Chapter 1

Simple Harmonic Motion and Applications

In this chapter we shall first define the physical terms that will be used throughout the book and then discuss the single concept most basic to the study of sound—simple harmonic motion—and its direct application to sound and resonance.

1.1 Fundamental Definitions

To understand physics, we must first understand its language. Its definitions are exact, formulated in the language of mathematics. For our purposes, however, more qualitative and operational definitions will suffice. The most fundamental quantities in physics are discussed next; they will be used regularly in our study of acoustics. Other important quantities will be defined throughout the book as the need arises.

Position, length, or distance (symbols x and y)

This quantity is simply a measure, in the normal everyday sense, of how far one point is from another. Units of length can be miles, feet, inches, or meters (m). We shall primarily study motion in one dimension, as shown in Figure 1-1. Position relative to the origin of coordinates ($x = 0$) can be positive (to the right) or negative (to the left); dots have been placed on the line at $x = 6$ centimeters (cm), $x = 2$ cm, and $x = -4$ cm. We shall regularly use the metric system; in this system the prefix *deci-* means 1/10, *centi-* means 1/100, *milli-* means 1/1000, *micro-* means 1/1,000,000, *kilo-* means 1000, and *mega-* means 1,000,000.

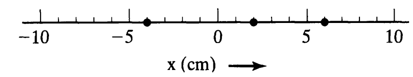

Figure 1-1 Graph showing points positioned along a line.

Some metric length units are meters (m), kilometers (km), centimeters (cm), and millimeters (mm), where 1 cm = 0.01 m, 1 mm = 0.001 m (so 1 m = 100 cm = 1000 mm), and 1 km = 1000 m. One meter is approximately 39.37 inches; 1 inch (in.) is 2.54 centimeters.

Time (symbol t)

For our purposes, we simply observe that time as we know it passes and is divided into seconds (s), hours, days, and so on. Units of time that will be used regularly in discussing sound include the millisecond (ms), where 1 ms = 0.001 s (so 1 s = 1000 ms), and the microsecond (μs), where 1 μs = 0.000001 s = 0.001 ms (so 1 s = 1,000,000 μs).

Velocity or speed (symbol v)

Velocity and speed, which should not be used synonymously, have different definitions in physics. *Speed* is distance traveled per unit of time. *Velocity*, a more general term, refers to both the speed and the direction of the motion, and is therefore a *vector* quantity. An object moving around a circle at a constant speed is continually changing its direction; therefore its velocity is continually changing. The examples that we use will be primarily one-dimensional; we will refer to velocity to include both the magnitude and the direction of the motion.

For example, if the dot in Figure 1-1 moves uniformly from $x = 2$ cm to $x = 6$ cm in 1 s, its velocity is +4 centimeters per second (+4 cm/s). If it moves from $x = 6$ cm to $x = -4$ cm in 2 s, $v = (-10\text{ cm})/(2\text{ s})$ or $v = -5$ cm/s. Convince yourself that 1 m/s = 1 mm/ms.

The speed of sound in air is approximately 345 m/s or 1100 ft/s and varies with temperature; the speed of light is approximately 300,000,000 m/s or 186,000 miles/s.

Acceleration (symbol a)

Acceleration is defined as change in velocity per unit of time. For example, suppose a ball rolls down a ramp starting with zero velocity (i.e., it starts at rest), and after 1 s its velocity is 1 cm/s, after 2 s its velocity is 2 cm/s, and after 3 s its velocity is 3 cm/s. The increase in velocity is 1 cm/s for each second it is rolling down the ramp, which can be written $a = (1\text{ cm/s})/(\text{s})$ or $a = 1\text{ cm/s}^2$. Technically, acceleration is a vector quantity too, because it points in a particular direction. Deceleration, or slowing down, is acceleration in the direction opposite to the motion. The acceleration of gravity is 9.8 m/s^2—that is, if you drop a heavy object near the surface of the earth, it will gain speed at a rate of 9.8 m/s for each second it falls.

Mass (symbol m)

Mass is a measure of the amount of matter in an object. A 1-gram mass of iron, for example, contains a certain number of iron atoms, which remains constant on the earth, on the moon, or in outer space. The most often used units of mass are the gram (g) and the kilogram (1 kg = 1000 g). One kilogram weighs about 2.2 pounds (lb) on the surface of the earth. A 5-cent coin has a mass of about 5 g.

Density (symbol ρ)

Density, or mass per unit volume, is represented by the symbol ρ, the Greek letter *rho*. When the pressure in the atmosphere or a wave increases, so does the density of the air through which the wave is propagating. The *mass per unit length,* or linear density, of stretched wires is important in the design of stringed instruments and the piano.

Force (symbol F)

Force can be thought of as a push or a pull. If you push or pull on a mass at rest with some net force, it will begin to move in the direction of the force. *Weight* is the *force of gravity* pulling a mass toward the center of the earth or other celestial body and will be different on the surface of the moon or some other planet; mass will not. In deep outer space, where there is almost no force of gravity, all objects are *weightless*, but they still have mass.

The metric unit of force is the newton (N), named after the physicist Isaac Newton (1642–1727). A 100-lb boy standing on the earth is held to the ground by a downward gravitational force, his weight, of about 445 N. The surface of the earth pushes up on the boy's feet with an equal force. These two forces balance, and the boy does not move up or down. On the moon, his weight would be only about 74 N, because the force of gravity is less on the surface of the moon than on the earth.

Pressure (symbol p)

Pressure is defined as force per unit area, and can be measured in units of pounds per square inch or in pascals (Pa), where a pascal is a newton per square meter. An example of pressure can be seen by considering an elephant walking on a sandy beach. Although the elephant weighs a great deal, its feet are very large, and the pressure, or force per unit area of its feet, exerted on the sand is relatively small. The weight is spread out over the large area. The elephant therefore leaves very shallow footprints in sand. On the other hand, a woman wearing high-heeled shoes exerts more pressure on the sand, because her smaller weight is exerted on the very small area of the heels. The prints of the heels in the sand would be deeper than the footprints of the elephant.

Air pressure is exerted on everything in contact with air; rapid changes in air pressure cause vibrations of the eardrum, which we hear as sound. Usually, air pressure on the outside of the eardrum is balanced by pressure in the ear, because air can flow into the mouth and then through the eustachian tube to the middle ear, behind the eardrum. Congestion of the eustachian tube restricts the airflow from the atmosphere to the space behind the eardrum. Slow changes of air pressure in the atmosphere can then create pressure differences between the atmosphere and the space behind the eardrum, causing large forces on the eardrum, which can be painful. Such an effect is often observed at the beginning and end of airplane flights, when the air pressure changes with changes in altitude. Atmospheric pressure on the surface of the earth is about 14.7 lb/in^2 or about 100,000 Pa.

