FACULDADE DE TECNOLOGIA DE SÃO PAULO
FATEC - SP

Área de Matemática

ASSUNTO :

EXERCÍCIOS DE CÁLCULO II

DISCIPLINAS:

1252 - MATEMÁTICA I - MAT I
1260 - MATEMÁTICA II - MAT II
1287 - MÉTODOS DE CÁLCULO II - CALC II
1503 - CÁLCULO PARA MECÂNICA DE PRECISÃO I - CALC I(F)
1562 - CÁLCULO PARA MECÂNICA DE PRECISÃO II - CALC II(F)

APOSTILA N ° 49

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IMPRESSO NA GRÁFICA DA FATEC-SP

FÓRMULAS IMPORTANTES

$y = ax^2 + bx + c \Rightarrow y = a(x - x')(x - x'')$

$A^3 + B^3 = (A + B)(A^2 - AB + B^2)$

$A^3 - B^3 = (A - B)(A^2 + AB + B^2)$

$\text{tg } x = \dfrac{\text{sen } x}{\cos x}$

$\text{cotg } x = \dfrac{\cos x}{\text{sen } x}$

$\sec x = \dfrac{1}{\cos x}$

$\text{cosec } x = \dfrac{1}{\text{sen } x}$

$\text{sen}^2 x + \cos^2 x = 1$

$\cos(\frac{\pi}{2} - x) = \text{sen } x$

$\text{sen}(\frac{\pi}{2} - x) = \cos x$

$\text{tg}(\frac{\pi}{2} - x) = \text{cotg } x$

$\text{cotg}(\frac{\pi}{2} - x) = \text{tg } x$

$\text{sen}(-x) = -\text{sen } x$

$\cos(-x) = \cos x$

$\text{sen } 2a = 2 \text{ sen } a \cos a$

$\cos 2a = \cos^2 a - \text{sen}^2 a$

$\text{sen}^2 a = \dfrac{1 - \cos 2a}{2}$

$\cos^2 a = \dfrac{1 + \cos 2a}{2}$

$\sec^2 x = \text{tg}^2 x + 1$

$\text{cosec}^2 x = \text{cotg}^2 x + 1$

$\cos(a - b) = \cos a \cos b + \text{sen } a \text{ sen } b$

$\cos(a + b) = \cos a \cos b - \text{sen } a \text{ sen } b$

$\text{sen}(a + b) = \text{sen } a \cos b + \text{sen } b \cos a$

$\text{sen}(a - b) = \text{sen } a \cos b - \text{sen } b \cos a$

$\text{sen } p + \text{sen } q = 2 \text{ sen } \frac{p+q}{2} \cos \frac{p-q}{2}$

$\text{sen } p - \text{sen } q = 2 \text{ sen } \frac{p-q}{2} \cos \frac{p+q}{2}$

$\cos p + \cos q = 2 \cos \frac{p+q}{2} \cos \frac{p-q}{2}$

$\cos p - \cos p = -2 \text{ sen } \frac{p+q}{2} \text{ sen } \frac{p-q}{2}$

Θ	sen Θ	cos Θ	tg Θ	cotg Θ	sec Θ	cosecΘ
$0 \equiv 2\pi$	0	1	0	*	1	*
$\pi/6$	$1/2$	$\sqrt{3}/2$	$\sqrt{3}/3$	$\sqrt{3}$	$2\sqrt{3}/3$	2
$\pi/4$	$\sqrt{2}/2$	$\sqrt{2}/2$	1	1	$\sqrt{2}$	$\sqrt{2}$
$\pi/3$	$\sqrt{3}/2$	$1/2$	$\sqrt{3}$	$\sqrt{3}/3$	2	$2\sqrt{3}/3$
$\pi/2$	1	0	*	0	*	1
π	0	-1	0	*	-1	*
$3\pi/2$	-1	0	*	0	*	-1

REGRAS DE DERIVAÇÃO

f(x)	f'(x)	f(x)	f'(x)
x^n	$n.x^{n-1}$	sec x	sec x.tg x
$m.g(x)$	$m.g'(x)$	cosec x	$-$ cosec x.cotg x
a^x	$a^x.\ln a$	arcsen x	$\frac{1}{\sqrt{1-x^2}}$
e^x	e^x	arccos x	$\frac{-1}{\sqrt{1-x^2}}$
$\log_a x$	$\frac{1}{x.\ell n\, a}$	arctg x	$\frac{1}{1+x^2}$
$\ln x$	$\frac{1}{x}$	arccotg x	$\frac{-1}{1+x^2}$
$u \pm v$	$u' \pm v'$	arcsec x	$\frac{1}{x\sqrt{x^2-1}}$
$u.v$	$u'v + uv'$	arccosec x	$\frac{-1}{x\sqrt{x^2-1}}$
$\frac{u}{v}$	$\frac{u'v - uv'}{v^2}$	$f[g(x)]$	$f'[g(x)].g'(x)$
sen x	cos x	u^v	$u^v\left[v'.\ln x + \frac{u'v}{u}\right]$
cos x	$-$ sen x		
tg x	$\sec^2 x$		
cotg x	$-\operatorname{cosec}^2 x$		

INTEGRAIS INDEFINIDAS

$$\boxed{F(x) + C} \xrightarrow{\text{derivação}} \boxed{F'(x) = f(x)}$$

$$\boxed{F(x) + C} \xleftarrow{\text{integração}} \boxed{F'(x) = f(x)}$$

PROPRIEDADES

$P_1)\quad d\int f(x)\,dx = f(x)dx$

$P_2)\quad \int d\left[F(x) + C\right] = F(x) + C$

$P_3)\quad \int M.f(x)dx = M\int f(x)\,dx\,,\quad M \in \mathbb{R}^*$

$P_4)\quad \int [u(x) \pm v(x)]\,dx = \int u(x)\,dx \pm \int v(x)\,dx$

INTEGRAIS IMEDIATAS

$I_1 = \int x^n\,dx$ .... $I_1 = \dfrac{x^{n+1}}{n+1} + C, \quad (n \neq -1)$

$I_2 = \int \frac{1}{x}\,dx = \int x^{-1}dx$ .... $I_2 = \ln|x| + C$

$I_3 = \int e^x\,dx$ .... $I_3 = e^x + C$

$I_4 = \int a^x\,dx$ .... $I_4 = \dfrac{a^x}{\ln a} + C$

$I_5 = \int \cos x\,dx$ .... $I_5 = \text{sen } x + C$

$I_6 = \int \text{sen } x\,dx$ .... $I_6 = -\cos x + C$

$I_7 = \int \sec^2 x\,dx = \int \dfrac{1}{\cos^2 x}\,dx$ .... $I_7 = \text{tg } x + C$

$I_8 = \int \text{cosec}^2 x\,dx = \int \dfrac{1}{\text{sen}^2 x}\,dx$ .... $I_8 = -\text{cotg } x + C$

$I_9 = \int \sec x.\text{tg } x\,dx$ .... $I_9 = \sec x + C$

$I_{10} = \int \text{cosec } x.\ \text{cotg } x\,dx$ .... $I_{10} = -\text{cosec } x + C$

$I_{11} = \int \dfrac{1}{\sqrt{1-x^2}}\,dx$ .... $I_{11} = \text{arcsen } x + C$

$I_{12} = \int \dfrac{1}{1+x^2}\,dx$ .... $I_{12} = \text{arctg } x + C$

$I_{13} = \int \dfrac{1}{x\sqrt{x^2-1}}\,dx$ .... $I_{13} = \text{arcsec } x + C$

ALGUMAS INTEGRAIS QUE APARECEM COM FREQÜÊNCIA

$I = \int \sec x \, dx$ .... $I = \ln|\sec x + tg\, x| + C$

$I = \int \text{cosec}\, x \, dx$ .... $I = -\ln|\text{cosec}\, x + \text{cotg}\, x| + C$

$I = \int \sec^3 x \, dx$ .... $I = \frac{1}{2}\left[\sec x . tg\, x + \ln|\sec x + tgx|\right] + C$

$I = \int \text{cosec}^3 x \, dx$ .... $I = -\frac{1}{2}\left[\text{cosec}\, x . \text{cotg}\, x + \ln|\text{cosec}\, x + \text{cotg}\, x|\right] + C$

F O R M U L Á R I O

1) $y_m = \frac{1}{b-a}\int_a^b f(x)\, dx$

2) $L = \int_a^b \sqrt{1 + y'^2}\, dx$

3) $S = 2\pi \int_a^b y \sqrt{1 + y'^2}\, dx$

4) $V = \pi \int_a^b y^2 \, dx$

5) $K = \frac{y''}{(1 + y'^2)^{3/2}}$

6) $L = \int_{t_1}^{t_2} \sqrt{\dot{x}^2 + \dot{y}^2}\, dt$

7) $S = 2\pi \int_{t_1}^{t_2} y \sqrt{\dot{x}^2 + \dot{y}^2}\, dt$

8) $V = \pi \int_{t_1}^{t_2} y^2 . \dot{x}\, dt$

9) $K = \frac{\ddot{y}\dot{x} - \dot{y}\ddot{x}}{(\dot{x}^2 + \dot{y}^2)^{3/2}}$

10) $A = \frac{1}{2}\int_{\theta_1}^{\theta_2} \rho^2 \, d\theta$

11) $L = \int_{\theta_1}^{\theta_2} \sqrt{\rho^2 + \rho'^2}\, d\theta$

12) $I(x) = e^{\int P(x)\, dx}$

13) $y = \frac{1}{I(x)} \int Q(x) . I(x)\, dx$

INTEGRAIS IMEDIATAS

EXEMPLOS RESOLVIDOS. Calcule as seguintes integrais

1) $I = \int x^3 \, dx$ ....(I_1)

$$I = \frac{x^{3+1}}{3+1} + C$$

$$I = \frac{x^4}{4} + C$$

.-.

2) $I = \int \frac{x^2 - 1}{x^4 - 1} \, dx$

$$I = \int \frac{\cancel{(x^2 - 1)}}{(x^2 + 1)\cancel{(x^2 - 1)}} \, dx$$

$I = \int \frac{1}{x^2 + 1} \, dx$ ....(I_{12})

$$\boxed{I = \text{arctg } x + C}$$

.-.

3) $I = \int 2a \cos x \, dx$

$I = 2a \int \cos x \, dx$ .... (I_5)

$$I = 2a(\text{sen } x + c_1)$$

$\boxed{I = 2a \text{ sen } x + C}$, onde $C = 2ac_1$

.-.

4) $I = \int (2 \text{ sen } x + 3\, e^x)\, dx$

$$I = 2\int \text{sen } x\, dx + 3\int e^x\, dx \quad \ldots\ldots\ldots\ldots\ldots\ldots (I_6),(I_3)$$

$$I = 2\left[-\cos x + c_1\right] + 3\left[e^x + c_2\right]$$

$$\boxed{I = -2\cos x + 3e^x + C}, \quad \text{onde } C = 2c_1 + 3c_2$$

.-.

5) $I = \int \left(\dfrac{\text{cosec}^2 x}{3} - \dfrac{x+1}{x^2}\right) dx$

$$I = \int \frac{\text{cosec}^2 x}{3}\, dx - \int \frac{x+1}{x^2}\, dx$$

$$I = \frac{1}{3}\int \text{cosec}^2 x\, dx - \int\left(\frac{1}{x} + \frac{1}{x^2}\right) dx$$

$$I = \frac{1}{3}\int \text{cosec}^2 x\, dx - \int \frac{1}{x}\, dx - \int x^{-2} dx \quad \ldots\ldots\ldots\ldots (I_8),(I_2),(I_1)$$

$$I = \frac{1}{3}\left[-\text{cotg } x + c_1\right] - \left[\ln|x| + c_2\right] - \left[\frac{x^{-1}}{-1} + c_3\right]$$

$$\boxed{I = -\frac{1}{3}\text{cotg } x - \ln|x| + \frac{1}{x} + C}, \quad \text{onde } C = \frac{1}{3}c_1 - c_2 - c_3$$

.-.

6) $I = \int \dfrac{x}{\sqrt{x^6 - x^4}}\, dx$

$$I = \int \frac{x}{x^2\sqrt{x^2 - 1}}\, dx$$

$$I = \int \frac{1}{x\sqrt{x^2 - 1}}\, dx \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (I_{13})$$

$$\boxed{I = \text{arcsec } x + C}$$

EXERCÍCIOS - LISTA I

Calcule as seguintes integrais

1) $I = \int x^3 \, dx$

2) $I = \int \frac{1}{x^2} \, dx$

3) $I = \int 2x \sqrt{x} \, dx$

4) $I = \int 3x \sqrt[3]{x} \, dx$

5) $I = \int \frac{2a \cos x}{3} \, dx$

6) $I = \int \frac{1}{5 \operatorname{sen}^2 x} \, dx$

7) $I = \int \frac{1}{5 \cos^2 x} \, dx$

8) $I = \int 3a \sec x \, . \, 2b \operatorname{tg} x \, dx$

9) $I = \int \frac{3}{7(1 + x^2)} \, dx$

10) $I = \int (e^x + 3x - \frac{5}{x}) \, dx$

11) $I = \int (3.2^x - 3 \operatorname{sen} x) \, dx$

12) $I = \int \frac{2}{\sqrt{1 + x} \, . \, \sqrt{1 - x}} \, dx$

13) $I = \int \frac{1}{\sqrt{4 - 4x^2}}\, dx$

14) $I = \int \frac{x^2}{\sqrt{x^8 - x^6}}\, dx$

15) $I = \int \frac{x^5 + 2x^3 + x - 1}{x}\, dx$

16) $I = \int \frac{x^3 - x}{x\sqrt{x}}\, dx$

17) $I = \int \frac{1 + tg^2 x}{tg^2 x}\, dx$

18) $I = \int \frac{1 + sen^2 x}{sen^2 x}\, dx$

19) $I = \int (3 + 3\, cotg^2 x)\, dx$

20) $I = \int \frac{x - x^3}{x - x^5}\, dx$

21) $I = \int tg^2 x\, dx$

22) $I = \int \frac{tg\, x}{sen\, 2x}\, dx$

23) $I = \int \frac{sen\, 2x}{cos^3 x}\, dx$

24) $I = \int \frac{4x^2 + 4x + 1}{4x + 2}\, dx$

25) $I = \int \frac{\sqrt{1 - x^2}}{1 - x^2}\, dx$

26) $I = \int \left(\frac{x}{2} - \frac{2}{x}\right)^2 dx$

27) $I = \int \frac{x - 1}{\sqrt{x} + 1}\, dx$

28) $I = \int \frac{x^2 - 3}{x - \sqrt{3}}\, dx$

29) $I = \int \operatorname{sen}^2\left(\frac{x}{2}\right)\, dx$

30) $I = \int \cos^2\left(\frac{x}{2}\right)\, dx$

31) $I = \int \frac{\cos\, 2x}{\cos^2 x - \frac{1}{2}}\, dx$

32) $I = \int \operatorname{sen}\, x\, .\, \sec\, x\, .\, \operatorname{tg}\, x\, dx$

33) $I = \int \cos\, x\, .\, \operatorname{cosec}\, x\, .\, \operatorname{cotg}\, x\, dx$

34) $I = \int \frac{x^3 + x + 1}{x^2 + 1}\, dx$

35) $I = \int \frac{x^3 - x^2 + x - 2}{x^2 + 1}\, dx$

36) $I = \int \frac{x^4 + 2x^2}{1 + x^2}\, dx$

37) $I = \int \frac{x^5 + 3x^3 + 2x - 1}{x^2 + 1}\, dx$

38) $I = \int \frac{2x^4 - 3x^3 + 2x^2 - 3x + 1}{x^2 + 1}\, dx$

INTEGRAÇÃO POR SUBSTITUIÇÃO DE VARIÁVEL

EXEMPLOS RESOLVIDOS. Calcule as seguintes integrais.

1) $I = \int (12x - 5)^{15} dx$ .... $12x - 5 = t$

$12\,dx = dt$

$I = \int t^{15} \cdot \frac{1}{12}\,dt$ $\qquad dx = \frac{1}{12}\,dt$

$$I = \frac{1}{12} \int t^{15} dt$$

$$I = \frac{1}{12}\left(\frac{t^{16}}{16} + c_1\right)$$

$$I = \frac{t^{16}}{192} + C$$

$$\boxed{I = \frac{(12x - 5)^{16}}{192} + C}$$

.-.

2) $I = \int \frac{e^{5x-8}}{e^{2x}}\,dx$

$$I = \int e^{5x-8-2x}\,dx$$

$I = \int e^{3x-8}\,dx$ .... $3x - 8 = t$

$3\,dx = dt$

$I = \int e^{t} \cdot \frac{1}{3}\,dt$ $\qquad dx = \frac{1}{3}\,dt$

$$I = \frac{1}{3} \int e^{t}\,dt$$

$$I = \frac{1}{3}(e^{t} + c_1)$$

$$I = \frac{1}{3} e^{t} + C \longrightarrow \boxed{I = \frac{1}{3} e^{3x-8} + C}$$

3) $I = \int \frac{tg^{50}3x}{\cos^2 3x}\, dx$

$I = \int tg^{50}3x \,.\, \sec^2 3x\, dx$ .... $tg\ 3x = t$

$3\ \sec^2 3x\ dx = dt$

$\sec^2 3x\ dx = \frac{1}{3}\ dt$

$I = \int t^{50} . \frac{1}{3}\ dt$

$I = \frac{1}{3} \int t^{50} dt$

$I = \frac{1}{3}\left[\frac{t^{51}}{51} + c_1\right]$

$I = \frac{t^{51}}{153} + C$

$$\boxed{I = \frac{tg^{51}3x}{153} + C}$$

.-.

4) $I = \int x^3(x^2 + 1)^8\ dx$ .... $x^2 + 1 = t \longrightarrow x^2 = t - 1$

$2x\ dx = dt$

$x\ dx = \frac{1}{2}\ dt$

$I = \int x^2(x^2 + 1)^8 x\ dx$

$I = \int (t - 1)t^8 . \frac{1}{2}\ dt$

$I = \frac{1}{2} \int (t^9 - t^8)\ dt$

$I = \frac{1}{2}\left[\frac{t^{10}}{10} - \frac{t^9}{9} + c_1\right]$

$$\boxed{I = \frac{1}{2}\left[\frac{(x^2 + 1)^{10}}{10} - \frac{(x^2 + 1)^9}{9}\right] + C}$$

EXERCÍCIOS - LISTA II

Calcule as seguintes integrais.

1) $I = \int (3x + 1)^{20} dx$

2) $I = \int (5x - 1)^{15} dx$

3) $I = \int \dfrac{3}{(2x - 3)^{10}} dx$

4) $I = \int \dfrac{1}{\sqrt{5 + x}} dx$

5) $I = \int \dfrac{x}{\sqrt{x^2 + 1}} dx$

6) $I = \int \dfrac{x^2}{\sqrt{5x^3 + 1}} dx$

7) $I = \int (2x + 1) \sqrt{x^2 + x - 3}\, dx$

8) $I = \int \dfrac{2x^2 + 1}{\sqrt{2x^3 + 3x + 1}} dx$

9) $I = \int \dfrac{1}{(x - a)^4} dx$

10) $I = \int \dfrac{(4x - 2)^2}{(2x - 1)^5} dx$

11) $I = \int \dfrac{1}{(2x - 5)^2 \sqrt{2x - 5}} dx$

12) $I = \int \dfrac{\sqrt[3]{4x - 16}}{(x - 4)^3} dx$

13) $I = \int \frac{\sqrt[5]{(4x+2)^7}}{8x+4}\, dx$

14) $I = \int \sqrt{\left(\frac{x^2-1}{x+1}\right)^5}\, dx$

15) $I = \int \frac{\sqrt{\left(4 - \frac{3}{x^2}\right)^3}}{x^3}\, dx$

16) $I = \int \frac{1}{x^2}\sqrt[3]{\left(\frac{1}{x} + b\right)^2}\, dx$

17) $I = \int \frac{1}{\sqrt[3]{x^2}\left(1 - 2\sqrt[3]{x}\right)}\, dx$

18) $I = \int e^{-\frac{x}{a}}\, dx$

19) $I = \int \frac{e^{3x-4}}{e^{x+2}}\, dx$

20) $I = \int \frac{x \cdot e^{x^2+x}}{e^{x+1}}\, dx$

21) $I = \int a^{4x} \cdot \ln a\, dx$

22) $I = \int \operatorname{sen} \frac{x}{2}\, dx$

23) $I = \int (x+1) \cdot \operatorname{sen}(x^2 + 2x)\, dx$

24) $I = \int x \cdot \operatorname{sen} \frac{6x^4 - 2x^2}{x^2}\, dx$

25) $I = \int \operatorname{tg}(2x) \cdot \cos(2x)\, dx$

26) $I = \int e^x . \cos(e^x - 1)\ dx$

27) $I = \int \text{sen}(2x) . \cos(2x)\ dx$

28) $I = \int \cos(3x) . \text{sen}(3x)\ dx$

29) $I = \int \text{tg}\ x\ dx$

30) (*) $I = \int \text{cotg}\ x\ dx$

31) $I = \int \text{tg}(2a + x)\ dx$

32) $I = \int a . \text{cotg}(ax + b)\ dx$

33) $I = \int \frac{\text{sen}\ x}{\cos^3 x}\ dx$

34) $I = \int \frac{1}{x . \ln x}\ dx$

35) $I = \int \frac{\ln^2 x}{x}\ dx$

36) $I = \int \frac{\ln(2x - 4)}{x - 2}\ dx$

37) $I = \int \frac{\ln^2(ax + 1)}{ax + 1}\ dx$

38) $I = \int \frac{\text{arctg}\ x}{1 + x^2}\ dx$

39) $I = \int \frac{\text{arccotg } x}{1 + x^2}\, dx$

40) $I = \int \frac{\text{arcsen } x}{\sqrt{1 - x^2}}\, dx$

41) $I = \int \frac{1}{x + x \cdot \ln^2 x}\, dx$

42) $I = \int \sqrt{x} \cdot \text{sen}(x\sqrt{x})\, dx$

43) $I = \int \sqrt{x} \cdot e^{2x\sqrt{x}}\, dx$

44) $I = \int \frac{\sqrt{x}}{(a + x\sqrt{x})^2}\, dx$

45) $I = \int \sqrt{x} \cdot \sqrt{1 + x\sqrt{x}}\, dx$

46) $I = \int \frac{x}{\sqrt{1 - x^4}}\, dx$

47) $I = \int \frac{a}{\sqrt{a^2 - x^2}}\, dx$

48) $I = \int x(x + 1)^{10}\, dx$

49) $I = \int x^2(x - 2)^4\, dx$

50) $I = \int \frac{x^3}{\sqrt{1 + x^2}}\, dx$

51) $I = \int \frac{x^2}{\sqrt{1 + x}}\, dx$

52) $I = \int x^3 \sqrt{1 - x^2}\, dx$

53) $I = \int x^5 \sqrt{1 - x^2}\, dx$

54) $I = \int x^5 (x^3 + 1)^{\frac{2}{3}}\, dx$

55) $I = \int x^7 \sqrt{x^4 + 2}\, dx$

56) $I = \int x^3 \sqrt{2x^2 + 4}\, dx$

57) $I = \int x^3 \sqrt{3x^2 - 6}\, dx$

58) $I = \int \frac{1}{1 + \sqrt{x}}\, dx$

59) $I = \int \frac{\sqrt{x}}{1 + \sqrt{x}}\, dx$

60) $I = \int \frac{\sqrt[3]{x}}{(2 + x\sqrt[3]{x})^2}\, dx$

61) $I = \int \frac{1}{\sqrt{x}(x + 1)}\, dx$

62) $I = \int \frac{1}{1 + x + \sqrt{x + 1}}\, dx$

63) $I = \int \frac{x^5}{\sqrt{x^3 + 4} - 2}\, dx$

64) $I = \int \frac{2x + \sqrt{x + 1}}{x^2 + 2x + 1}\, dx$

INTEGRAÇÃO POR MUDANÇA DE DIFERENCIAL

EXEMPLOS RESOLVIDOS. Calcule as seguintes integrais.

1) $I = \int e^{3x+1}\, dx$ .... $d(3x+1) = 3\, dx$

$\frac{d(3x+1)}{3} = dx$

$I = \int e^{3x+1}\, \frac{d(3x+1)}{3}$

$I = \frac{1}{3} \int e^{3x+1} d(3x+1)$ ....OBS: $\int e^x dx = e^x + C$

$$\boxed{I = \frac{1}{3}\, e^{3x+1} + C}$$

.-

2) $I = \int \cos(7x - 2)\, dx$ .... $d(7x - 2) = 7\, dx$

$\frac{d(7x - 2)}{7} = dx$

$I = \int \cos(7x - 2)\, \frac{d(7x - 2)}{7}$

$I = \frac{1}{7} \int \cos(7x - 2)\, d(7x - 2)$ ..... OBS: $\int \cos x\, dx = \operatorname{sen} x + C$

$I = \frac{1}{7} \operatorname{sen}(7x - 2) + C$

.-

3) $I = \int \frac{2x}{x^2 - 1}\, dx$ .... $d(x^2 - 1) = 2x\, dx$

$I = \int \frac{1}{x^2 - 1}\, d(x^2 - 1)$ .... OBS: $\int \frac{1}{x}\, dx = \ln|x| + C$

$$\boxed{I = \ln|x^2 - 1| + C}$$

4) $I = \int (2x - 7)^{12} dx$ .... $d(2x - 7) = 2\,dx$

$$\frac{d(2x - 7)}{2} = dx$$

$$I = \int (2x - 7)^{12} \cdot \frac{d(2x - 7)}{2}$$

$I = \frac{1}{2} \int (2x - 7)^{12} d(2x - 7)$ .....OBS: $\int x^{12} dx = \frac{x^{13}}{13} + C$

$$I = \frac{1}{2} \frac{(2x - 7)^{13}}{13} + C$$

$$\boxed{I = \frac{(2x - 7)^{13}}{26} + C}$$

.-

5) $I = \int \cos^7 x \,.\, \text{sen}\, x\, dx$ .... $d(\cos x) = -\,\text{sen}\, x\, dx$

$$-d(\cos x) = \text{sen}\, x\, dx$$

$$I = \int (\cos x)^7 [-d(\cos x)]$$

$I = -\int (\cos x)^7\, d(\cos x)$ ...., OBS: $\int x^7 dx = \frac{x^8}{8} + C$

$$\boxed{I = -\frac{\cos^8 x}{8} + C}$$

-.-

EXERCÍCIOS - LISTA III

Calcule as seguintes integrais.

1) $I = \int e^{7x+2}\, dx$

2) $I = \int e^{-x}\, dx$

3) $I = \int a \cdot e^{ax}\, dx$

4) $I = \int e^{tg\, x} \sec^2 x\, dx$

5) $I = \int \frac{1}{x - 1}\, dx$

6) $I = \int \frac{3}{2x + 5}\, dx$

7) $I = \int \frac{x^2}{1 + x^3}\, dx$

8) $I = \int \frac{3x^2 - 5}{x^3 - 5x + 7}\, dx$

9) $I = \int \frac{3x^2 - 10x + 6}{x^3 - 5x^2 + 6x - 8}\, dx$

10) $I = \int \frac{x}{(2x - 4)(2x + 4)}\, dx$

11) $I = \int \frac{8\sqrt{x}}{2x\sqrt{x} + 3\sqrt{x}}\, dx$

12) $I = \int \cos(2x - 5)\, dx$

13) $I = \int \operatorname{sen}(2abx - 1)\, dx$

14) $I = \int \sqrt{1 - \operatorname{sen}^2(3x + 1)}\, dx$

15) $I = \int \dfrac{\operatorname{cotg}(x - 1)}{\sqrt{1 + \operatorname{cotg}^2(x - 1)}}\, dx$

16) $I = \int \dfrac{1}{\cos^2(3x - 9)}\, dx$

17) $I = \int \dfrac{1}{1 - \operatorname{sen}^2(4x + 1)}\, dx$

18) $I = \int \dfrac{\operatorname{tg}^2(2x + 1)}{\operatorname{sen}^2(2x + 1)}\, dx$

19) $I = \int \dfrac{\sec^2 x}{2 + \operatorname{tg} x}\, dx$

20) $I = \int \dfrac{\operatorname{tg} x}{1 + \ln(\cos x)}\, dx$

21) $I = \int \dfrac{\operatorname{tg}^3(2x)}{\cos^2(2x)}\, dx$

22) $I = \int \dfrac{2 \operatorname{tg}(3x + a)}{\cos^2(3x + a)}\, dx$

23) $I = \int \dfrac{1}{\cos^2(ax) - \cos(2ax)}\, dx$

24) $I = \int \dfrac{1}{\cos(2x + 4) + 1}\, dx$

25) $I = \int \dfrac{1}{1 - \cos(6x - 8ab)}\, dx$

INTEGRAÇÃO DAS FUNÇÕES RACIONAIS

$$I = \int \frac{a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \ldots + a_n}{b_0x^m + b_1x^{m-1} + b_2x^{m-2} + \ldots + b_m}\,dx$$

EXEMPLOS RESOLVIDOS. Calcule as seguintes integrais.

