Exercícios – Lista V

Calcule as seguintes integrais

1-) $I = \int x \, sen(x) \, dx$

	D	I
+	x	$sen(x)$
−	1	$-cos(x)$
+	0	$-sen(x)$

$I = -x\cos(x) + sen(x) + C$

2-) $I = \int x \cos(x) \, dx$

	D	I
+	x	$cos(x)$
−	1	$sen(x)$
+	0	$-cos(x)$

$I = x \, sen(x) + cos(x) + C$

3-) $I = \int x \, e^x \, dx$

	D	I
+	x	e^x
−	1	e^x
+	0	e^x

$I = x e^x - e^x$

$I = e^x (x-1) + C$

4-) $I = \int x \sec^2(x) \, dx$

	D	I
+	x	$sec^2(x)$
−	1	$tg(x)$
+	0	$-\ln\lvert\cos(x)\rvert$

$I = x \, tg(x) + \ln\lvert\cos(x)\rvert + C$

5) $I = \int x \operatorname{cosec}^2(x)\, dx$

	D	I
$+$	x	$\operatorname{cosec}^2(x)$
$-$	1	$-\operatorname{cotg}(x)$
$+$	0	$-\ln\lvert \operatorname{sen}(x)\rvert$

$$I = -x \operatorname{cotg}(x) + \ln\lvert \operatorname{sen}(x)\rvert + C$$

6) $I = \int x \operatorname{sen}(2x)\, dx$

	D	I
$+$	x	$\operatorname{sen}(2x)$
$-$	1	$-\frac{1}{2}\cos(2x)$
$+$	0	$-\frac{1}{4}\operatorname{sen}(2x)$

$$I = -\frac{1}{2} x \cos(2x) + \frac{1}{4}\operatorname{sen}(2x) + C$$

7) $I = \int 3x \cos(5x)\, dx$

	D	I
$+$	$3x$	$\cos(5x)$
$-$	3	$\frac{1}{5}\operatorname{sen}(5x)$
$+$	0	$-\frac{1}{25}\cos(5x)$

$$I = \frac{3}{5} x \operatorname{sen}(5x) + \frac{3}{25}\cos(5x) + C$$

8) $I = \int x e^{3x}\, dx$

	D	I
$+$	x	e^{3x}
$-$	1	$\frac{1}{3}e^{3x}$
$+$	0	$\frac{1}{9}e^{3x}$

$$I = \frac{1}{3} x e^{3x} - \frac{1}{9} e^{3x}$$

$$I = \frac{1}{3} e^{3x}\left(x - \frac{1}{3}\right) + C$$

9-) $I = \int 2x\, e^{-2x}\, dx$

	D	I
$+$	$2x$	e^{-2x}
$-$	2	$-\frac{1}{2}e^{-2x}$
$+$	0	$\frac{1}{4}e^{-2x}$

$$I = -x e^{-2x} - \frac{1}{2}e^{-2x}$$

$$I = -e^{-2x}\left(x + \frac{1}{2}\right) + C$$

10-) $J = \int 2x\, \sec^2(3x)\, dx$

	D	I
$+$	$2x$	$\sec^2(3x)$
$-$	2	$\frac{1}{3}\operatorname{tg}(3x)$
$+$	0	$-\frac{1}{9}\ln\lvert\cos(3x)\rvert$

$$J = \frac{2}{3}x\operatorname{tg}(3x) + \frac{2}{9}\ln\lvert\cos(3x)\rvert + C$$

