Nome: Douglas Ribas de Mattos   nUSP: 11010930

Lista 1 - Parte 1

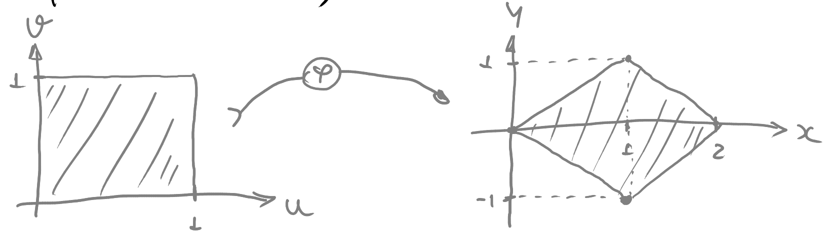
1 Exercícios 32.1

2) Considere a transformação φ de $\mathbb{R}^2$ em $\mathbb{R}^2$ dada por $x = u + v$ e $y = u - v$.

Desenhe $\varphi(B)$

(b) sendo B o quadrado $0 \leq u \leq 1$, $0 \leq v \leq 1$

$x = u + v$
$y = u - v$



3) Mostre que a transformação φ do exercício anterior transforma o círculo $u^2 + v^2 \leq r^2$ no círculo $x^2 + y^2 \leq 2r^2$.

$(x,y) = \varphi(u,v)$

$x = u + v$
$y = u - v$

$x^2 + y^2 = 2r^2$

$(u+v)^2 + (u-v)^2 = 2r^2$

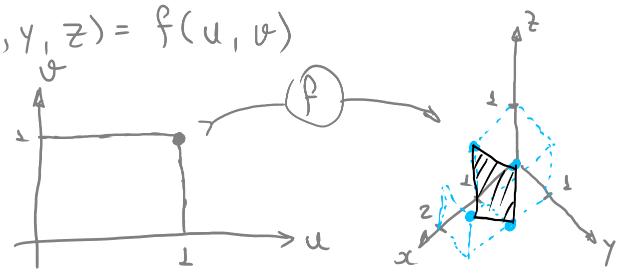
$u^2 + \cancel{2uv} + v^2 + u^2 \cancel{- 2uv} + v^2 = 2r^2$

$2u^2 + 2v^2 = 2r^2$

$2(u^2 + v^2) = 2r^2 \Rightarrow u^2 + v^2 = r^2$

4) Seja f a transformação de $\mathbb{R}^2$ em $\mathbb{R}^3$ dada por $(x,y,z) = (u+v, u, v)$. Mostre que f transforma o plano uv no plano $x - y - z = 0$.

$(x,y,z) = f(u,v)$



A: $(0,0) = (0,0,0)$
B: $(0,1) = (1,0,1)$
C: $(1,0) = (1,1,0)$
D: $(1,1) = (2,1,1)$

$n = y + z - x$

$n = (-1, 1, 1)$

$AB \Rightarrow (1,0,1)$

$AC \Rightarrow (1,1,0)$

$$\begin{vmatrix} i & j & k & i & j \\ 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 & 1 \end{vmatrix} = j + k - i$$

