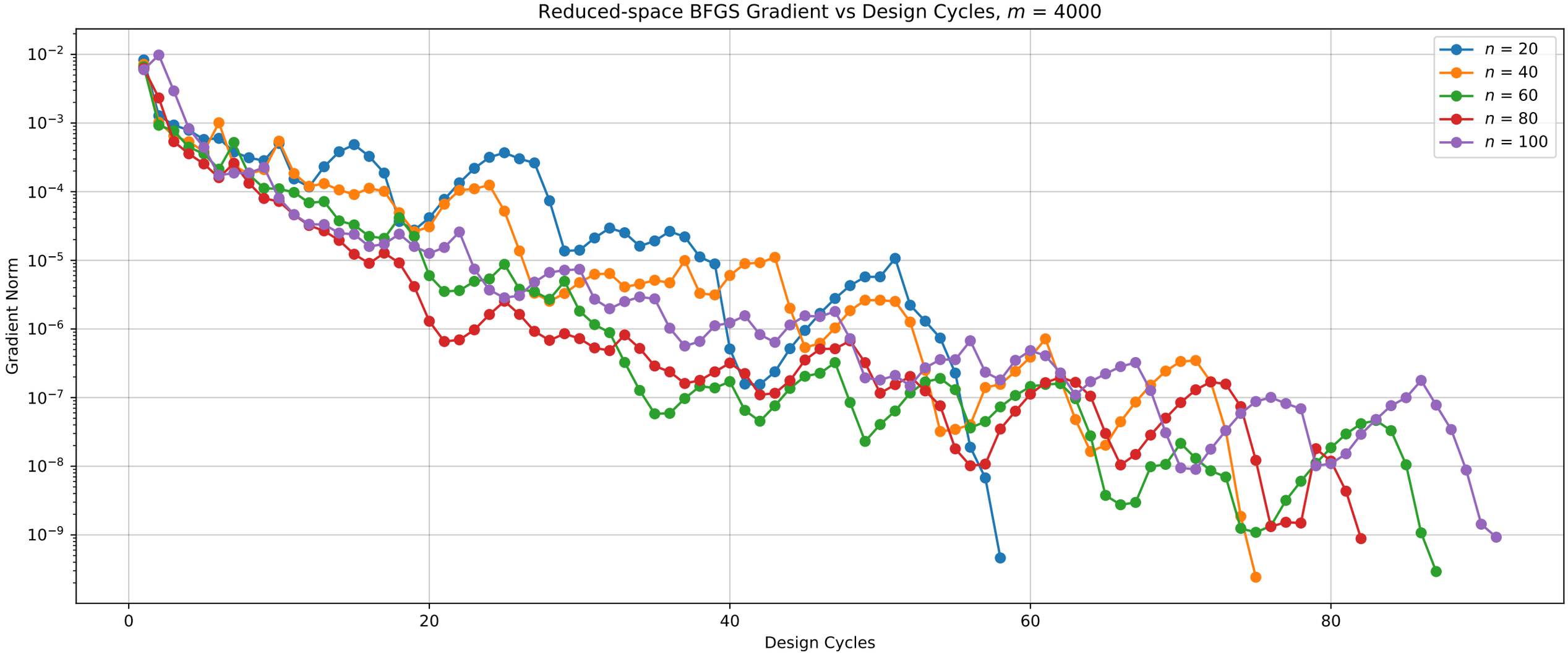
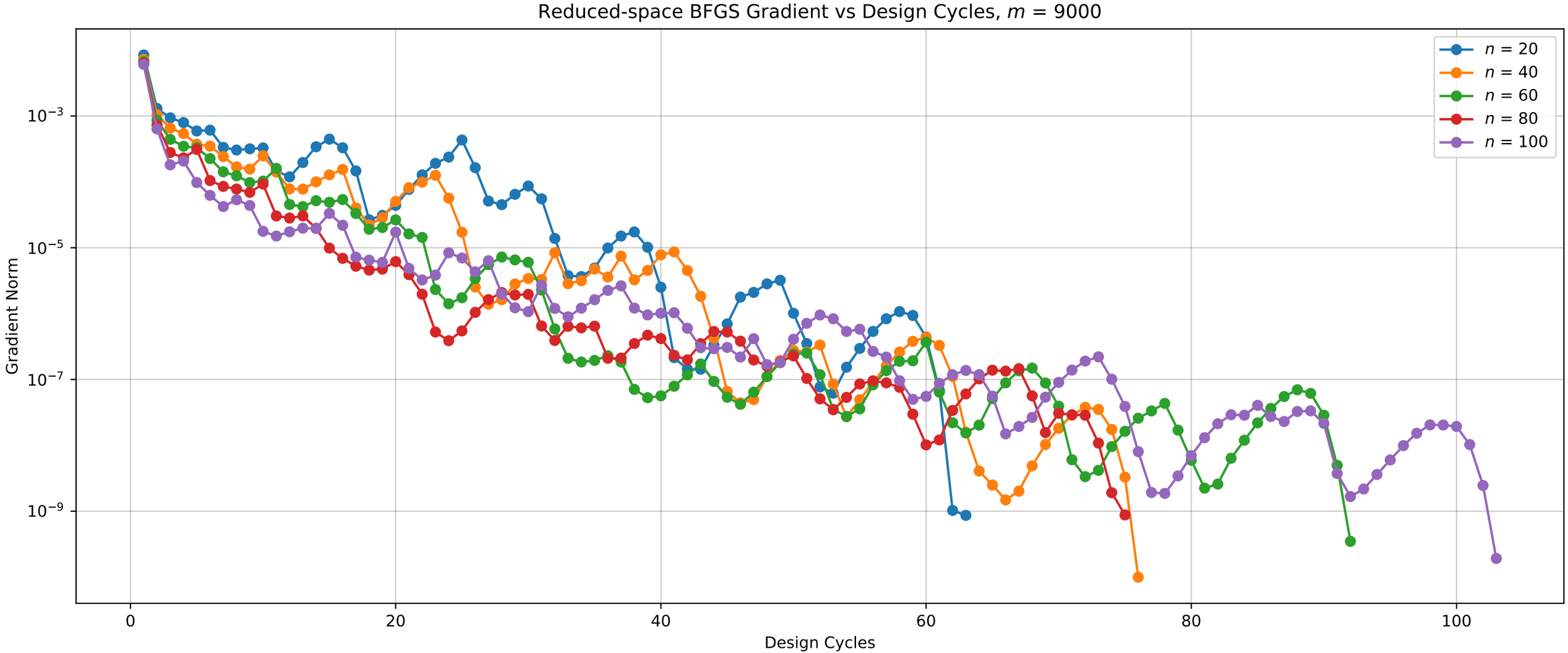
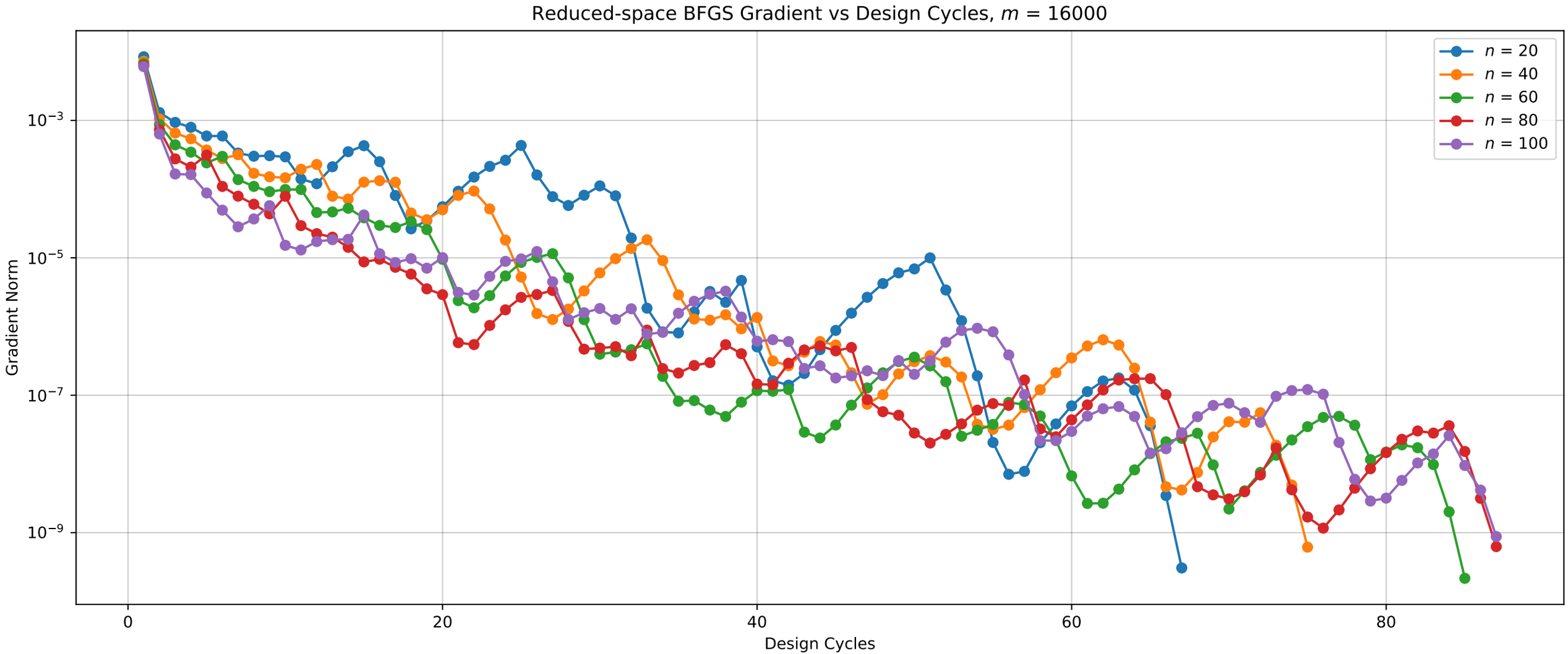
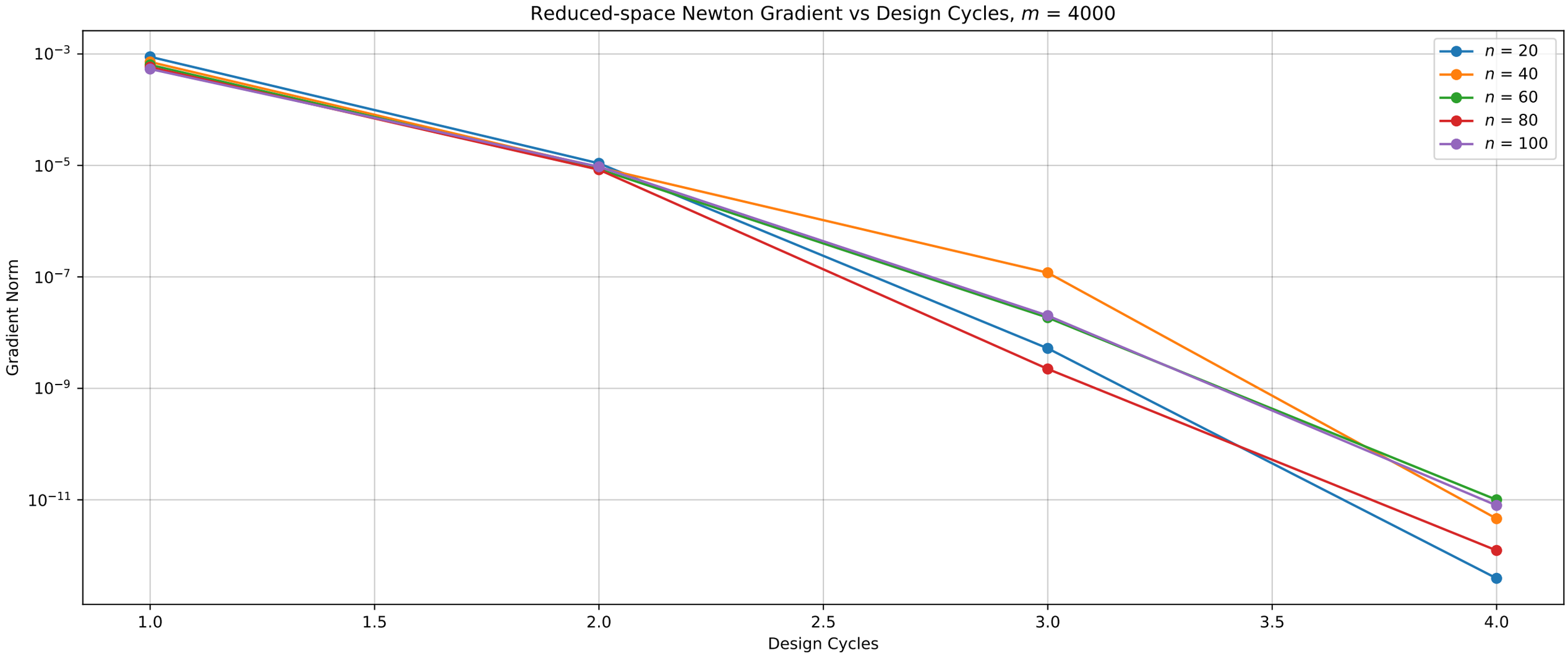
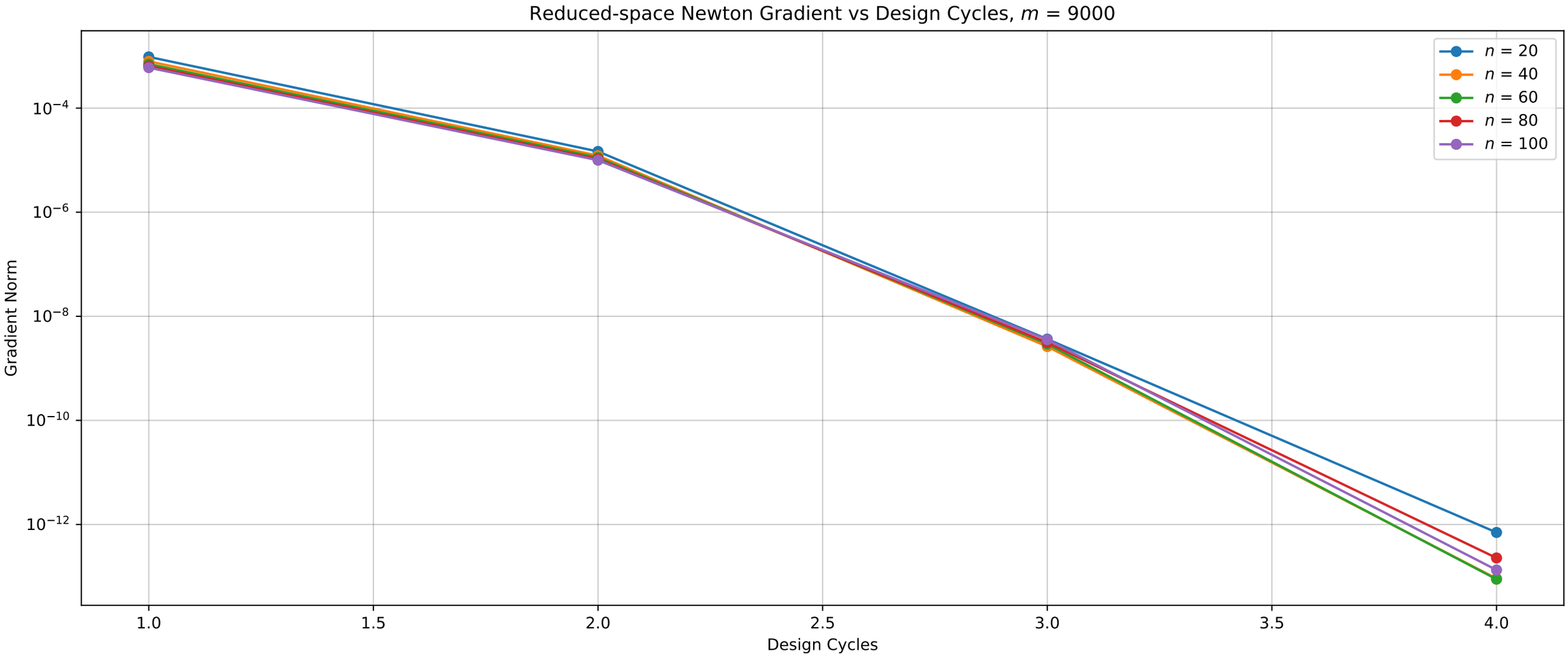
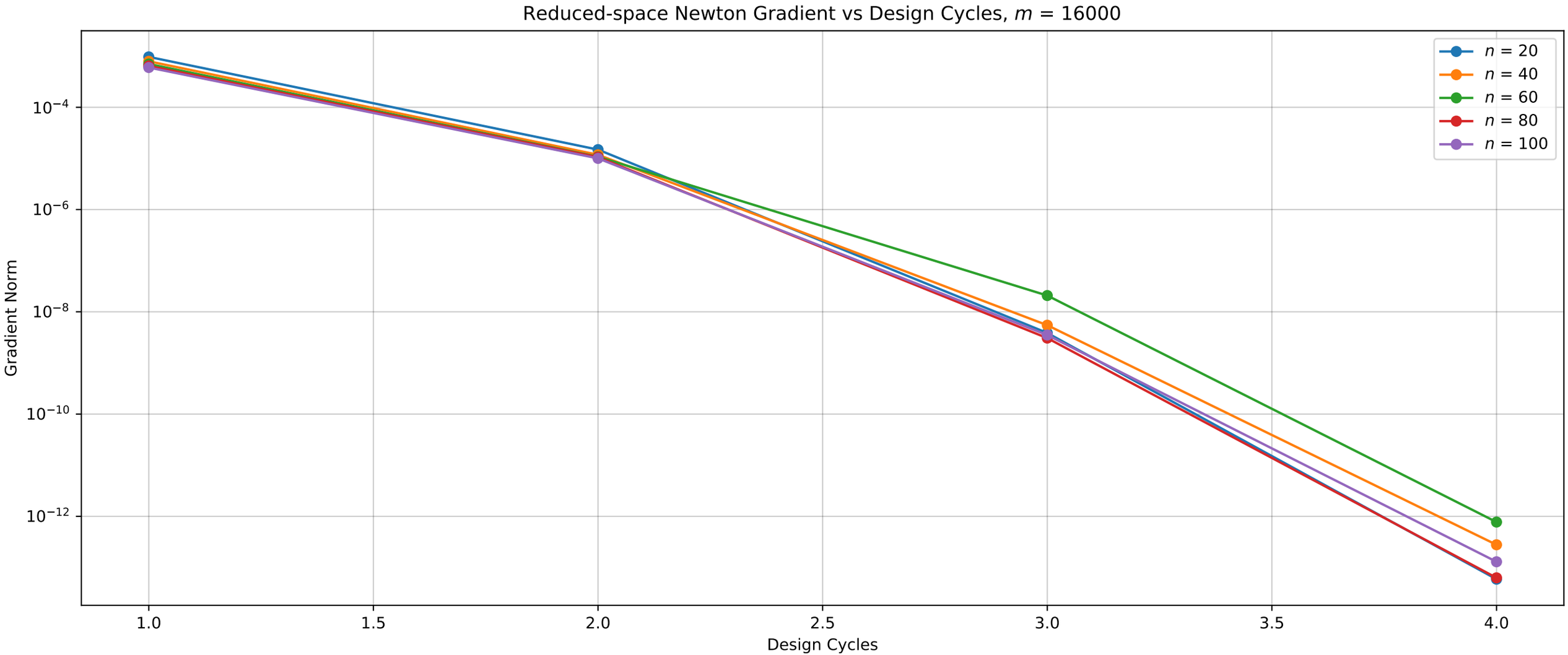

