the differences between strength and stiffness. One can relate the Young's modulus, *E,* for a material in bulk to the fine-scale stiffness of its chemical bonds with considerable accuracy. Generally speaking this is not true of strength. The iron to iron bond in steel is not especially strong, it is easily broken chemically when iron rusts. Rust, iron oxide, is weak mechanically although its chemical bonds are strong. Again, magnesium metal is stronger than magnesium oxide, magnesia, though the energy difference in the bonds is dramatically shown by burning magnesium ribbon in oxygen. Any attempt to relate chemical to mechanical strength works only in a vague and irregular way. About all one can say is that while it is only too easy to make a weak material (or indeed a material of no strength at all) from strong chemical bonds, it is not possible to make very strong materials from weak bonds.

The plastics and polymers which came into use between the wars were, or were claimed to be, the first man-made strong materials to come out of chemical laboratories and they rather went to the heads of the chemists, who supposed, not unnaturally, that these polymers were strong because they had put them together with strong chemical bonds. When the last war broke out, a very able young academic chemist came to work with me. He set to work straight away to make a stronger plastic. He explained to me that it must be stronger because it contained stronger bonds and more of them than any previous material. Since he really was a very competent chemist I expect it did. At any rate it took a long time to synthesize. When it was ready we removed this war-winning product from the mould with excitement. It was about as strong as stale hard cheese.

Griffith and energy

We must now go back to about 1920 when the whole subject could be described as pretty well bogged down. At this time A. A. Griffith (1893–1963) was a young man working at the Royal Aircraft Establishment at Farnborough. He had ideas which cut through the mass of tradition and very dull detail which hung around materials work everywhere but unfortunately nobody took them very seriously. Griffith asked in effect 'Why are there large variations between the strengths of different solids? Why don't all solids have the same strength? Why do they have any strength at all? Why aren't they much stronger? How strong "ought" they to be anyway?' Until fairly recently these questions were regarded as unfathomable or unimportant or just silly.

We now understand in a general way how strong any particular solid ought to be and why it falls short of that strength in practice. Furthermore we know more or less what to do to increase its strength. Much of this success is due directly and indirectly to Griffith. In what follows I have shortened and transposed Griffith's arguments.

To calculate how strong a material ought to be we need to make use of the concept of energy. Energy is officially defined as 'capacity for doing work' and it has the dimensions of force multiplied by distance. Thus if I raise a two-pound weight through a height of five feet I have increased its potential energy by ten foot pounds.[*] This energy (which comes from my dinner which comes ultimately from the sun and so on) can be transformed into any of the many alternative forms of energy but it cannot be destroyed. Potential energy is one convenient way of parking energy until it is wanted and this energy can be followed through its various subsequent transformations by a sort of accounting procedure which can be very revealing.

The stored or potential energy in a raised weight can be used, for instance, to drive the mechanism of a grandfather clock though in most clocks a spring is usually more convenient, if only because it stores the same amount of energy which ever way up it is.[†] The strain energy in a stretched material is very like the potential energy which is in a raised weight, except, of course, that the stress is changing as the material is strained whereas the weight of a weight is constant as it is raised to any normal height.

Because of Hooke's law, when a material is strained the stress in it varies from nothing at the beginning of the operation up to a maximum at the final strain. For this reason the strain energy in a material is:

$$\tfrac{1}{2} \text{ stress} \times \text{strain per unit volume}$$

That strain energy is more than a triviality was demonstrated by the bowmen at Agincourt and, incidentally, one is well advised to keep out of the way of a stretched hawser such as is used for checking a ship. The kinetic energy, or energy of motion, of the ship has been exchanged for strain energy in the rope. There is a lot of energy, and, if the rope breaks strain energy is reconverted to energy of motion in the rope and somebody may get killed.

All stressed solids thus contain strain energy and this strain energy can be converted by one means or another into any of the other forms of energy. Most commonly a relaxed stress simply reverts to heat but children have discovered that it is possible to convert the strain energy of catapult elastic into the fracture of, say, glass. Whether or not something of the sort put into Griffith's head the idea of fracture as an energy process, I have no idea.

