

EXERCÍCIOS - LISTA III

Calcule as seguintes integrais.

$$1) I = \int e^{7x+2} dx$$

$$\begin{aligned} d(7x+2) &= 7dx \\ dx &= \frac{1}{7} d(7x+2) \end{aligned} \quad I = \frac{1}{7} \int e^{7x+2} d(7x+2) \rightarrow I = \frac{1}{7} e^{7x+2} + C \quad \checkmark$$

$$2) I = \int e^{-x} dx$$

$$\begin{aligned} d(-x) &= -dx \\ dx &= -d(-x) \end{aligned} \quad I = - \int e^{-x} d(-x) \rightarrow I = -e^{-x} + C \quad \checkmark$$

$$3) I = \int a \cdot e^{ax} dx$$

$$\begin{aligned} d(ax) &= a dx \\ dx &= \frac{1}{a} d(ax) \end{aligned} \quad I = \frac{a}{a} \int e^{ax} d(ax) \rightarrow I = \int e^{ax} d(ax)$$
$$I = e^{ax} + C \quad \checkmark$$

$$4) I = \int e^{\operatorname{tg} x} \sec^2 x dx$$

$$\begin{aligned} d(\operatorname{tg}(x)) &= \sec^2(x) dx \\ dx &= \frac{1}{\sec^2(x)} d(\operatorname{tg}(x)) \end{aligned} \quad I = \int \frac{e^{\operatorname{tg}(x)} \sec^2(x)}{\sec^2(x)} d(\operatorname{tg}(x))$$
$$I = \int e^{\operatorname{tg}(x)} d(\operatorname{tg}(x)) \rightarrow I = e^{\operatorname{tg}(x)} + C \quad \checkmark$$

$$5) I = \int \frac{1}{x-1} dx$$

$$d(x-1) = dx \quad I = \int \frac{1}{x-1} d(x-1) \rightarrow I = \ln |x-1| + C \quad \checkmark$$

$$6) I = \int \frac{3}{2x+5} dx$$

$$d(2x+5) = 2dx \quad I = \frac{3}{2} \int \frac{1}{2x+5} d(2x+5) \rightarrow I = \frac{3}{2} \ln|2x+5| + C \quad \checkmark$$

$$dx = \frac{1}{2} d(2x+5)$$

$$7) I = \int \frac{x^2}{1+x^3} dx$$

$$d(1+x^3) = 3x^2 dx \quad I = \frac{1}{3} \int \frac{x^2}{x^2(1+x^3)} d(1+x^3) \rightarrow I = \frac{1}{3} \int \frac{1}{1+x^3} d(1+x^3)$$

$$dx = \frac{1}{3x^2} d(1+x^3)$$

$$I = \frac{1}{3} \ln|1+x^3| + C \quad \checkmark$$

$$8) I = \int \frac{3x^2 - 5}{x^3 - 5x + 7} dx$$

$$d(x^3 - 5x + 7) = 3x^2 - 5 dx \quad I = \int \frac{3x^2 - 5}{(3x^2 - 5)(x^3 - 5x + 7)} d(x^3 - 5x + 7)$$

$$dx = \frac{1}{3x^2 - 5} d(x^3 - 5x + 7)$$

$$I = \int \frac{1}{x^3 - 5x + 7} d(x^3 - 5x + 7)$$

$$I = \ln|x^3 - 5x + 7| + C \quad \checkmark$$

$$9) I = \int \frac{3x^2 - 10x + 6}{x^3 - 5x^2 + 6x - 8} dx$$

$$d(x^3 - 5x^2 + 6x - 8) = 3x^2 - 10x + 6 dx$$

$$dx = \frac{1}{3x^2 - 10x + 6} d(x^3 - 5x^2 + 6x - 8)$$

$$I = \int \frac{3x^2 - 10x + 6}{(3x^2 - 10x + 6)(x^3 - 5x^2 + 6x - 8)} d(x^3 - 5x^2 + 6x - 8)$$

