EXERCÍCIOS - LISTA I

Calcule as seguintes integrais

1) $I = \int x^3 \, dx$

$I = \int x^3 dx \to I = \frac{x^{3+1}}{3+1} \to \boxed{I = \frac{1}{4}x^4 + C}$ ✓

2) $I = \int \frac{1}{x^2} \, dx$

$I = \int \frac{1}{x^2} dx \to I = \int x^{-2} dx \to I = \frac{x^{-2+1}}{-2+1} \to I = \frac{x^{-1}}{-1}$

$\boxed{I = -\frac{1}{x} + C}$ ✓

3) $I = \int 2x\sqrt{x} \, dx$

$I = 2\int x\sqrt{x} dx \to I = 2\int x \, x^{1/2} dx \to I = 2\int x^{3/2} dx$

$I = 2\left[\frac{x^{3/2+1}}{3/2+1}\right] \to I = 2\left[\frac{x^{\frac{5}{2}}}{5/2}\right] \to \boxed{I = \frac{4}{5}\sqrt{x^5} + C}$ ✓

4) $I = \int 3x\sqrt[3]{x} \, dx$

$I = 3\int x \, x^{1/3} dx \to I = 3\int x^{4/3} dx \to I = 3\left[\frac{x^{7/3}}{7/3}\right]$

$\boxed{I = \frac{9}{7}\sqrt[3]{x^7} + C}$ ✓

5) $I = \int \frac{2a \cos x}{3} \, dx$

$I = \frac{2}{3}a\int \cos(x) \, dx \to \boxed{I = \frac{2}{3}a \, \text{sen}(x) + C}$ ✓

6) $I = \int \frac{1}{5\ \text{sen}^2 x}\, dx$

$I = \frac{1}{5}\int \frac{1}{\text{sen}^2(x)}\, dx \rightarrow I = \frac{1}{5}\int \text{cosec}^2(x)\, dx$

$\text{cotg}(x) = \frac{\cos(x)}{\text{sen}(x)} \rightarrow \text{cotg}'(x) = \frac{\cos'(x)\text{sen}(x) - \cos(x)\text{sen}'(x)}{\text{sen}^2(x)}$

$\text{cotg}'(x) = \frac{-\text{sen}^2(x) - \cos^2(x)}{\text{sen}^2(x)} \rightarrow \text{cotg}'(x) = -\frac{1}{\text{sen}^2(x)} \rightarrow \text{cotg}'(x) = -\text{cossec}^2(x)$

$I = \frac{1}{5}\left[-\text{cotg}(x)\right] \rightarrow \boxed{I = -\frac{1}{5}\text{cotg}(x) + C}$ ✓

7) $I = \int \frac{1}{5\cos^2 x}\, dx$

$I = \frac{1}{5}\int \frac{1}{\cos^2(x)}\, dx \rightarrow I = \frac{1}{5}\int \sec^2(x)\, dx$

$\text{tg}(x) = \frac{\text{sen}(x)}{\cos(x)} \rightarrow \text{tg}'(x) = \frac{\text{sen}'(x)\cos(x) - \text{sen}(x)\cos'(x)}{\cos^2(x)}$

$\text{tg}'(x) = \frac{\cos^2(x) + \text{sen}^2(x)}{\cos^2(x)} \rightarrow \text{tg}'(x) = \frac{1}{\cos^2(x)} \rightarrow \text{tg}'(x) = \sec^2(x)$

$\boxed{I = \frac{1}{5}\text{tg}(x) + C}$ ✓

8) $I = \int 3a \sec x \cdot 2b\ \text{tg}\ x\ dx$

$I = 3a2b\int \sec(x)\ \text{tg}(x)\, dx \rightarrow I = 6ab\int \sec(x)\text{tg}(x)\, dx$

$\sec(x) = \frac{1}{\cos(x)} \rightarrow \sec'(x) = \frac{1'\cos(x) - 1\cos'(x)}{\cos^2(x)} \rightarrow \sec'(x) = \frac{\text{sen}(x)}{\cos(x)}\,\frac{1}{\cos(x)}$

$\sec'(x) = \text{tg}(x)\sec(x)$

$\boxed{I = 6ab\sec(x) + C}$ ✓

9) $I = \int \frac{3}{7(1 + x^2)} \, dx$.

