Exercícios - Lista II

Calcule as seguintes integrais

1-) $I = \int (3x+1)^{20} dx$

$3x+1 = t$ $\quad I = \frac{1}{3}\int t^{20} dt \rightarrow I = \frac{1}{3}\left[\frac{t^{21}}{21}\right]$

$3dx = dt$

$dx = \frac{1}{3} dt$ $\quad I = \frac{1}{63} t^{21} \rightarrow I = \frac{1}{63}(3x+1)^{21} + C$ ✓

2-) $I = \int (5x-1)^{15} dx$

$5x-1 = t$ $\quad I = \frac{1}{5}\int t^{15} dt \rightarrow I = \frac{1}{5}\left[\frac{t^{16}}{16}\right] \rightarrow I = \frac{1}{80} t^{16}$

$5dx = dt$

$dx = \frac{1}{5} dt$ $\quad I = \frac{1}{80}(5x-1)^{16} + C$ ✓

3-) $I = \int \frac{3}{(2x-3)^{10}} dx$

$2x-3 = t$ $\quad I = \frac{3}{2}\int \frac{1}{t^{10}} dt \rightarrow I = \frac{3}{2}\int t^{-10} dt$

$2dx = dt$

$dx = \frac{1}{2} dt$ $\quad I = \frac{3}{2}\left[\frac{t^{-9}}{-9}\right] \rightarrow I = -\frac{1}{6}\,\frac{1}{t^9}$

$$I = -\frac{1}{6(2x-3)^9} + C$$ ✓

4-) $I = \int \frac{1}{\sqrt{5+x}} dx$

$5+x = t$ $\quad I = \int t^{-1/2} dt \rightarrow I = \frac{t^{1/2}}{1/2} \rightarrow I = 2\sqrt{t}$

$dx = dt$

$I = 2\sqrt{5+x} + C$ ✓

5-) $I=\int \frac{x}{\sqrt{x^2+1}}\,dx$

$x^2+1=t$ $\quad$ $I=\frac{1}{2}\int \frac{1}{\sqrt{t}}\,dt \rightarrow I=\frac{1}{2}\int t^{-1/2}\,dt$

$2x\,dx=dt$

$dx=\frac{1}{2x}\,dt$ $\quad$ $I=\frac{1}{2}\left[\frac{t^{1/2}}{1/2}\right] \rightarrow I=\sqrt{t} \rightarrow I=\sqrt{x^2+1}+C$

6-) $I=\int \frac{x^2}{\sqrt{5x^3+1}}\,dx$

$5x^3+1=t$ $\quad$ $I=\frac{1}{15}\int \frac{x^2}{x^2\sqrt{t}}\,dt \rightarrow I=\frac{1}{15}\int t^{-1/2}\,dt$

$15x^2\,dx=dt$

$dx=\frac{1}{15x^2}\,dt$ $\quad$ $I=\frac{1}{15}\left[\frac{t^{1/2}}{1/2}\right] \rightarrow I=\frac{2}{15}\sqrt{t} \rightarrow I=\frac{2}{15}\sqrt{5x^3+1}+C$

7-) $I=\int (2x+1)\sqrt{x^2+x-3}\,dx$

$x^2+x-3=t$ $\quad$ $I=\int \frac{\cancel{(2x+1)}\sqrt{t}}{\cancel{2x+1}}\,dt \rightarrow I=\int t^{1/2}\,dt$

$2x+1\,dx=dt$

$dx=\frac{1}{2x+1}\,dt$ $\quad$ $I=\left[\frac{t^{3/2}}{3/2}\right] \rightarrow I=\frac{2}{3}\sqrt{t^3}$

$I=\frac{2}{3}\sqrt{(x^2+x-3)^3}+C$

8-) $I=\int \frac{2x^2+1}{\sqrt{2x^3+3x+1}}\,dx$

$2x^3+3x+1=t$ $\quad$ $I=\frac{1}{3}\int \frac{2x^2+1}{(2x^2+1)\sqrt{t}}\,dt \rightarrow I=\frac{1}{3}\int t^{-1/2}\,dt$

$6x^2+3\,dx=dt$

$3(2x^2+1)dx=dt$ $\quad$ $I=\frac{1}{3}\left[\frac{t^{1/2}}{1/2}\right] \rightarrow I=\frac{2}{3}\sqrt{t}$

$dx=\frac{1}{3(2x^2+1)}\,dt$

$I=\frac{2}{3}\sqrt{2x^3+3x+1}+C$

9-) $I = \int \frac{1}{(x-a)^4} dx$

$x - a = t$ $\quad I = \int \frac{1}{t^4} dt \rightarrow I = \int t^{-4} dt \rightarrow I = \frac{t^{-3}}{-3}$

$dx = dt$

$I = -\frac{1}{3t^3} \rightarrow I = -\frac{1}{3(x-a)^3} + C$ ✓

10-) $I = \int \frac{(4x-2)^2}{(2x-1)^5} dx$

$I = \int \frac{[2(2x-1)]^2}{(2x-1)^5} dx \rightarrow I = 4\int \frac{(2x-1)^2}{(2x-1)^5} dx \rightarrow I = 4\int \frac{1}{(2x-1)^3} dx$

$2x - 1 = t$ $\quad I = 2\int \frac{1}{t^3} dt \rightarrow I = 2\int t^{-3} dt$

$2dx = dt$

$dx = \frac{1}{2} dt$ $\quad I = 2\left[\frac{t^{-2}}{-2}\right] \rightarrow I = -\frac{1}{t^2}$

