Exercícios - Lista IV

Calcule as seguintes integrais.

1-) $\int \frac{3x^2}{5x^3-1}dx$

$5x^3-1=t$

$15x^2\,dx=dt$

$dx=\frac{1}{15x^2}dt$

$I=\frac{3}{15}\int\frac{x^2}{x^2\,t}dt \rightarrow I=\frac{1}{5}\int\frac{1}{t}dt$

$I=\frac{1}{5}\ln|t| \rightarrow \underline{I=\frac{1}{5}\ln|5x^3-1|+C}$ ✓

2-) $I=\int\frac{x^2+\frac{2}{3}}{x^3+2x-13}dx$

$x^3+2x-13=t$

$3x^2+2\,dx=dt$

$dx=\frac{1}{3x^2+2}dt$

$I=\int\frac{x^2+\frac{2}{3}}{(3x^2+2)\,t}dt \rightarrow I=\int\frac{x^2+\frac{2}{3}}{3(x^2+\frac{2}{3})t}dt$

$I=\frac{1}{3}\int\frac{1}{t}dt \rightarrow I=\frac{1}{3}\ln|t|$

$\underline{I=\frac{1}{3}\ln|x^3+2x-13|+C}$ ✓

3-) $I=\int\frac{x+3}{x^2+4x+3}dx$

$x^2+4x+3=0$

$x^2+4x+2^2=-3+2^2$

$(x+2)^2=1$

$x+2=\pm1$

$x=\pm1-2$ → $x_1=-1$, $x_2=-3$

$I=\int\frac{\cancel{x+3}}{(x+1)\cancel{(x+3)}}dx$

$I=\int\frac{1}{x+1}dx \rightarrow I=\int\frac{1}{t}dt$

$x+1=t$

$dx=dt$

$\rightarrow I=\ln|t|$

$\underline{I=\ln|x+1|+C}$ ✓

4) $I = \int \frac{3}{x^2 - 3x - 4} dx$

$I = 3 \int \frac{1}{x^2 - 3x - 4} dx$

$\Delta = (-3)^2 - 4.1.(-4)$

$\Delta = 9 + 16$

$\Delta = 25$

$x = \frac{+3 \pm 5}{2}$

$x_1 = 4$

$x_2 = -1$

$I = 3 \int \frac{1}{(x-4)(x+1)} dx$

$(x-4)(x+1)$

$\frac{1}{(x-4)(x+1)} = \frac{A}{x-4} + \frac{B}{x+1} = \frac{A(x+1) + B(x-4)}{(x-4)(x+1)}$

$1 = Ax + A + Bx - 4B$

$1 = x(A+B) + A - 4B$

$\begin{cases} A + B = 0 \\ A - 4B = 1 \end{cases}$

$5B = -1$

$B = -\frac{1}{5}$ $\quad A = \frac{1}{5}$

$I = 3\left[\frac{1}{5}\int \frac{1}{x-4} dx - \frac{1}{5}\int \frac{1}{x+1} dx\right] \rightarrow I = \frac{3}{5} \ln|x-4| - \frac{3}{5} \ln|x+1| + C$ ✓

5) $I = \int \frac{2}{x^2 + 4x + 3} dx$

$x^2 + 4x + 3 = 0$

$x^2 + 4x + 2^2 = -3 + 2^2$

$(x+2)^2 = 1$

$x + 2 = \pm 1$

$x = \pm 1 - 2$

$x_1 = -1$

$x_2 = -3$

$I = 2 \int \frac{1}{(x+1)(x+3)} dx$

$\frac{1}{(x+1)(x+3)} = \frac{A}{x+1} + \frac{B}{x+3} = \frac{A(x+3) + B(x+1)}{(x+1)(x+3)}$

$1 = Ax + 3A + Bx + B$

$1 = x(A+B) + 3A + B$

$\begin{cases} A + B = 0 \\ 3A + B = 1 \end{cases}$

$-2A = -1$

$A = \frac{1}{2}$ $\quad B = -\frac{1}{2}$

$I = 2\left[\frac{1}{2}\int \frac{1}{x+1} dx - \frac{1}{2}\int \frac{1}{x+3} dx\right]$

$I = \ln|x+1| - \ln|x+3| + C$ ✓

6-) $I = \int \frac{2x+4}{x^2-4x+4}\,dx$

$x^2-4x+4=0$

$x^2-4x+2^2=-4$

$(x-2)^2=0$

$I = \int \frac{2x+4}{(x-2)^2}\,dx$ $\quad \frac{2x+4}{(x-2)^2} = \frac{A}{(x-2)^2} + \frac{B}{x-2} = \frac{A+B(x-2)}{(x-2)^2}$