1) $I = \displaystyle\int \frac{3x - 1}{3x^2 - 2x + 1}\,dx$ .... $3x^2 - 2x + 1 = t$

$(6x - 2)\,dx = dt$

$2(3x - 1)\,dx = dt$

$(3x - 1)\,dx = \frac{1}{2}\,dt$

$I = \displaystyle\frac{1}{2}\int \frac{1}{t}\,dt$

$I = \frac{1}{2}\ln|t| + C$

$$\boxed{I = \frac{1}{2}\ln|3x^2 - 2x + 1| + C}$$

-.-

2) $I = \displaystyle\int \frac{x + 1}{x^2 - 5x}\,dx$

$I = \displaystyle\int \frac{x + 1}{x(x - 5)}\,dx$

$I = \displaystyle\int \left(\frac{A}{x} + \frac{B}{x - 5}\right)dx$ .... OBS: $\dfrac{A}{x} + \dfrac{B}{x - 5} = \dfrac{x + 1}{x(x - 5)}$

$A(x - 5) + B\,x = x + 1$

$\begin{cases} x = 5 \Rightarrow 5\,B = 6 \Rightarrow B = \frac{6}{5} \\ x = 0 \Rightarrow -5\,A = 1 \Rightarrow A = -\frac{1}{5} \end{cases}$

$I = \displaystyle\int \left(\frac{-\frac{1}{5}}{x} + \frac{\frac{6}{5}}{x - 5}\right)dx$

$I = \displaystyle -\frac{1}{5}\int \frac{1}{x}\,dx + \frac{6}{5}\int \frac{1}{x - 5}\,dx$

$$\boxed{I = -\frac{1}{5}\ln|x| + \frac{6}{5}\ln|x - 5| + C}$$

3) $I = \int \frac{x}{(x-2)^2}\,dx$

$I = \int \left(\frac{A}{(x-2)^2} + \frac{B}{x-2} \right) dx$ ...... OBS: $\frac{A}{(x-2)^2} + \frac{B}{x-2} = \frac{x}{(x-2)^2}$

$A + B(x-2) = x$

$\begin{cases} x = 2 \Rightarrow A = 2 \\ x = 0 \Rightarrow A - 2B = 0 \end{cases}$

$-2B = -2 \Rightarrow B = 1$

$I = \int \left(\frac{1}{(x-2)^2} + \frac{2}{x-2} \right) dx$ .... $x - 2 = t$

$dx = dt$

$I = \int t^{-2}dt + 2\int \frac{1}{t}\,dt$

$I = \frac{t^{-1}}{-1} + 2\ln|t| + C \longrightarrow \boxed{I = -\frac{1}{x-2} + 2\ln|x-2| + C}$

-.-

4) $I = \int \frac{x^4 - 3x^2 + x}{x^2 - 3}\,dx$ ....

$$\begin{array}{l|l} x^4 - 3x^2 + x & x^2 - 3 \\ \cline{2-2} -x^4 + 3x^2 & x^2 \\ \cline{1-1} \qquad\qquad x & \end{array}$$

$I = \int \left(x^2 + \frac{x}{x^2 - 3} \right) dx$

$I = \int x^2\,dx + \int \frac{x}{x^2-3}\,dx$ .... $x^2 - 3 = t$

$2x\,dx = dt$

$x\,dx = \frac{1}{2}\,dt$

$I = \frac{x^3}{3} + \frac{1}{2}\int \frac{1}{t}\,dt$

$I = \frac{x^3}{3} + \frac{1}{2}\ln|t| + C$

$\boxed{I = \frac{x^3}{3} + \frac{1}{2}\ln|x^2 - 3| + C}$

EXERCÍCIOS - LISTA IV

Calcule as seguintes integrais.

1) $I = \int \frac{3x^2}{5x^3 - 1}\,dx$

2) $I = \int \frac{x^2 + \frac{2}{3}}{x^3 + 2x - 13}\,dx$

3) $I = \int \frac{x + 3}{x^2 + 4x + 3}\,dx$

4) $I = \int \frac{3}{x^2 - 3x - 4}\,dx$

5) $I = \int \frac{2}{x^2 + 4x + 3}\,dx$

6) $I = \int \frac{2x + 4}{x^2 - 4x + 4}\,dx$

7) $I = \int \frac{x - 1}{(x + 1)^2}\,dx$

8) $I = \int \frac{x + 2}{x^2 + x}\,dx$

9) $I = \int \frac{1}{x^2 + 3x}\,dx$

10) $I = \int \frac{1}{x^2 - 2x}\,dx$

11) $I = \int \frac{1}{x^2 - 1}\,dx$

12) $I = \int \frac{5x - 19}{x^2 - 7x + 10}\,dx$

13) $I = \displaystyle\int \frac{2x - 1}{x^2 - 5x + 6}\, dx$

14) $I = \displaystyle\int \frac{2x - 3}{(x - 1)(x^2 + 2x - 15)}\, dx$

15) $I = \displaystyle\int \frac{2x - 3}{(x + 1)(x - 1)^2}\, dx$

16) $I = \displaystyle\int \frac{x}{(x - 1)(x + 1)^2}\, dx$

17) $I = \displaystyle\int \frac{x^2 + x + 1}{x^2(x + 1)^2}\, dx$

18) $I = \displaystyle\int \frac{2x^2 + 3x}{x + 1}\, dx$

19) $I = \displaystyle\int \frac{x^3 + x^2 + 1}{x^2 + 1}\, dx$

20) $I = \displaystyle\int \frac{x^3 + 2x^2 - x + 2}{x^2 + 2x - 3}\, dx$

21) $I = \displaystyle\int \frac{2t^3 - 2t^2 - 1}{t^2 - t}\, dt$

22) $I = \displaystyle\int \frac{x^3 - x^2 - 5x - 8}{x^2 - x - 6}\, dx$

23) $I = \displaystyle\int \frac{x^3 + 3x^2 - 28x + 3}{x^2 + 3x - 28}\, dx$

24) $I = \displaystyle\int \frac{\sqrt{1 + 4x^2}}{x}\, dx$

25) $I = \displaystyle\int \frac{\sqrt{1 - x^2}}{x}\, dx$

INTEGRAÇÃO POR PARTES

$$\int u.v'\,dx = u.v - \int u'.v\,dx$$

EXEMPLOS RESOLVIDOS. Calcule as seguintes integrais.

1) $I = \int \underbrace{x}_{u}.\underbrace{\cos x}_{v'}\,dx$ .... $u = x \Rightarrow u' = 1$

$v = \int \cos x\,dx \Rightarrow v = \operatorname{sen} x$

$I = x.\operatorname{sen} x - \int 1.\operatorname{sen} x\,dx$

$I = x.\operatorname{sen} x - (-\cos x) + C$

$$\boxed{I = x.\operatorname{sen} x + \cos x + C}$$

.-

2) $I = \int \underbrace{x^2}_{u}.\underbrace{\ln(x+1)}_{v'}\,dx$ .... $u = \ln(x+1) \Rightarrow u' = \frac{1}{x+1}$

$v = \int x^2\,dx \Rightarrow v = \frac{x^3}{3}$

$I = \ln(x+1).\frac{x^3}{3} - \int \frac{1}{x+1}.\frac{x^3}{3}\,dx$

$I = \frac{x^3}{3}\ln(x+1) - \frac{1}{3}\int \frac{x^3}{x+1}\,dx$ ....

$$\begin{array}{r|l} x^3 & x+1 \\ \hline -x^3 - x^2 & x^2 - x + 1 \\ \hline -x^2 & \\ x^2 + x & \\ \hline x & \\ -x - 1 & \\ \hline -1 & \end{array}$$

$I = \frac{x^3}{3}\ln(x+1) - \frac{1}{3}\int \left(x^2 - x + 1 - \frac{1}{x+1}\right)dx$

$I = \frac{x^3}{3}\ln(x+1) - \frac{1}{3}\left(\frac{x^3}{3} - \frac{x^2}{2} + x - \ln(x+1)\right) + C$

$$\boxed{I = \frac{x^3}{3}\ln(x+1) - \frac{x^3}{9} + \frac{x^2}{6} - \frac{x}{3} + \frac{1}{3}\ln(x+1) + C}$$

EXERCÍCIOS - LISTA V

Calcule as seguintes integrais.

1) $I = \int x \operatorname{sen} x \, dx$

2) $I = \int x \cos x \, dx$

3) $I = \int x e^{x} \, dx$

4) $I = \int x \sec^{2} x \, dx$

5) $I = \int x \operatorname{cosec}^{2} x \, dx$

6) $I = \int x \operatorname{sen} 2x \, dx$

7) $I = \int 3x \cos 5x \, dx$

8) $I = \int x e^{3x} \, dx$

9) $I = \int 2x e^{-2x} \, dx$

10) $I = \int 2x \sec^{2} 3x \, dx$

11) $I = \int x \ln x \, dx$

12) $I = \int x^{3} \ln x \, dx$

13) $I = \int \sqrt{x} \ln x \, dx$

14) $I = \int \frac{\ln x}{x^4} \, dx$

15) $I = \int \operatorname{arctg} x \, dx$

16) $I = \int \operatorname{arccotg} x \, dx$

17) $I = \int \operatorname{arcsen} x \, dx$

18) $I = \int \arccos 3x \, dx$

19) $I = \int x^2 \operatorname{sen} x \, dx$

20) $I = \int x^2 \cos ax \, dx$

21) $I = \int x^2 e^x \, dx$

22) $I = \int x^2 e^{-x} \, dx$

23) $I = \int x^3 e^x \, dx$

24) $I = \int x^3 \cos x \, dx$

25) $I = \int \sec^3 x \, dx$

26) $I = \int \operatorname{cosec}^3 x \, dx$

27) $I = \int e^x \cos x \, dx$

28) $I = \int e^x \operatorname{sen} 2x \, dx$

29) $I = \int e^{-4x} \cos 3x \, dx$

30) $I = \int \operatorname{sen} x \operatorname{sen} 3x \, dx$

31) $I = \int \cos 4x \cos 7x \, dx$

32) $I = \int \operatorname{sen} 2x \cos 4x \, dx$

33) $I = \int x \cos^2 x \, dx$

34) $I = \int x \operatorname{sen}^2 x \, dx$

35) $I = \int x \ln(x + 1) \, dx$

36) $I = \int \ln(x^2 + 1) \, dx$

37) $I = \int x^2 \operatorname{arcsen} x \, dx$

38) $I = \int x^2 \operatorname{arctg} x \, dx$

INTEGRAÇÃO DE POTÊNCIAS DE FUNÇÕES TRIGONOMÉTRICAS

EXEMPLOS RESOLVIDOS. Calcule as seguintes integrais.

1) $I = \int \text{sen}^3 x \,.\, \cos^8 x \, dx$

$I = \int \text{sen}^2 x \,.\, \cos^8 x \,.\, \text{sen}\, x \, dx$ .... $\cos x = t$

$-\text{sen}\, x \, dx = dt$

$\text{sen}\, x \, dx = -dt$

$I = \int (1 - \cos^2 x).\, \cos^8 x \,.\, \text{sen}\, x \, dx$

$I = \int (1 - t^2)\, t^8 (-dt)$

$I = -\int (t^8 - t^{10}) \, dt$

$I = -\left[\frac{t^9}{9} - \frac{t^{11}}{11}\right] + C$

$$\boxed{I = -\left[\frac{\cos^9 x}{9} - \frac{\cos^{11} x}{11}\right] + C}$$

-.-

2) $I = \int \cos^3 x \, dx$

$I = \int \cos^2 x \,.\, \cos x \, dx$ .... $\text{sen}\, x = t$

$\cos x \, dx = dt$

$I = \int (1 - \text{sen}^2 x).\cos x \, dx$

$I = \int (1 - t^2) \, dt$

$I = t - \frac{t^3}{3} + C \longrightarrow \boxed{I = \text{sen}\, x - \frac{\text{sen}^3 x}{3} + C}$

3) $I = \int \text{sen}^2(3x)\ dx$ .... $\text{sen}^2(3x) = \dfrac{1 - \cos(6x)}{2}$

$$I = \int \frac{1 - \cos(6x)}{2}\ dx$$

$$I = \frac{1}{2}\int\left[1 - \cos(6x)\right]dx$$

$$I = \frac{1}{2}\left[\int dx - \int \cos(6x)\ dx\right]$$ .... $6x = t$

$6\ dx = dt$

$dx = \frac{1}{6}\ dt$

$$I = \frac{1}{2}\left[x - \frac{1}{6}\int \cos\ t\ dt\right]$$

$$I = \frac{1}{2}\left[x - \frac{1}{6}\ \text{sen}\ t\right] + C$$

$$\boxed{I = \frac{1}{2}\left[x - \frac{1}{6}\ \text{sen}(6x)\right] + C}$$

-.-

4) $I = \int \sec(5ax + b)\ dx$ .... $5ax + b = t$

$5a\ dx = dt$

$dx = \frac{1}{5a}\ dt$

$$I = \frac{1}{5a}\int \sec\ t\ dt$$

$$I = \frac{1}{5a}\ \ln\left|\sec\ t + tg\ t\right| + C$$

$$\boxed{I = \frac{1}{5a}\ \ln\left|\sec(5ax + b) + tg(5ax + b)\right| + C}$$

5) $I = \int \sec^4(2x)\ dx$

$I = \int \sec^2(2x).\sec^2(2x)\ dx$ .... $tg(2x) = t$

$2\sec^2(2x)\ dx = dt$

$\sec^2(2x)\ dx = \frac{1}{2}\ dt$

$I = \int \left[tg^2(2x) + 1\right].\sec^2(2x)\ dx$

$I = \int (t^2 + 1).\frac{1}{2}\ dt$

$I = \frac{1}{2}\int (t^2 + 1)\ dt$

$I = \frac{1}{2}\left[\frac{t^3}{3} + t\right] + C$

$$\boxed{I = \frac{1}{2}\left[\frac{tg^3(2x)}{3} + tg(2x)\right] + C}$$

.-

6) $I = \int cotg^3x\ dx$

$I = \int \frac{\cos^3 x}{sen^3 x}\ dx$

$I = \int sen^{-3}x.\cos^2x.\cos x\ dx$ .... $sen\ x = t$

$\cos x\ dx = dt$

$I = \int sen^{-3}x.(1 - sen^2x)\cos x\ dx$

$I = \int t^{-3}(1 - t^2)\ dt$

$I = \int (t^{-3} - t^{-1})\ dt$

$I = \frac{t^{-2}}{-2} - \ln t + C \longrightarrow \boxed{I = -\frac{1}{2\ sen^2x} - \ln sen\ x + C}$

7) $I = \int \cot g^6 x . \operatorname{cosec}^4 x \; dx$

$I = \int \cot g^6 x . \operatorname{cosec}^2 x . \operatorname{cosec}^2 x \; dx$ .... $\cot g\, x = t$

$-\operatorname{cosec}^2 x \; dx = dt$

$I = \int \cot g^6 x . (\cot g^2 x + 1) . \operatorname{cosec}^2 x \; dx$

$I = \int t^6 (t^2 + 1)(-dt)$

$I = -\int (t^8 + t^6) \; dt$

$I = -\left(\frac{t^9}{9} + \frac{t^7}{7}\right) + C \longrightarrow \boxed{I = -\frac{\cot g^9 x}{9} - \frac{\cot g^7 x}{7} + C}$

-.

8) $I = \int \sec^5 x . \operatorname{tg}^3 x \; dx$

$I = \int \sec^4 x . \operatorname{tg}^2 x . \sec x . \operatorname{tg} x \; dx$ .... $\sec x = t$

$\sec x . \operatorname{tg} x \; dx = dt$

$I = \int \sec^4 x . (\sec^2 x - 1) . \sec x . \operatorname{tg} x \; dx$

$I = \int t^4 (t^2 - 1) \; dt$

$I = \int (t^6 - t^4) \; dt$

$I = \frac{t^7}{7} - \frac{t^5}{5} + C$

$$\boxed{I = \frac{\sec^7 x}{7} - \frac{\sec^5 x}{5} + C}$$

EXERCÍCIOS - LISTA VI

Calcule as seguintes integrais.

1) $I = \int \operatorname{sen}^3 x \cos^2 x \, dx$

2) $I = \int \operatorname{sen}^2 x \cos^3 x \, dx$

3) $I = \int \cos^4 x \operatorname{sen}^3 x \, dx$

4) $I = \int \operatorname{sen}^3 x \cos^{12} x \, dx$

5) $I = \int \cos^5 x \operatorname{sen}^6 x \, dx$

6) $I = \int \cos^3 2x \operatorname{sen}^2 2x \, dx$

7) $I = \int \cos^5 3x \operatorname{sen}^{13} 3x \, dx$

8) $I = \int \cos^8 5x \operatorname{sen}^5 5x \, dx$

9) $I = \int \operatorname{sen}^3 x \, dx$

10) $I = \int \cos^3 2x \, dx$

11) $I = \int \operatorname{sen}^5 3x \, dx$

12) $I = \int \cos^5 5x \, dx$

13) $I = \int \text{sen}^7 x \, dx$

14) $I = \int \cos^7 x \, dx$

15) $I = \int \text{sen}^2 x \, dx$

16) $I = \int \cos^2 x \, dx$

17) $I = \int \cos^4 2x \, dx$

18) $I = \int \text{sen}^4 3x \, dx$

19) $I = \int \cos^6 x \, dx$

20) $I = \int \text{sen}^6 2x \, dx$

21) $I = \int \sec 3x \, dx$

22) $I = \int \text{cosec}\ 2ax \, dx$

23) $I = \int \sec(2x + 5) \, dx$

24) $I = \int \text{cosec}(3x + 2) \, dx$

25) $I = \int \sec^2(2abx + 7c) \, dx$

26) $I = \int \sec^4 x \, dx$

27) $I = \int \mathrm{cosec}^4 2x \, dx$

28) $I = \int \sec^6 2x \, dx$

29) $I = \int \mathrm{cosec}^6 x \, dx$

30) $I = \int \sec^8 x \, dx$

31) $I = \int \mathrm{tg}\ 2x \, dx$

32) $I = \int \mathrm{cotg}\ ax \, dx$

33) $I = \int \mathrm{tg}^3 4x \, dx$

34) $I = \int \mathrm{cotg}^3 2ax \, dx$

35) $I = \int \mathrm{cotg}^5 x \, dx$

36) $I = \int \mathrm{cotg}^7 x \, dx$

37) $I = \int \mathrm{tg}^2 x \, dx$

38) $I = \int \mathrm{cotg}^2\ 5x \, dx$

39) $I = \int tg^4 x \; dx$

40) $I = \int tg^6 x \; dx$

41) $I = \int sec^4 x \; tg^3 x \; dx$

42) $I = \int \frac{sec^4 x}{tg^3 x} \; dx$

43) $I = \int sec^4 2x \; tg \; 2x \; dx$

44) $I = \int sec^3 x . tg^3 x . dx$

45) $I = \int sec^3 x \; tg \; x \; dx$

46) $I = \int cotg \; \frac{x}{3} \; cosec^3 \; \frac{x}{3} \; dx$

47) $I = \int 3cos^3 x \; tg^2 x \; dx$

48) $I = \int sen \; 2x \; cos^3 x \; dx$

49) $I = \int cos \; 2x \; sen \; x \; dx$

50) $I = \int \frac{sen^{13}(3x + 1)}{sec^3(3x + 1)} \; dx$

51) $I = \int \frac{tg(5x + 1)}{cos^2(5x + 1)} \; dx$

INTEGRAÇÃO POR SUBSTITUIÇÃO TRIGONOMÉTRICA

EXEMPLOS RESOLVIDOS. Calcule as seguintes integrais.

1) $I = \int \sqrt{1 - x^2}\, dx$ .... $x = \text{sen } t$

$dx = \cos t\ dt$

$$I = \int \sqrt{1 - \text{sen}^2 t}\,.\cos t\ dt$$

$$I = \int \sqrt{\cos^2 t}\,.\cos t\ dt$$

$$I = \int \cos^2 t\ dt$$

$$I = \frac{1}{2}\left[t + \frac{1}{2}\,\text{sen } 2t\right] + C$$

$I = \frac{1}{2}\left[t + \text{sen } t \,.\, \cos t\right] + C$ .... $x = \text{sen } t \Rightarrow t = \text{arcsen } x$

$\cos t = \sqrt{1-\text{sen}^2 t} = \sqrt{1-x^2}$

$$\boxed{I = \frac{1}{2}\left[\text{arcsen } x + x\sqrt{1 - x^2}\right] + C}$$

-.-

2) $I = \int \sqrt{x^2 + 1}\, dx$ .... $x = \text{tg } t$

$dx = \sec^2 t\ dt$

$$I = \int \sqrt{\text{tg}^2 t + 1}\,.\sec^2 t\ dt$$

$$I = \int \sqrt{\sec^2 t}\,.\sec^2 t\ dt$$

$$I = \int \sec^3 t\ dt$$

$I = \frac{1}{2}\left[\sec t \,.\, \text{tg } t + \ln\left|\sec t + \text{tg } t\right|\right] + C$ .... $x = \text{tg } t$

$\sec t = \sqrt{\text{tg}^2 t+1}$

$\sec t = \sqrt{x^2 + 1}$

$$\boxed{I = \frac{1}{2}\left[x\sqrt{x^2 + 1} + \ln\left|\sqrt{x^2 + 1} + x\right|\right] + C}$$

3) $I = \int \sqrt{x^2 - 4}\, dx$

$$I = \int \sqrt{4(\frac{x^2}{4} - 1)}\ dx$$

$$I = 2 \int \sqrt{(\frac{x}{2})^2 - 1}\ dx \ \ldots\ldots\ldots\ldots\ \frac{x}{2} = \sec t$$

$$x = 2 \sec t$$

$$dx = 2 \sec t \,.\, tg\ t$$

$$I = 2 \int \sqrt{\sec^2 t - 1}.2 \sec t \,.\, tg\ t\ dt$$

$$I = 4 \int \sqrt{tg^2 t}.\sec t \,.\, tg\ t\ dt$$

$$I = 4 \int tg^2 t.\sec t\ dt$$

$$I = 4 \int (\sec^2 t - 1)\sec t\ dt$$

$$I = 4 \int (\sec^3 t - \sec t)\ dt$$

$$I = 4 \left[\int \sec^3 t\ dt - \int \sec t\ dt \right]$$

$$I = 4 \left[\frac{1}{2}(\sec t \,.\, tg\ t + \ln|\sec t + tg\ t|) - \ln|\sec t + tg\ t| \right] + C$$

$$I = 2 \sec t \,.\, tg\ t + 2 \ln|\sec t + tg\ t| - 4 \ln|\sec t + tg\ t| + C$$

$$I = 2 \sec t \,.\, tg\ t - 2 \ln|\sec t + tg\ t| + C$$

$$I = 2 \left[\sec t \,.\, tg\ t - \ln|\sec t + tg\ t| \right] + C$$

$$I = 2 \left[\frac{x}{2}\, \frac{\sqrt{x^2 - 4}}{2} - \ln\left|\frac{x}{2} + \frac{\sqrt{x^2 - 4}}{2}\right| \right] + C$$

$$\boxed{I = 2 \left[\frac{x\sqrt{x^2 - 4}}{4} - \ln\left| \frac{x + \sqrt{x^2 - 4}}{2} \right| \right] + C}$$

$$\frac{x}{2} = \sec t$$

$$tg\ t = \sqrt{\sec^2 t - 1}$$

$$tg\ t = \sqrt{\frac{x^2}{4} - 1}$$

$$tg\ t = \frac{\sqrt{x^2 - 4}}{2}$$

EXERCÍCIOS - LISTA VII

Calcule as seguintes integrais.

1) $I = \int \sqrt{1 - 4x^2}\, dx$

2) $I = \int \sqrt{x^2 + 1}\, dx$

3) $I = \int \sqrt{x^2 - 1}\, dx$

4) $I = \int \sqrt{25 - x^2}\, dx$

5) $I = \int \frac{\sqrt{1 - 3x^2}}{x^2}\, dx$

6) $I = \int \frac{\sqrt{4 - x^2}}{x^2}\, dx$

7) $I = \int \frac{\sqrt{9 - x^2}}{x^2}\, dx$

8) $I = \int \frac{x^2}{\sqrt{x^2 + 9}}\, dx$

9) $I = \int \frac{1}{\sqrt{x^2 + 9}}\, dx$

10) $I = \int \frac{1}{x\sqrt{x^2 + 1}}\, dx$

11) $I = \int \frac{1}{x\sqrt{4 - x^2}}\, dx$

12) $I = \int \frac{x^2}{\sqrt{9 - x^2}}\, dx$

13) $I = \int \frac{\sqrt{x^2 - 25}}{x^2}\, dx$

14) $I = \int \frac{x^2}{\sqrt{x^2 - 16}}\, dx$

15) $I = \int \frac{\sqrt{2x^2 - 1}}{x}\, dx$

16) $I = \int \frac{1}{x^3\sqrt{x^2 - 9}}\, dx$

17) $I = \int e^x \sqrt{1 + e^{2x}}\, dx$

18) $I = \int \frac{1}{(x^2 + 1)(x + \sqrt{x^2 + 1})}\, dx$

19) $I = \int \sqrt{x^2 + 2x + 2}\, dx$

20) $I = \int \sqrt{x^2 - 2x}\, dx$

21) $I = \int \frac{1}{(x^2 - 6x + 10)^{3/2}}\, dx$

22) $I = \int \sqrt{x^2 - 6x + 8}\, dx$

23) $I = \int \frac{x}{\sqrt{x^2 - 2x + 10}}\, dx$

INTEGRAIS DEFINIDAS

$$\boxed{\int_a^b f(x)\,dx = F(x)\Big|_a^b = F(b) - F(a)}$$

EXEMPLOS RESOLVIDOS. Calcule o valor das seguintes integrais.

1) $I = \displaystyle\int_0^1 \frac{3x}{2x^2 + 3}\,dx$ .... $d(2x^2 + 3) = 4x\,dx$

$$I = \frac{3}{4}\int_0^1 \frac{1}{2x^2 + 3}\,d(2x^2 + 3)$$

$$I = \frac{3}{4}\ln(2x^2 + 3)\Big|_0^1 = \frac{3}{4}\left[\ln(2.1^2 + 3) - \ln(2.0^2 + 3)\right]$$

$$\boxed{I = \frac{3}{4}\left[\ln 5 - \ln 3\right]} \quad \text{ou} \quad \boxed{I = \frac{3}{4}\ln\frac{5}{3}}$$

2) $I = \displaystyle\int_0^{\pi/2} \cos^3 x.\mathrm{sen}^2 x\,dx$

$I = \displaystyle\int_0^{\pi/2} \cos^2 x.\mathrm{sen}^2 x.\cos x\,dx$ .... $\mathrm{sen}\,x = t$

$\cos x\,dx = dt$

$I = \displaystyle\int_0^{\pi/2} (1 - \mathrm{sen}^2 x).\mathrm{sen}^2 x.\cos x\,dx$

Para $x = 0 \longrightarrow t = 0$

Para $x = \pi/2 \longrightarrow t = 1$

$$I = \int_0^1 (1 - t^2)\,t^2\,dt$$

$$I = \int_0^1 (t^2 - t^4)\,dt = \left(\frac{t^3}{3} - \frac{t^5}{5}\right)\Big|_0^1 = \left(\frac{1^3}{3} - \frac{1^5}{5}\right) - \left(\frac{0^3}{3} - \frac{0^5}{5}\right)$$

$$I = \frac{1}{3} - \frac{1}{5} - 0 = \frac{5 - 3}{15} \longrightarrow \boxed{I = \frac{2}{15}}$$

EXERCÍCIOS DE CÁLCULO II - LISTA VIII

Calcule o valor das seguintes integrais definidas

1) $I = \int_0^{\frac{\pi}{4}} \cos 2x \, dx$

2) $I = \int_1^2 \frac{x^2}{x^3 + 1} \, dx$

3) $I = \int_0^1 x^2\sqrt{6 - 2x^3} \, dx$

4) $I = \int_0^{\frac{\pi}{4}} \frac{\operatorname{sen} x}{\cos^2 x} \, dx$

5) $I = \int_0^1 e^x(3 + e^x)^4 \, dx$

6) $I = \int_0^1 \frac{x^3 + x + 1}{x^2 + 1} \, dx$

7) $I = \int_0^1 x \, e^{x+1} \, dx$

8) $I = \int_1^2 x \ln(x^2) \, dx$

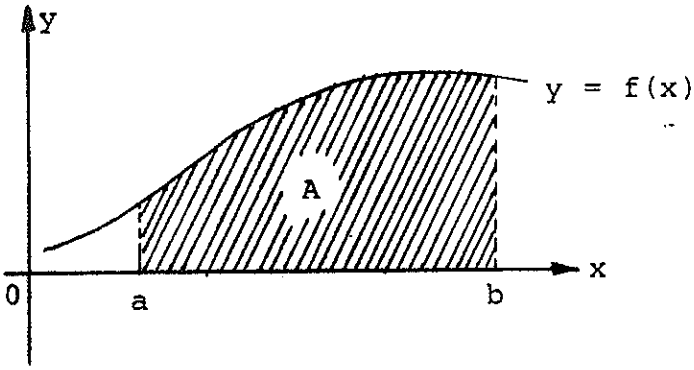
9) $I = \int_0^3 x \sqrt{x + 1} \, dx$

10) $I = \int_0^1 \frac{1}{x^2 - 2x - 3} \, dx$

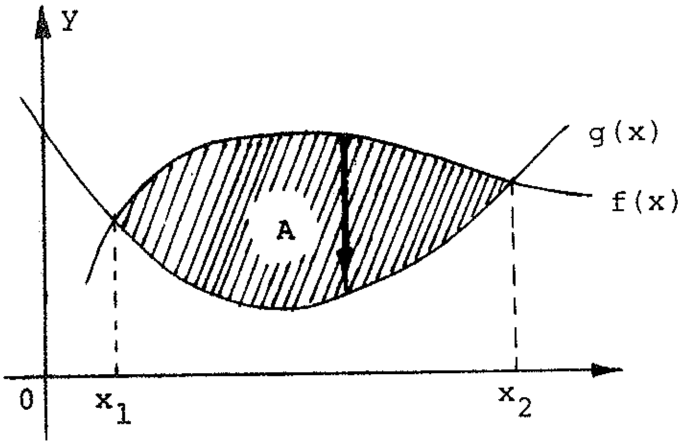
11) $I = \int_0^{\frac{\pi}{2}} \cos^3 x . \operatorname{sen}^8 x \, dx$

12) $I = \int_0^{\frac{\pi}{4}} \operatorname{tg}^3 x . \sec^5 x \, dx$

CÁLCULO DE ÁREAS



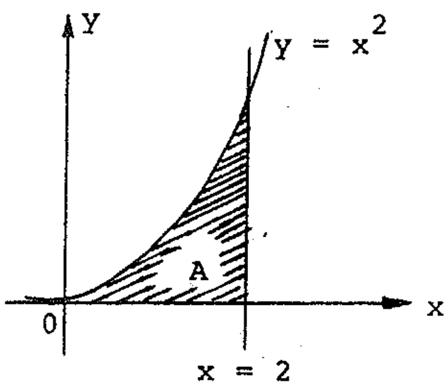
$$A = \int_a^b f(x)\,dx$$



$$A = \int_{x_1}^{x_2} \left[f(x) - g(x) \right] dx$$

EXEMPLOS RESOLVIDOS. Calcule a área da região hachurada das seguintes figuras.