Reduced-space BFGS Gradient vs Design Cycles, m = 4000
Gradient Norm
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-8}
10^{-9}
0
20
40
60
80
Design Cycles
n = 20
n = 40
n = 60
n = 80
n = 100

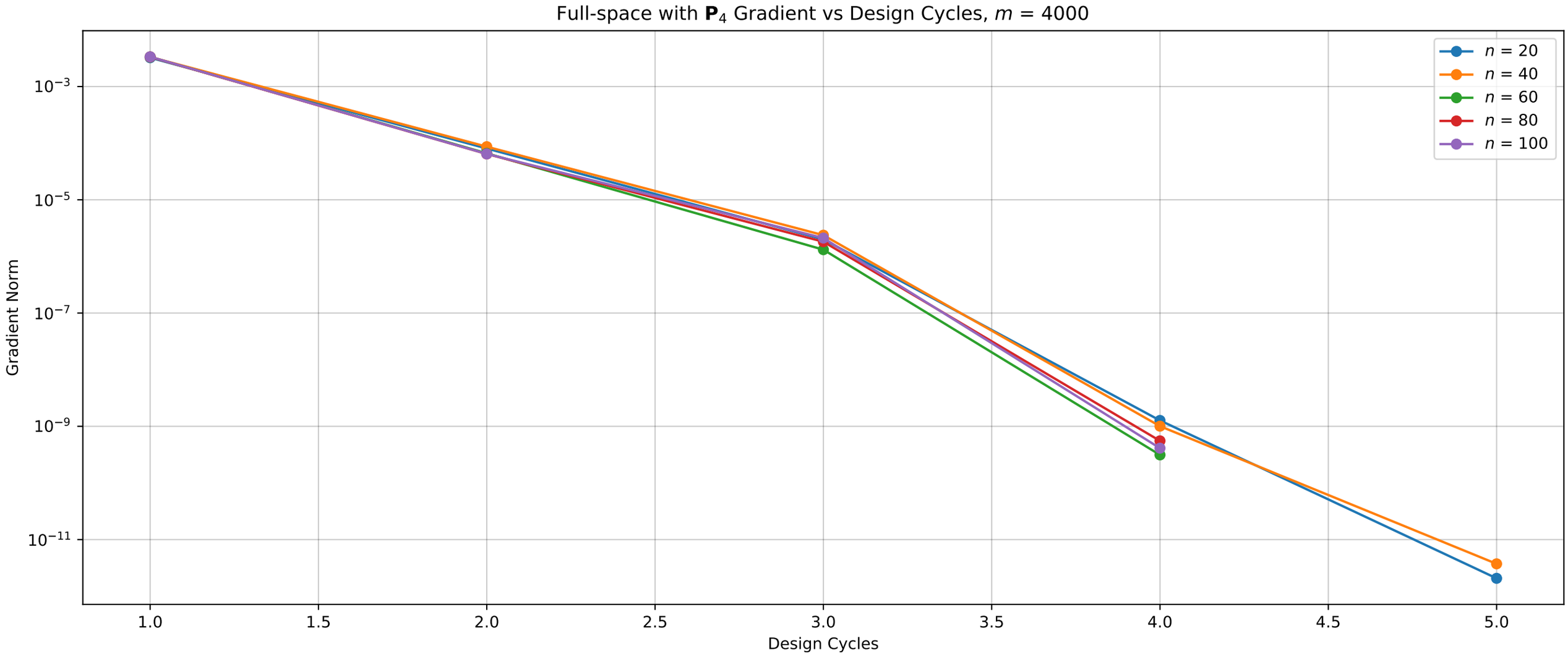

Reduced-space BFGS Gradient vs Design Cycles, m = 9000
Gradient Norm
10^{-3}
10^{-5}
10^{-7}
10^{-9}
0
20
40
60
80
100
Design Cycles
n = 20
n = 40
n = 60
n = 80
n = 100

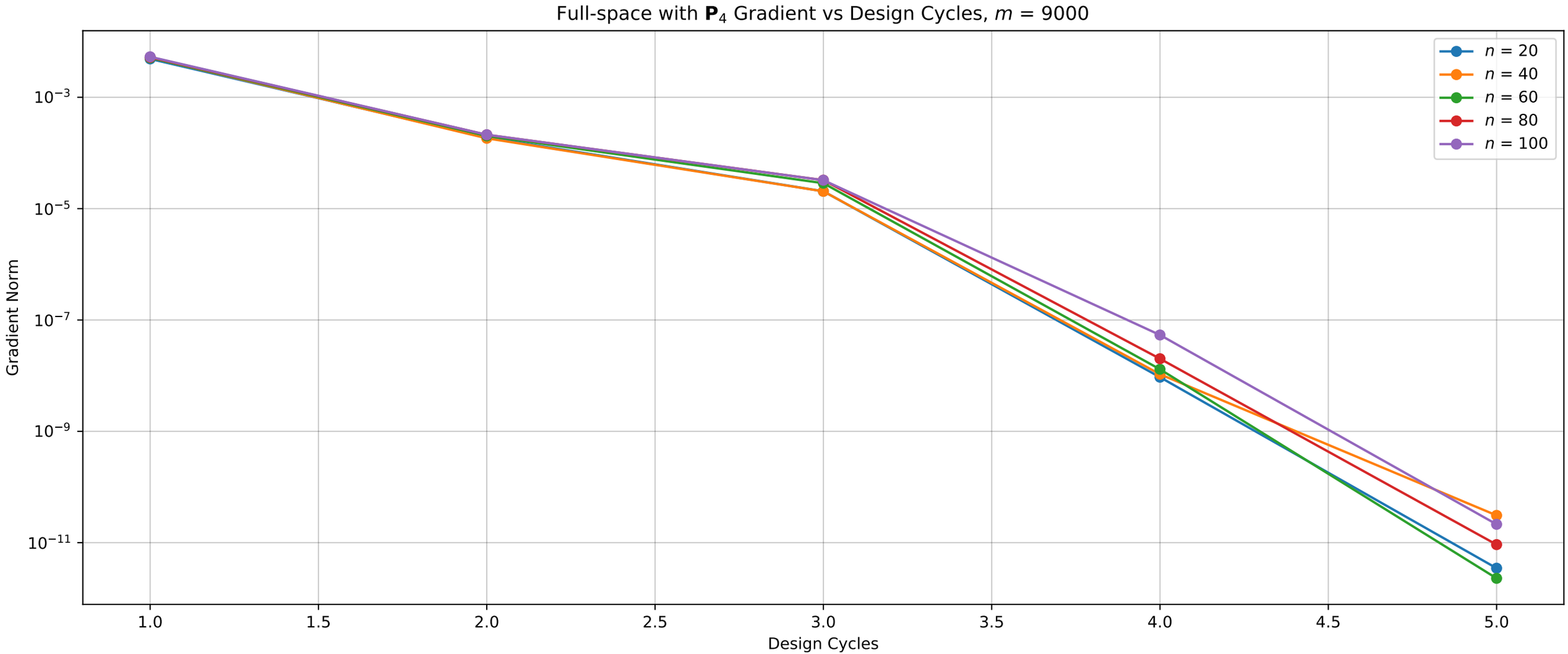

Reduced-space BFGS Gradient vs Design Cycles, m = 16000
Gradient Norm
10^{-3}
10^{-5}
10^{-7}
10^{-9}
0
20
40
60
80
Design Cycles
n = 20
n = 40
n = 60
n = 80
n = 100