$2x+4 = A+B(x-2)$

$2x+4 = A+Bx-2B$ $\quad 2=B$ $\quad A=8$

$2x+4 = xB + A-2B$ $\quad 4 = A-2B$

$I = 8\int \frac{1}{(x-2)^2}\,dx + 2\int \frac{1}{x-2}\,dx$

$x-2=t$ $\quad I = 8\int t^{-2}\,dt + 2\int \frac{1}{t}\,dt$

$dx=dt$

$I = 8\left[\frac{t^{-1}}{-1}\right] + 2\ln|t| \rightarrow I = -\frac{8}{x-2} + 2\ln|x-2| + C$ ✓

7-) $I = \int \frac{x-1}{(x+1)^2}\,dx$

$\frac{x-1}{(x+1)^2} = \frac{A}{(x+1)^2} + \frac{B}{x+1} = \frac{A+B(x+1)}{(x+1)^2}$

$x-1 = A+Bx+B$ $\quad \begin{cases} B=1 \\ A+B=-1 \end{cases}$ $\quad A=-2$

$I = -2\int \frac{1}{(x+1)^2}\,dx + \int \frac{1}{x+1}\,dx$

$x+1=t$ $\quad I = -2\int t^{-2}\,dt + \int \frac{1}{t}\,dt \rightarrow I = -2\left[\frac{t^{-1}}{-1}\right] + \ln|t|$

$dx=dt$

$I = \frac{2}{t} + \ln|t| \rightarrow I = \frac{2}{x+1} + \ln|x+1| + C$ ✓

8-) $I = \int \frac{x+2}{x^2+x} dx$

$x^2 + x = x(x+1)$

$I = \int \frac{x+2}{x(x+1)} dx \qquad \frac{x+2}{x(x+1)} = \frac{A}{x} + \frac{B}{x+1} = \frac{A(x+1)+Bx}{x(x+1)}$

$x+2 = Ax + A + Bx \qquad \begin{cases} A + B = 1 \\ A = 2 \end{cases} \qquad B = -1$

$x+2 = x(A+B) + A$

$I = 2\int \frac{1}{x} dx - \int \frac{1}{x+1} dx \rightarrow \underline{I = 2\ln|x| - \ln|x+1| + C}$ ✓

9-) $I = \int \frac{1}{x^2+3x} dx$

$x^2 + 3x = x(x+3) \qquad \frac{1}{x(x+3)} = \frac{A}{x} + \frac{B}{x+3} = \frac{A(x+3)+Bx}{x(x+3)}$

$I = \int \frac{1}{x(x+3)} dx$

$1 = Ax + 3A + Bx \qquad A + B = 0 \qquad A = \frac{1}{3} \qquad B = -\frac{1}{3}$

$1 = x(A+B) + 3A \qquad 3A = 1$

$I = \frac{1}{3}\int \frac{1}{x} dx - \frac{1}{3}\int \frac{1}{x+3} dx \rightarrow \underline{I = \frac{1}{3}\ln|x| - \frac{1}{3}\ln|x+3| + C}$ ✓

10-) $I = \int \frac{1}{x^2-2x} dx$

$\frac{1}{x(x-2)} = \frac{A}{x} + \frac{B}{x-2} = \frac{A(x-2)+Bx}{x(x-2)} \qquad 1 = Ax - 2A + Bx$

$1 = x(A+B) - 2A$

$\begin{cases} A + B = 0 \\ -2A = 1 \end{cases} \qquad A = -\frac{1}{2} \qquad B = \frac{1}{2}$

$I = -\frac{1}{2}\int \frac{1}{x} dx + \frac{1}{2}\int \frac{1}{x-2} dx$

$\underline{I = -\frac{1}{2}\ln|x| + \frac{1}{2}\ln|x-2| + C}$ ✓

11-) $I = \int \frac{1}{x^2-1} dx$

$\frac{1}{(x-1)(x+1)} = \frac{A}{x-1} + \frac{B}{x+1} = \frac{A(x+1)+B(x-1)}{(x-1)(x+1)}$

$1 = Ax + A + Bx - B$

$1 = x(A+B) + A - B$

$A + B = 0$

$+\ A - B = 1$

$2A = 1$

$A = \frac{1}{2}$ $\quad B = -\frac{1}{2}$

$I = \frac{1}{2}\int \frac{1}{x-1} dx - \frac{1}{2}\int \frac{1}{x+1} dx \to I = \frac{1}{2}\ln|x-1| - \frac{1}{2}\ln|x+1| + C$ ✓