Again, consider the 100-lb boy. Suppose the area on the bottom of each foot is 0.01 m^2. His weight, 445 N, is distributed over both feet, 0.02 m^2, and the pressure on the bottom of each foot is 22,250 Pa. When the boy stands on one foot, the area over which his weight is distributed is halved, so the pressure on his foot doubles to 44,500 Pa.

1.2 Simple Harmonic Motion

The single most important concept in the study of waves and sound is that of simple harmonic motion (SHM). We shall now study SHM and some of its properties, and in Chap. 2 demonstrate how SHM is basic to waves and sound.

Periodic motion is any type of motion that repeats itself after successive equal time intervals. Some examples of periodic events are twirling a rock on a string around your head, the rotation of the earth on its axis, the revolution of the moon around the earth, a mass bouncing up and down on the end of a spring, a swinging pendulum, a blinking warning light, the vibration of a tuning fork, and the vibration of the reed of a clarinet or a singer's vocal folds, producing a constant musical tone.

Simple harmonic motion is a specific type of periodic motion (for instance, a mass bouncing up and down on the end of a spring) that arises from the conditions described in the next paragraph and whose graph is a sine or cosine shape, as shown in Figure 1-2. This graph shows the position of one dot in Figure 1-1 over time. Notice that the motion repeats itself after 2 s in this example, and, as with all SHM, it has a particular characteristic smooth shape. Other names for SHM or waves of this character are simple, pure, sine, cosine, or sinusoidal motion or waves.

Two conditions must be met to produce SHM in a mechanical system. First, an *equilibrium position* must exist; that is, there must be one position at which the object executing the motion would remain at rest if placed there and to which it returns if displaced and released. Second, the force tending to pull the object back to its equilibrium position must be a *linear restoring force:* that is, the force must be linearly proportional to the distance of the object from the equilibrium position, as explained next.

As an example of these two conditions, let us consider a particular mass hanging on the end of a flexible spring attached to a fixed point, as shown in Figure 1-3. The equilibrium position is the position of the mass when it is hanging at rest directly below its suspension point. We consider only vertical motion of the mass along the direction of the extended spring, above and below the equilibrium position. If the mass is lifted above its equilibrium position, the force of gravity and perhaps also the compression of the spring will force the mass back down, toward the equilibrium position. Conversely, if the mass is pulled down, the spring tends to pull it back up, toward the equilibrium position. If it requires 1 N of force to lift the mass up 1 cm, it will require 2 N for a 2-cm

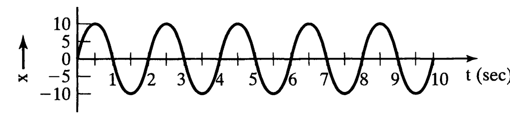

Figure 1-2 Graph of simple harmonic motion (SHM).

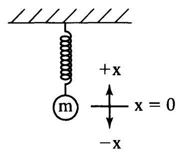

Figure 1-3 Mass hanging from a spring; equilibrium position is labeled $x = 0$.

displacement, 3 N for a 3-cm displacement, and so forth, because for this system the restoring force is linear. Likewise, if the restoring force is linear it will require 1 N of force to pull down the mass 1 cm from the equilibrium position as well. This is an example of a linear restoring force with a force constant (or force per unit of stretch) of 1 N/cm. Such a linear system is said to obey Hooke's law, named after the English physicist Robert Hooke (1635–1702).

When the mass on the spring in the example in Figure 1-3 is lifted 3 cm above its equilibrium position and released, it begins to accelerate downward, passing through its equilibrium position, and executes SHM as shown in Figure 1-4. At time $t = 0$ the mass is released from rest 3 cm above ($x = +3$ cm) the equilibrium position ($x = 0$). The mass starts moving, builds up speed as it passes through the equilibrium point, then slows down to zero velocity by the time it reaches the point $x = -3$ cm below the equilibrium point. The mass then starts upward, moving faster until it goes through the equilibrium point, then slows down to zero velocity by the time it reaches its original position, $x = 3$ cm. The motion then repeats itself every 2 s, as shown on the graph.

The maximum displacement A of the mass from the equilibrium position (in either direction) is called the *amplitude,* and the repetition time T is called the *period* of the SHM. For this example, $A = 3$ cm and $T = 2$ s. Note that the amplitude of any oscillation, including SHM, can be expressed in units appropriate to the system under consideration: for example, centimeters for a mass oscillating on a spring, pressure units like pascals for a sound wave traveling through air, and volts for an electrical signal in a stereo set.

The period T is related to the frequency f, which is the number of periods (or cycles) per second:

$$f = \frac{1}{T} \text{ or } T = \frac{1}{f}. \tag{1.1}$$

A high-frequency oscillation has many vibrations per second; to fit many vibrations into a 1-second time interval, the period must be short. Thus, a high-frequency oscillation has

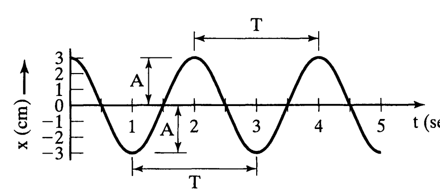

Figure 1-4 SHM with amplitude $A = 3$ cm and period $T = 2$ sec.

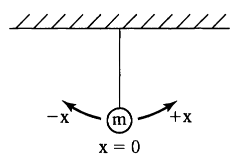

Figure 1-5 Pendulum consisting of mass hanging on end of string; mass swings left to right in the plane of the paper.

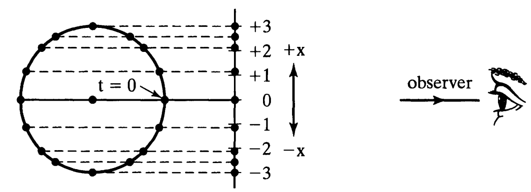

Figure 1-6 SHM as the projection of uniform circular motion along a line in the plane of the paper.

a short period; a low-frequency oscillation has a long period. If we know that the period is 2 s, the frequency f is

$$f = \frac{1}{T} = \frac{1 \text{ cycle}}{2\text{ s}} = 1/2 \text{ cycle per second} = 0.5 \text{ Hz}. \tag{1.2}$$

The unit of frequency is the hertz (Hz), which is 1 cycle/s, named after Heinrich Hertz (1857–1894). For a 500-Hz audio-frequency oscillation, the period is

$$T = \frac{1}{f} = \frac{1 \text{ cycle}}{500 \text{ cycles per second}} = \frac{1}{500/\text{s}} = 0.002 \text{ s} = 2 \text{ ms}. \tag{1.3}$$

Thus the period of a 500-Hz oscillation is 2 ms. Check to see that if the period of an audio-frequency tone is 0.5 ms, its frequency is 2000 Hz or 2 kilohertz (kHz).