1)



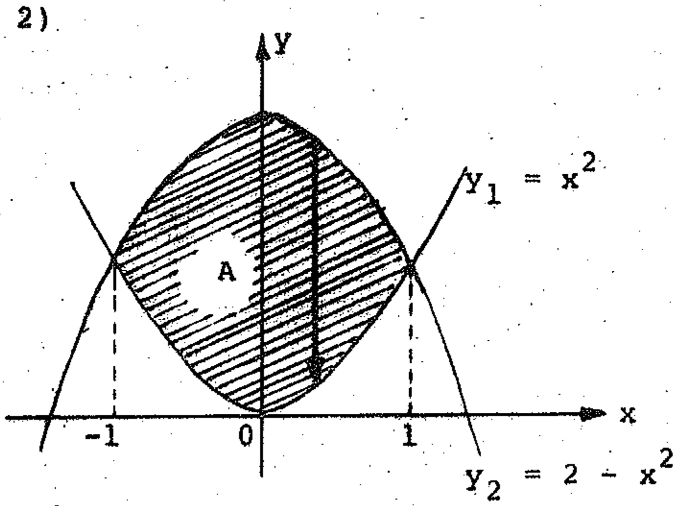
$$A = \int_0^2 x^2\,dx$$

$$A = \frac{x^3}{3}\Bigg|_0^2 = \frac{2^3}{3} - \frac{0^3}{3}$$

$$\boxed{A = \frac{8}{3}\ \text{u.a}}$$

u.a. = unidade de área

2)



Cálculo dos extremos

$$y_1 = y_2 \longrightarrow x^2 = 2 - x^2$$

$$x^2 + x^2 = 2$$

$$2x^2 = 2$$

$$x^2 = 1$$

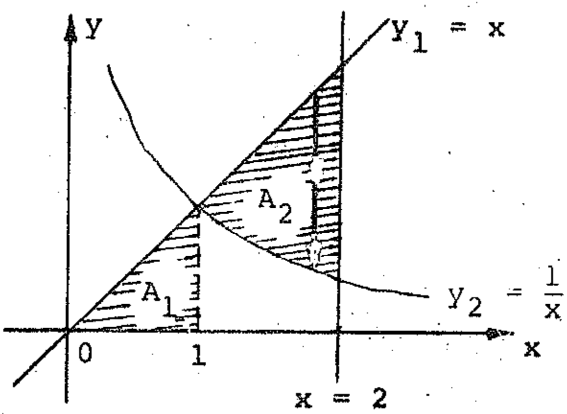
$$x = \pm 1 \begin{cases} x = 1 \\ x = -1 \end{cases}$$

$$A = \int_{-1}^{1} \left[y_2 - y_1\right] dx = \int_{-1}^{1} \left[(2 - x^2) - x^2\right] dx = \int_{-1}^{1} (2 - 2x^2)\, dx$$

$$A = \left(2x - \frac{2x^3}{3}\right)\Big|_{-1}^{1} = 2 - \frac{2}{3} - \left(-2 + \frac{2}{3}\right) = 2 - \frac{2}{3} + 2 - \frac{2}{3} = 4 - \frac{4}{3}$$

$$\boxed{A = \frac{8}{3}\ \text{u.a}}$$

3)



Cálculo dos extremos

$$y_1 = y_2 \longrightarrow x = \frac{1}{x}$$

$$x^2 = 1$$

$$x = \pm 1 \begin{cases} x = 1 \\ x = -1 \text{ NC} \end{cases}$$

$$A_T = A_1 + A_2$$

$$A_1 = \int_0^1 y_1 dx = \int_0^1 x\, dx = \frac{x^2}{2}\Big|_0^1 = \frac{1}{2} - 0 = \frac{1}{2} \longrightarrow \boxed{A_1 = \frac{1}{2}}$$

$$A_2 = \int_1^2 \left[y_1 - y_2\right] dx = \int_1^2 \left[x - \frac{1}{x}\right] dx = \left(\frac{x^2}{2} - \ln x\right)\Big|_1^2$$

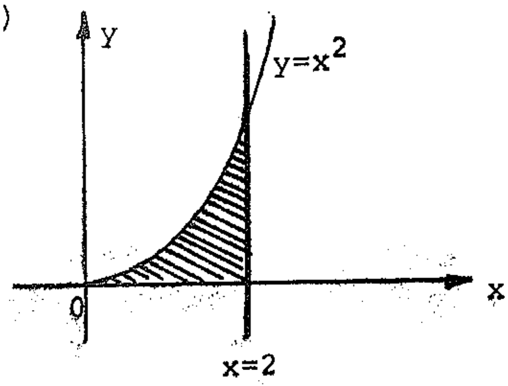
$$A_2 = \frac{4}{2} - \ell n\ 2 - \left(\frac{1}{2} - \ell n\ 1\right) = 2 - \ell n\ 2 - \frac{1}{2} \longrightarrow \boxed{A_2 = \frac{3}{2} - \ell n\ 2}$$

$$A_T = \frac{1}{2} + \frac{3}{2} - \ln 2 \qquad \boxed{A_T = (2 - \ln 2)\ \text{u.a.}}$$

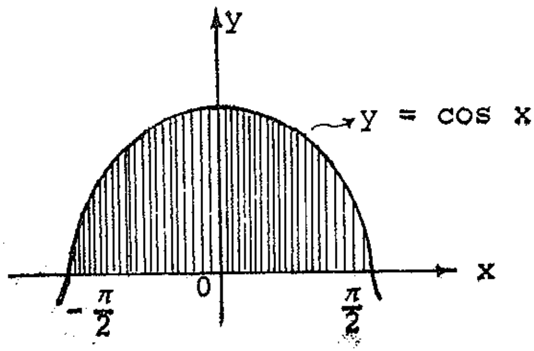
EXERCÍCIOS DE CÁLCULO II - LISTA IX

CALCULE A ÁREA DA PARTE HACHURADA DAS SEGUINTES FIGURAS.

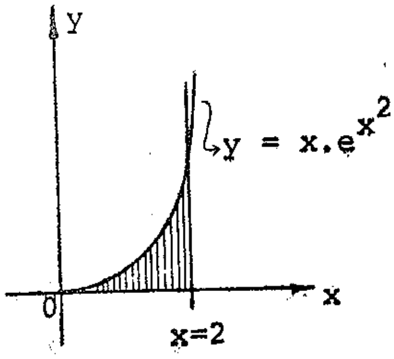
1)



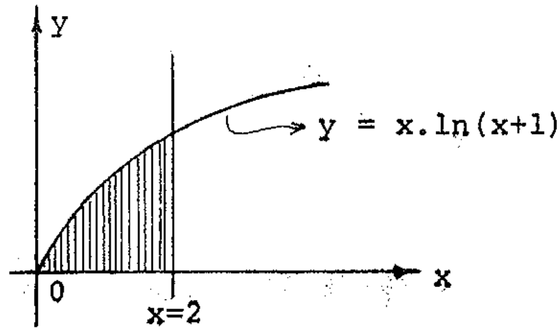
2)



3)



4)



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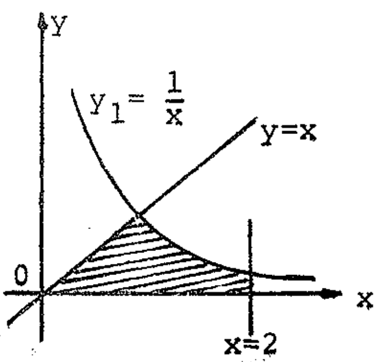

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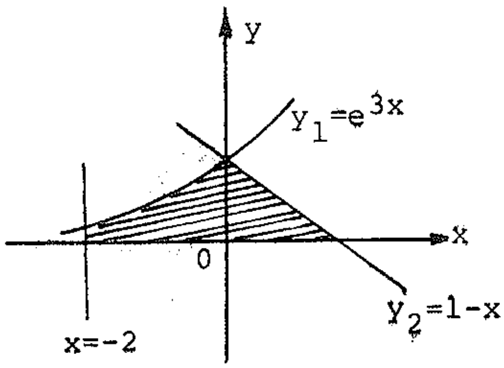
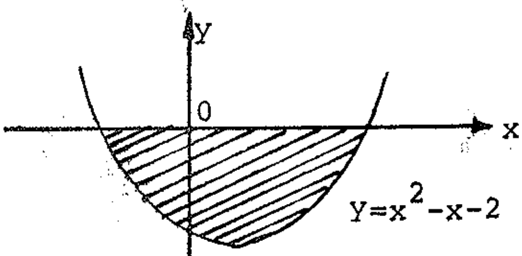


7)



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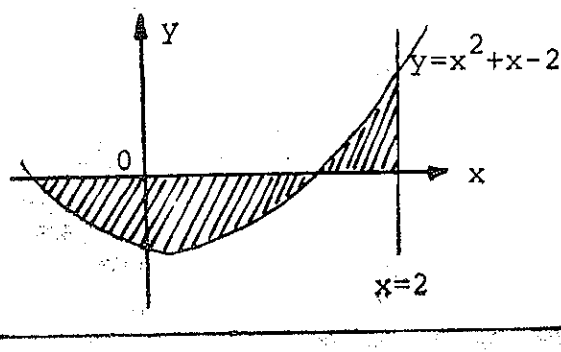
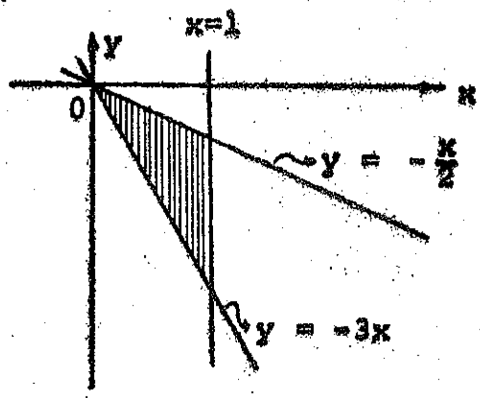
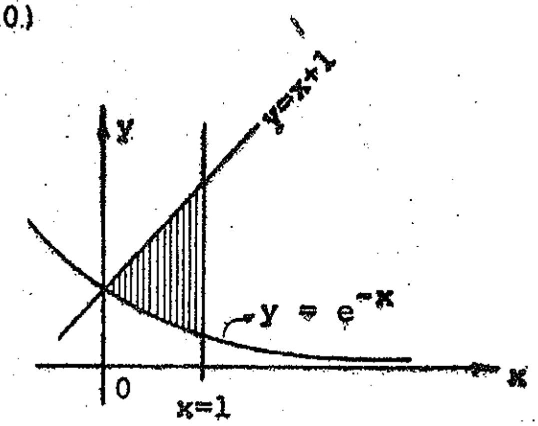


9)



10)



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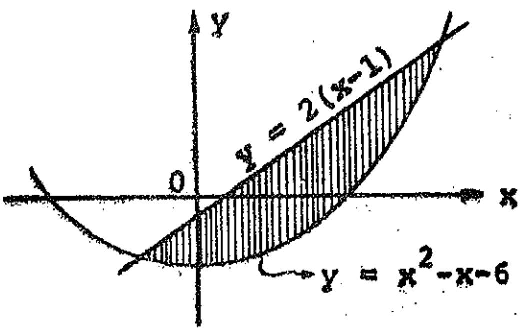

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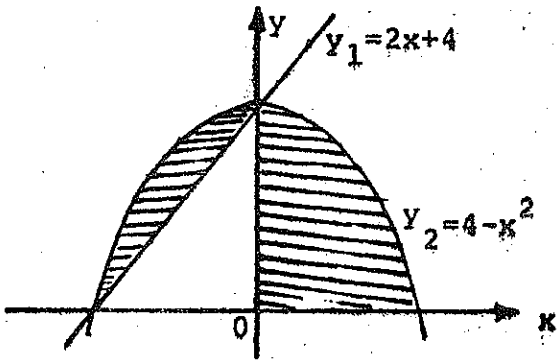
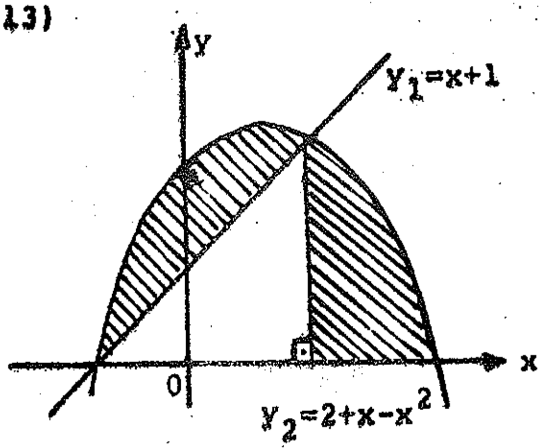
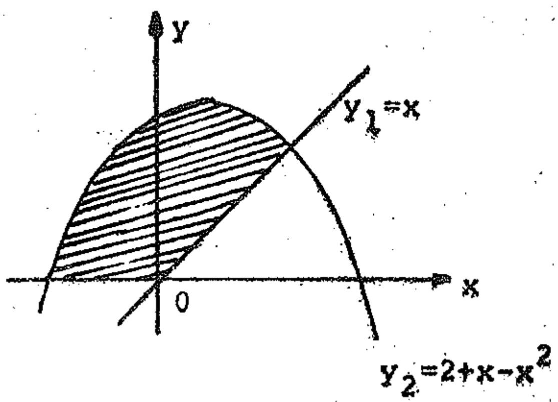


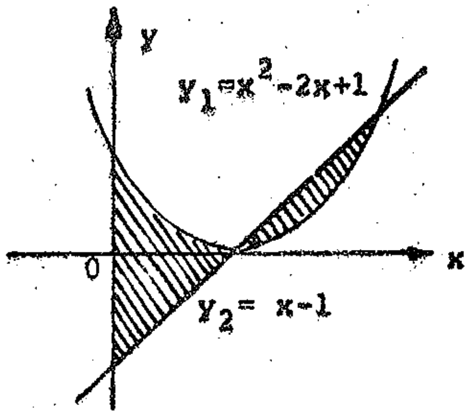
13)



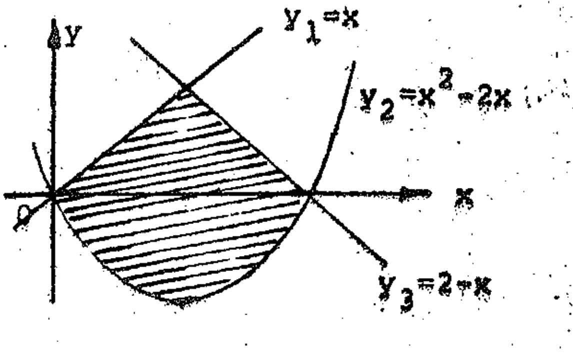
14)



15)



16)



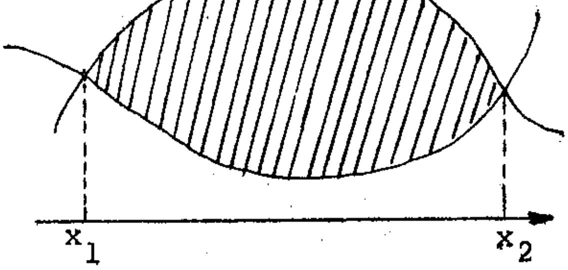
FORMULÁRIO DE DERIVADAS

DERIVADA DE ALGUMAS FUNÇÕES COMPOSTAS :

Se C é uma constante, real, $u = u(x)$ e $v = v(x)$ são funções deriváveis, então:

1. $f(x) = C \longrightarrow f'(x) = 0$
2. $f(x) = u^{\alpha},\ \alpha \in \mathbb{R} \longrightarrow f'(x) = \alpha\, u^{\alpha-1} . u'$
3. $f(x) = \sqrt{u} \longrightarrow f'(x) = \dfrac{u'}{2\sqrt{u}} = \dfrac{u'.\sqrt{u}}{2u}, (u > 0)$
4. $f(x) = \sqrt[m]{u} \longrightarrow f'(x) = \dfrac{u'}{m.\sqrt[m]{u^{m-1}}} = \dfrac{u'.\sqrt[m]{u}}{m.u}$, (m inteiro)
5. $f(x) = a^{u} \longrightarrow f'(x) = a^{u} . \ln a . u' \quad , (0 < a \neq 1)$
6. $f(x) = e^{u} \longrightarrow f'(x) = e^{u} . u'$
7. $f(x) = \log_a u \longrightarrow f'(x) = \dfrac{u'}{u.\ln a} = \dfrac{u'}{u}.\log_a e \ , (u > 0, 0 < a \neq 1)$
8. $f(x) = \ln u \longrightarrow f'(x) = \dfrac{u'}{u} \ , \ (u > 0)$
9. $f(x) = u^{v} \longrightarrow f'(x) = v . u^{v-1} . u' + u^{v} . \ln u . v'$
10. $f(x) = \operatorname{sen} u \longrightarrow f'(x) = \cos u \ . \ u'$
11. $f(x) = \cos u \longrightarrow f'(x) = -\operatorname{sen} u \ . \ u'$
12. $f(x) = \operatorname{tg} u \longrightarrow f'(x) = \sec^2 u \ . \ u'$
13. $f(x) = \operatorname{cotg} u \longrightarrow f'(x) = -\operatorname{cossec}^2 u \ . \ u'$
14. $f(x) = \sec u \longrightarrow f'(x) = \sec u \ . \ \operatorname{tg} u \ . \ u'$
15. $f(x) = \operatorname{cossec} u \longrightarrow f'(x) = -\operatorname{cossec} u \ . \ \operatorname{cotg} u \ . \ u'$
16. $f(x) = \operatorname{arc\,sen} u \longrightarrow f'(x) = \dfrac{u'}{\sqrt{1-u^2}}$
17. $f(x) = \operatorname{arc\,cos} u \longrightarrow f'(x) = \dfrac{-u'}{\sqrt{1-u^2}}$
18. $f(x) = \operatorname{arc\,tg} u \longrightarrow f'(x) = \dfrac{u'}{1+u^2}$
19. $f(x) = \operatorname{arc\,cotg} u \longrightarrow f'(x) = \dfrac{-u'}{1+u^2}$
20. $f(x) = \operatorname{arc\,sec} u \longrightarrow f'(x) = \dfrac{u'}{u\sqrt{u^2-1}}$
21. $f(x) = \operatorname{arc\,cossec} u \longrightarrow f'(x) = \dfrac{-u'}{u\sqrt{u^2-1}}$

ÁREA DA FIGURA LIMITADA PELAS CURVAS DE DUAS FUNÇÕES



$$A = \left| \int_{x_1}^{x_2} \left[f(x) - g(x) \right] dx \right|$$

EXEMPLO RESOLVIDO. Calcule a área da figura limitada pelas curvas das seguintes funções:

$$\begin{cases} y_1 = x^2 - 2x + 1 \\ y_2 = 3 - x \end{cases}$$

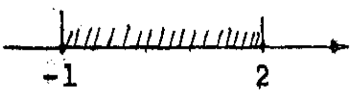
Cálculo dos extremos

$$y_1 = y_2 \longrightarrow x^2 - 2x + 1 = 3 - x$$

$$x^2 - x - 2 = 0$$

$$(x - 2)(x + 1) = 0$$

$$\begin{cases} x = 2 \\ x = -1 \end{cases}$$



$$A = \left| \int_{-1}^{2} \left[y_1 - y_2 \right] dx \right| = \left| \int_{-1}^{2} \left[(x^2 - 2x + 1) - (3 - x) \right] dx \right|$$

$$A = \left| \int_{-1}^{2} (x^2 - x - 2)\, dx \right| = \left| \left(\frac{x^3}{3} - \frac{x^2}{2} - 2x \right) \Big|_{-1}^{2} \right|$$

$$A = \left| \frac{8}{3} - \frac{4}{2} - 4 - \left(-\frac{1}{3} - \frac{1}{2} + 2 \right) \right| = \left| \frac{8}{3} - 2 - 4 + \frac{1}{3} + \frac{1}{2} - 2 \right|$$

$$A = \left| \frac{9}{3} - 8 + \frac{1}{2} \right| = \left| -5 + \frac{1}{2} \right| = \left| -\frac{9}{2} \right|$$

$$\boxed{A = \frac{9}{2} \text{ u.a.}}$$

EXERCÍCIOS DE CÁLCULO II - LISTA X

Calcule a área limitada pelas curvas das seguintes funções:

1) $\begin{cases} y_1 = x^2 - x \\ y_2 = x \end{cases}$

2) $\begin{cases} y_1 = x^2 - x + 6 \\ y_2 = 3x + 3 \end{cases}$

3) $\begin{cases} y_1 = x^2 - 3x + 1 \\ y_2 = -x^2 - 2x + 1 \end{cases}$

4) $\begin{cases} y_1 = x + 1 \\ y_2 = x^3 + 1 \end{cases}$

5) $\begin{cases} y_1 = x^3 - x \\ y_2 = x^2 + x \end{cases}$

6) $\begin{cases} y_1 = x^3 - x^2 + 5x \\ y_2 = 2x^2 + 3x \end{cases}$

7) $\begin{cases} y_1 = 2x^3 - 3x^2 + 5x - 2 \\ y_2 = x^3 - 3x^2 + 6x - 2 \end{cases}$

8) $\begin{cases} y_1 = \sqrt[3]{x} \\ y_2 = x^3 \end{cases}$

9) $\begin{cases} y_1 = x^4 + 1 \\ y_2 = 3x^2 + 5 \end{cases}$

10) $\begin{cases} y_1 = x^2(x - 1) \\ y_2 = (x - 1)(x + 2) \end{cases}$

11) $\begin{cases} y_1 = (x - 2)(2x^2 - 4x - 3) \\ y_2 = (x - 2)(x^2 - 3x - 3) \end{cases}$

CÁLCULO DO VALOR MÉDIO

$$\boxed{y_m = \frac{1}{b-a}\int_a^b f(x)\,dx}$$

EXEMPLO RESOLVIDO. Calcule o valor médio da função

$$f(x) = \frac{\sec^2 x}{(1 + 2\,tg\,x)^3}, \text{ no intevalo } \left[0, \frac{\pi}{4}\right].$$

$$y_m = \frac{1}{\frac{\pi}{4} - 0}\int_0^{\pi/4} \frac{\sec^2 x}{(1+2\,tg\,x)^3}\,dx \quad \ldots\ldots\ldots\ldots\ldots \quad \begin{array}{l} 1 + 2\,tg\,x = t \\ 2\sec^2 x\,dx = dt \end{array}$$

Para $x = 0 \longrightarrow t = 1$

Para $x = \pi/4 \longrightarrow t = 3$

$$y_m = \frac{4}{\pi}\cdot\frac{1}{2}\int_1^3 \frac{1}{t^3}\,dt$$

$$y_m = \frac{2}{\pi}\int_1^3 t^{-3}dt$$

$$y_m = \frac{2}{\pi}\left(\frac{t^{-2}}{-2}\right)\Bigg|_1^3 = -\frac{1}{\pi}\cdot\frac{1}{t^2}\Bigg|_1^3 = -\frac{1}{\pi}\left(\frac{1}{9} - \frac{1}{1}\right) = -\frac{1}{\pi}\left(\frac{1-9}{9}\right) = -\frac{1}{\pi}\left(-\frac{8}{9}\right)$$

$$\boxed{y_m = \frac{8}{9\pi}}$$

EXERCÍCIOS DE CÁLCULO II - LISTA XI

Calcule o valor médio das funções abaixo, nos intervalos dados

1) $y = x.\cos x^2$ .... $[0,\pi]$

2) $y = e^x(3 + e^x)$ .... $[0,1]$

3) $y = \dfrac{1}{x + x.\ln^2 x}$ .... $[1,e]$

4) $y = \dfrac{\text{sen } x}{(1 + 2\cos x)^3}$ .... $[0,\frac{\pi}{2}]$

5) $y = \dfrac{\text{sen } x}{\sqrt{1 + 3\cos x}}$ .... $[0,\frac{\pi}{2}]$

6) $y = \dfrac{1}{\cos^2 x.(1 + 2\,\text{tg } x)^3}$ .... $[0,\frac{\pi}{4}]$

7) $y = \dfrac{1}{x\,(1 + 2\ln x)^3}$ .... $[1,e]$

8) $y = \ln x^2$ .... $[1,e]$

9) $y = \ln(3x + 1)$ .... $[1,2]$

10) $y = x.\ln(x^2 + 1)$ .... $[0,1]$

11) $y = x^5.e^{x^3+1}$ .... $[0,1]$

12) $y = \dfrac{1}{\text{sen}^2 x.\cos^2 x}$ .... $[\frac{\pi}{4},\frac{\pi}{3}]$

COMPRIMENTO DO ARCO

$$\boxed{\ell = \int_a^b \sqrt{1 + y'^2}\, dx}$$

EXEMPLO RESOLVIDO. Calcule o comprimento do arco definido pela curva da função $f(x) = \frac{1}{3} x\sqrt{x} - \sqrt{x}$, no intervalo $[0, 3]$.

$$y = \frac{1}{3} x \sqrt{x} - \sqrt{x}$$

$$y = \frac{1}{3} x^{3/2} - x^{1/2} \longrightarrow y' = \frac{1}{3}\cdot\frac{3}{2} x^{1/2} - \frac{1}{2} x^{-1/2}$$

$$y' = \frac{1}{2} x^{1/2} - \frac{1}{2} x^{-1/2}$$

$$y' = \frac{1}{2}(x^{1/2} - x^{-1/2})$$

$$y'^2 = \frac{1}{4}(x - 2 + x^{-1})$$

Somando 1 em ambos os membros da igualdade temos

$$1 + y'^2 = 1 + \frac{x}{4} - \frac{2}{4} + \frac{x^{-1}}{4}$$

$$1 + y'^2 = \frac{x}{4} + \frac{2}{4} + \frac{x^{-1}}{4} = \frac{1}{4}(x + 2 + x^{-1})$$

$$1 + y'^2 = \frac{1}{4}(x^{1/2} + x^{-1/2})^2$$

$$\sqrt{1 + y'^2} = \frac{1}{2}(x^{1/2} + x^{-1/2})$$

$$\ell = \int_0^3 \frac{1}{2}(x^{1/2} + x^{-1/2})\, dx = \frac{1}{2}\left(\frac{x^{3/2}}{\frac{3}{2}} + \frac{x^{1/2}}{\frac{1}{2}}\right)\Bigg|_0^3 = \frac{1}{\cancel{2}}\left(\frac{\cancel{2}x\sqrt{x}}{3} + \cancel{2}\sqrt{x}\right)\Bigg|_0^3$$

$$\ell = \frac{3\sqrt{3}}{3} + \sqrt{3} - 0 = \sqrt{3} + \sqrt{3} = 2\sqrt{3}$$

$$\boxed{\ell = 2\sqrt{3}\ \text{u.c.}}$$

u.c. = unidade de comprimento

EXERCÍCIOS DE CÁLCULO II - LISTA XII

Calcule a medida do comprimento do arco, nos intervalos dados

1) $y = \frac{2}{3}\, x\sqrt{x}$ .... $[0,1]$

2) $y = x^{3/2}$ .... $[2,3]$

3) $y = \frac{2}{3}\,(2x + 1)^{3/2}$ .... $[0,1]$

4) $y = \frac{x^4}{8} + \frac{1}{4x^2}$ .... $[1,2]$

5) $y = \frac{x^3}{6} + \frac{1}{2x}$ .... $[1,2]$

6) $y = \frac{1}{3}\,(x-3)\sqrt{x}$ .... $[0,3]$

7) $y = \frac{1}{2}\,(e^x + e^{-x})$ .... $[0,1]$

8) $y = \ln(x^2 - 1)$ .... $[2,3]$

9) $y = \frac{1}{4}\,x^2 - \frac{1}{2}\ln x$ .... $[1,e]$

10) $y = \frac{1}{2}(\frac{x^3}{3} + x - \text{arctg}\ x)$ $[0,1]$

11) $y = \frac{1}{2}\,(\frac{1}{2}\,x^2 + x - \ln(x + 1)$ .. $[0,1]$

12) $y = \ln(\sec x)$ .... $[0,\frac{\pi}{3}]$

13) $y = \ln(\cos x)$ .... $[0,\frac{\pi}{4}]$

14) $y = \ln(\text{cosec}\ x)$ .... $[\frac{\pi}{4},\frac{\pi}{2}]$

15) $y = \frac{2}{3}(x^2 + 1)^{3/2}$.............. $[0,1]$

16) $y = \frac{\sqrt{3}}{9}(3x^2 + 2)^{3/2}$............ $[1,2]$

17) $y = \sqrt{1 - x^2}$ .... $[0,\frac{1}{2}]$

ÁREA DA SUPERFÍCIE DE REVOLUÇÃO

$$\boxed{S = 2\pi \int_a^b y \sqrt{1 + y'^2}\, dx}$$

EXEMPLO RESOLVIDO. Calcule a área de superfície de revolução definida pela curva da função $y = \sqrt{2x - 1}$, no intervalo $[1, 4]$, ao girar em torno do eixo dos x.

