EXERCÍCIOS - LISTA II

Calcule as seguintes integrais.

1) $I = \int (3x + 1)^{20} dx$

$3x + 1 = t$ $\quad I = \frac{1}{3}\int t^{20} dt \rightarrow I = \frac{1}{3} \frac{t^{21}}{21}$

$3dx = dt$

$dx = \frac{1}{3} dt$ $\quad \boxed{I = \frac{1}{63} (3x+1)^{21} + C}$ ✓

2) $I = \int (5x - 1)^{15} dx$

$5x - 1 = t$ $\quad I = \frac{1}{5}\int t^{15} dt \rightarrow I = \frac{1}{5} \frac{t^{16}}{16} \rightarrow I = \frac{1}{80} t^{16}$

$5dx = dt$

$dx = \frac{1}{5} dt$ $\quad \boxed{I = \frac{1}{80} (5x-1)^{16} + C}$ ✓

3) $I = \int \frac{3}{(2x - 3)^{10}} dx$

$2x - 3 = t$ $\quad I = \frac{1}{2}\int \frac{3}{t^{10}} dt \rightarrow I = \frac{3}{2}\int t^{-10} dt$

$2dx = dt$

$dx = \frac{1}{2} dt$ $\quad I = \frac{3}{2}\left[\frac{t^{-9}}{-9}\right] \rightarrow I = -\frac{3}{18} \frac{1}{t^9} \rightarrow \boxed{I = -\frac{1}{6(2x-3)^9} + C}$ ✓

4) $I = \int \frac{1}{\sqrt{5 + x}} dx$

$5 + x = t$ $\quad I = \int \frac{1}{\sqrt{t}} dt \rightarrow I = \int t^{-1/2} dt \rightarrow I = \frac{t^{1/2}}{1/2}$

$dx = dt$

$I = 2\sqrt{t} \rightarrow \boxed{I = 2\sqrt{5+x} + C}$ ✓

5) $I = \int \frac{x}{\sqrt{x^2+1}}\,dx$

$x^2 + 1 = t$

$2x\,dx = dt$

$dx = \frac{1}{2x}\,dt$

$I = \frac{1}{2}\int \frac{x}{x\sqrt{t}}\,dt \to I = \frac{1}{2}\int t^{-1/2}\,dt$

$I = \frac{1}{2}\,\frac{t^{1/2}}{1/2} \to I = \frac{1}{2}\,2\sqrt{t} \to \boxed{I = \sqrt{x^2+1} + C}$ ✓

6) $I = \int \frac{x^2}{\sqrt{5x^3+1}}\,dx$

$5x^3 + 1 = t$

$15x^2\,dx = dt$

$dx = \frac{1}{15x^2}\,dt$

$I = \frac{1}{15}\int \frac{x^2}{x^2\sqrt{t}}\,dt \to I = \frac{1}{15}\int \frac{1}{\sqrt{t}}\,dt$

$I = \frac{1}{15}\int t^{-1/2}\,dt \to I = \frac{1}{15}\,\frac{t^{1/2}}{1/2} \to I = \frac{2}{15}\sqrt{t}$

$\boxed{I = \frac{2}{15}\sqrt{5x^3+1} + C}$ ✓

7) $I = \int (2x+1)\sqrt{x^2+x-3}\,dx$

$x^2 + x - 3 = t$

$2x + 1\,dx = dt$

$dx = \frac{1}{2x+1}\,dt$

$I = \int \frac{\cancel{(2x+1)}\sqrt{t}}{\cancel{2x+1}}\,dt \to I = \int t^{1/2}\,dt$

$I = \frac{2}{3}t^{3/2} \to \boxed{I = \frac{2}{3}\sqrt{(x^2+x-3)^3} + C}$ ✓

8) $I = \int \frac{2x^2+1}{\sqrt{2x^3+3x+1}}\,dx$

$2x^3 + 3x + 1 = t$

$6x^2 + 3\,dx = dt$

$dx = \frac{1}{6x^2+3}\,dt$

$I = \int \frac{(2x^2+1)}{(6x^2+3)\sqrt{t}}\,dt \to I = \int \frac{2x^2+1}{3(2x^2+1)\sqrt{t}}\,dt$

$I = \frac{1}{3}\int \frac{1}{\sqrt{t}}\,dt \to I = \frac{1}{3}\int t^{-1/2}\,dt \to I = \frac{1}{3}\,\frac{t^{1/2}}{1/2}$

$\boxed{I = \frac{2}{3}\sqrt{2x^3+3x+1} + C}$ ✓

9) $I = \int \frac{1}{(x-a)^4}\, dx$

$x - a = t$

$dx = dt$

$I = \int \frac{1}{t^4}\, dt \rightarrow I = \int t^{-4}\, dt \rightarrow I = \frac{t^{-3}}{-3} \rightarrow I = -\frac{1}{3t^3}$

$$\boxed{I = -\frac{1}{3(x-a)^3} + C}$$

✓

10) $I = \int \frac{(4x-2)^2}{(2x-1)^5}\, dx$

$2x - 1 = t$

$2dx = dt$

$dx = \frac{1}{2} dt$

$I = \frac{1}{2} \int \frac{2^2 (2x-1)^2}{(2x-1)^5} dx \rightarrow I = \frac{1}{2} \int \frac{4t^2}{t^5}\, dt$

$I = \frac{4}{2} \int \frac{1}{t^3}\, dt \rightarrow I = 2 \int t^{-3}\, dt \rightarrow I = 2\, \frac{t^{-2}}{-2}$

$I = -\frac{1}{t^2} \rightarrow \boxed{I = -\frac{1}{(2x-1)^2} + C}$

✓

11) $I = \int \frac{1}{(2x-5)^2 \sqrt{2x-5}}\, dx$

$2x - 5 = t$

$2dx = dt$

$dx = \frac{1}{2} dt$

$I = \frac{1}{2} \int \frac{1}{t^2\, t^{1/2}}\, dt \rightarrow I = \frac{1}{2} \int t^{-5/2}\, dt \rightarrow I = \frac{1}{2} \left[\frac{t^{-3/2}}{-3/2} \right]$