Another example of SHM is a pendulum consisting of a mass or "bob" hanging by a string from a fixed point and free to oscillate back and forth in a plane, as shown in Figure 1-5. The equilibrium position is the point directly below the suspension point, where the pendulum will hang motionless. The force required to pull the pendulum away from its equilibrium position is linearly proportional to the displacement of the bob from equilibrium, as shown in Figure 1-5, so long as that distance is small. When displaced through some small angle and released from rest, the bob executes SHM in the plane of the paper. Its motion can be described by a graph similar to the one in Figure 1-4.

Some important details of SHM can be illustrated by comparison of SHM with uniform circular motion, which is the motion of an object around a circle with constant speed. Uniform circular motion, viewed from a distant point in the plane of the motion, is SHM, as can be seen using Figure 1-6; SHM is the projection of uniform circular motion on a line in the plane of the motion. Points are shown on the circle corresponding to the position of the object at 16 equal time intervals for one period of the motion (one complete circle). The projections of these points on the line are shown and give the position of an object, such as a mass on the end of a spring, executing SHM along the line; the dots represent equal time intervals. The radius of the circle is 3 units; thus the amplitude of the projected motion is 3 units.

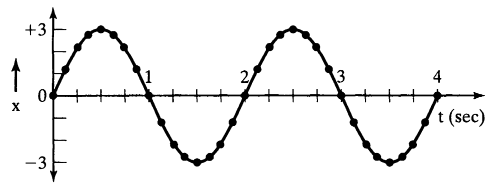

Figure 1-7 SHM position versus time at equal time intervals.

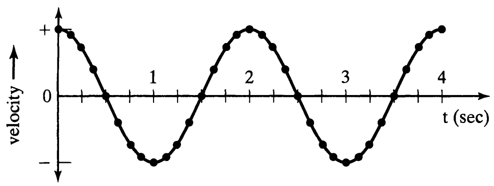

Figure 1-8 SHM velocity versus time at equal time intervals.

Notice that the projection of the distance traveled in successive equal time intervals is not the same and that therefore the velocity along the vertical line is continually changing. The velocity is least when the distance between two successive points is smallest, and, in fact, the velocity is zero when the projection is at either extreme of displacement. That is, for a brief instant of time, the point is moving neither up nor down, but is at rest. If the x position of the point in Figure 1-6 is plotted after each time interval, beginning with $x = 0$, the graph, shown in Figure 1-7, is a sinusoidal or SHM curve. Starting at $t = 0$, the point moves a large distance in the positive direction in the first time interval and less in each succeeding time interval until it reaches its maximum position. Its velocity is thus large and positive at $t = 0$ and decreases to 0 at $t = 0.5$ s, when the largest positive position is reached, as shown in Figure 1-8. The velocity of the point at any given time can be determined by examining the slope of the graph in Figure 1-7. When the slope of the graph is large, the velocity is high; a large distance is traversed in a given time. When the slope of the graph is 0, as at $t = 0.5$ s, the velocity is 0. (The slope is graphed in Figure 1-8.) The direction of the motion then changes; that is, the velocity becomes negative, and the point moves down through 0 toward its extreme negative position. Looking at the spacing of the points, one can see that at $t = 1$ s, as the point passes through $x = 0$, its velocity has the largest negative value, as shown in Figure 1-8. As time progresses, the position and velocity repeat once per period, as shown. Make sure you understand how position and velocity of SHM are correlated, as shown in the figures. Note specifically that velocity as a function of time for SHM also has the unique SHM shape.

Also by analogy between uniform circular motion and SHM, we can divide one period of SHM into 360° of phase ϕ (lowercase Greek letter *phi*), as shown in Figure 1-9. If one period is 360° of phase, the half-period is 180° and the quarter-period is 90°. Two curves are said to be *out of phase* if they differ in phase by 180° at all times, as shown in Figure 1-10. One of the curves would have to be moved forward or backward in time by one half-period, or 180° of phase, to make the two curves appear the same, or *in phase*. In

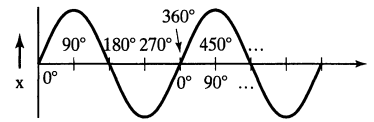

Figure 1-9 Phase of SHM curve.

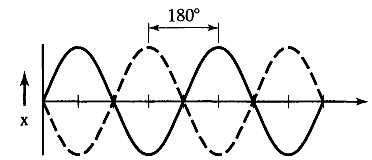

Figure 1-10 Two SHM curves differing in phase by 180° (out of phase).

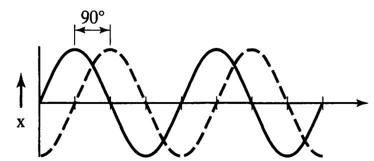

Figure 1-11 Solid curve 90° ahead of dashed curve.

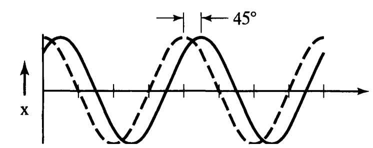

Figure 1-12 Solid curve 45° behind dashed curve.

Figure 1-11 the solid curve is 90° in phase ahead of the dashed curve; thus the peak of the solid curve occurs earlier in time by one quarter-period than the peak of the dashed curve. The solid curve in Figure 1-12 is 45° behind the dashed curve. Relative phase between two curves is only well defined when the two periods (and thus frequencies) are the same.

1.3 Application to Sound

Our knowledge of simple harmonic motion can be applied directly to sound. In our study of acoustics we shall generally deal with frequencies in the range from 20 Hz to 20 kHz, the "audible frequency range" that human ears can detect. Sound waves are changes in air pressure occurring at frequencies in the audible range. The normal variation in air pressure associated with a musical instrument played quietly is about 0.002 Pa. The smallest pressure variation that can be heard is about 0.00002 Pa, whereas the pressure variation that produces pain in the ear is about 20 Pa. Normal atmospheric pressure is about 100,000 (10^5) Pa. The maximum change in atmospheric pressure due to changes in weather is a few percent of this average value. Notice that the *variation* in pressure of a sound wave is a tiny fraction of the ambient pressure.

We can correlate the physical properties of sound waves with our perception of pitch, loudness, and tone quality; this can be readily demonstrated using an electronic wave generator and a loudspeaker. Changing the frequency of the oscillator varies the pitch; changing the amplitude of the oscillator's signal varies the loudness.

The higher the frequency of a wave, the shorter its period and the higher its pitch. For two waves of the same frequency, the one with the greater amplitude sounds louder. We shall discuss in Chap. 6 the variation in loudness with frequency for a constant-intensity wave and the slight variation in pitch with intensity. Shown in Figure 1-13 are waves that (a) remain at the same pitch but get softer, (b) get higher in pitch but remain approximately at the same volume, and (c) become simultaneously softer and lower in pitch.