$$y = \sqrt{2x - 1}$$

$$y = (2x - 1)^{1/2}$$

$$y' = \frac{1}{\cancel{2}} (2x - 1)^{-1/2} . \cancel{2}$$

$$y' = \frac{1}{\sqrt{2x - 1}}$$

$$y'^2 = \frac{1}{2x - 1}$$

$$1 + y'^2 = 1 + \frac{1}{2x - 1} = \frac{2x - 1 + 1}{2x - 1} = \frac{2x}{2x - 1}$$

$$\sqrt{1 + y'^2} = \sqrt{\frac{2x}{2x - 1}}$$

$$\sqrt{1 + y'^2} = \frac{\sqrt{2x}}{\sqrt{2x - 1}}$$

$$S = 2\pi \int_1^4 \cancel{\sqrt{2x - 1}} . \frac{\sqrt{2x}}{\cancel{\sqrt{2x - 1}}} dx = 2\pi \int_1^4 \sqrt{2x}\, dx = 2\sqrt{2}\,\pi \int_1^4 x^{1/2}\, dx$$

$$S = 2\sqrt{2}\,\pi \left. \frac{x^{3/2}}{3/2} \right|_1^4 = \left. \frac{4\sqrt{2}\pi}{3} x\sqrt{x} \right|_1^4 = \frac{4\sqrt{2}\pi}{3}(4.2 - 1.1) = \frac{28\sqrt{2}\pi}{3}$$

$$\boxed{S = \frac{28\sqrt{2}\,\pi}{3} \text{ u.a.}}$$

EXERCÍCIOS DE CÁLCULO II - LISTA XIII

Calcule a área da superfície de revolução, nos intervalos dados

1) $y = \sqrt{3}\,x + 1$ .... $[0, \sqrt{3}]$

2) $y = \frac{1}{3}x^3$ .... $[1, 2]$

3) $y = x^3$ .... $[0, 3]$

4) $y = \sqrt{2 + 5x}$ .... $[0, 1]$

5) $y = \sqrt{2 + 3x}$ .... $[-\frac{1}{12}, \frac{2}{3}]$

6) $y = \sqrt{1 + 2x}$ .... $[1, 7]$

7) $y = \sqrt{x + 1}$ .... $[1, 5]$

8) $y = \sqrt{2x + 4}$ .... $[2, 10]$

9) $y = \dfrac{e^x + e^{-x}}{2}$ .... $[0, 1]$

10) $y = \frac{1}{4}(x^2 - 2\ln x)$ .... $[1, 2]$

11) $y = \sqrt{1 + e^x}$ .... $[0, 1]$

12) $y = \sqrt{1 - x^2}$ .... $[0, \frac{1}{2}]$

13) $y = \frac{x^3}{6} + \frac{1}{2x}$ .... $[1, 2]$

14) $y = \frac{x^4}{8} + \frac{1}{4x^2}$ .... $[1, 2]$

CÁLCULO DO VOLUME DO SÓLIDO DE REVOLUÇÃO

$$\boxed{V = \pi \int_a^b y^2 . dx}$$

EXEMPLO RESOLVIDO. Calcule o volume do sólido de revolução gerado pela curva da funçao $f(x) = \sqrt{x . \ln(x + 1)}$ no intervalo $[0, 1]$

$$V = \pi \int_0^1 \left[\sqrt{x . \ln(x + 1)} \right]^2 dx$$

$$V = \pi \int_0^1 x . \ln(x + 1) \; dx \qquad \ldots\ldots \begin{cases} u = \ln(x + 1) \longrightarrow u' = \dfrac{1}{x + 1} \\ v = \int x \, dx \longrightarrow v = \dfrac{x^2}{2} \end{cases}$$

$$V = \pi \left[\ln(x + 1) . \frac{x^2}{2} \Big|_0^1 - \int_0^1 \frac{1}{x + 1} . \frac{x^2}{2} \, dx \right]$$

$$V = \pi \left[\frac{x^2}{2} . \ln(x + 1) \Big|_0^1 - \frac{1}{2} \int_0^1 \frac{x^2}{x + 1} \, dx \right] \quad \ldots\ldots$$

$$\begin{array}{r|l} x^2 & x + 1 \\ \cline{2-2} -x^2 - x & x - 1 \\ \cline{1-1} - x & \\ x - 1 & \\ \cline{1-1} - 1 & \end{array}$$

$$V = \pi \left[\frac{x^2}{2} . \ln(x + 1) \Big|_0^1 - \frac{1}{2} \int_0^1 (x - 1 + \frac{-1}{x + 1}) \, dx \right]$$

$$V = \pi \left[\frac{x^2}{2} . \ln(x + 1) - \frac{1}{2} \left[\frac{x^2}{2} - x - \ln(x + 1) \right] \right] \Big|_0^1$$

$$V = \frac{\pi}{2} \left[x^2 . \ln(x + 1) - \frac{x^2}{2} + x + \ln(x + 1) \right] \Big|_0^1$$

$$V = \frac{\pi}{2} \left[1 . \ln(1 + 1) - \frac{1}{2} + 1 + \ln(1 + 1) - (0 - 0 + 0 + \ln(0 + 1)) \right]$$

$$V = \frac{\pi}{2} \left[\ln 2 + \frac{1}{2} + \ln 2 \right]$$

$$V = \frac{\pi}{2} \left[2 \ln 2 + \frac{1}{2} \right]$$

$$\boxed{V = \frac{\pi}{4} \left[4 \ln 2 + 1 \right] \text{ u.v.}}$$ u.v. = unidade de volume

EXERCÍCIOS DE CÁLCULO II - LISTA XIV

Calcule o volume do sólido de revolução, nos intervalos dados

1) $y = (2x - 3)^6$ .... $[0,1]$

2) $y = e^x + 1$ .... $[0,1]$

3) $y = \sec x . \sqrt{\operatorname{tg} x}$ .... $\left[0,\frac{\pi}{4}\right]$

4) $y = x . e^{x^3}$ .... $[1,2]$

5) $y = \sqrt{x(e^x + 1)}$ .... $[0,1]$

6) $y = e^x \sqrt{e^x + 1}$ .... $[0,1]$

7) $y = x^4 (x^9 + 1)^{3/2}$ .... $[0,1]$

8) $y = \dfrac{1}{\sqrt[4]{x + x\sqrt{x}}}$ .... $[1,9]$

9) $y = \dfrac{1}{\sqrt[4]{x^2 + x^2 \ln x}}$ .... $[1,e^3]$

10) $y = \left(\dfrac{e^{3x} + e^{2x}}{9}\right)^{1/4}$ .... $[0,1]$

11) $y = x^{3/2}(x^2 + 1)^{1/4}$ .... $[0,\sqrt{3}]$

12) $y = \sqrt{x . \operatorname{arctg} x}$ .... $[0,1]$

13) $y = \sqrt{3x} . \sec(2x)$ .... $\left[0,\frac{\pi}{8}\right]$

14) $y = \dfrac{1}{\sqrt{x^2 - 16}}$ .... $[5,6]$

15) $y = \dfrac{1}{\sqrt[4]{x + 1} . \sqrt{x + 10}}$ .... $[2,8]$

16) $y = e^{\frac{1}{2}\sqrt{x}}$ .... $[0,1]$

17) $y = \dfrac{1}{x}\sqrt{\dfrac{x^2 + 1}{x + 1}}$ .... $[1,2]$

18) $y = \sqrt{\dfrac{2x^2 + 3}{(x-3)^2 . (x-1)}}$ .... $[4,5]$

CURVATURA E RAIO DE CURVATURA

$$\boxed{K = \frac{y''}{(1 + y'^2)^{3/2}}} \qquad \boxed{R = \left| \frac{1}{K} \right|}$$

EXEMPLO RESOLVIDO. Calcule a curvatura e o raio de curvatura no ponto $x_0 = 1$ da função $f(x) = \text{arctg}\, x^2$.

$$f(x) = \text{arctg}\, x^2$$

$$f'(x) = \frac{2x}{1 + x^4} \longrightarrow f'(1) = \frac{2.1}{1 + 1} = \frac{2}{2} = 1$$

$$f''(x) = \frac{2(1 + x^4) - 2x.4x^3}{(1 + x^4)^2} \longrightarrow f''(1) = \frac{2.2 - 2.4}{2^2} = \frac{4 - 8}{4} = -1$$

$$K = \frac{-1}{(1 + 1)^{3/2}} = \frac{-1}{2^{3/2}} = \frac{-1}{2\sqrt{2}} \longrightarrow \boxed{K = -\frac{1}{2\sqrt{2}}}$$

$$R = \left| \frac{1}{\frac{-1}{2\sqrt{2}}} \right| = 2\sqrt{2} \longrightarrow \boxed{R = 2\sqrt{2}}$$

EXERCÍCIOS DE CÁLCULO II - LISTA XV

Calcule a curvatura e o raio de curvatura, nos pontos dados

1) $y = e^{-x}$ .... $x = 1$

2) $y = \ln 2x$ .... $x = 1$

3) $y = x + \frac{1}{x}$ .... $x = 1$

4) $y = x.e^{\operatorname{sen} x}$ .... $x = \frac{\pi}{2}$

5) $y = e^{-x^2} + 3$ .... $x = 1$

6) $y = e^{2x}. \ln(\operatorname{sen} x)$ .... $x = \frac{\pi}{4}$

7) $y = 3^{\operatorname{arcsen} x}$ .... $x = 0$

8) $y = e^{x^2}$ .... $x = 0$

9) $y = e^{2^x}$ .... $x = 0$

10) $y = e^{\operatorname{sen} 3x}$ .... $x = \frac{\pi}{6}$

11) $y = \operatorname{arctg} 3x$ .... $x = 0$

12) $y = e^{\frac{x}{\pi}}. \operatorname{sen} \frac{x}{2}$ .... $x = \pi$

13) $y = x.\ln(2x + 1)$ .... $x = 0$

14) $y = \ln(\sec 3x)$ .... $x = \frac{\pi}{12}$

EQUAÇÃO DA RETA TANGENTE E NORMAL NA FORMA PARAMÉTRICA

Equação da reta tangente → $\boxed{y - y_0 = \frac{\dot{y}}{\dot{x}}(x - x_0)}$

Equação da reta normal → $\boxed{y - y_0 = -\frac{\dot{x}}{\dot{y}}(x - x_0)}$

EXEMPLO RESOLVIDO. Calcule a equação da reta tangente e reta normal no ponto $t_0 = 0$, a curva definida por

$$\begin{cases} x = t^4 + 2t + 1 \\ y = t.e^{2t} \end{cases}$$

$x = t^4 + 2t + 1$	$y = t.\ e^{2t}$
$\dot{x} = 4t^3 + 2$	$\dot{y} = 1.e^{2t} + t.e^{2t}.2$

No ponto $t_0 = 0$, temos

$x_0 = 0^4 + 2.0 + 1 = 1$

$y_0 = 0.e^0 = 0$

$\dot{x}(0) = 4.0^3 + 2 = 2$

$\dot{y}(0) = e^0 + 0.e^0.2 = 1$

Reta tangente $\quad y - 0 = \frac{1}{2}(x - 1) \longrightarrow \boxed{y = \frac{1}{2}(x - 1)}$

Reta normal $\quad y - 0 = -\frac{2}{1}(x - 1) \longrightarrow \boxed{y = -2(x - 1)}$

EXERCÍCIOS DE CÁLCULO II - LISTA XVI

Determine a equação da reta tangente e da reta normal, as seguintes curvas nos pontos dados

1) $\begin{cases} x = e^{t} \\ y = 2e^{-t}, \text{ no ponto } t = 0 \end{cases}$

2) $\begin{cases} x = t^{2} - 1 \\ y = 2e^{t}, \text{ no ponto } t = -1 \end{cases}$

3) $\begin{cases} x = e^{2t} \\ y = 2 + t^{2}, \text{ no ponto } (x,y) = (e^{2},3) \end{cases}$

4) $\begin{cases} x = t^{4} + 1 \\ y = t^{8} + t, \text{ no ponto } t = 1 \end{cases}$

COMPRIMENTO DO ARCO NA FORMA PARAMÉTRICA

$$\boxed{\ell = \int_{t_1}^{t_2} \sqrt{\dot{x}^2 + \dot{y}^2}\, dt}$$

EXEMPLO RESOLVIDO. Calcule o comprimento do arco definido pela curva

$$\begin{cases} x = \sqrt{2}\, t^2 + 1 \\ y = \frac{1}{3} t^3 - 2t + e \end{cases}, \text{ no intervalo } [1,\ 2]$$

$x = \sqrt{2}\, t^2 + 1$	$y = \frac{1}{3} t^3 - 2t + e$
$\dot{x} = 2\sqrt{2}\, t$	$\dot{y} = t^2 - 2$
$\dot{x}^2 = 8\, t^2$	$\dot{y}^2 = t^4 - 4\, t^2 + 4$

$$\dot{x}^2 + \dot{y}^2 = 8\, t^2 + t^4 - 4\, t^2 + 4$$

$$\dot{x}^2 + \dot{y}^2 = t^4 + 4\, t^2 + 4$$

$$\dot{x}^2 + \dot{y}^2 = (t^2 + 2)^2$$

$$\sqrt{\dot{x}^2 + \dot{y}^2} = (t^2 + 2)$$

$$\ell = \int_1^2 (t^2 + 2)\, dt$$

$$\ell = \left(\frac{t^3}{3} + 2\, t\right)\Big|_1^2$$

$$\ell = \frac{8}{3} + 4 - \left(\frac{1}{3} + 2\right) = \frac{8}{3} + 4 - \frac{1}{3} - 2 = \frac{7}{3} + 2 = \frac{13}{3}$$

$$\boxed{\ell = \frac{13}{3} \text{ u.c.}}$$

EXERCÍCIOS DE CÁLCULO II - LISTA XVII

Calcule o comprimento do arco das seguintes curvas, nos intervalos dados

1) $\begin{cases} x = e^t - t + 3 \\ y = 4(e^{1/2\, t} - 1) \end{cases}$ .... $t \in [0,1]$

2) $\begin{cases} x = t^3 \\ y = 2t^2 \end{cases}$ .... $t \in [0,1]$

3) $\begin{cases} x = \frac{1}{5}t^5 - 2t + 7 \\ y = \frac{2\sqrt{2}}{3}t^3 + 1 \end{cases}$ .... $t \in [0,1]$

4) $\begin{cases} x = \frac{1}{3}t^3 - 2t \\ y = \sqrt{2}\, t^2 + 1 \end{cases}$ .... $t \in [0,1]$

5) $\begin{cases} x = \frac{1}{3}t^3 - 3t + 1 \\ y = \sqrt{3}\, t^2 \end{cases}$ .... $t \in [0,1]$

6) $\begin{cases} x = \sqrt{2}\, t^2 + 1 \\ y = \frac{1}{3}t^3 - 2t + 3e \end{cases}$ .... $t \in [1,2]$

7) $\begin{cases} x = t^2 - et + 1 \\ y = \frac{4}{3}\sqrt{2e}\, t^{3/2} + 5 \end{cases}$ .... $t \in [0,1]$

8) $\begin{cases} x = e^t \operatorname{sen} t \\ y = e^t \cos t \end{cases}$ .... $t \in [0,\frac{\pi}{2}]$

9) $\begin{cases} x = 3(\cos t + t \operatorname{sen} t) \\ y = 3(\operatorname{sen} t - t \cos t) \end{cases}$ .... $t \in [0,3]$

10) $\begin{cases} x = e^t(\operatorname{sen} t + \cos t) \\ y = e^t(\operatorname{sen} t - \cos t) \end{cases}$ .... $t \in [0,\frac{\pi}{2}]$

11) $\begin{cases} x = 2 - \operatorname{sen} \frac{t}{2} \\ y = \cos \frac{t}{2} - 1 \end{cases}$ .... $t \in [0,8]$

12) $\begin{cases} x = 2 + \operatorname{sen} 2t \\ y = 4 - \cos 2t \end{cases}$ .... $t \in [0,\frac{\pi}{2}]$

13) $\begin{cases} x = r(t - \text{sen } t) \\ y = r(1 - \cos t) \end{cases}$ (CICLÓIDE) .... $t \in [0,2\pi]$

14) $\begin{cases} x = 2 \cos^3 t \\ y = 2 \text{ sen}^3 t \end{cases}$ (ASTRÓIDE) .... Comprimento total

15) $\begin{cases} x = a \cos^3 t \\ y = a \text{ sen}^3 t \end{cases}$ (ASTRÓIDE) .... Comprimento total

16) $\begin{cases} x = \frac{1}{7} t^7 - t \\ y = \frac{t^4 + 3e}{2} \end{cases}$ .... $t \in [0,1]$

17) $\begin{cases} x = 2 \ln t \\ y = t + \frac{1}{t} \end{cases}$ .... $t \in [1,2]$

18) $\begin{cases} x = 2t \\ y = \ln(\cos 2t) \end{cases}$ .... $t \in [0,\frac{\pi}{8}]$

19) $\begin{cases} x = \frac{2}{5} t^{5/2} + 1 \\ y = \frac{1}{2} t^2 + 2 \end{cases}$ .... $t \in [0,3]$

20) $\begin{cases} x = t^2 + 5 \\ y = \frac{1}{6}(4t + 1)\sqrt{4t + 1} \end{cases}$ .... $t \in [0,1]$

21) $\begin{cases} x = \ln t - \frac{1}{2} t^2 \\ y = 2t \end{cases}$ .... $t \in [1,2]$

ÁREA DA SUPERFÍCIE DE REVOLUÇÃO NA FORMA PARAMÉTRICA

$$\boxed{S = 2\pi\int_{t_1}^{t_2} y.\sqrt{\dot{x}^2 + \dot{y}^2}\,dt}$$

EXEMPLO RESOLVIDO. Calcule a área da superfície de revolução definida por $\begin{cases} x = \frac{1}{7}t^7 - t + 2 \\ y = \frac{1}{2}t^4 \end{cases}$, no intervalo $[0, 1]$.

$$x = \frac{1}{7}t^7 - t + 2 \qquad y = \frac{1}{2}t^4$$

$$\dot{x} = t^6 - 1 \qquad \dot{y} = 2t^3$$

$$\dot{x}^2 = t^{12} - 2t^6 + 1 \qquad \dot{y}^2 = 4t^6$$

$$\dot{x}^2 + \dot{y}^2 = t^{12} - 2t^6 + 1 + 4t^6$$

$$\dot{x}^2 + \dot{y}^2 = t^{12} + 2t^6 + 1$$

$$\dot{x}^2 + \dot{y}^2 = (t^6 + 1)^2$$

$$\sqrt{\dot{x}^2 + \dot{y}^2} = (t^6 + 1)$$

$$S = 2\pi\int_0^1 \frac{1}{2}t^4.(t^6 + 1)dt$$

$$S = \pi\int_0^1 (t^{10} + t^4)dt$$

$$S = \pi\left(\frac{t^{11}}{11} + \frac{t^5}{5}\right)\Big|_0^1 = \pi\left(\frac{1}{11} + \frac{1}{5} - 0\right) = \pi\left(\frac{5 + 11}{55}\right) = \frac{16\pi}{55}$$

$$\boxed{S = \frac{16\pi}{55}\ \text{u.a.}}$$

EXERCÍCIOS DE CÁLCULO II - LISTA XVIII

Calcule a área da superfície de revolução, nos intervalos dados

1) $\begin{cases} x = 4 + 3t^2 \\ y = 6 - 4t^2 \end{cases}$ .... $t \in [0,1]$

2) $\begin{cases} x = 6 + 4 \operatorname{sen} \frac{t}{2} \\ y = 1 + 4 \cos \frac{t}{2} \end{cases}$ .... $t \in [0,\frac{\pi}{2}]$

3) $\begin{cases} x = \frac{1}{7} t^7 - t + 3e \\ y = \frac{1}{2} t^4 \end{cases}$ .... $t \in [0,1]$

4) $\begin{cases} x = e^t \operatorname{sen} t \\ y = e^t \cos t \end{cases}$ .... $t \in [0,\frac{\pi}{2}]$

5) $\begin{cases} x = \frac{1}{3} t^3 - t \\ y = t^2 \end{cases}$ .... $t \in [1,2]$

6) $\begin{cases} x = a \cos t \\ y = a \operatorname{sen} t \end{cases}$ .... $t \in [0,\frac{\pi}{2}]$

7) $\begin{cases} x = 2 \ln t \\ y = t + t^{-1} \end{cases}$ .... $t \in [1,e]$

8) $\begin{cases} x = t + 1 \\ y = t^3 \end{cases}$ .... $t \in [1,2]$

9) $\begin{cases} x = \frac{1}{3} t^3 - 1 \\ y = \frac{1}{2} t^2 \end{cases}$ .... $t \in [0,\sqrt{3}]$

10) $\begin{cases} x = e^t \\ y = e^t + 1 \end{cases}$ .... $t \in [0,1]$

11) $\begin{cases} x = t - \frac{1}{2} e^{2t} \\ y = 2 e^t \end{cases}$ .... $t \in [0,1]$

12) $\begin{cases} x = \frac{2}{3} t^{3/2} \\ y = t \end{cases}$ .... $t \in [0,5]$

VOLUME DO SÔLIDO DE REVOLUÇÃO NA FORMA PARAMÉTRICA

$$\boxed{V = \pi \int_{t_1}^{t_2} y^2 . \dot{x} \; dx}$$

EXEMPLO RESOLVIDO. Calcule o volume do sôlido de revolução definido

$$\text{por} \quad \begin{cases} x = t + \frac{1}{2}\, e^{2t} \\ y = 2\, e^{t} \end{cases} \quad , \text{ para } t \in [0,\ 1]$$

$$x = t + \frac{1}{2}\, e^{2t}$$

$$\dot{x} = 1 + e^{2t}$$

$$V = \pi \int_0^1 (2e^t)^2 . (1 + e^{2t}) dt = \pi \int_0^1 4\, e^{2t}(1 + e^{2t})\; dt$$

$$V = 4\pi \int_0^1 (e^{2t} + e^{4t})\; dt = 4\pi \left[\int_0^1 e^{2t}\; dt + \int_0^1 e^{4t} dt \right] \quad \begin{cases} d(2t) = 2\; dt \\ d(4t) = 4\; dt \end{cases}$$

$$V = 4\pi \left[\frac{1}{2} \int_0^1 e^{2t} d(2t) + \frac{1}{4} \int_0^1 e^{4t} d(4t) \right]$$

$$V = 4\pi \left[\frac{1}{2}\, e^{2t} + \frac{1}{4}\, e^{4t} \right] \Bigg|_0^1$$

$$V = 4\pi \left[\frac{1}{2}\, e^2 + \frac{1}{4}\, e^4 - \left(\frac{1}{2} + \frac{1}{4}\right) \right] = 4\pi \left[\frac{1}{2}\, e^2 + \frac{1}{4}\, e^4 - \frac{1}{2} - \frac{1}{4} \right]$$

$$V = 4\pi \left[\frac{e^2}{2} - \frac{e^4}{4} - \frac{3}{4} \right] = 4\pi \left[\frac{2e^2 - e^4 - 3}{4} \right]$$

$$\boxed{V = \pi (e^4 + 2e^2 - 3) \text{ u.v.}}$$

EXERCÍCIOS DE CÁLCULO II - LISTA XIX

Calcule o volume do sólido de revolução, nos intervalos dados

1) $\begin{cases} x = \text{arctg}\, t \\ y = \sqrt{t} \end{cases}$ .... $t \in [0,1]$

2) $\begin{cases} x = 2t - t^2 \\ y = \dfrac{1}{\sqrt{1 - t^2}} \end{cases}$ .... $t \in [0,\frac{1}{2}]$

3) $\begin{cases} x = \text{arctg}\, t \\ y = t \end{cases}$ .... $t \in [0,1]$

4) $\begin{cases} x = t^2 + 5 \\ y = \sqrt{\cos t} \end{cases}$ .... $t \in [0,\frac{\pi}{2}]$

5) $\begin{cases} x = \sqrt{e^t + 1} \\ y = 1 \end{cases}$ .... $t \in [0,1]$

6) $\begin{cases} x = \frac{1}{3} t^3 + 1 \\ y = \frac{1}{t} e^t \end{cases}$ .... $t \in [1,2]$

7) $\begin{cases} x = 2\sqrt{t + 1} \\ y = t \end{cases}$ .... $t \in [0,3]$

8) $\begin{cases} x = t.\text{sen}\, t + \cos t \\ y = \sqrt{\sec t} \end{cases}$ .... $t \in [0,\frac{\pi}{4}]$

9) $\begin{cases} x = \frac{1}{2} t^2 + 3 \\ y = \sqrt[4]{2t + 1} \end{cases}$ .... $t \in [0,4]$

10) $\begin{cases} x = \text{sen}\, 2t + 7 \\ y = (\text{sen}\, 2t)^{3/2} \end{cases}$ .... $t \in [0,\frac{\pi}{4}]$

11) $\begin{cases} x = t^2 \\ y = \sqrt{\dfrac{t + 1}{2t^2 + t}} \end{cases}$ .... $t \in [1,2]$

12) $\begin{cases} x = \ln(\sec t) \\ y = \sec t \end{cases}$ .... $t \in [0,\frac{\pi}{4}]$

CURVATURA E RAIO DE CURVATURA NA FORMA PARAMÉTRICA

$$\boxed{K = \frac{\ddot{y}\,\dot{x} - \dot{y}\,\ddot{x}}{(\dot{x}^2 + \dot{y}^2)^{3/2}}} \qquad \boxed{R = \left| \frac{1}{K} \right|}$$

EXEMPLO RESOLVIDO. Calcule a curvatura e o raio de curvatura da função $\begin{cases} x = t^2.e^t \\ y = 2 \text{ sen } 2t \end{cases}$, no ponto $t_0 = 0$

$x = t^2.e^t$

$\dot{x} = 2t.e^t + t^2.e^t$ .... $\dot{x}(0) = 0$

$\ddot{x} = 2\,e^t + 2t.e^t + 2t.e^t + t^2.e^t$ .... $\ddot{x}(0) = 2$

$y = 2 \text{ sen } 2t$

$\dot{y} = 4 \cos 2t$ .... $\dot{y}(0) = 4$

$\ddot{y} = -\,8 \text{ sen } 2t$ .... $\ddot{y}(0) = 0$

$$K = \frac{0.0 - 4.2}{(0 + 16)^{3/2}} = \frac{-8}{(4^2)^{3/2}} = \frac{-8}{4^3} = \frac{-8}{64} = -\frac{1}{8}$$

$$\boxed{K = -\frac{1}{8}}$$

$$R = \left| \frac{1}{K} \right| \longrightarrow \boxed{R = 8}$$

EXERCÍCIOS DE CÁLCULO II - LISTA XX

Calcule a curvatura e o raio de curvatura, nos pontos dados

1) $\begin{cases} x = t - \operatorname{sen} t \\ y = 1 - \cos t \end{cases}$ .... no ponto $t = \frac{\pi}{3}$

2) $\begin{cases} x = e^{t} \\ y = t^{2} \end{cases}$ .... no ponto $t = -2$

3) $\begin{cases} x = t^{2} - 1 \\ y = t + \frac{1}{t} \end{cases}$ .... no ponto $t = 1$

4) $\begin{cases} x = e^{2t} \\ y = \ln t \end{cases}$ .... no ponto $t = 2$

5) $\begin{cases} x = e^{t^{2}} + 1 \\ y = t.\ln t + 2 \end{cases}$ .... no ponto $t = 1$

6) $\begin{cases} x = t^{3} + 1 \\ y = t.e^{t} \end{cases}$ .... no ponto $t = 1$

7) $\begin{cases} x = r(t + \operatorname{sen} t) \\ y = r(1 + \cos t) \end{cases}$ .... no ponto $t = 0$

8) $\begin{cases} x = 2^{\operatorname{sen} t} \\ y = 2^{\cos t} \end{cases}$ .... no ponto $t = \frac{\pi}{2}$

CÁLCULO DE ÁREA NA FORMA POLAR

$$\boxed{A = \frac{1}{2}\int_{\theta_1}^{\theta_2} \rho^2 d\theta}$$

EXEMPLO RESOLVIDO. Calcule a área definida pela curva da função

$$\rho = \frac{1}{\sqrt{\theta^2 - 16}} \quad , \text{ para } \quad \theta \in [5, 6]$$

$$A = \frac{1}{2}\int_5^6 \left[\frac{1}{\sqrt{\theta^2 - 16}}\right]^2 d\theta = \frac{1}{2}\int_5^6 \frac{1}{\theta^2 - 16}\, d\theta = \frac{1}{2}\int_5^6 \frac{1}{(\theta + 4)(\theta - 4)}\, d\theta$$

$$A = \frac{1}{2}\int_5^6 \left[\frac{M}{(\theta + 4)} + \frac{N}{(\theta - 4)}\right] d\theta \; \ldots\ldots \; \frac{M}{\theta + 4} + \frac{N}{\theta - 4} = \frac{1}{(\theta + 4)(\theta - 4)}$$

$$M(\theta - 4) + N(\theta + 4) = 1$$

Para $\theta = 4 \longrightarrow 8N = 1 \quad N = \frac{1}{8}$

Para $\theta = -4 \longrightarrow -8M = 1 \quad M = -\frac{1}{8}$

$$A = \frac{1}{2}\int_5^6 \left[\frac{-\frac{1}{8}}{\theta + 4} + \frac{\frac{1}{8}}{\theta - 4}\right] d\theta = \frac{1}{2}\left[\int_5^6 -\frac{1}{8}\left(\frac{1}{\theta + 4}\right) d\theta + \int_5^6 \frac{1}{8}\left(\frac{1}{\theta - 4}\right) d\theta\right]$$

$$A = \frac{1}{16}\left[-\int_5^6 \frac{1}{\theta + 4}\, d\theta + \int_5^6 \frac{1}{\theta - 4}\, d\theta\right]$$

$$A = \frac{1}{16}\left[-\ln(\theta + 4) + \ln(\theta - 4)\right]\Big|_5^6$$

$$A = \frac{1}{16}\left[-\ln 10 + \ln 2 - (-\ln 9 + \ln 1)\right]$$

$$A = \frac{1}{16}\left[-\ln 10 + \ln 2 + \ln 9\right]$$

$$A = \frac{1}{16}\left[\ln \frac{2.9}{10}\right] \longrightarrow \boxed{A = \frac{1}{16}\ln\frac{9}{5} \text{ u.a.}}$$

