Introdução à Física Computacional (4300218)

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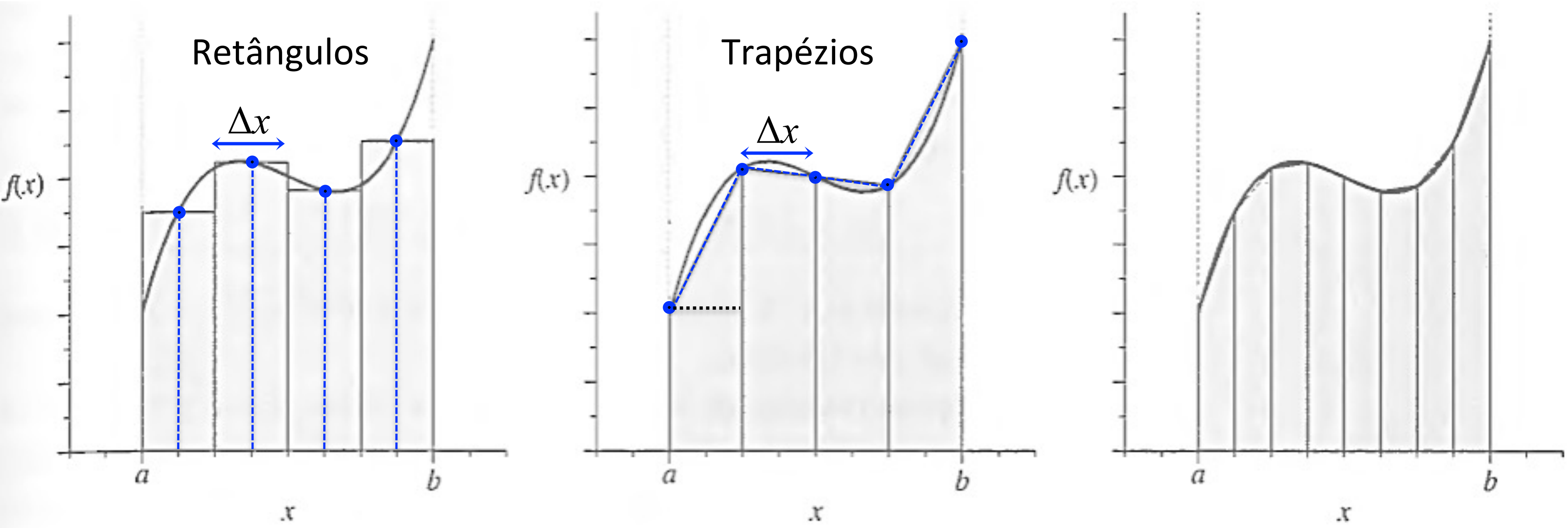
Aula 8

Programação em Pythom para físicos:
Integração: Trapézio e Simpson

Integração por Trapézios



- O método do trapézio é um pouco mais preciso que o método do retângulo.

$$\text{Área} = \frac{1}{2}\Delta x\left(f(x_{i+1}) + f(x_i)\right)$$

$$I = \int_a^b f(x)dx = \frac{1}{2}\Delta x\left(f(a) + f(b)\right) + \Delta x \sum_{k=1}^{N-1} f(x_i + k\Delta x)$$

Exemplo 1:

Realizar a integral abaixo usando o método dos trapézios com N= 10, 20, 30, ..., 200, onde N é o número de trapézios no interval de integração.

$$\int_0^2 (x^4 - 2x + 1)dx = \left[\frac{1}{5}x^5 - x^2 + x\right]_0^2 = 4.4$$

Faça o grafíco de f(x) entre x= 0 e 2, e o gráfico do valor da intergral variando com N.