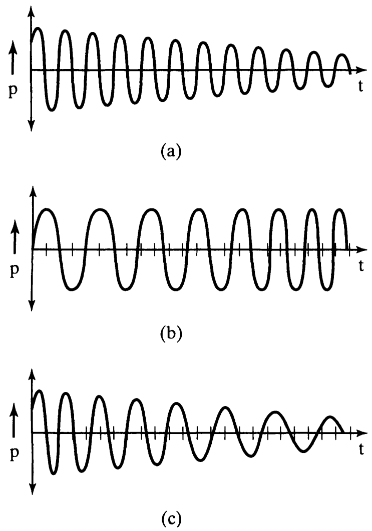

Figure 1-13 Audible waves that (a) remain constant in pitch but become softer; (b) become higher in pitch but remain at nearly the same volume; and (c) become simultaneously softer and lower in pitch.

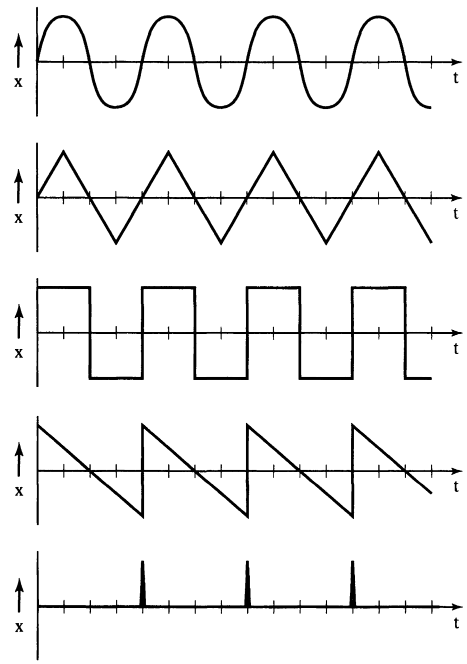

Figure 1-14 Sine wave, triangular wave, square wave, sawtooth (ramp) wave, and pulse train (series of pulses) of the same frequency.

Waveforms other than the sine wave will play an important part in our study of sound. Shown in Figure 1-14 are a sine wave, a triangular wave, a square wave, a sawtooth (or ramp) wave, and a pulse train (series of pulses). These waves all have the same period, so they all have the same frequency and pitch. The important difference between them is that they sound different; they differ in tone quality, or timbre. The variation in tone quality for different wave shapes can be demonstrated using an oscillator that can produce different wave shapes. In Chap. 4 we shall study what makes these tones sound different and under what conditions two waves with different shapes might sound very similar.

Wave shapes can be displayed using an *oscilloscope*. An oscilloscope, or "scope," is a device for displaying an electronic signal—that is, translating it into visible form on a screen. The signal from a microphone or any other electronic signal is traced onto the oscilloscope screen by a beam of electrons produced in the scope. If no signal is fed in, the beam moves at a constant velocity from left to right in a straight line, taking a specified time to cross the screen. Input of a signal causes vertical motion of the beam as it moves left to right and thus traces out the graph of the signal voltage as a function of time. Both axes can be calibrated, so we can observe the amplitude of the signal and how rapidly it varies. The graphs of sounds and other waves obtained using an oscilloscope are like the

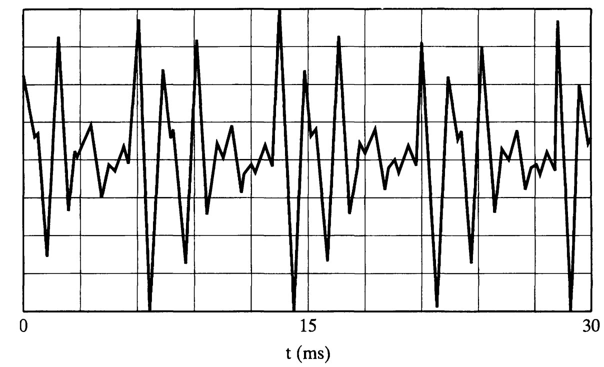

Figure 1-15 The wave shape of a male voice singing the vowel "ah" on the note $C_3 = 130.81$ Hz.

graphs previously shown in the various figures. Computer software has been developed as an alternative to using an oscilloscope. The signal from a microphone can be digitized using a standard computer sound card and plotted on the computer screen using readily available software.

Just as each electronic wave has a particular shape and tone, the sound wave of a note played on a musical instrument has a shape, called its *waveform*. A musical tone, which consists of periodic oscillations of air pressure, is converted into an electrical signal by a microphone. This electrical signal, or voltage, which is proportional to the pressure of the air in the sound wave, can be displayed using an oscilloscope as voltage versus time. The waveform of a male voice singing the vowel sound "ah" at $C_3 = 130.81$ Hz is shown in Figure 1-15. From the graph, its period is about 7.5 ms, so its frequency would be about 133 Hz.

All the preceding waveforms have one very important common feature: they are periodic. Waves do not have to be periodic; Figure 1-16 shows a wave shape of "noise,"

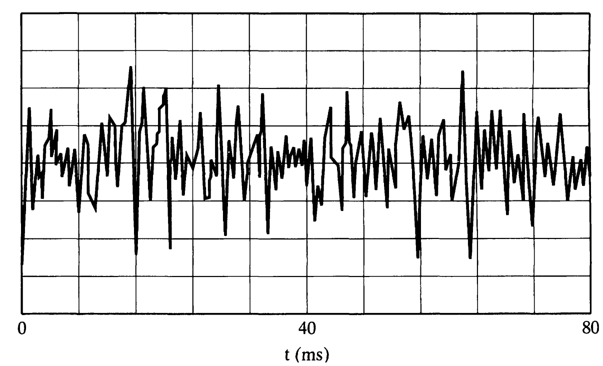

Figure 1-16 The wave shape of "noise" from a Moog Rogue analog synthesizer.

which contains no observable periodicity. This is the difference between musical sounds, which are periodic, and nonmusical sounds, or noise, which are nonperiodic. In Chaps. 4 and 5 we shall discuss the nature of several types of noise and the uses of noise in musical instruments and electronic music synthesizers.

1.4 Damped and Driven Oscillations

Consider a pendulum as in the example in Sec. 1.2. If it is started into motion by moving the bob to one side and releasing it from rest, it will not continue to oscillate forever, but will slowly decrease in amplitude, losing energy due to air resistance and friction in the suspension, until it stops. This process, called *damping,* is graphed in Figure 1-17. In this case, the damping is very slow (try a pendulum), but under certain circumstances it can be very rapid, as in the damping of a pendulum under water or a plucked guitar string. For the guitar, while the amplitude decreases, the period remains constant until the motion stops; that is, the pitch remains almost constant while the note becomes continuously softer. In damped harmonic motion the period remains constant while the amplitude decreases.