$$I = \int \frac{1}{x^3 - 5x^2 + 6x - 8} d(x^3 - 5x^2 + 6x - 8) \rightarrow I = \ln|x^3 - 5x^2 + 6x - 8| + C \quad \checkmark$$

$$10) I = \int \frac{x}{(2x-4)(2x+4)} dx$$

$$I = \int \frac{x}{4x^2-4^2} dx \rightarrow I = \int \frac{x}{4(x^2-4)} dx \rightarrow I = \frac{1}{4} \int \frac{x}{x^2-4} dx$$

$$d(x^2-4) = 2x dx \quad I = \frac{1}{8} \int \frac{x}{x(x^2-4)} d(x^2-4) \rightarrow I = \frac{1}{8} \ln|x^2-4| + C \quad \checkmark$$

$$dx = \frac{1}{2x} d(x^2-4)$$

$$11) I = \int \frac{8\sqrt{x}}{2x\sqrt{x} + 3\sqrt{x}} dx$$

$$I = 8 \int \frac{\sqrt{x}}{\sqrt{x}(2x+3)} dx \rightarrow I = 8 \int \frac{1}{2x+3} dx$$

$$d(2x+3) = 2dx \quad I = \frac{8}{2} \int \frac{1}{2x+3} d(2x+3) \rightarrow I = 4 \ln|2x+3| + C \quad \checkmark$$

$$dx = \frac{1}{2} d(2x+3)$$

$$12) I = \int \cos(2x-5) dx$$

$$d(2x-5) = 2dx \quad I = \frac{1}{2} \int \cos(2x-5) d(2x-5)$$

$$dx = \frac{1}{2} d(2x-5)$$

$$I = \frac{1}{2} \sin(2x-5) + C \quad \checkmark$$

$$13) I = \int \sin(2abx-1) dx$$

$$d(2abx-1) = 2ab dx \quad I = \frac{1}{2ab} \int \sin(2abx-1) d(2abx-1)$$

$$dx = \frac{1}{2ab} d(2abx-1)$$

$$I = -\frac{1}{2ab} \cos(2abx-1) + C \quad \checkmark$$

$$14) I = \int \sqrt{1 - \sin^2(3x+1)} \, dx$$

$$d(3x+1) = 3 \, dx$$

$$dx = \frac{1}{3} d(3x+1)$$

$$\sin^2(3x+1) + \cos^2(3x+1) = 1$$

$$\cos^2(3x+1) = 1 - \sin^2(3x+1)$$

$$I = \frac{1}{3} \int \sqrt{1 - \sin^2(3x+1)} \, d(3x+1)$$

$$I = \frac{1}{3} \int \sqrt{\cos^2(3x+1)} \, d(3x+1)$$

$$I = \frac{1}{3} \int \cos(3x+1) \, d(3x+1)$$

$$I = \frac{1}{3} \sin(3x+1) + C$$

✓

$$15) I = \int \frac{\cotg(x-1)}{\sqrt{1 + \cotg^2(x-1)}} \, dx$$

$$d(x-1) = dx \quad I = \int \frac{\cotg(x-1)}{\sqrt{1 + \cotg^2(x-1)}} \, d(x-1)$$

$$\frac{\sin^2(x-1)}{\sin^2(x-1)} + \frac{\cos^2(x-1)}{\sin^2(x-1)} = \frac{1}{\sin^2(x-1)}$$

$$1 + \cotg^2(x-1) = \operatorname{cosec}^2(x-1)$$

$$I = \int \frac{\cotg(x-1)}{\operatorname{cosec}(x-1)} \, d(x-1)$$

$$I = \int \frac{\cos(x-1)}{\sin(x-1)} \cancel{\sin(x-1)} \, d(x-1) \rightarrow I = \int \cos(x-1) \, d(x-1)$$