$I = -\frac{1}{(2x-1)^2} + C$ ✓

11-) $I = \int \frac{1}{(2x-5)^2 \sqrt{2x-5}} dx$

$I = \int \frac{1}{(2x-5)^2 (2x-5)^{1/2}} dx \rightarrow I = \int \frac{1}{(2x-5)^{5/2}} dx \rightarrow I = \int (2x-5)^{-5/2} dx$

$2x - 5 = t$ $\quad I = \frac{1}{2}\int t^{-5/2} dt \rightarrow I = \frac{1}{2}\left[\frac{t^{-3/2}}{-3/2}\right]$

$2dx = dt$

$dx = \frac{1}{2} dt$ $\quad I = -\frac{1}{3\sqrt{t^3}} \rightarrow I = -\frac{1}{3\sqrt{(2x-5)^3}} + C$ ✓

12-) $I=\int \frac{\sqrt[3]{4x-16}}{(x-4)^3}\,dx$

$I=\int \frac{\sqrt[3]{4(x-4)}}{(x-4)^3}\,dx \rightarrow I=\sqrt[3]{4}\int (x-4)^{1/3}(x-4)^{-3}\,dx$

$I=\sqrt[3]{4}\int (x-4)^{-8/3}\,dx \qquad I=\sqrt[3]{4}\int t^{-8/3}\,dt$

$x-4=t$

$dx=dt$

$I=\sqrt[3]{4}\left[\frac{t^{-\frac{5}{3}}}{-5/3}\right] \rightarrow I=-\frac{3}{5}\frac{\sqrt[3]{4}}{\sqrt[3]{t^5}} \rightarrow I=-\frac{3}{5}\sqrt[3]{\frac{4}{(x-4)^5}}+C$

13-) $I=\int \frac{\sqrt[5]{(4x+2)^7}}{8x+4}\,dx$

$I=\int \frac{(4x+2)^{7/5}}{2(4x+2)}\,dx \rightarrow I=\frac{1}{2}\int (4x+2)^{7/5}(4x+2)^{-1}\,dx$

$I=\frac{1}{2}\int (4x+2)^{2/5}\,dx$

$4x+2=t$

$4\,dx=dt$

$dx=\frac{1}{4}dt$

$I=\frac{1}{8}\int t^{2/5}\,dt \rightarrow I=\frac{1}{8}\left[\frac{t^{7/5}}{7/5}\right]$

$I=\frac{5}{56}\sqrt[5]{(4x+2)^7}+C$

14-) $I=\int \sqrt{\left(\frac{x^2-1}{x+1}\right)^5}\,dx$

$I=\int \sqrt{\left(\frac{(x-1)\cancel{(x+1)}}{\cancel{x+1}}\right)^5}\,dx \rightarrow I=\int (x-1)^{5/2}\,dx$

$x-1=t$

$dx=dt$

$I=\int t^{5/2}\,dt \rightarrow I=\frac{t^{7/2}}{7/2} \rightarrow I=\frac{2}{7}\sqrt{t^7}$

$I=\frac{2}{7}\sqrt{(x-1)^7}+C$

15-) $$I=\int \frac{\sqrt{\left(4-\frac{3}{x^2}\right)^3}}{x^3}\,dx$$

$$I=\int \frac{\left(4-\frac{3}{x^2}\right)^{3/2}}{x^3}\,dx$$

$$4-\frac{3}{x^2}=t$$

$$4-3x^{-2}=t$$

$$6x^{-3}\,dx=dt$$

$$dx=\frac{x^3}{6}\,dt$$

$$I=\frac{1}{6}\int \frac{t^{3/2}}{x^3}\,x^3\,dt$$

$$I=\frac{1}{6}\int t^{3/2}\,dt$$

$$I=\frac{1}{6}\left[\frac{t^{5/2}}{5/2}\right]$$

$$I=\frac{1}{15}\sqrt{t^5} \rightarrow I=\frac{1}{15}\sqrt{\left(4-\frac{3}{x^2}\right)^5}+C$$

16-) $$I=\int \frac{1}{x^2}\sqrt[3]{\left(\frac{1}{x}+b\right)^2}\,dx$$

$$I=\int \frac{1}{x^2}\left(\frac{1}{x}+b\right)^{2/3}dx$$

$$\frac{1}{x}+b=t$$

$$x^{-1}+b=t$$

$$-x^{-2}\,dx=dt$$

$$dx=-x^2\,dt$$

$$I=-\int \frac{x^2\,t^{2/3}}{x^2}\,dt$$

$$I=-\int t^{2/3}\,dt$$

$$I=-\left[\frac{t^{5/3}}{5/3}\right] \rightarrow I=-\frac{3}{5}\sqrt[3]{\left(\frac{1}{x}+b\right)^5}+C$$

17-) $$I=\int \frac{1}{\sqrt[3]{x^2}\left(1-2\sqrt[3]{x}\right)}\,dx$$

$$1-2x^{1/3}=t$$

$$-\frac{2}{3}x^{-2/3}\,dx=dt$$

$$dx=-\frac{3\sqrt[3]{x^2}}{2}\,dt$$

$$I=-\frac{3}{2}\int \frac{\sqrt[3]{x^2}}{\sqrt[3]{x^2}\,t}\,dt \rightarrow I=-\frac{3}{2}\int \frac{1}{t}\,dt$$

$$I=-\frac{3}{2}\ln|t| \rightarrow I=-\frac{3}{2}\ln\left|1-2\sqrt[3]{x}\right|+C$$

