EXERCÍCIOS - LISTA III

Calcule as seguintes integrais.

1) $I = \int e^{7x+2}\,dx$

$d(7x+2) = 7dx$

$dx = \frac{1}{7}d(7x+2)$

$I = \frac{1}{7}\int e^{7x+2}\,d(7x+2 \rightarrow \boxed{I = \frac{1}{7}e^{7x+2} + C}$

2) $I = \int e^{-x}\,dx$

$d(-x) = -dx$

$dx = -d(-x)$

$I = -\int e^{-x}\,d(-x) \rightarrow \boxed{I = -e^{-x} + C}$

3) $I = \int a \cdot e^{ax}\,dx$

$d(ax) = a\,dx$

$dx = \frac{1}{a}\,d(ax)$

$I = \frac{a}{a}\int e^{ax}\,d(ax) \rightarrow I = \int e^{ax}\,d(ax)$

$\boxed{I = e^{ax} + C}$

4) $I = \int e^{tg\,x}\sec^2 x\,dx$

$d(tg(x)) = sec^2(x)\,dx$

$dx = \frac{1}{sec^2(x)}\,d(tg(x))$

$I = \int \frac{e^{tg(x)}\,\cancel{sec^2(x)}}{\cancel{sec^2(x)}}\,d(tg(x))$

$I = \int e^{tg(x)}\,d(tg(x)) \rightarrow \boxed{I = e^{tg(x)} + C}$

5) $I = \int \frac{1}{x-1}\,dx$

$d(x-1) = dx$

$I = \int \frac{1}{x-1}\,d(x-1) \rightarrow \boxed{I = \ln|x-1| + C}$

6) $I = \int \frac{3}{2x + 5}\, dx$

$d(2x+5) = 2dx$

$dx = \frac{1}{2} d(2x+5)$

$I = \frac{3}{2} \int \frac{1}{2x+5}\, d(2x+5) \rightarrow \boxed{I = \frac{3}{2} \ln|2x+5| + C}$ ✓

7) $I = \int \frac{x^2}{1 + x^3}\, dx$

$d(1+x^3) = 3x^2 dx$

$dx = \frac{1}{3x^2} d(1+x^3)$

$I = \frac{1}{3} \int \frac{x^2}{x^2(1+x^3)}\, d(1+x^3) \rightarrow I = \frac{1}{3} \int \frac{1}{1+x^3}\, d(1+x^3)$

$\boxed{I = \frac{1}{3} \ln|1+x^3| + C}$ ✓

8) $I = \int \frac{3x^2 - 5}{x^3 - 5x + 7}\, dx$

$d(x^3 - 5x + 7) = 3x^2 - 5\, dx$

$dx = \frac{1}{3x^2 - 5} d(x^3 - 5x + 7)$

$I = \int \frac{3x^2 - 5}{(3x^2-5)(x^3 - 5x + 7)}\, d(x^3 - 5x + 7)$

$I = \int \frac{1}{x^3 - 5x + 7}\, d(x^3 - 5x + 7)$

$\boxed{I = \ln|x^3 - 5x + 7| + C}$ ✓

9) $I = \int \frac{3x^2 - 10x + 6}{x^3 - 5x^2 + 6x - 8}\, dx$

$d(x^3 - 5x^2 + 6x - 8) = 3x^2 - 10x + 6\, dx$

$dx = \frac{1}{3x^2 - 10x + 6} d(x^3 - 5x^2 + 6x - 8)$

$I = \int \frac{3x^2 - 10x + 6}{\cancel{(3x^2 - 10x + 6)}(x^3 - 5x^2 + 6x - 8)}\, d(x^3 - 5x^2 + 6x - 8)$

$I = \int \frac{1}{x^3 - 5x^2 + 6x - 8}\, d(x^3 - 5x^2 + 6x - 8) \rightarrow \boxed{I = \ln|x^3 - 5x^2 + 6x - 8| + C}$ ✓