12-) $I = \int \frac{5x-19}{x^2-7x+10} dx$

$\Delta = (-7)^2 - 4.1.10$

$\Delta = 49-40$

$\Delta = 9$

$x = \frac{7 \pm 3}{2}$ $\to x_1 = 5$, $x_2 = 2$

$I = \int \frac{5x-19}{(x-5)(x-2)} dx$

$\frac{5x-19}{(x-5)(x-2)} = \frac{A}{x-5} + \frac{B}{x-2} = \frac{A(x-2)+B(x-5)}{(x-5)(x-2)}$

$5x - 19 = Ax - 2A + Bx - 5B$

$5x - 19 = x(A+B) - 2A - 5B$

$\begin{cases} A + B = 5 \\ -2A - 5B = -19 \end{cases}$

$\begin{cases} 2A + 2B = 10 \\ -2A - 5B = -19 \end{cases}$

$-3B = -9$

$A = 2$

$B = 3$

$I = 2\int \frac{1}{x-5} dx + 3\int \frac{1}{x-2} dx \to I = 2\ln|x-5| + 3\ln|x-2| + C$ ✓

13) $I = \int \dfrac{2x-1}{x^2-5x+6}\,dx$

$\Delta = (-5)^2 - 4.1.6$ $\qquad x = \dfrac{5 \pm 1}{2} \to x_1 = 3,\ x_2 = 2$ $\qquad I = \int \dfrac{2x-1}{(x-2)(x-3)}\,dx$

$\Delta = 25 - 24$

$\Delta = 1$

$$\frac{2x-1}{(x-2)(x-3)} = \frac{A}{x-2} + \frac{B}{x-3} = \frac{A(x-3)+B(x-2)}{(x-2)(x-3)}$$

$2x-1 = Ax - 3A + Bx - 2B$

$2x-1 = x(A+B) - 3A - 2B$

$$\begin{cases} A+B = 2 \\ -3A-2B = -1 \end{cases} \qquad \begin{cases} 2A+2B = 4 \\ -3A-2B = -1 \end{cases} \qquad \begin{matrix} A = -3 \\ B = 5 \end{matrix}$$

$-A = 3$

$I = -3\int \dfrac{1}{x-2}\,dx + 5\int \dfrac{1}{x-3}\,dx \to$ $\quad I = -3\ln|x-2| + 5\ln|x-3| + C$

14) $I = \int \dfrac{2x-3}{(x-1)(x^2+2x-15)}\,dx$

$\Delta = 2^2 - 4.1.-15$ $\qquad x = \dfrac{-2 \pm 8}{2} \to x_1 = 3,\ x_2 = -5$ $\qquad I = \int \dfrac{2x-3}{(x-1)(x-3)(x+5)}\,dx$

$\Delta = 4 + 60$

$\Delta = 64$

$$\frac{2x-3}{(x-1)(x-3)(x+5)} = \frac{A}{x-1} + \frac{B}{x-3} + \frac{C}{x+5} = \frac{A(x-3)(x+5) + B(x-1)(x+5) + C(x-1)(x-3)}{(x-1)(x-3)(x+5)}$$

$2x-3 = A(x^2+5x-3x-15) + B(x^2+5x-x-5) + C(x^2-3x-x+3)$

$2x-3 = A(x^2+2x-15) + B(x^2+4x-5) + C(x^2-4x+3)$

$2x-3 = \cancel{Ax^2} + 2A\cancel{x} - 15A + \cancel{Bx^2} + 4B\cancel{x} - 5B + \cancel{Cx^2} - 4C\cancel{x} + 3C$

$2x-3 = x^2(A+B+C) + x(2A+4B-4C) - 15A - 5B + 3C$

$$\begin{cases} A+B+C = 0 \\ 2A+4B-4C = 2 \\ -15A-5B+3C = -3 \end{cases} \qquad \begin{bmatrix} 1 & 1 & 1 \\ 2 & 4 & -4 \\ -15 & -5 & +3 \end{bmatrix} \times \begin{bmatrix} A \\ B \\ C \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \\ -3 \end{bmatrix}$$

$$\begin{bmatrix} A \\ B \\ C \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 4 & -4 \\ -15 & -5 & 3 \end{bmatrix}^{-1} \times \begin{bmatrix} 0 \\ 2 \\ -3 \end{bmatrix}$$

$$\begin{vmatrix} A \\ B \\ C \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 4 & -4 \\ -15 & -5 & 3 \end{vmatrix}^{-1} \times \begin{vmatrix} 0 \\ 2 \\ -3 \end{vmatrix}$$

$$\text{Det} = \left|\begin{array}{ccc|cc} 1 & 1 & 1 & 1 & 1 \\ 2 & 4 & -4 & 2 & 4 \\ -15 & -5 & 3 & -15 & -5 \end{array}\right| = 62 - (-34) = 96$$