18) $I = \int e^{-\frac{x}{a}} dx$

$-\frac{x}{a} = t$ $\qquad I = -a\int e^{t} dt \rightarrow I = -ae^{t}$

$-\frac{1}{a}dx = dt$ $\qquad \underline{I = -a\, e^{-x/a} + C}$ ✓

$dx = -a\,dt$

19) $I = \int \frac{e^{3x-4}}{e^{x+2}} dx$

$I = \int e^{3x-4}\, e^{-(x+2)} dx \rightarrow I = \int e^{3x-4-x-2} dx$

$I = \int e^{2x-6} dx$ $\qquad 2x-6 = t$

$2dx = dt$

$I = \frac{1}{2}\int e^{t} dt \rightarrow I = \frac{1}{2}e^{t}$ $\qquad dx = \frac{1}{2}dt$

$\underline{I = \frac{1}{2}e^{2x-6} + C}$ ✓

20) $I = \int \frac{x\, e^{x^2+x}}{e^{x+1}} dx$

$I = \int x\, e^{x^2+x}\, e^{-(x+1)} dx \rightarrow I = \int x\, e^{x^2+x-x-1} dx$

$I = \int x\, e^{x^2-1} dx$ $\qquad x^2-1 = t$

$2x\,dx = dt$

$I = \frac{1}{2}\int \frac{x\, e^{t}}{x} dt \rightarrow I = \frac{1}{2}\int e^{t} dt$ $\qquad dx = \frac{1}{2x}dt$

$I = \frac{1}{2}e^{t} \rightarrow \underline{I = \frac{1}{2}e^{x^2-1} + C}$ ✓

21-> $I = \int a^{4x} \ln(a)\, dx$

$I = \ln(a) \int a^{4x}\, dx$ $\qquad 4x = t$

$4dx = dt$

$I = \frac{1}{4} \ln(a) \int a^{t}\, dt$ $\qquad dx = \frac{1}{4} dt$

$I = \frac{1}{4} \ln(a) \left[\frac{a^{t}}{\ln(a)} \right] \rightarrow I = \frac{1}{4} a^{t} \rightarrow \underline{I = \frac{1}{4} a^{4x} + C}$ ✓

22-> $I = \int \operatorname{sen}\left(\frac{x}{2}\right) dx$

$\frac{x}{2} = t$ $\qquad I = 2 \int \operatorname{sen}(t)\, dt$

$\frac{1}{2} dx = dt$ $\qquad I = -2\cos(t) \rightarrow \underline{I = -2\cos\left(\frac{x}{2}\right) + C}$ ✓

$dx = 2dt$

23-> $I = \int (x+1) \operatorname{sen}(x^2 + 2x)\, dx$

$x^2 + 2x = t$ $\qquad I = \frac{1}{2} \int \frac{\cancel{(x+1)} \operatorname{sen}(t)}{\cancel{x+1}}\, dt$

$2x + 2\, dx = dt$

$2(x+1) dx = dt$ $\qquad I = \frac{1}{2} \int \operatorname{sen}(t)\, dt \rightarrow I = -\frac{1}{2} \cos(t)$

$dx = \frac{1}{2(x+1)} dt$

$\underline{I = -\frac{1}{2} \cos(x^2 + 2x) + C}$ ✓

24-) $I = \int x \operatorname{sen}\left(\frac{6x^4 - 2x^2}{x^2}\right) dx$

$I = \int x \operatorname{sen}\left(\frac{x^2(6x^2-2)}{x^2}\right) dx \rightarrow I = \int x \operatorname{sen}(6x^2-2)\, dx$

$6x^2 - 2 = t$ $\quad I = \frac{1}{12}\int \frac{x \operatorname{sen}(t)}{x} dt \rightarrow I = \frac{1}{12}\int \operatorname{sen}(t)\, dt$

$12x\, dx = dt$

$dx = \frac{1}{12x} dt$ $\quad I = -\frac{1}{12}\cos(t) \rightarrow I = -\frac{1}{12}\cos(6x^2-2) + C$ ✓

25-) $I = \int tg(2x)\cos(2x)\, dx$

$I = \int \frac{\operatorname{sen}(2x)}{\cancel{\cos(2x)}} \cdot \cancel{\cos(2x)}\, dx \rightarrow I = \int \operatorname{sen}(2x)\, dx$

$2x = t$ $\quad I = \frac{1}{2}\int \operatorname{sen}(t)\, dt \rightarrow I = -\frac{1}{2}\cos(t)$

$2dx = dt$

$dx = \frac{1}{2} dt$ $\quad I = -\frac{1}{2}\cos(2x) + C$ ✓

26-) $I = \int e^x \cos(e^x - 1)\, dx$

$e^x - 1 = t$ $\quad I = \int \frac{e^x \cos(t)}{e^x} dt \rightarrow I = \int \cos(t)\, dt$

$e^x\, dx = dt$

$dx = \frac{1}{e^x} dt$ $\quad I = \operatorname{sen}(t) \rightarrow I = \operatorname{sen}(e^x - 1) + C$ ✓

27-) $I = \int \operatorname{sen}(2x)\cos(2x)\,dx$

$\operatorname{sen}(2x) = t$

$2\cos(2x)\,dx = dt$

$dx = \frac{1}{2\cos(2x)}\,dt$

$I = \frac{1}{2}\int \frac{t\cos(2x)}{\cos(2x)}\,dt$

$I = \frac{1}{2}\int t\,dt \rightarrow I = \frac{1}{4}t^2 \rightarrow I = \frac{1}{4}\operatorname{sen}^2(2x) + C$ ✓