10) $I = \int \frac{x}{(2x - 4)(2x + 4)} dx$

$I = \int \frac{x}{4x^2 - 4^2} dx \to I = \frac{1}{4} \int \frac{x}{x^2 - 4} dx$

$d(x^2-4) = 2x\,dx$

$dx = \frac{1}{2x} d(x^2-4)$

$I = \frac{1}{8} \int \frac{x}{x(x^2-4)} d(x^2-4) \to \boxed{I = \frac{1}{8} \ln|x^2-4| + C}$

11) $I = \int \frac{8\sqrt{x}}{2x\sqrt{x} + 3\sqrt{x}} dx$

$I = 8 \int \frac{\sqrt{x}}{\sqrt{x}(2x+3)} dx \to I = 8 \int \frac{1}{2x+3} dx$

$d(2x+3) = 2dx$

$dx = \frac{1}{2} d(2x+3)$

$I = \frac{8}{2} \int \frac{1}{2x+3} d(2x+3) \to \boxed{I = 4 \ln|2x+3| + C}$

12) $I = \int \cos(2x - 5)\, dx$

$d(2x-5) = 2dx$

$dx = \frac{1}{2} d(2x-5)$

$I = \frac{1}{2} \int \cos(2x-5)\, d(2x-5)$

$\boxed{I = \frac{1}{2} \operatorname{sen}(2x-5) + C}$

13) $I = \int \operatorname{sen}(2abx - 1)\, dx$

$d(2abx - 1) = 2ab\, dx$

$dx = \frac{1}{2ab} d(2abx-1)$

$I = \frac{1}{2ab} \int \operatorname{sen}(2abx-1)\, d(2abx-1)$

$\boxed{I = -\frac{1}{2ab} \cos(2abx-1) + C}$

14) $I = \int \sqrt{1 - sen^2(3x + 1)}\, dx$

$d(3x+1) = 3dx$

$dx = \frac{1}{3} d(3x+1)$

$I = \frac{1}{3}\int \sqrt{1 - sen^2(3x+1)}\, d(3x+1)$

$I = \frac{1}{3}\int \sqrt{\cos^2(3x+1)}\, d(3x+1)$

$sen^2(3x+1) + \cos^2(3x+1) = 1$

$\cos^2(3x+1) = 1 - sen^2(3x+1)$

$I = \frac{1}{3}\int \cos(3x+1)\, d(3x+1)$

$\boxed{I = \frac{1}{3} sen(3x+1) + C}$ ✓

15) $I = \int \frac{cotg(x - 1)}{\sqrt{1 + cotg^2(x - 1)}}\, dx$

$d(x-1) = dx$

$I = \int \frac{cotg(x-1)}{\sqrt{1 + cotg^2(x-1)}}\, d(x-1)$

$\frac{sen^2(x-1)}{sen^2(x-1)} + \frac{\cos^2(x-1)}{sen^2(x-1)} = \frac{1}{sen^2(x-1)}$

$1 + cotg^2(x-1) = cossec^2(x-1)$

$I = \int \frac{cotg(x-1)}{cossec(x-1)}\, d(x-1)$

$I = \int \frac{\cos(x-1)}{\cancel{sen(x-1)}}\, \cancel{sen(x-1)}\, d(x-1) \rightarrow I = \int \cos(x-1)\, d(x-1)$

$\boxed{I = sen(x-1) + C}$ ✓

16) $I = \int \frac{1}{\cos^2(3x - 9)}\, dx$

$d(3x-9) = 3dx$

$dx = \frac{1}{3} d(3x-9)$

$I = \frac{1}{3}\int \frac{1}{\cos^2(3x-9)}\, d(3x-9) \rightarrow I = \frac{1}{3}\int sec^2(3x-9)\, d(3x-9)$