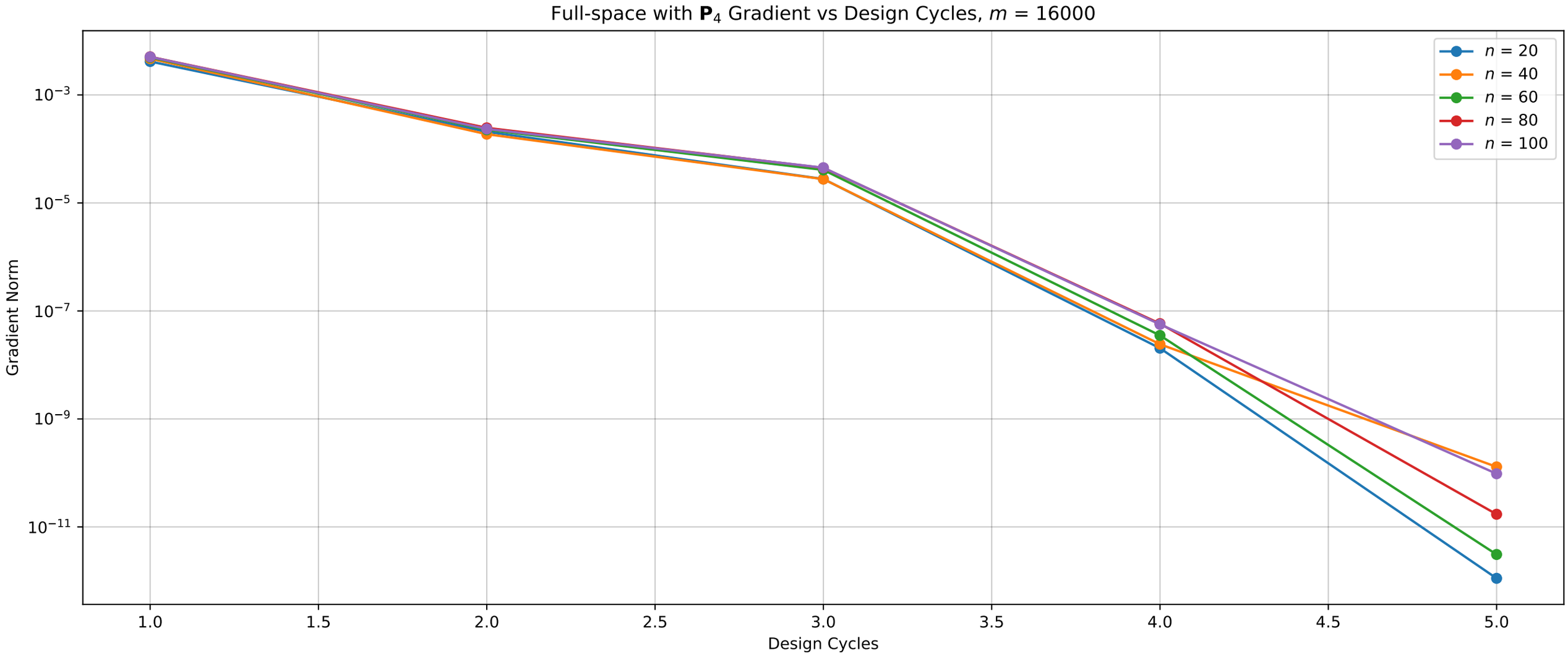

Reduced-space Newton Gradient vs Design Cycles, m = 4000
Gradient Norm
10^{-3}
10^{-5}
10^{-7}
10^{-9}
10^{-11}
1.0
1.5
2.0
2.5
3.0
3.5
4.0
Design Cycles
n = 20
n = 40
n = 60
n = 80
n = 100

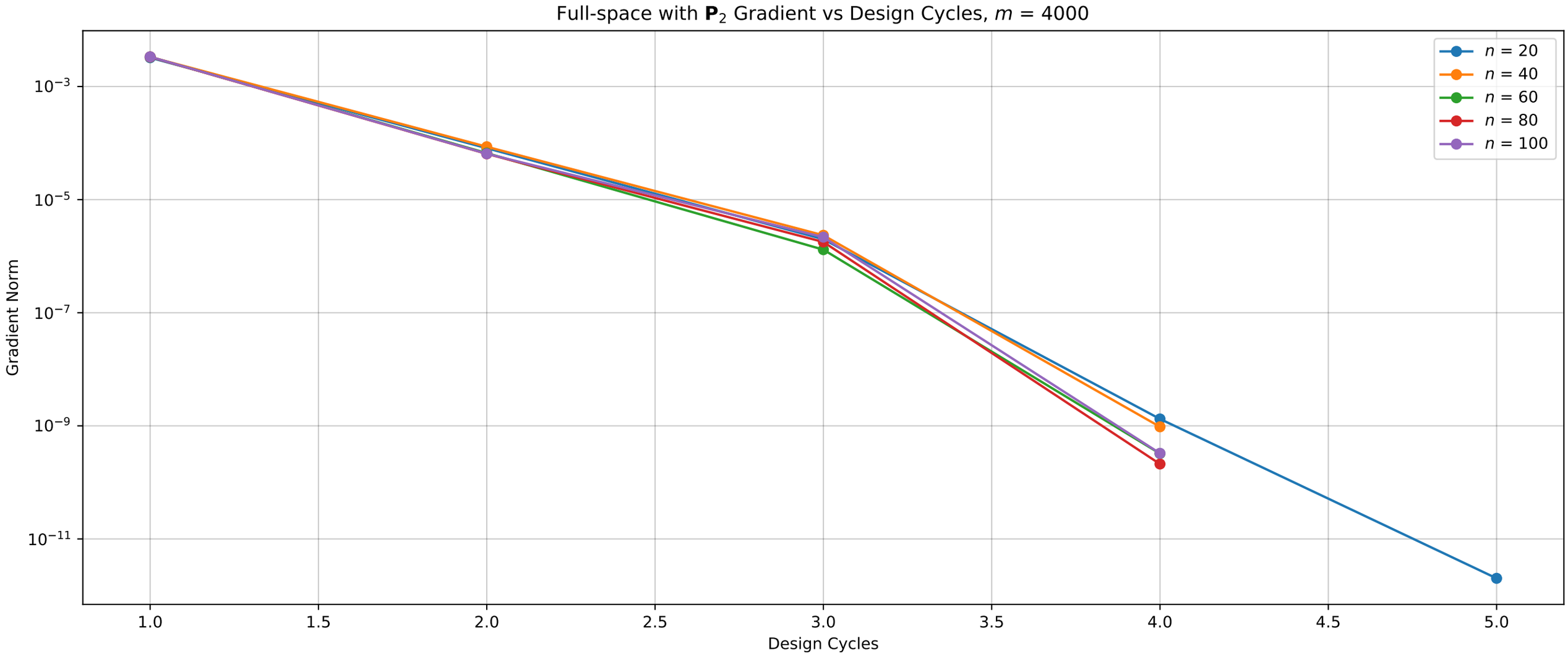

Reduced-space Newton Gradient vs Design Cycles, m = 9000
Gradient Norm
10^{-4}
10^{-6}
10^{-8}
10^{-10}
10^{-12}
1.0
1.5
2.0
2.5
3.0
3.5
4.0
Design Cycles
n = 20
n = 40
n = 60
n = 80
n = 100

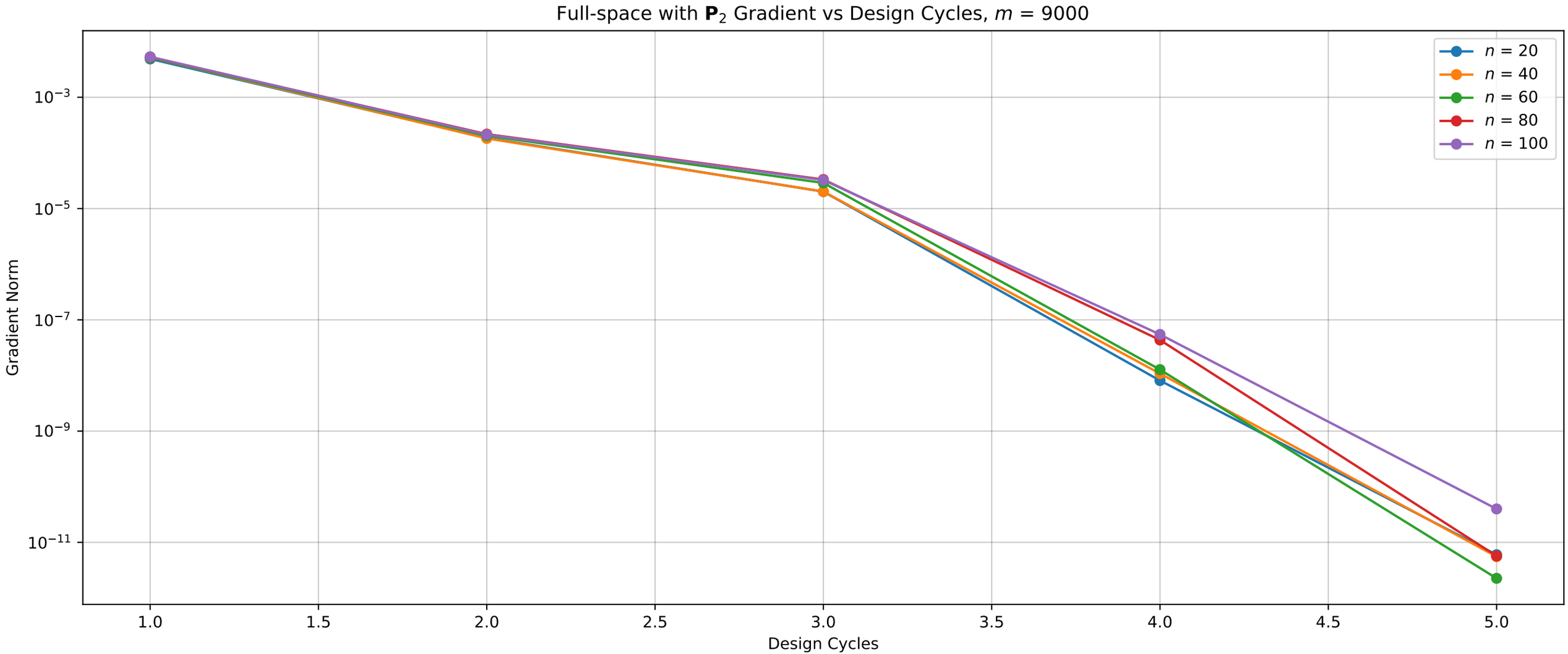

Reduced-space Newton Gradient vs Design Cycles, m = 16000
Gradient Norm
10^{-4}
10^{-6}
10^{-8}
10^{-10}
10^{-12}
1.0
1.5
2.0
2.5
3.0
3.5
4.0
Design Cycles
n = 20
n = 40
n = 60
n = 80
n = 100

