Exercícios - Lista III

Calcule as seguintes integrais.

1-) $I = \int e^{7x+2}\, dx$

$d(7x+2) = 7\,dx$ $\quad dx = \frac{1}{7} d(7x+2)$ $\quad I = \frac{1}{7}\int e^{7x+2} d(7x+2) \rightarrow I = \frac{1}{7} e^{7x+2} + C$ ✓

2-) $I = \int e^{-x}\, dx$

$d(-x) = -1\,dx$ $\quad dx = -d(-x)$ $\quad I = -\int e^{-x} d(-x) \rightarrow I = -e^{-x} + C$ ✓

3-) $I = \int a\, e^{ax}\, dx$

$d(ax) = a\,dx$ $\quad dx = \frac{1}{a} d(ax)$ $\quad I = \frac{a}{a}\int e^{ax} d(ax) \rightarrow I = e^{ax} + C$ ✓

4-) $I = \int e^{tg(x)} \sec^2(x)\, dx$

$d(tg(x)) = \sec^2(x)\, dx$ $\quad I = \int e^{tg(x)} d(tg(x)) \rightarrow I = e^{tg(x)} + C$ ✓

5-) $I = \int \frac{1}{x-1}\, dx$

$d(x-1) = dx$ $\quad I = \int \frac{1}{x-1} d(x-1) \rightarrow I = \ln|x-1| + C$ ✓

6-) $I=\int \frac{3}{2x+5}\,dx$

$d(2x+5)=2\,dx$

$dx=\frac{1}{2}\,d(2x+5)$

$I=\frac{3}{2}\int \frac{1}{2x+5}\,d(2x+5)$

$I=\frac{3}{2}\ln|2x+5|+C$ ✓

7-) $I=\int \frac{x^2}{1+x^3}\,dx$

$d(1+x^3)=3x^2\,dx$

$dx=\frac{1}{3x^2}$

$I=\frac{1}{3}\int \frac{x^2}{x^2(1+x^3)}\,d(1+x^3)$

$I=\frac{1}{3}\int \frac{1}{1+x^3}\,d(1+x^3) \rightarrow I=\frac{1}{3}\ln|1+x^3|+C$ ✓

8-) $I=\int \frac{3x^2-5}{x^3-5x+7}\,dx$

$d(x^3-5x+7)=3x^2-5\,dx$

$I=\int \frac{1}{x^3-5x+7}\,d(x^3-5x+7)$ $\quad I=\ln|x^3-5x+7|+C$ ✓

9-) $I=\int \frac{3x^2-10x+6}{x^3-5x^2+6x-8}\,dx$

$d(x^3-5x^2+6x-8)=3x^2-10x+6\,dx$

$I=\int \frac{1}{x^3-5x^2+6x-8}\,d(x^3-5x^2+6x-8)$

$I=\ln|x^3-5x^2+6x-8|+C$ ✓

10-) $I = \int \frac{x}{(2x-4)(2x+4)}\,dx$

$I = \int \frac{x}{(2x)^2 - 4^2}\,dx \rightarrow I = \int \frac{x}{4x^2 - 4^2}\,dx \rightarrow I = \int \frac{x}{4(x^2-4)}\,dx$

$I = \frac{1}{4}\int \frac{x}{x^2-4}\,dx \qquad I = \frac{1}{8}\int \frac{x}{x(x^2-4)}\,d(x^2-4)$

$d(x^2-4) = 2x\,dx$

$dx = \frac{1}{2x}\,d(x^2-4)$

$I = \frac{1}{8}\ln|x^2-4| + C$ ✓

11-) $I = \int \frac{8\sqrt{x}}{2x\sqrt{x} + 3\sqrt{x}}\,dx$

$I = 8\int \frac{\sqrt{x}}{\sqrt{x}(2x+3)}\,dx \rightarrow I = 8\int \frac{1}{2x+3}\,dx$