28-) $I = \int \cos(3x)\operatorname{sen}(3x)\,dx$

$\operatorname{sen}(3x) = t$

$3\cos(3x)\,dx = dt$

$dx = \frac{1}{3\cos(3x)}\,dt$

$I = \frac{1}{3}\int \frac{\cos(3x)\,t}{\cos(3x)}\,dt \rightarrow I = \frac{1}{3}\int t\,dt$

$I = \frac{1}{3}\left[\frac{t^2}{2}\right] \rightarrow I = \frac{1}{6}\operatorname{sen}^2(3x) + C$ ✓

29-) $I = \int tg(x)\,dx$

$I = \int \frac{\operatorname{sen}(x)}{\cos(x)}\,dx$

$\cos(x) = t$

$-\operatorname{sen}(x)\,dx = dt$

$dx = -\frac{1}{\operatorname{sen}(x)}\,dt$

$I = -\int \frac{\operatorname{sen}(x)}{t\operatorname{sen}(x)}\,dt \rightarrow I = -\int \frac{1}{t}\,dt$

$I = -\ln|t|$

$I = -\ln|\cos(x)| + C$ ✓

30-) $I = \int cotg(x)\,dx$

$I = \int \frac{\cos(x)}{\operatorname{sen}(x)}\,dx$

$\operatorname{sen}(x) = t$

$\cos(x)\,dx = dt$

$dx = \frac{1}{\cos(x)}\,dt$

$I = \int \frac{\cos(x)}{t\cos(x)}\,dt \rightarrow I = \int \frac{1}{t}\,dt$

$I = \ln|t| \rightarrow I = \ln|\operatorname{sen}(x)| + C$ ✓

31-) $I = \int tg(2a+x)\,dx$

$2a + x = t$

$dx = dt$

$I = \int \frac{sen(t)}{cos(t)} dt \to I = -\int \frac{sen(t)}{u\, sen(t)} du$

$cos(t) = u$

$-sen(t)dt = du$

$I = -\int \frac{1}{u} du \to I = -\ln|u|$

$dt = -\frac{1}{sen(t)} du$

$I = -\ln|cos(t)| \to I = -\ln|cos(2a+x)| + C$ ✓

32-) $I = \int a \cdot cotg(ax+b)\,dx$

$ax + b = t$

$a\,dx = dt$

$dx = \frac{1}{a} dt$

$I = \int cotg(t)\,dt \to I = \int \frac{cos(t)}{sen(t)} dt$

$sen(t) = u$

$cos(t)\,dt = du$

$dt = \frac{1}{cos(t)} du$

$I = \int \frac{cos(t)}{u\, cos(t)} du \to I = \int \frac{1}{u} du$

$I = \ln|u| \to I = \ln|sen(ax+b)| + C$ ✓

33-) $I = \int \frac{sen(x)}{cos^3(x)} dx$

$I = \int tg(x)\, sec(x)\, sec(x)\, dx$

$sec(x) = t$

$sec(x)\, tg(x)\, dx = dt$

$dx = \frac{1}{sec(x)\, tg(x)} dt$

$I = \int \frac{\cancel{tg(x)\, sec(x)}\, t}{\cancel{tg(x)\, sec(x)}} dt$

$I = \int t\,dt \to I = \frac{1}{2} t^2$

$I = \frac{1}{2} sec^2(x) + C$ ✓

34-) $J = \int \frac{1}{x \ln(x)} dx$

$\ln(x) = t$

$\frac{1}{x} dx = dt$

$dx = x\, dt$

$I = \int \frac{x}{x\, t} dt \rightarrow J = \int \frac{1}{t} dt$

$I = \ln|t| \rightarrow I = \ln|\ln(x)| + C$ ✓

35-) $I = \int \frac{\ln^2(x)}{x} dx$

$\ln(x) = t$

$\frac{1}{x} dx = dt$

$dx = x\, dt$

$I = \int \frac{t^2 x}{x} dt \rightarrow I = \int t^2 dt$

$I = \frac{1}{3} t^3 \rightarrow I = \frac{1}{3} \ln^3(x) + C$ ✓

36-) $I = \int \frac{\ln(2x-4)}{x-2} dx$

$\ln(2x-4) = t$

$\frac{2}{2x-4} dx = dt$

$dx = (x-2)\, dt$

$I = \int \frac{t(x-2)}{x-2} dt$

$I = \int t\, dt \rightarrow I = \frac{1}{2} t^2$

$I = \frac{1}{2} \ln^2(2x-4) + C$ ✓

37-) $I = \int \frac{\ln^2(ax+1)}{ax+1} dx$

$\ln(ax+1) = t$

$\frac{a}{ax+1} dx = dt$

$dx = \frac{ax+1}{a} dt$

$I = \frac{1}{a} \int \frac{(ax+1)\, t^2}{ax+1} dt \rightarrow I = \frac{1}{a} \int t^2 dt$

$I = \frac{1}{a} \left[\frac{t^3}{3} \right] \rightarrow I = \frac{1}{3a} \ln^3(ax+1) + C$ ✓