$$-60 \quad 20 \quad 6 \qquad 12 \quad 60 \quad -10$$

$$-34 \qquad\qquad 62$$

$$\begin{array}{ccccc} \cancel{1} & \cancel{1} & \cancel{1} & \cancel{1} & \cancel{1} \\ 2 & 4 & -4 & 2 & 4 \\ -15 & -5 & 3 & -15 & -5 \\ 1 & 1 & 1 & 1 & 1 \\ 2 & 4 & -4 & 2 & 4 \end{array}$$

$$\text{Adj} = \begin{pmatrix} -8 & -8 & -8 \\ 54 & 18 & 6 \\ 50 & -10 & 2 \end{pmatrix} \qquad \text{Inv} = \begin{pmatrix} -8/96 & -8/96 & -8/96 \\ 54/96 & 18/96 & 6/96 \\ 50/96 & -10/96 & 2/96 \end{pmatrix}$$

$$\text{Inv} = \begin{pmatrix} -\frac{1}{12} & -\frac{1}{12} & -\frac{1}{12} \\ \frac{9}{16} & \frac{3}{16} & \frac{1}{16} \\ \frac{25}{48} & -\frac{5}{48} & \frac{1}{48} \end{pmatrix}$$

$$\begin{vmatrix} A \\ B \\ C \end{vmatrix} = \begin{pmatrix} -\frac{1}{12} & -\frac{1}{12} & -\frac{1}{12} \\ \frac{9}{16} & \frac{3}{16} & \frac{1}{16} \\ \frac{25}{48} & -\frac{5}{48} & \frac{1}{48} \end{pmatrix} \times \begin{pmatrix} 0 \\ 2 \\ -3 \end{pmatrix} = \begin{vmatrix} -\frac{1}{12}\cdot 2 + -\frac{1}{12}\cdot(-3) \\ \frac{3}{16}\cdot 2 + \frac{1}{16}(-3) \\ -\frac{5}{48}\cdot 2 + \frac{1}{48}(-3) \end{vmatrix} = \begin{vmatrix} \frac{1}{12} \\ \frac{3}{16} \\ -\frac{13}{48} \end{vmatrix}$$

$$I = \frac{1}{12}\int \frac{1}{x-1}\,dx + \frac{3}{16}\int \frac{1}{x-3}\,dx - \frac{13}{48}\int \frac{1}{x+5}\,dx$$

$$I = \frac{1}{12}\ln|x-1| + \frac{3}{16}\ln|x-3| - \frac{13}{48}\ln|x+5| + C$$ ✓

15-) $I = \int \frac{2x-3}{(x+1)(x-1)^2}\,dx$

$$\frac{2x-3}{(x+1)(x-1)^2} = \frac{A}{x+1} + \frac{B}{(x-1)^2} + \frac{C}{x-1} = \frac{A(x-1)^2 + B(x+1) + C(x+1)(x-1)}{(x+1)(x-1)^2}$$

$$2x-3 = A(x^2-2x+1) + Bx + B + C(x^2-1)$$

$$2x-3 = \cancel{Ax^2} - \cancel{2Ax} + A + Bx + B + \cancel{Cx^2} - C$$

$$2x-3 = x^2(A+0B+C) + x(-2A+B+0C) + A+B-C$$

$$\begin{cases} A + 0B + C = 0 \\ -2A + B + 0C = 2 \\ A + B - C = -3 \end{cases}$$

$$\text{Det} = \begin{vmatrix} 1 & 0 & 1 \\ -2 & 1 & 0 \\ 1 & 1 & -1 \end{vmatrix} \begin{matrix} 1 & 0 \\ -2 & 1 \\ 1 & 1 \end{matrix} = -4$$

$$\begin{matrix} 1 & 0 & 0 & & -1 & 0 & -2 \\ & 1 & & & & -3 & \end{matrix}$$

$$\begin{matrix} \cancel{1} & \cancel{0} & \cancel{1} & \cancel{1} & \cancel{0} \\ -2 & 1 & 0 & -2 & 1 \\ 1 & 1 & -1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 \\ -2 & 1 & 0 & -2 & 1 \end{matrix}$$

$$\text{Adj} = \begin{pmatrix} -1 & 1 & -1 \\ -2 & -2 & -2 \\ -3 & -1 & 1 \end{pmatrix} \qquad I_{nv} = \begin{pmatrix} \frac{1}{4} & -\frac{1}{4} & \frac{1}{4} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{3}{4} & \frac{1}{4} & -\frac{1}{4} \end{pmatrix}$$