EXERCÍCIOS DE CÁLCULO II - LISTA XXI

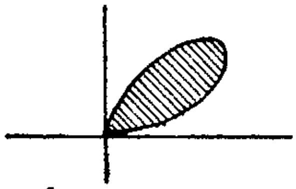
Calcule a área definida pelas curvas das funções abaixo

1) $\rho = \text{sen } 2\theta + \cos 2\theta$ .... $\theta \in [0, \frac{\pi}{4}]$

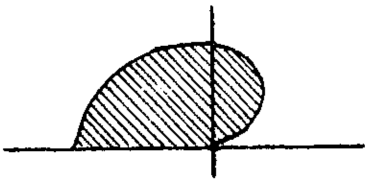
2) $\rho = \cos 3\theta + \text{sen } 3\theta$ .... $\theta \in [0, \frac{\pi}{6}]$

3) $\rho = \text{sen } \theta$ .... $\theta \in [0, \pi]$

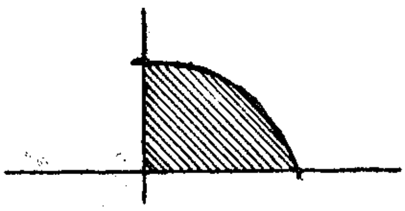
4) $\rho = \sqrt{\text{sen } 2\theta} . e^{\theta/2}$



5) $\rho = \sqrt{\theta} . e^{\theta}$



6) $\rho = \dfrac{e^{\theta/2}}{\sqrt{1 + e^{2\theta}}}$



7) $\rho^2 = a^2 \cos 2\theta$ ...(Leminiscata de Bernoulli)

8) $\rho = a \cos 3\theta$ ...(Rosácea de 3 folhas)

9) $\rho = 2 \text{ tg } \theta$ .... $\theta \in [0, \frac{\pi}{3}]$

10) $\rho = \sec^2 2\theta$ .... $\theta \in [0, \frac{\pi}{8}]$

11) $\rho = \sqrt[4]{e^{3\theta} + e^{2\theta}}$ .... $\theta \in [0, 1]$

CÁLCULO DO COMPRIMENTO DO ARCO NA FORMA POLAR

$$\boxed{\ell = \int_{\theta_1}^{\theta_2} \sqrt{\rho^2 + \rho'^2}\, d\theta}$$

EXEMPLO RESOLVIDO. Calcule o comprimento do arco definido por

$$\rho = 2\ \text{sen}^3(\theta/3), \quad \text{para} \quad \theta \in [0,\ 3\pi]$$

$$\rho = 2\ \text{sen}^3(\theta/3)$$

$$\rho' = 2.\ \cancel{3}\ \text{sen}^2(\theta/3).\cos(\theta/3).\ \frac{1}{\cancel{3}}$$

$$\rho' = 2\ \text{sen}^2(\theta/3).\cos(\theta/3)$$

$$\rho^2 + \rho'^2 = 4.\text{sen}^6(\theta/3) + 4.\text{sen}^4(\theta/3).\cos^2(\theta/3)$$

$$\rho^2 + \rho'^2 = 4.\text{sen}^4(\theta/3)\left[\text{sen}^2(\theta/3) + \cos^2(\theta/3)\right]$$

$$\rho^2 + \rho'^2 = 4.\text{sen}^4(\theta/3)$$

$$\sqrt{\rho^2 + \rho'^2} = 2.\text{sen}^2(\theta/3)$$

$$\ell = \int_0^{3\pi} 2\ \text{sen}^2(\theta/3)d\theta = \cancel{2}\int_0^{3\pi} \frac{1 - \cos(2\theta/3)}{\cancel{2}}\, d\theta = \int_0^{3\pi}\left[1 - \cos(2\theta/3)\right] d\theta$$

$$\ell = \int_0^{3\pi} d\theta - \int_0^{3\pi} \cos(2\theta/3)d\theta \longrightarrow d(2\theta/3) = \frac{2}{3}\, d\theta$$

$$\ell = \theta\Big|_0^{3\pi} - \frac{3}{2}\int_0^{3\pi} \cos(2\theta/3)d(2\theta/3) = \left[\theta - \frac{3}{2}\ \text{sen}(2\theta/3)\right]\Bigg|_0^{3\pi}$$

$$\ell = 3\pi - 0 \qquad \boxed{\ell = 3\pi \text{ u.c.}}$$

EXERCÍCIOS DE CÁLCULO II - LISTA XXII

Calcule o comprimento do arco, nos intervalos dados

1) $\rho = 2 \sec \theta$ .... $\theta \in [-\frac{\pi}{4}, \frac{\pi}{4}]$

2) $\rho = 4 \text{ sen } \theta$ .... $\theta \in [0, \pi]$

3) $\rho = a(1 - \cos \theta)$ ....(Cardióide) ...... (comprimento total)

4) $\rho = e^{\theta}$ .... $\theta \in [0,1]$

5) $\rho = a\, e^{2\theta}$ .... $\theta \in [0, \frac{1}{2}]$

6) $\rho = \theta^2$ .... $\theta \in [0, \sqrt{5}]$

7) $\rho = (\theta + 1)^2$ .... $\theta \in [0,1]$

DERIVADAS PARCIAIS

EXEMPLOS RESOLVIDOS.

1) Calcule as derivadas parciais da $z = 3x.e^{2y} + \ell n(2xy)$ em relação a x e a y.

a) em relação a x

$$\frac{\partial z}{\partial x} = 3.e^{2y} + \frac{\cancel{2}\cancel{y}}{\cancel{2}x\cancel{y}} = 3.e^{2y} + \frac{1}{x}$$

b) em relção a y

$$\frac{\partial z}{\partial y} = 3x.e^{2y}.2 + \frac{\cancel{2}\cancel{x}}{\cancel{2}\cancel{x}y} = 6x.e^{2y} + \frac{1}{y}$$

.-.

2) Calcule as derivadas parciais da $w = sen(\pi x^2 y) + e^{y^2 z^3}$, no ponto (1, 1, 1).

a) em relação a x

$$\frac{\partial w}{\partial x} = \cos(\pi x^2 y).2\pi xy \Big|_{(1,1,1)} = -1.2\pi = -2\pi$$

b) em relação a y

$$\frac{\partial w}{\partial y} = (\cos(\pi x^2 y).\pi x^2 + e^{y^2 z^3}.2yz^3) \Big|_{(1,1,1)} = -1.\pi + e.2 = 2e - \pi$$

c) em relação a z

$$\frac{\partial w}{\partial z} = e^{y^2 z^3}.3y^2 z^2 \Big|_{(1,1,1)} = e.3 = 3e$$

EXERCÍCIOS DE CÁLCULO II - LISTA XXIII

CALCULAR AS DERIVADAS PARCIAIS DAS SEGUINTES FUNÇÕES

1) $z = x^2y + 2xy^3 - 2x$

2) $z = e^{xy^2} + \cos(2xy^4)$

3) $z = \ln \dfrac{3xy}{x^2 + y^2}$

4) $z = e^{\operatorname{sen} \frac{x}{y}}$

5) $z = \dfrac{(x - 6y)^5}{xy}$, no ponto (5, 1)

6) $z = y.e^{\cos(\pi x)}$, no ponto (1, 1)

7) $z = x.\operatorname{arctg} \dfrac{2y}{x^2}$

8) $z = \dfrac{3x^2y^3}{x^2 + y^2}$

9) $z = e^{\operatorname{sen} x}.\operatorname{tg}(3x^3y^2)$

10) $z = 3^{2xy^2} + \ln(x^2y^2)$, no ponto (2, 1)

11) $z = 3xy.e^{x^2 - y^2}$, no ponto (1, 1)

12) $w = 3x^2yz + \dfrac{x^3y}{z^2} + \ln(yz^2)$, no ponto (1,2,1)

13) $z = \ln\sqrt{x^2 + y^2}$, no ponto (-3, 4)

14) $w = e^{xz}.\operatorname{tg}(3xy^2z^2)$

15) $w = y^{x^2z} + y\sqrt{2x - z}$, no ponto (1,e,1)

16) $z = e^{x^y} + \ln(\operatorname{tg}\dfrac{x}{y} + 1)$, no ponto $(\frac{\pi}{4}, 1)$

DERIVADAS PARCIAIS DE 2ª ORDEM

EXEMPLO RESOLVIDO. Calcule as derivadas parciais de 2ª ordem da

$$z = \operatorname{sen}(x^2y)$$

a) 1ª ordem em relação a x

$$\frac{\partial z}{\partial x} = 2xy.\cos(x^2y)$$

2ª ordem em relação a x

$$\frac{\partial^2 z}{\partial x^2} = 2y.\cos(x^2y) + 2xy.(-\operatorname{sen}(x^2y).2xy) =$$

$$= 2y\left[\cos(x^2y) - 2x^2y.\operatorname{sen}(x^2y)\right]$$

2ª ordem em relação a y

$$\frac{\partial^2 z}{\partial y \partial x} = 2x.\cos(x^2y) + 2xy.(-\operatorname{sen}(x^2y).x^2) =$$

$$= 2x\left[\cos(x^2y) - x^2y.\operatorname{sen}(x^2y)\right]$$

b) 1ª ordem em relação a y

$$\frac{\partial z}{\partial y} = x^2.\cos(x^2y)$$

2ª ordem em relação a y

$$\frac{\partial^2 z}{\partial y^2} = x^2.(-\operatorname{sen}(x^2y)).x^2 = -x^4.\operatorname{sen}(x^2y)$$

são iguais.

2ª ordem em relação a x

$$\frac{\partial^2 z}{\partial x \partial y} = 2x.\cos(x^2y) + x^2.(-\operatorname{sen}(x^2y)).2xy =$$

$$= 2x\left[\cos(x^2y) - x^2y.\operatorname{sen}(x^2y)\right]$$

OBSERVAÇÃO: $\dfrac{\partial^2 z}{\partial y \partial x} = \dfrac{\partial^2 z}{\partial x \partial y}$

EXERCÍCIOS DE CÁLCULO II - LISTA XXIV

Dados

1) $z = x^2 + 3xy + 2y^2 + 2x - 3y + 5$, calcule as derivadas parciais de segunda ordem.

2) $z = e^x.\cos y$, calcule $\dfrac{\partial^2 z}{\partial y \partial x}$.

3) $z = x^2 y e^{y^2} + \dfrac{1}{x^2 y^3}$, calcule $\dfrac{\partial^2 z}{\partial x \partial y}$ no ponto (1,1).

4) $z = e^{x^2 y^3}$, calcule $\dfrac{\partial^2 z}{\partial x^2}$.

5) $z = \ln xy + 2xy - x^3 + y^4$, calcule $\dfrac{\partial^2 z}{\partial x^2}$ e $\dfrac{\partial^2 z}{\partial y^2}$.

6) $z = \sqrt{x^2 + y^2}$, calcule $\dfrac{\partial^2 z}{\partial x^2} + \dfrac{\partial^2 z}{\partial y^2}$.

CÁLCULO DO GRADIENTE, DERIVADA DIRECIONAL E DERIVADA DIRECIONAL MÁXIMA

EXEMPLO RESOLVIDO. Calcule o vetor gradiente, a derivada direcional e a derivada direcional máxima na direção do vetor $\vec{v} = 3\vec{i} - 4\vec{j}$ da função $z = y^{x^2} + y\sqrt{2x - 1}$, no ponto A(1, 1).

a) Vetor gradiente $\quad \vec{\nabla}z = \frac{\partial z}{\partial x}\vec{i} + \frac{\partial z}{\partial y}\vec{j}$

$$\frac{\partial z}{\partial x} = y^{x^2}.\ln y.2x + y.\frac{1}{\cancel{2}}(2x - 1)^{-1/2}.\cancel{2} =$$

$$= \left(2x.y^{x^2}.\ln y + \frac{y}{\sqrt{2x - 1}}\right)\Bigg|_{(1,\ 1)} = 2.1.\ln 1 + \frac{1}{1} = 1$$

$$\frac{\partial z}{\partial y} = (x^2.y^{x^2 - 1} + 1.\sqrt{2x - 1})\Bigg|_{(1,\ 1)} = 1 + 1 = 2$$

$$\boxed{\vec{\nabla}z = \vec{i} + 2\vec{j}}$$

b) Derivada direcional $\quad D_{\vec{v}}z = \vec{\nabla}z.\vec{u}$, onde $\vec{u} = \frac{\vec{v}}{|\vec{v}|}$

$$\vec{u} = \frac{3}{\sqrt{9 + 16}}\vec{i} - \frac{4}{\sqrt{9 + 16}}\vec{j} = \frac{3}{5}\vec{i} - \frac{4}{5}\vec{j}$$

$$D_{\vec{v}}z = (\vec{i} + 2\vec{j}).\left(\frac{3}{5}\vec{i} - \frac{4}{5}\vec{j}\right) = \frac{3}{5} - \frac{8}{5} = -\frac{5}{5} = -1 \qquad \boxed{D_{\vec{v}}z = -1}$$

c) Derivada direcional máxima $\quad D_{f_{máx}} = |\vec{\nabla}z|$

$$D_{f_{máx}} = \sqrt{1 + 4} = \sqrt{5}$$

$$\boxed{D_{f_{máx}} = \sqrt{5}}$$

EXERCÍCIOS DE CÁLCULI II - LISTA XXV

Calcule o vetor gradiente, a derivada direcional e a derivada direcional máxima das seguintes funções:

1) $z = x^2 + 4xy + y^3 - 1$, no ponto $(1, 2)$ e na direção do vetor $\vec{v} = -\vec{i} + 3\vec{j}$

2) $z = x^{\text{sen}\, y} + x^2$, no ponto $(1, \frac{\pi}{2})$ e na direção do vetor $\vec{v} = 2\,\vec{i} - \vec{j}$

3) $w = \ln xyz + 2yz - z^2$, no ponto $(1, 1, 1)$ e na direção do vetor $\vec{v} = \vec{i} + \vec{j} + \vec{k}$

4) $w = e^{x^2 y} - \dfrac{z}{x^3} + y\,\text{sen}(x^2 - 2z)$, no ponto $(2, 0, 2)$ e na direção do vetor $\vec{v} = 2\,\vec{i} + \vec{j} + 2\,\vec{k}$

5) $z = e^{xy^2}.\cos(\pi x^3)$, no ponto $(1, 1)$ e na direção do vetor $\vec{v} = \sqrt{5}\,\vec{i} - 2\,\vec{j}$

6) $z = e^{y\,\text{sen}\,x} + \ln(2x + y)$, no ponto $(0, 1)$ e na direção do vetor $\vec{v} = 4\vec{i} - 3\vec{j}$

7) $z = x^2 y.\cos(2xy)$, no ponto $(1, \pi)$ e na direção do vetor $\vec{v} = \vec{i} + \vec{j}$

8) $z = y^{\text{tg}\,x}$, no ponto $(\frac{\pi}{4}, e)$ e na direção do vetor $\vec{v} = \sqrt{2}\,\vec{i} + \sqrt{2}\,\vec{j}$

9) $w = 2^{x^2 yz} + \text{sen}(xy^2 z) + \dfrac{1}{yz^2}$, no ponto $(0, 1, -1)$ e na direção do vetor $\vec{v} = 6\vec{i} - 2\vec{j} + 3\vec{k}$

10) $w = 3x^2 yz + \dfrac{x^3 y}{z^2} + \ln(yz^2)$, no ponto $(1, 1, 1)$ e na direção do vetor $\vec{v} = \vec{i} - 2\vec{j} + 2\vec{k}$

11) $z = e^{x^2 y} - \text{arctg}\, x^2 y^2$, no ponto $(1, 1)$ e na direção do vetor $\vec{v} = 4\,\vec{i} - 3\vec{j}$

12) $z = y^{x^2} + \text{sen}(\pi xy^2)$, no ponto $(1, 1)$ e na direção do vetor $\vec{v} = 3\vec{i} - 4\vec{j}$

13) $z = \dfrac{100xy}{x^2 + y^2}$, no ponto $(2, 1)$ e na direção que faz um angulo de 60° com o eixo do x.

14) $w = \ln(2x + yz) + \text{sen}(xyz)$, no ponto $(0, 1, 1)$ e na direção da normal do plano $\pi: x + 2y - 2z = 5$

DIFERENCIAL TOTAL

Se $z = f(x,y) \longrightarrow \begin{cases} dz = \frac{\partial z}{\partial x}\,dx + \frac{\partial z}{\partial y}\,dy & \text{ou} \\ \Delta z = \frac{\partial z}{\partial x}\,\Delta x + \frac{\partial z}{\partial y}\,\Delta y \end{cases}$

Se $w = f(x,y,z) \longrightarrow \begin{cases} dw = \frac{\partial w}{\partial x}\,dx + \frac{\partial w}{\partial y}\,dy + \frac{\partial w}{\partial z}\,dz & \text{ou} \\ \Delta w = \frac{\partial w}{\partial x}\,\Delta x + \frac{\partial w}{\partial y}\,\Delta y + \frac{\partial w}{\partial z}\,\Delta z \end{cases}$

EXEMPLOS RESOLVIDOS.

1) Calcule a expressão da diferencial total da $z = \operatorname{sen}(x^2y^3)$,

$$\frac{\partial z}{\partial x} = 2xy^3.\cos(x^2y^3)$$

$$\frac{\partial z}{\partial y} = 3x^2y^2.\cos(x^2y^3)$$

$$\boxed{dz = 2xy^3.\cos(x^2y^3)\,dx + 3x^2y^2.\cos(x^2y^3)\,dy}$$

-.-

2) Calcule o valor de $M = (1,01)^3.(2,99)^2 + (5,02)^3$ com o auxílio da diferencial total.

$$M = \underbrace{(1,01)}_{x}{}^3.\underbrace{(2,99)}_{y}{}^2 - \underbrace{(5,02)}_{z}{}^3$$

$$M = x^3.y^2 - z^3 \longrightarrow \begin{cases} x_0 = 1 \longrightarrow \Delta x = 0,01 \\ y_0 = 3 \longrightarrow \Delta y = -0,01 \\ z_0 = 5 \longrightarrow \Delta z = 0,02 \end{cases}$$

$$M = x_0^3.y_0^2 - z_0^3 + \Delta M$$

$$\Delta M = \frac{\partial M}{\partial x}\Delta x + \frac{\partial M}{\partial y}\Delta y + \frac{\partial M}{\partial z}\Delta z$$

$$\frac{\partial M}{\partial x} = 3x^2y^2\Big|_{(1,3,5)} = 3.1^2.3^2 = 27$$

$$\frac{\partial M}{\partial y} = 2x^3y\Big|_{(1,3,5)} = 2.1.3 = 6$$

$$\frac{\partial M}{\partial z} = -3z^2\Big|_{(1,3,5)} = -3.5^2 = -75$$

$$\Delta M = 27.(0,01) + 6.(-0,01) - 75(0,02)$$

$$\Delta M = 0,27 - 0,06 - 1,50$$

$$\Delta M = -1,29$$

$$M = 1^3.3^2 - 5^3 - 1,29$$

$$M = 9 - 125 - 1,29$$

$$\boxed{M = -117,29}$$

EXERCÍCIOS DE CÁLCULO II - LISTA XXVI

1) Dar a expressão da diferencial total

a) $z = \text{sen}(x\,e^{y})$

b) $z = e^{x} + \ell n(\cos y)$

c) $z = 2^{y^2} + 3^{x^3}$

d) $w = \sqrt{x} - 2y + \frac{1}{z^2}$

e) $w = x^{y} + y^{x} + z^{xy}$

f) $w = \ell n(xy^3z + 2)$

g) $z = e^{xy}\text{sen}(x^2y)$

h) $w = 3x^2y^3z^2.e^{xyz}$

i) $z = \text{tg}(x.2^{y})$

2) Calcule com o auxílio da diferencial total

a) $M = (1{,}98)^3 + (1{,}98).(2{,}01)^3$

b) $N = (2{,}99)^3 - (2{,}99)^2.(1{,}99) + (3{,}02)^3$

3) Resolver:

a) Seja m = 1000 gramas e v = 10 cm/s. Calcule a variação da energia cinética para uma perda de massa de 0,01 g e um aumento de velocidade de 0,002 cm/s.

b) Determinar com o auxílio da diferencial total a variação do volume de um prisma de base quadrada, sabendo-se que os lados da base diminuiram de 0,01 cm e a altura aumentou de 0,02 cm. Estado inicial $\ell = 2.10^3$ cm e $h = 4.10^3$ cm.

c) Determinar a variação do volume de um cilindro de raio r = 10 cm e altura h = 18 cm, sabendo-se que submetido à tensões, h diminui de 0,01 cm e r cresce de 0,02 cm.

MÁXIMOS E MÍNIMOS RELATIVOS (FUNÇÕES DE DUAS VARIÁVEIS)

EXEMPLO RESOLVIDO. Testar os pontos críticos da seguinte função

$$z = x^2y - 2x^2 + 2y^2 - 12y + 5$$

a) Determinação dos pontos críticos.

$$\frac{\partial z}{\partial x} = 2xy - 4x$$

$$\frac{\partial z}{\partial y} = x^2 + 4y - 12$$

$$\begin{cases} 2xy - 4x = 0 & \text{(I)} \\ x^2 + 4y - 12 = 0 & \text{(II)} \end{cases}$$

De (I) $\quad 2xy - 4x = 0$

$$2x(y-2) = 0 \Longrightarrow \begin{cases} 2x = 0 \Longrightarrow x = 0 \\ y - 2 = 0 \Longrightarrow y = 2 \end{cases}$$

Substituindo em (II) $\quad x^2 + 4y - 12 = 0$

Para $x = 0 \Longrightarrow 4y - 12 = 0 \Longrightarrow 4y = 12 \Longrightarrow y = 3 \longrightarrow (0, 3)$

Para $y = 2 \Longrightarrow x^2 + 4.2 - 12 = 0 \Longrightarrow x^2 = 4 \Longrightarrow \begin{cases} x = 2 \longrightarrow (2, 2) \\ x = -2 \longrightarrow (-2, 2) \end{cases}$

b) Determinação de A, B e C

$$A = \frac{\partial^2 z}{\partial x^2} = 2y - 4$$

$$B = \frac{\partial^2 z}{\partial y^2} = 4$$

$$C = \frac{\partial^2 z}{\partial x \partial y} = 2x$$

	(0, 3)	(2, 2)	(-2, 2)
A	2	0	0
B	4	4	4
C	0	4	-4
$A.B - C^2$	8	-16	-16
	MIN	NSC	NSC

EXERCÍCIOS DE CÁLCULO II - LISTA XXVII

TESTAR OS PONTOS CRÍTICOS DAS FUNÇÕES

1) $z = x^2 + 2y^2 - 4x + 4y - 3$

2) $z = -\frac{1}{2} y^2 - 12 x^2 + 3xy + 30x + 5$

3) $z = 6xy - x^3 - y^3 + 5$

4) $z = x^3 + y^2 - y - xy + 5$

5) $z = \frac{16}{3} x^3 + x^2 - 6xy + y^2$

6) $z = 2x^3 + 3y^2 - 12xy + 1$

7) $z = x^3 - 2xy^2 + 4y^3 - 4x + 2$

8) $z = y^3x - 3x^2y^2 + 2x^4 + 8x - 2$

9) $z = x^4 - 2x^2y + 2y^2 - 8y$

10) $z = x^3 + y^2 + 2x^2 - yx^2 - 3$

11) $z = x^3 - 2xy + y - y^2$

12) $z = \frac{1}{3} x^3 + y^3 - x^2y + 5y - 1$

13) $z = \frac{1}{4} x^4 + \frac{1}{4} y^4 - xy + 1$

14) $z = \frac{1}{9} x^4 - x^2y + 3y^3 - x^2 + 4$

15) $z = y^3 - 3x^2y + 2x^2$

16) $z = 2x^8 - x^2y + y^2$

17) $z = 2x^2y - 2x^2 + 8y^2 - 24y + 11$

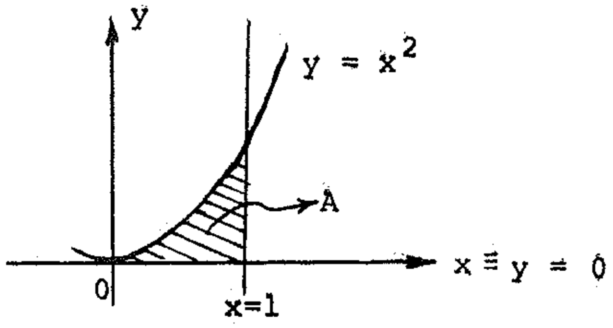
18) $z = x^3 + 3xy^2 - 3x^2 - 3y^2 + 4$

19) $z = xy^2 - 2y^2 + 2x^2 - 12x + 2$

INTEGRAIS DUPLAS

$$\boxed{\int_a^b \int_c^d f(x,y)\,dx.dy = \int_a^b \left[\int_c^d f(x,y)\,dx \right] dy}$$

EXEMPLO RESOLVIDO. Calcule a integral $I = \iint_A (y + 2x)\, dA$, onde A é a região limitada por $x = 1$, $y = 0$ e $y = x^2$.



$$I = \int_0^1 \left[\int_0^{x^2} (y + 2x)\; dy \right] dx$$

$$I = \int_0^1 \left[\left(\frac{y^2}{2} + 2xy\right) \Big|_{y=0}^{y=x^2} \right] dx$$

$$I = \int_0^1 \left[\frac{(x^2)^2}{2} + 2x.(x^2) - 0 \right] dx$$

$$I = \int_0^1 \left(\frac{x^4}{2} + 2x^3\right) dx$$

$$I = \left(\frac{x^5}{10} + \frac{2x^4}{4}\right) \Big|_0^1 = \frac{1}{10} + \frac{1}{2} - 0 = \frac{1+5}{10} = \frac{6}{10} = \frac{3}{5}$$

$$\boxed{I = \frac{3}{5}}$$

EXERCÍCIOS DE CÁLCULO II - LISTA XXVIII

1) Calcule o valor das seguintes integrais:

a) $I = \int_0^1 \int_0^2 (x + 2y)\, dy\, dx$

b) $I = \int_0^1 \int_1^2 (x^2 + y^2)\, dx\, dy$

c) $I = \int_0^1 \int_x^1 (x + y)\, dy\, dx$

d) $I = \int_0^1 \int_0^{y^2} (2x + y^2)\, dx\, dy$

2) Calcule a integral $I = \iint_A xy\, dA$, onde A é a região limitada por $x = 2$, $y = 0$ e $y = 2x$.

3) Calcule a integral $I = \iint_M (x^2 + 2xy)\, dM$, onde M é a região limitada por $x = 0$, $x = 2$, $y = 0$ e $y = 1$.

4) Calcule a integral $I = \iint_R x \cos y\, dR$, onde R é a região limitada por $x = \sqrt{\pi/2}$, $y = 0$ e $y = x^2$.

5) Calcule a integral $I = \iint_S \sqrt{1 + x^2}\, dS$, onde S é a região limitada por $x = 2$, $y = 0$ e $y = x$.

6) Calcule a integral $I = \iint_R (2x + y)\, dR$, onde R é a região limitada por $x = 0$, $y = 1$ e $y = x^2$.

7) Calcule a integral $I = \iint_M dM$, onde M é a região limitada por $y = x$ e $y = x\sqrt{x}$.

8) Calcule a integral $I = \iint_A (x + 2y)\, dA$, onde A é a região limitada por $0 \leqslant x \leqslant 2$ e $0 \leqslant y \leqslant x^2$.

9) Calcule a integral $I = \iint_S (x + y)\, dS$, onde S é a região limitada por $0 \leqslant x \leqslant 2$ e $x^3 \leqslant y \leqslant 8$.

10) Calcule a integral $I = \iint_R \frac{x^2}{1 + y^2}\, dR$, onde R é a região limitada por $0 \leqslant x \leqslant 1$ e $0 \leqslant y \leqslant 1$.

11) Calcule a integral $I = \iint_A (2x + y)\, dA$, onde A é a região limitada por $x = 2$, $y = 1$ e $y = x^2$.

12) Calcule a integral $I = \iint_M (3x - y^2)\, dM$, onde M é a região limitada por $x = 1$, $y = 0$ e $y = x^2$.

13) Calcule a integral $I = \iint_D \sqrt{1 + y}\, dD$, onde D é a região limitada por $x = 1$, $y = x$ e $y = 3x$.

EQUAÇÕES DIFERENCIAIS IMEDIATAS

EXEMPLOS RESOLVIDOS. Resolver as seguintes equações diferenciais.