When a brittle material breaks, two new surfaces are created at the point of fracture which were not there before fracture, and Griffith's very brilliant idea was to relate the surface energy of the fracture surfaces to the strain energy in the material before it broke. Energy has many forms – heat, electrical energy, mechanical energy, strain energy and so on – but it is not immediately clear that the surface of a solid has energy, merely by virtue of its existence as a surface.

From watching raindrops, bubbles and insects walking on ponds it is obvious that water and other liquids have a surface tension. This tension is a perfectly real physical force which is quite easily measured. Consequently, when the surface of a liquid is extended, as by inflating a soap bubble, work is done against this tension and energy is stored in the new surface. In the accountancy of energy, surface energy counts in the balance just as much as any other kind of energy. When an insect alights on water, the surface is dimpled by its legs and thus extended and so the surface energy is increased. The insect sinks until the increase of surface energy just balances the decrease in its potential energy when it sinks no further and is, presumably, happy. Liquids tend, if they can, to minimize their surface energy. For instance a thin stream of liquid, from a tap which is being turned off, will reach a diameter at which it pays it to break up into separate drops simply because these have less aggregate area then the cylindrical stream.

When a liquid freezes, the molecular character of its surface is not too greatly changed and the energy of the surface remains much the same although the surface tension is no longer able to change the shape of small particles by rounding them off into drops. With a number of solids the interatomic forces are stronger and stiffer than they are in common liquids and so the surface energies are higher, often ten or twenty times the values for ordinary liquids.* The reason why we do not notice surface tensions in solids is not that the surface tensions are weak but rather that solids are too rigid to be visibly distorted by them.

Just as we could, perhaps, calculate the weight of the largest insect which could walk on a given liquid, so we can use these concepts to calculate how strong we ought to expect materials to be. As calculations go, this one turns out to be surprisingly simple, once somebody has had the original idea.

What we want to do is to calculate the stress which will just separate two adjacent layers of atoms inside the material. At this stage we need not worry too much whether the material is glassy or crystalline and all we really need to know about the solid is the Young's modulus and the surface energy. The two layers of atoms are initially x metres apart and so the strain energy per square metre for a stress s causing a strain e will be:

½ stress × strain × volume = ½ $s.e.x.$

But Hooke's law says:

$$E = \frac{s}{e} \text{ so } e = \frac{s}{E}$$

So, putting in $\frac{s}{E}$ for e:

Strain energy per square metre =

If G is the surface energy of the solid per square metre, then the total surface energy of the two new fracture surfaces would be $2G$ per sq. metre.

We now suppose that, at our theoretical strength, the whole of the strain energy between any two layers of atoms is potentially convertible to surface energy, then:

$$\frac{s^2.x}{2E} = 2G$$

so: $$s = 2\sqrt{\frac{G.E}{x}}$$

Actually, this is a bit optimistic because we have assumed that the material will go on obeying Hooke's law right up to failure. As we saw in the last chapter, Hooke's law is really only true for small strains and at large strains the interatomic force curve bends over so that the strain energy is less than we have calculated, very roughly about half. We can allow for this effect by dropping the two from the strength equation which we have just derived, bearing in mind that we are in no position to quibble about exact values. Thus a reasonable expectation for the strength of a material would be:

$$s = \sqrt{\frac{G.E}{x}}$$

which could hardly be much simpler.

For steel some typical values in S.I. units would be:

Surface energy G = 1J per square metre

Young's modulus $E = 2 \times 10^{11}$ Newtons per square metre (note *not* Meganewtons)

Distance between atoms x = 2 Ångström units

$$= 2 \times 10^{-10} \text{ metre}$$

Putting in these values gives us a strength of about 3×10^4 MN/m^2 or about five million pounds per square inch. Say (E/6). This is rather over 2000 tons per square inch. The strength of ordinary commercial steels is usually about 60,000 p.s.i. or 400 MN/m^2 while very strong wires may reach about 400,000 p.s.i. or 3,000 MN/m^2.