$$\begin{vmatrix} A \\ B \\ C \end{vmatrix} = \begin{pmatrix} \frac{1}{4} & -\frac{1}{4} & \frac{1}{4} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{3}{4} & \frac{1}{4} & -\frac{1}{4} \end{pmatrix} \times \begin{vmatrix} 0 \\ 2 \\ -3 \end{vmatrix} = \begin{vmatrix} -\frac{1}{4}\cdot 2 + \frac{1}{4}(-3) \\ \frac{1}{2}\cdot 2 + \frac{1}{2}(-3) \\ \frac{1}{4}2 + \left(-\frac{1}{4}\right)(-3) \end{vmatrix} = \begin{vmatrix} -5/4 \\ -1/2 \\ 5/4 \end{vmatrix}$$

$$I = -\frac{5}{4}\int \frac{1}{x+1}dx - \frac{1}{2}\int \frac{1}{(x-1)^2}dx + \frac{5}{4}\int \frac{1}{x-1}dx$$

$$I = -\frac{5}{4}\ln|x+1| - \frac{1}{2}\int t^{-2}\,dt + \frac{5}{4}\ln|x-1|$$

$$I = -\frac{5}{4}\ln|x+1| + \frac{1}{2(x-1)} + \frac{5}{4}\ln|x-1| + C$$ ✓

16-) $I = \int \dfrac{x}{(x-1)(x+1)^2}\,dx$

$$\frac{x}{(x-1)(x+1)^2} = \frac{A}{x-1} + \frac{B}{(x+1)^2} + \frac{C}{x+1} = \frac{A(x+1)^2 + B(x-1) + C(x-1)(x+1)}{(x-1)(x+1)^2}$$

$$x = A(x^2+2x+1) + Bx - B + C(x^2-1)$$

$$x = \cancel{Ax^2} + \cancel{2Ax} + A + \cancel{Bx} - B + \cancel{Cx^2} - C$$

$$x = x^2(A + 0B + C) + x(2A + B + 0C) + A - B - C$$

$$\begin{cases} A + 0B + C = 0 \\ 2A + B + 0C = 1 \\ A - B - C = 0 \end{cases}$$

$$\text{Det} = \left|\begin{matrix} 1 & 0 & 1 \\ 2 & 1 & 0 \\ 1 & -1 & -1 \end{matrix}\right| \begin{matrix} 1 & 0 \\ 2 & 1 \\ 1 & -1 \end{matrix} = -4$$

$$\begin{matrix} 1 & 0 & 0 \\ & 1 & \end{matrix} \qquad \begin{matrix} -1 & 0 & -2 \\ & -3 & \end{matrix}$$

$$\begin{matrix} \cancel{1} & \cancel{0} & \cancel{1} & \cancel{1} & \cancel{0} \\ 2 & 1 & 0 & 2 & 1 \\ 1 & -1 & -1 & 1 & -1 \\ 1 & 0 & 1 & 1 & 0 \\ 2 & 1 & 0 & 2 & 1 \end{matrix}$$

$$\text{Adj} = \begin{bmatrix} -1 & -1 & -1 \\ 2 & -2 & 2 \\ -3 & 1 & 1 \end{bmatrix} \qquad I_{nv} = \begin{bmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ -\frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \\ \frac{3}{4} & -\frac{1}{4} & -\frac{1}{4} \end{bmatrix}$$

$$\begin{bmatrix} A \\ B \\ C \end{bmatrix} = \begin{bmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ -\frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \\ \frac{3}{4} & -\frac{1}{4} & -\frac{1}{4} \end{bmatrix} \times \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{1}{4}\cdot 1 \\ \frac{1}{2}\cdot 1 \\ -\frac{1}{4}\cdot 1 \end{bmatrix}$$

$$I = \frac{1}{4}\int \frac{1}{x-1}\,dx + \frac{1}{2}\int \frac{1}{(x+1)^2}\,dx - \frac{1}{4}\int \frac{1}{x+1}\,dx$$

$$\boxed{I = \frac{1}{4}\ln|x-1| - \frac{1}{2(x+1)} - \frac{1}{4}\ln|x+1| + C}$$ ✓