1) $y' = \dfrac{x}{2x^2 + 1}$

$$y = \int \frac{x}{2x^2 + 1}\, dx \quad \ldots\ldots\ldots\ldots \quad d(2x^2 + 1) = 4x\, dx$$

$$y = \frac{1}{4} \int \frac{1}{2x^2 + 1}\, d(2x^2 + 1)$$

$$\boxed{y = \frac{1}{4}\left[\ell n\,(2x^2 + 1) + C\right]}$$

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2) $y'' = e^{2x}$

$$y' = \int e^{2x} dx \quad \ldots\ldots\ldots\ldots \quad d(2x) = 2\, dx$$

$$y' = \frac{1}{2} \int e^{2x} d(2x)$$

$$y' = \frac{1}{2}\left[e^{2x} + C_1\right]$$

$$y = \int \frac{1}{2}\left[e^{2x} + C_1\right] dx$$

$$y = \frac{1}{2} \int \left[e^{2x} + C_1\right] dx$$

$$y = \frac{1}{2}\left[\frac{1}{2} \int e^{2x} d(2x) + C_1 \int dx\right]$$

$$\boxed{y = \frac{1}{2}\left[\frac{1}{2}\, e^{2x} + C_1 x + C_2\right]}$$

EXERCÍCIOS DE CÁLCULO II - LISTA XXIX

RESOLVER AS SEGUINTES EQUAÇÕES DIFERENCIAIS

1) $y' = \dfrac{1}{x^2}$

2) $dx = \dfrac{1}{2y + 3}\, dy$

3) $y' = \dfrac{x^2}{2x^3 + 1}$. Dê a solução particular para $x = 1$ e $y = 2$.

4) $\dfrac{dy}{dx} = x.\ln x$. Dê a solução particular para $x = 1$ e $y = -2$.

5) $y''' = x$

6) $y' = 3x^2 - 2x + 5$

7) $y' = x.\sec^2 x^2$

8) $y'' = e^{-2x}$

9) $y' = x^3.\operatorname{sen}(5x^2 - 1)$

10) $y' = x^3.\cos(3x^2 - 12)$. Dê a solução particular no ponto $P(2, \frac{19}{18})$

11) $y' = \dfrac{\ln(\ln x)}{x}$. Dê a solução particular no ponto $A(2,2)$

12) $y'' = \operatorname{sen} 2x - \cos 4x$. Dê a solução particular para $f'(0) = 2$ e $f(\frac{\pi}{2}) = 3$.

13) $y' = x^3(x^2 + 1)^8$

14) $y' = \dfrac{x^2}{\sqrt{x + 1}}$

15) $y' = \dfrac{1 + \sqrt{x}}{x - \sqrt{x}}$

EQUAÇÃO DIFERENCIAL DE 1ª ORDEM A VARIÁVEL SEPARÁVEL

$$\boxed{P(x,y)\,dx + Q(x,y)\,dy = 0 \longrightarrow f(x)\,dx = g(y)\,dy}$$

Solução geral $\boxed{F(x) = G(y) + C}$

EXEMPLO RESOLVIDO. Resolver a equação

$$(x + 2)(y^2 + 1)dx - (x^2y - 4y)\,dy = 0$$

$$(x + 2)(y^2 + 1)\,dx - (x^2y - 4y)\,dy = 0$$

$$(x + 2)(y^2 + 1)\,dx = (x^2y - 4y)\,dy$$

$$(x + 2)(y^2 + 1)\,dx = y(x^2 - 4)\,dy$$

$$\frac{x + 2}{x^2 - 4}\,dx = \frac{y}{y^2 + 1}\,dy$$

$$\frac{(x \cancel{+} 2)}{(x \cancel{+} 2)(x - 2)}\,dx = \frac{y}{y^2 + 1}\,dy$$

$$\frac{1}{x - 2}\,dx = \frac{y}{y^2 + 1}\,dy$$

$$\int \frac{1}{x - 2}\,dx = \int \frac{y}{y^2 + 1}\,dy \longrightarrow d(y^2 + 1) = 2y\,dy$$

$$\int \frac{1}{x - 2}\,dx = \frac{1}{2}\int \frac{y}{y^2 + 1}\,d(y^2 + 1)$$

$$\boxed{\ln(x - 2) = \frac{1}{2}\ln(y^2 + 1) + C}$$

EXERCÍCIOS DE CÁLCULO II - LISTA XXX

RESOLVER AS SEGUINTES EQUAÇÕES DIFERENCIAIS

1) $x^2 dx - e^y dy = 0$

2) $\cos^2 y \, dx - x^2 \, dy = 0$

3) $x\sqrt{1 - y^2} \, dx - \sqrt{1 - x^2} \, dy = 0$

4) $(1 - x)y^2 \, dx + x \, dy = 0$

5) $(x + xy^2) dx + (1 + x) \, dy = 0$

6) $(y^2 + 1) \, dx + 2(y + xy) \, dy = 0$

7) $(3xy + 3x - y - 1) \, dx - xy \, dy = 0$

8) $(xy - 2x - y + 2) \, dx + xy \, dy = 0$

9) $(3x^2y - xy) \, dx + (2x^3y^2 + x^3y^4) \, dy = 0$

10) $4x \, dy - y \, dx = x^2 \, dy$

11) $(x^2y + 2y) \, dx = (xy + x + y + 1) \, dy$

12) $e^{2x - 3y} \, dy = dx$

13) $(x + xy^2) \, dx + (x^2 + 2) \, \text{arctg } y \, dy = 0$

14) $(y^2 + 1) \text{arctg } x \, dx = \frac{y}{x} \, dy$

15) $y^2 \cos(\ln x) \, dx = x.e^{1/y} \, dy$

16) $\sqrt{y}(y - 1) \ln(\text{sen } x) \, dx = \sec x \, dy$

17) $\sec^2 x \, dx - y(1 + \text{tg } x) \sec y^2 \, dy = 0$

18) $(1 - x) \, dx - (1 + x) \sec y \, dy = 0$

19) $(1 - x^2)\ dx - y(1 + x)\sec^2 y\ dy = 0$

20) $y^2 . e^{1/y} \operatorname{sen} x\ dx = dy$

21) $(x^2 + x)\ dy = (x + 2)\ dx$

EQUAÇÕES DIFERENCIAIS DE 1ª ORDEM - EXATAS

$$\boxed{P(x,y)\,dx + Q(x,y)\,dy = 0 \longrightarrow \frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}}$$

EXEMPLO RESOLVIDO. Resolver a equação diferencial

$$\underbrace{(2xy + 2y^2 + 3x^2 + 2)}_{P(x,y)}dx + \underbrace{(x^2 + 4xy + 3y^2)}_{Q(x,y)}dy = 0$$

1º) $\begin{cases} \dfrac{\partial P}{\partial y} = 2x + 4y \\ \dfrac{\partial Q}{\partial x} = 2x + 4y \end{cases} \longrightarrow$ são iguais, portanto é exata.

2º) $P(x,y) = \dfrac{\partial f}{\partial x} \longrightarrow f(x,y) = \displaystyle\int P(x,y)\,dx$

$$f(x,y) = \int (2xy + 2y^2 + 3x^2 + 2)\,dx$$

$$f(x,y) = \frac{\cancel{2}x^2y}{\cancel{2}} + 2xy^2 + \frac{\cancel{3}x^3}{\cancel{3}} + 2x + k(y) = c_1$$

$$f(x,y) = x^2y + 2xy^2 + x^3 + 2x + k(y) = c_1$$

3º)

$$\begin{cases} \dfrac{\partial f}{\partial y} = Q(x,y) = \cancel{x^2} + \cancel{4xy} + 3y^2 \\ \dfrac{\partial f}{\partial y} = \cancel{x^2} + \cancel{4xy} + k'(y) \end{cases} \Longrightarrow k'(y) = 3y^2$$

$$k(y) = \int 3y^2\,dy$$

$$k(y) = \frac{\cancel{3}y^3}{\cancel{3}} + c_2$$

$$k(y) = y^3 + c_2$$

4º) $f(x,y) = x^2y + 2xy^2 + x^3 + 2x + (y^3 + c_2) = c_1$

$$\boxed{x^2y + 2xy^2 + x^3 + 2x + y^3 = C}$$

EXERCÍCIOS DE CÁLCULO II - LISTA XXXI

RESOLVER AS SEGUINTES EQUAÇÕES DIFERENCIAIS

1) $(x + 2y - 2)\,dx + (2x - y + 3)\,dy = 0$

2) $(3x^2y + 5y^2 + 14x)\,dx + (x^3 + 10xy + 12y^2)\,dy = 0$

3) $\left(\frac{\operatorname{sen} 2x}{y} + x\right)dx + \left(y - \frac{\operatorname{sen}^2 x}{y^2}\right)dy = 0$

4) $\left(y - \frac{y}{x^2}\right)dx + \left(\frac{1}{x} + x\right)dy = 0$

5) $\frac{xy + 1}{y}\,dx + \frac{2y - x}{y^2}\,dy = 0$

6) $(\operatorname{sen} x + x \operatorname{sen} y)\,dy - (\cos y - y \cos x)\,dx = \operatorname{sen} y\,dy$

7) $(\ln y + 2xy^3 + e^y)\,dx + \left(\frac{x}{y} + 3x^2y^2 + x.e^y + 3y^2\right)dy = 0$

Determine o valor de α e β de modo que as equações diferenciais sejam exatas, e resolva-as.

8) $(2xy^3 + \alpha x^2y - 4y^3 + 4x)\,dx + (3x^2y^2 - 2x^3 + \beta xy^2 - 9y^2)\,dy = 0$

9) $(3x^2y + \alpha xy + 2y^3 + 7)\,dx + (x^3 - 5x^2 + \beta xy^2 - 16y)\,dy = 0$

10) $(y^3 + \alpha xy + 15x^2y^2 - 4)\,dx + (3xy^2 - 2x^2 + \beta x^3y + 7)\,dy = 0$

11) $(\alpha xy^3 + 4y^2 + 21x^2y + 5)\,dx + (3x^2y^2 + \beta xy + 7x^3 - 24y^2)\,dy = 0$

12) $\left(\alpha x^2y^2 - \frac{1}{y} - 3x^2 + y^2e^x\right)dx + \left(\frac{2}{3}x^3y + \frac{x}{y^2} + \beta ye^x - 2y + 1\right)dy = 0$

13) $[(\alpha+\beta)\operatorname{sen} y + y\cos x + \cos x]\,dx + [(\alpha-\beta)x\cos y + \alpha \operatorname{sen} x - \operatorname{sen} y]\,dy = 0$

EQUAÇÕES DIFERENCIAIS LINEARES DE 1ª ORDEM

$$\boxed{y' + P(x)\, y = Q(x)}$$

$$\boxed{I(x) = e^{\int P(x)dx}} \qquad \boxed{y = \frac{1}{I(x)} \int Q(x).I(x).dx}$$

EXEMPLO RESOLVIDO. Resolver a equação $y' + \frac{7}{x}\, y = \frac{2\cos x^3}{x^5}$

$$\begin{cases} P(x) = \frac{7}{x} \\ Q(x) = \frac{2\cos x^3}{x^5} \end{cases}$$

$$I(x) = e^{\int P(x)dx} = e^{\int \frac{7}{x}\, dx} = e^{7\int \frac{1}{x}\, dx} = e^{7.\ell n\, x} = e^{\ell n\, x^7}$$

$$\boxed{I(x) = x^7}$$

$$y = \frac{1}{I(x)}\int Q(x).I(x)\, dx = \frac{1}{x^7}\int \frac{2\cos x^3}{x^5}.\, x^7\, dx$$

$$y = \frac{1}{x^7}\int 2\cos x^3 . x^2\, dx \longrightarrow d(x^3) = 3x^2 dx$$

$$y = \frac{1}{x^7}\cdot\frac{2}{3}\int \cos x^3\, d(x^3)$$

$$\boxed{y = \frac{2}{3x^7}\,(\text{sen}\, x^3 + C)}$$

EXERCÍCIOS DE CÁLCULO II - LISTA XXXII

RESOLVER AS SEGUINTES EQUAÇÕES DIFERENCIAIS

1) $y' + y = \frac{1}{2}$

2) $y' - \frac{3}{x}y = x^4$

3) $y' + (4x - 3x^2)y = 4x - 3x^2$

4) $(1 + x^2)y' - 2xy = x(1 + x^2)$

5) $(2x + 1)y' + (2x + 3)y = (2x + 1)e^{x^2}$

6) $y' + \frac{7 - 6x}{6x - 1}y = e^{3x^2}$

7) $(x^2 + 1)y' + xy = x\sqrt{x^2 + 1}$

8) $(x + 2)y' + y = x^2 - x + 2$

9) $(1 - x)y' + y = x(1 - x)$

10) $xy' + (2x^2 + 1)y = x$

11) $(x^2 + 1)y' + y = e^{\operatorname{arctg} x}$

12) $x^3y' + (2 - 3x^2)y = x^3$

EQUAÇÕES DIFERENCIAIS DE BERNOULLI

$$\boxed{y' + P(x)\, y = Q(x) . y^n}$$

EXEMPLO RESOLVIDO. Resolver a equação $x\, y' - y = x^2 y^3$

$$x\, y' - y = x^2 y^3 \qquad (: x)$$

$$y' - \frac{1}{x}\, y = x y^3 \qquad (: y^3)$$

$$\frac{y'}{y^3} - \frac{1}{x}\,\frac{y}{y^3} = x$$

$$y^{-3} y' - \frac{1}{x}\, y^{-2} = x \qquad \ldots\ldots\ldots\ldots \text{ Fazendo } \quad y^{-2} = t$$

$$-2\, y^{-3} y' = t'$$

$$y^{-3} y' = -\frac{1}{2}\, t'$$

$$-\frac{1}{2}\, t' - \frac{1}{x}\, t = x \qquad (.-2)$$

$$t' + \frac{2}{x}\, t = -\, 2x$$

$$\begin{cases} P(x) = \dfrac{2}{x} \\ Q(x) = -\, 2x \end{cases}$$

$$I(x) = e^{\int P(x) dx} = e^{\int \frac{2}{x}\, dx} = e^{2 \int \frac{1}{x}\, dx} = e^{2\, \ell n\, x} = e^{\ell n\, x^2} = x^2$$

$$t = \frac{1}{x^2} \int (-2x) . x^2\, dx = \frac{1}{x^2} \int (-2x^3)\, dx = -\frac{2}{x^2}\left(\frac{x^4}{4} + C\right)$$

$$t = -\frac{2}{x^2}\left(\frac{x^4}{4} + C\right) \quad , \quad \text{como} \quad t = y^{-2}$$

$$\boxed{y^{-2} = -\frac{2}{x^2}\left(\frac{x^4}{4} + C\right)}$$

EXERCÍCIOS DE CÁLCULO II - LISTA XXXIII

RESOLVER AS SEGUINTES EQUAÇÕES DIFERENCIAIS

1) $xy' + y = x^2y^3$

2) $2xy' - 5(2x^2 - 1)y = 5x^3y^{7/5}$

3) $3x^5y' + 14x^4y = \dfrac{2\cos x^3}{\sqrt{y}}$

4) $y' - \dfrac{2x}{1 + x^2}\,y = 2x\sqrt{y}$

5) $y' - \dfrac{4}{2x + 3}\,y = 2x\sqrt{y}$

6) $y' + \dfrac{1}{6x}\,y = \dfrac{y^4}{3x + 3}$

7) $y' + 2\left(\dfrac{3 - 2x}{2x - 1}\right)y = 2\sqrt{y}.e^{x^2}$

8) $(x - 1)(x^2 + 1)y' + 3(x^2 + 1)y = 3(x - 1)y^{2/3}$

9) $xy' + 2(1 + x.\mathrm{tg}\ x)y = 2x.\cos x.\sqrt{y}$

10) $3(2x + 1)y' + (2x + 3)y = e^{x^2}(2x + 1)y^{-2}$

11) $3xy' - 2y = 2x.\ln x.y^{5/2}$

12) $xy' - y = xy^3(1 + \ln x)$

13) $y' + \dfrac{1}{2x}\,y = (xy)^3 \mathrm{sen}\ x^3$

14) $y' - \dfrac{1}{2x}\,y = \dfrac{\cos(\ln x)}{2y}$

EQUAÇÕES DIFERENCIAIS LINEARES DE ORDEM N

$$\boxed{A_0y^{(n)} + A_1y^{(n-1)} + A_2y^{(n-2)} + \dots + A_{n-2}y'' + A_{n-1}y' + A_ny = B(x)}$$

SOLUÇÃO

a) Se $B(x) = 0 \longrightarrow$ Homogênea

P(D) admite raízes reais simples

$$\boxed{y = c_1e^{D_1x} + c_2e^{D_2x} + c_3e^{D_3x} + \dots + c_re^{D_rx}}$$

P(D) admite raízes reais múltiplas

$$\boxed{y = e^{Dx}(c_1 + c_2x + c_3x^2 + \dots + c_\alpha x^{\alpha-1}}$$

b) Se $B(x) \neq 0 \longrightarrow$ NÃO Homogênea

1º caso: $B(x) = K$ (K constante)

$$\boxed{y_2 = \frac{K}{A_n}}$$

2º caso: $B(x) = a\ e^{rx}$

$$\boxed{y_2 = \frac{a\ e^{rx}}{P(D=r)}}$$

3º caso: $B(x) = a_0x^r + a_1x^{r-1} + a_2x^{r-2} + \dots + a_r$

$$\boxed{y_2 = ax^r + bx^{r-1} + cx^{r-2} + \dots}$$

4º caso: $B(x) = a\ \text{sen}(px)$ ou $B(x) = b\cos(qx)$

$$\boxed{y_2 = \alpha\ \text{sen}(px) + \beta\cos(px)}$$ ou

$$\boxed{y_2 = \alpha\ \text{sen}(qx) + \beta\cos(qx)}$$

EXEMPLOS RESOLVIDOS. Resolver as equações

1) $y''' - 2y'' - 4y' + 8y = 0$

$$P(D) = D^3 - 2D^2 - 4D + 8$$

Raízes: $D^3 - 2D^2 - 4D + 8 = 0$

$$D^2(D-2) - 4(D-2) = 0$$

$$(D-2)(D^2-4) = 0$$

$$(D-2)(D+2)(D-2) = 0$$

$$(D-2)^2(D+2) = 0$$

$$\begin{cases} D_1 = D_2 = 2 \\ D_3 = -2 \end{cases}$$

$$\boxed{y = c_1 e^{-2x} + e^{2x}(c_2 + c_3 x)}$$

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2) $y'' - 13y' + 42y = 12$

a) Homogênea

$$P(D) = D^2 - 13D + 42$$

Raízes: $D^2 - 13D + 42 = 0$

$$(D-7)(D-6) = 0$$

$$\begin{cases} D_1 = 7 \\ D_2 = 6 \end{cases}$$

$$\boxed{y_1 = c_1 e^{7x} + c_2 e^{6x}}$$

b) NÃO Homogênea

$$y_2 = \frac{K}{A_n}$$

$$y_2 = \frac{12}{42}$$

$$\boxed{y_2 = \frac{2}{7}}$$

$$\boxed{y = c_1 e^{7x} + c_2 e^{6x} + \frac{2}{7}}$$

3) $y'' + y' - 2y = 2e^{3x}$

a) Homogênea

$P(D) = D^2 + D - 2$

Raízes: $D^2 + D - 2 = 0$

$(D + 2)(D - 1) = 0$

$\begin{cases} D_1 = -2 \\ D_2 = 1 \end{cases}$

$$\boxed{y_1 = c_1 e^{-2x} + c_2 e^{x}}$$

b) NÃO homogênea

$y_2 = \dfrac{a\, e^{rx}}{P(D=r)}$

$P(D=3) = 3^2 + 3 - 2 = 10$

$y_2 = \dfrac{2\, e^{3x}}{10}$

$$\boxed{y_2 = \frac{1}{5} e^{3x}}$$

$$\boxed{y = c_1 e^{-2x} + c_2 e^{x} + \frac{1}{5} e^{3x}}$$

-.-

4) $y'' - 3y' + 2y = 1 - 2x^2$

a) Homogênea

$P(D) = D^2 - 3D + 2$

Raízes: $D^2 - 3D + 2 = 0$

$(D - 2)(D - 1) = 0$

$\begin{cases} D_1 = 2 \\ D_2 = 1 \end{cases}$

$$\boxed{y_1 = c_1 e^{2x} + c_2 e^{x}}$$

b) NÃO homogênea

$y_2 = a x^2 + b x + c$

$y_2' = 2a x + b$

$y_2'' = 2a$

Substituindo no equação

$2a - 3(2a x + b) + 2(a x^2 + b x + c) = 1 - 2x^2$

$2a - 6ax - 3b + 2ax^2 + 2bx + 2c = 1 - 2x^2$

$$2ax^2 + (-6a + 2b)\,x + (2a - 3b + 2c) = 1 - 2x^2$$

$$\begin{cases} 2a = -2 & \text{(I)} \\ -6a + 2b = 0 & \text{(II)} \\ 2a - 3b + 2c = 1 & \text{(III)} \end{cases}$$

De (I): $2a = -2 \Longrightarrow \boxed{a = -1}$

De (II): $-6(-1) + 2b = 0 \Longrightarrow 2b = -6 \Longrightarrow \boxed{b = -3}$

De (III): $2(-1) - 3(-3) + 2c = 1 \Longrightarrow -2 + 9 + 2c = 1 \Longrightarrow 2c = -6$

$$\boxed{c = -3}$$

$$\boxed{y_2 = -x^2 - 3x - 3}$$

$$\boxed{y = c_1e^{2x} + c_2e^{x} - x^2 - 3x - 3}$$

-.-

5) $y'' - 2y' - 3y = 4\,\text{sen}\,2x$

a) Homogênea

$P(D) = D^2 - 2D - 3$

Raízes: $D^2 - 2D - 3 = 0$

$$(D - 3)(D + 1) = 0 \Longrightarrow \begin{cases} D_1 = 3 \\ D_2 = -1 \end{cases}$$

$$\boxed{y_1 = c_1e^{3x} + c_2e^{-x}}$$

b) NÃO homogênea

$y_2 = \alpha\text{sen}2x + \beta\cos 2x$

$y_2' = 2\alpha\cos 2x - 2\beta\text{sen}2x$

$y_2'' = -4\alpha\text{sen}2x - 4\beta\cos 2x$

$$(-4\alpha\text{sen}2x - 4\beta\cos 2x) - 2(2\alpha\cos 2x - 2\beta\text{sen}2x) - 3(\alpha\text{sen}2x + \beta\cos 2x) =$$
$$= 4\,\text{sen}2x$$

$$-4\alpha\text{sen}2x - 4\beta\cos 2x - 4\alpha\cos 2x + 4\beta\text{sen}2x - 3\alpha\text{sen}2x - 3\beta\cos 2x = 4\text{sen}2x$$

$$(-4\alpha + 4\beta - 3\alpha)\text{sen}2x + (-4\beta - 4\alpha - 3\beta)\cos 2x = 4\ \text{sen}2x$$

$$(-7\alpha + 4\beta)\text{sen}2x + (-4\alpha - 7\beta)\cos 2x = 4\ \text{sen}2x$$

$$\begin{cases} -7\alpha + 4\beta = 4 & (7) \\ -4\alpha - 7\beta = 0 & (4) \end{cases}$$

$$\begin{cases} -49\alpha + 28\beta = 28 \\ -16\alpha - 28\beta = 0 \end{cases}$$

$$-65\alpha = 28 \Longrightarrow \boxed{\alpha = -\frac{28}{65}}$$

$$-4\alpha - 7\beta = 0 \Longrightarrow -7\beta = 4\alpha \Longrightarrow 7\beta = -4\alpha$$

$$7\beta = -4\left(-\frac{28}{65}\right) \Longrightarrow \boxed{\beta = \frac{112}{455}}$$

$$\boxed{y_2 = -\frac{28}{65}\ \text{sen}2x + \frac{112}{455}\cos 2x}$$

$$\boxed{y = c_1e^{3x} + c_2e^{-x} - \frac{28}{65}\ \text{sen}2x + \frac{112}{455}\cos 2x}$$

-.-

EXERCÍCIOS DE CÁLCULO II - LISTA XXXIV

RESOLVER AS SEGUINTES EQUAÇÕES DIFERENCIAIS.

1) $y'' - 4y' + 3y = 0$

2) $y'' - 3y' + 2y = 0$

3) $y'' + y' - 6y = 0$

4) $y''' - y'' - 12y' = 0$

5) $y''' - 4y' = 0$

6) $y'' + 6y' + 9y = 0$

7) $y'' - 4y' + 4 = 0$

8) $y''' - 3y'' + 3y' - y = 0$

9) $y''' - 2y'' - 4y' + 8y = 0$

10) $y''' + 5y'' - y' - 5y = 0$

11) $y''' + 2y'' - 9y' - 18y = 0$

12) $y^{IV} - 5y''' + 6y'' = 0$

13) $y^{V} - 6y^{IV} + 9y''' = 0$

14) $y^{(5)} - 4y^{(4)} - 16y''' + 64y'' = 0$

15) $y'' - 4y' + 3y = 9$

16) $y'' - 8y' + 16y = -2$

17) $y''' - y' = 4$

18) $y''' - 4y' = 5$

19) $y^{IV} - y'' = 5$

20) $y^{IV} - y'' = 12$

21) $y''' - 2y'' + y' = 1$

22) $y''' - 6y'' + 9y' = 8$

23) $y'' - 4y' - 5y = 3e^{2x}$

24) $y'' - 4y' + 4y = 5e^{3x}$

25) $y'' - 2y' + y = 2e^{3x}$

26) $y'' - 10y' + 24y = 2e^{-x}$

27) $y'' - 6y' + 8y = 3e^{x} + 4$

28) $y''' + 4y'' - 5y' = 4e^{2x} - 5$

29) $y''' + y'' - 4\,y' - 4\,y = 20\,e^{x} + 16$

30) $y''' - 3\,y'' + 3\,y' - y = 5 + 4\,e^{3x}$

31) $y'' - 4\,y = 8x + 4$

32) $y'' - 2\,y' - 3\,y = 27x^2 - 1$

33) $y''' - y' = x^2 + 1$

34) $y'' - y = 2x^2 - 3x + 1$

35) $y'' - 2\,y' + y = 5x^2 + 6x + 13$

36) $y'' - 3\,y' + 2\,y = 8x^2 + e^{3x}$

37) $y'' - y = e^{2x} - 3x^2 + 5x - 1$

38) $y'' - y' - 2\,y = 10\,e^{x} + 30\,x^2 - 76\,x - 80$

39) $y'' - 4\,y' + 3\,y = 3\,\text{sen}\,x$

40) $y'' - 3\,y' + 2\,y = 3\,\cos 2x$

41) $y'' - y' - 6\,y = 4\,\cos 2x - 28\,\text{sen}\,2x$

42) $y'' - y' - 2\,y = 12 - 30\,\text{sen}\,3x - 6\,\cos 3x$

43) $y'' + 2\,y' - 15\,y = 84\,\cos 2x - 22\,\text{sen}\,2x - 45$

44) $y''' - 9\,y' = 12\,e^{4x} - \text{sen}\,2x + 20$

45) $y'' - 5\,y' - 14\,y = 4x^2 + 18\,e^{-x} + 10\,\cos x$

RESPOSTAS

LISTA I

1) $I = \frac{x^4}{4} + C$

2) $I = -\frac{1}{x} + C$

3) $I = \frac{4}{5} x^2\sqrt{x} + C$

4) $I = \frac{9}{7} x^2\sqrt[3]{x} + C$

5) $I = \frac{2a}{3} \operatorname{sen} x + C$

6) $I = -\frac{1}{5} \operatorname{cotg} x + C$

7) $I = \frac{1}{5} \operatorname{tg} x + C$

8) $I = 6ab \sec x + C$

9) $I = \frac{3}{7} \operatorname{arctg} x + C$

10) $I = e^x + \frac{3}{2} x^2 - 5 \ln x + C$

11) $I = \frac{3}{\ln 2} 2^x + 3 \cos x + C$

12) $I = 2 \operatorname{arcsen} x + C$

13) $I = \frac{1}{2} \operatorname{arcsen} x + C$

14) $I = \operatorname{arcsec} x + C$

15) $I = \frac{1}{5} x^5 + \frac{2}{3} x^3 + x - \ln x + C$

16) $I = \frac{2}{5} x^2\sqrt{x} - 2\sqrt{x} + C$

17) $I = -\operatorname{cotg} x + C$

18) $I = -\operatorname{cotg} x + x + C$

19) $I = -3 \operatorname{cotg} x + C$

20) $I = \operatorname{arctg} x + C$

21) $I = \operatorname{tg} x - x + C$

22) $I = \frac{1}{2} \operatorname{tg} x + C$

23) $I = 2 \sec x + C$

24) $I = \frac{1}{2}(x^2 + x) + C$

25) $I = \operatorname{arcsen} x + C$

26) $I = \frac{1}{12} x^3 - 2x - \frac{4}{x} + C$

27) $I = \frac{2}{3} x\sqrt{x} - x + C$

28) $I = \frac{1}{2} x^2 + \sqrt{3} x + C$

29) $I = \frac{1}{2}(x - \operatorname{sen} x) + C$

30) $I = \frac{1}{2}(x + \operatorname{sen} x) + C$

31) $I = 2x + C$

32) $I = \operatorname{tg} x - x + C$

33) $I = -\operatorname{cotg} x - x + C$

34) $I = \frac{1}{2} x^2 + \operatorname{arctg} x + C$

35) $I = \frac{1}{2} x^2 - x - \operatorname{arctg} x + C$

36) $I = \frac{1}{3} x^3 + x - \operatorname{arctg} x + C$

37) $I = \frac{1}{4} x^4 + x^2 - \operatorname{arctg} x + C$

38) $I = \frac{2}{3} x^3 - \frac{3}{2} x^2 + \operatorname{arctg} x + C$

LISTA II

1) $I = \frac{1}{63}(3x + 1)^{21} + C$

2) $I = \frac{1}{80}(5x - 1)^{16} + C$

3) $I = \frac{-1}{6(2x - 3)^9} + C$

4) $I = 2\sqrt{x + 5} + C$

5) $I = \sqrt{x^2 + 1} + C$

6) $I = \frac{2}{15}\sqrt{5x^3 + 1} + C$

7) $I = \frac{2}{3}(x^2 + x - 3)^{3/2} + C$

8) $I = \frac{2}{3}\sqrt{2x^3 + 3x + 1} + C$

9) $I = -\frac{1}{3(x-a)^3} + C$

10) $I = -\frac{1}{(2x-1)^2} + C$

11) $I = -\frac{1}{3(2x-5)^{3/2}} + C$

12) $I = -\frac{3\sqrt[3]{4}}{5(x-4)^{5/3}} + C$

13) $I = \frac{5}{56}(4x + 2)^{7/5} + C$

14) $I = \frac{2}{7}(x - 1)^{7/2} + C$

15) $I = \frac{1}{15}(4 - \frac{3}{x^2})^{5/2} + C$

16) $I = -\frac{3}{5}(\frac{1}{x} + b)^{5/3} + C$

17) $I = -\frac{3}{2}\ln(1 - 2\sqrt[3]{x}) + C$

18) $I = -a\,e^{-x/a} + C$

19) $I = \frac{1}{2}e^{2x-6} + C$

20) $I = \frac{1}{2}e^{x^2-1} + C$

21) $I = \frac{1}{4}a^{4x} + C$

22) $I = -2\cos\frac{x}{2} + C$

23) $I = -\frac{1}{2}\cos(x^2 + 2x) + C$

24) $I = -\frac{1}{12}\cos(6x^2 - 2) + C$

25) $I = -\frac{1}{2}\cos 2x + C$

26) $I = \text{sen}(e^x - 1) + C$

27) $I = \frac{1}{4}\text{sen}^2(2x) + C$ ou

$I = -\frac{1}{4}\cos^2(2x) + C$

28) $I = \frac{1}{6}\text{sen}^2(3x) + C$ ou

$I = -\frac{1}{6}\cos^2(3x) + C$

29) $I = -\ln(\cos x) + C$

30) $I = \ln(\text{sen}\,x) + C$

31) $I = -\ln[\cos(2a + x)] + C$

32) $I = \ln[\text{sen}(ax + b)] + C$

33) $I = \frac{1}{2\cos^2 x} + C$

34) $I = \ln(\ln x) + C$

35) $I = \frac{1}{3}\ln^3 x + C$

36) $I = \frac{1}{2}\ln^2(2x - 4) + C$

37) $I = \frac{1}{3a}\ln^3(ax + 1) + C$

38) $I = \frac{1}{2}\text{arctg}^2 x + C$

39) $I = -\frac{1}{2}\text{arccotg}^2 x + C$

40) $I = \frac{1}{2}\text{arcsen}^2 x + C$

41) $I = \text{arctg}(\ln x) + C$

42) $I = -\frac{2}{3}\cos(x\sqrt{x}) + C$

43) $I = \frac{1}{3}e^{2x\sqrt{x}} + C$

44) $I = -\frac{2}{3(a + x\sqrt{x})} + C$

45) $I = \frac{4}{9}(1 + x\sqrt{x})^{3/2} + C$

46) $I = \frac{1}{2}\text{arcsen } x^2 + C$

47) $I = a \text{ arcsen}\frac{x}{a} + C$

48) $I = \frac{(x + 1)^{12}}{12} - \frac{(x + 1)^{11}}{11} + C$

49) $I = \frac{(x - 2)^7}{7} + \frac{2(x - 2)^6}{3} + \frac{4(x - 2)^5}{5} + C$

50) $I = \frac{1}{3}(1 + x^2)^{3/2} - \sqrt{1 + x^2} + C$

51) $I = \frac{2}{5}(1 + x)^{5/2} - \frac{4}{3}(1 + x)^{3/2} + 2\sqrt{1 + x} + C$