$I = \frac{1}{2} \left[-\frac{2}{3} (2x-5)^{-3/2} \right] \rightarrow I = -\frac{1}{3} \frac{1}{\sqrt{(2x-5)^3}}$

$$\boxed{I = -\frac{1}{3\sqrt{(2x-5)^3}} + C}$$

✓

12) $I = \int \frac{\sqrt[3]{4x - 16}}{(x-4)^3}\, dx$

$x - 4 = t$

$dx = dt$

$I = \int \frac{\sqrt[3]{4(x-4)}}{(x-4)^3}\, dx \to I = \int \frac{(4t)^{1/3}}{t^3}\, dt \to I = \int (4t)^{1/3} t^{-3}\, dt$

$I = 4^{1/3} \int t^{1/3} t^{-3}\, dt \to I = \sqrt[3]{4} \int t^{-8/3}\, dt \to I = \sqrt[3]{4}\, \frac{t^{-5/3}}{-5/3}$

$I = \frac{3\sqrt[3]{4}}{5(x-4)^{5/3}} \to \boxed{I = -\frac{3\sqrt[3]{4}}{5\sqrt[3]{(x-4)^5}} + C}$ ✓

13) $I = \int \frac{\sqrt[5]{(4x+2)^7}}{8x+4}\, dx$

$I = \int \frac{\sqrt[5]{(4x+2)^7}}{2(4x+2)}\, dx \to I = \frac{1}{2 \cdot 4} \int \frac{\sqrt[5]{t^7}}{t}\, dt \to I = \frac{1}{8} \int t^{7/5} t^{-1}\, dt$

$4x + 2 = t$

$4\, dx = dt$

$dx = \frac{1}{4} dt$

$I = \frac{1}{8} \int t^{2/5}\, dt \to I = \frac{1}{8} \frac{t^{7/5}}{7/5} \to I = \frac{1}{8} \frac{5}{7} \sqrt[5]{t^7}$

$\boxed{I = \frac{5}{56} \sqrt[5]{(4x+2)^7} + C}$ ✓

14) $I = \int \sqrt{\left(\frac{x^2 - 1}{x + 1}\right)^5}\, dx$

$I = \int \sqrt{\left[\frac{(x-1)\cancel{(x+1)}}{\cancel{x+1}}\right]^5}\, dx \to I = \int \sqrt{(x-1)^5}\, dx$

$x - 1 = t$

$dx = dt$

$I = \int t^{5/2}\, dt \to I = \frac{t^{7/2}}{7/2} \to I = \frac{2}{7} \sqrt{t^7}$

$\boxed{I = \frac{2}{7} \sqrt{(x-1)^7} + C}$ ✓

15) $I = \int \frac{\sqrt{\left(4 - \frac{3}{x^2}\right)^3}}{x^3} dx$

$4 - \frac{3}{x^2} = t$

$4 - 3x^{-2} = t$

$6x^{-3} dx = dt$

$dx = \frac{x^3 dt}{6}$

$I = \frac{1}{6} \int \frac{x^3 \sqrt{t^3}}{x^3} dt \rightarrow I = \frac{1}{6} \int t^{3/2} dt \rightarrow I = \frac{1}{6} \left[\frac{t^{5/2}}{5/2} \right]$

$I = \frac{1}{6} \frac{2}{5} \sqrt{t^5} \rightarrow \boxed{I = \frac{1}{15} \sqrt{\left(4 - \frac{3}{x^2}\right)^5} + C}$ ✓

16) $I = \int \frac{1}{x^2} \sqrt[3]{\left(\frac{1}{x} + b\right)^2} dx$

$\frac{1}{x} + b = t$

$x^{-1} + b = t$

$-x^{-2} dx = dt$

$dx = -x^2 dt$

$I = -\int \frac{x^2}{x^2} \sqrt[3]{(t)^2} dt \rightarrow I = -\int t^{2/3} dt$

$I = -\left[\frac{t^{5/3}}{5/3} \right] \rightarrow \boxed{I = -\frac{3}{5} \sqrt[3]{\left(\frac{1}{x} + b\right)^5} + C}$ ✓

17) $I = \int \frac{1}{\sqrt[3]{x^2}\left(1 - 2\sqrt[3]{x}\right)} dx$

$1 - 2\sqrt[3]{x} = t$

$1 - 2x^{1/3} = t$

$-\frac{2}{3} x^{-2/3} dx = dt$

$-\frac{2}{3\sqrt[3]{x^2}} dx = dt$

$dx = -\frac{3\sqrt[3]{x^2}}{2} dt$

$I = -\frac{3}{2} \int \frac{\sqrt[3]{x^2}}{\sqrt[3]{x^2}} \frac{1}{t} dt \rightarrow I = -\frac{3}{2} \int \frac{1}{t} dt$

$\boxed{I = -\frac{3}{2} \ln \left|1 - 2\sqrt[3]{x}\right| + C}$ ✓

18) $I = \int e^{-\frac{x}{a}}\, dx$

$-\frac{x}{a} = t$

$-\frac{1}{a} dx = dt$

$dx = -a\, dt$

$I = -a \int e^{t} dt \rightarrow I = -a e^{t}$

$\boxed{I = -a e^{-\frac{x}{a}} + C}$ ✓

19) $I = \int \frac{e^{3x-4}}{e^{x+2}}\, dx$

$I = \int e^{3x-4} e^{-(x+2)} dx \rightarrow I = \int e^{3x-4-x-2} dx$

$I = \int e^{2x-6} dx \rightarrow I = \frac{1}{2}\int e^{t} dt \rightarrow I = \frac{1}{2} e^{t}$

$2x-6 = t \rightarrow \boxed{I = \frac{1}{2} e^{2x-6} + C}$ ✓

$2dx = dt$

$dx = \frac{1}{2} dt$

20) $I = \int \frac{x \cdot e^{x^2+x}}{e^{x+1}}\, dx$

$I = \int x \cdot e^{x^2+x} e^{-(x+1)} dx \rightarrow I = \int x e^{x^2+x-x-1} dx$

$I = \int x e^{x^2-1} dx \qquad I = \frac{1}{2}\int \frac{x e^{t}}{x} dt \rightarrow I = \frac{1}{2}\int e^{t} dt$