$I = \frac{3}{7} \int \frac{1}{1+x^2} \, dx$

$f(x) = \operatorname{arc} tg(x)$

$x' = tg'(f(x))$

$1 = \sec^2 f(x) \, f'(x)$

$f'(x) = \frac{1}{\sec^2 f(x)}$

$\frac{sen^2 f(x)}{\cos^2 f(x)} + \frac{\cos^2 f(x)}{\cos^2 f(x)} = \frac{1}{\cos^2 f(x)}$

$tg^2(f(x)) + 1 = \sec^2 f(x)$

$f'(x) = \frac{1}{x^2+1}$

$\boxed{I = \frac{3}{7} \operatorname{arc} tg(x) + C}$ ✓

10) $I = \int (e^x + 3x - \frac{5}{x}) \, dx$

$I = \int \left(e^x + 3x - \frac{5}{x}\right) dx \rightarrow I = \int e^x dx + 3\int x \, dx - 5\int \frac{1}{x} dx$

$\boxed{I = e^x + \frac{3}{2}x^2 - 5\ln|x| + C}$ ✓

11) $I = \int (3.2^x - 3 \, sen \, x) \, dx$

$I = 3\int 2^x - sen(x) \, dx \rightarrow \boxed{I = 3\left[\frac{2^x}{\ln(2)} + \cos(x)\right] + C}$ ✓

12) $I = \int \frac{2}{\sqrt{1+x} \cdot \sqrt{1-x}} dx$

$$I = 2\int \frac{1}{\sqrt{(1+x)(1-x)}} dx \rightarrow I = 2\int \frac{1}{\sqrt{1-x+x-x^2}} dx$$

$$I = 2\int \frac{1}{\sqrt{1-x^2}} dx$$

$f(x) = \text{arc sen}(x)$

$x = \text{sen } f(x)$

$1 = \cos f(x) \; f'(x)$

$f'(x) = \frac{1}{\cos f(x)}$

$\text{sen}^2 f(x) + \cos^2 f(x) = 1$

$\cos^2 f(x) = 1 - \text{sen}^2 f(x)$

$\cos f(x) = \sqrt{1-x^2}$

$f'(x) = \frac{1}{\sqrt{1-x^2}}$

$$I = 2\int \frac{1}{\sqrt{1-x^2}} dx$$

$$\boxed{I = 2 \text{ arc sen}(x) + C}$$ ✓

13) $I = \int \frac{1}{\sqrt{4 - 4x^2}} dx$

$$I = \int \frac{1}{\sqrt{4(1-x^2)}} dx \rightarrow I = \int \frac{1}{2\sqrt{1-x^2}} dx \rightarrow I = \frac{1}{2}\int \frac{1}{\sqrt{1-x^2}} dx$$

$f(x) = \text{arc sen}(x)$

$x = \text{sen } f(x)$

$1 = \cos f(x) \; f'(x)$

$f'(x) = \frac{1}{\cos f(x)}$

$\text{sen}^2 f(x) + \cos^2 f(x) = 1$

$\cos^2 f(x) = 1 - \text{sen}^2 f(x)$

$\cos f(x) = \sqrt{1-x^2}$

$f'(x) = \frac{1}{\sqrt{1-x^2}}$

$$\boxed{I = \frac{1}{2} \text{ arc sen}(x) + C}$$ ✓

14) $I = \int \frac{x^2}{\sqrt{x^8 - x^6}} dx$

$$I = \int \frac{x^2}{\sqrt{x^6(x^2-1)}} dx \to I = \int \frac{x^2}{\sqrt{(x^3)^2(x^2-1)}} dx \to I = \int \frac{x^2}{x^3\sqrt{x^2-1}} dx$$

$$I = \int \frac{1}{x\sqrt{x^2-1}} dx$$

$f(x) = \text{arc sec}(x)$

$x = \sec f(x)$

$1 = \sec f(x)\ tg\ f(x)\ f'(x)$

$f'(x) = \frac{1}{x\ tg(x)}$

$\frac{\text{sen}^2 f(x)}{\cos^2 f(x)} + \frac{\cos^2 f(x)}{\cos^2 f(x)} = \frac{1}{\cos^2 f(x)}$