$(0 + i + 0) - (0 + j + k)$

$ax + by + cz + d = 0$ $d = 0$

$-x + y + z + d = 0$

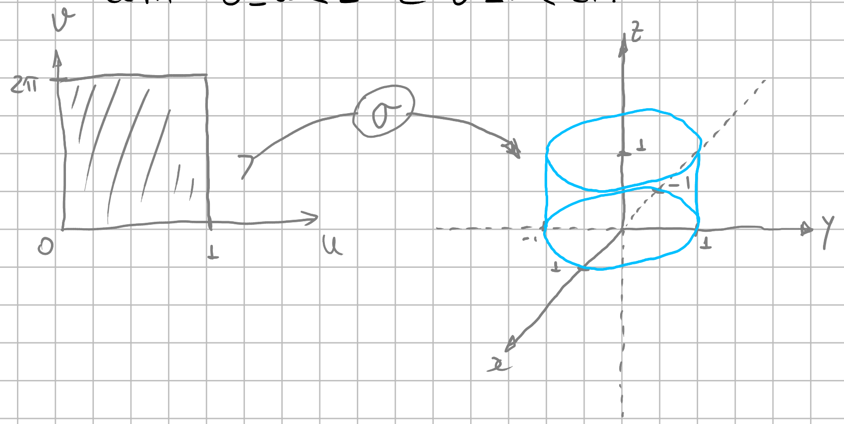
$-0 + 0 + 0 + d = 0$

$-x + y + z = 0$ ou

$x - y - z = 0$

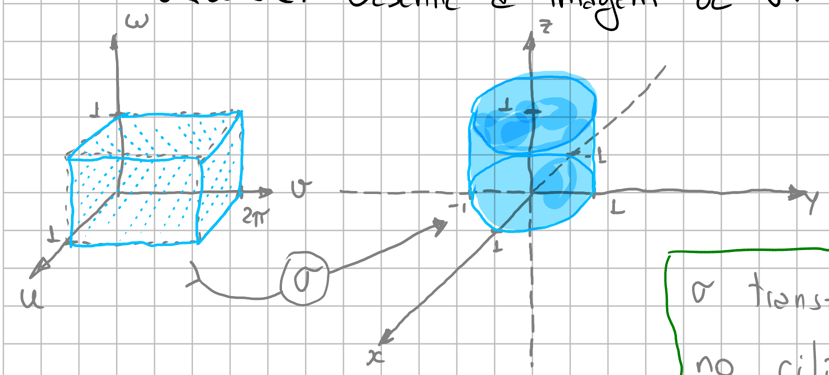
8) Desenhe a imagem de $\sigma(u,\vartheta) = (\cos(\vartheta), \text{sen}(\vartheta), u)$,

com $0 \leq u \leq 1$ e $0 \leq \vartheta \leq 2\pi$.



Superfície do cilindro

12) Seja $\sigma(u,\vartheta,\omega) = (u\cos(\vartheta), u\,\text{sen}(\vartheta), \omega)$, $0 \leq u \leq 1$, $0 \leq \vartheta \leq 2\pi$ e $0 \leq \omega \leq 1$. Desenhe a imagem de σ.



$x = u\cos(\vartheta)$

$y = u\,\text{sen}(\vartheta)$

$z = \omega$

σ transforma o paralelepípedo no cilindro

② Exercícios 33.6

1) Seja A o retângulo $1 \leq x \leq 2$, $0 \leq y \leq 1$. Calcule $\iint_A f(x,y)\,dx\,dy$, sendo $f(x,y)$ igual a

(c) $\sqrt{x+y}$

$$\int_0^1\int_1^2 \sqrt{x+y}\,dx\,dy \Rightarrow \int_0^1\int_{1+y}^{2+y} t^{1/2}\,dt\,dy \Rightarrow \int_0^1 \frac{2}{3}t^{3/2}\Big|_{1+y}^{2+y} dy$$

$x+y = t$
$dx = dt$

$2+y = u$
$dy = du$

$1+y = \vartheta$
$dy = d\vartheta$

$$\int_0^1 \frac{2}{3}\left[(2+y)^{3/2} - (1+y)^{3/2}\right] dy \Rightarrow \frac{2}{3}\int_0^1 (2+y)^{3/2} - (1+y)^{3/2}\,dy$$

$$\Rightarrow \frac{2}{3}\left[\int_2^3 u^{3/2}\,du - \int_1^2 \vartheta^{3/2}\,d\vartheta\right] \Rightarrow \frac{2}{3}\left[\frac{2}{5}u^{5/2}\Big|_2^3 - \frac{2}{5}\vartheta^{5/2}\Big|_1^2\right]$$

