in the inverse sense as

$$Q_i(i\omega) = \frac{Q_o(i\omega)}{(q_o/q_i)(i\omega)}$$

to find $Q_i(i\omega)$.

3. Inverse-transform $Q_i(i\omega)$ to $q_i(t)$.

This procedure theoretically will give the exact $q_i(t)$ whether the measurement system meets the $K\angle(-\omega\tau_{dt})$ requirements or not. In actual practice, of course, while the measurement system does not have to meet $K\angle(-\omega\tau_{dt})$, it *does* have to respond fairly strongly to all frequencies present in q_i; otherwise, some parts of the q_o frequency spectrum will be so small as to be submerged in the unavoidable "noise" present in all systems and thus be unrecoverable by the above mathematical process. As general-purpose digital computers are used more in data processing and as computing power is built into more "instruments," such dynamic correction becomes increasingly practical and is a usable alternative in those situations where measurement systems meeting $K\angle(-\omega\tau_{dt})$ cannot be constructed with the present state of the art. Many FFT signal/system analyzers have this capability built into their software.

An important variation of the above process has been successfully applied in cases where the primary sensor is inadequate but can be cascaded with frequency-sensitive analog elements whose transfer functions make up the deficiencies in the primary sensor. The above computations are then, in a sense, automatically and continuously carried out by the compensating equipment to reconstruct $q_i(t)$. This subject is discussed in detail later under the topic Dynamic Compensation.

Experimental Determination of Measurement-System Parameters

While theoretical analysis of instruments is vital to reveal the basic relationships involved in the operation of a device, it is rarely accurate enough to provide usable numerical values for critical parameters such as sensitivity, time constant, natural frequency, etc. Thus calibration of instrument systems is a necessity. We discussed static calibration; here we concentrate on dynamic characteristics.

For zero-order instruments, the response is instantaneous and so no dynamic characteristics exist. The only parameter to be determined is the static sensitivity K, which is found by static calibration.

For first-order instruments, the static sensitivity K also is found by static calibration. There is only one parameter pertinent to dynamic response, the time constant τ, and this may be found by a variety of methods. One common method applies a step input and measures τ as the time to achieve 63.2 percent of the final value. This method is influenced by inaccuracies in the determination of the $t = 0$ point and also gives no check as to whether the instrument is really first-order. A preferred method uses the data from a step-function test replotted semilogarithmically to get a better estimate of τ and to check conformity to true first-order response. This method goes as follows: From Eq. (3.143) we can write

$$\frac{q_o - Kq_{is}}{Kq_{is}} = -e^{-t/\tau} \tag{3.315}$$

$$1 - \frac{q_o}{Kq_{is}} = e^{-t/\tau} \tag{3.316}$$

Now we define

$$Z \triangleq \log_e\left(1 - \frac{q_o}{Kq_{is}}\right) \tag{3.317}$$

and then

$$Z = \frac{-t}{\tau} \qquad \frac{dZ}{dt} = \frac{-1}{\tau} \tag{3.318}$$

Thus if we plot Z versus t, we get a straight line whose slope is numerically $-1/\tau$. Figure 3.99 illustrates the procedure. This gives a more accurate value of τ since the best line through *all* the data points is used rather than just two points, as in the 63.2 percent method. Furthermore, if the data points fall nearly on a straight line, we are assured that the instrument is behaving as a first-order type. If the data deviate considerably from a straight line, we know the instrument is not truly first-order and a τ value obtained by the 63.2 percent method would be quite misleading.

An even stronger verification (or refutation) of first-order dynamic characteristics is available from frequency-response testing, although at considerable cost of time and money if the system is not completely electrical, since nonelectrical sine-wave generators are neither common nor necessarily cheap. If the equipment is available, the system is subjected to sinusoidal inputs over a wide frequency

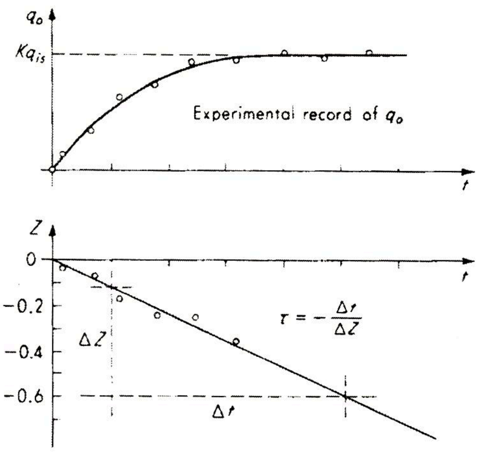

Figure 3.99 Step-function test of first-order system.