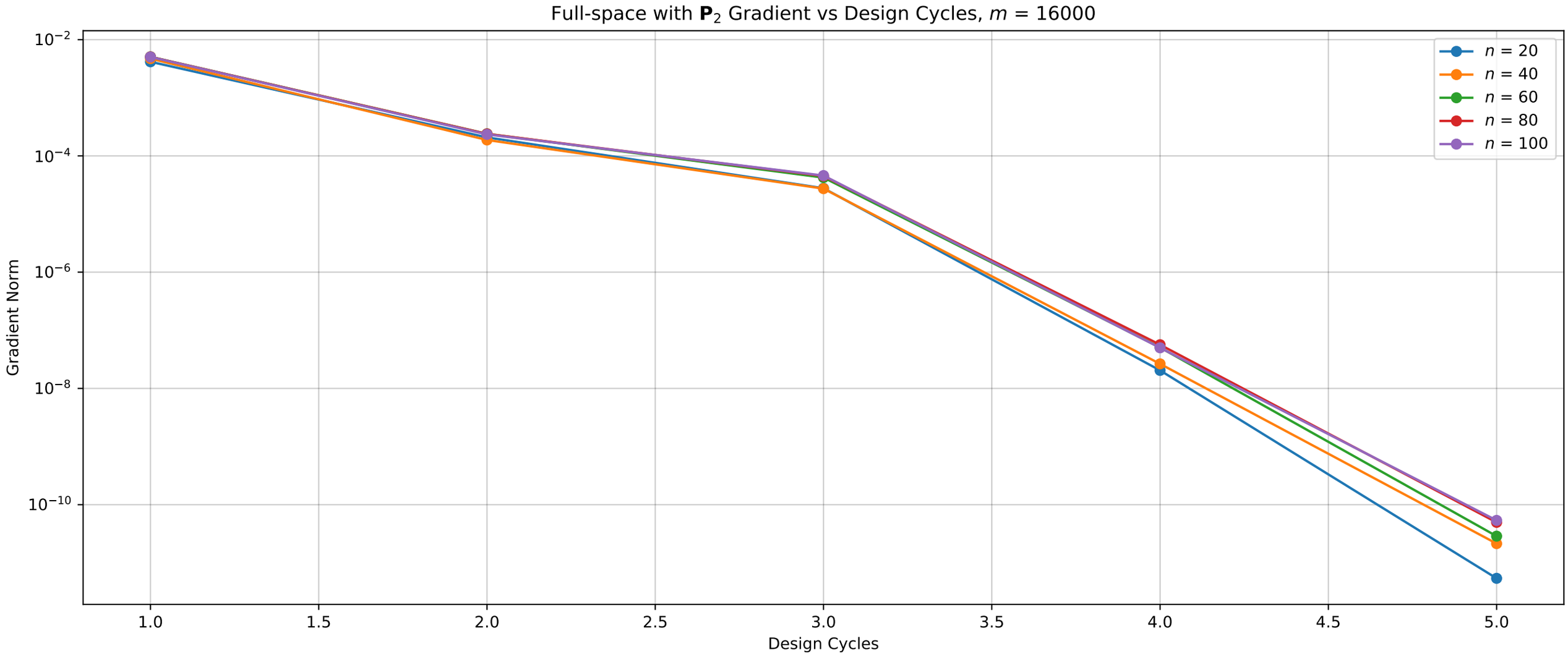

Full-space with $\mathbf{P}_4$ Gradient vs Design Cycles, m = 4000
Gradient Norm
10^{-3}
10^{-5}
10^{-7}
10^{-9}
10^{-11}
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0
Design Cycles
n = 20
n = 40
n = 60
n = 80
n = 100

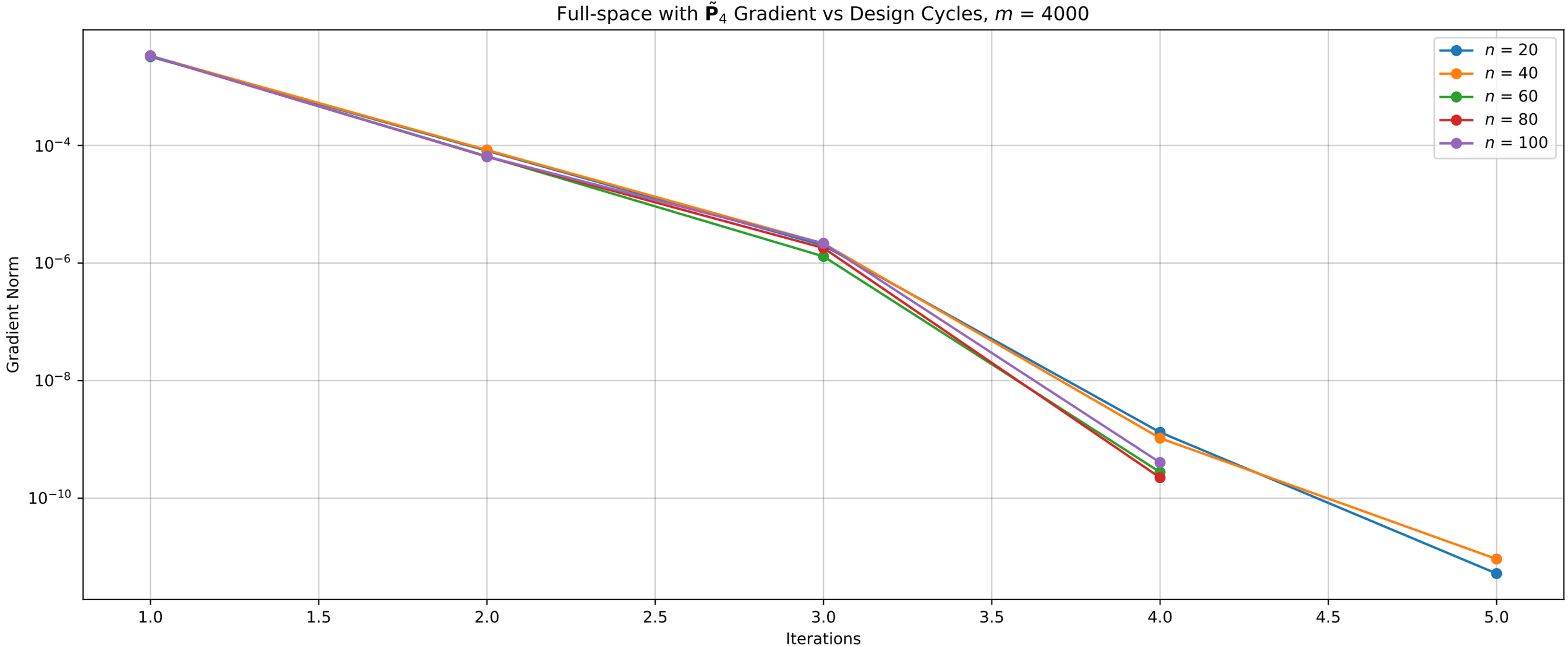

Full-space with $\mathbf{P}_4$ Gradient vs Design Cycles, m = 9000
Gradient Norm
10^{-3}
10^{-5}
10^{-7}
10^{-9}
10^{-11}
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0
Design Cycles
n = 20
n = 40
n = 60
n = 80
n = 100

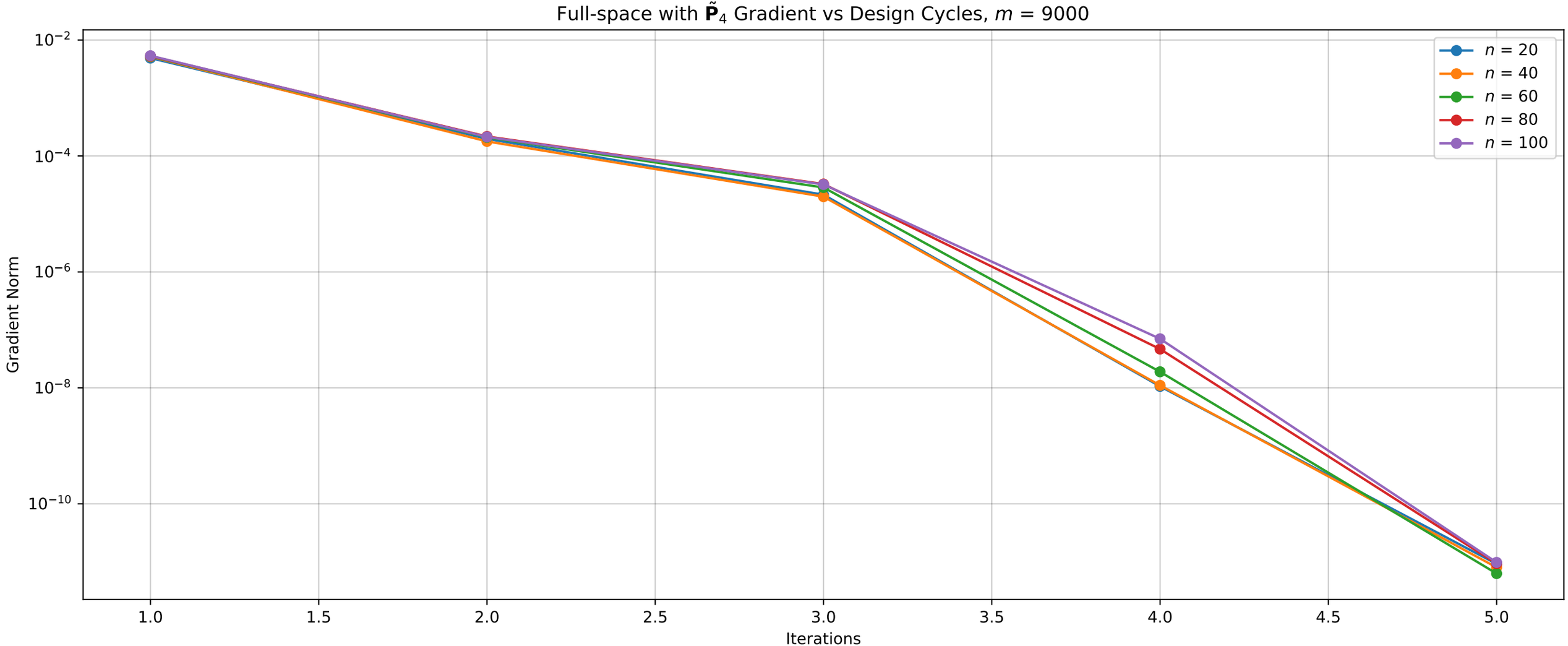

Full-space with $\mathbf{P}_4$ Gradient vs Design Cycles, m = 16000
Gradient Norm
10^{-3}
10^{-5}
10^{-7}
10^{-9}
10^{-11}
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0
Design Cycles
n = 20
n = 40
n = 60
n = 80
n = 100

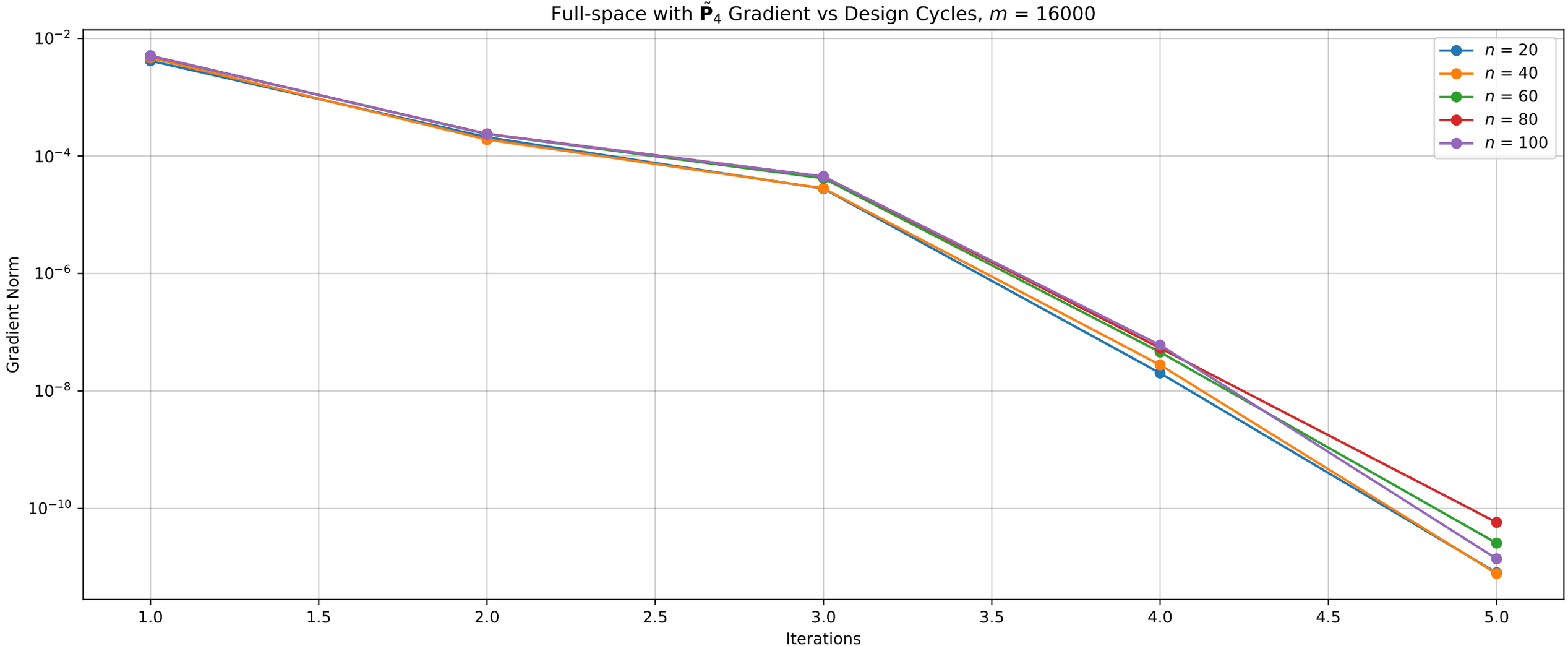

Full-space with $\mathbf{P}_2$ Gradient vs Design Cycles, m = 4000
Gradient Norm
10^{-3}
10^{-5}
10^{-7}
10^{-9}
10^{-11}
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0
Design Cycles
n = 20
n = 40
n = 60
n = 80
n = 100