38-) $I=\int \frac{arctg(x)}{1+x^2}\,dx$

$arctg(x) = t$

$\frac{1}{1+x^2}\,dx = dt$

$dx = (1+x^2)\,dt$

$I=\int \frac{t(1+x^2)}{1+x^2}\,dt \rightarrow I=\int t\,dt$

$I=\frac{1}{2}t^2 \rightarrow \underline{I=\frac{1}{2}\,arctg^2(x)+C}$

39-) $I=\int \frac{arccotg(x)}{1+x^2}\,dx$

$arccotg(x) = t$

$-\frac{1}{1+x^2}\,dx = dt$

$dx = -(1+x^2)\,dt$

$I=-\int \frac{t(1+x^2)}{1+x^2}\,dt \rightarrow I=-\int t\,dt$

$I=-\frac{1}{2}t^2 \rightarrow \underline{I=-\frac{1}{2}\,arccotg^2(x)+C}$

40-) $I=\int \frac{arcsen(x)}{\sqrt{1-x^2}}\,dx$

$arcsen(x) = t$

$\frac{1}{\sqrt{1-x^2}}\,dx = dt$

$dx = \sqrt{1-x^2}\,dt$

$I=\int \frac{t\sqrt{1-x^2}}{\sqrt{1-x^2}}\,dt \rightarrow I=\int t\,dt$

$I=\frac{1}{2}t^2 \rightarrow \underline{I=\frac{1}{2}\,arcsen^2(x)+C}$

41-) $I=\int \frac{1}{x+x\,ln^2(x)}\,dx$

$I=\int \frac{1}{x(1+ln^2(x))}\,dx$

$ln(x) = t$

$\frac{1}{x}\,dx = dt$

$dx = x\,dt$

$I=\int \frac{x}{x(1+t^2)}\,dt$

$I=\int \frac{1}{1+t^2}\,dt$

$I = arctg(t)$

$\underline{I = arctg(ln(x))+C}$

42-) $I = \int \sqrt{x}\ \text{sen}(x\sqrt{x})\ dx$

$x\sqrt{x} = t$ $\qquad I = \frac{2}{3}\int \frac{\sqrt{x}\ \text{sen}(t)}{\sqrt{x}}\ dt \rightarrow I = \frac{2}{3}\int \text{sen}(t)\ dt$

$x^{3/2} = t$

$\frac{3}{2}x^{\frac{1}{2}}dx = dt$ $\qquad I = -\frac{2}{3}\cos(t) \rightarrow I = -\frac{2}{3}\cos(x\sqrt{x}) + C$ ✓

$dx = \frac{2}{3\sqrt{x}}\ dt$

43-) $I = \int \sqrt{x}\ e^{2x\sqrt{x}}\ dx$

$2x\sqrt{x} = t$ $\qquad I = \frac{1}{3}\int \frac{\sqrt{x}\ e^{t}}{\sqrt{x}}\ dt \rightarrow I = \frac{1}{3}\int e^{t}\ dt$

$2x^{3/2} = t$

$3x^{\frac{1}{2}}dx = dt$ $\qquad I = \frac{1}{3}e^{t} \rightarrow I = \frac{1}{3}e^{2x\sqrt{x}} + C$ ✓

$dx = \frac{1}{3\sqrt{x}}dt$

44-) $I = \int \frac{\sqrt{x}}{(a + x\sqrt{x})^2}\ dx$

$a + x\sqrt{x} = t$ $\qquad I = \frac{2}{3}\int \frac{\sqrt{x}}{\sqrt{x}\ t^2}\ dt \rightarrow I = \frac{2}{3}\int \frac{1}{t^2}\ dt$

$a + x^{3/2} = t$

$\frac{3}{2}x^{\frac{1}{2}}dx = dt$ $\qquad I = \frac{2}{3}\int t^{-2}\ dt \rightarrow I = \frac{2}{3}\left[\frac{t^{-1}}{-1}\right]$

$dx = \frac{2}{3\sqrt{x}}dt$ $\qquad I = -\frac{2}{3t} \rightarrow I = -\frac{2}{3(a + x\sqrt{x})} + C$ ✓

45-) $I = \int \sqrt{x}\sqrt{1 + x\sqrt{x}}\, dx$

$1 + x\sqrt{x} = t$ $\qquad I = \frac{2}{3}\int \frac{\sqrt{x}\sqrt{t}}{\sqrt{x}}\, dt \to I = \frac{2}{3}\int t^{1/2}\, dt$

$1 + x^{3/2} = t$

$\frac{3}{2}x^{1/2}\, dx = dt$ $\qquad I = \frac{2}{3}\left[\frac{t^{3/2}}{3/2}\right] \to I = \frac{4}{9}\sqrt{(t)^3}$

$dx = \frac{2}{3\sqrt{x}}\, dt$ $\qquad I = \frac{4}{9}\sqrt{(1 + x\sqrt{x})^3} + C$ ✓

46-) $I = \int \frac{x}{\sqrt{1 - x^4}}\, dx$

$x^2 = t$ $\qquad I = \frac{1}{2}\int \frac{x}{x\sqrt{1 - t^2}}\, dt \to I = \frac{1}{2}\int \frac{1}{\sqrt{1 - t^2}}\, dt$

$2x\, dx = dt$

$dx = \frac{1}{2x}\, dt$ $\qquad I = \frac{1}{2}\operatorname{arcsen}(t) \to I = \frac{1}{2}\operatorname{arcsen}(x^2) + C$ ✓

47-) $I = \int \frac{a}{\sqrt{a^2 - x^2}}\, dx$

$I = a\int \frac{1}{\sqrt{a^2\left(1 - \left(\frac{x}{a}\right)^2\right)}}\, dx \to I = \int \frac{1}{\sqrt{1 - \left(\frac{x}{a}\right)^2}}\, dx$

$\frac{x}{a} = t$ $\qquad I = a\int \frac{1}{\sqrt{1 - t^2}}\, dt$ $\qquad I = a \operatorname{arcsen}(t)$

$\frac{1}{a}\, dx = dt$ $\qquad I = a \operatorname{arcsen}\left(\frac{x}{a}\right) + C$ ✓