$$\Rightarrow \frac{4}{15}\left(\sqrt{3^5} - \sqrt{2^5}\right) - \frac{4}{15}\left(\sqrt{2^5} - \sqrt{1^5}\right) \Rightarrow \frac{4}{15}\left(9\sqrt{3} - 4\sqrt{2} - 4\sqrt{2} + 1\right)$$

$$\Rightarrow \frac{4}{15}\left(9\sqrt{3} - 8\sqrt{2} + 1\right)$$

(h) $\dfrac{1}{(x+y)^2}$

$$\int_0^1\int_1^2 \frac{1}{(x+y)^2}\,dx\,dy \Rightarrow \int_0^1\int_{1+y}^{2+y} \frac{1}{t^2}\,dt\,dy \Rightarrow \int_0^1\int_{1+y}^{2+y} t^{-2}\,dt\,dy$$

$x+y=t$
$dx=dt$

$$\Rightarrow \int_0^1 \frac{t^{-1}}{-1}\Big|_{1+y}^{2+y} dy \Rightarrow -\int_0^1 \frac{1}{t}\Big|_{1+y}^{2+y} dy \Rightarrow -\int_0^1 \frac{1}{2+y}-\frac{1}{1+y}\,dy \Rightarrow -\left[\ln|2+y|\Big|_0^1 - \ln|1+y|\Big|_0^1\right]$$

$$\Rightarrow -\left[\left(\ln|3|-\ln|2|\right)-\left(\ln|2|-\ln|1|\right)\right] \Rightarrow -\left[\ln|3|-\ln|2|-\ln|2|+\ln|1|\right]$$

$$\Rightarrow -\left[\ln|3|-2\ln|2|+0\right] \Rightarrow -\ln|3|+\ln|2|^2 \Rightarrow \ln\left|\frac{4}{3}\right|$$

(l) $x\,\mathrm{sen}(\pi y)$

$$\int_0^1\int_1^2 x\,\mathrm{sen}(\pi y)\,dx\,dy \Rightarrow \int_0^1 \mathrm{sen}(\pi y)\,dy \cdot \int_1^2 x\,dx$$

$\pi y = t$
$\pi\,dy = dt$

$dy = \frac{1}{\pi}dt$

$$\Rightarrow \frac{1}{\pi}\int_0^{\pi} \mathrm{sen}(t)\,dt \cdot \int_1^2 x\,dx \Rightarrow -\frac{1}{\pi}\left[\cos(t)\Big|_0^{\pi}\right]\cdot\left[\frac{1}{2}x^2\Big|_1^2\right]$$

$$\Rightarrow -\frac{1}{\pi}\left(\cos(\pi)-\cos(0)\right)\cdot\left[\frac{1}{2}2^2-\frac{1}{2}1^2\right] \Rightarrow -\frac{1}{\pi}(-1-1)\cdot\left(2-\frac{1}{2}\right)$$

$$\Rightarrow \frac{2}{\pi}\left(\frac{4-1}{2}\right) \Rightarrow \frac{3}{\pi}$$

(m) $\dfrac{1}{1+x^2+2xy+y^2}$ $\quad 1\le x\le 2,\ 0\le y\le 1.$ Calcule $\iint f(x,y)\,dx\,dy$

$$\int_0^1\int_1^2 \frac{1}{1+x^2+2xy+y^2}\,dx\,dy \Rightarrow \int_0^1\int_1^2 \frac{1}{1+(x+y)^2}\,dx\,dy$$

$x+y=t$
$dx=dt$

$$\Rightarrow \int_0^1\int_{1+y}^{2+y} \frac{1}{1+t^2}\,dt\,dy \Rightarrow \int_0^1 \mathrm{arctg}(t)\Big|_{1+y}^{2+y} dy \Rightarrow \int_0^1 \mathrm{arctg}(2+y)-\mathrm{arctg}(1+y)\,dy$$