17-) $I=\int \dfrac{x^2+x+1}{x^2(x+1)^2}\,dx$

$$\frac{x^2+x+1}{x^2(x+1)^2}=\frac{A}{x^2}+\frac{B}{x}+\frac{C}{(x+1)^2}+\frac{D}{x+1}=\frac{A(x+1)^2+Bx(x+1)^2+Cx^2+Dx^2(x+1)}{x^2(x+1)^2}$$

$$x^2+x+1=A(x^2+2x+1)+Bx(x^2+2x+1)+Cx^2+D(x^3+x^2)$$

$$x^2+x+1=\cancel{Ax^2}+\cancel{2Ax}+\cancel{A}+\cancel{Bx^3}+\cancel{2Bx^2}+\cancel{Bx}+\cancel{Cx^2}+\cancel{Dx^3}+\cancel{Dx^2}$$

$$x^2+x+1=x^3(0A+B+0C+D)+x^2(A+2B+C+D)$$
$$+x(2A+B+0C+0D)+A+0B+0C+0D$$

$$\begin{cases} 0A+B+0C+D=0 \\ A+2B+C+D=1 \\ 2A+B+0C+0D=1 \\ A+0B+0C+0D=1 \end{cases}$$

$B+D=0$ $\quad$ $D=1$

$2A+B=1$ $\quad$ $B=-1$

$A=1$ $\quad$ $1-2+C+1=1$

$C=1$

$$I=\int\frac{1}{x^2}dx-\int\frac{1}{x}dx+\int\frac{1}{(x+1)^2}dx+\int\frac{1}{x+1}dx$$

$$I=-\frac{1}{x}-\ln|x|-\frac{1}{x+1}+\ln|x+1|+C$$ ✓

18-) $I=\int \dfrac{2x^2+3x}{x+1}\,dx$

$$\begin{array}{l|l} 2x^2+3x+0 & x+1 \\ \hline \underline{2x^2+2x} & 2x+1 \\ \quad x+0 & \\ \quad \underline{x+1} & \\ \quad\ -1 & \end{array}$$

$$I=\int\frac{(2x+1)\cancel{(x+1)}}{\cancel{x+1}}-\frac{1}{x+1}\,dx$$

$$I=2\int x\,dx+\int dx-\ln|x+1|$$

$$I=x^2+x-\ln|x+1|+C$$ ✓

19-) $I = \int \frac{x^3 + x^2 + 1}{x^2 + 1}\,dx$

$$\begin{array}{l|l} x^3 + x^2 + 0x + 1 & x^2+1 \\ \hline \underline{x^3 \qquad x} & x + 1 \\ \quad x^2 - x + 1 & \\ \quad \underline{x^2 \qquad + 1} & \\ \quad\quad -x & \end{array}$$

$I = \int \frac{(x+1)\cancel{(x^2+1)}}{\cancel{x^2+1}} - \frac{x}{x^2+1}\,dx$

$I = \int x + 1\,dx - \underbrace{\int \frac{x}{x^2+1}\,dx}_{I_1}$

$I_1 = \int \frac{x}{x^2+1}\,dx \rightarrow I_1 = \frac{1}{2}\int \frac{x}{xt}\,dt \rightarrow I_1 = \frac{1}{2}\ln|x^2+1|$

$x^2 + 1 = t$

$2x\,dx = dt$

$dx = \frac{1}{2x}dt$

$I = \frac{1}{2}x^2 + x - \frac{1}{2}\ln|x^2+1| + C$ ✓

20-) $I = \int \frac{x^3 + 2x^2 - x + 2}{x^2 + 2x - 3}\,dx$

$$\begin{array}{l|l} x^3 + 2x^2 - x + 2 & x^2 + 2x - 3 \\ \hline \underline{x^3 + 2x^2 - 3x} & x \\ \qquad 2x + 2 & \end{array}$$

$I = \int \frac{x\cancel{(x^2+2x-3)}}{\cancel{x^2+2x-3}} + \frac{2x+2}{x^2+2x-3}\,dx$

$I = \int x\,dx + \int \frac{\overbrace{2x+2}^{S_1}}{x^2+2x-3}\,dx$

$\Delta = 2^2 - 4.1.(-3)$ $\qquad x = \frac{-2 \pm 4}{2}$ $\quad x_1 = 1$, $x_2 = -3$

$\Delta = 4 + 12$

$\Delta = 16$

$I_1 = \int \frac{2x+2}{(x-1)(x+3)}\,dx$

$\frac{2x+2}{(x-1)(x+3)} = \frac{A}{x-1} + \frac{B}{x+3} = \frac{A(x+3) + B(x-1)}{(x-1)(x+3)}$

$2x + 2 = Ax + 3A + Bx - B$

$2x + 2 = x(A+B) + 3A - B$

$\begin{cases} A + B = 2 \\ 3A - B = 2 \end{cases}$ $\qquad A = 1 \qquad B = 1$

$4A = 4$

$I_1 = \int \frac{1}{x-1}\,dx + \int \frac{1}{x+3}\,dx \rightarrow I = \frac{1}{2}x^2 + \ln|x-1| + \ln|x+3| + C$ ✓