11-) $J = \int x \ln(x)\, dx$

	D	I
$+$	$\ln(x)$	x
$-$	$\frac{1}{x}$	$\frac{1}{2}x^2$

$$I = \frac{1}{2}x^2 \ln(x) - \int \frac{1}{x}\cdot\frac{1}{2}x^2\, dx$$

$$I = \frac{1}{2}x^2 \ln(x) - \frac{1}{2}\int x\, dx$$

$$I = \frac{1}{2}x^2 \ln(x) - \frac{1}{4}x^2$$

$$J = \frac{1}{2}x^2\left(\ln(x) - \frac{1}{2}\right) + C$$

12-) $I = \int x^3 \ln(x)\, dx$

	D	I
+	$\ln(x)$	x^3
−	$\frac{1}{x}$	$\frac{1}{4}x^4$

$$I = \frac{1}{4}x^4 \ln(x) - \int \frac{1}{x} \cdot \frac{1}{4}x^4\, dx$$

$$I = \frac{1}{4}x^4 \ln(x) - \frac{1}{4}\int x^3\, dx$$

$$I = \frac{1}{4}x^4 \ln(x) - \frac{1}{4} \cdot \frac{1}{4}x^4$$

$$I = \frac{1}{4}x^4 \left(\ln(x) - \frac{1}{4}\right) + C$$

✓

13-) $I = \int \sqrt{x} \ln(x)\, dx$

	D	I
+	$\ln(x)$	$x^{1/2}$
−	$\frac{1}{x}$	$\frac{2}{3}x^{3/2}$

$$I = \frac{2}{3}\sqrt{x^3} \ln(x) - \int \frac{1}{x} \frac{2}{3} x^{3/2}\, dx$$

$$I = \frac{2}{3}\sqrt{x^3} \ln(x) - \frac{2}{3}\int x^{1/2}\, dx$$

$$I = \frac{2}{3}\sqrt{x^3} \ln(x) - \frac{2}{3}\left[\frac{x^{3/2}}{3/2}\right]$$

$$I = \frac{2}{3}\sqrt{x^3} \ln(x) - \frac{2}{3} \cdot \frac{2}{3}\sqrt{x^3} \rightarrow I = \frac{2}{3}\sqrt{x^3}\left(\ln(x) - \frac{2}{3}\right) + C$$

✓

14-) $I = \int \frac{\ln(x)}{x^4}\, dx$

	D	I
+	$\ln(x)$	x^{-4}
−	$\frac{1}{x}$	$\frac{x^{-3}}{-3}$

$$I = -\frac{\ln(x)}{3x^3} + \frac{1}{3}\int \frac{1}{x} x^{-3}\, dx$$

$$I = -\frac{1}{3}\frac{\ln(x)}{x^3} + \frac{1}{3}\int x^{-4}\, dx$$

$$I = -\frac{1}{3}\frac{\ln(x)}{x^3} + \frac{1}{3}\left[\frac{x^{-3}}{-3}\right] \rightarrow I = -\frac{1}{3x^3}\left[\ln(x) + \frac{1}{3}\right] + C$$

✓

15-) $I = \int \operatorname{arctg}(x)\, dx$

	D	I
+	$\operatorname{arctg}(x)$	1
−	$\frac{1}{1+x^2}$	x

$$I = x \operatorname{arctg}(x) - \int \underbrace{\frac{x}{1+x^2}\, dx}_{I_1}$$

$1 + x^2 = t$

$2x\, dx = dt$

$dx = \frac{1}{2x} dt$

$I_1 = \frac{1}{2} \int \frac{x}{x\ t} dt$

$I_1 = \frac{1}{2} \ln |1 + x^2|$

$$I = x \operatorname{arctg}(x) - \frac{1}{2} \ln |1 + x^2| + C$$

16-) $I = \int \operatorname{arccotg}(x)\, dx$

	D	I
+	$\operatorname{arcotg}(x)$	1
−	$-\frac{1}{1+x^2}$	x

$$I = x \operatorname{arcotg}(x) + \int \underbrace{\frac{x}{1+x^2}\, dx}_{I_1}$$

$I_1 = \frac{1}{2} \int \frac{x}{x\ t} dt \to I = \frac{1}{2} \ln |t|$

$1 + x^2 = t$

$2x\, dx = dt$

$dx = \frac{1}{2x} dt$

$$I = x \operatorname{arcotg}(x) + \frac{1}{2} \ln |1 + x^2| + C$$

17-) $I = \int \operatorname{arcsen}(x)\, dx$

	D	I
+	$\operatorname{arcsen}(x)$	1
−	$\frac{1}{\sqrt{1-x^2}}$	x

$$I = x \operatorname{arcsen}(x) - \int \underbrace{\frac{x}{1\sqrt{1-t}}\, dx}_{I_1}$$