Since the values for E and G vary, of course, for each solid the values we get for the theoretical strengths will vary too. The only thing they have in common is that they are all very much above any strength normally realized in ordinary experiments. In fact steel is exceptional in sometimes reaching strengths as high as a tenth of its calculated strength; the great majority of common solids can show only a hundredth or a thousandth of what theory indicates.

As a matter of fact, forty or fifty years ago, nobody actually and openly disbelieved this calculation. If they had, they would have had to provide an alternative explanation of where the surface energy of a newly broken surface came from, but somehow nobody took it very seriously. There was a discrepancy somewhere and perhaps the less said about it the better.

If we confine our calculation simply to strength as such, we get a different figure for the theoretical strength of each material. However, we can nearly as easily do the sum for the theoretical elastic breaking strain, and if we do this, we are apt to find that the answer we get is very roughly the same for any solid, almost irrespective of its chemical entity. Generally speaking, this strain is something like 10 or 20 per cent.* If this is more or less true, then the strength of any solid should lie between E/10 and E/5. Hence, although we cannot say that every material ought to have the same strength, we can say that, very approximately, all materials ought to have the same elastic breaking strain. In everyday practice it is palpable that not only do materials not have the same breaking strain but also that the calculated strengths are, without exception, far above any commonly realized practical strength.

Griffith set out to find some physical theory which would bridge this gap between theory and practice. I never knew Griffith himself but Sir Ben Lockspeiser, who acted as Griffith's assistant at this time, told me something about the circumstances under which the work was done. In those days research workers were expected to earn their money by being practical, and in the case of materials they were expected to confine their experiments to proper engineering materials like wood and steel. Griffiths wanted a much simpler experimental material than wood or steel and one which would have an uncomplicated brittle fracture, for these reasons he chose glass as what is now called a 'model' material. In those days models were all very well in the wind tunnel for aerodynamic experiments but, damn it, who ever heard of a model *material*?

These things being so, Griffith and Lockspeiser took care not to bring the details of their experiments too much to the notice of the authorities. The experiments, however, involved drawing fibres and blowing bubbles of molten glass and one day, after the work had been going on for some months, Lockspeiser went home leaving the gas torch used for melting the glass still burning. After the inquiry into the resulting fire, Griffith and Lockspeiser were commanded to cease wasting their time. Griffith was transferred to other work and became a very famous engine designer. The feeling about glass died hard. Many years later, about 1943. I introduced a distinguished Air Marshal to one of the first of the airborne glass-fibre radomes, a biggish thing intended to be bolted under a Lancaster bomber. ‘What’s it made of?’ ‘Glass sir.’ ‘GLASS! – GLASS! I won’t have you putting glass on any of my bloody aeroplanes, blast you!’ The turnover of the fibreglass industry passed the £100,000,000 mark about 1959 I believe.

To return to Griffith’s experiments, Griffith was not the first man to draw strong glass fibres but he was probably the first man to do it in a systematic way and to provide a plausible explanation of the results.

Griffith had first to determine, at least approximately, the theoretical strength of the glass he was using. The Young’s modulus was easily found by a simple mechanical experiment and two or three Ångström units is a fair guess for the interatomic spacing and cannot be far out.[*] It remained to measure the surface energy. It was here that one of the advantages of glass as an experimental material lay. Glass, like toffee, has no sharp melting point but changes gradually, as it is heated, from a brittle solid to a viscous liquid and during this process there is no important change of molecular structure. For this reason one might expect there to be no large change in surface energy between liquid and solid glass so that surface tension and therefore surface energy, measured quite easily on molten glass, ought to be approximately applicable to the same glass when hardened. When the end of a glass rod is heated in a flame the glass softens and tends to round off into a blob because surface tension remains active long after permanent mechanical resistance to deformation has disappeared. The force, which is easily measured, needed slowly to extend the rod under these conditions is therefore that which will just overcome the surface tension. From experiments of this type, done with very simple apparatus, Griffith could deduce that the strength of the glass he was using (at room temperature) ought to be nearly 2,000,000 p.s.i. or about 14,000 MN/m^2.