$d(2x+3) = 2\,dx$

$dx = \frac{1}{2}\,d(2x+3)$

$I = 4\int \frac{1}{2x+3}\,d(2x+3)$

$I = 4\ln|2x+3| + C$ ✓

12-) $I = \int \cos(2x-5)\,dx$

$d(2x-5) = 2\,dx$

$dx = \frac{1}{2}\,d(2x-5)$

$I = \frac{1}{2}\int \cos(2x-5)\,d(2x-5)$

$I = \frac{1}{2}\operatorname{sen}(2x-5) + C$ ✓

13-) $I = \int \operatorname{sen}(2abx - 1)\,dx$

$d(2abx-1) = 2ab\,dx$

$dx = \frac{1}{2ab}\,d(2abx-1)$

$I = \frac{1}{2ab}\int \operatorname{sen}(2abx-1)\,d(2abx-1)$

$I = \frac{-1}{2ab}\cos(2abx-1) + C$ ✓

14) $I = \int \sqrt{1 - \operatorname{sen}^2(3x+1)}\, dx$

$\operatorname{sen}^2(x) + \cos^2(x) = 1$

$\cos^2(x) = 1 - \operatorname{sen}^2(x)$

$\cos(x) = \sqrt{1 - \operatorname{sen}^2(x)}$

$I = \int \cos(3x+1)\, dx$

$d(3x+1) = 3dx$

$dx = \frac{1}{3} d(3x+1)$

$I = \frac{1}{3} \int \cos(3x+1)\, d(3x+1)$

$I = \frac{1}{3} \operatorname{sen}(3x+1) + C$ ✓

15) $I = \int \frac{\operatorname{cotg}(x-1)}{\sqrt{1 + \operatorname{cotg}^2(x-1)}}\, dx$

$\frac{\operatorname{sen}^2(x)}{\operatorname{sen}^2(x)} + \frac{\cos^2(x)}{\operatorname{sen}^2(x)} = \frac{1}{\operatorname{sen}^2(x)}$

$1 + \operatorname{cotg}^2(x) = \operatorname{cosec}^2(x)$

$\operatorname{cosec}(x) = \sqrt{1 + \operatorname{cotg}^2(x)}$

$I = \int \frac{\cos(x-1)}{\operatorname{sen}(x-1)} \cdot \operatorname{sen}(x-1)\, dx$

$I = \int \cos(x-1)\, dx$

$d(x-1) = dx$

$I = \int \cos(x-1)\, d(x-1)$

$I = \operatorname{sen}(x-1) + C$ ✓

16) $I = \int \frac{1}{\cos^2(3x-9)}\, dx$

$I = \int \sec^2(3x-9)\, dx$

$d(3x-9) = 3dx$

$dx = \frac{1}{3} d(3x-9)$

$I = \frac{1}{3} \int \sec^2(3x-9)\, d(3x-9)$

$I = \frac{1}{3} \operatorname{tg}(3x-9) + C$ ✓

17-) $I = \int \frac{1}{1 - \operatorname{sen}^2(4x+1)}\, dx$

$\operatorname{sen}^2(x) + \cos^2(x) = 1$

$\cos^2(x) = 1 - \operatorname{sen}^2(x)$

$I = \int \frac{1}{\cos^2(4x+1)}\, dx \rightarrow I = \int \sec^2(4x+1)\, dx$

$d(4x+1) = 4\,dx$

$dx = \frac{1}{4} d(4x+1)$

$I = \frac{1}{4} \int \sec^2(4x+1)\, d(4x+1)$

$I = \frac{1}{4} \operatorname{tg}(4x+1) + C$ ✓

18-) $I = \int \frac{\operatorname{tg}^2(2x+1)}{\operatorname{sen}^2(2x+1)}\, dx$

$I = \int \frac{\operatorname{sen}^2(2x+1)}{\cos^2(2x+1)} \cdot \frac{1}{\operatorname{sen}^2(2x+1)}\, dx \rightarrow I = \int \sec^2(2x+1)\, dx$

