Exercícios - Lista I

Calcule as seguintes integrais

1-) $I = \int x^3 \, dx$

$I = \left[\frac{x^{3+1}}{3+1} \right] \Rightarrow I = \frac{1}{4} x^4 + C$ //

2-) $I = \int \frac{1}{x^2} \, dx$

$I = \int x^{-2} \, dx \rightarrow I = \left[\frac{x^{-2+1}}{-2+1} \right] \Rightarrow I = \frac{x^{-1}}{-1} \rightarrow I = -\frac{1}{x} + C$ //

3-) $I = \int 2x\sqrt{x} \, dx$

$I = 2\int x \, x^{1/2} \, dx \rightarrow I = 2\int x^{3/2} \, dx \rightarrow I = 2\left[\frac{x^{5/2}}{5/2} \right]$

$I = \frac{4}{5} \sqrt{x^5} \rightarrow I = \frac{4}{5} \sqrt{x^2 x^2 x} \rightarrow I = \frac{4}{5} x^2 \sqrt{x} + C$ //

4-) $I = \int 3x\sqrt[3]{x} \, dx$

$I = 3\int x \, x^{1/3} \, dx \rightarrow I = 3\int x^{4/3} \, dx \rightarrow I = 3\left[\frac{x^{7/3}}{7/3} \right]$

$I = \frac{9}{7} \sqrt[3]{x^7} \rightarrow I = \frac{9}{7} \sqrt[3]{x^3 x^3 x} \Rightarrow I = \frac{9}{7} x^2 \sqrt[3]{x} + C$ //

5-) $I = \int \frac{2a\cos(x)}{3} \, dx$

$I = \frac{2a}{3} \int \cos(x) \, dx \rightarrow I = \frac{2}{3} a \operatorname{sen}(x) + C$ //

6) $I = \int \frac{1}{5\,\mathrm{sen}^2(x)}\,dx$

$I = \frac{1}{5}\int \mathrm{cosec}^2(x)\,dx \rightarrow \underline{I = -\frac{1}{5}\,\mathrm{cotg}(x) + C}$ ✓

7) $I = \int \frac{1}{5\cos^2(x)}\,dx$

$I = \frac{1}{5}\int \sec^2(x)\,dx \rightarrow \underline{I = \frac{1}{5}\,\mathrm{tg}(x) + C}$ ✓

8) $I = \int 3a\,\sec(x)\,2b\,\mathrm{tg}(x)\,dx$

$I = 6ab\int \sec(x)\,\mathrm{tg}(x)\,dx \rightarrow \underline{I = 6ab\,\sec(x) + C}$ ✓

9) $I = \int \frac{3}{7(1+x^2)}\,dx$

$I = \frac{3}{7}\int \frac{1}{1+x^2}\,dx \rightarrow \underline{I = \frac{3}{7}\,\mathrm{arctg}(x) + C}$ ✓

10) $I = \int \left(e^x + 3x - \frac{5}{x}\right)dx$

$\underline{I = e^x + \frac{3}{2}x^2 - 5\ln|x| + C}$ ✓

11) $I = \int (3.2^x - 3.\mathrm{sen}(x))\,dx$

$\underline{I = 3\left[\frac{2^x}{\ln(2)} + \cos(x)\right] + C}$ ✓

12-) $I = \int \frac{2}{\sqrt{1+x}\,\sqrt{1-x}}\,dx$

$I = 2\int \frac{1}{\sqrt{(1+x)(1-x)}}\,dx \rightarrow I = 2\int \frac{1}{\sqrt{1-x^2}}\,dx$

$\underline{I = 2\,\mathrm{arcsen}(x) + C}$ ✓

13-) $I = \int \frac{1}{\sqrt{4-4x^2}}\,dx$

$I = \int \frac{1}{\sqrt{4(1-x^2)}}\,dx \rightarrow I = \frac{1}{2}\int \frac{1}{\sqrt{1-x^2}}\,dx$

$\underline{I = \frac{1}{2}\,\mathrm{arcsen}(x) + C}$ ✓

14-) $I = \int \frac{x^2}{\sqrt{x^8 - x^6}}\,dx$

$I = \int \frac{x^2}{\sqrt{x^6(x^2-1)}}\,dx \rightarrow I = \int \frac{x^2}{x^3\sqrt{x^2-1}}\,dx$

$I = \int \frac{1}{x\sqrt{x^2-1}}\,dx \rightarrow \underline{I = \mathrm{arcsec}(x) + C}$ ✓