Programa

```python
from pylab import *
from numpy import *

def f(x):
    return x**4 - 2*x + 1

a  = 0.0
b  = 2.0
x  = []
y  = []
fxi= []
xi = []
xmin= 10
xmax= 200
xint= 10
x0= [0,xmax]
y0= [4.4,4.4]

s0 = (f(a) + f(b))*0.5
for N in arange(xmin,xmax+xint,xint):
    s = 0.00
    dx = (b-a)/N
    for k in range(1,N):
        fk = f(a+k*dx)
        s += fk
    s = (s0 + s)*dx
    x.append(N)
    y.append(s)
    print(N,s)

dx = (b-a)/xmax
for k in arange(a,b+dx,dx):
    fxi.append(f(k))
    xi.append(k)
```

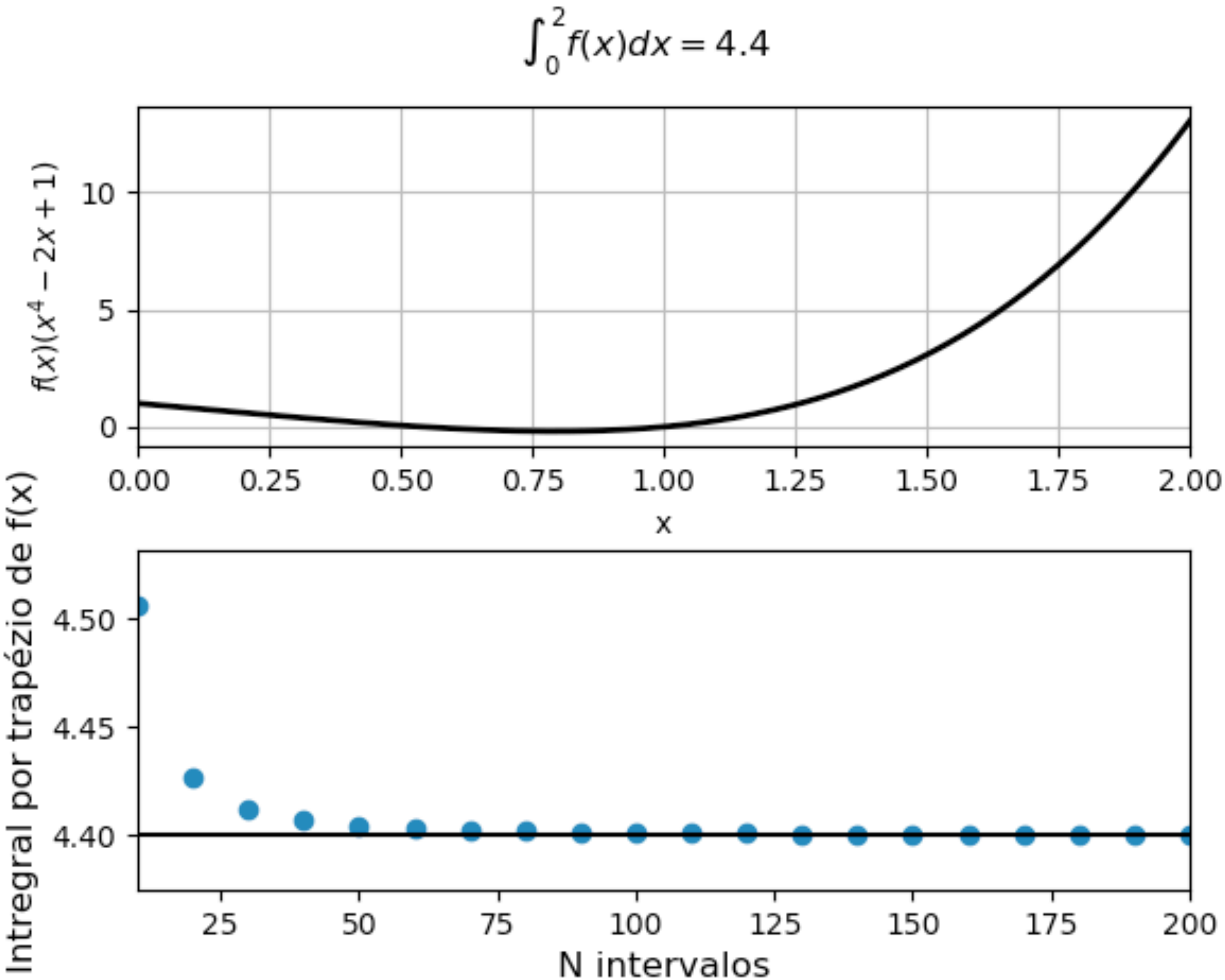
$$I = \int_a^b f(x)dx$$

$$I = \frac{1}{2}\Delta x \left(f(a) + f(b)\right)$$