Griffith then took ordinary cold rods of the same glass about a millimetre thick and broke them in tension, finding that they had a tensile strength of about 25,000 p.s.i. or 170 MN/m^2 which is round about the average for laboratory glassware, window panes, beer bottles and most of the other common forms of glass but was something between a fiftieth and a hundredth of what he reckoned it ought to be.

Griffith now heated his rods in the middle and drew them down to thinner and thinner fibres which after cooling he also broke in tension. As the fibres got thinner so they got stronger, slowly at first and then, when they got really thin, very rapidly. Fibres about one ten thousandth of an inch (2·5 mμ) thick showed strengths up to about 900,000 p.s.i. or 6,000 MN/m^2 when they were newly drawn, falling to about 500,000 p.s.i. or 3,500 MN/m^2 after a few hours. The curve of size against strength was rising so rapidly (Figure 1) that it was difficult to ascertain a maximum or upper limit to the strength. The increase of strength with thinness was not entirely smooth but showed a certain amount of scatter or variability. However, there was absolutely no doubt about the general trend.

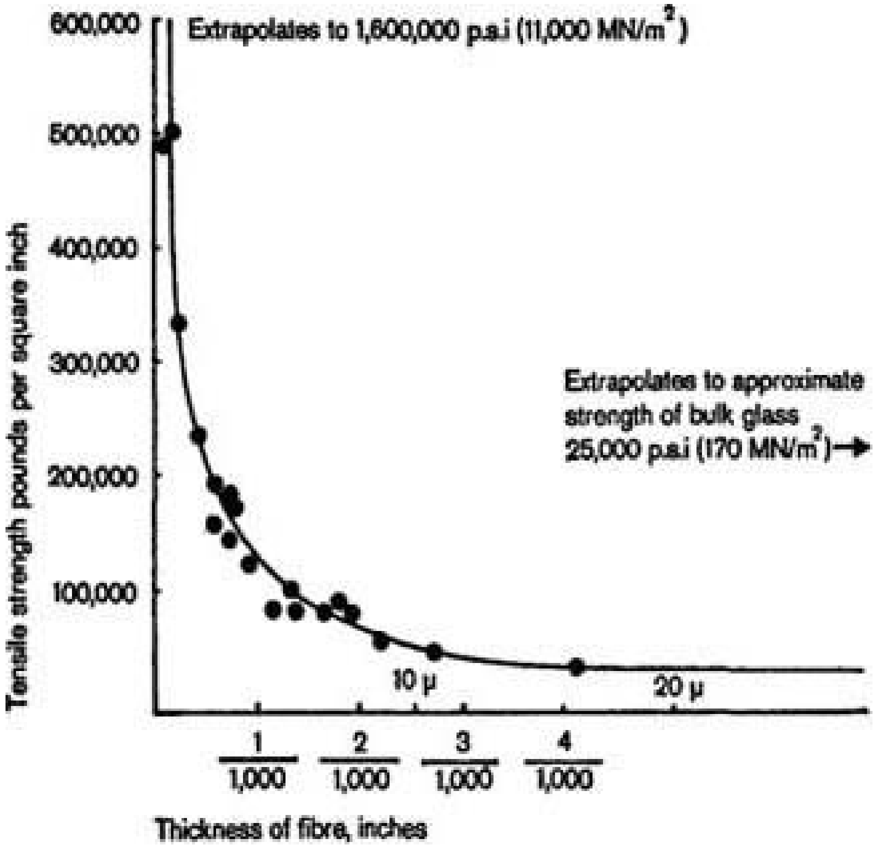


Figure 1. Griffith made and tested thinner and thinner glass fibres. As the fibres got thinner so they got stronger until the strength of the thinnest fibres approached the calculated theoretical strength.