$dx = a\, dt$

48-) $I = \int x(x+1)^{10}\,dx$

$x+1 = t$
$dx = dt$
$\Rightarrow x = t-1$

$I = \int (t-1)\,t^{10}\,dt \Rightarrow I = \int t^{11} - t^{10}\,dt$

$I = \frac{1}{12}t^{12} - \frac{1}{11}t^{11} \Rightarrow \underline{I = \frac{1}{12}(x+1)^{12} - \frac{1}{11}(x+1)^{11} + C}$ ✓

49-) $I = \int x^2 (x-2)^4\,dx$

$x-2 = t$
$dx = dt$
$\Rightarrow x = t+2$

$I = \int (t+2)^2\,t^4\,dt \Rightarrow I = \int (t^2+4t+4)\,t^4\,dt$

$I = \int t^6 + 4t^5 + 4t^4\,dt \Rightarrow I = \frac{1}{7}t^7 + \frac{4}{6}t^6 + \frac{4}{5}t^5$

$\underline{I = \frac{1}{7}(x-2)^7 + \frac{2}{3}(x-2)^6 + \frac{4}{5}(x-2)^5 + C}$ ✓

50-) $I = \int \frac{x^3}{\sqrt{1+x^2}}\,dx$

$1+x^2 = t$
$2x\,dx = dt$
$dx = \frac{1}{2x}dt$
$\Rightarrow x^2 = t-1$

$I = \frac{1}{2}\int \frac{x^3}{x\sqrt{t}}\,dt \Rightarrow I = \frac{1}{2}\int \frac{x^2}{\sqrt{t}}\,dt$

$I = \frac{1}{2}\int (t-1)\,t^{-1/2}\,dt \Rightarrow I = \frac{1}{2}\int t^{\frac{1}{2}} - t^{-1/2}\,dt$

$I = \frac{1}{2}\left[\frac{t^{3/2}}{3/2} - \frac{t^{1/2}}{1/2}\right] \Rightarrow I = \frac{1}{3}\sqrt{t^3} - \sqrt{t}$

$\underline{I = \frac{1}{3}\sqrt{(1+x^2)^3} - \sqrt{1+x^2} + C}$ ✓

S1 $\Rightarrow$ $$I=\int \frac{x^2}{\sqrt{1+x}}\,dx$$

$$\begin{cases} 1+x=t \\ dx=dt \\ \Rightarrow x=t-1 \end{cases}$$

$$I=\int \frac{(t-1)^2}{\sqrt{t}}\,dt \Rightarrow I=\int (t^2-2t+1)\,t^{-1/2}\,dt$$

$$I=\int t^{3/2}-2t^{1/2}+t^{-1/2}\,dt \Rightarrow I=\frac{t^{5/2}}{5/2}-2\left[\frac{t^{3/2}}{3/2}\right]+\frac{t^{1/2}}{1/2}$$

$$I=\frac{2}{5}\sqrt{t^5}-\frac{4}{3}\sqrt{t^3}+2\sqrt{t}$$

$$I=\frac{2}{5}\sqrt{(1+x)^5}-\frac{4}{3}\sqrt{(1+x)^3}+2\sqrt{1+x}+C$$

✓

S2 $\Rightarrow$ $$I=\int x^3\sqrt{1-x^2}\,dx$$

$$\begin{cases} 1-x^2=t \\ -2x\,dx=dt \\ dx=-\frac{1}{2x}\,dt \\ \Rightarrow x^2=1-t \end{cases}$$

$$I=-\frac{1}{2}\int \frac{x^3\sqrt{t}}{x}\,dt \Rightarrow I=-\frac{1}{2}\int x^2\,t^{1/2}\,dt$$

$$I=-\frac{1}{2}\int (1-t)\,t^{1/2}\,dt \Rightarrow I=-\frac{1}{2}\int t^{1/2}-t^{3/2}\,dt$$

$$I=-\frac{1}{2}\left[\frac{t^{3/2}}{3/2}-\frac{t^{5/2}}{5/2}\right] \Rightarrow I=-\frac{1}{3}\sqrt{t^3}+\frac{1}{5}\sqrt{t^5}$$

$$I=-\frac{1}{3}\sqrt{(1-x^2)^3}+\frac{1}{5}\sqrt{(1-x^2)^5}+C$$

✓

53-) $I=\int x^5\sqrt{1-x^2}\,dx$

$1-x^2=t$

$-2x\,dx=dt$

$dx=-\frac{1}{2x}dt$

$\to 1-t=x^2$

$I=-\frac{1}{2}\int\frac{x^5\sqrt{t}\,dt}{x} \to I=-\frac{1}{2}\int x^4 t^{1/2}dt$

$I=-\frac{1}{2}\int(1-t)^2 t^{1/2}dt \to I=-\frac{1}{2}\int(1-2t+t^2)t^{1/2}dt \to I=-\frac{1}{2}\int t^{1/2}-2t^{3/2}+t^{5/2}dt$

$I=-\frac{1}{2}\left[\frac{t^{3/2}}{3/2}-2\left\{\frac{t^{5/2}}{5/2}\right\}+\frac{t^{7/2}}{7/2}\right]$

$I=-\frac{1}{2}\left[\frac{2}{3}\sqrt{t^3}-\frac{4}{5}\sqrt{t^5}+\frac{2}{7}\sqrt{t^7}\right] \to I=-\frac{1}{3}\sqrt{t^3}+\frac{2}{5}\sqrt{t^5}-\frac{1}{7}\sqrt{t^7}$

$I=-\frac{1}{3}\sqrt{(1-x^2)^3}+\frac{2}{5}\sqrt{(1-x^2)^5}-\frac{1}{7}\sqrt{(1-x^2)^7}+C$ ✓