$1 - x^2 = t$

$-2x\, dx = dt$

$dx = -\frac{1}{2x} dt$

$I_1 = -\frac{1}{2} \int t^{-1/2}\, dt$

$I_1 = -\frac{1}{2} \left[\frac{t^{1/2}}{1/2} \right] \to I_1 = -\sqrt{t}$

$$I = x \operatorname{arcsen}(x) + \sqrt{1 - x^2} + C$$

18-) $I=\int \arccos(3x)\,dx$

	D	I
+	$\arccos(3x)$	1
−	$-\dfrac{3}{\sqrt{1-9x^2}}$	x

$f(x) = \arccos(3x)$

$3x = \cos(f(x))$

$3dx = -\,\text{sen}(f(x))\,f'(x)$

$f'(x) = -\dfrac{3}{\text{sen}\,f(x)}$

$\text{sen}^2 f(x) + \cos^2 f(x) = 1$

$\text{sen}\,f(x) = \sqrt{1-\cos^2 f(x)}$

$\text{sen}\,f(x) = \sqrt{1-9x^2}$

$f'(x) = -\dfrac{3}{\sqrt{1-9x^2}}$

$$I = x\arccos(3x) + 3\underbrace{\int \frac{x}{\sqrt{1-9x^2}}\,dx}_{I_1}$$

$$I_1 = \int \frac{x}{\sqrt{1-9x^2}}\,dx$$

$1-9x^2 = t$

$-18x\,dx = dt$

$dx = -\dfrac{1}{18x}\,dt$

$I_1 = -\dfrac{1}{18}\int t^{-1/2}\,dt$

$I_1 = -\dfrac{1}{18}\left[\dfrac{t^{1/2}}{1/2}\right] \rightarrow I_1 = -\dfrac{1}{9}\sqrt{1-9x^2}$

$$I = x\arccos(3x) - \frac{1}{3}\sqrt{1-9x^2} + C$$ ✓

19-) $I=\int x^2\,\text{sen}(x)\,dx$

	D	I
+	x^2	$\text{sen}(x)$
−	$2x$	$-\cos(x)$
+	2	$-\text{sen}(x)$
−	0	$\cos(x)$

$$I = -x^2\cos(x) + 2x\,\text{sen}(x) + 2\cos(x) + C$$

$$I = \cos(x)(2-x^2) + 2x\,\text{sen}(x) + C$$ ✓

20-) $I = \int x^2 \cos(ax)\,dx$

	D	I
$+$	x^2	$\cos(ax)$
$-$	$2x$	$\frac{1}{a}\,\text{sen}(ax)$
$+$	2	$-\frac{1}{a^2}\cos(ax)$
$-$	0	$-\frac{1}{a^3}\,\text{sen}(ax)$

$$I = \frac{1}{a}x^2\,\text{sen}(ax) + \frac{2}{a^2}x\cos(ax) - \frac{2}{a^3}\,\text{sen}(ax) + C$$

21-) $I = \int x^2 e^x\,dx$

	D	I
$+$	x^2	e^x
$-$	$2x$	e^x
$+$	2	e^x
$-$	0	e^x

$$I = x^2e^x - 2xe^x + 2e^x$$
$$I = e^x(x^2 - 2x + 2) + C$$

22-) $I = \int x^2 e^{-x}\,dx$

	D	I
$+$	x^2	e^{-x}
$-$	$2x$	$-e^{-x}$
$+$	2	e^{-x}
$-$	0	$-e^{-x}$

$$I = -x^2e^{-x} - 2xe^{-x} - 2e^{-x}$$
$$I = -e^{-x}(x^2 + 2x + 2) + C$$

23-) $I = \int x^3 e^x \, dx$

	D	I
$+$	x^3	e^x
$-$	$3x^2$	e^x
$+$	$6x$	e^x
$-$	6	e^x
$+$	0	e^x

$I = x^3 e^x - 3x^2 e^x + 6x e^x - 6e^x$

$I = e^x (x^3 - 3x^2 + 6x - 6) + C$

24-) $I = \int x^3 \cos(x) \, dx$

	D	I
$+$	x^3	$\cos(x)$
$-$	$3x^2$	$\text{sen}(x)$
$+$	$6x$	$-\cos(x)$
$-$	6	$-\text{sen}(x)$
$+$	0	$\cos(x)$

