Chapter 11 - Mathematical Tools for Quantum Computing I

11.1 Exercise Self-Test

1. Is the function

$$T: \mathbb{R} \to \mathbb{R}$$

$$T(x) := x+1$$

a linear transformation?

2. Does a binary operation have anything to do with binary code?

3. Wich space has a bigger dimension: $\mathbb{R}^4$ or $\mathbb{C}^2$? the same dimension ✓

4. Of these expressions:

a-> $\langle 0|1\rangle$ a-> $|0\rangle = \begin{bmatrix}1\\0\end{bmatrix}$ $|1\rangle = \begin{bmatrix}0\\1\end{bmatrix}$ $1\times 2 \times 2\times 1 = 1\times 1$ number ✓

b-> $|0\rangle\langle 1|$

c-> $\langle 1|0\rangle|0\rangle$

d-> $\langle i|A|j\rangle$ where A is a matrix and i and j are numbers

Wich is a number? A vector? A matrix?

b-> $2\times 1 \times 1\times 2 = 2\times 2$ matrices ✓

c-> $\begin{bmatrix}0 & 1\end{bmatrix} \times \begin{bmatrix}1\\0\end{bmatrix} = 1\times 1 \times |0\rangle$ is a vector ✓

d->

11.7 Exercise Find the sum of vectors $\begin{pmatrix}1\\2\end{pmatrix}$ and $\begin{pmatrix}4\\2\end{pmatrix}$

$$\begin{pmatrix}1\\2\end{pmatrix}+\begin{pmatrix}4\\2\end{pmatrix}=\begin{pmatrix}4+1\\2+2\end{pmatrix} \rightarrow \begin{pmatrix}5\\4\end{pmatrix}$$ ✓

11.10 Exercise Perform the scalar multiplication $4.\begin{pmatrix}5\\6\end{pmatrix}$

$$4.\begin{pmatrix}5\\6\end{pmatrix}=\begin{pmatrix}4.5\\4.6\end{pmatrix} \rightarrow \begin{pmatrix}20\\24\end{pmatrix}$$

11.16 Exercise You're invited to verify expression 11.15 is a bona fide superposition of states in the sense that, as per Born's rule, the sum of the squares of the absolute values of the coeficients (or amplitudes, in the language of quantum mechanics) $\frac{3}{5}$ and $\frac{4}{5}$ is in fact 1:

$$\frac{3}{5}|0\rangle + \frac{4}{5}|1\rangle \qquad (11.15)$$

$$\left|\frac{3}{5}\right|^2+\left|\frac{4}{5}\right|^2=1 \rightarrow \left|\frac{9}{25}\right|+\left|\frac{16}{25}\right|$$

$$\rightarrow \frac{25}{25}=1$$ ✓

11.17 Exercise