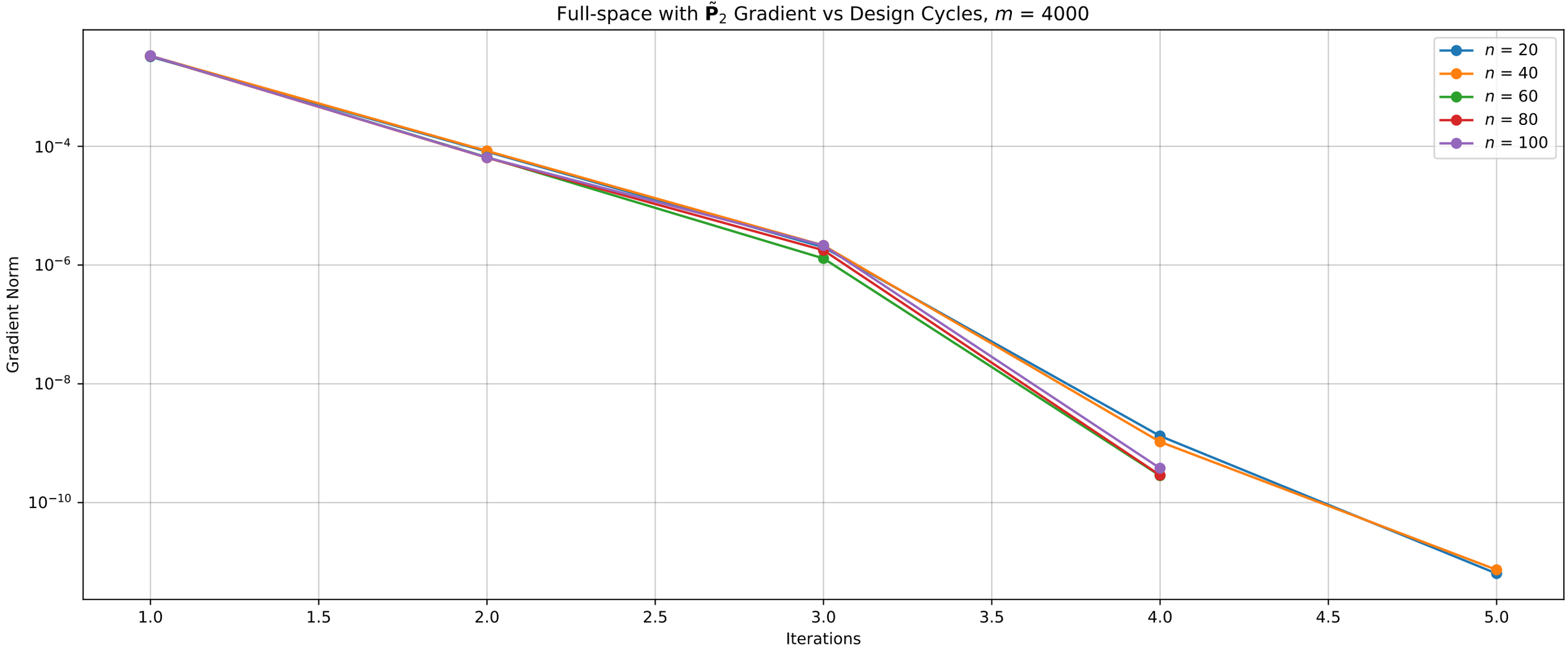

Full-space with $\mathbf{P}_2$ Gradient vs Design Cycles, m = 9000
Gradient Norm
10^{-3}
10^{-5}
10^{-7}
10^{-9}
10^{-11}
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0
Design Cycles
n = 20
n = 40
n = 60
n = 80
n = 100

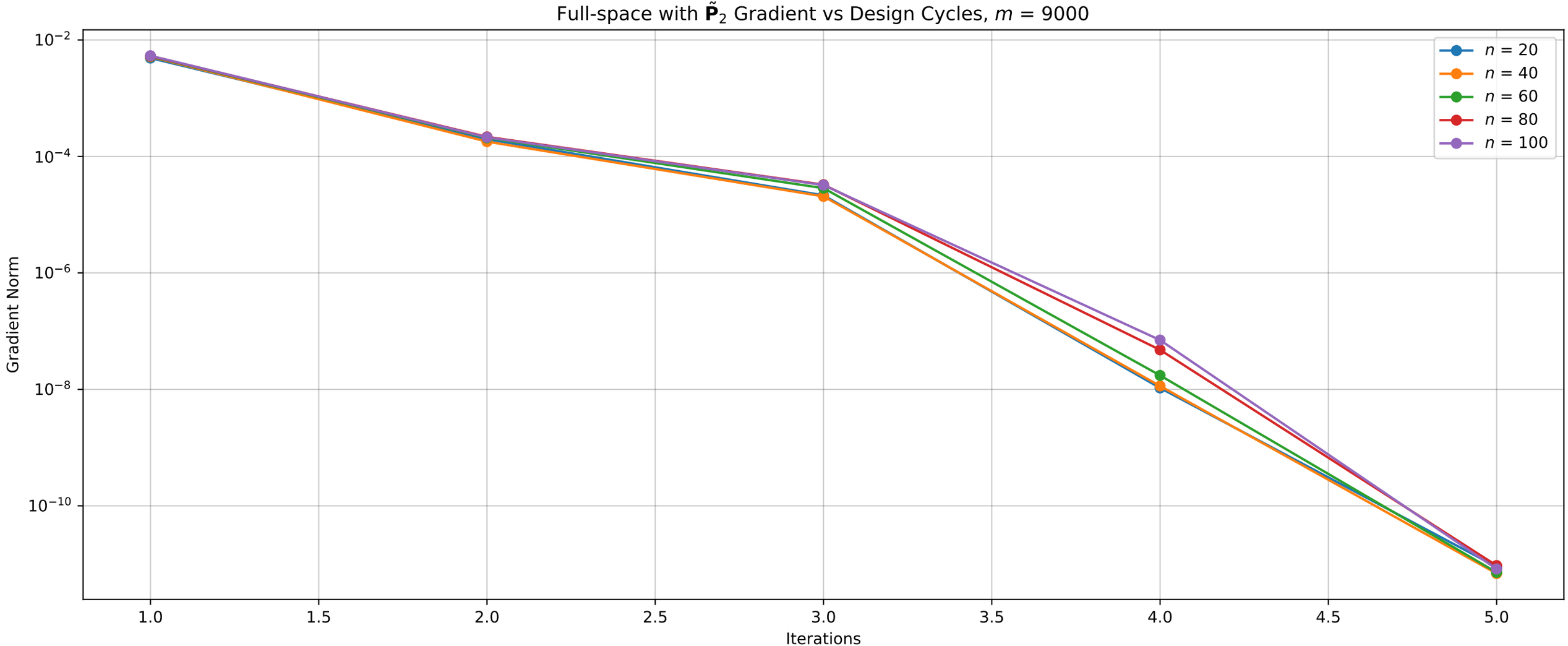

Full-space with $\mathbf{P}_2$ Gradient vs Design Cycles, m = 16000
Gradient Norm
10^{-2}
10^{-4}
10^{-6}
10^{-8}
10^{-10}
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0
Design Cycles
n = 20
n = 40
n = 60
n = 80
n = 100

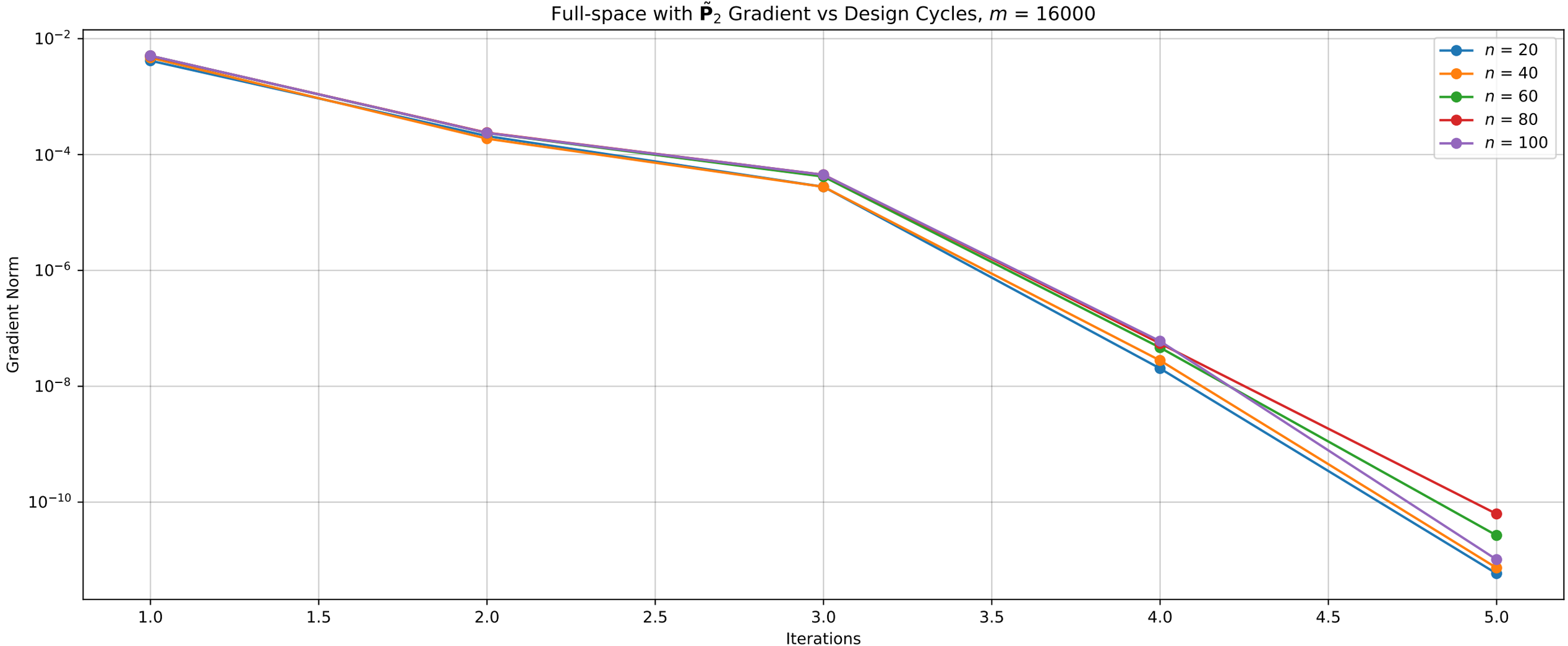

Full-space with $\tilde{\mathbf{P}}_4$ Gradient vs Design Cycles, m = 4000
Gradient Norm
10^{-4}
10^{-6}
10^{-8}
10^{-10}
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0
Iterations
n = 20
n = 40
n = 60
n = 80
n = 100

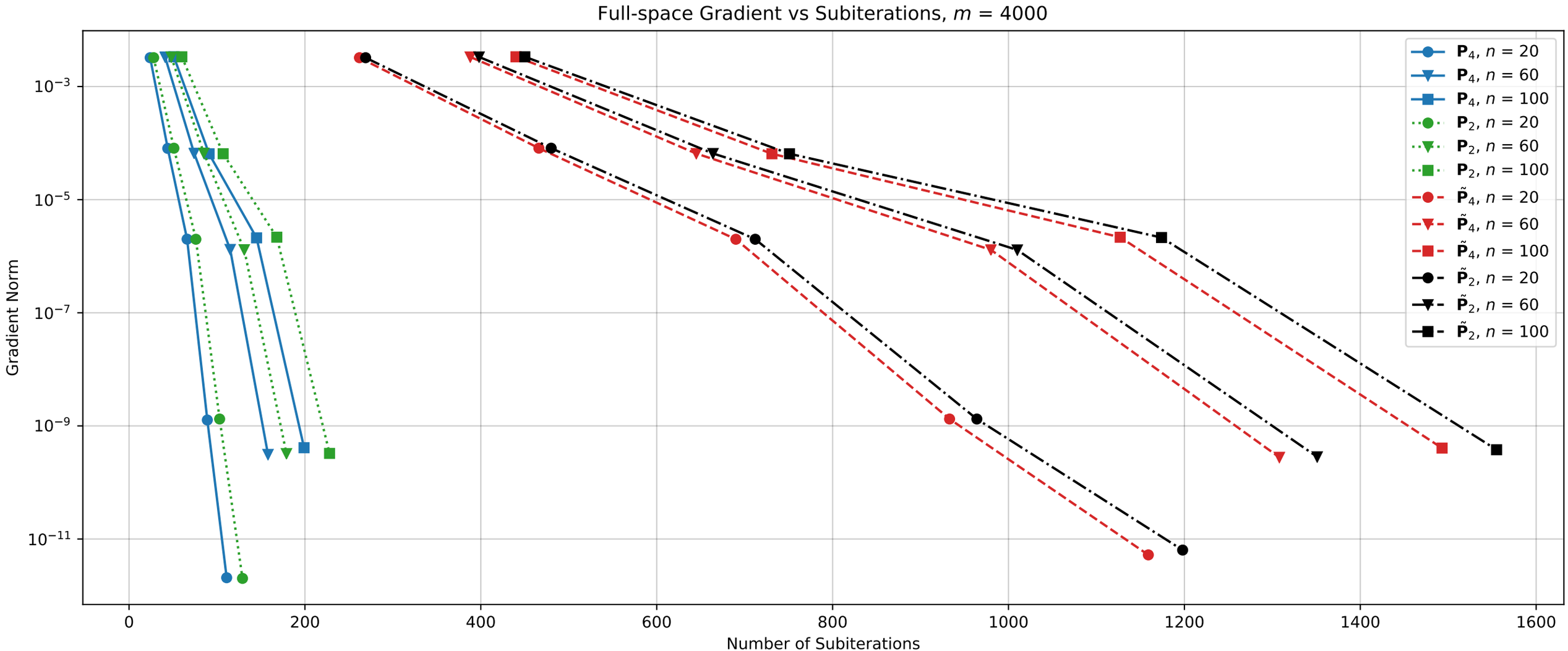

Full-space with $\tilde{\mathbf{P}}_4$ Gradient vs Design Cycles, m = 9000
Gradient Norm
10^{-2}
10^{-4}
10^{-6}
10^{-8}
10^{-10}
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0
Iterations
n = 20
n = 40
n = 60
n = 80
n = 100