$\int \mathrm{arctg}(x)\,dx$

	D	I
+	$\mathrm{arctg}(x)$	1
−	$\frac{1}{1+x^2}$	x

$$\int \mathrm{arctg}(x)\,dx = x\,\mathrm{arctg}(x) - \int \frac{x}{1+x^2}\,dx$$

$$\Rightarrow x\,\mathrm{arctg}(x) - \frac{1}{2}\int \frac{x}{x\,u}\,du$$

$1+x^2=u$
$2x\,dx=du$
$dx=\frac{1}{2x}du$

$$\int \mathrm{arctg}(x)\,dx = x\,\mathrm{arctg}(x) - \frac{1}{2}\ln|1+x^2|$$

$$\int_0^1 arctg(2+y) - arctg(1+y)\,dy \Rightarrow \int_0^1 arctg(2+y)\,dy - \int_0^1 arctg(1+y)\,dy$$

$2+y = \vartheta$
$dy = d\vartheta$

$1+y = \omega$
$dy = d\omega$

$$\int_2^3 arctg(\vartheta)\,d\vartheta - \int_1^2 arctg(\omega)\,d\omega$$

$$\Rightarrow \left(\vartheta\, arctg(\vartheta) - \frac{1}{2}\ln|1+\vartheta^2| \Big|_2^3\right) - \left(\omega\, arctg(\omega) - \frac{1}{2}\ln|1+\omega^2| \Big|_1^2\right)$$

$$\Rightarrow \left(3\,arctg(3) - \frac{1}{2}\ln|1+3^2| - 2arctg(2) + \frac{1}{2}\ln|1+2^2|\right) -$$

$$\left(2arctg(2) - \frac{1}{2}\ln|1+2^2| - arctg(1) + \frac{1}{2}\ln|1+1^2|\right)$$

$$\Rightarrow 3\cancel{arctg}(3) - \cancel{\frac{1}{2}\ln|10|} - 2\cancel{arctg}(2) + \cancel{\frac{1}{2}\ln|5|} - 2\cancel{arctg}(2) + \cancel{\frac{1}{2}\ln|5|} + \cancel{arctg}(1) - \frac{1}{2}\ln|2|$$

$$\Rightarrow 3\,arctg(3) - 4\,arctg(2) + \frac{\pi}{4} - \ln\sqrt{10} + 2\ln\sqrt{5} - \ln\sqrt{2}$$

$$\Rightarrow 3\,arctg(3) - 4\,arctg(2) - \ln\sqrt{10} - \ln\sqrt{2} + \ln(5) + \frac{\pi}{4}$$

$$\Rightarrow 3\,arctg(3) - 4\,arctg(2) - \left(\ln\sqrt{10} + \ln\sqrt{2}\right) + \ln(5) + \frac{\pi}{4}$$

$$\Rightarrow 3\,arctg(3) - 4\,arctg(2) - \ln\sqrt{20} + \ln(5) + \frac{\pi}{4}$$

$$\Rightarrow 3\,arctg(3) - 4\,arctg(2) + \ln\left(\frac{5}{\sqrt{20}}\right) + \frac{\pi}{4}$$

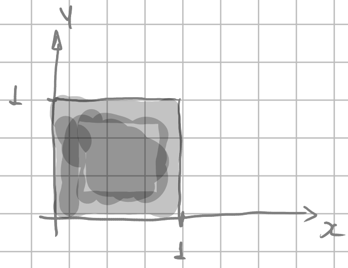
$$\Rightarrow 3\,arctg(3) - 4\,arctg(2) + \ln\left(\frac{5}{2\sqrt{5}}\right) + \frac{\pi}{4}$$

$$\Rightarrow 3\,arctg(3) - 4\,arctg(2) + \ln\left(\frac{5\sqrt{5}}{10}\right) + \frac{\pi}{4}$$

$$\Rightarrow 3\,arctg(3) - 4\,arctg(2) + \ln\left(\frac{\sqrt{5}}{2}\right) + \frac{\pi}{4}$$