Griffith could not prepare or test fibres thinner than about a ten thousandth of an inch (2·5 mμ) and, if he had, it would have been difficult at that time to measure the thickness with any sort of accuracy. However, by the simple mathematical device of ploting reciprocals it was possible to extend or extrapolate the size-strength curve fairly reliably so as to ascertain the strength of a fibre of negligible thickness. This turned out to be 1,600,000 p.s.i. or 11,000 MN/m^2. It will be remembered that Griffith had calculated a value a little under 2,000,000 p.s.i. or 14,000 MN/m^2 for the glass he was using. He therefore concluded that he had approached the theoretical strength quite closely enough to satisfy most people, and that if thinner fibres could actually be made, their strength would be very near to the theoretical value. The achievement by experiment of an approximation to the theoretical strength was of course a triumph, especially when one considers the conditions under which the work was done.

During the last few years, John Morley, of Rolls Royce, has prepared silica glass fibres (with a composition different from Griffith's glass) with strengths rather over 2,000,000 p.s.i. (14,000 MN/m^2) (Plate 4). As we shall see in the next chapter these very high strengths are not in fact confined to glass fibres but can be got from almost any solid, glassy or crystalline.

Griffith had demonstrated that the theoretical strength could be approximated experimentally in at least one case, he had now to show why the great majority of solids fell so far below it.

Chapter 4 Cracks and dislocations
or why things are weak

‘The fault that leaves six thousand ton a log upon the sea.’

Rudyard Kipling, ‘McAndrew’s Hymn’.

Griffith wrote a classic Royal Society paper about his experiments which was published in 1920. In this paper he pointed out that the problem was not to explain why his thin fibres were strong, since a single chain of atoms must, inescapably, have either the theoretical strength or none at all, but rather to explain why the thicker fibres were weak.