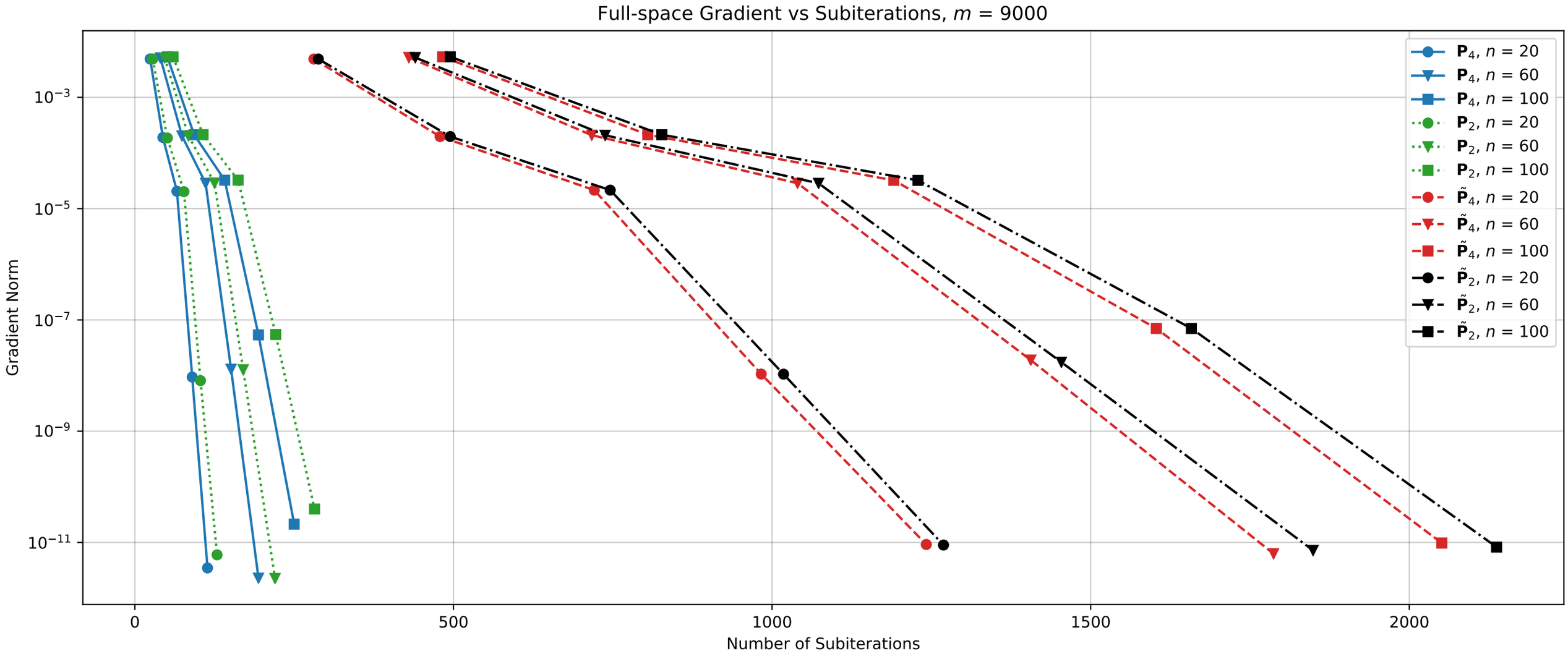

Full-space with $\tilde{\mathbf{P}}_4$ Gradient vs Design Cycles, m = 16000
Gradient Norm
10^{-2}
10^{-4}
10^{-6}
10^{-8}
10^{-10}
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0
Iterations
n = 20
n = 40
n = 60
n = 80
n = 100

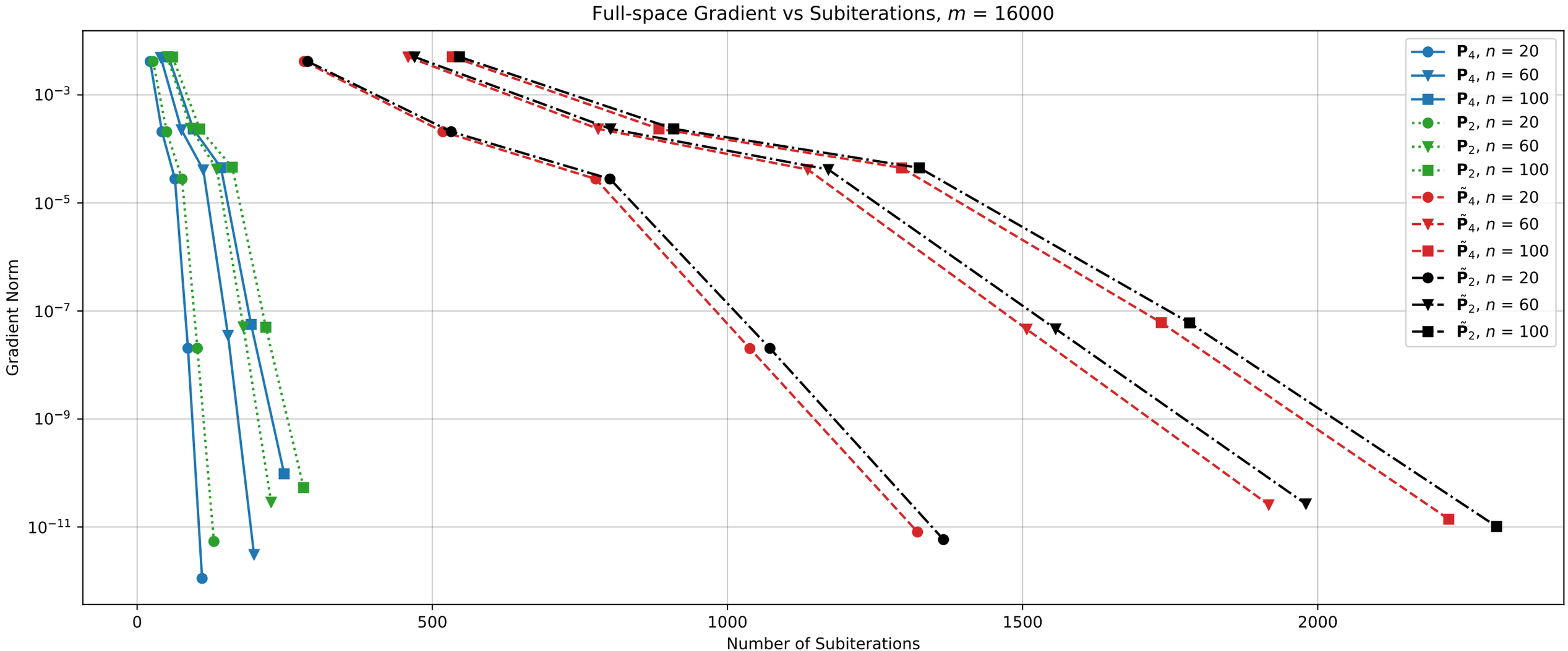

Full-space with $\tilde{\mathbf{P}}_2$ Gradient vs Design Cycles, m = 4000
Gradient Norm
10^{-4}
10^{-6}
10^{-8}
10^{-10}
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0
Iterations
n = 20
n = 40
n = 60
n = 80
n = 100


Full-space with $\tilde{\mathbf{P}}_2$ Gradient vs Design Cycles, m = 9000
Gradient Norm
10^{-2}
10^{-4}
10^{-6}
10^{-8}
10^{-10}
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0
Iterations
n = 20
n = 40
n = 60
n = 80
n = 100


Full-space with $\tilde{\mathbf{P}}_2$ Gradient vs Design Cycles, m = 16000
Gradient Norm
10^{-2}
10^{-4}
10^{-6}
10^{-8}
10^{-10}
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0
Iterations
n = 20
n = 40
n = 60
n = 80
n = 100


Full-space Gradient vs Subiterations, $m = 4000$
Gradient Norm
10^{-3}
10^{-5}
10^{-7}
10^{-9}
10^{-11}
0
200
400
600
800
1000
1200
1400
1600
Number of Subiterations
$\mathbf{P}_4$, $n = 20$
$\mathbf{P}_4$, $n = 60$
$\mathbf{P}_4$, $n = 100$
$\mathbf{P}_2$, $n = 20$
$\mathbf{P}_2$, $n = 60$
$\mathbf{P}_2$, $n = 100$
$\tilde{\mathbf{P}}_4$, $n = 20$
$\tilde{\mathbf{P}}_4$, $n = 60$
$\tilde{\mathbf{P}}_4$, $n = 100$
$\tilde{\mathbf{P}}_2$, $n = 20$
$\tilde{\mathbf{P}}_2$, $n = 60$
$\tilde{\mathbf{P}}_2$, $n = 100$


Full-space Gradient vs Subiterations, m = 9000
Gradient Norm
10^{-3}
10^{-5}
10^{-7}
10^{-9}
10^{-11}
0
500
1000
1500
2000
Number of Subiterations
$\mathbf{P}_4$, n = 20
$\mathbf{P}_4$, n = 60
$\mathbf{P}_4$, n = 100
$\mathbf{P}_2$, n = 20
$\mathbf{P}_2$, n = 60
$\mathbf{P}_2$, n = 100
$\tilde{\mathbf{P}}_4$, n = 20
$\tilde{\mathbf{P}}_4$, n = 60
$\tilde{\mathbf{P}}_4$, n = 100
$\tilde{\mathbf{P}}_2$, n = 20
$\tilde{\mathbf{P}}_2$, n = 60
$\tilde{\mathbf{P}}_2$, n = 100


Full-space Gradient vs Subiterations, m = 16000
Gradient Norm
10^{-3}
10^{-5}
10^{-7}
10^{-9}
10^{-11}
0
500
1000
1500
2000
Number of Subiterations
$\mathbf{P}_4$, n = 20
$\mathbf{P}_4$, n = 60
$\mathbf{P}_4$, n = 100
$\mathbf{P}_2$, n = 20
$\mathbf{P}_2$, n = 60
$\mathbf{P}_2$, n = 100
$\tilde{\mathbf{P}}_4$, n = 20
$\tilde{\mathbf{P}}_4$, n = 60
$\tilde{\mathbf{P}}_4$, n = 100
$\tilde{\mathbf{P}}_2$, n = 20
$\tilde{\mathbf{P}}_2$, n = 60
$\tilde{\mathbf{P}}_2$, n = 100