4) Calcule o volume do conjunto dado

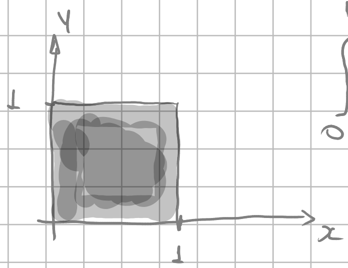
(a) $\{(x,y,z) \in \mathbb{R}^3 \mid 0 \le x \le 1,\ 0 \le y \le 1,\ 0 \le z \le x+2y\}$



$$\int_0^1\int_0^1 x+2y \, dx\, dy \Rightarrow \int_0^1 \left. \frac{x^2}{2} + 2yx \right|_0^1 dy$$

$$\int_0^1 \frac{1}{2} + 2y \, dy \Rightarrow \left. \frac{1}{2}y + y^2 \right|_0^1 \Rightarrow \frac{1}{2} + 1 \Rightarrow \boxed{\frac{1+2}{2} = \frac{3}{2}}$$

(c) $\{(x,y,z) \in \mathbb{R}^3 \mid 0 \le x \le 1,\ 0 \le y \le 1,\ 0 \le z \le xy\, e^{x^2-y^2}\}$



$$\int_0^1\int_0^1 xy\, e^{x^2-y^2} dx\, dy \Rightarrow \frac{1}{2}\int_0^1\int_{-y^2}^{1-y^2} \frac{xy\, e^t}{x} dt\, dy$$

$x^2-y^2 = t$

$2x\,dx = dt$

$dx = \frac{1}{2x} dt$

$$\Rightarrow \int_0^1 y \int_{0-y^2}^{1-y^2} e^t dt\, dy \Rightarrow \int_0^1 y\left[e^t \Big|_{0-y^2}^{1-y^2} \right] dy$$

$$\Rightarrow \int_0^1 y\left[e^{1-y^2} - e^{0-y^2}\right] dy \Rightarrow \int_0^1 y e^{1-y^2} dy - \int_0^1 y e^{0-y^2} dy$$

$1-y^2 = u$

$-2y\,dy = du$

$dy = -\frac{1}{2y} du$

$$\Rightarrow -\frac{1}{2}\int_1^0 \frac{y\, e^u}{y} du + \frac{1}{2}\int_0^{-1} \frac{y\, e^\vartheta}{y} d\vartheta \Rightarrow \frac{1}{2}\int_0^1 e^u du - \frac{1}{2}\int_{-1}^0 e^\vartheta d\vartheta$$

$0-y^2 = \vartheta$

$-2y\,dy = d\vartheta$

$dy = -\frac{1}{2y} d\vartheta$

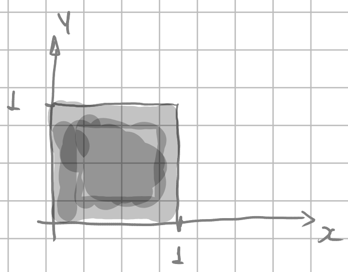
$$\Rightarrow \frac{1}{2}(e^1 - e^0) - \frac{1}{2}(e^0 - e^{-1}) \Rightarrow \frac{1}{2}(e-1) - \frac{1}{2}(1-e^{-1})$$

$$\Rightarrow \frac{1}{2}\left((e-1) - (1-e^{-1})\right) \Rightarrow \frac{1}{2}\left(e-1-1+\frac{1}{e}\right)$$

$$\Rightarrow \frac{1}{2}\left(e-2+\frac{1}{e}\right) \Rightarrow \frac{1}{2}\left(\frac{e^2-2e+1}{e}\right) \Rightarrow \frac{1}{2e}(e^2-2e+1)$$

$$\Rightarrow \boxed{\frac{(e-1)^2}{2e}}$$

(f) $\{(x,y,z)\in\mathbb{R}^3 \mid 0\leq x\leq 1,\ 0\leq y\leq 1,\ 1\leq z\leq e^{x+y}\}$



$$\int_0^1\int_0^1 e^{x+y}-1\,dx\,dy \Rightarrow \int_0^1\int_y^{1+y} e^{t}-1\,dt\,dy$$