52) $I = \frac{1}{5}(1 - x^2)^{5/2} - \frac{1}{3}(1 - x^2)^{3/2} + C$

53) $I = -\frac{1}{3}(1 - x^2)^{3/2} + \frac{2}{5}(1 - x^2)^{5/2} - \frac{1}{7}(1 - x^2)^{7/2} + C$

54) $I = \frac{1}{8}(x^3 + 1)^{8/3} - \frac{1}{5}(x^3 + 1)^{5/3} + C$

55) $I = \frac{1}{10}(x^4 + 2)^{5/2} - \frac{1}{3}(x^4 + 2)^{3/2} + C$

56) $I = \frac{1}{20}(2x^2 + 4)^{5/2} - \frac{1}{3}(2x^2 + 4)^{3/2} + C$

57) $I = \frac{1}{45}(3x^2 - 6)^{5/2} + \frac{2}{9}(3x^2 - 6)^{3/2} + C$

58) $I = 2\sqrt{x} - 2\ln(1 + \sqrt{x}) + C$

59) $I = x - 2\sqrt{x} + 2\ln(\sqrt{x} + 1)\ C$

60) $I = -\dfrac{3}{4(2 + x\sqrt[3]{x})} + C$

61) $I = 2\ \mathrm{arctg}\sqrt{x} + C$

62) $I = 2\ln(\sqrt{x+1} + 1) + C$

63) $I = \frac{2}{3}\left[\dfrac{(x^3+4)\sqrt{x^3+4}}{3} + (x^3+4)\right] + C$

64) $I = 4\ln\sqrt{x+1} + \dfrac{2}{x+1} - \dfrac{2}{\sqrt{x+1}} + C$

LISTA III

1) $I = \frac{1}{7}e^{7x+2} + C$

2) $I = -e^{-x} + C$

3) $I = e^{ax} + C$

4) $I = e^{\mathrm{tg}\,x} + C$

5) $I = \ln(x - 1) + C$

6) $I = \frac{3}{2}\ln(2x + 5) + C$

7) $I = \frac{1}{3}\ln(1 + x^3) + C$

8) $I = \ln(x^3 - 5x + 7) + C$

9) $I = \ln(x^3 - 5x^2 + 6x - 8) + C$

10) $I = \frac{1}{8}\ln(x^2 - 4) + C$

11) $I = 4\ln(2x + 3) + C$

12) $I = \frac{1}{2}\,\mathrm{sen}(2x - 5) + C$

13) $I = -\frac{1}{2ab}\cos(2abx - 1) + C$

14) $I = \frac{1}{3}\,\mathrm{sen}(3x + 1) + C$

15) $I = \mathrm{sen}(x - 1) + C$

16) $I = \frac{1}{3}\,\mathrm{tg}(3x - 9) + C$

17) $I = \frac{1}{4}\,\mathrm{tg}(4x + 1) + C$

18) $I = \frac{1}{2}\,\mathrm{tg}(2x + 1) + C$

19) $I = \ln(2 + \mathrm{tg}\,x) + C$

20) $I = -\ln\left[1 + \ln(\cos x)\right] + C$

21) $I = \frac{1}{8}\,\mathrm{tg}^4(2x) + C$

22) $I = \frac{1}{3}\sec^2(3x + a) + C$

23) $I = -\frac{1}{a}\,\mathrm{cotg}(ax) + C$

24) $I = \frac{1}{2}\,\mathrm{tg}(x + 2) + C$

25) $I = -\frac{1}{6}\,\mathrm{cotg}(3x - 4ab) + C$

LISTA IV

1) $I = \frac{1}{5} \ln(5x^3 - 1) + C$

2) $I = \frac{1}{3} \ln(x^3 + 2x - 13) + C$

3) $I = \ln(x + 1) + C$

4) $I = \frac{3}{5} \ln(x-4) - \frac{3}{5} \ln(x+1) + C$

5) $I = \ln(x + 1) - \ln(x + 3) + C$

6) $I = 2 \ln(x - 2) - \frac{8}{x - 2} + C$

7) $I = \ln(x + 1) + \frac{2}{x + 1} + C$

8) $I = 2 \ln x - \ln(x + 1) + C$

9) $I = \frac{1}{3} \ln x - \frac{1}{3} \ln(x + 3) + C$

10) $I = -\frac{1}{2} \ln x + \frac{1}{2} \ln(x - 2) + C$

11) $I = -\frac{1}{2} \ln(x+1) + \frac{1}{2} \ln(x-1) + C$

12) $I = 2 \ln(x-5) + 3 \ln(x-2) + C$

13) $I = -3 \ln(x-2) + 5 \ln(x-3) + C$

14) $I = \frac{1}{12} \ln(x-1) + \frac{3}{16} \ln(x-3) - \frac{13}{48} \ln(x-5) + C$

15) $I = \frac{5}{4} \ln(x-1) - \frac{5}{4} \ln(x+1) + \frac{1}{2(x-1)} + C$

16) $I = \frac{1}{4} \ln(x-1) - \frac{1}{4} \ln(x+1) - \frac{1}{2(x+1)} + C$

17) $I = \ln(x+1) - \ln x - \frac{1}{x} - \frac{1}{x + 1} + C$

18) $I = x^2 + x - \ln(x + 1) + C$

19) $I = \frac{1}{2} x^2 + x - \frac{1}{2} \ln(x^2 + 1) + C$

20) $I = \frac{1}{2} x^2 + \ln(x^2 + 2x - 3) + C$

21) $I = t^2 + \ln t - \ln(t - 1) + C$

22) $I = \frac{1}{2} x^2 - \ln(x-3) + 2 \ln(x+2) + C$

23) $I = \frac{1}{2} x^2 - \frac{3}{11} \ln(x+7) + \frac{3}{11} \ln(x-4) + C$

24) $I = \sqrt{1 + 4x^2} - \frac{1}{2} \ln(\sqrt{1 + 4x^2} + 1) + \frac{1}{2} \ln(\sqrt{1 + 4x^2} - 1) + C$

25) $I = \sqrt{1 - x^2} + \frac{1}{2} \ln(1 + \sqrt{1 - x^2}) - \frac{1}{2} \ln(1 - \sqrt{1 - x^2}) + C$

LISTA V

1) $I = -x\cos x + \operatorname{sen} x + C$

2) $I = x\operatorname{sen} x + \cos x + C$

3) $I = e^{x}(x-1) + C$

4) $I = x\operatorname{tg} x + \ln(\cos x) + C$

5) $I = -x\operatorname{cotg} x + \ln(\operatorname{sen} x) + C$

6) $I = \frac{1}{4}\operatorname{sen}(2x) - \frac{1}{2}x\cos(2x) + C$

7) $I = \frac{3}{5}x\operatorname{sen}(5x) + \frac{3}{25}\cos(5x) + C$

8) $I = \frac{1}{3}e^{3x}(x - \frac{1}{3}) + C$

9) $I = -e^{-2x}(x + \frac{1}{2}) + C$

10) $I = \frac{2}{3}x\operatorname{tg}(3x) + \frac{2}{9}\ln[\cos(3x)] + C$

11) $I = \frac{1}{2}x^2(\ln x - \frac{1}{2}) + C$

12) $I = \frac{1}{4}x^4(\ln x - \frac{1}{4}) + C$

13) $I = \frac{2}{3}x\sqrt{x}(\ln x - \frac{2}{3}) + C$

14) $I = -\frac{1}{3x^3}(\ln x + \frac{1}{3}) + C$

15) $I = x\operatorname{arctg} x - \frac{1}{2}\ln(1 + x^2) + C$

16) $I = x\operatorname{arccotg} x + \frac{1}{2}\ln(1 + x^2) + C$

17) $I = x\operatorname{arcsen} x + \sqrt{1 - x^2} + C$

18) $I = x\arccos(3x) - \frac{1}{3}\sqrt{1 - 9x^2} + C$

19) $I = -x^2\cos x + 2x\operatorname{sen} x + 2\cos x + C$

20) $I = \frac{1}{a}x^2\operatorname{sen}(ax) + \frac{2}{a^2}x\cos(ax) - \frac{2}{a^3}\operatorname{sen}(ax) + C$

21) $I = e^{x}(x^2 - 2x + 2) + C$

22) $I = -e^{-x}(x^2 + 2x + 2) + C$

23) $I = e^{x}(x^3 - 3x^2 + 6x - 6) + C$

24) $I = x^3\operatorname{sen} x + 3x^2\cos x - 6x\operatorname{sen} x - 6\cos x + C$

25) $I = \frac{1}{2}\left[\sec x.\operatorname{tg} x + \ln(\sec x + \operatorname{tg} x)\right] + C$

26) $I = -\frac{1}{2}\left[\operatorname{cosec} x . \operatorname{cotg} x + \ln(\operatorname{cosec} x + \operatorname{cotg} x)\right] + C$

27) $I = \frac{1}{2} e^{x}(\operatorname{sen} x + \cos x) + C$

28) $I = -\frac{1}{5} e^{x}\left[2 \cos(2x) - \operatorname{sen}(2x)\right] + C$

29) $I = \frac{1}{25} e^{-4x}\left[3 \operatorname{sen}(3x) - 4 \cos(3x)\right] + C$

30) $I = \frac{-3}{8} \operatorname{sen} x . \cos(3x) + \frac{1}{8} \cos x . \operatorname{sen}(3x) + C$

31) $I = \frac{7}{33} \cos(4x) . \operatorname{sen}(7x) - \frac{4}{33} \cos(7x) . \operatorname{sen}(4x) + C$

32) $I = \frac{1}{3}\left[\operatorname{sen}(4x) . \operatorname{sen}(2x) + \frac{1}{2} \cos(4x) . \cos(2x)\right] + C$

33) $I = \frac{1}{4} x^2 + \frac{1}{4} x \operatorname{sen}(2x) + \frac{1}{8} \cos(2x) + C$

34) $I = \frac{1}{4} x^2 - \frac{1}{4} x \operatorname{sen}(2x) - \frac{1}{8} \cos(2x) + C$

35) $I = \frac{1}{2} x^2 \ln(x + 1) - \frac{1}{4} x^2 + \frac{1}{2} x - \frac{1}{2} \ln(x + 1) + C$

36) $I = x \ln(x^2 + 1) - 2x + 2 \operatorname{arctg} x + C$

37) $I = \frac{1}{3} x^3 \operatorname{arcsen} x + \frac{1}{3}\sqrt{1 - x^2} - \frac{1}{9}(1 - x^2)^{3/2} + C$

38) $I = \frac{1}{3} x^3 \operatorname{arctg} x - \frac{1}{6} x^2 + \frac{1}{6} \ln(1 + x^2) + C$

LISTA VI

1) $I = -\frac{1}{3} \cos^3 x + \frac{1}{5} \cos^5 x + C$

2) $I = \frac{1}{3} \operatorname{sen}^3 x - \frac{1}{5} \operatorname{sen}^5 x + C$

3) $I = -\frac{1}{5} \cos^5 x + \frac{1}{7} \cos^7 x + C$

4) $I = -\frac{1}{13} \cos^{13} x + \frac{1}{15} \cos^{15} x + C$

5) $I = \frac{1}{7} \operatorname{sen}^7 x - \frac{2}{9} \operatorname{sen}^9 x + \frac{1}{11} \operatorname{sen}^{11} x + C$

6) $I = \frac{1}{2}\left[\frac{1}{3} \operatorname{sen}^3(2x) - \frac{1}{5} \operatorname{sen}^5(2x)\right] + C$

7) $I = \frac{1}{3}\left[\frac{1}{14} \operatorname{sen}^{14}(3x) - \frac{1}{8} \operatorname{sen}^{16}(3x) + \frac{1}{18} \operatorname{sen}^{18}(3x)\right] + C$

8) $I = -\frac{1}{5}\left[\frac{1}{9} \cos^9(5x) - \frac{2}{11} \cos^{11}(5x) + \frac{1}{13} \cos^{13}(5x)\right] + C$

9) $I = \frac{1}{3} \cos^3 x - \cos x + C$

10) $I = \frac{1}{2} \operatorname{sen}(2x) - \frac{1}{6} \operatorname{sen}^3(2x) + C$

11) $I = -\frac{1}{3}\left[\cos(3x) - \frac{2}{3}\cos^3(3x) + \frac{1}{5}\cos^5(3x)\right] + C$

12) $I = \frac{1}{5}\left[\text{sen}(5x) - \frac{2}{3}\text{sen}^3(5x) + \frac{1}{5}\text{sen}^5(5x)\right] + C$

13) $I = -\cos x + \cos^3 x - \frac{3}{5}\cos^5 x + \frac{1}{7}\cos^7 x + C$

14) $I = \text{sen}\, x - \text{sen}^3 x + \frac{3}{5}\text{sen}^5 x - \frac{1}{7}\text{sen}^7 x + C$

15) $I = \frac{1}{2}\left[x - \frac{1}{2}\text{sen}(2x)\right] + C$

16) $I = \frac{1}{2}\left[x + \frac{1}{2}\text{sen}(2x)\right] + C$

17) $I = \frac{3}{8}x + \frac{1}{8}\text{sen}(4x) + \frac{1}{64}\text{sen}(8x) + C$

18) $I = \frac{3}{8}x - \frac{1}{12}\text{sen}(6x) + \frac{1}{96}\text{sen}(12x) + C$

19) $I = \frac{5}{16}x + \frac{1}{4}\text{sen}(2x) + \frac{3}{64}\text{sen}(4x) - \frac{1}{48}\text{sen}^3(2x) + C$

20) $I = \frac{5}{16}x - \frac{1}{8}\text{sen}(4x) + \frac{3}{128}\text{sen}(8x) + \frac{1}{96}\text{sen}^3(4x) + C$

21) $I = \frac{1}{3}\ln\left[\sec(3x) + \text{tg}(3x)\right] + C$

22) $I = -\frac{1}{2a}\ln\left[\text{cosec}(2ax) + \text{cotg}(2ax)\right] + C$

23) $I = \frac{1}{2}\ln\left[\sec(2x+5) + \text{tg}(2x+5)\right] + C$

24) $I = -\frac{1}{3}\ln\left[\text{cosec}(3x+2) + \text{cotg}(3x+2)\right] + C$

25) $I = \frac{1}{2ab}\text{tg}(2abx + 7c) + C$

26) $I = \text{tg}\, x + \frac{1}{3}\text{tg}^3 x + C$

27) $I = -\frac{1}{2}\left[\text{cotg}(2x) + \frac{1}{3}\text{cotg}^3(2x)\right] + C$

28) $I = \frac{1}{2}\text{tg}(2x) + \frac{1}{3}\text{tg}^3(2x) + \frac{1}{10}\text{tg}^5(2x) + C$

29) $I = -\text{cotg}\, x - \frac{2}{3}\text{cotg}^3 x - \frac{1}{5}\text{cotg}^5 x + C$

30) $I = \text{tg}\, x + \text{tg}^3 x + \frac{3}{5}\text{tg}^5 x + \frac{1}{7}\text{tg}^7 x + C$

31) $I = -\frac{1}{2}\ln\left[\cos(2x)\right] + C$

32) $I = \frac{1}{a}\ln\left[\text{sen}(ax)\right] + C$

33) $I = \frac{1}{8\cos^2(4x)} + \frac{1}{4}\ln\left[\cos(4x)\right] + C$

34) $I = -\dfrac{1}{4a\ \text{sen}^2(2ax)} - \ln\left[\text{sen}(2ax)\right] + C$

35) $I = \dfrac{-1}{4\ \text{sen}^4 x} + \dfrac{1}{\text{sen}^2 x} + \ln(\text{sen}\, x) + C$

36) $I = \dfrac{-1}{6\ \text{sen}^6 x} + \dfrac{3}{4\ \text{sen}^4 x} - \dfrac{3}{2\ \text{sen}^2 x} - \ln(\text{sen}\, x) + C$

37) $I = \text{tg}\, x - x + C$

38) $I = -\frac{1}{5}\ \text{cotg}(5x) - x + C$

39) $I = \frac{1}{3}\ \text{tg}^3 x - \text{tg}\, x + x + C$

40) $I = \frac{1}{5}\ \text{tg}^5 x - \frac{1}{3}\ \text{tg}^3 x + \text{tg}\, x - x + C$

41) $I = \frac{1}{4}\ \text{tg}^4 x + \frac{1}{6}\ \text{tg}^6 x + C$

42) $I = \ln(\text{tg}\, x) - \dfrac{1}{2\ \text{tg}^2 x} + C$

43) $I = \frac{1}{4}\ \text{tg}(2x) + \frac{1}{8}\ \text{tg}^4(2x) + C$

44) $I = \dfrac{1}{5\ \cos^5 x} - \dfrac{1}{3\ \cos^3 x} + C$

45) $I = \dfrac{1}{\cos^3 x} + C$

46) $I = -\dfrac{1}{\text{sen}^3(\frac{x}{3})} + C$

47) $I = \text{sen}^3 x + C$

48) $I = -\frac{2}{5}\ \cos^5 x + C$

49) $I = -\cos x + \frac{2}{3}\ \cos^3 x + C$

50) $I = \frac{1}{3}\left[\frac{1}{14}\ \text{sen}^{14}(3x+1) - \frac{1}{16}\ \text{sen}^{16}(3x+1)\right] + C$

51) $I = \frac{1}{10}\ \text{tg}^2(5x+1) + C$

LISTA VII

1) $I = \frac{1}{4}\left[\text{arcsen}(2x) + 2x\sqrt{1 - 4x^2}\right] + C$

2) $I = \frac{1}{2}\left[x\sqrt{x^2+1} + \ln(\sqrt{x^2+1} + x\right] + C$

3) $I = \frac{1}{2}\left[x\sqrt{x^2-1} - \ln(x + \sqrt{x^2-1})\right] + C$

4) $I = \frac{25}{2}\left[\operatorname{arcsen}\frac{x}{5} + \frac{1}{25}\,x\sqrt{25 - x^2}\right] + C$

5) $I = -\sqrt{3}\left[\frac{\sqrt{1 - 3x^2}}{\sqrt{3}\,x} + \operatorname{arcsen}(\sqrt{3}\,x)\right] + C$

6) $I = -\frac{\sqrt{4 - x^2}}{x} - \operatorname{arcsen}(\frac{x}{2}) + C$

7) $I = -\frac{\sqrt{9 - x^2}}{x} - \operatorname{arcsen}(\frac{x}{3}) + C$

8) $I = \frac{9}{2}\left[\frac{x\sqrt{x^2+9}}{9} - \ln(\frac{\sqrt{x^2 + 9} + x}{3})\right] + C$

9) $I = \ln(\frac{\sqrt{x^2 + 9}}{3} + \frac{x}{3}) + C$

10) $I = -\ln(\frac{\sqrt{1 + x^2} + 1}{x}) + C$

11) $I = -\frac{1}{2}\ln(\frac{\sqrt{4 - x^2} + 2}{x}) + C$

12) $I = \frac{9}{2}\left[\operatorname{arcsen}(\frac{x}{3}) - \frac{1}{9}\,x\sqrt{9 - x^2}\right] + C$

13) $I = \ln(\frac{x + \sqrt{x^2 - 25}}{5}) - \frac{\sqrt{x^2 - 25}}{x} + C$

14) $I = 8\left[\frac{x\sqrt{x^2 - 16}}{16} + \ln(\frac{x + \sqrt{x^2 - 16}}{4})\right] + C$

15) $I = \sqrt{2x^2 - 1} - \operatorname{arcsec}(\sqrt{2}\,x) + C$

16) $I = \frac{1}{54}\left[\operatorname{arcsec}(\frac{x}{3}) + \frac{3\sqrt{x^2 - 9}}{x^2}\right] + C$

17) $I = \frac{1}{2}\left[e^x\sqrt{e^{2x} + 1} + \ln(\sqrt{e^{2x} + 1} + e^x)\right] + C$

18) $I = \ln(\frac{x}{\sqrt{x^2 + 1}} + 1) + C$

19) $I = \frac{1}{2}\left[(x + 1)\sqrt{x^2 + 2x + 2} + \ln(\sqrt{x^2 + 2x + 2} + x + 1)\right] + C$

20) $I = \frac{1}{2}\left[(x - 1)\sqrt{x^2 - 2x} - \ln(x - 1 + \sqrt{x^2 - 2x})\right] + C$

21) $I = \frac{x - 3}{\sqrt{x^2 - 6x + 10}} + C$

22) $I = \frac{1}{2}\left[(x - 3)\sqrt{x^2 - 6x + 8} - \ln(x - 3 + \sqrt{x^2 - 6x + 8})\right] + C$

23) $I = \sqrt{x^2 - 2x + 10} + \ln(\frac{\sqrt{x^2 - 2x + 10} + x - 1}{3}) + C$

LISTA VIII

1) $\frac{1}{2}$

2) $\frac{1}{3}\ln\frac{9}{2}$

3) $\frac{2}{9}(3\sqrt{6}-4)$

4) $\sqrt{2}-1$

5) $\frac{1}{5}\{(3+e)^5-4^5\}$

6) $\frac{2+\pi}{4}$

7) e

8) $\frac{1}{2}(8\ln 2-3)$

9) $\frac{116}{15}$

10) $-\frac{1}{4}\ln 3$

11) $\frac{2}{99}$

12) $\frac{2}{35}(6\sqrt{2}+1)$

LISTA IX

1) $\frac{8}{3}$

2) 2

3) $\frac{1}{2}(e^4-1)$

4) $\frac{3}{2}\ln 3$

5) $\frac{1}{2}+\ln 2$

6) $\frac{1}{6e^6}(5e^6-2)$

7) $\frac{9}{2}$

8) $\frac{19}{3}$

9) $\frac{5}{4}$

10) $\frac{e+2}{2e}$

11) $\frac{125}{6}$

12) $\frac{20}{3}$

13) $\frac{5}{2}$

14) $\frac{8\sqrt{2}+7}{6}$

15) 1

16) $\frac{7}{3}$

LISTA X

1) $\frac{4}{3}$

2) $\frac{4}{3}$

3) $\frac{1}{24}$

4) $\frac{1}{2}$

5) $\frac{37}{12}$

6) $\frac{1}{2}$

7) $\frac{1}{2}$

8) 1

9) $\frac{96}{5}$

10) $\frac{37}{12}$

11) $\frac{1}{2}$

LISTA XI

1) $\frac{\operatorname{sen}\pi^2}{2\pi}$

2) $\frac{e^2+6e-7}{2}$

3) $\frac{\pi}{4(e-1)}$

4) $\frac{4}{9\pi}$

5) $\frac{4}{3\pi}$

6) $\frac{8}{9\pi}$

7) $\frac{2}{9(e-1)}$

8) $\frac{2}{e-1}$

9) $\frac{7\ln 7-4\ln 4-3}{3}$

10) $\frac{2\ln 2-1}{2}$

11) $\frac{e}{3}$

12) $\frac{8\sqrt{3}}{\pi}$

LISTA XII

1) $\frac{2}{3}(2\sqrt{2} - 1)$
2) $\frac{1}{27}(31\sqrt{31} - 22\sqrt{22})$
3) $\frac{1}{12}(13\sqrt{13} - 5\sqrt{5})$
4) $\frac{33}{16}$
5) $\frac{17}{12}$
6) $2\sqrt{3}$
7) $\frac{e^2 - 1}{2e}$
8) $1 + \ln\frac{3}{2}$
9) $\frac{1}{4}(e^2 + 1)$
10) $\frac{1}{24}(16 + 3\pi)$
11) $\frac{1}{2}(\frac{3}{2} + \ln 2)$
12) $\ln(\sqrt{3} + 2)$
13) $\ln(\sqrt{2} + 1)$
14) $\ln(\sqrt{2} + 1)$
15) $\frac{5}{3}$
16) 8
17) $\frac{\pi}{6}$

LISTA XIII

1) $10\pi\sqrt{3}$
2) $\frac{\pi}{9}(17\sqrt{17} - 2\sqrt{2})$
3) $\frac{\pi}{27}(730\sqrt{730} - 1)$
4) $\frac{\pi}{30}(53\sqrt{53} - 33\sqrt{33})$
5) $\frac{61\pi}{18}$
6) $\frac{112\pi}{3}$
7) $\frac{49\pi}{3}$
8) $\frac{196\pi}{3}$
9) $\frac{\pi}{4}(e^2 + 4 - e^{-2})$
10) $\frac{\pi}{4}(\frac{27}{4} - 4\ln 2 - \ln^2 2)$
11) $\pi(e + 1)$
12) π
13) $\frac{47\pi}{16}$
14) $\frac{1179\pi}{256}$

LISTA XIV

1) $\frac{\pi}{26}(3^{13} - 1)$
2) $\pi(\frac{e^2}{2} + 2e - \frac{3}{2})$
3) $\frac{\pi}{2}$
4) $\frac{\pi e^2}{6}(e^{14} - 1)$
5) $\frac{3\pi}{2}$
6) $\frac{\pi}{6}(2e^3 + 3e^2 - 5)$
7) $\frac{5\pi}{12}$
8) $4\pi(2 - \sqrt{2})$
9) 2π
10) $\frac{2\pi}{9}\{(e+1)\sqrt{e+1} - 2\sqrt{2})\}$
11) $\frac{58\pi}{15}$
12) $\frac{\pi}{4}(\pi - 2)$
13) $\frac{3\pi}{4}(\frac{\pi}{4} + \ln\frac{\sqrt{2}}{2})$
14) $\frac{\pi}{8}\ln\frac{9}{5}$
15) $\frac{\pi^2}{18}$
16) 2π
17) $\pi(\frac{1}{2} + 2\ln 3 - 3\ln 2)$
18) $\frac{\pi}{4}(21 + 13\ln 2 - 5\ln 3)$

LISTA XV

1) $R = \dfrac{(e^2 + 1)^{3/2}}{e^2}$

2) $R = 2\sqrt{2}$

3) $R = 1/2$

4) $R = \dfrac{2(1 + e^2)^{3/2}}{e\pi}$

5) $R = \dfrac{1}{2e^2}(e^2 + 4)^{3/2}$

6) $R = \dfrac{\{1 + e^{\pi}(2 \ln \sqrt{2}/2 + 1)^2\}^{3/2}}{2e^{\pi/2}(2 \ln \sqrt{2}/2 + 1)}$