$I = \frac{1}{2}x^2 + \ln|x^2 + 3x - x - 3| + C$

$I = \frac{1}{2}x^2 + \ln|x^2 + 2x - 3| + C$ ✓

21-) $I = \int \dfrac{2t^3 - 2t^2 - 1}{t^2 - t}\, dt$

$$\begin{array}{r|l} 2t^3 - 2t^2 - 1 & t^2 - t \\ \hline 2t^3 - 2t^2 & 2t \\ \hline -1 & \end{array}$$

$I = \int \dfrac{2t\cancel{(t^2 - t)}}{\cancel{t^2 - t}} - \dfrac{1}{t^2 - t}\, dt$

$I = 2\int t\, dt - \underbrace{\int \dfrac{1}{t^2 - t}\, dt}_{I_1}$

$I_1 = \int \dfrac{1}{t(t-1)}\, dt$

$\dfrac{1}{t(t-1)} = \dfrac{A}{t} + \dfrac{B}{t-1} = \dfrac{A(t-1) + Bt}{t(t-1)}$

$1 = At - A + Bt$

$1 = t(A+B) - A$

$A + B = 0$

$-A = 1$

$A = -1$

$B = 1$

$I_1 = -\int \dfrac{1}{t}\, dt + \int \dfrac{1}{t-1}\, dt \rightarrow I_1 = -\ln|t| + \ln|t-1|$

$I = t^2 - \left[-\ln|t| + \ln|t-1|\right] \rightarrow \underline{I = t^2 + \ln|t| - \ln|t-1| + C}$ ✓

22-) $I=\int \dfrac{x^3-x^2-5x-8}{x^2-x-6}\,dx$

$$\begin{array}{l|l} x^3-x^2-5x-8 & x^2-x-6 \\ \underline{x^3-x^2-6x} & x \\ \quad x-8 & \end{array}$$

$I=\int \dfrac{x\cancel{(x^2-x-6)}}{\cancel{x^2-x-6}}+\dfrac{x-8}{x^2-x-6}\,dx$

$I=\int x\,dx+\int \underbrace{\dfrac{x-8}{x^2-x-6}}_{I_1}dx$

$\Delta=(-1)^2-4.1.(-6)$

$\Delta=25$

$x=\dfrac{1\pm5}{2}$ $\nearrow x_1=3$ $\searrow x_2=-2$

$I_1=\int \dfrac{x-8}{(x-3)(x+2)}\,dx$

$\dfrac{x-8}{(x-3)(x+2)}=\dfrac{A}{x-3}+\dfrac{B}{x+2}=\dfrac{A(x+2)+B(x-3)}{(x-3)(x+2)}$

$x-8=Ax+2A+Bx-3B$

$x-8=x(A+B)+2A-3B$

$\begin{cases} A+B=1 \\ 2A-3B=-8 \end{cases}$

$2A+2B=2$

$\ominus\ 2A-3B=-8$

$5B=10$

$A=-1 \qquad B=2$

$I_1=-\int \dfrac{1}{x-3}\,dx+2\int \dfrac{1}{x+2}\,dx \rightarrow I_1=-\ln|x-3|+2\ln|x+2|$

$I=\int x\,dx+I_1 \rightarrow \underline{I=\dfrac{1}{2}x^2-\ln|x-3|+2\ln|x+2|+C}$

23) $I = \int \frac{x^3+3x^2-28x+3}{x^2+3x-28}\,dx$

$$\begin{array}{l|l} x^3+3x^2-28x+3 & x^2+3x-28 \\ \underline{x^3+3x^2-28x} & x \\ \qquad\qquad +3 & \end{array}$$

$I = \int \frac{x(x^2+3x-28)}{x^2+3x-28} + \frac{3}{x^2+3x-28}\,dx$

$I = \int x\,dx + 3\int \underbrace{\frac{1}{x^2+3x-28}}_{I_1}\,dx$

$\Delta = 3^2 - 4.1.(-28)$ $\qquad x = \frac{-3 \pm 11}{2}$

$\Delta = 9 + 112$

$\Delta = 121$ $\qquad x_1 = 4 \qquad x_2 = -7$

$I_1 = \int \frac{1}{(x-4)(x+7)}\,dx$

$\frac{1}{(x-4)(x+7)} = \frac{A}{x-4} + \frac{B}{x+7} = \frac{A(x+7)+B(x-4)}{(x-4)(x+7)}$

$1 = Ax + 7A + Bx - 4B$

$1 = x(A+B) + 7A - 4B$

$\begin{cases} A + B = 0 \\ 7A - 4B = 1 \end{cases}$ $\qquad \begin{cases} 4A + 4B = 0 \\ 7A - 4B = 1 \end{cases}$