$x+y=t$

$dx=dt$

$$\int_0^1\int_y^{1+y} e^{t}\,dt-\int_y^{1+y}dt\,dy \Rightarrow \int_0^1 e^{t}\Big|_y^{1+y}-t\Big|_y^{1+y}\,dy$$

$$\Rightarrow \int_0^1 e^{1+y}-e^{y}-(1+y)+y\,dy \Rightarrow \int_0^1 e^{1+y}-e^{y}-1-y+y\,dy \Rightarrow \int_0^1 e^{1+y}-e^{y}-1\,dy$$

$$\Rightarrow \int_0^1 e^{1+y}dy-\int_0^1 e^{y}dy-\int_0^1 dy \Rightarrow \int_1^2 e^{u}du-\Big[e^{y}\Big]_0^1-\Big[y\Big]_0^1$$

$1+y=u$

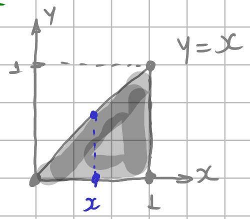
$dy=du$

$$\Rightarrow e^{u}\Big|_1^2-(e^{1}-e^{0})-(1-0) \Rightarrow (e^2-e)-(e-1)-(1-0)$$

$$\Rightarrow e^2-e-e+1-1 \Rightarrow e^2-2e+1 \Rightarrow (e-1)^2$$

5) Calcule $\iint_B y\,dx\,dy$ onde B é o conjunto dado.

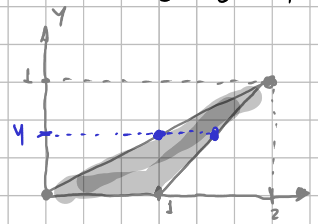
(a) B é o triangulo de vértices $(0,0)$, $(1,0)$ e $(1,1)$.



$$\int_0^1\int_0^x y\,dy\,dx \Rightarrow \int_0^1 \frac{1}{2}y^2\Big|_0^x dx \Rightarrow \int_0^1 \frac{1}{2}x^2\,dx$$

$$\Rightarrow \frac{1}{2}\int_0^1 x^2\,dx \Rightarrow \frac{1}{2}\left[1^2-0^2\right] \Rightarrow \frac{1}{2}$$

(d) B é o triangulo de vértices $(0,0)$, $(1,0)$ e $(2,1)$.



$$\int_0^1\int_{2y}^{y+1} y\,dx\,dy \Rightarrow \int_0^1 yx\Big|_{2y}^{y+1} dy \Rightarrow \int_0^1 y(y+1)-y(2y)\,dy$$

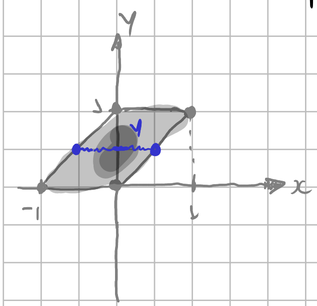
$y=\frac{1}{2}x \Rightarrow x=2y$

$y=x-1 \Rightarrow x=y+1$

$$\Rightarrow \int_0^1 y^2+y-2y^2\,dy \Rightarrow \int_0^1 -y^2+y\,dy \Rightarrow -\frac{y^3}{3}+\frac{y^2}{2}\Big|_0^1$$

$$\Rightarrow -\frac{1^3}{3}+\frac{1^2}{2} \Rightarrow -\frac{1}{3}+\frac{1}{2} \Rightarrow \frac{-2+3}{6} \Rightarrow \frac{1}{6}$$

(f) B é o paralelogramo de vértices $(-1,0)$, $(0,0)$, $(1,1)$ e $(0,1)$.



