Comments/Corrections for Semigroup congruences: computation techniques and theoretical applications

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I agree with all comments by Martyn Quick (MRQ). I will not repeat these points, except to add further clarifications or requests.

A * indicates that a change might have consequences throughout the thesis. Critical sections should be identified and checked.

Page 12: The results from [ABG18] are not cited correctly.

*Page 18, Def 1.6 describes subuniverses, instead of subsemigroups. It also does not distinguish between an operation and its restriction.

Page 19, top three points: the definite article is not justified without further elaboration.

*Page 21 (further to MRQ): note that the change to the definition of a monoid homomorphism has consequences. Where relevant it needs to be identified which type of homomorphism is under consideration.

Page 21, Prop 1.22: to serve as an alternative definition (as claimed in the preceding paragraph), an "if and only if" result is needed.

*Page 23, Definition 1.28 (MRQ): check for consequences

Page 24: relations are assumed to be binary throughout the thesis. This should be stated somewhere suitable.

*Page 28: Def 1.43, $X \neq \emptyset$ in the semigroup case.

*Page 29: Def 1.47: check consequences (!)

Page 30, Ex 1.49: Equality notation not yet defined

Page 35, Theorem 1.63: no citation. Also s needs to be in S^1 (in addition to MRQ's remarks)

Page 35, Th. 1.66: citation missing.

*Page 39, Def 1.77, Ex 1.78(see MRQ): check for consequences

*Page 44ff (MRQ), and pseudocode algorithms in general: it will greatly help confirm (partial) correctness if you provide loop-invariants for your algorithms.

*Page 51-52, section 2.2: In addition to MRQ's remarks about "concrete semigroups" and the consequences to the rest of the thesis, I think this section should be rewritten to explain "concrete" both abstractly and by example.

Page 52, -3: strictly speaking, X is not a subset of X^+. You need to make an identification.

Page 53, Section 2.4: The fact that some of the algorithms do not truly answer the tasks is a mayor problem. These limitations should be addressed, and discussed both in this summary and also in connection to the individual algorithms. Potential improvements to the data structures should at least be mentioned. Also, the "class number of an element" is dependent on some ordering of the classes and not only on the congruences themselves. This requirement should be reformulated.

Page 54, proof of Method 2.4: You also need that π is surjective. Also, there is a formatting error in $\bar{x_1}$, and similar expressions.

Page 56, second point: "confluent rewriting system" not defined yet.

*Page 59 top: it seems you need S^1 to stand for the semigroup with added identity, even if S is already a monoid. Similar problems throughout the thesis, for example page 59, -9 and in interpreting Alg. 2.11

Page 61, Alg. 2.11: ε's should be 0.

Page 74, l-2: There are additional reasons that guarantee termination. For a (simple) example, the system could already be confluent terminating (see example 2.46)

*Page 92, def 3.3-3.5 (MRQ): Check consequences.

Page 93, def 3.6: the last point should be under the definition part, not under "where"; 0 cannot be in any of the other sets, and depending on your changes to the previous definitions, I and λ might need to be non-empty.

Page 97 3.1.5: How do you represent a Rees congruence?

Page 101ff, section 3.2.4: Several results in this section require the congruence to be non-universal.

Page 104, Alg. 3.26: You need the semigroup itself as input for the algorithm.

Page 106, Alg. 3.28, line 18: the double bracket notation is only defined for groups.

Page 113, def. 3.34: this definition is very difficult to read. It would help to define π_K separably.

Page 118, l-13: With the description given, the algorithm would need to run once with a reflexive pair to generate Δ. Of course, it would be faster to just add Δ without calculation.

Page 127, paragraphs 3/4: Please describe more precisely your individual contribution to [EMRT18].

Page 129; Def 5.2: "can be represented" in this definition is imprecise. For example, β in Example 5.3 "can" be represented without crossing edges if we order the points differently. You need to make clear which representation are allowed.

Page 131, Def. 5.11: "a" liftable ....

Page 132, proof of Th 5.15: There is a left/right clash between the displayed line (sx), and the line before (xs).

Page 134: Proposition 5.22: This is not yet a characterisation, as you have not yet shown the absence of other congruences.

Page 135, Theorem 5.23: part (iv) holds (more or less trivially) in any semigroup (actually any algebra).

Page 135ff: As pointed out Theorem 5.23 requires $n \geq 2$. The formulation of Lemma 5.26, 5.27, ect, also make sense for $n = 1$, but the proofs might be affected. Please check, and adapt if necessary.

*Page 136, Figure 5.34 and similar figures later: The figures need to have a certain minimal value of n to represent the congruence lattice. These values should be mentioned in the caption of each figure.

Page 137, line 13: A_1 " or B_1".

Page 138f, Proof of 5.27: There are quite a few instances (too many to list) of typos, or were the calculated expression is only valid in special subcases (MRQ gives an exampe). All expressions should be rechecked.

Page 140, line 6: The definition of x does not work if $r = 0$. Of course, this case is easily dismissed, but needs to be mentioned.

Page 146, line 2: A_2 will not be used either.

Page 152: definition of $\circ$: first case needs condition $a, b, ab \in J$.

Page 152, line 10: The 0 is not necessarily an appended zero. An appended 0 would only have $ab = 0$ if $a = 0$ or $b = 0$.

Page 153, line 2: The permutations on $\{1, \ldots, n\}$ are not assignments of kernels to images.

*Page 155ff: "1" is used as both an element of the underlying set of T_n, as the identity of the group G, and as standing for a trivial group. Different symbols should be used for all cases.

Page 156, line-2: "codomain" should be "domain".

Page 165, Figure 6.19: Notation M not introduced.

Page 167ff, 6.3: As pointed out during the viva, using the number of semigroups up to isomorphism and anti-isomorphism is problematic because it "double-counts" commutative semigroups. Consider revising the counts.

Page 168, last paragraph: I believe that there was a problem with the result on 3-nilpotent semigroups. Please research and revise, if necessary.

*Page 176: Note that the list of notations and the index might be affected by other changes.