$11A = 1 \qquad A = \frac{1}{11}$

$B = -\frac{1}{11}$

$I_1 = \frac{1}{11}\int \frac{1}{x-4}\,dx - \frac{1}{11}\int \frac{1}{x+7}$

$I_1 = \frac{1}{11}\ln|x-4| - \frac{1}{11}\ln|x+7|$

$I = \int x\,dx + 3I_1 \rightarrow I = \frac{1}{2}x^2 + \frac{3}{11}\ln|x-4| - \frac{3}{11}\ln|x+7| + C$

24-) $I=\int \frac{\sqrt{1+4x^2}}{x}\,dx$

$\sqrt{1+4x^2}=t$

$(1+4x^2)^{1/2}=t$

$\frac{1}{2}(1+4x^2)^{-1/2}\,8x\,dx=dt$

$dx=\frac{2\sqrt{1+4x^2}}{8x}$

$1+4x^2=t^2$

$4x^2=t^2-1$

$x^2=\frac{t^2-1}{4}$

$I=\frac{1}{4}\int \frac{t^2}{x^2}\,dt$

$I=\frac{1}{4}\int \frac{t^2}{\frac{t^2-1}{4}}\,dt \rightarrow I=\int \frac{t^2}{t^2-1}\,dt$

$$\begin{array}{r|l} t^2+0 & t^2-1 \\ \hline t^2-1 & 1 \\ 1 & \end{array}$$

$I=\int \frac{1(t^2-1)+1}{t^2-1}\,dt$

$I=\int dt+\int \frac{1}{t^2-1}\,dt$

$\frac{1}{(t-1)(t+1)}=\frac{A}{t-1}+\frac{B}{t+1}=\frac{A(t+1)+B(t-1)}{(t-1)(t+1)}$

$1=At+A+Bt-B$

$1=t(A+B)+A-B$

$\begin{cases} A+B=0 \\ A-B=1 \end{cases}$ $\quad A=\frac{1}{2}$

$2A=1 \quad B=-\frac{1}{2}$

$I=\int dt+\frac{1}{2}\int \frac{1}{t-1}\,dt-\frac{1}{2}\int \frac{1}{t+1}\,dt$

$I=t+\frac{1}{2}\ln|t-1|-\frac{1}{2}\ln|t+1|+C$

$I=\sqrt{1+4x^2}+\frac{1}{2}\ln\left|\sqrt{1+4x^2}-1\right|-\frac{1}{2}\ln\left|\sqrt{1+4x^2}+1\right|+C$ ✓

25-) $I = \int \frac{\sqrt{1-x^2}}{x}\, dx$

$\sqrt{1-x^2} = t$

$1 - x^2 = t^2$

$-2x\,dx = 2t\,dt$

$dx = -\frac{t}{x}\,dt$

$\rightarrow 1 - t^2 = x^2$

$I = -\int \frac{t^2}{x^2}\,dt \rightarrow I = -\int \frac{t^2}{1-t^2}\,dt$

$t^2 + 0 \;\big|\; 1 - t^2$

$\underline{t^2 - 1} \quad -1$

1

$I = -\int \frac{-1(1-t^2)}{1-t^2} + \frac{1}{1-t^2}\,dt$

$I = -\left[-\int dt + \underbrace{\int \frac{1}{1-t^2}\,dt}_{I_1}\right]$

$\frac{1}{(1-t)(1+t)} = \frac{A}{1-t} + \frac{B}{1+t} = \frac{A(1+t)+B(1-t)}{(1-t)(1+t)}$

$1 = A + At + B - Bt$

$1 = t(A-B) + A + B$

$\begin{cases} A - B = 0 \\ A + B = 1 \end{cases}$

$2A = 1$

$A = \frac{1}{2}$

$B = \frac{1}{2}$

$I_1 = \frac{1}{2}\int \frac{1}{1-t}\,dt + \frac{1}{2}\int \frac{1}{1+t}\,dt$

$I_1 = -\frac{1}{2}\ln|1-t| + \frac{1}{2}\ln|1+t|$

$I = -\left[-t - \frac{1}{2}\ln|1-t| + \frac{1}{2}\ln|1+t|\right] \rightarrow I = t + \frac{1}{2}\ln|1-t| - \frac{1}{2}\ln|1+t|$

$I = \sqrt{1-x^2} + \frac{1}{2}\ln\left|1 - \sqrt{1-x^2}\right| - \frac{1}{2}\ln\left|1 + \sqrt{1-x^2}\right| + C$

???