$I = x^3 \,\text{sen}(x) + 3x^2 \cos(x) - 6x \,\text{sen}(x) - 6 \cos(x) + C$

25-) $I = \int \sec^3(x)\,dx$

	D	I
+	$\sec(x)$	$\sec^2(x)$
−	$\sec(x)tg(x)$	$tg(x)$

$$I = \sec(x)\,tg(x) - \underbrace{\int \sec(x)\,tg^2(x)\,dx}_{I_1}$$

$$I_1 = \int \sec(x)\left(\sec^2(x) - 1\right)dx$$

$$\frac{sen^2(x)}{\cos^2(x)} + \frac{\cos^2(x)}{\cos^2(x)} = \frac{1}{\cos^2(x)}$$

$$I_1 = \int \sec^3(x) - \sec(x)\,dx$$

$$tg^2(x) + 1 = \sec^2(x)$$

$$tg^2(x) = \sec^2(x) - 1$$

$$I_1 = \int \sec^3(x)\,dx - \int \sec(x)\,dx$$

$$I = \sec(x)\,tg(x) - \left[\underbrace{\int \sec^3(x)\,dx}_{I} - \int \sec(x)\,dx\right]$$

$$2I = \sec(x)\,tg(x) + \underbrace{\int \sec(x)\,dx}_{I_2}$$

$$I_2 = \int \sec(x)\,dx$$

$$2I = \sec(x)\,tg(x) + \ln|\sec(x) + tg(x)|$$

$$I = \frac{1}{2}\left[\sec(x)\,tg(x) + \ln|\sec(x) + tg(x)|\right] + C$$

$$I_2 = \int \frac{\sec(x)\left(\sec(x) + tg(x)\right)}{\sec(x) + tg(x)}\,dx$$

$$I_2 = \int \frac{\sec^2(x) + \sec(x)\,tg(x)}{\sec(x) + tg(x)}\,dx$$

$$u = \sec(x) + tg(x)$$

$$du = \sec(x)\,tg(x) + \sec^2(x)\,dx$$

$$dx = \frac{1}{\sec(x)\,tg(x) + \sec^2(x)}\,du$$

$$I_2 = \int \frac{\sec^2(x) + \sec(x)\,tg(x)}{\left(\sec^2(x) + \sec(x)\,tg(x)\right)u}\,du$$

$$I_2 = \int \frac{1}{u}\,du \rightarrow I_2 = \ln|u|$$

26-> $I = \int \operatorname{cosec}^3(x)\,dx$

	D	I
+	$\operatorname{cosec}(x)$	$\operatorname{cosec}^2(x)$
−	$-\operatorname{cosec}(x)\operatorname{cotg}(x)$	$-\operatorname{cotg}(x)$

$$I = -\operatorname{cosec}(x)\operatorname{cotg}(x) - \underbrace{\int \operatorname{cosec}(x)\operatorname{cotg}^2(x)\,dx}_{I_1}$$

$$I_1 = \int \operatorname{cosec}(x)\left(\operatorname{cosec}^2(x) - 1\right)dx$$

$$\frac{\operatorname{sen}^2(x)}{\operatorname{sen}^2(x)} + \frac{\cos^2(x)}{\operatorname{sen}^2(x)} = \frac{1}{\operatorname{sen}^2(x)}$$