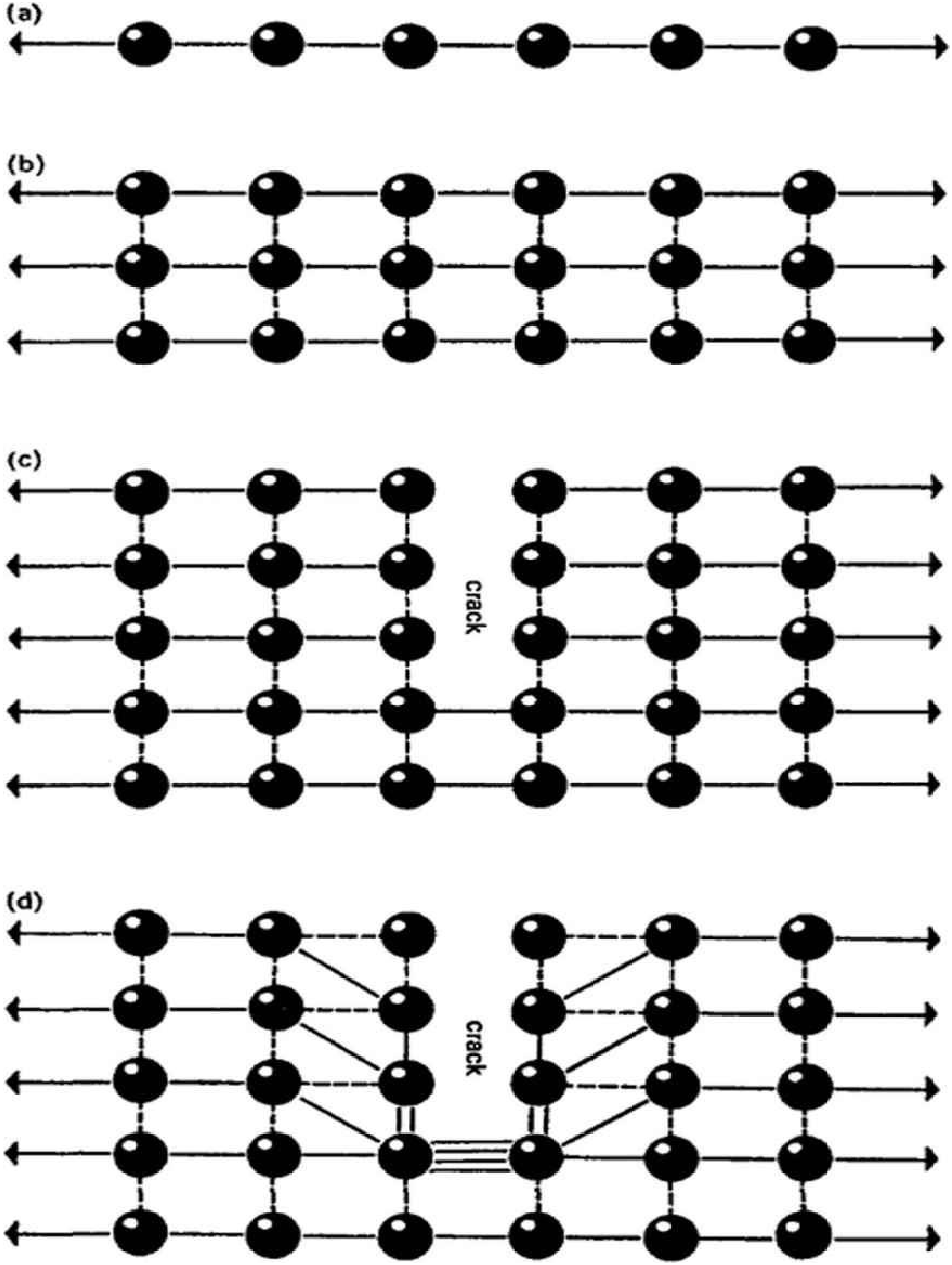
It was becoming clear, at any rate to Griffith if to nobody else, that in a world where practical materials only reached a small and highly irregular fraction of the strength of their chemical bonds, the weakening mechanism, rather than the bond strength, was what really controlled mechanical strength. It is only quite lately and now that we are able regularly to get strengths which are a large fraction of the theoretical value, that it has become really important and worthwhile to make materials with very strong chemical bonds.

The weakness of glass fibres brings us to the question of Griffith cracks and it also brings us back to Professor Inglis, whom we left in Chapter 2 worrying about why ships broke in two at sea when simple calculation showed them to be amply strong enough. Inglis made calculations about the effect of hatchways and other openings in large structures like ships. Griffith had the wit to apply Inglis’s mathematics on a far finer scale, to ‘openings’ of almost molecular size and too fine to see with an optical microscope.

Stress concentrations

Whatever the scale, the practical importance of stress concentrations is enormous. The idea which Inglis expounded is that *any* hole or sharp re-entrant in a material causes the stress in that material to be increased locally. The increase in local stress, which can be calculated, depends solely upon the *shape* of the hole and has nothing at all to do with its *size.* All engineers know about stress concentrations but a good many don’t really in their hearts believe in them since it is clearly contrary to common sense that a tiny hole should weaken a material just as much as a great big one.[*] The root cause of the Comet aircraft disasters was a rivet hole perhaps an eighth of an inch in diameter. Small holes and notches are particularly good at starting fatigue failures but they also do very well for starting ordinary static fracture. When a glass cutter wants to cut glass, he does not bother to cut right through but makes a shallow scratch on the surface after which the glass breaks easily along the line of the scratch. (By the way, so-called ‘cut glass’ is ground to shape, not cut.) The weakening effect of the scratch has very little to do with the amount of material removed, a shallow scratch will do nearly as well as a deep one, it is the sharpness of the re-entrant that increases the stress.

