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Prova 1   26/04/2023

① Seja r_1: $3x-4y=7$;

r_2: $x-y=-3$;

Achar $D\in r_2$, tal que $dist(D, r_1)=2$

$D(p,q)=2$ $p-q=-3$ (1)

$3p-4q-7=0$ $\frac{|3p-4q-7|}{\sqrt{3^2+(-4)^2}}=2 \Rightarrow |3p-4q-7|=2\sqrt{25}$

$\Rightarrow |3p-4q-7|=10$

$\boxed{1}$ $3p-4q-7=10 \Rightarrow 3p-10-7=4q \Rightarrow q=\frac{3p-17}{4}$

de (1): $p-\frac{3p-17}{4}+3=0 \Rightarrow \frac{4p-3p+17+12}{4}=0$

$\Rightarrow \frac{p+29}{4}=0 \Rightarrow \boxed{p=-29}$

$q=\frac{3(-29)-17}{4} \Rightarrow q=\frac{-104}{4} \Rightarrow \boxed{q=-26}$ $\boxed{D_1=(-29,-26)}$

$\boxed{2}$ $-(3p-4q-7)=10 \Rightarrow -3p+4q+7=10 \Rightarrow q=\frac{10+3p-7}{4} \Rightarrow q=\frac{3p+3}{4}$

de (1): $p-\frac{3p+3}{4}+3=0 \Rightarrow \frac{4p-3p-3+12}{4}=0$

$\Rightarrow \frac{p+9}{4}=0 \Rightarrow \boxed{p=-9}$

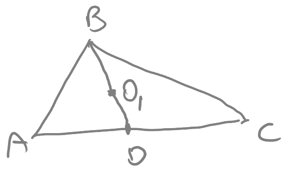
$q=\frac{3(-9)+3}{4} \Rightarrow q=\frac{-24}{4} \Rightarrow \boxed{q=-6}$

$\boxed{D_2=(-9,-6)}$

② Seja $\Delta = \Delta ABC$, $A=(-1,2)$, $B=(1,5)$, $C=(4,2)$

Achar

a) baricentro;



$$D=\left(\frac{-1+4}{2}, \frac{2+2}{2}\right) \Rightarrow D=\left(\frac{3}{2}, 2\right)$$

$$O_1=(p,q) \qquad \frac{DO}{OB}=2 \Rightarrow p=\left(\frac{1+2.3/2}{2+1}\right)$$

$$p=\frac{1+2.\frac{3}{2}}{2+1} \Rightarrow p=\frac{4}{3}$$

$$O_1=\left(\frac{4}{3}, 3\right)$$

$$q=\frac{5+2.2}{2+1} \Rightarrow q=3$$

b) ortocentro;

equação da reta AB

$$\frac{x+1}{1+1}=\frac{y-2}{5-2} \Rightarrow (x+1)3 = 2(y-2) \Rightarrow 3x+3=2y-4$$

$$\Rightarrow 3x+3-2y+4=0 \Rightarrow 3x-2y+7=0$$

$$\Rightarrow 3x+7=2y \Rightarrow \boxed{y=\frac{3x}{2}+\frac{7}{2}}$$

$$y=-\frac{2}{3}x+b \Rightarrow 2=-\frac{2}{3}(4)+b \Rightarrow 2=-\frac{8}{3}+b \Rightarrow b=\frac{14}{3}$$

$$\boxed{y=-\frac{2}{3}x+\frac{14}{3}} \quad (1)$$

equação da reta BC

$$\frac{x-1}{4-1}=\frac{y-5}{2-5} \Rightarrow -\cancel{3}(x-1)=\cancel{3}(y-5) \Rightarrow -x+1=y-5 \Rightarrow y=-x+6$$

$$y=x+b \Rightarrow 2=-1+b \Rightarrow 3=b$$

$$\boxed{y=x+3} \quad (2)$$

de (1) e (2):

$$-\frac{2}{3}x+\frac{14}{3}=x+3 \Rightarrow -\frac{2}{3}x-x=3-\frac{14}{3}$$

$$-\frac{5x}{3}=\frac{9-14}{3} \Rightarrow -5x=-5 \Rightarrow x=1$$

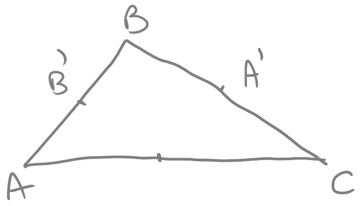
de (2):

$$y=1+3 \Rightarrow y=4$$

$\therefore$ ortocentro $(1,4)$

c) centro de circunferência circunscrita;

$O_3 = (p, q)$ $\qquad O_3 = A'S \cap B'T$



$A' = \left(\frac{1+4}{2}, \frac{5+2}{2}\right) \Rightarrow A' = \left(\frac{5}{2}, \frac{7}{2}\right)$