$$+\Delta x \sum_{k=1}^{N-1} f(x_i + k\Delta x)$$

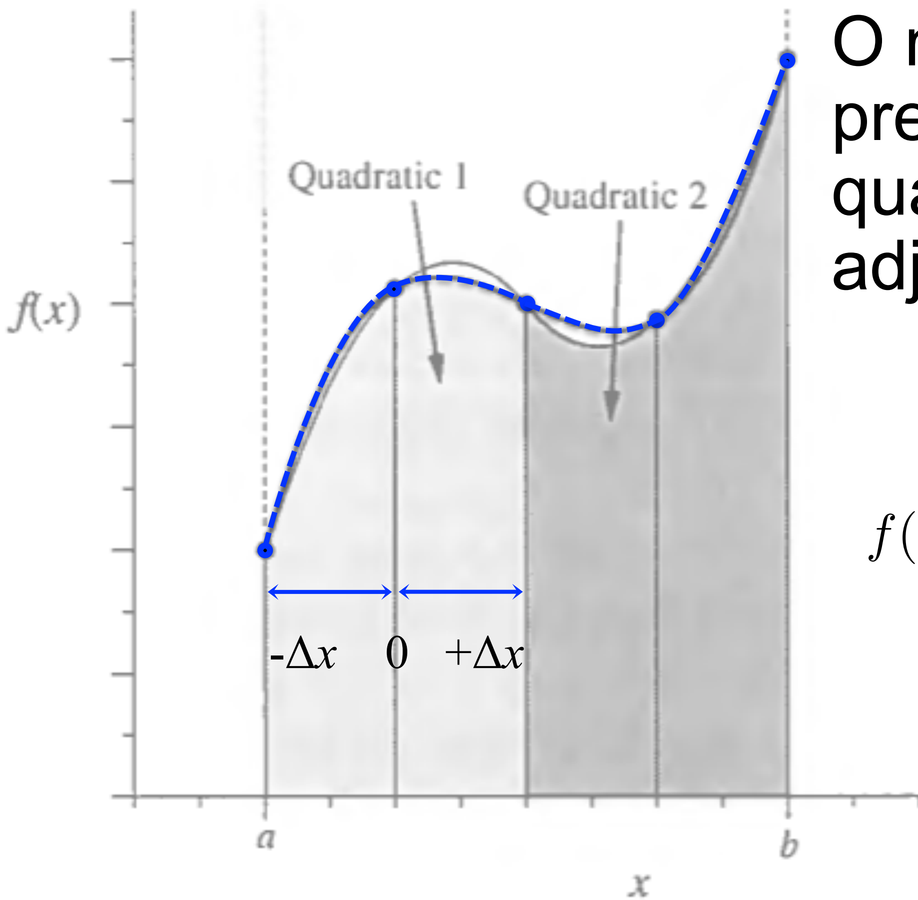
N	I(N)
10	4.50656
20	4.42666
30	4.41185
40	4.40666
50	4.40426
60	4.40296
70	4.40217

```python

subplots_adjust(hspace=0.3,wspace=0.2)
subplot(2,1,1)
plot(xi,fxi,color='black',linewidth=2)
xlabel("x",fontsize=10)
ylabel(r'$f(x)(x^4 -2x + 1)$',fontsize=10)
grid(True)
xlim(a,b)

subplot(2,1,2)
scatter(x,y)
plot(x0,y0, color='black')
suptitle(r'$\int_{0}^{2} f(x) dx = 4.4$',fontsize=12)
xlabel("N intervalos",fontsize=12)
ylabel("Intregral por trapézio de f(x)",fontsize=12)
xlim(xmin,xmax)
show()
```