15-) $I = \int \frac{x^5 + 2x^3 + x - 1}{x}\,dx$

$I = \int x^4 + 2x^2 + 1 - \frac{1}{x}\,dx \rightarrow \underline{I = \frac{1}{5}x^5 + \frac{2}{3}x^3 + x - \ln|x| + C}$ ✓

16-) $I = \int \frac{x^3 - x}{x\sqrt{x}}\,dx$

$I = \int \frac{x(x^2-1)}{x\sqrt{x}}\,dx \rightarrow I = \int (x^2-1)\,x^{-1/2}\,dx$

$I = \int x^{3/2} - x^{-1/2}\,dx \rightarrow I = \frac{x^{5/2}}{5/2} - \frac{x^{1/2}}{1/2} \rightarrow \underline{I = \frac{2}{5}\sqrt{x^5} - 2\sqrt{x} + C}$ ✓

17-) $I = \int \frac{1 + tg^2(x)}{tg^2(x)} dx$

$I = \int cotg^2(x) + 1 \, dx \rightarrow I = \int cosec^2(x) \, dx$

$\frac{sen^2(x)}{sen^2(x)} + \frac{cos^2(x)}{sen^2(x)} = \frac{1}{sen^2(x)} \rightarrow \underline{I = -cotg(x) + C}$ ✓

$1 + cotg^2(x) = cosec^2(x)$

18-) $I = \int \frac{1 + sen^2(x)}{sen^2(x)} dx$

$I = \int cosec^2(x) + 1 \, dx \rightarrow \underline{I = -cotg(x) + x + C}$ ✓

19-) $I = \int (3 + 3cotg^2(x)) \, dx$

$I = 3\int 1 + cotg^2(x) \, dx \rightarrow I = 3\int cosec^2(x) \, dx$

$\underline{I = -3\,cotg(x) + C}$ ✓

20-) $I = \int \frac{x - x^3}{x - x^5} dx$

$I = \int \frac{x(1-x^2)}{x(1-x^4)} dx \rightarrow I = \int \frac{\cancel{1-x^2}}{\cancel{(1-x^2)}(1+x^2)} dx$

$I = \int \frac{1}{1+x^2} dx \rightarrow \underline{I = arctg(x) + C}$ ✓

21-) $I = \int tg^2(x)\, dx$

$$\frac{sen^2(x)}{cos^2(x)} + \frac{cos^2(x)}{cos^2(x)} = \frac{1}{cos^2(x)}$$

$tg^2(x) + 1 = sec^2(x)$

$tg^2(x) = sec^2(x) - 1$

$I = \int sec^2(x) - 1\, dx$

$I = tg(x) - x + C$ ✓

22-) $I = \int \frac{tg(x)}{sen(2x)}\, dx$

$sen(2x) = 2sen(x)\cos(x)$

$I = \int \frac{\cancel{sen(x)}}{2\,\cancel{sen(x)}\cos^2(x)}\, dx \rightarrow I = \frac{1}{2}\int \frac{1}{\cos^2(x)}\, dx$

$I = \frac{1}{2}\int sec^2(x)\, dx \rightarrow I = \frac{1}{2} tg(x) + C$ ✓

23-) $I = \int \frac{sen(2x)}{cos^3(x)}\, dx$

$I = 2\int \frac{sen(x)\,\cancel{cos(x)}}{cos^{\cancel{3}\,2}(x)}\, dx \rightarrow I = 2\int tg(x)\, sec(x)\, dx$

$I = 2\, sec(x) + C$ ✓

24-) $I = \int \frac{4x^2 + 4x + 1}{4x + 2}\, dx$

$$\begin{array}{r|l} 4x^2 + 4x + 1 & 4x + 2 \\ \underline{4x^2 + 2x} & x + \frac{1}{2} \\ 2x + 1 & \\ \underline{2x + 1} & \\ 0 & \end{array}$$

$I = \int \frac{\left(x + \frac{1}{2}\right)\cancel{(4x+2)}}{\cancel{4x+2}}\, dx$

$I = \int x + \frac{1}{2}\, dx$

$I = \frac{1}{2}x^2 + \frac{1}{2}x + C$ ✓

25-) $I=\int \frac{\sqrt{1-x^2}}{1-x^2}\,dx$

$I=\int \frac{\sqrt{1-x^2}\,\sqrt{1-x^2}}{(1-x^2)\sqrt{1-x^2}}\,dx \rightarrow I=\int \frac{\cancel{1-x^2}}{(\cancel{1-x^2})\sqrt{1-x^2}}\,dx$

$I=\int \frac{1}{\sqrt{1-x^2}}\,dx \rightarrow \underline{I=\operatorname{arcsen}(x)+C}$ ✓