$$I = \sin(x-1) + C$$

✓

$$16) I = \int \frac{1}{\cos^2(3x-9)} \, dx$$

$$d(3x-9) = 3 \, dx$$

$$dx = \frac{1}{3} d(3x-9)$$

$$I = \frac{1}{3} \int \frac{1}{\cos^2(3x-9)} \, d(3x-9) \rightarrow I = \frac{1}{3} \int \sec^2(3x-9) \, d(3x-9)$$

$$I = \frac{1}{3} \tan(3x-9) + C$$

✓

$$17) I = \int \frac{1}{1 - \sin^2(4x + 1)} dx$$

$$d(4x+1) = 4dx$$

$$dx = \frac{1}{4} d(4x+1)$$

$$I = \frac{1}{4} \int \frac{1}{1 - \sin^2(4x+1)} d(4x+1)$$

$$\sin^2(4x+1) + \cos^2(4x+1) = 1$$

$$\cos^2(4x+1) = 1 - \sin^2(4x+1)$$

$$I = \frac{1}{4} \int \frac{1}{\cos^2(4x+1)} d(4x+1) \rightarrow I = \frac{1}{4} \int \sec^2(4x+1) d(4x+1)$$

$$I = \frac{1}{4} \operatorname{tg}(4x+1) + C \quad \checkmark$$

$$18) I = \int \frac{\operatorname{tg}^2(2x+1)}{\sin^2(2x+1)} dx$$

$$I = \int \frac{\sin^2(2x+1)}{\cos^2(2x+1)} \cdot \frac{1}{\sin^2(2x+1)} dx \rightarrow I = \int \frac{1}{\cos^2(2x+1)} dx$$

$$d(2x+1) = 2dx$$

$$dx = \frac{1}{2} d(2x+1)$$

$$I = \frac{1}{2} \int \sec^2(2x+1) d(2x+1)$$

$$I = \frac{1}{2} \operatorname{tg}(2x+1) + C \quad \checkmark$$

$$19) I = \int \frac{\sec^2 x}{2 + \operatorname{tg} x} dx$$

$$d(2 + \operatorname{tg}(x)) = \sec^2(x) dx$$

$$dx = \frac{1}{\sec^2(x)} d(2 + \operatorname{tg}(x))$$

$$I = \int \frac{\sec^2(x)}{\sec^2(x)(2 + \operatorname{tg}(x))} d(2 + \operatorname{tg}(x))$$

$$I = \int \frac{1}{2 + \operatorname{tg}(x)} d(2 + \operatorname{tg}(x))$$

$$I = \ln |2 + \operatorname{tg}(x)| + C \quad \checkmark$$

$$20) I = \int \frac{\operatorname{tg} x}{1 + \ln(\cos x)} dx$$

$$d(1 + \ln(\cos(x))) = - \frac{\sin(x)}{\cos(x)} dx$$

$$dx = - \frac{\cos(x)}{\sin(x)} d(1 + \ln(\cos(x)))$$

$$I = - \int \frac{\cancel{\cos(x)} \cdot \cancel{\sin(x)} \cdot \frac{1}{1 + \ln(\cos(x))}}{\cancel{\sin(x)} \cdot \cancel{\cos(x)}} d(1 + \ln(\cos(x)))$$

$$I = - \int \frac{1}{1 + \ln(\cos(x))} d(1 + \ln(\cos(x))) \Rightarrow I = - \ln |1 + \ln(\cos(x))| + C \quad \checkmark$$

$$21) I = \int \frac{\operatorname{tg}^3(2x)}{\cos^2(2x)} dx$$

$$I = \int \operatorname{tg}^3(2x) \sec^2(2x) dx$$

$$d(\operatorname{tg}(2x)) = 2 \sec^2(2x) dx$$

$$dx = \frac{1}{2 \sec^2(2x)} d(\operatorname{tg}(2x))$$

$$I = \frac{1}{2} \int \frac{\operatorname{tg}^3(2x) \sec^2(2x)}{\sec^2(2x)} d(\operatorname{tg}(2x))$$