$$1 + \operatorname{cotg}^2(x) = \operatorname{cosec}^2(x)$$

$$\operatorname{cotg}^2(x) = \operatorname{cosec}^2(x) - 1$$

$$I_1 = \int \operatorname{cosec}^3(x)\,dx - \int \operatorname{cosec}(x)\,dx$$

$$I_1 = I - \underbrace{\int \operatorname{cosec}(x)\,dx}_{I_2}$$

$$I_2 = \int \operatorname{cosec}(x)\,dx \to I_2 = \int \frac{\operatorname{cosec}(x)\left(\operatorname{cosec}(x) + \operatorname{cotg}(x)\right)}{\operatorname{cosec}(x) + \operatorname{cotg}(x)}\,dx$$

$$I_2 = \int \frac{\operatorname{cosec}^2(x) + \operatorname{cosec}(x)\operatorname{cotg}(x)}{\operatorname{cosec}(x) + \operatorname{cotg}(x)}\,dx$$

$$u = \operatorname{cosec}(x) + \operatorname{cotg}(x)$$

$$du = -\operatorname{cosec}(x)\operatorname{cotg}(x) - \operatorname{cosec}^2(x)\,dx$$

$$dx = -\frac{1}{\operatorname{cosec}(x)\operatorname{cotg}(x) + \operatorname{cosec}^2(x)}\,du$$

$$I_2 = -\int \frac{\operatorname{cosec}^2(x) + \operatorname{cosec}(x)\operatorname{cotg}(x)}{u\left(\operatorname{cosec}(x)\operatorname{cotg}(x) + \operatorname{cosec}^2(x)\right)}\,du$$

$$I_2 = -\int \frac{1}{u}\,du \to I_2 = -\ln|u|$$

$$I = -\operatorname{cosec}(x)\operatorname{cotg}(x) - \left[I - \left[-\ln|\operatorname{cosec}(x) + \operatorname{cotg}(x)|\right]\right]$$

$$I = -\operatorname{cosec}(x)\operatorname{cotg}(x) - I - \ln|\operatorname{cosec}(x) + \operatorname{cotg}(x)|$$

$$2I = -\operatorname{cosec}(x)\operatorname{cotg}(x) - \ln|\operatorname{cosec}(x) + \operatorname{cotg}(x)|$$

$$I = -\frac{1}{2}\left[\operatorname{cosec}(x)\operatorname{cotg}(x) + \ln|\operatorname{cosec}(x) + \operatorname{cotg}(x)|\right] + C$$

✓

27-) $I = \int e^{x} \cos(x)\, dx$

	D	I
+	$\cos(x)$	e^{x}
−	$-\text{sen}(x)$	e^{x}
+	$-\cos(x)$	e^{x}

$I = e^{x}\cos(x) + e^{x}\text{sen}(x) - \int e^{x}\cos(x)\,dx$

$2I = e^{x}\cos(x) + e^{x}\text{sen}(x)$

$I = \frac{e^{x}}{2}(\cos(x) + \text{sen}(x)) + C$ ✓

28-) $I = \int e^{x} \text{sen}(2x)\, dx$

	D	I
+	$\text{sen}(2x)$	e^{x}
−	$2\cos(2x)$	e^{x}
+	$-4\,\text{sen}(2x)$	e^{x}

$I = e^{x}\text{sen}(2x) - 2e^{x}\cos(2x) - 4\int e^{x}\text{sen}(2x)\,dx$

$5I = e^{x}(\text{sen}(2x) - 2\cos(2x))$

$I = \frac{1}{5}e^{x}(\text{sen}(2x) - 2\cos(2x)) + C$ ✓

$I = -\frac{1}{5}e^{x}(2\cos(2x) - \text{sen}(2x)) + C$ ✓

29-) $I = \int e^{-4x} \cos(3x)\, dx$

	D	I
+	e^{-4x}	$\cos(3x)$
−	$-4e^{-4x}$	$\frac{1}{3}\text{sen}(3x)$
+	$16e^{-4x}$	$-\frac{1}{9}\cos(3x)$

$I = \frac{1}{3}e^{-4x}\text{sen}(3x) - \frac{4}{9}e^{-4x}\cos(3x) - \frac{16}{9}\int e^{-4x}\cos(3x)\,dx$