$tg^2 f(x) + 1 = \sec^2 f(x)$

$tg\ f(x) = \sqrt{x^2-1}$

$f'(x) = \frac{1}{x\sqrt{x^2-1}}$

$\boxed{I = \text{arc sec}(x) + C}$ ✓

15) $I = \int \frac{x^5 + 2x^3 + x - 1}{x} dx$

$$I = \int \frac{x^5}{x} dx + 2\int \frac{x^3}{x} dx + \int \frac{x}{x} dx - \int \frac{1}{x} dx$$

$$I = \int x^4 dx + 2\int x^2 dx + \int dx - \int \frac{1}{x} dx \to \boxed{I = \frac{1}{5}x^5 + \frac{2x^3}{3} + x - \ln|x| + C}$$ ✓

16) $I = \int \frac{x^3 - x}{x\sqrt{x}} dx$

$$I = \int \frac{(x^3 - x)}{x\, x^{1/2}} dx \to I = \int (x^3 - x)\, x^{-3/2} dx \to I = \int x^{3/2} - x^{-1/2} dx$$

$$I = \frac{2}{5} x^{5/2} - \frac{x^{1/2}}{1/2} \to \boxed{I = \frac{2}{5}\sqrt{x^5} - 2\sqrt{x} + C}$$ ✓

17) $I = \int \frac{1 + tg^2 x}{tg^2 x} dx$

$$I = \int \frac{1}{tg^2(x)} + \frac{tg^2(x)}{tg^2(x)} dx \rightarrow I = \int cotg^2(x) + 1 \, dx$$

$$\frac{sen^2(x)}{sen^2(x)} + \frac{cos^2(x)}{sen^2(x)} = \frac{1}{sen^2(x)} \rightarrow 1 + cotg^2(x) = cossec^2(x)$$

$$cotg^2(x) = cossec^2(x) - 1$$

$$I = \int cossec^2(x) - 1 + 1 \, dx \rightarrow I = \int cossec^2(x) \, dx$$

$$cotg(x) = \frac{cos(x)}{sen(x)} \rightarrow cotg'(x) = \frac{cos'(x)sen(x) - cos(x)sen'(x)}{sen^2(x)}$$

$$cotg'(x) = \frac{-sen^2(x) - cos^2(x)}{sen^2(x)} \rightarrow cotg'(x) = -\frac{1}{sen^2(x)} \rightarrow cotg'(x) = -cossec^2(x)$$

$$I = \int cossec^2(x) \, dx \rightarrow \boxed{I = -cotg(x) + C}$$ ✓

18) $I = \int \frac{1 + sen^2 x}{sen^2 x} dx$

$$I = \int \frac{1}{sen^2(x)} + \frac{sen^2(x)}{sen^2(x)} dx \rightarrow I = \int cossec^2(x) + 1 \, dx$$

$$cotg(x) = \frac{cos(x)}{sen(x)} \rightarrow cotg'(x) = \frac{cos'(x)sen(x) - cos(x)sen'(x)}{sen^2(x)}$$

$$cotg'(x) = \frac{-sen^2(x) - cos^2(x)}{sen^2(x)} \rightarrow cotg'(x) = -\frac{1}{sen^2(x)} \rightarrow cotg'(x) = -cossec^2(x)$$

$$I = \int cossec^2(x) + 1 \, dx \rightarrow \boxed{I = -cotg(x) + x + C}$$ ✓

19) $I = \int (3 + 3\cot g^2 x)\, dx$

$I = 3\int 1 + \cot g^2(x)\, dx$

$I = 3\int \operatorname{cossec}^2(x)\, dx$

$\frac{\operatorname{sen}^2(x)}{\operatorname{sen}^2(x)} + \frac{\cos^2(x)}{\operatorname{sen}^2(x)} = \frac{1}{\operatorname{sen}^2(x)}$