$y = x \Rightarrow x = y$

$y = x+1 \Rightarrow x = y-1$

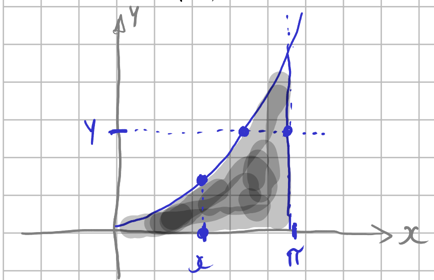
$$\int_0^1 \int_{y-1}^{y} y \, dx \, dy$$

$$\int_0^1 yx \Big|_{y-1}^{y} dy \Rightarrow \int_0^1 y(y) - y(y-1) \, dy \Rightarrow \int_0^1 y^2 - y^2 + y \, dy$$

$$\Rightarrow \int_0^1 y \, dy \Rightarrow \frac{1}{2} y^2 \Big|_0^1 \Rightarrow \frac{1}{2}$$

6) Calcule $\iint_B f(x,y) \, dx \, dy$ sendo dados:

(a) $f(x,y) = x \cos(y)$ e $B = \{(x,y) \in \mathbb{R}^2 \mid x \geq 0, \; x^2 \leq y \leq \pi\}$.



$$\int_0^{\pi} \int_0^{x^2} x \cos(y) \, dy \, dx \Rightarrow \int_0^{\pi} x \int_0^{x^2} \cos(y) \, dy \, dx$$

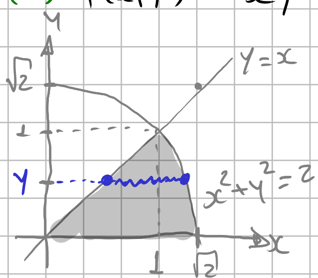
$$\Rightarrow \int_0^{\pi} x \, \text{sen}(y) \Big|_0^{x^2} dx \Rightarrow \int_0^{\pi} x \, \text{sen}(x^2) - x \, \text{sen}(0^2) \, dx$$

$$\Rightarrow \int_0^{\pi} x \, \text{sen}(x^2) \, dx \Rightarrow \frac{1}{2} \int_0^{\pi^2} \frac{x \, \text{sen}(t)}{x} \, dt \Rightarrow \frac{1}{2} \int_0^{\pi^2} \text{sen}(t) \, dt$$

$x^2 = t$
$2x \, dx = dt$
$dx = \frac{1}{2x} dt$

$$\Rightarrow \frac{1}{2} \left[-\cos(t) \Big|_0^{\pi^2} \right] \Rightarrow -\frac{1}{2} \left[\cos(\pi^2) - \cos(0^2) \right] \Rightarrow -\frac{1}{2} \left(\cos(\pi^2) - 1 \right)$$

(b) $f(x,y) = xy$ e $B = \{(x,y) \in \mathbb{R}^2 \mid x^2 + y^2 \leq 2, \; y \leq x \text{ e } x \geq 0\}$



$$\int_0^1 \int_y^{\sqrt{2-y^2}} xy \, dx \, dy$$

$$\Rightarrow \int_0^1 \frac{x^2 y}{2} \Big|_y^{\sqrt{2-y^2}} dy \Rightarrow \int_0^1 \frac{(2-y^2)y}{2} - \frac{y^3}{2} \, dy$$

$x = y$
$x^2 + y^2 = 2$
$x = \sqrt{2-y^2}$

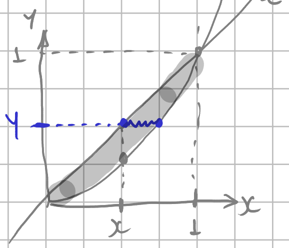
$y = \sqrt{2-y^2}$
$y^2 = 2 - y^2$
$2y^2 = 2 \Rightarrow y^2 = 1$
$y = \pm 1$

$$\Rightarrow \int_0^1 \frac{2y - y^3 - y^3}{2} \, dy \Rightarrow \int_0^1 2y - 2y^3 \, dy \Rightarrow 2 \int_0^1 y - y^3 \, dy$$

$$\Rightarrow 2 \left[\frac{y^2}{2} - \frac{y^4}{4} \right] \Big|_0^1 \Rightarrow 2 \left[\frac{1}{2} - \frac{1}{4} \right] \Rightarrow 2 \left[\frac{4-2}{8} \right] \Rightarrow \frac{4}{8} \Rightarrow \frac{1}{2}$$