$I + \frac{16}{9}I = \frac{1}{3}e^{-4x}\text{sen}(3x) - \frac{4}{9}e^{-4x}\cos(3x)$

$\frac{25I}{9} = e^{-4x}\left(\frac{1}{3}\text{sen}(3x) - \frac{4}{9}\cos(3x)\right)$

$I = \frac{9}{25}e^{-4x}\left(\frac{1}{3}\text{sen}(3x) - \frac{4}{9}\cos(3x)\right) \rightarrow I = \frac{1}{25}e^{-4x}\left(3\,\text{sen}(3x) - 4\cos(3x)\right) + C$ ✓

30-) $I = \int \operatorname{sen}(x)\operatorname{sen}(3x)\,dx$

	D	I
+	$\operatorname{sen}(x)$	$\operatorname{sen}(3x)$
−	$\cos(x)$	$-\frac{1}{3}\cos(3x)$
+	$-\operatorname{sen}(x)$	$-\frac{1}{9}\operatorname{sen}(3x)$

$I = -\frac{1}{3}\operatorname{sen}(x)\cos(3x) + \frac{1}{9}\cos(x)\operatorname{sen}(3x) + \cdots$

$\cdots + \frac{1}{9}\int \operatorname{sen}(x)\operatorname{sen}(3x)\,dx$

$I - \frac{1}{9}I = -\frac{1}{3}\operatorname{sen}(x)\cos(3x) + \frac{1}{9}\cos(x)\operatorname{sen}(3x)$

$\frac{8}{9}I = -\frac{1}{3}\operatorname{sen}(x)\cos(3x) + \frac{1}{9}\cos(x)\operatorname{sen}(3x)$

$I = \frac{9}{8}\left[-\frac{1}{3}\operatorname{sen}(x)\cos(3x) + \frac{1}{9}\cos(x)\operatorname{sen}(3x)\right] + C$ ✓

$I = -\frac{3}{8}\operatorname{sen}(x)\cos(3x) + \frac{1}{8}\cos(x)\operatorname{sen}(3x) + C$ ✓

31-) $I = \int \cos(4x)\cos(7x)\,dx$

	D	I
+	$\cos(4x)$	$\cos(7x)$
−	$-4\operatorname{sen}(4x)$	$\frac{1}{7}\operatorname{sen}(7x)$
+	$-16\cos(4x)$	$-\frac{1}{49}\cos(7x)$

$I = \frac{1}{7}\cos(4x)\operatorname{sen}(7x) - \frac{4}{49}\operatorname{sen}(4x)\cos(7x) + \cdots$

$\cdots + \frac{16}{49}\int \cos(4x)\cos(7x)\,dx$

$I = \frac{1}{7}\cos(4x)\operatorname{sen}(7x) - \frac{4}{49}\operatorname{sen}(4x)\cos(7x) + \frac{16}{49}I$

$I - \frac{16}{49}I = \frac{1}{7}\cos(4x)\operatorname{sen}(7x) - \frac{4}{49}\operatorname{sen}(4x)\cos(7x)$

$I = \frac{49}{33}\left(\frac{1}{7}\cos(4x)\operatorname{sen}(7x) - \frac{4}{49}\operatorname{sen}(4x)\cos(7x)\right)$

$I = \frac{7}{33}\cos(4x)\operatorname{sen}(7x) - \frac{4}{33}\operatorname{sen}(4x)\cos(7x) + C$ ✓