$1 + \cot g^2(x) = \operatorname{cossec}^2(x)$

$\cot g(x) = \frac{\cos(x)}{\operatorname{sen}(x)} \rightarrow \cot g'(x) = \frac{\cos'(x)\operatorname{sen}(x) - \cos(x)\operatorname{sen}'(x)}{\operatorname{sen}^2(x)}$

$\cot g'(x) = \frac{-\operatorname{sen}^2(x) - \cos^2(x)}{\operatorname{sen}^2(x)} \rightarrow \cot g'(x) = -\frac{1}{\operatorname{sen}^2(x)} \rightarrow \cot g'(x) = -\operatorname{cossec}^2(x)$

$\boxed{I = -3\cot g(x) + C}$ ✓

20) $I = \int \frac{x - x^3}{x - x^5}\, dx$

$I = \int \frac{x(1-x^2)}{x(1-x^4)}\, dx \rightarrow I = \int \frac{\cancel{(1-x^2)}}{(1+x^2)\cancel{(1-x^2)}}\, dx \rightarrow I = \int \frac{1}{1+x^2}\, dx$

$f(x) = \operatorname{arc} tg(x)$

$x' = tg'(f(x))$

$1 = \sec^2 f(x)\, f'(x)$

$f'(x) = \frac{1}{\sec^2 f(x)}$

$\frac{\operatorname{sen}^2 f(x)}{\cos^2 f(x)} + \frac{\cos^2 f(x)}{\cos^2 f(x)} = \frac{1}{\cos^2 f(x)}$

$tg^2(f(x)) + 1 = \sec^2 f(x)$

$f'(x) = \frac{1}{x^2+1}$

$I = \int \frac{1}{1+x^2}\, dx \rightarrow \boxed{I = \operatorname{arc} tg(x) + C}$ ✓

21) $I = \int tg^2x \, dx$

$$I = \int tg^2(x)\, dx$$

$$\frac{sen^2(x)}{cos^2(x)} + \frac{cos^2(x)}{cos^2(x)} = \frac{1}{cos^2(x)}$$

$$tg^2(x) + 1 = sec^2(x)$$

$$I = \int sec^2(x) - 1 \, dx$$

$$\boxed{I = tg(x) - x + C}$$ ✓

22) $I = \int \frac{tg\ x}{sen\ 2x} dx$

$$I = \int \frac{tg(x)}{sen(2x)} dx \rightarrow I = \int \frac{\cancel{sen(x)}}{cos(x)} \cdot \frac{1}{2\,\cancel{sen(x)}\,cos(x)} dx$$

$$sen(2x) = 2sen(x)\,cos(x)$$

$$I = \frac{1}{2}\int \frac{1}{cos^2(x)} dx \rightarrow I = \frac{1}{2}\int sec^2(x)\,dx \rightarrow \boxed{I = \frac{1}{2} tg(x) + C}$$ ✓

23) $I = \int \frac{sen\ 2x}{cos^3 x} dx$

$$I = \int \frac{sen(2x)}{cos^3(x)} dx$$

$$sen(2x) = 2sen(x)\,cos(x)$$

$$I = 2\int \frac{sen(x)\,\cancel{cos(x)}}{cos^{\cancel{3}\,2}(x)} dx \rightarrow I = 2\int \frac{sen(x)}{cos(x)} \frac{1}{cos(x)} dx$$

$$sec(x) = \frac{1}{cos(x)} \rightarrow sec'(x) = \frac{1'cos(x) - 1cos'(x)}{cos^2(x)} \rightarrow sec'(x) = \frac{sen(x)}{cos(x)}\frac{1}{cos(x)}$$

$$sec'(x) = tg(x)\,sec(x)$$

$$I = 2\int tg(x)\,sec(x)\,dx \rightarrow \boxed{I = 2\,sec(x) + C}$$ ✓

24) $I = \int \frac{4x^2 + 4x + 1}{4x + 2}\, dx$

$$\begin{array}{l|l} 4x^2+4x+1 & 4x+2 \\ \underline{4x^2+2x} & x+\frac{1}{2} \\ \quad 2x+1 & \\ \quad \underline{2x+1} & \\ \quad\quad 0 & \end{array}$$