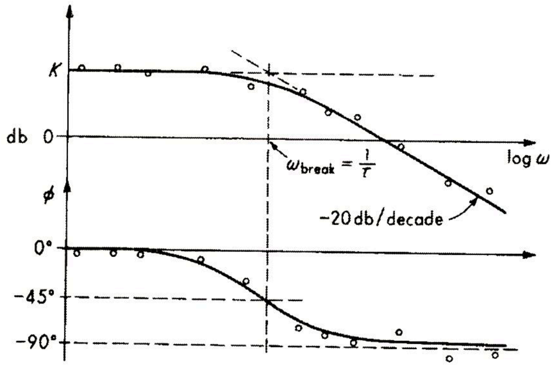

Figure 3.100 Frequency-response test of first-order system.

range, and the input and output are recorded. Amplitude ratio and phase angle are plotted on the logarithmic scales. If the system is truly first-order, the amplitude ratio follows the typical low- and high-frequency asymptotes (slope 0 and -20 dB/decade) and the phase angle approaches $-90°$ asymptotically. If these characteristics are present, the numerical value of τ is found by determining ω at the breakpoint and using $\tau = 1/\omega_{break}$ (see Fig. 3.100). Deviations from the above amplitude and/or phase characteristics indicate non-first-order behavior.

For second-order systems, K is found from static calibration, and ζ and ω_n can be obtained in a number of ways from step or frequency-response tests. Figure 3.101*a* shows a typical step-function response for an underdamped second-order system. The values of ζ and ω_n may be found from the relations

$$\zeta = \sqrt{\frac{1}{[\pi/\log_e (a/A)]^2 + 1}} \tag{3.319}$$

$$\omega_n = \frac{2\pi}{T\sqrt{1-\zeta^2}} \tag{3.320}$$

When a system is lightly damped, any fast transient input will produce a response similar to Fig. 3.101*b*. Then ζ can be closely approximated by

$$\zeta \approx \frac{\log_e (x_1/x_n)}{2\pi n} \tag{3.321}$$

This approximation assumes $\sqrt{1-\zeta^2} \approx 1.0$, which is quite accurate when $\zeta < 0.1$, and again ω_n can be found from Eq. (3.320). In applying Eq. (3.320), if several cycles of oscillation appear in the record, it is more accurate to determine the period T as the average of as many distinct cycles as are available rather than from a single cycle. If a system is strictly linear and second-order, the value of n in Eq. (3.321) is immaterial; the same value of ζ will be found for any number of cycles. Thus if ζ is calculated for, say, $n = 1, 2, 4$, and 6 and *different* numerical