$x^2-1 = t \qquad \boxed{I = \frac{1}{2} e^{x^2-1} + C}$ ✓

$2x\, dx = dt$

$dx = \frac{1}{2x} dt$

21) $I = \int a^{4x} \cdot \ln a \; dx$

$I = \ln a \int a^{4x} dx \rightarrow$

$4x = t$ $\qquad I = \frac{1}{4} \ln a \int a^{t} dt \rightarrow \qquad I = \frac{1}{4} \ln a \; \frac{a^{t}}{\ln a}$

$4dx = dt$

$dx = \frac{1}{4} dt$ $\qquad \boxed{I = \frac{1}{4} a^{4x} + C}$ ✓

22) $I = \int \text{sen} \frac{x}{2} \; dx$

$\frac{x}{2} = t$ $\qquad I = 2 \int \text{sen}(t) dt \rightarrow \quad I = -2 \cos(t)$

$\frac{1}{2} dx = dt$

$dx = 2dt$ $\qquad \boxed{I = -2 \cos\left(\frac{x}{2}\right) + C}$ ✓

23) $I = \int (x + 1) \cdot \text{sen}(x^2 + 2x) \; dx$

$x^2 + 2x = t$ $\qquad I = \frac{1}{2} \int \frac{\cancel{(x+1)} \text{sen}(t)}{\cancel{x+1}} dt$

$2x + 2 \; dx = dt$

$dx = \frac{1}{2(x+1)} dt$ $\qquad I = \frac{1}{2} \int \text{sen}(t) dt \rightarrow \quad I = -\frac{1}{2} \cos(t)$

$\boxed{I = -\frac{1}{2} \cos(x^2 + 2x) + C}$ ✓

24) $I = \int x \cdot \text{sen}\frac{6x^4-2x^2}{x^2}\,dx$

$$I=\int x\cdot \text{sen}\left(\frac{\cancel{x^2}(6x^2-2)}{\cancel{x^2}}\right)dx \rightarrow I=\int x\,\text{sen}\,(6x^2-2)\,dx$$

$6x^2-2=t$

$12x\,dx=dt$

$dx=\frac{1}{12x}dt$

$$I=\frac{1}{12}\int\frac{x\,\text{sen}(t)}{x}dt \rightarrow I=\frac{1}{12}\int \text{sen}(t)\,dt$$

$$I=-\frac{1}{12}\cos(t) \rightarrow \boxed{I=-\frac{1}{12}\cos(6x^2-2)+C}$$

25) $I = \int \text{tg}(2x).\cos\,(2x)\,dx$

$2x=t$

$2dx=dt$

$dx=\frac{1}{2}dt$

$$I=\frac{1}{2}\int\frac{\text{sen}(t)}{\cos(t)}.\cos(t)\,dt \rightarrow I=\frac{1}{2}\int \text{sen}(t)\,dt$$

$$I=\frac{-1}{2}\cos(t) \rightarrow \boxed{I=-\frac{1}{2}\cos(2x)+C}$$

26) $I = \int e^x.\cos(e^x-1)\,dx$

$e^x-1=t$

$e^x\,dx=dt$

$dx=\frac{1}{e^x}dt$

$$I=\int\frac{e^x\cos(t)}{e^x}dt \rightarrow I=\int\cos(t)\,dt$$

$$I=\text{sen}(t) \rightarrow \boxed{I=\text{sen}(e^x-1)+C}$$

27) $I = \int \operatorname{sen}(2x) . \cos(2x) \, dx$

$\operatorname{sen}(2x) = t$

$2\cos(2x)\,dx = dt$

$dx = \dfrac{1}{2\cos(2x)} dt$

$I = \dfrac{1}{2}\int \dfrac{t \cancel{\cos(2x)}}{\cancel{\cos(2x)}} dt \rightarrow I = \dfrac{1}{2}\int t \, dt$

$I = \dfrac{1}{2} \dfrac{t^2}{2} \rightarrow \boxed{I = \dfrac{1}{4} \operatorname{sen}^2(2x) + C}$

28) $I = \int \cos(3x) . \operatorname{sen}(3x) \, dx$

$\operatorname{sen}(3x) = t$

$3\cos(3x)\,dx = dt$

$dx = \dfrac{1}{3\cos(3x)} dt$

$I = \dfrac{1}{3}\int \dfrac{\cancel{\cos(3x)}\, t}{\cancel{\cos(3x)}} dt \rightarrow I = \dfrac{1}{3}\int t \, dt$

$I = \dfrac{1}{3} \dfrac{t^2}{2} \rightarrow \boxed{I = \dfrac{1}{6} \operatorname{sen}^2(3x) + C}$

29) $I = \int \operatorname{tg} x \, dx$

$I = \int \operatorname{tg}(x) \, dx \rightarrow I = \int \dfrac{\operatorname{sen}(x)}{\cos(x)} dx \rightarrow$

$\cos(x) = t$

$-\operatorname{sen}(x)\,dx = dt$

$dx = -\dfrac{1}{\operatorname{sen}(x)} dt$

$I = -\int \dfrac{\cancel{\operatorname{sen}(x)}}{t \cancel{\operatorname{sen}(x)}} dt \rightarrow I = -\int \dfrac{1}{t} dt$

$I = -\ln|t| \rightarrow \boxed{I = -\ln|\cos(x)| + C}$

30) (*) $I = \int \operatorname{cotg} x \, dx$

$I = \int \operatorname{cotg}(x) \, dx \rightarrow I = \int \dfrac{\cos(x)}{\operatorname{sen}(x)} dx$

$\operatorname{sen}(x) = t$

$\cos(x)\,dx = dt$

$dx = \dfrac{1}{\cos(x)} dt$

$I = \int \dfrac{\cancel{\cos(x)}}{t \cancel{\cos(x)}} dt \rightarrow I = \int \dfrac{1}{t} dt \rightarrow I = \ln|t|$

$\boxed{I = \ln|\operatorname{sen}(x)| + C}$

31) $I = \int tg(2a + x)\, dx$

$2a + x = t$

$dx = dt$

$I = \int \frac{sen(t)}{cos(t)}\, dt$

$cos(t) = u$

$-sen(t)dt = du$

$du = -\frac{1}{sen(t)} du$

$I = -\int \frac{\cancel{sen(t)}}{u\ \cancel{sen(t)}}\, du \to I = -\int \frac{1}{u}\, du$