$4x^2+4x+1 = \left(x+\frac{1}{2}\right)(4x+2)$

$$I = \int \frac{\left(x+\frac{1}{2}\right)(4x+2)}{4x+2}\, dx \to I = \int x + \frac{1}{2}\, dx \to I = \frac{1}{2}x^2 + \frac{1}{2}x$$

$$\boxed{I = \frac{1}{2}(x^2+x) + C}$$ ✓

25) $I = \int \frac{\sqrt{1-x^2}}{1-x^2}\, dx$

$$I = \int \frac{\sqrt{1-x^2}}{1-x^2} \cdot \frac{\sqrt{1-x^2}}{\sqrt{1-x^2}}\, dx \to I = \int \frac{\cancel{(1-x^2)}}{\cancel{(1-x^2)}\sqrt{1-x^2}}\, dx \to I = \int \frac{1}{\sqrt{1-x^2}}\, dx$$

$f(x) = \text{arc sen}(x)$

$x = \text{sen } f(x)$

$1 = \cos f(x)\, f'(x)$

$f'(x) = \frac{1}{\cos f(x)}$

$\text{sen}^2 f(x) + \cos^2 f(x) = 1$

$\cos^2 f(x) = 1 - \text{sen}^2 f(x)$

$\cos f(x) = \sqrt{1-x^2}$

$f'(x) = \frac{1}{\sqrt{1-x^2}}$

$$\boxed{I = \text{arc sen}(x) + C}$$ ✓

26) $I = \int \left(\frac{x}{2} - \frac{2}{x}\right)^2 dx$

$$I = \int \left(\frac{x}{2} - \frac{2}{x}\right)^2 dx \to I = \int \frac{x^2}{4} - 2\cdot\left(\frac{x}{2}\right)\left(\frac{2}{x}\right) + \frac{4}{x^2}\, dx \to I = \int \frac{x^2}{4} - 2 + \frac{4}{x^2}\, dx$$

$$I = \frac{1}{4}\int x^2 dx - 2\int dx + 4\int \frac{1}{x^2}\, dx \to I = \frac{1}{12}x^3 - 2x + 4\int x^{-2} dx$$

$$I = \frac{1}{12}x^3 - 2x + 4\left[\frac{x^{-2+1}}{-2+1}\right] \to I = \frac{1}{12}x^3 - 2x + 4\left[\frac{x^{-1}}{-1}\right]$$

$$\boxed{I = \frac{1}{12}x^3 - 2x - \frac{4}{x} + C}$$ ✓

27) $I = \int \frac{x - 1}{\sqrt{x} + 1}\, dx$

$I = \int \frac{(x-1)(\sqrt{x} - 1)}{(\sqrt{x} + 1)(\sqrt{x} - 1)}\, dx \rightarrow I = \int \frac{\cancel{(x-1)}(\sqrt{x} - 1)}{\cancel{(x-1)}}\, dx$

$I = \int \sqrt{x} - 1\, dx \rightarrow I = \int x^{1/2} dx - \int dx \rightarrow I = \frac{x^{3/2}}{3/2} - x$

$\boxed{I = \frac{2}{3}\sqrt{x^3} - x + C}$ ✓

28) $I = \int \frac{x^2 - 3}{x - \sqrt{3}}\, dx$

$I = \int \frac{x^2 - 3}{(x - \sqrt{3})} \frac{(x + \sqrt{3})}{(x + \sqrt{3})}\, dx \rightarrow I = \int \frac{\cancel{(x^2 - 3)}(x + \sqrt{3})}{\cancel{x^2 - 3}}\, dx$

$I = \int x + \sqrt{3}\, dx \rightarrow \boxed{I = \frac{1}{2}x^2 + \sqrt{3}x + C}$ ✓