Now consider the case of driven, or forced, oscillations of a pendulum, as illustrated in Figure 1-18. If we pull the pendulum through a small displacement in the $-x$ direction and release it from rest at $t = 0$, it will undergo SHM. Suppose each time the pendulum reaches its extreme $-x$ position, we give it an additional sharp force, or push, in the $+x$ direction; this will cause the amplitude of the motion to increase slightly each cycle, as shown; this is similar to pushing a child on a swing. Arrows indicate the

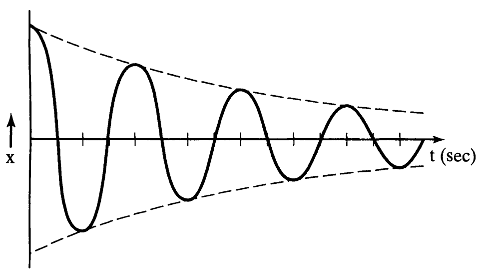

Figure 1-17 Damped harmonic motion.

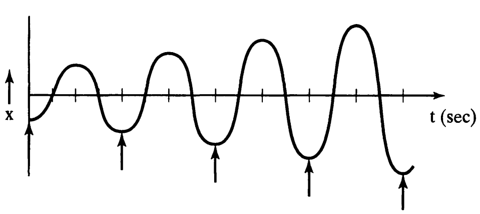

Figure 1-18 Driven harmonic motion with the frequency of applied force the same as the natural frequency of the pendulum, so that the force remains in the same phase relationship to the motion. Arrows denote times when the driving force is applied.

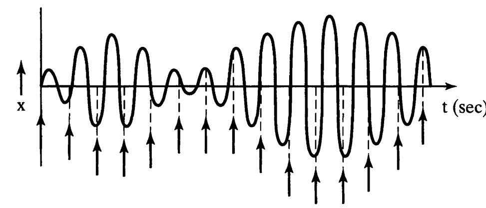

Figure 1-19 Driven harmonic motion with the frequency of applied force different from the natural frequency of the oscillator, resulting in continuous phase change between the oscillation and the force. Arrows denote times when the driving force is applied.

times at which the force is applied. This growth in amplitude resulting from application of a periodic force is called *driven* or *forced harmonic motion*. Although the amplitude increases with time, the period remains constant.

It is important to recognize that this continual increase in amplitude can occur only if the applied force continually comes at the right time, or at the proper phase, with respect to the motion of the pendulum bob; the force and the motion of the bob are then said to be *resonant* or *in resonance* with each other. In general, resonance can occur whenever the frequency of the driving force is the same as the *natural*, or normal, frequency of the oscillating system. If started at the proper phase, this phase relationship will continue. If the frequencies are the same, the system will adjust to the proper phase relationship for a resonance to occur.

Suppose now that the frequency of the driving force is slightly different from the natural frequency of oscillation of the pendulum. If the sharp force starts in phase with the swing of the pendulum, after some time the force and oscillation will become out of phase. The force, which originally was causing an increase in the amplitude of the motion, will now be opposing, or tending to decrease, the motion. This situation is shown in Figure 1-19; again, arrows indicate the force. The continual phase change between the force and the motion can easily be observed.

Another type of resonance, called a *coupled* or *coupling resonance,* occurs when two mechanical oscillatory systems with the same (or simply related) natural frequency are connected, or *coupled*, mechanically, so that vibrational energy can be transferred from one oscillator to the other.

As an example of coupled resonance, consider a set of pendula attached to a rod that can rock slightly in loosely fitting holes in the frame in which it is mounted, as shown in Figure 1-20. If the pendulum at the left is started in motion (by lifting the bob out of the plane of the paper and releasing it), its swinging will start the rod rocking back and forth at the same frequency. The rocking motion of the rod will then act as the driving force for the other five pendula. The middle pendulum of the five on the right has the same length and thus the same natural frequency as the moving pendulum on the left; it will therefore be driven into oscillation by the pendulum on the left. The other four, having different lengths, and therefore different frequencies, are not driven

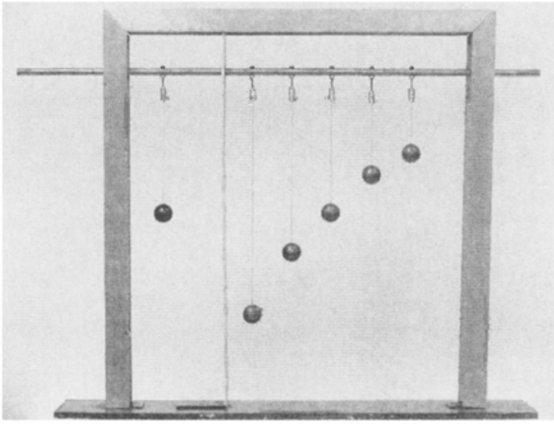

Figure 1-20 Coupled pendulum system. Suspension rod is free to rock in holes in the support plates.

to large amplitudes. In fact, in this demonstration the motion transfers almost completely back and forth between the two identical pendula, while the other four pendula, not being in resonance, execute smaller oscillations similar to those shown in Figure 1-19. Their motion goes in and out of phase with the motion of the driving pendulum at the left, so the amplitudes of their oscillations will alternately increase to a small level and then decrease to zero.

One of the most dramatic mechanical resonances arose when, owing to the action of the wind, certain oscillations built up in the Tacoma Narrows Bridge. Eventually the amplitude of the oscillations exceeded the elastic limit (breaking point) of the material in the bridge and caused its collapse on November 7, 1940.

Another example of resonance can be illustrated by using two identical tuning bars, say at 440 Hz. One is struck, and then held near the other. A resonance condition exists between the two tuning bars, with air providing the coupling, and *sympathetic vibrations* of the second tuning bar result. If either of the tuning bars is tuned slightly higher or lower, at 441 Hz for instance, the two do not have the same natural frequencies; no resonance condition exists, and therefore no significant sympathetic vibrations result.

Pendula and simple harmonic motion have found their way into magic and the occult. One of the topics for charlatans throughout the ages has been in communication using "brain waves" or other indirect forms of communication, or sending out "brainwave vibrations" to make something move, as in the case of the Ouija® board. Mental control over physical motion, whether or not it exists, is called *psychokinesis*. Debunking this concept has been the subject of serious scientific research for over 100 years. A simple psychokinetic device, the *psychoacoustic vibration transducer*, consists of a straw with four pendula of different lengths made from thread and paper clips, as shown in

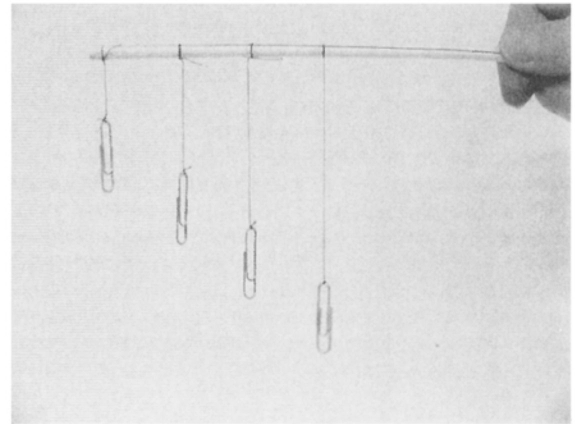

Figure 1-21 A carefully crafted "psychoacoustic vibration transducer."