7) Inverta a ordem de integração

(b) $\displaystyle\int_0^1 \left[\int_{x^2}^{x} f(x,y)\,dy\right]dx$

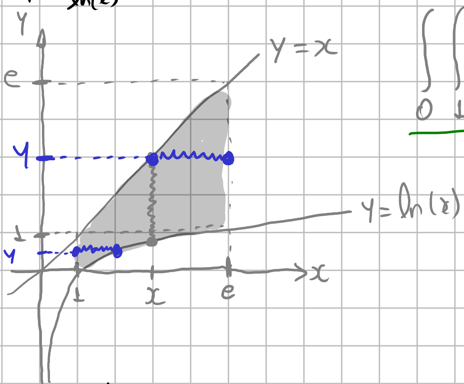


$$\int_0^1 \int_y^{\sqrt{y}} f(x,y)\,dx\,dy$$

$y = x^2 \Rightarrow x = \sqrt{y}$

$y = x \Rightarrow x = y$

(d) $\displaystyle\int_1^e \left[\int_{\ln(x)}^{x} f(x,y)\,dy\right]dx$



$$\int_0^1 \int_1^{e^y} f(x,y)\,dx\,dy + \int_1^e \int_y^e f(x,y)\,dx\,dy$$

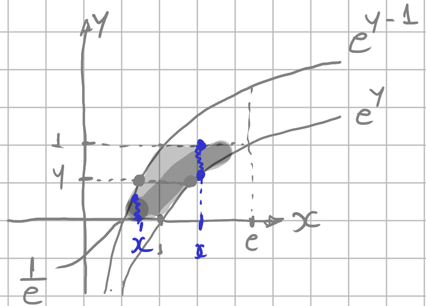
$y = x \to x = y$

$y = \ln(x) \Rightarrow e^y = e^{\ln(x)}$

$\Rightarrow e^y = x$

$\Rightarrow x = e^y$

(j) $\displaystyle\int_0^1 \left[\int_{e^{y-1}}^{e^y} f(x,y)\,dx\right]dy$



$x = e^{y-1} \Rightarrow \ln(x) = \ln(e^{y-1})$

$\Rightarrow \ln(x) = (y-1)\ln(e)$

$\Rightarrow y-1 = \ln(x) \Rightarrow y = \ln(x) + 1$

$x = e^y \Rightarrow \ln(x) = \ln(e^y)$

$\Rightarrow \ln(x) = y\ln(e) \Rightarrow y = \ln(x)$

$$\int_{\frac{1}{e}}^{1} \int_0^{\ln(x)+1} f(x,y)\,dy\,dx$$

$y = \ln(x) + 1$

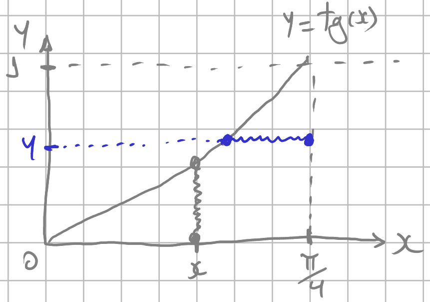
$0 = \ln(x) + 1$

$-1 = \ln(x)$

$e^{-1} = e^{\ln(x)}$

$e^{-1} = x$

(m) $\int_0^{\pi/4}\left[\int_0^{tg(x)} f(x,y)\,dy\right]dx$



$$\boxed{\int_0^1 \int_{arctg(y)}^{\pi/4} f(x,y)\,dx\,dy}$$

$y = tg(x) \Rightarrow x = arctg(y)$