29) $I = \int \text{sen}^2 (\frac{x}{2})\, dx$

$\cos(2x) = \cos^2(x) - \text{sen}^2(x)$

$-1 = -\text{sen}^2(x) - \cos^2(x)$

$\cos(2x) - 1 = -2\,\text{sen}^2(x)$

$\text{sen}^2(x) = \frac{1 - \cos(2x)}{2}$

$I = \int \frac{1 - \cos(x)}{2}\, dx$

$I = \frac{1}{2}\int 1 - \cos(x)\, dx$

$\boxed{I = \frac{1}{2}\left[x - \text{sen}(x)\right] + C}$ ✓

30) $I = \int \cos^2\left(\frac{x}{2}\right) dx$

$\cos(2x) = \cos^2(x) - \sin^2(x)$

$+ \quad 1 \quad = \sin^2(x) + \cos^2(x)$

$\cos(2x) + 1 = 2\cos^2(x)$

$\cos^2(x) = \frac{1 + \cos(2x)}{2}$

$I = \int \frac{1 + \cos(x)}{2} dx$

$I = \frac{1}{2} \int 1 + \cos(x)\, dx$

$\boxed{I = \frac{1}{2}\left[x + \text{sen}(x)\right] + C}$ ✓

31) $I = \int \frac{\cos 2x}{\cos^2 x - \frac{1}{2}} dx$

$I = \int \frac{\cos(2x)}{\cos^2(x) - \frac{3}{2}} dx \rightarrow I = \int \frac{\cos(2x)}{\frac{1 + \cos(2x)}{2} - \frac{1}{2}} dx$

$\cos(2x) = \cos^2(x) - \text{sen}^2(x)$

$+ \quad 1 \quad = \text{sen}^2(x) + \cos^2(x)$

$\cos(2x) + 1 = 2\cos^2(x)$

$\cos^2(x) = \frac{1 + \cos(2x)}{2}$

$I = \int \frac{\cos(2x)}{\frac{\cos(2x)}{2}} dx$

$I = 2\int dx \rightarrow \boxed{I = 2x + C}$ ✓

32) $I = \int \text{sen } x \cdot \sec x \cdot \text{tg } x \, dx$

$I = \int \frac{\text{sen}(x)}{\cos(x)} \cdot \frac{\text{sen}(x)}{\cos(x)} dx \rightarrow I = \int \text{tg}^2(x)\, dx$

$\frac{\text{sen}^2(x)}{\cos^2(x)} + \frac{\cos^2(x)}{\cos^2(x)} = \frac{1}{\cos^2(x)}$

$\text{tg}^2(x) + 1 = \sec^2(x)$

$I = \int \sec^2(x) - 1\, dx$

$\boxed{I = \text{tg}(x) - x + C}$ ✓

33) $I = \int \cos x \cdot \operatorname{cosec} x \cdot \operatorname{cotg} x \, dx$

$I = \int \frac{\cos(x)}{\operatorname{sen}(x)} \frac{\cos(x)}{\operatorname{sen}(x)} dx \rightarrow I = \int \operatorname{cotg}^2(x) \, dx$

$\frac{\operatorname{sen}^2(x)}{\operatorname{sen}^2(x)} + \frac{\cos^2(x)}{\operatorname{sen}^2(x)} = \frac{1}{\operatorname{sen}^2(x)}$

$1 + \operatorname{cotg}^2(x) = \operatorname{cossec}^2(x)$

$\operatorname{cotg}^2(x) = \operatorname{cossec}^2(x) - 1$

$I = \int \operatorname{cossec}^2(x) - 1 \, dx \rightarrow \boxed{I = -\operatorname{cotg}(x) - x + C}$ ✓

34) $I = \int \frac{x^3 + x + 1}{x^2 + 1} dx$

$$\begin{array}{l|l} x^3 + x + 1 & x^2 + 1 \\ \hline \underline{x^3 + x} & x \\ 0 + 1 & \end{array}$$