$I = -\ln|u| \to I = -\ln|cos(t)| \to \boxed{I = -\ln|cos(2a + x)| + C}$ ✓

32) $I = \int a \cdot cotg(ax + b)\, dx$

$ax + b = t$

$a\, dx = dt$

$dx = \frac{1}{a} dt$

$sen(t) = u$

$cos(t)\, dt = du$

$dt = \frac{1}{cos(t)} du$

$I = \frac{\cancel{a}}{a} \int cotg(t)\, dt \to I = \int \frac{cos(t)}{sen(t)}\, dt$

$I = \int \frac{\cancel{cos(t)}}{u\ \cancel{cos(t)}}\, du \to I = \int \frac{1}{u}\, du$

$I = \ln|u| \to I = \ln|sen(t)|$

$\boxed{I = \ln|sen(ax + b)| + C}$ ✓

33) $I = \int \frac{sen\ x}{cos^3 x}\, dx$

$cos(x) = t$

$-sen(x)\, dx = dt$

$dx = -\frac{1}{sen(x)} dt$

$I = -\int \frac{\cancel{sen(x)}}{\cancel{sen(x)}\ t^3}\, dt \to I = -\int \frac{1}{t^3}\, dt \to I = -\int t^{-3}\, dt$

$I = -\left[\frac{t^{-2}}{-2}\right] \to I = \frac{1}{2t^2} \to \boxed{I = \frac{1}{2 cos^2(x)} + C}$ ✓

34) $I = \int \frac{1}{x \cdot \ln x} dx$

$\ln(x) = t$

$\frac{1}{x} dx = dt$

$dx = x\,dt$

$I = \int \frac{x}{x\,t} dt \rightarrow I = \int \frac{1}{t} dt$

$I = \ln|t| \rightarrow \boxed{I = \ln|\ln(x)| + C}$ ✓

35) $I = \int \frac{\ln^2 x}{x} dx$

$\ln(x) = t$

$\frac{1}{x} dx = dt$

$dx = x\,dt$

$I = \int \frac{t^2 x}{x} dt \rightarrow I = \int t^2 dt$

$I = \frac{1}{3} t^3 \rightarrow \boxed{I = \frac{1}{3} \ln^3(x) + C}$ ✓

36) $I = \int \frac{\ln(2x - 4)}{x - 2} dx$

$2x - 4 = t$

$2dx = dt$

$dx = \frac{1}{2} dt$

$\rightarrow 2(x-2) = t$

$(x-2) = \frac{t}{2}$

$I = \frac{1}{2} \int \frac{\ln(t)}{t/2} dt \rightarrow I = \frac{2}{2} \int \frac{\ln(t)}{t} dt$

$\ln(t) = u$

$\frac{1}{t} dt = du$

$dt = t\,du$

$I = \int \frac{u}{t} t\,du \rightarrow I = \int u\,du$

$I = \frac{1}{2} u^2 \rightarrow I = \frac{1}{2} \ln^2(t)$

$\boxed{I = \frac{1}{2} \ln^2(2x - 4) + C}$ ✓

37) $I = \displaystyle\int \frac{\ln^2(ax+1)}{ax+1}\,dx$

$\ln(ax+1) = t$

$\dfrac{a}{ax+1}\,dx = dt$

$dx = \dfrac{ax+1}{a}\,dt$

$I = \dfrac{1}{a}\displaystyle\int \frac{t^2\,\cancel{(ax+1)}}{\cancel{ax+1}}\,dt \rightarrow I = \frac{1}{a}\int t^2\,dt$

$I = \dfrac{1}{a}\,\dfrac{1}{3}t^3 \rightarrow \boxed{I = \dfrac{1}{3a}\ln^3(ax+1) + C}$ ✓

38) $I = \displaystyle\int \frac{\operatorname{arctg} x}{1+x^2}\,dx$

$f(x) = \operatorname{arctg}(x)$

$x = \operatorname{tg}(f(x))$

$1 = \sec^2(f(x))\,f'(x)$

$f'(x) = \dfrac{1}{\sec^2 f(x)}$

$\dfrac{\operatorname{sen}^2 f(x)}{\cos^2 f(x)} + \dfrac{\cos^2 f(x)}{\cos^2 f(x)} = \dfrac{1}{\cos^2 f(x)}$

$\operatorname{tg}^2 f(x) + 1 = \sec^2 f(x)$

$x^2 + 1 = \sec^2 f(x)$

$f'(x) = \dfrac{1}{x^2+1}$

$\operatorname{arctg}(x) = t$

$\dfrac{1}{x^2+1}\,dx = dt$

$dx = x^2+1\,dt$

$I = \displaystyle\int \frac{t\,\cancel{(x^2+1)}}{\cancel{1+x^2}}\,dt \rightarrow I = \int t\,dt$

$I = \dfrac{1}{2}t^2 \rightarrow \boxed{I = \dfrac{1}{2}\operatorname{arctg}^2(x) + C}$ ✓

39) $I = \displaystyle\int \frac{\operatorname{arccotg} x}{1+x^2}\,dx$

$f(x) = \operatorname{arccotg}(x)$

$x = \operatorname{cotg}(f(x))$

$1 = -\operatorname{cossec}^2 f(x)\,f'(x)$

$f'(x) = -\dfrac{1}{\operatorname{cossec}^2 f(x)}$

$\dfrac{\operatorname{sen}^2 f(x)}{\operatorname{sen}^2 f(x)} + \dfrac{\cos^2 f(x)}{\operatorname{sen}^2 f(x)} = \dfrac{1}{\operatorname{sen}^2 f(x)}$

$1 + \operatorname{cotg}^2 f(x) = \operatorname{cossec}^2 f(x)$

$1 + x^2 = \operatorname{cossec}^2 f(x)$

$f'(x) = -\dfrac{1}{1+x^2}$

$\operatorname{arccotg}(x) = t$

$-\dfrac{1}{1+x^2}\,dx = dt$

$dx = -(1+x^2)\,dt$

$I = -\displaystyle\int \frac{t\,\cancel{(1+x^2)}}{\cancel{1+x^2}}\,dt \rightarrow I = -\int t\,dt \rightarrow I = -\frac{1}{2}t^2$