$\boxed{I = \frac{1}{3} tg(3x-9) + C}$ ✓

17) $$I = \int \frac{1}{1 - \operatorname{sen}^2(4x+1)}\,dx$$

$d(4x+1) = 4dx$

$dx = \frac{1}{4}d(4x+1)$

$$I = \frac{1}{4}\int \frac{1}{1-\operatorname{sen}^2(4x+1)}\,d(4x+1)$$

$\operatorname{sen}^2(4x+1) + \cos^2(4x+1) = 1$

$\cos^2(4x+1) = 1 - \operatorname{sen}^2(4x+1)$

$$I = \frac{1}{4}\int \frac{1}{\cos^2(4x+1)}\,d(4x+1) \to I = \frac{1}{4}\int \sec^2(4x+1)\,d(4x+1)$$

$$\boxed{I = \frac{1}{4}\operatorname{tg}(4x+1) + C}$$ ✓

18) $$I = \int \frac{\operatorname{tg}^2(2x+1)}{\operatorname{sen}^2(2x+1)}\,dx$$

$$I = \int \frac{\cancel{\operatorname{sen}^2(2x+1)}}{\cos^2(2x+1)} \cdot \frac{1}{\cancel{\operatorname{sen}^2(2x+1)}}\,dx \to I = \int \frac{1}{\cos^2(2x+1)}\,dx$$

$d(2x+1) = 2dx$

$dx = \frac{1}{2}d(2x+1)$

$$I = \frac{1}{2}\int \sec^2(2x+1)\,d(2x+1)$$

$$\boxed{I = \frac{1}{2}\operatorname{tg}(2x+1) + C}$$ ✓

19) $$I = \int \frac{\sec^2 x}{2 + \operatorname{tg} x}\,dx$$

$d(2+\operatorname{tg}(x)) = \sec^2(x)\,dx$

$dx = \frac{1}{\sec^2(x)}\,d(2+\operatorname{tg}(x))$

$$I = \int \frac{\cancel{\sec^2(x)}}{\cancel{\sec^2(x)}(2+\operatorname{tg}(x))}\,d(2+\operatorname{tg}(x))$$

$$I = \int \frac{1}{2+\operatorname{tg}(x)}\,d(2+\operatorname{tg}(x))$$

$$\boxed{I = \ln|2+\operatorname{tg}(x)| + C}$$ ✓

20) $I = \int \frac{\operatorname{tg} x}{1 + \ln(\cos x)}\, dx$

$$d(1 + \ln(\cos(x))) = -\frac{\operatorname{sen}(x)}{\cos(x)}\, dx$$

$$dx = -\frac{\cos(x)}{\operatorname{sen}(x)}\, d(1 + \ln(\cos(x)))$$

$$I = -\int \frac{\cancel{\cos(x)}}{\cancel{\operatorname{sen}(x)}} \cdot \frac{\cancel{\operatorname{sen}(x)}}{\cancel{\cos(x)}} \cdot \frac{1}{(1 + \ln|\cos(x)|)}\, d(1 + \ln(\cos(x)))$$

$$I = -\int \frac{1}{1 + \ln|\cos(x)|}\, d(1 + \ln(\cos(x))) \rightarrow \boxed{I = -\ln|1 + \ln(\cos(x))| + C}$$

21) $I = \int \frac{\operatorname{tg}^3(2x)}{\cos^2(2x)}\, dx$

$$I = \int \operatorname{tg}^3(2x)\, \sec^2(2x)\, dx$$

$$d(\operatorname{tg}(2x)) = 2\sec^2(2x)\, dx$$

$$dx = \frac{1}{2\sec^2(2x)}\, d(\operatorname{tg}(2x))$$

$$I = \frac{1}{2}\int \frac{\operatorname{tg}^3(2x)\, \cancel{\sec^2(2x)}}{\cancel{\sec^2(2x)}}\, d(\operatorname{tg}(2x))$$

$$I = \frac{1}{2}\int \operatorname{tg}^3(2x)\, d(\operatorname{tg}(2x))$$

$$I = \frac{1}{2}\left[\frac{\operatorname{tg}^4(2x)}{4}\right] \rightarrow \boxed{I = \frac{1}{8}\operatorname{tg}^4(2x) + C}$$