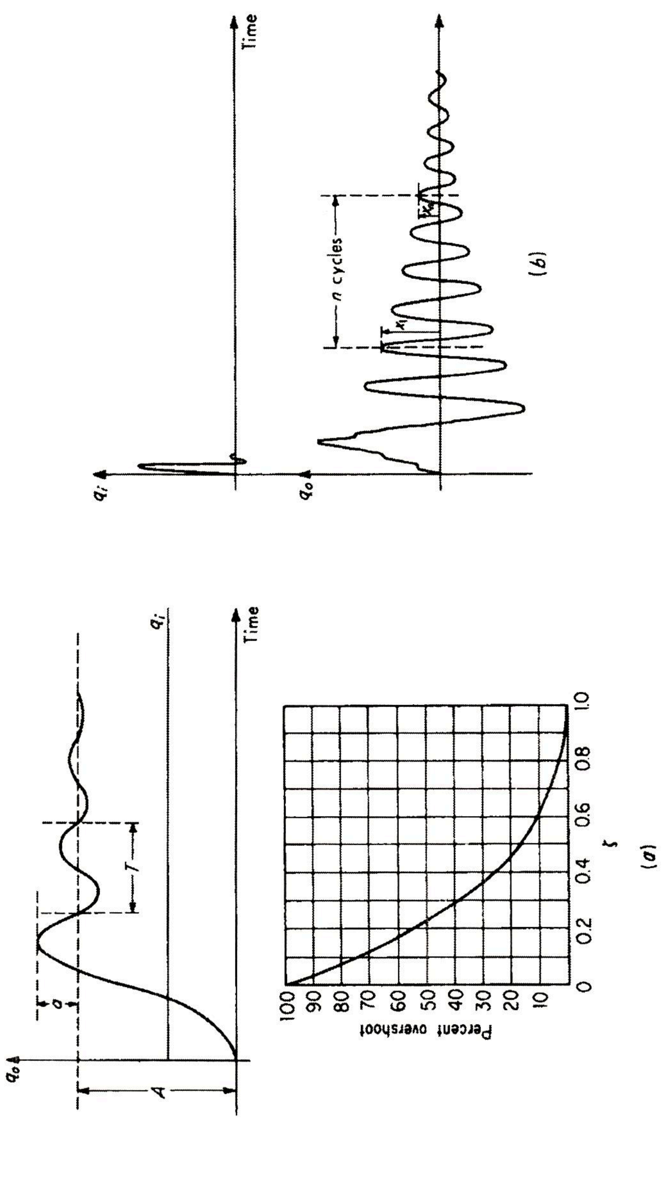

Figure 3.101 Step and pulse tests for second-order system.

values of ζ are obtained, we know the system is not following the postulated mathematical model. For overdamped systems ($\zeta > 1.0$), no oscillations exist, and the determination of ζ and ω_n becomes more difficult. Usually it is easier to express the system response in terms of two time constants τ_1 and τ_2, rather than ζ and ω_n. From Eq. (3.202) we can write

$$\frac{q_o}{Kq_{is}} = \frac{\tau_1}{\tau_2 - \tau_1} e^{-t/\tau_1} - \frac{\tau_2}{\tau_2 - \tau_1} e^{-t/\tau_2} + 1 \tag{3.322}$$

where

$$\tau_1 \triangleq \frac{1}{(\zeta - \sqrt{\zeta^2 - 1})\omega_n} \tag{3.323}$$

$$\tau_2 \triangleq \frac{1}{(\zeta + \sqrt{\zeta^2 - 1})\omega_n} \tag{3.324}$$

To find τ_1 and τ_2 from a step-function response curve, we may proceed as follows[1]:

1. Define the *percent incomplete response* R_{pi} as

$$R_{pi} \triangleq \left(1 - \frac{q_o}{Kq_{is}}\right) 100$$

2. Plot R_{pi} on a logarithmic scale versus time t on a linear scale. This curve will approach a straight line for large t if the system is second-order. Extend this line back to $t = 0$, and note the value P_1 where this line intersects the R_{pi} scale. Now, τ_1 is the time at which the straight-line asymptote has the value $0.368P_1$.
3. Now plot on the same graph a new curve which is the difference between the straight-line asymptote and R_{pi}. If this new curve is not a straight line, the system is not second-order. If it is a straight line, the time at which this line has the value $0.368(P_1 - 100)$ is numerically equal to τ_2.

Figure 3.102 illustrates this procedure. Once τ_1 and τ_2 are found, ζ and ω_n can be determined from Eqs. (3.323) and (3.324) if desired. Other methods[2] for finding τ_1 and τ_2 are available in the literature. Frequency-response methods also may be used to find ζ and ω_n or τ_1 and τ_2. Figure 3.103 shows the application of these techniques. The methods shown use the amplitude-ratio curve only. If phase-angle curves are available, they constitute a valuable check on conformance to the postulated model.

For measurement systems of arbitrary form (as contrasted to first- and second-order types), description of the dynamic behavior in terms of frequency response usually is desired. This information may be obtained by sinusoidal,

[1] N. A. Anderson, Step-Analysis Method of Finding Time Constant, *Instr. Contr. Syst.*, p. 130, November 1963.

[2] G. M. Hoerner, Second-Order System Characteristics from Initial Step Response, *Cont. Eng.*, p. 93, December 1962.