$\boxed{I = -\dfrac{1}{2}\operatorname{arccotg}^2(x) + C}$ ✓

40) $I = \int \frac{\text{arcsen } x}{\sqrt{1 - x^2}} dx$

$f(x) = \text{arc sen}(x)$

$x = \text{sen } f(x)$

$1 = \cos f(x) \, f'(x)$

$f'(x) = \frac{1}{\cos f(x)}$

$\text{sen}^2 f(x) + \cos^2 f(x) = 1$

$\cos^2 f(x) = 1 - \text{sen}^2 f(x)$

$\cos f(x) = \sqrt{1 - \text{sen}^2 f(x)}$

$\cos f(x) = \sqrt{1 - x^2}$

$f'(x) = \frac{1}{\sqrt{1 - x^2}}$

$\text{arc sen}(x) = t$

$\frac{1}{\sqrt{1-x^2}} dx = dt$

$dx = \sqrt{1-x^2} \, dt$

$I = \int \frac{t \cancel{\sqrt{1-x^2}}}{\cancel{\sqrt{1-x^2}}} dt \rightarrow I = \int t \, dt$

$I = \frac{1}{2} t^2 \rightarrow \boxed{I = \frac{1}{2} \text{arc sen}^2(x) + C}$ ✓

41) $I = \int \frac{1}{x + x \cdot \ln^2 x} dx$

$I = \int \frac{1}{x(1 + \ln^2 x)} dx$

$\ln(x) = t$

$\frac{1}{x} dx = dt$

$dx = x \, dt$

$I = \int \frac{x}{x(1 + t^2)} dt \rightarrow I = \int \frac{1}{1 + t^2} dt$

$f(x) = \text{arc tg}(x)$

$x = \text{tg}(f(x))$

$1 = \sec^2(f(x)) \, f'(x)$

$f'(x) = \frac{1}{\sec^2 f(x)}$

$\frac{\text{sen}^2 f(x)}{\cos^2 f(x)} + \frac{\cos^2 f(x)}{\cos^2 f(x)} = \frac{1}{\cos^2 f(x)}$

$\text{tg}^2 f(x) + 1 = \sec^2 f(x)$

$x^2 + 1 = \sec^2 f(x)$

$f'(x) = \frac{1}{x^2 + 1}$

$I = \text{arc tg}(t) \rightarrow \boxed{I = \text{arc tg}(\ln(x)) + C}$ ✓

42) $I = \int \sqrt{x} \cdot \operatorname{sen}(x\sqrt{x})\, dx$

$x\sqrt{x} = t$

$x^1 x^{1/2} = t$

$x^{3/2} = t$

$\frac{3}{2} x^{1/2}\, dx = dt$

$dx = \frac{2}{3\sqrt{x}}\, dt$

$I = \frac{2}{3}\int \frac{\sqrt{x}\,\operatorname{sen}(t)\,dt}{\sqrt{x}} \to I = \frac{2}{3}\int \operatorname{sen}(t)\, dt$

$I = -\frac{2}{3}\cos(t) \to \boxed{I = -\frac{2}{3}\cos(x\sqrt{x}) + C}$ ✓

43) $I = \int \sqrt{x} \cdot e^{2x\sqrt{x}}\, dx$

$2x\sqrt{x} = t$

$2x\, x^{1/2} = t$

$2x^{3/2} = t$

$\frac{6}{2} x^{1/2}\, dx = dt$

$dx = \frac{1}{3\sqrt{x}}\, dt$

$I = \frac{1}{3}\int \frac{\sqrt{x}\; e^{t}\, dt}{\sqrt{x}} \to I = \frac{1}{3}\int e^{t}\, dt$

$I = \frac{1}{3} e^{t} \to \boxed{I = \frac{1}{3} e^{2x\sqrt{x}} + C}$ ✓

44) $I = \int \frac{\sqrt{x}}{(a + x\sqrt{x})^2}\, dx$

$a + x\sqrt{x} = t$

$a + x^{3/2} = t$

$\frac{3}{2} x^{1/2}\, dx = dt$

$dx = \frac{2}{3\sqrt{x}}\, dt$

$I = \frac{2}{3}\int \frac{\sqrt{x}}{\sqrt{x}\; t^2}\, dt \to I = \frac{2}{3}\int \frac{1}{t^2}\, dt$

$I = \frac{2}{3}\int t^{-2}\, dt \to I = \frac{2}{3}\left[\frac{t^{-1}}{-1}\right]$

$\boxed{I = -\frac{2}{3(a + x\sqrt{x})} + C}$ ✓

45) $I = \int \sqrt{x}\cdot\sqrt{1 + x\sqrt{x}}\, dx$

$1 + x\sqrt{x} = t$

$1 + x^{3/2} = t$

$\frac{3}{2} x^{\frac{1}{2}} dx = dt$

$dx = \frac{2\, dt}{3\sqrt{x}}$

$I = \frac{2}{3}\int \frac{\sqrt{x}\,\sqrt{t}}{\sqrt{x}}\, dt \rightarrow I = \frac{2}{3}\int t^{1/2}\, dt \rightarrow I = \frac{2}{3}\left[\frac{t^{3/2}}{3/2}\right]$

$I = \frac{4}{9}\sqrt{t^3} \rightarrow \boxed{I = \frac{4}{9}\sqrt{(1+x\sqrt{x})^3} + C}$ ✓

46) $I = \int \frac{x}{\sqrt{1 - x^4}}\, dx$

$I = \int \frac{x}{\sqrt{1-(x^2)^2}}\, dx$

$x^2 = t$

$2x\, dx = dt$

$dx = \frac{1}{2x}\, dt$

$I = \frac{1}{2}\int \frac{x}{x\sqrt{1-t^2}}\, dt \rightarrow I = \frac{1}{2}\int \frac{1}{\sqrt{1-t^2}}\, dt$

$f(x) = \text{arc sen}(x)$

$x = \text{sen } f(x)$

$1 = \cos f(x)\, f'(x)$

$f'(x) = \frac{1}{\cos f(x)}$

$\text{sen}^2 f(x) + \cos^2 f(x) = 1$

$\cos^2 f(x) = 1 - \text{sen}^2 f(x)$

$\cos f(x) = \sqrt{1 - \text{sen}^2 f(x)}$

$\cos f(x) = \sqrt{1 - x^2}$

$f'(x) = \frac{1}{\sqrt{1-x^2}}$

$I = \frac{1}{2}\int \frac{1}{\sqrt{1-t^2}}\, dt \rightarrow I = \frac{1}{2}\text{arcsen}(t) \rightarrow \boxed{I = \frac{1}{2}\text{arcsen}(x^2) + C}$ ✓