7) $R = \dfrac{(1 + \ln^2 3)^{3/2}}{\ln^2 3}$

8) $R = 1/2$

9) $R = \dfrac{(1 + e^2 \ln^2 2)^{3/2}}{2e \ln^2 2}$

10) $R = 1/9e$

11) $K = 0$

12) $R = \dfrac{4(\pi^2 + e^2)^{3/2}}{e\pi(4 - \pi^2)}$

13) $R = 1/4$

14) $R = \dfrac{5\sqrt{10}}{9}$

LISTA XVI

1) $y - 2 = -2(x - 1)$ e $y - 2 = \frac{1}{2}(x - 1)$

2) $y - \frac{2}{e} = -\frac{1}{e}(x - 0)$ e $y - \frac{2}{e} = e(x - 0)$

3) $y - 3 = \frac{1}{e}(x - e^2)$ e $y - 3 = -e(x - e^2)$

4) $y - 2 = \frac{9}{4}(x - 2)$ e $y - 2 = -\frac{4}{9}(x - 2)$

LISTA XVII

1) e

2) $61/27$

3) $11/5$

4) $7/3$

5) $10/3$

6) $13/3$

7) $e + 1$

8) $\sqrt{2}(e^{\pi/2} - 1)$

9) $27/2$

10) $2(e^{\pi/2} - 1)$

11) 4

12) π

13) $8r$

14) 12

15) $6a$

16) $8/7$

17) $3/2$

18) $\ln(\sqrt{2} + 1)$

19) $116/15$

20) 2

21) $\ln 2 + \frac{3}{2}$

LISTA XVIII

1) 40π
2) $2\pi(\pi + 8\sqrt{2})$
3) $\frac{16\pi}{55}$
4) $\frac{2\sqrt{2}\pi}{5}(e^{\pi} - 2)$
5) $\frac{256\pi}{15}$
6) $2a^2\pi$
7) $\frac{\pi}{e^2}(e^4 + 4e^2 - 1)$
8) $\frac{\pi}{27}(145\sqrt{145} - 10\sqrt{10})$
9) $\frac{58\pi}{15}$
10) $\sqrt{2}\pi(e^2 + 2e - 3)$
11) $\frac{4\pi}{3}(e^3 + 3e - 4)$
12) $\frac{232\pi}{15}$

LISTA XIX

1) $\frac{\pi}{2}\ln 2$
2) $2\pi \ln\frac{3}{2}$
3) $\frac{\pi}{4}(4 - \pi)$
4) $\pi(\pi - 2)$
5) $\pi(\sqrt{e + 1} - \sqrt{2})$
6) $\frac{\pi e^2}{2}(e^2 - 1)$
7) $\frac{76\pi}{15}$
8) $\frac{\pi^3}{32}$
9) $\frac{298\pi}{15}$
10) $\frac{\pi}{4}$
11) $\frac{\pi}{2}(2 + \ln 5 - \ln 3)$
12) $\frac{\pi}{2}$

LISTA XX

1) $R = 2$
2) $R = \frac{(1 + 16e^4)^{3/2}}{6e^4}$
3) $R = 2$
4) $R = \frac{(16e^4 + 1)^{3/2}}{20e^4}$
5) $R = \frac{(4e^2 + 1)^{3/2}}{4e}$
6) $R = \frac{(9 + 4e^2)^{3/2}}{3e}$
7) $R = 4r$
8) $R = \frac{\ln 2}{2}$

LISTA XXI

1) $\frac{1}{8}(\pi + 2)$
2) $\frac{1}{12}(\pi + 2)$
3) $\frac{\pi}{4}$
4) $\frac{1}{5}(e^{\pi/2} + 1)$
5) $\frac{1}{8}(2\pi e^{2\pi} - e^{2\pi} + 1)$
6) $\frac{1}{2}(\text{arctg } e^{\pi/2} - \frac{\pi}{4})$
7) a^2
8) $\frac{\pi a^2}{4}$
9) $\frac{2}{3}(3\sqrt{3} - \pi)$
10) $\frac{1}{3}$
11) $\frac{1}{3}\left[(e+1)\sqrt{e+1} - 2\sqrt{2}\right]$

LISTA XXII

1) 4

2) 4π

3) $8a$

4) $\sqrt{2}(e - 1)$

5) $\frac{\sqrt{5}}{2}\, a(e - 1)$

6) $\frac{19}{3}$

7) $\frac{1}{3}(16\sqrt{2} - 5\sqrt{5})$

LISTA XXIII

1) $\begin{cases} \frac{\partial z}{\partial x} = 2xy + 2y^3 - 2 \\ \frac{\partial z}{\partial y} = x(x + 6y^2) \end{cases}$

2) $\begin{cases} \frac{\partial z}{\partial x} = y^2(e^{xy^2} - 2y^2 \operatorname{sen}(2xy^4)) \\ \frac{\partial z}{\partial y} = 2xy(e^{xy^2} - 4y^2 \operatorname{sen}(2xy^4)) \end{cases}$

3) $\begin{cases} \frac{\partial z}{\partial x} = \frac{y^2 - x^2}{x(x^2 + y^2)} \\ \frac{\partial z}{\partial y} = \frac{x^2 - y^2}{y(x^2 + y^2)} \end{cases}$

4) $\begin{cases} \frac{\partial z}{\partial x} = \frac{1}{y}\, e^{\operatorname{sen}(x/y)}.\cos(x/y) \\ \frac{\partial z}{\partial y} = -\frac{x}{y^2}\, e^{\operatorname{sen}(x/y)}.\cos(x/y) \end{cases}$

5) $\begin{cases} \frac{\partial z}{\partial x} = \frac{26}{25} \\ \frac{\partial z}{\partial y} = -\frac{29}{5} \end{cases}$

6) $\begin{cases} \frac{\partial z}{\partial x} = 0 \\ \frac{\partial z}{\partial y} = \frac{1}{e} \end{cases}$

7) $\begin{cases} \frac{\partial z}{\partial x} = \operatorname{arctg}\frac{2y}{x^2} - \frac{4x^2y}{x^4 + 4y^2} \\ \frac{\partial z}{\partial y} = \frac{2x^3}{x^4 + 4y^2} \end{cases}$

8) $\begin{cases} \frac{\partial z}{\partial x} = \frac{6xy^5}{(x^2 + y^2)^2} \\ \frac{\partial z}{\partial y} = \frac{3x^2y^2(3x^2 + y^2)}{(x^2 + y^2)^2} \end{cases}$

9) $\begin{cases} \frac{\partial z}{\partial x} = e^{\operatorname{sen}x}(\cos x.\operatorname{tg}(3x^3y^2) + 9x^2y^2. \\ \qquad .\sec^2(3x^3y^2)) \\ \frac{\partial z}{\partial y} = 6x^3y\, e^{\operatorname{sen}x}.\sec^2(3x^3y^2) \end{cases}$

10) $\begin{cases} \frac{\partial z}{\partial x} = 2.3^4 \ln 3 + 1 \\ \frac{\partial z}{\partial y} = 8.3^4 \ln 3 + 2 \end{cases}$

11) $\frac{\partial z}{\partial x} = 9$ e $\frac{\partial z}{\partial y} = -3$

12) $\frac{\partial w}{\partial x} = 18$; $\frac{\partial w}{\partial y} = \frac{9}{2}$ e $\frac{\partial w}{\partial z} = 4$

13) $\frac{\partial z}{\partial x} = -\frac{3}{25}$ e $\frac{\partial z}{\partial y} = \frac{4}{25}$

14) $\begin{cases} \frac{\partial w}{\partial x} = ze^{xz}(\operatorname{tg}(3xy^2z^2) + 3y^2z\,. \\ \qquad .\sec^2(3xy^2z^2)) \\ \frac{\partial w}{\partial y} = 6xyz^2e^{xz}\sec^2(3xy^2z^2) \\ \frac{\partial w}{\partial z} = xe^{xz}(\operatorname{tg}(3xy^2z^2) + 6y^2z. \\ \qquad .\sec^2(3xy^2z^2)) \end{cases}$

15) $\frac{\partial w}{\partial x} = 3e$; $\frac{\partial w}{\partial y} = 2$ e $\frac{\partial w}{\partial z} = \frac{e}{2}$

16) $\begin{cases} \frac{\partial z}{\partial x} = e^{\pi/4} + 1 \\ \frac{\partial z}{\partial y} = \frac{\pi}{4}(e^{\pi/4}.\ln\frac{\pi}{4} - 1) \end{cases}$

LISTA XXIV

1) $\begin{cases} \dfrac{\partial^2 z}{\partial x^2} = 2 \\ \dfrac{\partial^2 z}{\partial y^2} = 4 \\ \dfrac{\partial^2 z}{\partial x \partial y} = \dfrac{\partial^2 z}{\partial y \partial x} = 3 \end{cases}$

2) $\dfrac{\partial^2 z}{\partial y \partial x} = -\, e^x \operatorname{sen} y$

3) $\dfrac{\partial^2 z}{\partial x \partial y} = 6(e + 1)$

4) $\dfrac{\partial^2 z}{\partial x^2} = 2y^3 e^{x^2 y^2} . (2x^2 y^3 + 1)$

5) $\begin{cases} \dfrac{\partial^2 z}{\partial x^2} = -\dfrac{1}{x^2} - 6x \\ \dfrac{\partial^2 z}{\partial y^2} = -\dfrac{1}{y^2} + 12y^2 \end{cases}$

6) $\dfrac{\partial^2 z}{\partial x^2} + \dfrac{\partial^2 z}{\partial y^2} = \dfrac{1}{\sqrt{x^2 + y^2}}$

LISTA XXV

1) $\begin{cases} \vec{\nabla} z = 10\vec{i} + 16\vec{j} \\ D_{\vec{v}} z = \dfrac{19\sqrt{10}}{5} \\ D_{fmax} = 2\sqrt{89} \end{cases}$

2) $\begin{cases} \vec{\nabla} z = 3\vec{i} \\ D_{\vec{v}} z = \dfrac{6\sqrt{5}}{5} \\ D_{fmax} = 3 \end{cases}$

3) $\begin{cases} \vec{\nabla} w = \vec{i} + 3\vec{j} + \vec{k} \\ D_{\vec{v}} w = \dfrac{5\sqrt{3}}{3} \\ D_{fmax} = \sqrt{11} \end{cases}$

4) $\begin{cases} \vec{\nabla} w = \dfrac{3}{8}\vec{i} + 4\vec{j} - \dfrac{1}{8}\vec{k} \\ D_{\vec{v}} w = \dfrac{3}{2} \\ D_{fmax} = \dfrac{1}{8}\sqrt{1034} \end{cases}$

5) $\begin{cases} \vec{\nabla} z = -\, e\vec{i} - 2e\vec{j} \\ D_{\vec{v}} z = \dfrac{e}{3}(4 - \sqrt{5}) \\ D_{fmax} = \sqrt{5}e \end{cases}$

6) $\begin{cases} \vec{\nabla} z = 3\vec{i} + \vec{j} \\ D_{\vec{v}} z = \dfrac{9}{5} \\ D_{fmax} = \sqrt{10} \end{cases}$

7) $\begin{cases} \vec{\nabla} z = 2\pi\vec{i} + \vec{j} \\ D_{\vec{v}} z = \sqrt{2}(\pi + \dfrac{1}{2}) \\ D_{fmax} = \sqrt{4\pi^2 + 1} \end{cases}$

8) $\begin{cases} \vec{\nabla} z = 2e\vec{i} + \vec{j} \\ D_{\vec{v}} z = \sqrt{2}(e + \dfrac{1}{2}) \\ D_{fmax} = \sqrt{4e^2 + 1} \end{cases}$

9) $\begin{cases} \vec{\nabla} w = -\, \vec{i} - \vec{j} + 2\,\vec{k} \\ D_{\vec{v}} z = \dfrac{2}{7} \\ D_{fmax} = \sqrt{6} \end{cases}$

10) $\begin{cases} \vec{\nabla} w = 9\vec{i} + 5\vec{j} + 3\vec{k} \\ D_{\vec{v}} w = \frac{5}{3} \\ D_{fmax} = \sqrt{115} \end{cases}$

11) $\begin{cases} \vec{\nabla} z = (2e - 1)\vec{i} + (e - 1)\vec{j} \\ D_{\vec{v}} z = \frac{1}{5}(5e + 1) \\ D_{fmax} = \sqrt{5e^2 - 6e + 2} \end{cases}$

12) $\begin{cases} \vec{\nabla} z = -\pi\vec{i} + (1 - 2\pi)\vec{j} \\ D_{\vec{v}} z = \pi - \frac{4}{5} \\ D_{fmax} = \sqrt{5\pi^2 - 4\pi + 1} \end{cases}$

13) $\begin{cases} \vec{\nabla} z = -12\vec{i} + 24\vec{j} \\ D_{\vec{v}} z = 6(2\sqrt{3} - 1) \\ D_{fmax} = 12\sqrt{5} \end{cases}$

14) $\begin{cases} \vec{\nabla} w = 3\vec{i} + \vec{j} + \vec{k} \\ D_{\vec{v}} w = 1 \\ D_{fmax} = \sqrt{11} \end{cases}$

LISTA XXVI

1a) $dz = e^y\cos(xe^y)\,dx + xe^y\cos(xe^y)\,dy$

1b) $dz = e^x dx - tgy\ dy$

1c) $dz = 3x^2.3^{x^3}\ln 3\,dx + 2y.2^{y^2}.\ln 2\,dy$

1d) $dw = \frac{1}{2\sqrt{x}}\,dx - 2\,dy - \frac{2}{z^3}\,dz$

1e) $dw = (yx^{y-1} + y^x\ln y + y\,z^{xy}\ln z)\,dx + (x^y\ln x + xy^{x-1} + x\,z^{xy}\ln z)\,dy +$
$+ xy\,z^{xy-1}\,dz$

1f) $dw = \frac{y^3z}{xy^3z + 2}\,dx + \frac{3xy^2z}{xy^3z + 2}\,dy + \frac{xy^3}{xy^3z + 2}\,dz$

1g) $dz = ye^{xy}(\mathrm{sen}(x^2y) + 2x\cos(x^2y))\,dx + xe^{xy}(\mathrm{sen}(x^2y) + x\cos(x^2y))\,dy$

1h) $dw = 3xy^3z^2e^{xyz}(2 + xyz)\,dx + 3x^2y^2z^2e^{xyz}(3 + xyz)\,dy + 3x^2y^3ze^{xyz}(2+xyz)\,dz$

1i) $dz = 2^y\sec^2(x\,2^y)\,dx + x\,2^y\sec^2(x\,2^y)\ln 2\,dy$

2a) 23,84

2b) 36,48

3a) 19,5 erg

3b) $-8.10^4 cm^3$

3c) $6,2\,\pi\,cm^3$

LISTA XXVII

1)

$(2, -1)$
min

2)

$(2, 6)$
max

3)

$(0, 0)$	$(2, 2)$
nsc	max

4)

$(-\frac{1}{3}, \frac{1}{3})$	$(\frac{1}{2}, \frac{3}{4})$
nsc	min

5)

$(0, 0)$	$(1, 3)$
nsc	min

6)

$(0, 0)$	$(4, 8)$
nsc	min

15)

$(0, 0)$	$(\frac{2}{3}, \frac{2}{3})$	$(\frac{-2}{3}, \frac{2}{3})$
nsc	nsc	nsc

16)

$(0, 0)$	$(\frac{1}{2}, \frac{1}{8})$	$(\frac{-1}{2}, \frac{1}{8})$
nsc	min	min

17)

$(0, \frac{3}{2})$	$(2, 1)$	$(-2, 1)$
min	nsc	nsc

18)

$(0, 0)$	$(2, 0)$	$(1, 1)$	$(1, -1)$
max	min	nsc	nsc

19)

$(3, 0)$	$(2, 2)$	$(2, -2)$
min	nsc	nsc

7)

$(\frac{2\sqrt{3}}{3}, 0)$	$(\frac{-2\sqrt{3}}{3}, 0)$	$(\frac{6}{5}, \frac{2}{5})$	$(\frac{-6}{5}, \frac{-2}{5})$
nsc	nsc	min	max

8)

$(0, 2)$	$(-1, 0)$	$(1, 2)$
nsc	nsc	nsc

9)

$(0, 2)$	$(2, 4)$	$(-2, 4)$
nsc	min	min

10)

$(0, 0)$	$(4, 8)$	$(-1, \frac{1}{2})$
min	nsc	nsc

11)

$(-1, \frac{3}{2})$	$(\frac{1}{3}, \frac{1}{6})$
max	nsc

12)

$(2\sqrt{5}, \sqrt{5})$	$(-2\sqrt{5}, -\sqrt{5})$
nsc	nsc

13)

$(0, 0)$	$(1, 1)$	$(-1, -1)$
nsc	min	min

14)

$(0, 0)$	$(3, 1)$	$(-3, 1)$	$(\frac{-3}{2}, \frac{-1}{2})$	$(\frac{3}{2}, \frac{-1}{2})$
nsc	min	min	nsc	nsc

LISTA XXVIII

1a) 5	2) 8	6) $\frac{9}{10}$	10) $\frac{\pi}{12}$
1b) $\frac{8}{3}$	3) $\frac{14}{3}$	7) $\frac{1}{10}$	11) $\frac{53}{5}$
1c) $\frac{1}{2}$	4) $\frac{1}{2}$	8) $\frac{52}{5}$	12) $\frac{59}{84}$
1d) $\frac{2}{5}$	5) $\frac{1}{3}(5\sqrt{5} - 1)$	9) $\frac{2256}{35}$	13) $\frac{8}{45}(17 - 6\sqrt{2})$

LISTA XXIX

1) $y = -\frac{1}{x} + c$

2) $x = \frac{1}{2}(\ln(2y + 3) + c)$

3) $y = \frac{1}{6}(\ln(2x^3 + 1) + \frac{12 - \ln 3}{6})$

4) $y = \frac{x^2}{2}\ln x - \frac{x^2}{4} - \frac{7}{4}$

5) $y = \frac{x^4}{24} + \frac{c_1 x^2}{2} + c_2 x + c_3$

6) $y = x^3 - x^2 + 5x + c$

7) $y = \frac{1}{2}(\mathrm{tg}\, x^2 + c)$

8) $y = -\frac{1}{2}(-\frac{1}{2}e^{-2x} + c_1 x + c_2)$

9) $y = \frac{1}{50}(-5x^2\cos(5x^2 - 1) + \mathrm{sen}(5x^2 - 1) + c)$

10) $y = \frac{1}{18}(3x^2\mathrm{sen}(3x^2 - 12) + \cos(3x^2 - 12) + 18)$

11) $y = \ln x . \ln(\ln x) - \ln x + 2 - \ln 2(\ln(\ln 2) - 1)$

12) $y = -\frac{1}{4}\mathrm{sen}2x + \frac{1}{16}\cos 4x + \frac{5x}{2} - \frac{47 - 20\pi}{16}$

13) $y = \frac{1}{2}(\frac{(x^2 + 1)^{10}}{10} - \frac{(x^2 + 1)^9}{9} + c)$

14) $y = 2(\frac{1}{5}(x + 1)^{5/2} - \frac{2}{3}(x + 1)^{3/2} + (x + 1)^{1/2} + c)$

15) $y = 2(\sqrt{x} + 2\ln(\sqrt{x} - 1) + c)$

LISTA XXX

1) $\frac{x^3}{3} = e^y + c$

2) $-\frac{1}{x} = \operatorname{tg} y + c$

3) $-\sqrt{1 - x^2} = \operatorname{arctg} y + c$

4) $\ln x - x = \frac{1}{y} + c$

5) $x - \ln(x + 1) = -\operatorname{arctg} y + c$

6) $\ln(1 + x) = -\ln(y^2 + 1) + c$

7) $3x - \ln x = y - \ln(y + 1) + c$

8) $-x + \ln x = y + 2\ln(y - 2) + c$

9) $3\ln x + \frac{1}{x} = -y^2 - \frac{y^4}{4} + c$

10) $\frac{1}{4}\ln\frac{x}{4 - x} = \ln y + c$

11) $\frac{x^2}{2} - x + 3\ln(x + 1) = y + \ln y + c$

12) $\frac{e^{-3y}}{3} = \frac{e^{-2x}}{2} + c$

13) $\frac{1}{2}\ln(x^2 + 2) = -\frac{1}{2}\operatorname{arctg}^2 y + c$

14) $x^2 \operatorname{arctg} x - x + \operatorname{arctg} x = \ln(y^2 + 1) + c$

15) $\operatorname{sen}(\ln x) = -e^{1/y} + c$

16) $\operatorname{sen} x \, .(\ln(\operatorname{sen} x) - 1) = \ln(\frac{\sqrt{y} - 1}{\sqrt{y} + 1}) + c$

17) $\ln(1 + \operatorname{tg} x) = \frac{1}{2}\ln(\sec y^2 + \operatorname{tg} y^2) + c$

18) $-x + 2\ln(x + 1) = \ln(\sec y + \operatorname{tg} y) + c$

19) $-\frac{x^2}{2} + x = y \operatorname{tg} y + \ln(\cos y) + c$

20) $-\cos x = e^{-1/y} + c$

21) $y = 2\ln x - \ln(x + 1) + c$

LISTA XXXI

1) $\frac{x^2}{2} + 2xy - 2x - \frac{y^2}{2} + 3y = c$

2) $x^3y + 5xy^2 + 7x^2 + 4y^3 = c$

3) $\frac{y^2}{2} + \frac{\mathrm{sen}^2 x}{2} + \frac{x^2}{2} = c$

4) $\frac{y}{x} + xy = c$

5) $\frac{x^2}{2} + \frac{x}{y} + 2 \ln y = c$

6) $y \,\mathrm{sen}\, x - x \cos y + \cos y = c$

7) $x \ln y + x^2y^3 + x e^y + y^3 = c$

8) $\alpha = -6$, $\beta = -12$ $\quad x^2y^3 - 2x^3y - 4y^3x + 2x^2 - 3y^3 = c$

9) $\alpha = -10$ $\quad \beta = 6$ $\quad x^3y - 5x^2y + 2xy^3 + 7x - 8y^2 = c$

10) $\alpha = -4$ $\quad \beta = 10$ $\quad xy^3 - 2x^2y + 5x^3y^2 - 4x + 7y = c$

11) $\alpha = 2$ $\quad \beta = 8$ $\quad x^2y^3 + 4xy^2 + 7x^3y + 5x - 8y^3 = c$

12) $\alpha = 2$ $\quad \beta = 8$ $\quad x^2y^3 + 4y^2 + 7x^3y + 5x - 8y^3 = c$

13) $\alpha = 1$ $\quad \beta = 0$ $\quad x \,\mathrm{sen}\, y + y \,\mathrm{sen}\, x + \mathrm{sen}\, x + \cos y = c$

LISTA XXXII

1) $y = \frac{1}{2e^x}(e^x + c)$

2) $y = x^3(\frac{x^2}{2} + c)$

3) $y = \frac{1}{e^{2x^2 - x^3}}(e^{2x^2} + x^3 + c)$

4) $y = \frac{1 + x^2}{2}(\ln(1 + x^2) + c)$

5) $y = \frac{1}{e^x(2x + 1)}(e^{x^2} + x + c)$

6) $y = \frac{e^x}{6x - 1}(e^{3x^2} - x + c)$

7) $\frac{1}{2\sqrt{x^2 + 1}}(x^2 + 2c)$

8) $\frac{1}{6(x + 2)}(2x^3 - 3x^2 + 12x + 6c)$

9) $(x - 1)(x + \ln(x - 1) + c)$

10) $\frac{1}{2xe^{x^2}}(e^{x^2} + c)$

11) $\frac{1}{2e^{\mathrm{arctg}\, x}}(e^{2\mathrm{arctg}\, x} + c)$

12) $\frac{1}{2}x^3e^{x^{-2}}(e^{-x^{-2}} + c)$

LISTA XXXIII

1) $y^2 = \dfrac{-1}{2x^2(\ln x + c)}$

2) $y^{2/5} = \dfrac{1}{x(\frac{1}{2} + c/_{e}x^2)}$

3) $y^{3/2} = \dfrac{2(\text{sen } x^3 + c)}{3x^7}$

4) $y = (1 + x^2 + c\sqrt{1 + x^2})^2$

5) $\sqrt{y} = (2x + 3)(\frac{x}{2} - \frac{3}{4}\ln(2x + 3) + c)$

6) $y^3 = \dfrac{-1}{2\sqrt{x}(\text{arctg}\sqrt{x} + c)}$

7) $y = (\dfrac{e^{x^2} + c\,e^x}{2x - 1})^2$

8) $y = \dfrac{1}{(x - 1)^3}(\frac{1}{2}\ln(x^2 + 1) - \text{arctg } x + c)^3$

9) $y = \dfrac{(x^2 + 2c)^2\cos^2 x}{4x^2}$

10) $y^3 = \dfrac{1}{(2x + 1)e^x}(e^{x^2} + x + c)$

11) $y^{3/2} = \dfrac{4x}{-2x^2\ln x + x^2 + 4c}$

12) $y^2 = \dfrac{-9x^2}{2(2x^3 + 3x^3\ln x + 9c)}$

13) $y^2 = \dfrac{3}{2x(\cos x^3 + c)}$

14) $y^2 = x(\text{sen}(\ln x) + c)$

LISTA XXXIV

1) $y = c_1e^{x} + c_2e^{3x}$

2) $y = c_1e^{x} + c_2e^{2x}$

3) $y = c_1e^{-3x} + c_2e^{2x}$

4) $y = c_1 + c_2e^{4x} + c_3e^{-3x}$

5) $y = c_1 + c_2e^{2x} + c_3e^{-2x}$

6) $y = e^{-3x}(c_1 + c_2x)$

7) $y = e^{2x}(c_1 + c_2x)$

8) $y = e^{x}(c_1 + c_2x + c_3x^2)$

9) $y = c_1e^{-2x} + e^{2x}(c_2 + c_3x)$

10) $y = c_1e^{-5x} + c_2e^{x} + c_3e^{-x}$

11) $y = c_1e^{-2x} + c_2e^{-3x} + c_3e^{3x}$

12) $y = c_1e^{3x} + c_2e^{2x} + c_4 + c_5x$

13) $y = c_1 + c_2x + c_3x^2 + e^{3x}(c_4 + c_5x)$

14) $y = c_1 + c_2x + e^{4x}(c_3 + c_4x) + c_5e^{-4x}$

15) $y = c_1e^{3x} + c_2e^{x} + 3$

16) $y = e^{4x}(c_1 + c_2x) - \frac{1}{8}$

17) $y = c_1e^{x} - c_2e^{-x} - 4x + c_3$

18) $y = -\frac{1}{2}c_1e^{-2x} + \frac{1}{2}c_2e^{2x} - \frac{5}{4}x + c_3$

19) $y = c_1e^{x} + c_2e^{-x} - \frac{5}{2}x^2 + c_3x + c_4$

20) $y = c_1e^{x} + c_2e^{-x} - 6x^2 + c_3x + c_4$

21) $y = c_1e^{x} + c_2(xe^{x} - e^{x}) + x + c_3$

22) $y = \frac{1}{3}c_1e^{3x} + \frac{1}{3}c_2(xe^{3x} - \frac{1}{3}e^{x}) + \frac{9}{8} + c_3$

23) $y = c_1e^{-x} + c_2e^{5x} - \frac{1}{3}e^{2x}$

24) $y = e^{2x}(c_1 + c_2x) + 5e^{3x}$

25) $y = e^{x}(c_1 + c_2x) + \frac{1}{2}e^{3x}$

26) $y = c_1e^{6x} + c_2e^{4x} + \frac{2}{35}e^{-x}$

27) $y = c_1e^{4x} + c_2e^{2x} + e^{x} + \frac{1}{2}$

28) $y = -\frac{1}{5}c_1e^{-5x} + c_2e^{x} + \frac{2}{7}e^{2x} + x + c_3$

29) $y = c_1e^{2x} + c_2e^{-2x} + c_3e^{-x} + \frac{10}{3}e^{x} - 4$

30) $y = e^{x}(c_1 + c_2x + c_3x^2) - 5 + \frac{1}{2}e^{3x}$

31) $y = c_1e^{2x} + c_2e^{-2x} - 2x - 1$

32) $y = c_1e^{3x} + c_2e^{-x} - 9x^2 + 12x - \frac{41}{3}$

33) $y = c_1e^{x} - c_2e^{-x} - \frac{1}{3}x^3 - 3x + c_3$

34) $y = c_1e^{x} + c_2e^{-x} - 2x^2 + 3x - 5$

35) $y = e^{x}(c_1 + c_2x) + 5x^2 + 26x + 55$

36) $y = c_1e^{2x} + c_2e^{x} + \frac{1}{2}e^{3x} + 4x^2 + 12x + 14$

37) $y = c_1e^{x} + c_2e^{-x} + 3x^2 - 5x + 7 + \frac{1}{3}e^{2x}$

38) $y = c_1e^{2x} + c_2e^{-x} - 5e^{x} - 15x^2 + 53x - \frac{3}{2}$

39) $y = c_1e^{3x} + c_2e^{x} + \frac{3}{10}\operatorname{sen}x + \frac{3}{5}\cos x$

40) $y = c_1e^{2x} + c_2e^{x} - \frac{9}{20}\operatorname{sen}2x - \frac{3}{20}\cos 2x$

41) $y = c_1e^{3x} + c_2e^{-2x} - \frac{12}{13}\cos 2x + \frac{34}{13}\operatorname{sen}2x$

42) $y = c_1e^{2x} + c_2e^{-x} - 6 + \frac{86}{65}\operatorname{sen}3x - \frac{12}{65}\cos 3x$

43) $y = c_1e^{-5x} + c_2e^{3x} + 3 + \frac{161}{4}\cos 2x + 7\operatorname{sen}2x$

44) $y = \frac{1}{3}c_1e^{3x} - \frac{1}{3}c_2e^{-3x} + \frac{3}{7}e^{4x} - \frac{20}{9}x - \frac{1}{26}\cos 2x + c_3$

45) $y = c_1e^{7x} + c_2e^{-2x} + \frac{9}{4}e^{-x} - \frac{2}{7}x^2 + \frac{10}{49}x - \frac{39}{343} - \frac{1}{5}\operatorname{sen}x - \frac{3}{5}\cos x$