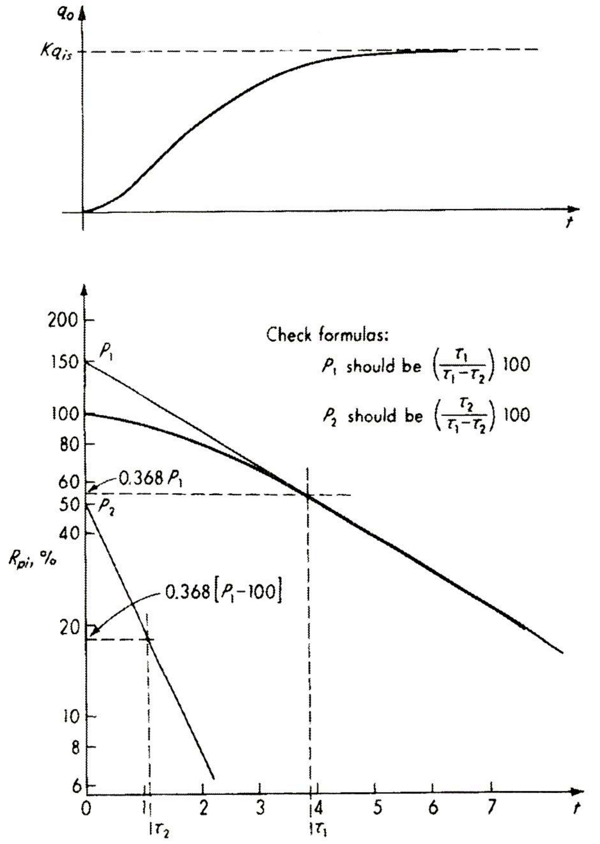

Figure 3.102 Step test for overdamped second-order systems.

pulse, or random signal testing, following the general methods[1] used to experimentally determine mathematical models for physical systems. When the physical system being studied is a *measurement* system, the output signal q_o is itself generally useful and no separate output sensor is required. However, we do usually need to measure the input signal q_i with a separate sensor, which serves as the calibration standard and whose accuracy is known, and, if possible, is about 10 times better than that of the system being calibrated. If we can obtain $(q_o/q_i)(i\omega)$ thus for the measurement system, this defines the frequency range over which corrections are negligible and provides the data needed to make dynamic corrections (using the transform methods of the previous section) if we wish to use the instrument in its nonflat range of frequency response.

[1] Doebelin, "System Modeling and Response," chap. 6.

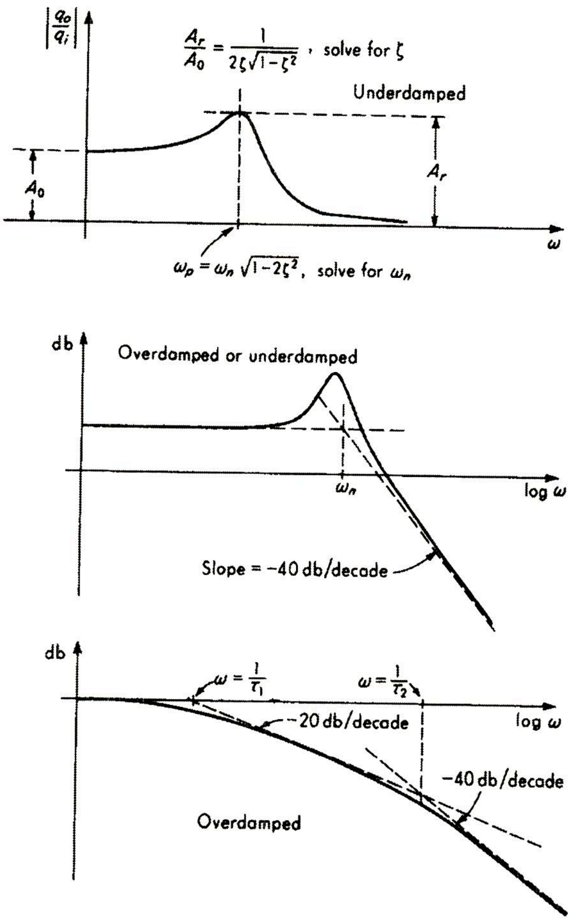

Figure 3.103 Frequency-response test of second-order system.

Loading Effects under Dynamic Conditions

The treatment of loading effects by means of impedance, admittance, etc., was discussed in Sec. 3.2 for static conditions. All these results can be immediately transferred to the case of dynamic operation by generalizing the definitions in terms of transfer functions. The basic equations relating the undisturbed value q_{i1u} and the actual measured value q_{i1m} at the input of a device are

$$q_{i1m} = \frac{1}{Z_{go}/Z_{gi} + 1} q_{i1u} \tag{3.55}$$

$$q_{i1m} = \frac{1}{Y_{go}/Y_{gi} + 1} q_{i1u} \tag{3.61}$$