54-) $I=\int x^5(x^3+1)^{2/3}dx$

$x^3+1=t$

$3x^2dx=dt$

$dx=\frac{1}{3x^2}dt$

$\to x^3=t-1$

$I=\frac{1}{3}\int\frac{x^5 t^{2/3}}{x^2}dt \to I=\frac{1}{3}\int x^3 t^{2/3}dt \to I=\frac{1}{3}\int(t-1)t^{2/3}dt$

$I=\frac{1}{3}\int t^{5/3}-t^{2/3}dt \to I=\frac{1}{3}\left[\frac{t^{8/3}}{8/3}-\frac{t^{5/3}}{5/3}\right]$

$I=\frac{1}{3}\left[\frac{3}{8}t^{8/3}-\frac{3}{5}t^{5/3}\right] \to I=\frac{1}{8}t^{8/3}-\frac{1}{5}t^{5/3}$

$I=\frac{1}{8}(x^3+1)^{8/3}-\frac{1}{5}(x^3+1)^{5/3}+C$ ✓

55-) $I = \int x^7 \sqrt{x^4+2}\, dx$

$x^4 + 2 = t$

$4x^3\, dx = dt$

$dx = \frac{1}{4x^3}\, dt$

$\to x^4 = t - 2$

$$I = \frac{1}{4}\int \frac{x^7\, t^{1/2}}{x^3}\, dt \to I = \frac{1}{4}\int x^4\, t^{1/2}\, dt \to I = \frac{1}{4}\int (t-2)\, t^{1/2}\, dt$$

$$I = \frac{1}{4}\int t^{3/2} - 2t^{1/2}\, dt \to I = \frac{1}{4}\left[\frac{t^{5/2}}{5/2} - 2\left[\frac{t^{3/2}}{3/2}\right]\right]$$

$$I = \frac{1}{4}\left[\frac{2}{5}\sqrt{t^5} - \frac{4}{3}\sqrt{t^3}\right] \to I = \frac{1}{10}\sqrt{t^5} - \frac{1}{3}\sqrt{t^3}$$

$$I = \frac{1}{10}\sqrt{(x^4+2)^5} - \frac{1}{3}\sqrt{(x^4+2)^3} + C$$

✓

56-) $I = \int x^3 \sqrt{2x^2+4}\, dx$

$2x^2 + 4 = t$

$4x\, dx = dt$

$dx = \frac{1}{4x}\, dt$

$\to 2x^2 = t - 4$

$x^2 = \frac{t-4}{2}$

$$I = \int \frac{x^3\, t^{1/2}}{4x}\, dt \to I = \frac{1}{4}\int x^2\, t^{1/2}\, dt \to I = \frac{1}{4}\,\frac{1}{2}\int (t-4)\, t^{1/2}\, dt$$

$$I = \frac{1}{8}\int t^{3/2} - 4t^{1/2}\, dt \to I = \frac{1}{8}\left[\frac{t^{5/2}}{5/2} - 4\left[\frac{t^{3/2}}{3/2}\right]\right]$$

$$I = \frac{1}{8}\left[\frac{2}{5}\sqrt{t^5} - \frac{8}{3}\sqrt{t^3}\right] \to I = \frac{1}{20}\sqrt{t^5} - \frac{1}{3}\sqrt{t^3}$$

$$I = \frac{1}{20}\sqrt{(2x^2+4)^5} - \frac{1}{3}\sqrt{(2x^2+4)^3} + C$$

✓

57-) $I=\int x^3\sqrt{3x^2-6}\,dx$

$3x^2-6=t$

$6x\,dx=dt$

$dx=\frac{1}{6x}dt$

$\to x^2=\frac{t+6}{3}$

$I=\frac{1}{6}\int x^2 t^{1/2}dt \to I=\frac{1}{6}\int\frac{(t+6)}{3}t^{1/2}dt$

$I=\frac{1}{18}\int t^{3/2}+6t^{1/2}dt \to J=\frac{1}{18}\left[\frac{t^{5/2}}{5/2}+6\left[\frac{t^{3/2}}{3/2}\right]\right]$

$J=\frac{1}{18}\left[\frac{2}{5}\sqrt{t^5}+\frac{12}{3}\sqrt{t^3}\right] \to J=\frac{1}{45}\sqrt{t^5}+\frac{2}{9}\sqrt{t^3}$

$I=\frac{1}{45}\sqrt{(3x^2-6)^5}+\frac{2}{9}\sqrt{(3x^2-6)^3}+C$ ✓

58-) $I=\int\frac{1}{1+\sqrt{x}}dx$

$1+\sqrt{x}=t$

$1+x^{1/2}=t$

$\frac{1}{2}x^{-1/2}dx=dt$

$dx=2\sqrt{x}\,dt$

$\to \sqrt{x}=t-1$

$I=2\int\frac{t-1}{t}dt \to I=2\int 1-\frac{1}{t}dt$

$I=2\left[t-\ln|t|\right] \to I=2(1+\sqrt{x})-2\ln|1+\sqrt{x}|$

$I=2+2\sqrt{x}-2\ln|1+\sqrt{x}|$

$I=2\sqrt{x}-2\ln|1+\sqrt{x}|+C$ ✓

59-) $I=\int\frac{\sqrt{x}}{1+\sqrt{x}}dx$

$1+\sqrt{x}=t$

$1+x^{1/2}=t$

$\frac{1}{2\sqrt{x}}dx=dt$

$dx=2\sqrt{x}\,dt$

$\to\sqrt{x}=t-1$

$I=2\int\frac{(t-1)^2}{t}dt \to I=2\int\frac{t^2-2t+1}{t}dt$

$I=2\int t-2+\frac{1}{t}dt \to I=t^2-4t+2\ln|t|$

$I=(1+\sqrt{x})^2-4(1+\sqrt{x})+2\ln|1+\sqrt{x}|$

$I=1+2\sqrt{x}+x-4-4\sqrt{x}+2\ln|1+\sqrt{x}|$

$I=x-2\sqrt{x}+2\ln|1+\sqrt{x}|+C$ ✓

60-) $I = \int \frac{\sqrt[3]{x}}{(2 + x\sqrt[3]{x})^2} dx$

$2 + x x^{1/3} = t$ $\quad I = \frac{3}{4}\int \frac{\sqrt[3]{x}}{\sqrt[3]{x}\; t^2} dt \to I = \frac{3}{4}\int t^{-2} dt$