47) $I = \int \frac{a}{\sqrt{a^2 - x^2}}\, dx$

$$I = \int \frac{a}{\sqrt{a^2\left(1 - \left(\frac{x}{a}\right)^2\right)}}\, dx \rightarrow I = \int \frac{a}{a\sqrt{1-\left(\frac{x}{a}\right)^2}}\, dx$$

$$\frac{x}{a} = t \rightarrow \frac{1}{a}\, dx = dt \rightarrow dx = a\, dt$$

$$I = a\int \frac{1}{\sqrt{1-t^2}}\, dt \rightarrow I = a \operatorname{arc\,sen}(t)$$

$$\boxed{I = a.\operatorname{arc\,sen}\left(\frac{x}{a}\right) + C}$$ ✓

48) $I = \int x(x+1)^{10}\, dx$

$x + 1 = t$

$dx = dt$

$x = t - 1$

$$I = \int (t-1)\, t^{10}\, dt \rightarrow I = \int t^{11} - t^{10}\, dt$$

$$I = \frac{1}{12}t^{12} - \frac{1}{11}t^{11} \rightarrow \boxed{I = \frac{1}{12}(x+1)^{12} - \frac{1}{11}(x+1)^{11} + C}$$ ✓

49) $I = \int x^2 (x-2)^4\, dx$

$x - 2 = t$

$dx = dt$

$x - 2 = t$

$x = t + 2$

$$I = \int (t+2)^2\, t^4\, dt \rightarrow I = \int (t^2 + 2.2.t + 2^2)\, t^4\, dt$$

$$I = \int (t^2 + 4t + 4)\, t^4\, dt \rightarrow I = \int t^6 + 4t^5 + 4t^4\, dt$$

$$I = \frac{1}{7}t^7 + \frac{4}{6}t^6 + \frac{4t^5}{5} \rightarrow \boxed{I = \frac{1}{7}(x-2)^7 + \frac{2}{3}(x-2)^6 + \frac{4}{5}(x-2)^5 + C}$$ ✓

50) $I = \int \frac{x^3}{\sqrt{1+x^2}}\,dx$

$1+x^2 = t$

$2x\,dx = dt$

$dx = \frac{1}{2x}\,dt$

$\to x^2 = t-1$

$I = \frac{1}{2}\int \frac{x^3}{x\sqrt{t}}\,dt \to I = \frac{1}{2}\int \frac{t-1}{\sqrt{t}}\,dt$

$I = \frac{1}{2}\int (t-1)\,t^{-\frac{1}{2}}\,dt \to I = \frac{1}{2}\int t\,t^{-1/2} - t^{-1/2}\,dt$

$I = \frac{1}{2}\int t^{1/2} - t^{-1/2}\,dt \to I = \frac{1}{2}\left[\frac{t^{3/2}}{3/2} - \frac{t^{1/2}}{1/2}\right]$

$I = \frac{1}{2}\left[\frac{2}{3}\sqrt{t^3} - 2\sqrt{t}\right] \to \boxed{I = \frac{1}{3}\sqrt{(1+x^2)^3} - \sqrt{(1+x^2)} + C}$ ✓

51) $I = \int \frac{x^2}{\sqrt{1+x}}\,dx$

$\sqrt{1+x} = t$

$(1+x)^{1/2} = t$

$\frac{1}{2}(1+x)^{-1/2}\,dx = dt$

$dx = 2\sqrt{1+x}\,dt$

$\to 1+x = t^2$

$x = t^2-1$

$I = 2\int \frac{x^2 t}{t}\,dt \to I = 2\int (t^2-1)^2\,dt$

$I = 2\int t^4 - 2t^2 + 1\,dt \to I = 2\left[\frac{1}{5}t^5 - \frac{2}{3}t^3 + t\right]$

$\boxed{I = \frac{2}{5}\sqrt{(1+x)^5} - \frac{4}{3}\sqrt{(1+x)^3} + 2\sqrt{1+x} + C}$ ✓

52) $I = \int x^3 \sqrt{1 - x^2}\, dx$

$$\left[\begin{array}{l} 1 - x^2 = t \\ -2x\, dx = dt \\ dx = -\frac{1}{2x} dt \\ \to 1 - t = x^2 \end{array}\right.$$

$$I = -\frac{1}{2}\int \frac{x^3 \sqrt{t}}{x} dt \to I = -\frac{1}{2}\int x^2 \sqrt{t}\, dt \to I = -\frac{1}{2}\int (1-t)\sqrt{t}\, dt$$

$$I = -\frac{1}{2}\int t^{1/2} - t t^{1/2} dt \to I = -\frac{1}{2}\int t^{1/2} - t^{3/2} dt \to I = -\frac{1}{2}\left[\frac{t^{3/2}}{3/2} - \frac{t^{5/2}}{5/2}\right]$$

$$I = -\frac{1}{2}\left[\frac{2}{3}\sqrt{t^3} - \frac{2}{5}\sqrt{t^5}\right] \to I = -\frac{1}{3}\sqrt{t^3} + \frac{1}{5}\sqrt{t^5}$$

$$\boxed{I = -\frac{1}{3}\sqrt{(1-x^2)^3} + \frac{1}{5}\sqrt{(1-x^2)^5} + C}$$ ✓

53) $I = \int x^5 \sqrt{1 - x^2}\, dx$

$$\left[\begin{array}{l} 1 - x^2 = t \\ -2x\, dx = dt \\ dx = -\frac{1}{2x} dt \\ \to 1 - t = x^2 \\ \quad x^4 = (1-t)^2 \end{array}\right.$$

$$I = -\frac{1}{2}\int \frac{x^5 \sqrt{t}}{x} dt \to I = -\frac{1}{2}\int x^4 \sqrt{t}\, dt$$

$$I = -\frac{1}{2}\int (1-t)^2 t^{1/2} dt \to I = -\frac{1}{2}\int (1 - 2t + t^2)\, t^{1/2} dt$$

$$I = -\frac{1}{2}\int t^{1/2} - 2t^{3/2} + t^{5/2} dt \to I = -\frac{1}{2}\left[\frac{t^{3/2}}{3/2} - 2\frac{t^{5/2}}{5/2} + \frac{t^{7/2}}{7/2}\right]$$

$$I = -\frac{1}{2}\left[\frac{2}{3}\sqrt{t^3} - 2\cdot\frac{2}{5}\sqrt{t^5} + \frac{2}{7}\sqrt{t^7}\right]$$

$$\boxed{I = -\frac{1}{3}\sqrt{(1-x^2)^3} + \frac{2}{5}\sqrt{(1-x^2)^5} - \frac{1}{7}\sqrt{(1-x^2)^7} + C}$$ ✓