It is not difficult to form a physical picture of what is actually happening at a re-entrant such as a crack, especially if we consider the matter upon a molecular scale. Referring to Figure 1 it is obvious that a single chain of atoms in tension must be uniformly stressed and should reach the theoretical strength (1a). The mere multiplication of such chains, side by side, to constitute a crystal, does not prevent each separate chain from still carrying its full theoretical stress (1b). Suppose now that we cut a number of adjacent bonds so as to constitute a crack, then of course we have interrupted the flow of stress in the broken chains and the load in these broken chains has got to go somewhere (1c). In fact it does the most natural thing, which is to go round the end of the opening. Thus the load in the whole of the cut chains may well have to pass through the single bond which closes the tip of the crack (1d). It is obvious that in these circumstances this bond will fail long before its companions. When this over-tried bond has broken the situation is no better. Indeed it is worse, for the next bond has to bear, not only the load in ail the initially broken chains but also that in the chain which has newly snapped. Thus the situation goes from bad to worse. In this way a crack is really a mechanism which enables a weak external force to break even the strongest bonds one by one. And so the crack runs through the material until total fracture occurs.


(a)
(b)
(c)
crack
(d)
crack

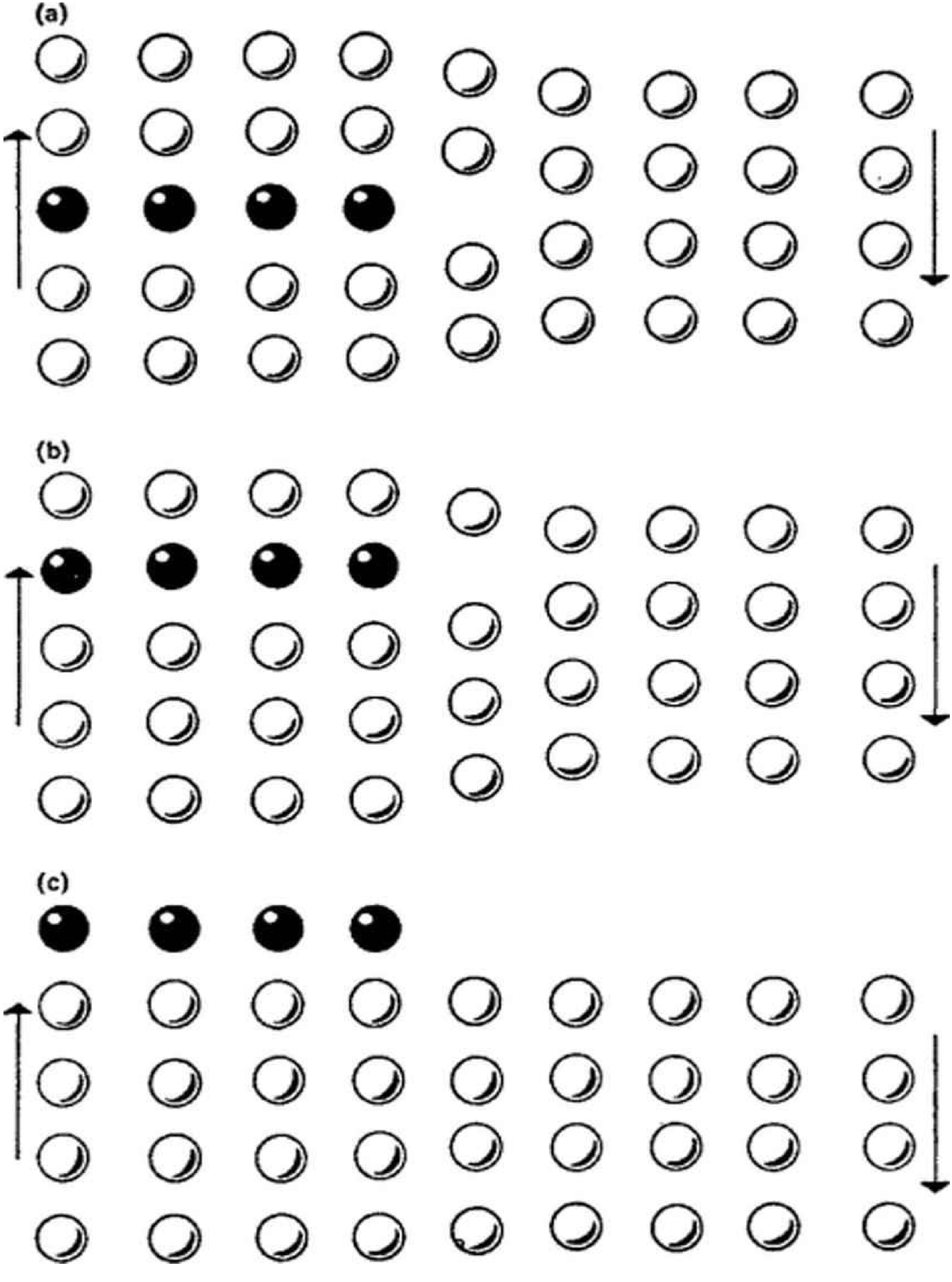

(a)
(b)
(c)

Figure 5. Shearing by means of an edge dislocation. For the reality of this phenomenon see Plate 14.

(a) Edge dislocation (diagrammatic).

(b) Dislocation sheared one lattice spacing.

(c) Dislocation sheared out of crystal altogether.

Note. The shaded atoms are not of course the *same* individual atoms in each of these diagrams. The shading merely indicates the position of the extra half sheet of atoms. During dislocation movement no individual atom has to move more than a fraction of an Ångström from its original position.

Engineers and some metallurgists resisted the idea with the whole force of their emotions and even today some of them are still making growling noises in caves in the backwoods. Academic physicists, on the whole, however, fell upon dislocations with glee. For many years nobody saw a dislocation in the flesh, or perhaps ever expected to, but their hypothetical movements (dislocations of like sign repel each other etc.) and breeding habits (when the union of two dislocations is blessed about five hundred new dislocations are suddenly released upon the crystal) could be theoretically predicted and provided a superb intellectual exercise like three dimensional chess.