22) $I = \displaystyle\int \frac{2\ tg(3x+a)}{\cos^2(3x+a)}\,dx$

$$I = 2\int tg(3x+a)\sec(3x+a)\sec(3x+a)\,dx$$

$$d(\sec(3x+a)) = 3\sec(3x+a)\,tg(3x+a)\,dx$$

$$dx = \frac{1}{3\sec(3x+a)\,tg(3x+a)}\,d(\sec(3x+a))$$

$$I = \frac{2}{3}\int \frac{\cancel{tg(3x+a)}\ \cancel{\sec(3x+a)}\ \sec(3x+a)}{\cancel{\sec(3x+a)}\ \cancel{tg(3x+a)}}\,d(\sec(3x+a))$$

$$I = \frac{2}{3}\int \sec(3x+a)\,d(3x+a) \rightarrow I = \frac{2}{3}\,\frac{\sec^2(3x+a)}{2} \rightarrow \boxed{I = \frac{1}{3}\sec^2(3x+a) + C}$$ ✓

23) $I = \displaystyle\int \frac{1}{\cos^2(ax) - \cos(2ax)}\,dx$

$$\boxed{\cos(2ax) = \cos^2(ax) - sen^2(ax)}$$

$$I = \int \frac{1}{\cos^2(ax) - (\cos^2(ax) - sen^2(ax))}\,dx \rightarrow I = \int \frac{1}{\cos^2(ax) - \cos^2(ax) + sen^2(ax)}\,dx$$

$$I = \int \frac{1}{sen^2(ax)}\,dx \rightarrow I = \int cossec^2(ax)\,dx$$

$$d(ax) = a\,dx \qquad I = \frac{1}{a}\int cossec^2(ax)\,d(ax)$$

$$dx = \frac{1}{a}\,d(ax)$$

$$\boxed{I = -\frac{1}{a}\,cotg(ax) + C}$$ ✓

24) $I = \displaystyle\int \frac{1}{\cos(2x+4)+1}\,dx$

$\cos(2x+4) = \cos(2(x+2))$

$\cos(2(x+2)) = \cos^2(x+2) - \text{sen}^2(x+2)$

$1 = \text{sen}^2(x+2) + \cos^2(x+2)$

$\cos(2(x+2)) + 1 = 2\cos^2(x+2)$

$I = \dfrac{1}{2}\displaystyle\int \frac{1}{\cos^2(x+2)}\,dx$

$I = \dfrac{1}{2}\displaystyle\int \sec^2(x+2)\,dx$

$d(x+2) = dx$

$I = \dfrac{1}{2}\displaystyle\int \sec^2(x+2)\,d(x+2) \rightarrow \boxed{I = \dfrac{1}{2}\,\text{tg}(x+2) + C}$

25) $I = \displaystyle\int \frac{1}{1-\cos(6x-8ab)}\,dx$

$\cos(2(3x-4ab)) = \cos^2(3x-4ab) - \text{sen}^2(3x-4ab)$

$-1 = -\text{sen}^2(3x-4ab) - \cos^2(3x-4ab)$

$\cos(2(3x-4ab)) - 1 = -2\,\text{sen}^2(3x-4ab)$

$1 - \cos(2(3x-4ab)) = 2\,\text{sen}^2(3x-4ab)$

$d(3x-4ab) = 3dx$

$dx = \dfrac{1}{3}\,d(3x-4ab)$

$I = \dfrac{1}{2}\displaystyle\int \frac{1}{\text{sen}^2(3x-4ab)}\,dx \rightarrow I = \dfrac{1}{6}\displaystyle\int \frac{1}{\text{sen}^2(3x-4ab)}\,d(3x-4ab$

$I = \dfrac{1}{6}\displaystyle\int \text{cossec}^2(3x-4ab)\,d(3x-4ab) \rightarrow \boxed{I = -\dfrac{1}{6}\,\text{cotg}(3x-4ab) + C}$