54) $I = \int x^5 (x^3 + 1)^{\frac{2}{3}} dx$

$$\begin{cases} x^3 + 1 = t \\ 3x^2 dx = dt \\ dx = \frac{1}{3x^2} dt \\ \to x^3 = t - 1 \end{cases}$$

$$I = \frac{1}{3}\int \frac{x^5 (t)^{2/3}}{x^2} dt \to I = \frac{1}{3}\int x^3 t^{2/3} dt$$

$$I = \frac{1}{3}\int (t-1) t^{2/3} dt \to I = \frac{1}{3}\int t^{5/3} - t^{2/3} dt$$

$$I = \frac{1}{3}\left[\frac{t^{8/3}}{8/3} - \frac{t^{5/3}}{5/3}\right] \to I = \frac{1}{3}\left[\frac{3}{8}\sqrt[3]{t^8} - \frac{3}{5}\sqrt[3]{t^5}\right]$$

$$\boxed{I = \frac{1}{8}\sqrt[3]{(x^3-1)^8} - \frac{1}{5}\sqrt[3]{(x^3-1)^5} + C}$$

✓

55) $I = \int x^7 \sqrt{x^4 + 2}\, dx$

$$\begin{cases} x^4 + 2 = t \\ 4x^3 dx = dt \\ dx = \frac{1}{4x^3} dt \\ \to x^4 = t - 2 \end{cases}$$

$$I = \frac{1}{4}\int \frac{x^7 \sqrt{t}}{x^3} dt \to I = \frac{1}{4}\int x^4 \sqrt{t}\, dt \to I = \frac{1}{4}\int (t-2) t^{1/2} dt$$

$$I = \frac{1}{4}\int t^{3/2} - 2t^{1/2} dt \to I = \frac{1}{4}\left[\frac{t^{5/2}}{5/2} - 2\frac{t^{3/2}}{3/2}\right]$$

$$I = \frac{1}{4}\left[\frac{2}{5}\sqrt{t^5} - \frac{4}{3}\sqrt{t^3}\right] \to I = \frac{1}{10}\sqrt{t^5} - \frac{1}{3}\sqrt{t^3}$$

$$\boxed{I = \frac{1}{10}\sqrt{(x^4+2)^5} - \frac{1}{3}\sqrt{(x^4+2)^3} + C}$$

✓

56) $I = \int x^3 \sqrt{2x^2 + 4}\, dx$

$2x^2+4=t$

$4x\,dx = dt$

$dx = \frac{1}{4x} dt$

$I = \frac{1}{4}\int \frac{x^3\sqrt{t}}{x} dt \to I = \frac{1}{4}\int x^2 \sqrt{t}\, dt$

$I = \frac{1}{4}\int \frac{t-4}{2}\sqrt{t}\, dt \to I = \frac{1}{8}\int (t-4)\, t^{1/2}\, dt$

$2x^2+4=t$

$2x^2 = t-4$

$x^2 = \frac{t-4}{2}$

$I = \frac{1}{8}\int t^{3/2} - 4t^{1/2}\, dt \to I = \frac{1}{8}\left[\frac{t^{5/2}}{5/2} - \frac{4t^{3/2}}{3/2}\right]$

$I = \frac{1}{8}\left[\frac{2}{5}\sqrt{t^5} - \frac{8}{3}\sqrt{t^3}\right] \to I = \frac{1}{20}\sqrt{t^5} - \frac{1}{3}\sqrt{t^3}$

$$\boxed{I = \frac{1}{20}\sqrt{(2x^2+4)^5} - \frac{1}{3}\sqrt{(2x^2+4)^3} + C}$$ ✓

57) $I = \int x^3 \sqrt{3x^2 - 6}\, dx$

$3x^2-6=t$

$6x\,dx = dt$

$dx = \frac{1}{6x} dt$

$I = \frac{1}{6}\int \frac{x^3\sqrt{t}}{x} dt \to I = \frac{1}{6}\int x^2\sqrt{t}\, dt$

$I = \frac{1}{6.3}\int (t+6)\, t^{1/2}\, dt \to I = \frac{1}{18}\int t^{3/2} + 6t^{1/2}\, dt$

$3x^2-6=t$

$3x^2 = t+6$

$x^2 = \frac{t+6}{3}$

$I = \frac{1}{18}\left[\frac{t^{5/2}}{5/2} + 6\frac{t^{3/2}}{3/2}\right] \to I = \frac{1}{18}\left[\frac{2}{5}\sqrt{t^5} + \frac{12}{3}\sqrt{t^3}\right]$

$I = \frac{1}{45}\sqrt{(3x^2-6)^5} + \frac{4}{18}\sqrt{(3x^2-6)^3}$

$$\boxed{I = \frac{1}{45}\sqrt{(3x^2-6)^5} + \frac{2}{9}\sqrt{(3x^2-6)^3} + C}$$ ✓

58) $I = \int \frac{1}{1 + \sqrt{x}}\, dx$

$1 + \sqrt{x} = t$

$1 + x^{1/2} = t$

$\frac{1}{2} x^{-1/2}\, dx = dt$

$dx = 2\sqrt{x}\, dt$

$\rightarrow \sqrt{x} = t - 1$

$I = 2\int \frac{\sqrt{x}}{t}\, dt \rightarrow I = 2\int (t-1)t^{-1}\, dt \rightarrow I = 2\int t t^{-1} - t^{-1}\, dt$

$I = 2\int 1 - \frac{1}{t}\, dt \rightarrow I = 2\left[t - \ln|t|\right] \rightarrow I = 2(1+\sqrt{x}) - \ln|(1+\sqrt{x})| + C$