$x^3 + x + 1 = x(x^2+1)$

$I = \int \frac{x(x^2+1) + 1}{x^2+1} dx$

$I = \int x \, dx + \int \frac{1}{x^2+1} dx \rightarrow \boxed{I = \frac{1}{2} x^2 + \operatorname{arctg}(x) + C}$ ✓

35) $I = \int \frac{x^3 - x^2 + x - 2}{x^2 + 1} dx$

$$\begin{array}{l|l} x^3 - x^2 + x - 2 & x^2 + 1 \\ \hline \underline{x^3 + 0x^2 + x} & x - 1 \\ -x^2 + 0 - 2 & \\ \underline{-x^2 + 0 - 1} & \\ 0 \quad 0 - 1 & \end{array}$$

$x^3 - x^2 + x - 2 = (x-1)(x^2+1) - 1$

$I = \int \frac{(x-1)(x^2+1) - 1}{x^2+1} dx \rightarrow I = \int (x-1) dx - \int \frac{1}{x^2+1} dx$

$f(x) = \operatorname{arc} tg(x)$

$x = tg(f(x))$

$1 = \sec^2 f(x) \, f'(x)$

$f'(x) = \frac{1}{\sec^2 f(x)}$

$\frac{\operatorname{sen}^2 f(x)}{\cos^2 f(x)} + \frac{\cos^2 f(x)}{\cos^2 f(x)} = \frac{1}{\cos^2 f(x)}$

$tg^2(f(x)) + 1 = \sec^2 f(x)$

$f'(x) = \frac{1}{x^2 + 1}$

$\boxed{I = \frac{1}{2} x^2 - x - \operatorname{arc} tg(x) + C}$ ✓

36) $I = \int \frac{x^4 + 2x^2}{1 + x^2}\,dx$

$$\begin{array}{l|l} x^4 + 2x^2 & 1 + x^2 \\ \hline \underline{x^4 + x^2} & x^2 + 1 \\ x^2 + 0 & \\ \underline{x^2 + 1} & \\ -1 & \end{array}$$

$x^4 + 2x^2 = (x^2+1)(x^2+1) - 1$

$I = \int \frac{(x^2+1)^2 - 1}{1 + x^2}\,dx \to I = \int (x^2+1)\,dx - \int \frac{1}{1+x^2}\,dx$

$$\boxed{I = \frac{1}{3}x^3 + x - \operatorname{arctg}(x) + C}$$ ✓

37) $I = \int \frac{x^5 + 3x^3 + 2x - 1}{x^2 + 1}\,dx$

$$\begin{array}{l|l} x^5 + 3x^3 + 2x - 1 & x^2 + 1 \\ \hline \underline{x^5 + x^3} & x^3 + 2x \\ 2x^3 + 2x - 1 & \\ \underline{2x^3 + 2x + 0} & \\ -1 & \end{array}$$

$x^5 + 3x^3 + 2x - 1 = (x^3 + 2x)(x^2 + 1) - 1$

$I = \int \frac{(x^3+2x)(x^2+1) - 1}{x^2+1}\,dx \to I = \int x^3 + 2x\,dx - \int \frac{1}{x^2+1}\,dx$

$$\boxed{I = \frac{1}{4}x^4 + x^2 - \operatorname{arctg}(x) + C}$$ ✓

38) $I = \int \frac{2x^4 - 3x^3 + 2x^2 - 3x + 1}{x^2 + 1}\,dx$

$$\begin{array}{l|l} 2x^4 - 3x^3 + 2x^2 - 3x + 1 & x^2 + 1 \\ \hline \underline{2x^4 - 0x^3 + 2x^2} & 2x^2 - 3x \\ -3x^3 + 0x^2 - 3x + 1 & \\ \underline{-3x^3 + 0x^2 - 3x} & \\ 0 + 1 & \end{array}$$

$I = \int \frac{(2x^2 - 3x)(x^2+1) + 1}{x^2 + 1}\,dx$

$I = \int 2x^2 - 3x\,dx + \int \frac{1}{x^2+1}\,dx$

$$\boxed{I = \frac{2}{3}x^3 - \frac{3}{2}x^2 + \operatorname{arctg}(x) + C}$$ ✓