As a matter of fact nearly all these academic predictions turn out to be true. Taylor supposed originally that slip in ductile crystals was due entirely to those dislocations which were present initially in the crystal due to the accidents of imperfect growth. It turns out that there are generally not enough dislocations originally present in most crystals to account for the very extensive slip which can take place in a ductile material. Large families of new dislocations can however be nucleated either by dislocation interaction (known as a Frank-Read source), or, more frequently, by severe stress concentrations, such as occur at crack tips. These mechanisms enable a stressed metal to be rapidly filled with dislocations (something like 10^{12} per square centimetre) and thus to flow under a steady load or the blow of a hammer quite easily.

It will be recalled that the dislocation is essentially a line defect which can move about in the crystal fairly freely. When there are many dislocations they do not have to move far before two or more dislocations meet. In rather special circumstances this can result in the creation of new dislocations but the much more usual effect is for them to repel each other. As more and more dislocations are born and move about they impede each other and get tangled up, like so much string. The result is that after a period of free movement the material begins to harden and if one goes on deforming it it will become brittle.

The most familiar example of this is when one wishes to break off a piece of metal such as a wire or the opened lid of a tin can. This can usually be done by bending it backwards and forwards a few times. The metal yields easily at first, hardens somewhat and then breaks off in a brittle fashion. Metal hardened by deformation can be returned to its initial soft condition by ‘annealing’, that is to say by heating it until total or partial recrystallization occurs, in which case most of the excess dislocations vanish. Thus copper tubes must be annealed after bending to shape or they will be brittle.

Altogether, the dislocation mechanism has been found to explain the mechanical properties of metals very well indeed. Although dislocations do exist in non-metallic crystals they are not usually very mobile and they seldom breed, thus dislocation movement does not play any important part in the way non-metals behave. It is the mobility of dislocations which accounts for the mechanical differences between metals and non-metals. Needless to say, dislocations cannot exist in glasses because glasses are not crystalline.