$B' = \left(\frac{-1+1}{2}, \frac{2+5}{2}\right) \Rightarrow B' = \left(0, \frac{7}{2}\right)$

$A'S \perp BC \quad y = -x+6 \qquad y = x+b \Rightarrow \frac{7}{2} = \frac{5}{2} + b \Rightarrow b = 1$

$y = x+1 \quad (1)$

$B'T \perp AB \quad y = \frac{3x}{2} + \frac{7}{2} \qquad y = -\frac{2}{3}x + b \Rightarrow \frac{7}{2} = -\frac{2}{3}.0 + b$

$\Rightarrow b = \frac{7}{2}$

$y = -\frac{2}{3}x + \frac{7}{2} \quad (2)$

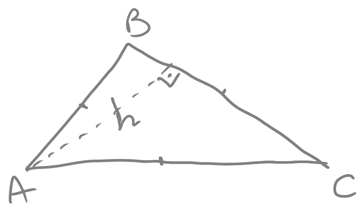
De (1) e (2): $\quad x+1 = -\frac{2}{3}x + \frac{7}{2} \Rightarrow x + \frac{2}{3}x = \frac{7}{2} - 1 \Rightarrow \frac{5x}{3} = \frac{5}{2}$

$\Rightarrow 10x = 15 \Rightarrow x = \frac{15}{10} \Rightarrow x = \frac{3}{2}$

De (1): $\quad y = \frac{3}{2} + 1 \Rightarrow y = \frac{5}{2}$

$O_3 = \left(\frac{3}{2}, \frac{5}{2}\right)$

d) área de Δ;



$h = dist(A, BC) \Rightarrow h = \frac{|q+p-6|}{\sqrt{1^2+1^2}} \Rightarrow h = \frac{|q+p-6|}{\sqrt{2}}$

$\overline{BC}: \; y = -x+6$

$y + x - 6 = 0$

$q + p - 6 = 0$

$\sqrt{2}\,h = |q+p-6|$

$\sqrt{2}\,h = q+p-6$

$p = \sqrt{2}\,h - q + 6$

$q = -p+6$

$p = \sqrt{2}\,h - (-p+6)$

$p = p - 6 + h\sqrt{2} \Rightarrow 0 = -6 + h\sqrt{2}$

$h = \frac{6}{\sqrt{2}}$

$\text{área } \Delta = \frac{\frac{6}{\sqrt{2}} \cdot 5}{2} \Rightarrow \text{área } \Delta = \frac{30}{\sqrt{2}} \cdot \frac{1}{2}$

$\text{área } \Delta = \frac{15}{\sqrt{2}} \Rightarrow \text{área } \Delta = \frac{15\sqrt{2}}{2}$

e) Raio da circunferência inscrita;

$$\text{Area } \Delta = r \frac{(|AB| + |BC| + |AC|)}{2}$$

$$|AB| = \sqrt{2^2+3^2} \Rightarrow |AB| = \sqrt{13}$$

$$|BC| = \sqrt{3^2+(-3)^2} \Rightarrow |BC| = \sqrt{18}$$

$$|AC| = \sqrt{5^2+0^2} \Rightarrow |AC| = 5$$

$$AB = (2,3)$$

$$BC = (3,-3)$$

$$AC = (5,0)$$

$$\frac{15\sqrt{2}}{2} = r \frac{(\sqrt{13} + \sqrt{18} + 5)}{2}$$

$$r = \frac{15\sqrt{2}}{\sqrt{13} + \sqrt{18} + 5} \Rightarrow r = \frac{15\sqrt{2}}{\sqrt{13} + 3\sqrt{2} + 5}$$

f) $\text{sen}(\alpha)$, $\alpha = \angle A$;

$$AB \to B - A \to (2,3)$$
$$AC \to C - A \to (5,0)$$

$$\cos(\alpha) = \frac{\vec{v}\ \vec{w}}{|\vec{v}||\vec{w}|}$$

$$\cos(\alpha) = \frac{(2,3) * (5,0)}{\sqrt{2^2+3^2} * \sqrt{5^2+0^2}} \Rightarrow \cos(\alpha) = \frac{10+0}{\sqrt{13} * 5} \to \cos(\alpha) = \frac{10}{5\sqrt{13}}$$

$$\cos(\alpha) = \frac{2}{\sqrt{13}} \to \cos(\alpha) = \frac{2\sqrt{13}}{13} \qquad \text{sen}(\alpha) = \sqrt{1 - \frac{2\sqrt{13}}{13}}$$

$$\text{sen}(\alpha) = \sqrt{\frac{13 - 2\sqrt{13}}{13}}$$