32-) $I = \int sen(2x)\cos(4x)\,dx$

	D	I
+	$sen(2x)$	$\cos(4x)$
−	$2\cos(2x)$	$\frac{1}{4}sen(4x)$
+	$-4sen(2x)$	$-\frac{1}{16}\cos(4x)$

$$I = \frac{1}{4}sen(2x)sen(4x) + \frac{2}{16}\cos(2x)\cos(4x) + \cdots$$

$$\cdots + \frac{4}{16}\int sen(2x)\cos(4x)\,dx$$

$$I = \frac{1}{4}sen(2x)sen(4x) + \frac{2}{16}\cos(2x)\cos(4x) + \frac{4}{16}I$$

$$I - \frac{4}{16}I = \frac{1}{4}sen(2x)sen(4x) + \frac{2}{16}\cos(2x)\cos(4x)$$

$$\frac{12}{16}I = \frac{1}{4}sen(2x)sen(4x) + \frac{2}{16}\cos(2x)\cos(4x)$$

$$I = \frac{16}{12}\left(\frac{1}{4}sen(2x)sen(4x) + \frac{2}{16}\cos(2x)\cos(4x)\right)$$

$$I = \frac{1}{3}sen(2x)sen(4x) + \frac{1}{6}\cos(2x)\cos(4x) + C$$

$$I = \frac{1}{3}\left[sen(2x)sen(4x) + \frac{1}{2}\cos(2x)\cos(4x)\right] + C$$

33-) $I = \int x\cos^2(x)\,dx$

	D	I
+	x	$\cos^2(x)$
−	1	$\frac{1}{2}x + \frac{1}{4}sen(2x)$
+	0	$\frac{1}{4}x^2 - \frac{1}{8}\cos(2x)$

$$I = \frac{1}{2}x^2 + \frac{1}{4}x\,sen(2x) - \cdots$$

$$\cdots - 1\left(\frac{1}{4}x^2 - \frac{1}{8}\cos(2x)\right)$$

$$I = \cancel{\frac{1}{2}x^2} + \frac{1}{4}x\,sen(2x) - \cancel{\frac{1}{4}x^2} + \frac{1}{8}\cos(2x)$$

$$I = \frac{1}{4}x^2 + \frac{1}{4}x\,sen(2x) + \frac{1}{8}\cos(2x) + C$$

34.) $I = \int x \operatorname{sen}^2(x)\, dx$

	D	I
+	x	$\operatorname{sen}^2(x)$
−	1	$\frac{1}{2}x - \frac{1}{4}\operatorname{sen}(2x)$
+	0	$\frac{1}{4}x^2 + \frac{1}{8}\cos(2x)$

$$I = \frac{1}{2}x^2 - \frac{1}{4}x\operatorname{sen}(2x) - 1\left(\frac{1}{4}x^2 + \frac{1}{8}\cos(2x)\right)$$

$$I = \frac{1}{\cancel{2}}\cancel{x^2} - \frac{1}{4}x\operatorname{sen}(2x) - \frac{1}{\cancel{4}}\cancel{x^2} - \frac{1}{8}\cos(2x)$$

$$\underline{I = \frac{1}{4}x^2 - \frac{1}{4}x\operatorname{sen}(2x) - \frac{1}{8}\cos(2x) + C}$$ ✓

35.) $I = \int x \ln(x+1)\, dx$

	D	I
+	$\ln(x+1)$	x
−	$\frac{1}{x+1}$	$\frac{1}{2}x^2$

$$I = \frac{1}{2}x^2\ln(x+1) - \frac{1}{2}\overbrace{\int \frac{x^2}{x+1}\,dx}^{I_1}$$

$$\begin{array}{r|l} x^2 + 0x + 0 & x+1 \\ \hline x^2 + x & x - 1 \\ \hline -x + 0 & \\ -x - 1 & \\ \hline 1 & \end{array}$$

$$I_1 = \int \frac{(x-1)\cancel{(x+1)}}{\cancel{x+1}} + \frac{1}{x+1}\,dx \qquad I_1 = \frac{1}{2}x^2 - x + \ln|x+1|$$

$$I = \frac{1}{2}x^2\ln(x+1) - \frac{1}{2}\left[\frac{1}{2}x^2 - x + \ln(x+1)\right]$$

$$\underline{I = \frac{1}{2}x^2\ln(x+1) - \frac{1}{4}x^2 + \frac{1}{2}x - \frac{1}{2}\ln(x+1) + C}$$ ✓