Figure 1-21. It is claimed to convert low frequency "brain waves" or *psychoacoustic* vibrations from the participants into vibrations of one of the pendula. The demonstrator holds the device up in front of a group of people, stops all motion of the pendula, and asks the group to concentrate their "psychic" energy on a pre-selected pendulum known to the entire group as well as the demonstrator. Shortly after the group begins to focus on that pendulum, it begins to move, attaining a large amplitude of oscillation, while the others either remain at rest or oscillate with much less amplitude.

In fact, there is no such thing as psychoacoustic waves, and this explanation is entirely without scientific basis. Although members of the group may be inherently skeptical, unless they understand the concept of driven mechanical resonance they are compelled to accept the demonstrator's phony explanation. What is really happening is a driven mechanical resonance. The demonstrator moves the straw back and forth almost imperceptibly at the natural frequency of the selected pendulum. That pendulum experiences a *resonance* and the amplitude of its oscillation rapidly begins to increase, like the pendulum shown in Figure 1-18. The other pendula react like that shown in Figure 1-19; if they get in phase with the driving force they will begin to oscillate, but a short time later the motion becomes out of phase with the driving force and is rapidly reduced.

If you have any doubts that people actually believe this nonsense, type "psychic pendulum" or "psychokinesis pendulum" into your Web browser's search engine.

1.5 Combinations of Simple Harmonic Oscillations

Several interesting applications of simple harmonic motion involve two or more simple harmonic oscillations occurring simultaneously. One particularly beautiful example of this is the children's drawing toy called the *spirograph*®, which uses a circle rotating within a larger circle to create beautiful patterns made up of two mutually perpendicular

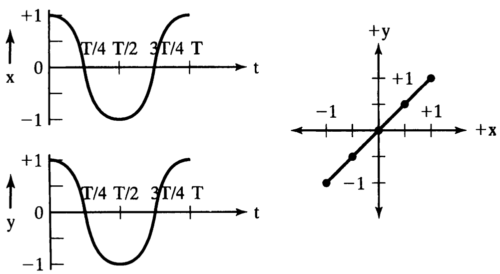

Figure 1-22 Reference graphs (left) and Lissajous figure with $A_x = A_y$ and $f_x = f_y$. The x and y oscillations are in phase.

oscillations. A number of Web sites discuss the mathematical nature of these oscillations, and the "Spirograph Nebula" is so named for its similarity to the drawings produced by this toy. Laser light shows often employ this technique to create fascinating patterns of laser light on clouds or walls. These patterns, called *Lissajous figures* after the French mathematician Jules Antoine Lissajous (1822–1880), have been used by physicists in comparing frequencies and phases of oscillations, among a host of other applications.

How these patterns are created can be understood by referring to Figures 1-22 and 1-23. Consider first the combination of x and y motion with equal amplitudes, in phase, as shown in Figure 1-22. One period T is shown in the *reference graphs* at the left; the motion repeats itself after each period. Shown at the right of the graphs of x and y is the curve traced out by a point executing these motions simultaneously. Following the motion, we notice that at $t = 0$, $x = 1$ and $y = 1$; at $t = T/4$, $x = 0$ and $y = 0$; at $t = T/2$, $x = -1$ and $y = -1$; at $t = 3T/4$, $x = 0$ and $y = 0$; and at $t = T$, $x = 1$ and $y = 1$ (the same as $t = 0$). In fact, $y = x$ for any time t, which is simply the equation of a line at an angle of 45° with respect to the axes, as seen. Now suppose that the relative phase of the oscillations is changed so that the x oscillation is 90° in phase *ahead* of the y oscillation. The oscillations, along with the resultant Lissajous figure, are shown in Figure 1-23. In this case, the point moves counterclockwise in a circle of radius 1: at $t = 0$, $x = 0$ and $y = -1$; at $t = T/4$, $x = 1$ and $y = 0$; at $t = T/2$, $x = 0$ and $y = 1$; at $t = 3T/4$, $x = -1$ and $y = 0$; and at $t = T$, $x = 0$ and $y = -1$ (the same as $t = 0$). Two other phase relationships are interesting: the x motion *out of phase* with the y motion, and the x motion 90° behind the y motion. These two cases are the subject of an exercise.

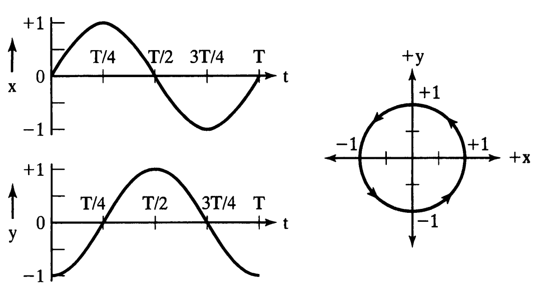

Figure 1-23 Reference graphs (left) and Lissajous figure with $A_x = A_y$ and $f_x = f_y$. The x oscillation is 90° in phase ahead of the y oscillation.

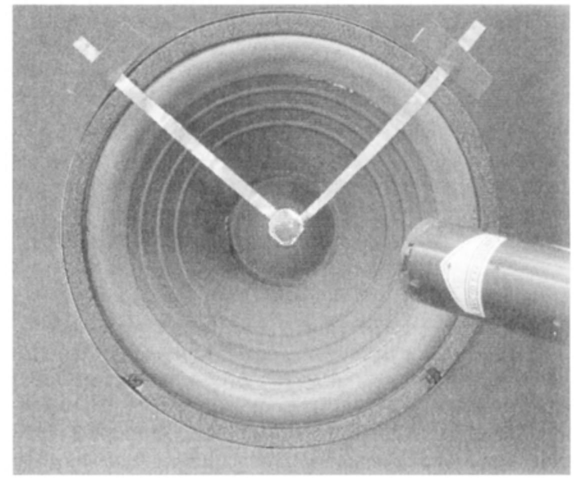

Figure 1-24 Nine-inch woofer with mirror assembly and laser.

A well-known example of Lissajous figures is the laser sound-and-light show. Music is played on a speaker that has been fitted with a mirror suspended by two perpendicular copper strips, as seen in Figure 1-24. A laser beam is reflected off the mirror onto a projection screen. The suspension creates some twisting motion that can be enhanced by displacing the mirror slightly to one side of the axis of the speaker. Because the motion of the mirror then contains two perpendicular components, the reflected light moves in two-dimensional patterns, as seen in Figure 1-25. The pattern at the left is for a simple wave, and the pattern at the right is for a more complicated musical tone.