$2 + x^{4/3} = t$

$\frac{4}{3} x^{\frac{1}{3}} dx = dt$ $\quad I = \frac{3}{4}\left[\frac{t^{-1}}{-1}\right] \to I = -\frac{3}{4t} \to I = -\frac{3}{4(2 + x\sqrt[3]{x})} + C$ ✓

$dx = \frac{3}{4\sqrt[3]{x}} dt$

61-) $I = \int \frac{1}{\sqrt{x}(x+1)} dx$

$x^{1/2} = t$ $\quad I = 2\int \frac{\sqrt{x}}{\sqrt{x}(t^2+1)} dt \to I = 2\int \frac{1}{t^2+1} dt$

$\frac{1}{2\sqrt{x}} dx = dt$

$I = 2\,\mathrm{arctg}(t) \to I = 2\,\mathrm{arctg}(\sqrt{x}) + C$

$dx = 2\sqrt{x}\, dt$

$\to \sqrt{x} = t$

$x = t^2$

62-) $I = \int \frac{1}{1 + x + \sqrt{x+1}} dx$

$1 + \sqrt{x+1} = t$ $\quad I = 2\int \frac{t-1}{x+t} dt \to I = 2\int \frac{t-1}{(t-1)^2 - 1 + t} dt$

$1 + (x+1)^{1/2} = t$

$\frac{1}{2}(x+1)^{-1/2} dx = dt$ $\quad I = 2\int \frac{t-1}{t^2 - 2t + 1 - 1 + t} dt \to I = 2\int \frac{t-1}{t^2 - t} dt$

$dx = 2\sqrt{x+1}\, dt$ $\quad I = 2\int \frac{t-1}{t(t-1)} dt \to I = 2\int \frac{1}{t} dt$

$\to \sqrt{x+1} = t - 1$

$x + 1 = (t-1)^2$ $\quad I = 2\ln|t| \to I = 2\ln|1 + \sqrt{x+1}| + C$ ✓

$x = (t-1)^2 - 1$

63-) $I = \int \frac{x^5}{\sqrt{x^3+4}-2}\,dx$

$x^3+4 = t$

$3x^2\,dx = dt$

$dx = \frac{1}{3x^2}\,dt$

$\to x^3 = t-4$

$I = \frac{1}{3}\int \frac{x^5}{x^2(\sqrt{t}-2)}\,dt \to I = \frac{1}{3}\int \frac{x^3}{\sqrt{t}-2}\,dt$

$I = \frac{1}{3}\int \frac{t-4}{\sqrt{t}-2}\,dt \to I = \frac{1}{3}\int \frac{(t-4)}{(\sqrt{t}-2)}\frac{(\sqrt{t}+2)}{(\sqrt{t}+2)}\,dt$

$I = \frac{1}{3}\int \frac{\cancel{(t-4)}(\sqrt{t}+2)}{\cancel{t-4}}\,dt \to I = \frac{1}{3}\int \sqrt{t}+2\,dt$

$I = \frac{1}{3}\left[\frac{t^{3/2}}{3/2} + 2t\right] \to I = \frac{2}{9}\sqrt{t^3} + \frac{2}{3}(x^3+4)$

$I = \frac{2}{9}\sqrt{(x^3+4)^3} + \frac{2}{3}(x^3+4) + C$ ✓

64-) $I = \int \frac{2x+\sqrt{x+1}}{x^2+2x+1}\,dx$

$x^2+2x+1 = 0$

$x^2+2x+1^2 = -1+1^2$

$(x+1)^2 = 0$

$x+1 = t$

$dx = dt$

$\to x = t-1$

$I = \int \frac{2x+\sqrt{x+1}}{(x+1)^2}\,dx$

$I = \int \frac{2(t-1)+\sqrt{t}}{t^2}\,dt \to I = \int \frac{2t-2+\sqrt{t}}{t^2}\,dt$

$I = \int \frac{2t}{t^2} - \frac{2}{t^2} + \frac{t^{1/2}}{t^2}\,dt \to I = 2\int \frac{1}{t}\,dt - 2\int t^{-2}\,dt + \int t^{-3/2}\,dt$

$I = 2\ln|t| - 2\left[\frac{t^{-1}}{-1}\right] + \left[\frac{t^{-1/2}}{-1/2}\right] \to I = 2\ln|t| + \frac{2}{t} - \frac{2}{\sqrt{t}}$

$I = \frac{2\cdot 2}{2}\ln|t| + \frac{2}{t} - \frac{2}{\sqrt{t}} \to I = 4\ln|t|^{1/2} + \frac{2}{t} - \frac{2}{\sqrt{t}}$

$I = 4\ln|\sqrt{x+1}| + \frac{2}{x+1} - \frac{2}{\sqrt{x+1}} + C$ ✓