26-) $I=\int \left(\frac{x}{2}-\frac{2}{x}\right)^2 dx$

$I=\int \frac{x^2}{4}-2\,\frac{\cancel{x}}{\cancel{2}}\,\frac{\cancel{2}}{\cancel{x}}+\frac{4}{x^2}\,dx \rightarrow I=\frac{1}{4}\int x^2\,dx-2\int dx+4\int x^{-2}\,dx$

$I=\frac{1}{12}x^3-2x+4\left[\frac{x^{-1}}{-1}\right] \rightarrow \underline{I=\frac{1}{12}x^3-2x-\frac{4}{x}+C}$

27-) $I=\int \frac{x-1}{\sqrt{x}+1}\,dx$

$I=\int \frac{(x-1)(\sqrt{x}-1)}{(\sqrt{x}+1)(\sqrt{x}-1)}\,dx \rightarrow I=\int \frac{(\cancel{x-1})(\sqrt{x}-1)}{\cancel{x-1}}\,dx$

$I=\int \sqrt{x}-1\,dx \rightarrow I=\int x^{1/2}\,dx-\int dx$

$I=\left[\frac{x^{3/2}}{3/2}\right]-x \rightarrow \underline{I=\frac{2}{3}\sqrt{x^3}-x+C}$ ✓

28-) $I=\int \frac{x^2-3}{x-\sqrt{3}}\,dx$

$I=\int \frac{(x^2-3)(x+\sqrt{3})}{(x-\sqrt{3})(x+\sqrt{3})}\,dx \rightarrow I=\int \frac{(\cancel{x^2-3})(x+\sqrt{3})}{\cancel{x^2-3}}\,dx$

$I=\int x+\sqrt{3}\,dx \rightarrow \underline{I=\frac{1}{2}x^2+\sqrt{3}\,x+C}$ ✓

29-) $I = \int \operatorname{sen}^2\left(\frac{x}{2}\right) dx$

$\cos(2x) = \cos^2(x) - \operatorname{sen}^2(x)$

$-1 \quad = -\operatorname{sen}^2(x) - \cos^2(x)$

$\cos(2x) - 1 = -2\operatorname{sen}^2(x)$

$\operatorname{sen}^2(x) = \frac{1 - \cos(2x)}{2}$

$I = \frac{1}{2}\int 1 - \cos\left(\frac{2x}{2}\right) dx$

$I = \frac{1}{2}\int 1 - \cos(x)\, dx$

$I = \frac{1}{2}\left[x - \operatorname{sen}(x)\right] + C$

30-) $I = \int \cos^2\left(\frac{x}{2}\right) dx$

$\cos(2x) = \cos^2(x) - \operatorname{sen}^2(x)$

$+ \quad 1 \quad = \operatorname{sen}^2(x) + \cos^2(x)$

$\cos(2x) + 1 = 2\cos^2(x)$

$\cos^2(x) = \frac{1 + \cos(2x)}{2}$

$I = \frac{1}{2}\int 1 + \cos\left(\frac{2x}{2}\right) dx$

$I = \frac{1}{2}\left[x + \operatorname{sen}(x)\right] + C$

31-) $I = \int \frac{\cos(2x)}{\cos^2(x) - \frac{1}{2}} dx$

$\cos(2x) = \cos^2(x) - \operatorname{sen}^2(x)$

$+ \quad 1 \quad = \operatorname{sen}^2(x) + \cos^2(x)$

$\cos(2x) + 1 = 2\cos^2(x)$

$\cos^2(x) = \frac{1}{2} + \frac{\cos(2x)}{2}$

$I = \int \frac{\cos(2x)}{\frac{1}{2} + \frac{\cos(2x)}{2} - \frac{1}{2}} dx$

$I = \int \frac{\cos(2x)}{\frac{\cos(2x)}{2}} dx \Rightarrow I = 2\int dx$

$I = 2x + C$

32-) $I = \int \operatorname{sen}(x)\, \sec(x)\, tg(x)\, dx$

$I = \int \frac{\operatorname{sen}(x)}{\cos(x)} \cdot tg(x)\, dx \rightarrow I = \int tg^2(x)\, dx$

$\frac{\operatorname{sen}^2(x)}{\cos^2(x)} + \frac{\cos^2(x)}{\cos^2(x)} = \frac{1}{\cos^2(x)}$ $\qquad I = \int \sec^2(x) - 1\, dx$

$tg^2(x) + 1 = \sec^2(x)$ $\qquad I = tg(x) - x + C$ ✓

$tg^2(x) = \sec^2(x) - 1$

33-) $I = \int \cos(x)\, \operatorname{cosec}(x)\, \operatorname{cotg}(x)\, dx$

$I = \int \frac{\cos(x)}{\operatorname{sen}(x)} \operatorname{cotg}(x)\, dx \rightarrow I = \int \operatorname{cotg}^2(x)\, dx$