$\int_0^2 f(x)dx = 4.4$
$f(x)(x^4 - 2x + 1)$
0
5
10
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00
x
Intregral por trapézio de f(x)
4.40
4.45
4.50
25
50
75
100
125
150
175
200
N intervalos

Integração por Simpson



O método de Simpson é mais preciso e utiliza função quadrática entre dois intervalos adjacentes.

$$f(x) = Ax^2 + Bx + C$$

$$\text{onde } x = -\Delta x, 0, +\Delta x$$

$$f(\pm\Delta x) = A\Delta x^2 \pm B\Delta x + C \text{ e } f(0) = C$$

$$A = \frac{1}{\Delta x^2}\left[\frac{1}{2}f(-\Delta x) - f(0) + \frac{1}{2}f(\Delta x)\right]$$

$$B = \frac{1}{2}\Delta x\left[f(\Delta x) - f(-\Delta x)\right] \text{ e } C = f(0)$$

$$\text{Área} = \int_{-\Delta x}^{\Delta x}(Ax^2 + Bx + C)dx = \frac{2}{3}A\Delta x^3 + 2C\Delta x = \frac{1}{3}\Delta x\left[f(-\Delta x) + 4f(0) + f(\Delta x)\right]$$

Integração por Simpson

$$I = \int_a^b f(x)dx = \frac{1}{3}\Delta x \left(f(a) + f(b)\right)$$

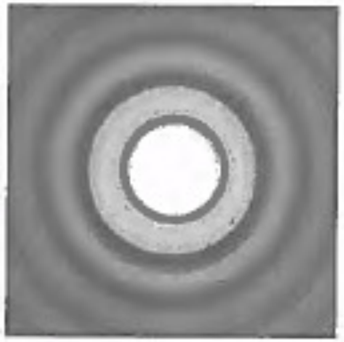
$$+\frac{4}{3}\Delta x \sum_{\substack{k=1 \\ impar}}^{N-1} f(a + k\Delta x) + \frac{2}{3}\Delta x \sum_{\substack{k=2 \\ par}}^{N-2} f(a + k\Delta x)$$

Exemplo 2:

Repetir o Exemplo 1 com Simpson e comparar os valores com ambos os métodos.

Exercise 5.4: The diffraction limit of a telescope

Our ability to resolve detail in astronomical observations is limited by the diffraction of light in our telescopes. Light from stars can be treated effectively as coming from a point source at infinity. When such light, with wavelength λ, passes through the circular aperture of a telescope (which we'll assume to have unit radius) and is focused by the telescope in the focal plane, it produces not a single dot, but a circular diffraction pattern consisting of central spot surrounded by a series of concentric rings. The intensity of the light in this diffraction pattern is given by



The diffraction pattern produced by a point source of light when viewed through a telescope.

$$I(r) = \left(\frac{J_1(kr)}{kr}\right)^2,$$

where r is the distance in the focal plane from the center of the diffraction pattern, $k = 2\pi/\lambda$, and $J_1(x)$ is a Bessel function. The Bessel functions $J_m(x)$ are given by

$$J_m(x) = \frac{1}{\pi}\int_0^{\pi} \cos(m\theta - x\sin\theta)\,d\theta,$$

where m is a nonnegative integer and $x \geq 0$.

a) Write a Python function `J(m,x)` that calculates the value of $J_m(x)$ using Simpson's rule with $N = 1000$ points. Use your function in a program to make a plot, on a single graph, of the Bessel functions J_0, J_1, and J_2 as a function of x from $x = 0$ to $x = 20$.

b) Make a second program that makes a density plot of the intensity of the circular diffraction pattern of a point light source with $\lambda = 500\,\text{nm}$, in a square region of the focal plane, using the formula given above. Your picture should cover values of r from zero up to about $1\,\mu\text{m}$.

Hint 1: You may find it useful to know that $\lim_{x\to 0} J_1(x)/x = \frac{1}{2}$. Hint 2: The central spot in the diffraction pattern is so bright that it may be difficult to see the rings around it on the computer screen. If you run into this problem a simple way to deal with it is to use one of the other color schemes for density plots described in Section 3.3. The "`hot`" scheme works well.