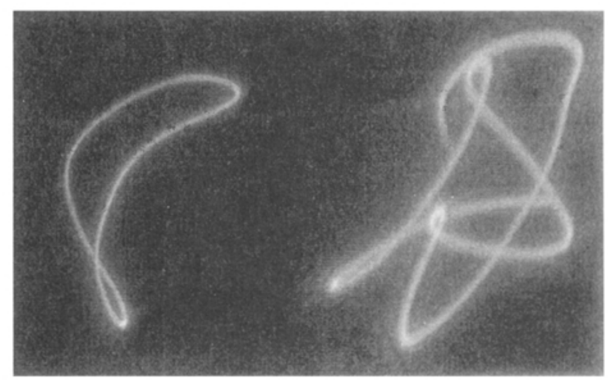

Figure 1-25 Laser light pattern from a 100 Hz tone (left) and music (right).

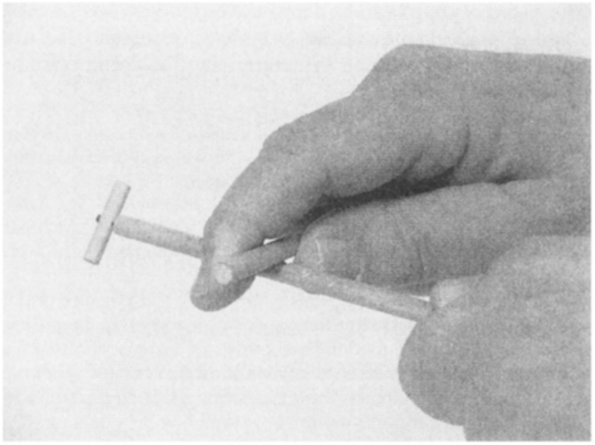

Figure 1-26 A notched stick with rotor with hand carefully positioned to produce "psychokinetic" propeller reversal when the notched stick is stroked.

An interesting toy using two mutually perpendicular oscillations, sometimes also claimed to demonstrate psychokinetic effects, is a *notched stick with rotor*, photographed in Figure 1-26. One stick has six or eight uniformly spaced notches and a well-balanced, loose-fitting propeller on the end. When you hold your hand as shown in the photograph and stroke the notches on top of the long stick with the shorter stick, the propeller rotates. If you push your thumb against the side of the stick as you stroke the rod back and forth, the propeller will rotate counterclockwise, as viewed by the operator. In this case, you are using your thumb to couple the vertical oscillations of your hand as you stroke the notches into horizontal oscillations of the notched stick. The horizontal oscillation is 90° in phase ahead of the vertical oscillation of the end of the notched stick, causing the end of the stick to rotate rapidly counterclockwise and thereby imparting a counterclockwise circulation to the propeller. If you contact the *opposite* side of the notched stick with your *forefinger*, you reverse the phase of the horizontal oscillation with respect to the vertical oscillation, so the horizontal oscillation is 90° in phase behind the vertical oscillation of the end of the notched stick. This reverses the rotation of the end of the stick and thus the circulation of the propeller. Sliding your hand back and forth a very small amount as you stroke the notched stick, so that either your thumb or your forefinger touches its side of the notched stick, the propeller can be made to reverse rotation direction apparently at will, and the reversal can (erroneously) be attributed to psychokinetic powers.

An old Appalachian folk toy called the "Gee Haw Whimmey Diddle"® uses the sound of the stick scraping over the notches as a rhythm instrument for folk music. If you use it to answer questions, you might call it a "Ouija® windmill." A "hooey stick" reverses direction when a selected observer shouts "hooey!"

The speed of sound can be determined by a very beautiful experiment using Lissajous figures; this technique is the subject of one of the questions at the end of the chapter. A more basic method of determining the speed of sound in air will be discussed in detail in Section 2.1.

SUMMARY

(***Section 1.1***) The physical description of motion in physics uses a well-defined system known as **metric units**. Fundamental metric units include the **kilogram** (kg) for **mass**, the **meter** (m) for **distance**, and the **second** (s) for **time**. Other quantities are derived from these fundamental units, such as **velocity** (m/s), **acceleration** (m/s^2), **force** (N), and **pressure** (Pa). (***Section 1.2***) **Periodic motion** is motion that repeats after some specific time interval. **Simple harmonic motion (SHM)** is a specific type of periodic motion that consists of a single vibration frequency, and is mathematically the **projection** of **uniform circular motion**. Such simple systems as the mass on a spring and the pendulum execute simple harmonic motion. The **frequency** f (in Hertz) and the **period** T (in seconds) of simple harmonic motion are related by the equation $fT = 1$. The **amplitude** A represents the maximum change from equilibrium. **Phase** ϕ is a quantity used to define the time relationship between two waves. (***Section 1.3***) The physical properties of waves are related to the psychophysical responses perceived by our brains. The **amplitude** A of a wave is related to its **intensity** or **loudness**, the **frequency** of a wave is related to its **pitch**, and the **wave shape** is related to the **timbre**, or **tone quality**, of the wave. (***Section 1.4***) **Damping** refers to the loss of energy in a vibrating system, causing a decrease in the amplitude of the oscillation. A **driven resonance** occurs when a system with some **natural frequency** is driven by an external periodic force with the same frequency. Two vibrating systems with the same natural frequency transfer energy between each other by means of a **coupling resonance**. **Psychokinesis** is the mythical claim that waves from the brain can actually make physical objects, such as pendulums, move. (***Section 1.5***) Combinations of simple harmonic motions such as **Lissajous figures** have applications in physics, music, and art.

QUESTIONS

1. Define and state the units of the following fundamental physical quantities:

a. position
b. time
c. velocity
d. acceleration
e. mass
f. force
g. weight
h. pressure
i. density

2. a. What is a linear restoring force?
   b. How is SHM related to a linear restoring force?

3. a. Give examples of motion that is periodic but not simple harmonic.
   b. Give examples of SHM.
   c. Is the brightness of a flashing automobile turn-signal light an example of simple harmonic motion? Is it periodic?

4. Draw graphs of the following:
   a. a triangular wave of period $T = 5$ ms and amplitude $A = 2$ V
   b. a square wave of period $T = 10$ ms and amplitude $A = 3$ V
   c. a sawtooth wave of period $T = 8$ ms and amplitude $A = 1$ V

   Calculate the frequencies of these waves.

5. Draw a 100 Hz sine wave with a 3 V amplitude. For each of the following parts, draw a wave that differs from this sine wave in the characteristic specified:
   a. greater in frequency
   b. lower in amplitude
   c. different in wave shape
   d. greater in period
   e. different in phase
   f. greater in intensity
   g. different in tone quality
   h. lower in pitch
   i. louder

6. a. Draw a graph of motion that decreases in amplitude with time but remains constant in period.
   b. Draw a graph of motion that decreases in *both* period and amplitude as time progresses. Which is damped harmonic motion? Which is not? Explain.