$\frac{\operatorname{sen}^2(x)}{\operatorname{sen}^2(x)} + \frac{\cos^2(x)}{\operatorname{sen}^2(x)} = \frac{1}{\operatorname{sen}^2(x)}$ $\qquad I = \int \operatorname{cosec}^2(x) - 1\, dx$

$1 + \operatorname{cotg}^2(x) = \operatorname{cosec}^2(x)$ $\qquad I = -\operatorname{cotg}(x) - x + C$ ✓

$\operatorname{cotg}^2(x) = \operatorname{cosec}^2(x) - 1$

34-) $I = \int \frac{x^3 + x + 1}{x^2 + 1}\, dx$

$$\begin{array}{l|l} x^3 + 0x^2 + x + 1 & x^2 + 1 \\ \hline x^3 \qquad\quad + x & x \\ \hline \qquad\qquad\quad + 1 & \end{array}$$

$I = \int \frac{x(x^2+1)}{x^2+1} + \frac{1}{x^2+1}\, dx$

$I = \int x + \frac{1}{x^2+1}\, dx$

$I = \frac{1}{2}x^2 + \operatorname{arctg}(x) + C$ ✓

35) $I = \int \frac{x^3 - x^2 + x - 2}{x^2 + 1}\, dx$

$$\begin{array}{l|l} x^3 - x^2 + x - 2 & x^2 + 1 \\ \hline \underline{x^3 \quad\quad + x} & x - 1 \\ \quad -x^2 + 0x - 2 & \\ \quad \underline{-x^2 \quad\quad - 1} & \\ \quad\quad\quad -1 & \end{array}$$

$I = \int \frac{(x-1)\cancel{(x^2+1)}}{\cancel{x^2+1}} - \frac{1}{x^2+1}\, dx$

$I = \frac{1}{2}x^2 - x - \operatorname{arctg}(x) + C$ ✓

36) $I = \int \frac{x^4 + 2x^2}{1 + x^2}\, dx$

$$\begin{array}{l|l} x^4 + 0x^3 + 2x^2 + 0x + 0 & x^2 + 1 \\ \hline \underline{x^4 \quad\quad + x^2} & x^2 + 1 \\ \quad\quad x^2 + 0x + 0 & \\ \quad\quad \underline{x^2 \quad\quad 1} & \\ \quad\quad\quad -1 & \end{array}$$

$I = \int \frac{(x^2+1)^2}{x^2+1} - \frac{1}{x^2+1}\, dx$

$I = \int x^2 + 1 - \frac{1}{x^2+1}\, dx$

$I = \frac{1}{3}x^3 + x - \operatorname{arctg}(x) + C$ ✓

37) $I = \int \frac{x^5 + 3x^3 + 2x - 1}{x^2 + 1}\, dx$

$$\begin{array}{l|l} x^5 + 0x^4 + 3x^3 + 0x^2 + 2x - 1 & x^2 + 1 \\ \hline \underline{x^5 \quad\quad + x^3} & x^3 + 2x \\ \quad\quad 2x^3 + 0x^2 + 2x - 1 & \\ \quad\quad \underline{2x^3 \quad\quad + 2x} & \\ \quad\quad\quad -1 & \end{array}$$

$I = \int \frac{(x^3+2x)\cancel{(x^2+1)}}{\cancel{x^2+1}} - \frac{1}{x^2+1}\, dx$

$I = \frac{1}{4}x^4 + x^2 - \operatorname{arctg}(x) + C$ ✓

38) $I = \int \frac{2x^4 - 3x^3 + 2x^2 - 3x + 1}{x^2+1} dx$

$$\begin{array}{l|l} 2x^4 - 3x^3 + 2x^2 - 3x + 1 & x^2+1 \\ \underline{2x^4 \quad\quad + 2x^2} & 2x^2 - 3x \\ \quad -3x^3 + 0x^2 - 3x + 1 & \\ \quad \underline{-3x^3 \quad\quad - 3x} & \\ \quad\quad\quad\quad + 1 & \end{array}$$

$I = \int \frac{(2x^2 - 3x)(x^2+1)}{x^2+1} + \frac{1}{x^2+1} dx$

$I = \int 2x^2 - 3x + \frac{1}{x^2+1} dx$

$\boxed{I = \frac{2}{3}x^3 - \frac{3}{2}x^2 + \mathrm{arctg}(x) + C}$ ✓