Full-space Gradient vs Subiterations

$m = 4000$

$m = 9000$

$m = 16000$

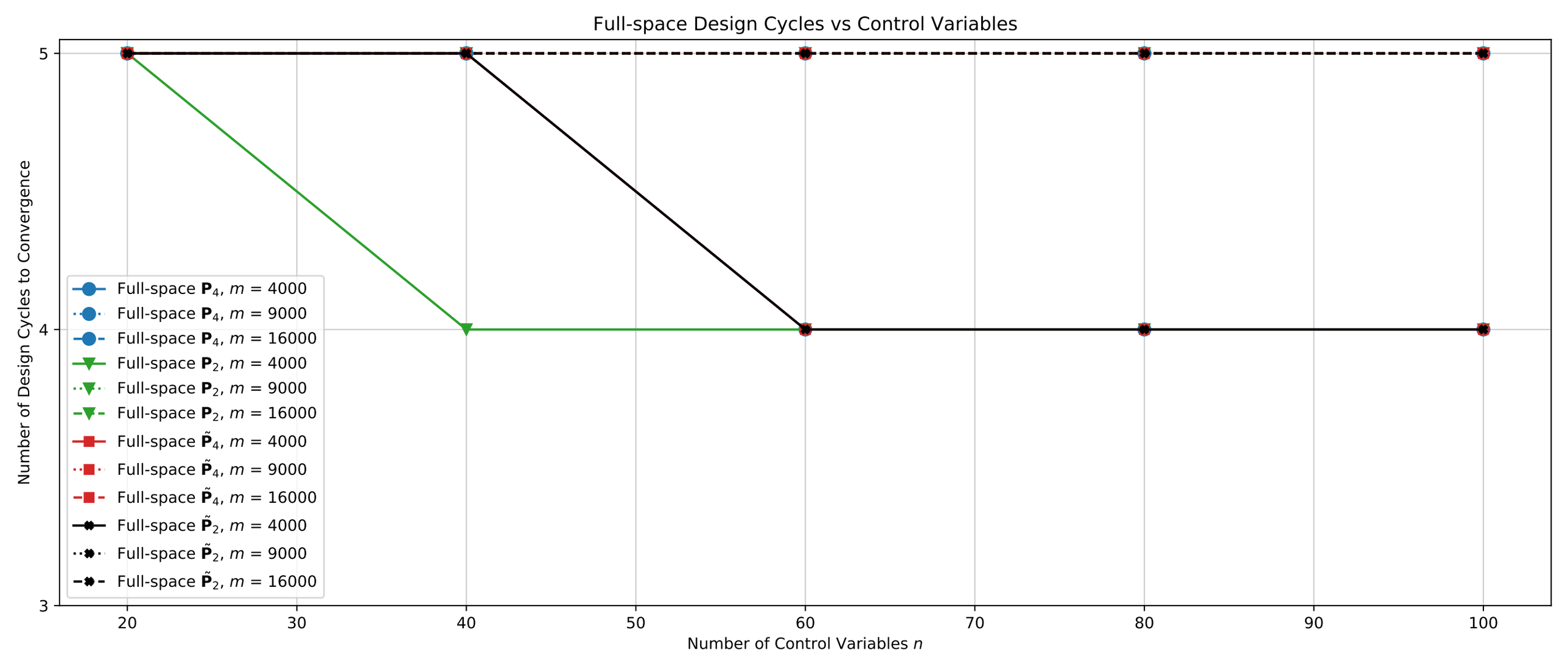
Gradient Norm

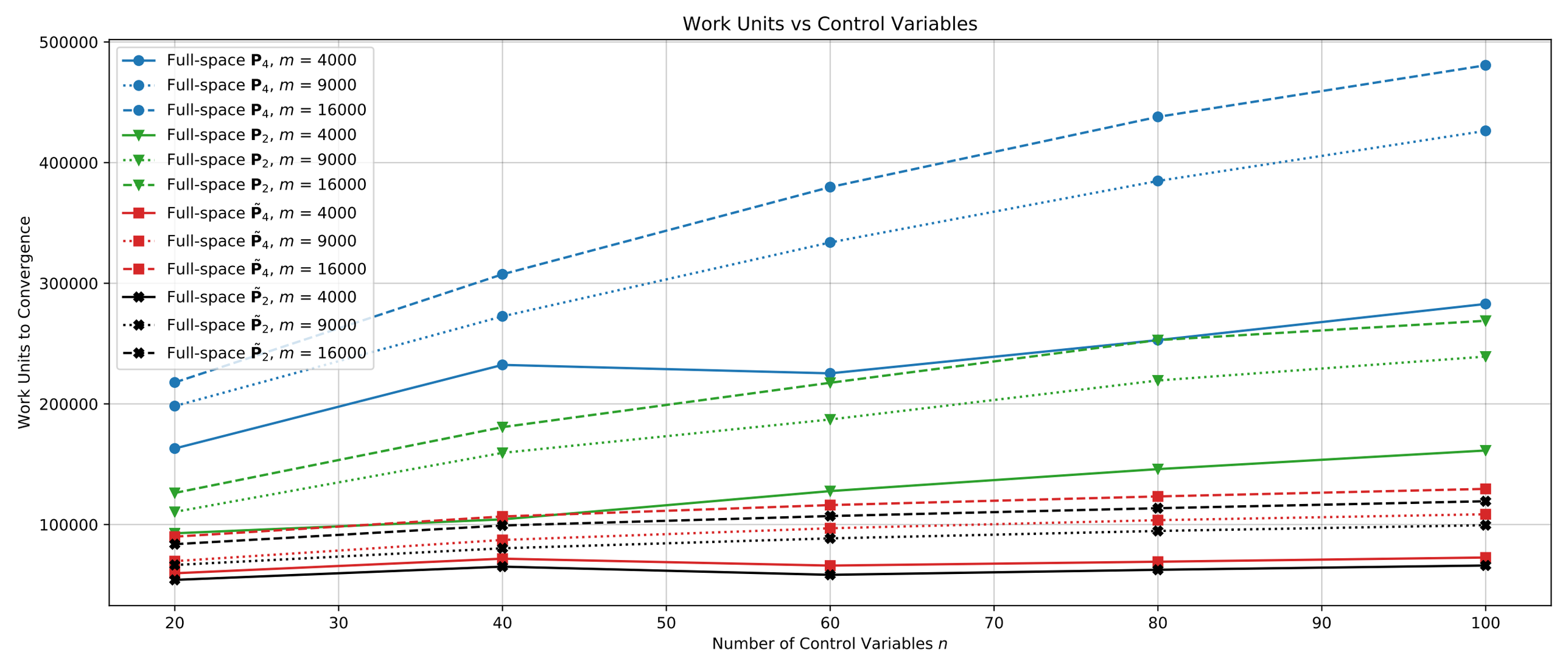
10^{-3}
10^{-5}
10^{-7}
10^{-9}
10^{-11}