36-) $I = \int \ln(x^2+1)\,dx$

	D	I
+	$\ln\lvert x^2+1\rvert$	1
−	$\frac{2x}{x^2+1}$	x

$$I = x\ln(x^2+1) - 2\overbrace{\int \frac{x^2}{x^2+1}\,dx}^{I_1}$$

$$\begin{array}{r|l} x^2+0 & x^2+1 \\ \hline x^2+1 & 1 \\ \hline -1 & \end{array}$$

$$I_1 = \int \frac{1(x^2+1)}{x^2+1} - \frac{1}{x^2+1}\,dx \rightarrow I_1 = x - \operatorname{arctg}(x)$$

$$\underline{I = x\ln\lvert x^2+1\rvert - 2(x - \operatorname{arctg}(x)) + C}$$

37-) $I = \int x^2 \operatorname{arcsen}(x)\,dx$

	D	I
+	$\operatorname{arcsen}(x)$	x^2
−	$\frac{1}{\sqrt{1-x^2}}$	$\frac{1}{3}x^3$

$$I = \frac{1}{3}x^3\operatorname{arcsen}(x) - \frac{1}{3}\overbrace{\int \frac{x^3}{\sqrt{1-x^2}}\,dx}^{I_1}$$

$$I_1 = \int \frac{x^3}{\sqrt{1-x^2}}\,dx$$

$1-x^2 = t$

$x^2 = t-1$

$2x\,dx = dt$

$dx = \frac{1}{2x}dt$

$$I_1 = \frac{1}{2}\int \frac{t-1}{\sqrt{t}}\,dt \rightarrow I_1 = \frac{1}{2}\int (t-1)t^{-1/2}\,dt \rightarrow I_1 = \frac{1}{2}\int t^{1/2} - t^{-1/2}\,dt$$

$$I_1 = \frac{1}{2}\left[\frac{t^{3/2}}{3/2} - \frac{t^{1/2}}{1/2}\right] \rightarrow I_1 = \frac{1}{3}\sqrt{t^3} - \sqrt{t} \rightarrow I_1 = \frac{1}{3}\sqrt{(1-x^2)^3} - \sqrt{1-x^2}$$

$$I = \frac{1}{3}x^3\operatorname{arcsen}(x) - \frac{1}{3}\left[\frac{1}{3}\sqrt{(1-x^2)^3} - \sqrt{1-x^2}\right]$$

$$\underline{I = \frac{1}{3}x^3\operatorname{arcsen}(x) - \frac{1}{9}\sqrt{(1-x^2)^3} + \frac{1}{3}\sqrt{1-x^2} + C}$$

38-) $I = \int x^2 \operatorname{arctg}(x)\, dx$

	D	I
$+$	$\operatorname{arctg}(x)$	x^2
$-$	$\frac{1}{1+x^2}$	$\frac{1}{3}x^3$

$$I = \frac{1}{3}x^3 \operatorname{arctg}(x) - \frac{1}{3}\int \frac{x^3}{1+x^2}\, dx$$

$1 + x^2 = t$

$x^2 = t - 1$

$2x\, dx = dt$

$dx = \frac{1}{2x} dt$

$$I = \frac{1}{3}x^3 \operatorname{arctg}(x) - \frac{1}{6}\int \frac{t-1}{t}\, dt$$

$$I = \frac{1}{3}x^3 \operatorname{arctg}(x) - \frac{1}{6}\int 1 - \frac{1}{t}\, dt$$

$$I = \frac{1}{3}x^3 \operatorname{arctg}(x) - \frac{1}{6}\left[(1+x^2) - \ln|1+x^2|\right]$$

$$I = \frac{1}{3}x^3 \operatorname{arctg}(x) - \frac{1}{6}(1+x^2) + \frac{1}{6}\ln|1+x^2|$$

$$I = \frac{1}{3}x^3 \operatorname{arctg}(x) - \frac{1}{6} - \frac{1}{6}x^2 + \frac{1}{6}\ln|1+x^2|$$

$$\underline{I = \frac{1}{3}x^3 \operatorname{arctg}(x) - \frac{1}{6}x^2 + \frac{1}{6}\ln|1+x^2| + C}$$