$\boxed{I = 2\sqrt{x} - 2\ln|1+\sqrt{x}| + C}$ ✓

59) $I = \int \frac{\sqrt{x}}{1 + \sqrt{x}}\, dx$

$1 + \sqrt{x} = t$

$1 + x^{1/2} = t$

$\frac{1}{2} x^{-1/2}\, dx = dt$

$dx = 2\sqrt{x}\, dt$

$\rightarrow \sqrt{x} = t - 1$

$x = (t-1)^2$

$I = 2\int \frac{x}{t}\, dt \rightarrow I = 2\int (t-1)^2 t^{-1}\, dt$

$I = 2\int (t^2 - 2t + 1)t^{-1}\, dt \rightarrow I = 2\int t - 2 + \frac{1}{t}\, dt$

$I = 2\left[\frac{1}{2}t^2 - 2t + \ln|t|\right] \rightarrow I = t^2 - 4t + 2\ln|t|$

$I = (1+\sqrt{x})^2 - 4(1+\sqrt{x}) + 2\ln|1+\sqrt{x}|$

$I = 1 + 2\sqrt{x} + x - 4 - 4\sqrt{x} + 2\ln|1+\sqrt{x}|$

$\boxed{I = x - 2\sqrt{x} + 2\ln|1+\sqrt{x}| + C}$ ✓

60) $I = \int \frac{\sqrt[3]{x}}{(2 + x\sqrt[3]{x})^2}\,dx$

$2 + x\sqrt[3]{x} = t$

$2 + x\,x^{1/3} = t$

$2 + x^{4/3} = t$

$\frac{4}{3}x^{1/3}\,dx = dt$

$dx = \frac{3\,dt}{4\sqrt[3]{x}}$

$I = \frac{3}{4}\int \frac{\sqrt[3]{x}}{\sqrt[3]{x}\,(t)^2}\,dt$

$I = \frac{3}{4}\int \frac{1}{t^2}\,dt \rightarrow I = \frac{3}{4}\int t^{-2}\,dt$

$I = \frac{3}{4}\,\frac{t^{-1}}{-1} \rightarrow I = -\frac{3}{4t}$

$$\boxed{I = -\frac{3}{4(2+x\sqrt[3]{x})} + C}$$ ✓

61) $I = \int \frac{1}{\sqrt{x}\,(x+1)}\,dx$

$\sqrt{x} = t$

$x^{1/2} = t$

$\frac{1}{2}x^{-1/2}\,dx = dt$

$dx = 2\sqrt{x}\,dt$

$x = t^2$

$I = 2\int \frac{t}{t\,(x+1)}\,dt \rightarrow I = 2\int \frac{1}{t^2+1}\,dt$

$I = 2\,\mathrm{arctg}\,(t) \rightarrow \boxed{I = 2\,\mathrm{arctg}\,(\sqrt{x}) + C}$ ✓

$$62)\quad I = \int \frac{1}{1 + x + \sqrt{x+1}}\,dx$$

$\sqrt{x+1} = t$

$(x+1)^{1/2} = t$

$\frac{1}{2}(x+1)^{-1/2}\,dx = dt$

$dx = 2\sqrt{x+1}\,dt$

$\rightarrow \sqrt{x+1} = t$

$x+1 = t^2$

$I = 2\int \frac{t}{t^2+t}\,dt \rightarrow I = 2\int \frac{t}{t(t+1)}\,dt$

$I = 2\int \frac{1}{t+1}\,dt \rightarrow I = 2\int \frac{1}{u}\,du$

$t + 1 = u$

$dt = du$

$I = 2\ln|u|$

$I = 2\ln|t+1|$

$$\boxed{I = 2\ln\left|\sqrt{x+1}+1\right| + C}$$

✓

$$63)\quad I = \int \frac{x^5}{\sqrt{x^3+4} - 2}\,dx$$

$\sqrt{x^3+4} = t$

$(x^3+4)^{1/2} = t$

$\frac{1}{2}(x^3+4)^{-1/2}(3x^2)\,dx = dt$

$dx = \frac{2\sqrt{x^3+4}}{3x^2}$

$\rightarrow x^3+4 = t^2$

$x^3 = t^2 - 4$

$I = \frac{2}{3}\int \frac{t x^5}{x^2(t-2)}\,dt \rightarrow I = \frac{2}{3}\int \frac{x^3\,t}{t-2}\,dt$

$I = \frac{2}{3}\int \frac{(t^2-4)t}{t-2}\,dt \rightarrow I = \frac{2}{3}\int \frac{\cancel{(t-2)}(t+2)\,t}{\cancel{t-2}}\,dt$

$I = \frac{2}{3}\int (t+2)t\,dt \rightarrow I = \frac{2}{3}\int t^2 + 2t\,dt$

$I = \frac{2}{3}\left[\frac{1}{3}t^3 + t^2\right] \rightarrow I = \frac{2}{3}\left[\frac{1}{3}\sqrt{(x^3+4)^3} + \sqrt{(x^3+4)^2}\right]$

$$\boxed{I = \frac{2}{3}\left(\frac{1}{3}\sqrt{(x^3+4)^3} + (x^3+4)\right) + C}$$

✓

64) $I = \int \frac{2x + \sqrt{x+1}}{x^2 + 2x + 1}\,dx$

$(x+1)^{1/2} = t$

$\frac{1}{2}(x+1)^{-1/2}\,dx = dt$

$dx = 2\sqrt{x+1}\,dt$

$\to \sqrt{x+1} = t$

$x + 1 = t^2$

$x = t^2 - 1$

$I = 2\int \frac{t(2x+t)\,dt}{(x+1)^2} \to I = 2\int \frac{2xt + t^2}{t^4}\,dt$

$I = 2\int \frac{2(t^2-1)t + t^2}{t^4}\,dt \to I = 2\int \frac{(2t^2-2)t + t^2}{t^4}\,dt$

$I = 2\int (2t^3 - 2t + t^2)\,t^{-4}\,dt \to I = 2\int 2t^{-1} - 2t^{-3} + t^{-2}\,dt$

$I = 2\left[2\ln|t| - \frac{2}{-2}t^{-2} - \frac{1}{t}\right] \to I = 4\ln|t| + \frac{2}{t^2} - \frac{2}{t}$

$$\boxed{I = 4\ln|\sqrt{x+1}| + \frac{2}{x+1} - \frac{2}{\sqrt{x+1}} + C}$$

✓