$d(2x+1) = 2\,dx$

$dx = \frac{1}{2} d(2x+1)$

$I = \frac{1}{2} \int \sec^2(2x+1)\, d(2x+1)$

$I = \frac{1}{2} \operatorname{tg}(2x+1) + C$ ✓

19-) $I = \int \frac{\sec^2(x)}{2 + \operatorname{tg}(x)}\, dx$

$d(2 + \operatorname{tg}(x)) = \sec^2(x)\, dx$

$I = \int \frac{1}{2 + \operatorname{tg}(x)}\, d(2 + \operatorname{tg}(x))$

$I = \ln|2 + \operatorname{tg}(x)| + C$ ✓

20-) $I = \int \frac{\operatorname{tg}(x)}{1 + \ln(\cos(x))}\, dx$

$d(1 + \ln(\cos(x))) = \frac{-\operatorname{sen}(x)}{\cos(x)}\, dx$

$I = -\int \frac{1}{1 + \ln(\cos(x))}\, dx$

$d(1 + \ln(\cos(x))) = -\operatorname{tg}(x)\, dx$

$I = -\ln|1 + \ln(\cos(x))| + C$ ✓

21-) $I = \int \frac{tg^3(2x)}{\cos^2(2x)}\,dx$

$I = \int tg^3(2x)\sec^2(2x)\,dx$

$d(tg(2x)) = 2\sec^2(2x)\,dx$

$dx = \frac{1}{2\sec^2(2x)}\,d(tg(2x))$

$I = \frac{1}{2}\int \frac{tg^3(2x)\sec^2(2x)}{\sec^2(2x)}\,d(tg(2x))$

$I = \frac{1}{2}\int tg^3(2x)\,d(tg(2x))$

$I = \frac{1}{2}\cdot\frac{1}{4}tg^4(2x) \rightarrow I = \frac{1}{8}tg^4(2x) + C$ ✓

22-) $I = \int \frac{2\,tg(3x+a)}{\cos^2(3x+a)}\,dx$

$I = 2\int tg(3x+a)\sec(3x+a)\sec(3x+a)\,dx$

$d(\sec(3x+a)) = 3\sec(3x+a)tg(3x+a)\,dx$

$dx = \frac{1}{3\sec(3x+a)tg(3x+a)}\,d(\sec(3x+a))$

$I = \frac{2}{3}\int \frac{tg(3x+a)\sec(3x+a)\sec(3x+a)}{\sec(3x+a)tg(3x+a)}\,d(\sec(3x+a))$

$I = \frac{2}{3}\int \sec(3x+a)\,d(\sec(3x+a)) \rightarrow I = \frac{2}{3}\cdot\frac{1}{2}\sec^2(3x+a)$

$I = \frac{1}{3}\sec^2(3x+a) + C$ ✓

23-) $I = \int \frac{1}{\cos^2(ax) - \cos(2ax)}\, dx$

$\cos(2x) = \cos^2(x) - \operatorname{sen}^2(x)$

$I = \int \frac{1}{\cos^2(ax) - (\cos^2(ax) - \operatorname{sen}^2(ax))}\, dx$

$I = \int \frac{1}{\cos^2(ax) - \cos^2(ax) + \operatorname{sen}^2(ax)}\, dx \rightarrow I = \int \operatorname{cosec}^2(ax)\, dx$

$d(ax) = a\, dx$

$dx = \frac{1}{a} d(ax)$

$I = \frac{1}{a} \int \operatorname{cosec}^2(ax)\, d(ax)$

$I = -\frac{1}{a} \operatorname{cotg}(ax) + C$ ✓

24-) $I = \int \frac{1}{\cos(2x+4) + 1}\, dx$

$I = \int \frac{1}{\cos(2(x+2)) + 1}\, dx \rightarrow I = \int \frac{1}{\cos^2(x+2) - \operatorname{sen}^2(x+2) + 1}\, dx$

$\cos(2x) = \cos^2(x) - \operatorname{sen}^2(x)$

$\operatorname{sen}^2(x) + \cos^2(x) = 1$

$\cos^2(x) = 1 - \operatorname{sen}^2(x)$

$d(x+2) = dx$

$I = \int \frac{1}{\cos^2(x+2) + \cos^2(x+2)}\, dx$

$I = \frac{1}{2} \int \sec^2(x+2)\, dx$

$I = \frac{1}{2} \int \sec^2(x+2)\, d(x+2) \rightarrow I = \frac{1}{2} \operatorname{tg}(x+2) + C$ ✓

25-) $I = \int \frac{1}{1 - \cos(6x - 8ab)}\, dx$

$I = \int \frac{1}{1 - \cos(2(3x - 4ab))}\, dx \rightarrow I = \int \frac{1}{1 - (\cos^2(3x - 4ab) - \operatorname{sen}^2(3x - 4ab))}\, dx$

$I = \int \frac{1}{1 - \cos^2(3x - 4ab) + \operatorname{sen}^2(3x - 4ab)}\, dx \rightarrow I = \int \frac{1}{\operatorname{sen}^2(3x - 4ab) + \operatorname{sen}^2(3x - 4ab)}\, dx$

$I = \frac{1}{2} \int \operatorname{cossec}^2(3x - 4ab)\, dx$

$d(3x - 4ab) = 3\, dx$

$dx = \frac{1}{3} d(3x - 4ab)$

$I = \frac{1}{6} \int \operatorname{cossec}^2(3x - 4ab)\, d(3x - 4ab)$

$I = -\frac{1}{6} \operatorname{cotg}(3x - 4ab) + C$