$$I = \frac{1}{2} \int \operatorname{tg}^3(2x) d(\operatorname{tg}(2x))$$

$$I = \frac{1}{2} \left[\frac{\operatorname{tg}^4(2x)}{4} \right] \Rightarrow I = \frac{1}{8} \operatorname{tg}^4(2x) + C \quad \checkmark$$

$$22) I = \int \frac{2 \operatorname{tg}(3x + a)}{\cos^2(3x + a)} dx$$

$$I = 2 \int \operatorname{tg}(3x + a) \sec(3x + a) \sec(3x + a) dx$$

$$d(\sec(3x + a)) = 3 \sec(3x + a) \operatorname{tg}(3x + a) dx$$

$$dx = \frac{1}{3 \sec(3x + a) \operatorname{tg}(3x + a)} d(\sec(3x + a))$$

$$I = \frac{2}{3} \int \frac{\operatorname{tg}(3x + a) \sec(3x + a) \sec(3x + a)}{\sec(3x + a) \operatorname{tg}(3x + a)} d(\sec(3x + a))$$

$$I = \frac{2}{3} \int \sec(3x + a) d(\sec(3x + a)) \rightarrow I = \frac{2}{3} \frac{\sec^2(3x + a)}{2} \rightarrow I = \frac{1}{3} \sec^2(3x + a) + C \quad \checkmark$$

$$23) I = \int \frac{1}{\cos^2(ax) - \cos(2ax)} dx$$

$$\cos(2ax) = \cos^2(ax) - \sin^2(ax)$$

$$I = \int \frac{1}{\cos^2(ax) - (\cos^2(ax) - \sin^2(ax))} dx \rightarrow I = \int \frac{1}{\cos^2(ax) - \cos^2(ax) + \sin^2(ax)} dx$$

$$I = \int \frac{1}{\sin^2(ax)} dx \rightarrow I = \int \operatorname{cosec}^2(ax) dx$$

$$d(ax) = a dx$$

$$I = \frac{1}{a} \int \operatorname{cosec}^2(ax) d(ax)$$

$$dx = \frac{1}{a} d(ax)$$

$$I = -\frac{1}{a} \cot(ax) + C \quad \checkmark$$

$$24) I = \int \frac{1}{\cos(2x+4) + 1} dx$$

$$\begin{aligned} \cos(2x+4) &= \cos(2(x+2)) \\ \cos(2(x+2)) &= \cos^2(x+2) - \sin^2(x+2) \\ 1 &= \sin^2(x+2) + \cos^2(x+2) \\ \hline \cos(2(x+2)) + 1 &= 2\cos^2(x+2) \end{aligned}$$

$$I = \frac{1}{2} \int \frac{1}{\cos^2(x+2)} dx$$

$$I = \frac{1}{2} \int \sec^2(x+2) dx$$

$$d(x+2) = dx \quad I = \frac{1}{2} \int \sec^2(x+2) d(x+2) \rightarrow I = \frac{1}{2} \tan(x+2) + C \quad \checkmark$$

$$25) I = \int \frac{1}{1 - \cos(6x - 8ab)} dx$$

$$\begin{aligned} \cos(2(3x-4ab)) &= \cos^2(3x-4ab) - \sin^2(3x-4ab) \\ -1 &= -\sin^2(3x-4ab) - \cos^2(3x-4ab) \\ \hline \cos(2(3x-4ab)) - 1 &= -2\sin^2(3x-4ab) \\ 1 - \cos(2(3x-4ab)) &= 2\sin^2(3x-4ab) \end{aligned}$$

$$d(3x-4ab) = 3dx$$

$$dx = \frac{1}{3} d(3x-4ab)$$

$$I = \frac{1}{2} \int \frac{1}{\sin^2(3x-4ab)} dx \rightarrow I = \frac{1}{6} \int \frac{1}{\sin^2(3x-4ab)} d(3x-4ab)$$

$$I = \frac{1}{6} \int \operatorname{cosec}^2(3x-4ab) d(3x-4ab) \rightarrow I = -\frac{1}{6} \cot(3x-4ab) + C \quad \checkmark$$