7. a. Draw a graph of two sinusoidal waves of the same frequency and amplitude that differ in phase by 180°.
   b. What term do we apply to two waves that have this phase relationship?
   c. Draw a graph of two sinusoidal waves of the same frequency but with a phase difference of 90°.
   d. Identify which wave is ahead in phase.
   e. Do the same for two waves that differ in phase by 45°.

8. Describe physically the relationship between the motions of two pendula whose oscillations
   a. are in phase
   b. are out of phase
   c. differ in phase by 90°

9. A mass is suspended on an ideal spring. It is lifted up 5 cm and released from rest at $t = 0$, executing simple harmonic motion with a period of 1 s. Draw a graph of the motion beginning at $t = 0$ and including two full periods of the oscillation. Assume that the equilibrium position for the weight on the spring at rest is $x = 0$, and that up is positive.

10. Define *resonance*. Give some examples of resonance from music and other fields.

11. A pendulum, initially at rest, can be placed into motion in a plane by a series of small pushes at equal time intervals, similar to the way you would push a small child on a swing. Describe what happens as the swing of the pendulum becomes bigger *using the vocabulary of physics*. What is the name for this phenomenon?

12. Draw graphs of a musical tone that:
    a. becomes louder with time
    b. becomes lower in pitch with time
    c. becomes simultaneously softer and higher in pitch

13. Construct a "psychoacoustic vibration transducer" as described in Section 1.4 of this book and learn how to make it work. Try it on people of various levels of physics sophistication, including children, and record their reactions. Ask people why they do or do not believe it. Explain the responses in terms of how people understand physics.

14. Figures 1-22 and 1-23 show Lissajous figures for x and y oscillations of equal amplitude with the x oscillation *in phase* with the y oscillation and with the x oscillation 90° *ahead* of the y oscillation. Draw the reference graphs and the Lissajous figures for
    a. the case in which the x oscillation is *out of phase* with the y oscillation
    b. the case in which the x oscillation is 90° *behind* the y oscillation

15. Make a "notched stick and rotor" using a $\frac{1}{4}$" wooden dowel rod, as described in Section 1-5 of this book and learn how to make it reverse when a subject says "hooey!" Try it on people of various levels of physics sophistication, including children, and record their reactions. Ask people why they do or do not believe it. Explain the responses in terms of how people understand physics.

16. Set up an oscilloscope in the xy mode to show Lissajous figures with sound waves as follows: connect an oscillator, set at about 5000 Hertz, to both a loudspeaker and the horizontal input of an oscilloscope, and connect a microphone to the vertical input of the oscilloscope. (You may need to use an amplifier to increase the sound level for the loudspeaker or to boost the microphone output for the oscilloscope.) Position the microphone in front of the loudspeaker and vary their spacing to change the shape of the pattern. Explain why the pattern is changing. Determine one wavelength λ of the sound wave by changing the Lissajous pattern through one cycle of its pattern (for example, starting in phase and moving the microphone until the pattern becomes in phase again). Use $S = f\lambda$ to determine the speed of sound S.

17. Stretch a string tightly between two points at the same height. Hang two pendula of slightly different lengths from the stretched string. Start one of the pendula in motion perpendicular to the stretched string and notice what happens. Change the length of one of the pendula and repeat the experiment. Summarize your results as to how the length of the second pendulum relative to the first pendulum affects their interaction.

PROBLEMS

1. Determine the frequencies of oscillations with the following periods: 1 s, 0.2 s, 10 ms, 0.0002 s, and 20 μs. (Use kilohertz and megahertz where appropriate to keep the numbers simple.)

2. Determine the frequencies of oscillations with the following periods: 0.5 s, 0.1 s, 20 ms, 0.001 s, and 40 μs. (Use kilohertz and megahertz where appropriate to keep the numbers simple.)

3. Determine the periods of oscillations with the following frequencies: 0.5 Hz, 2 Hz, 100 Hz, 5000 Hz, and 10,000 Hz. (Use milliseconds and microseconds where appropriate to keep the numbers simple.)

4. Determine the periods of oscillations with the following frequencies: 1 Hz, 20 Hz, 250 Hz, 10,000 Hz, and 50,000 Hz. (Use milliseconds and microseconds where appropriate to keep the numbers simple.)

5. Convert the following to Hertz: 1.5 kHz, 500 kHz, 2 MHz, and 1000 MHz. Why is it helpful to use the *kilo*- and *Mega*- prefixes?

6. The frequency range of human hearing is about 20 Hz to 20 kHz. What are the periods of these oscillations?

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PRINT DEMONSTRATION REFERENCES

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CARPENTER, D. RAE, and RICHARD B. MINNIX, *The Dick and Rae Demo Notebook*. Lexington, VA: Virginia Military Institute, 1993.

EDGE, R. D., *String and Sticky Tape Experiments*. College Park, MD: American Association of Physics Teachers, 1987.

These three books contain a large number of physics demonstration experiments ideal for either use in class or construction by students, and form an excellent nucleus for developing neat, easy-to-make classroom experiments. See especially their sections on vibrations, waves, and sound.

WEB DEMONSTRATION REFERENCES

Physics Instructional Resource Association (PIRA): Physics Resources on the Web:
http://www.wfu.edu/physics/pira/Resources.htm

PIRA is an umbrella group for educators involved in developing physics classroom demonstrations, and virtually any experiment used in a physics class can be discovered on this site.

The University of Maryland Physics Lecture-Demonstration Facility Web site:
http://www.physics.umd.edu/lecdem/
contains a library of over 1500 demonstration descriptions. The page:
http://www.physics.umd.edu/lecdem/services/demouse/phys102sugg.htm
is a list of over 200 demonstrations used with the course for which this textbook was written and organized by chapter and section in this textbook, all linked to the demonstration pictures and descriptions.

AUDIOVISUAL DEMONSTRATION REFERENCES

Physics Demonstrations in Sound and Waves, Parts I, II, and III, Physics Curriculum and Instruction, three videotapes, each with eight segments about 3 minutes in length, for a total of less than 30 minutes apiece. The tapes are of excellent quality and cover several areas of vibrations and sound.

Berg, Richard E., and David G. Stork, *Demonstrations in Acoustics.* College Park, MD: University of Maryland, Department of Physics, 1980. This is a set of four 1-hour color videos that show many of the experiments described in the present text being performed and gives brief explanations of them.

The Science of Sound, Folkways Record Album No. FX6136, Descriptive Literature by Bell Telephone Laboratories. Available from Frey Scientific Co., Mansfield, OH. This is an excellent recording, covering many areas of acoustics, that can be readily used in class lectures. The records contain short segments illustrating such topics as the overtone series, tone quality, filtering, distortion, reverberation, the Doppler effect, and others.