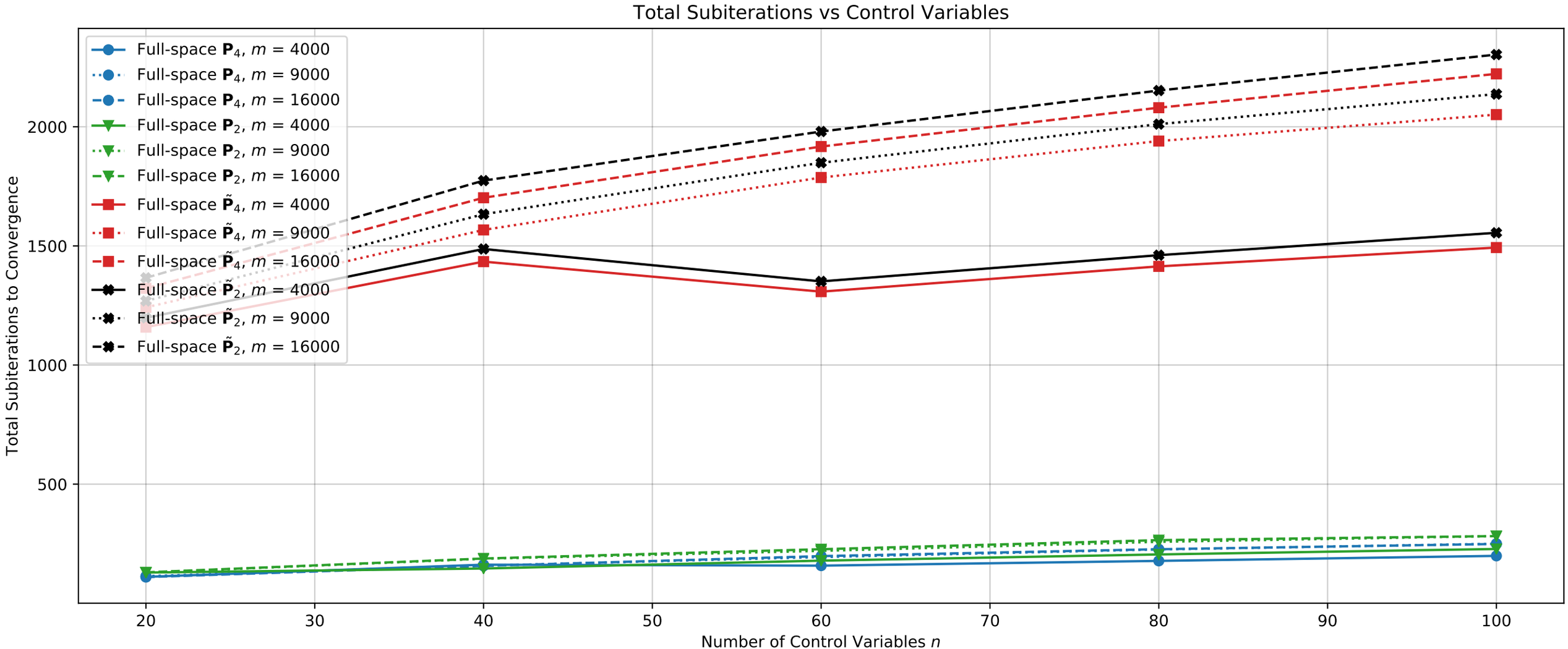
0
500
1000
1500
2000

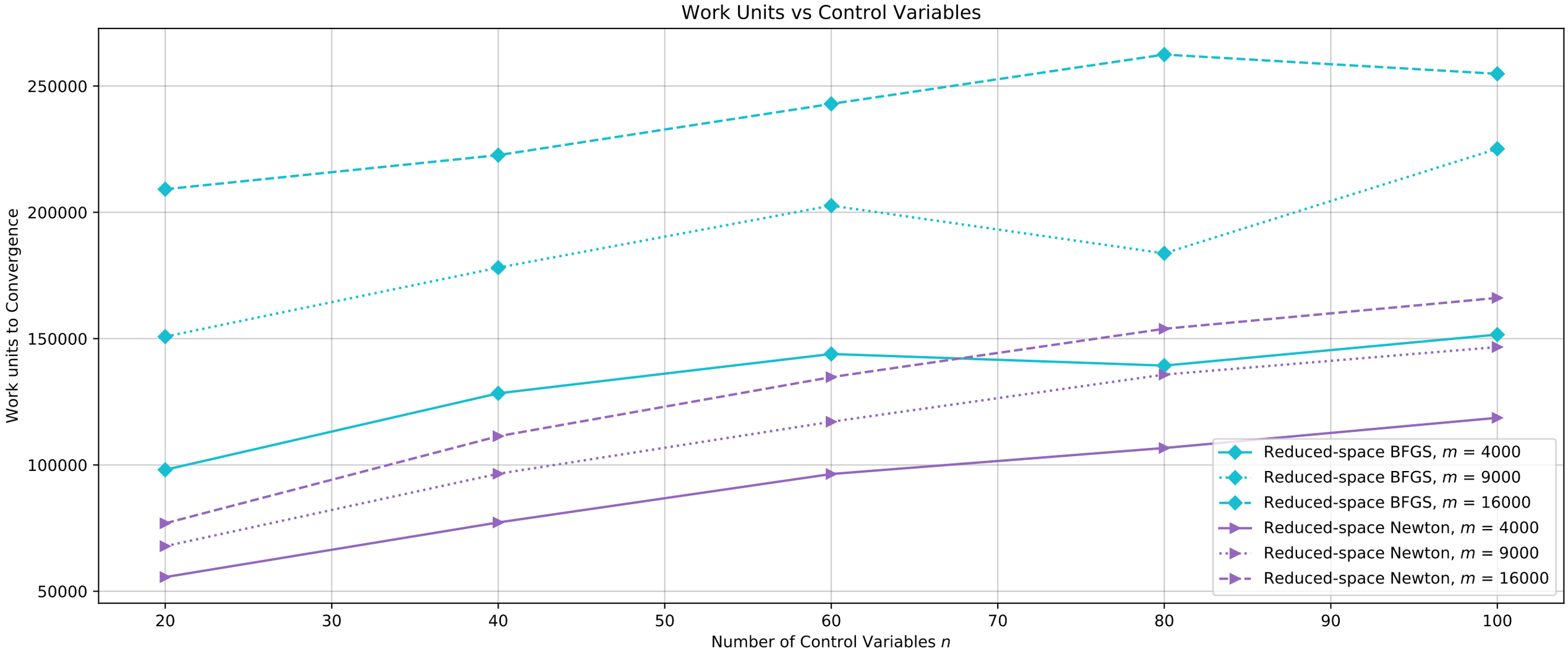
Number of subiterations

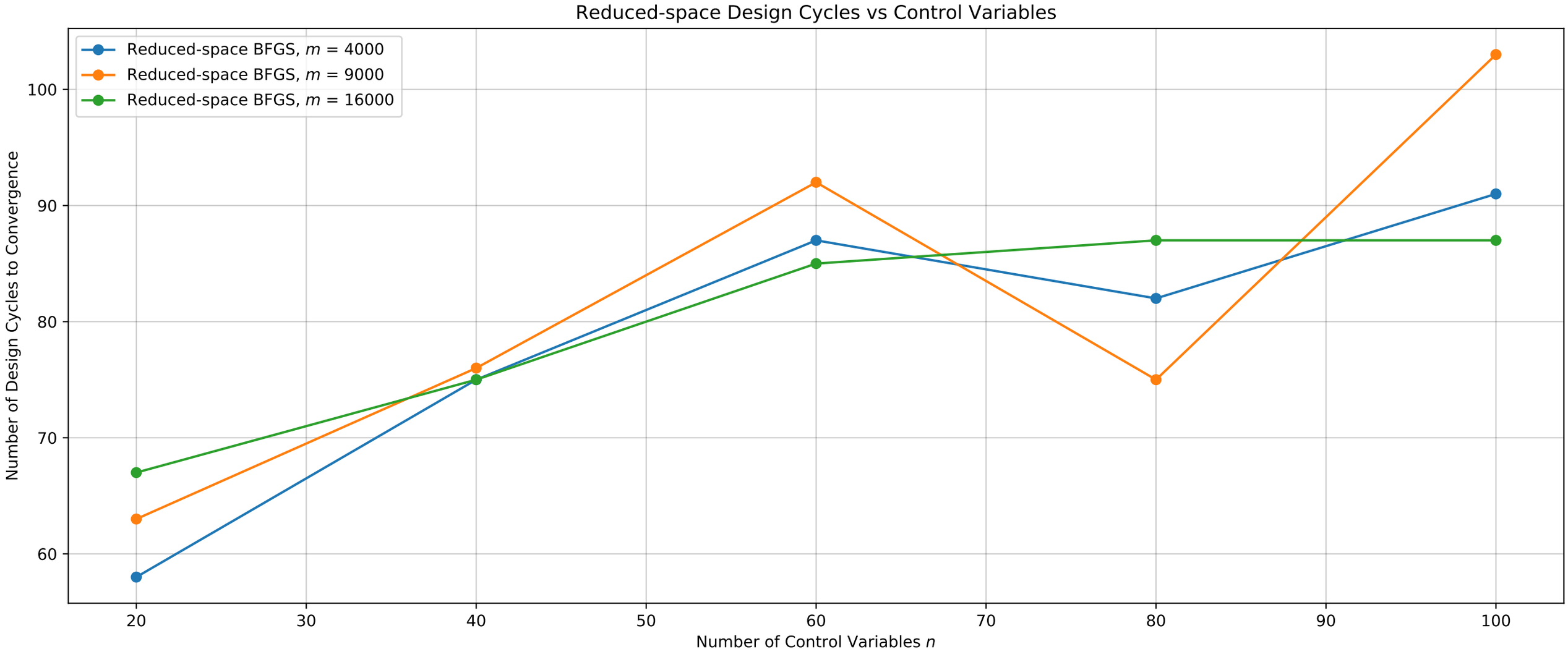
$\mathbf{P}_4$, $n = 20$
$\mathbf{P}_4$, $n = 60$
$\mathbf{P}_4$, $n = 100$
$\mathbf{P}_2$, $n = 20$
$\mathbf{P}_2$, $n = 60$
$\mathbf{P}_2$, $n = 100$
$\tilde{\mathbf{P}}_4$, $n = 20$
$\tilde{\mathbf{P}}_4$, $n = 60$
$\tilde{\mathbf{P}}_4$, $n = 100$
$\tilde{\mathbf{P}}_2$, $n = 20$
$\tilde{\mathbf{P}}_2$, $n = 60$
$\tilde{\mathbf{P}}_2$, $n = 100$


Full-space Design Cycles vs Control Variables
Number of Design Cycles to Convergence
Number of Control Variables n
5
4
3
20
30
40
50
60
70
80
90
100
Full-space $\mathbf{P}_4$, m = 4000
Full-space $\mathbf{P}_4$, m = 9000
Full-space $\mathbf{P}_4$, m = 16000
Full-space $\mathbf{P}_2$, m = 4000
Full-space $\mathbf{P}_2$, m = 9000
Full-space $\mathbf{P}_2$, m = 16000
Full-space $\tilde{\mathbf{P}}_4$, m = 4000
Full-space $\tilde{\mathbf{P}}_4$, m = 9000
Full-space $\tilde{\mathbf{P}}_4$, m = 16000
Full-space $\tilde{\mathbf{P}}_2$, m = 4000
Full-space $\tilde{\mathbf{P}}_2$, m = 9000
Full-space $\tilde{\mathbf{P}}_2$, m = 16000


Work Units vs Control Variables
Work Units to Convergence
Number of Control Variables n
500000
400000
300000
200000
100000
20
30
40
50
60
70
80
90
100
Full-space $\mathbf{P}_4$, $m = 4000$
Full-space $\mathbf{P}_4$, $m = 9000$
Full-space $\mathbf{P}_4$, $m = 16000$
Full-space $\mathbf{P}_2$, $m = 4000$
Full-space $\mathbf{P}_2$, $m = 9000$
Full-space $\mathbf{P}_2$, $m = 16000$
Full-space $\tilde{\mathbf{P}}_4$, $m = 4000$
Full-space $\tilde{\mathbf{P}}_4$, $m = 9000$
Full-space $\tilde{\mathbf{P}}_4$, $m = 16000$
Full-space $\tilde{\mathbf{P}}_2$, $m = 4000$
Full-space $\tilde{\mathbf{P}}_2$, $m = 9000$
Full-space $\tilde{\mathbf{P}}_2$, $m = 16000$


Total Subiterations vs Control Variables
Total Subiterations to Convergence
Number of Control Variables n
Full-space $\mathbf{P}_4$, $m = 4000$
Full-space $\mathbf{P}_4$, $m = 9000$
Full-space $\mathbf{P}_4$, $m = 16000$
Full-space $\mathbf{P}_2$, $m = 4000$
Full-space $\mathbf{P}_2$, $m = 9000$
Full-space $\mathbf{P}_2$, $m = 16000$
Full-space $\tilde{\mathbf{P}}_4$, $m = 4000$
Full-space $\tilde{\mathbf{P}}_4$, $m = 9000$
Full-space $\tilde{\mathbf{P}}_4$, $m = 16000$
Full-space $\tilde{\mathbf{P}}_2$, $m = 4000$
Full-space $\tilde{\mathbf{P}}_2$, $m = 9000$
Full-space $\tilde{\mathbf{P}}_2$, $m = 16000$
2000
1500
1000
500
20
30
40
50
60
70
80
90
100


Work Units vs Control Variables
Work units to Convergence
250000
200000
150000
100000
50000
20
30
40
50
60
70
80
90
100
Number of Control Variables n
Reduced-space BFGS, m = 4000
Reduced-space BFGS, m = 9000
Reduced-space BFGS, m = 16000
Reduced-space Newton, m = 4000
Reduced-space Newton, m = 9000
Reduced-space Newton, m = 16000


Reduced-space Design Cycles vs Control Variables
Reduced-space BFGS, m = 4000
Reduced-space BFGS, m = 9000
Reduced-space BFGS, m = 16000
Number of Design Cycles to Convergence
Number of Control Variables n
60
70
80
90
100
20
30
40
50
60
70
80
90
100

Reduced-space Gradient vs Design Cycles

m = 4000

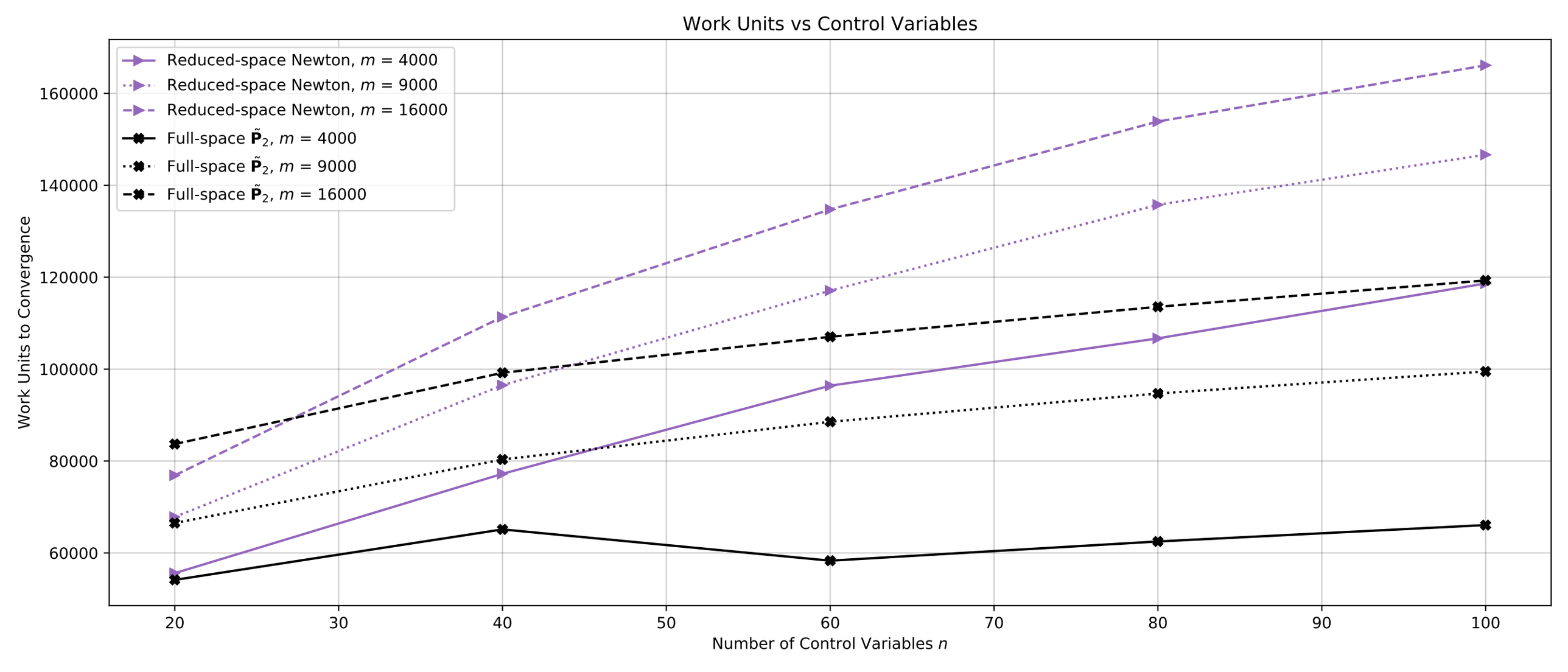
m = 9000

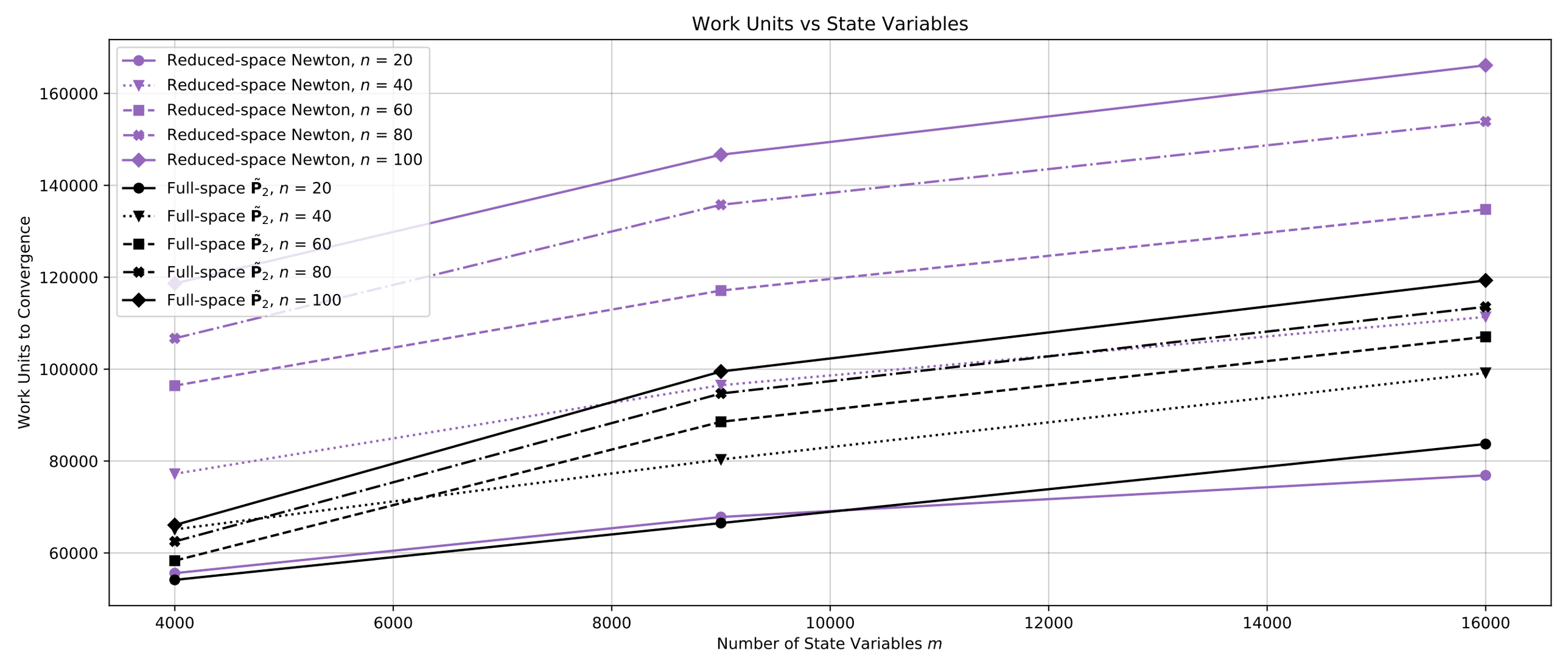
m = 16000

Gradient Norm

Design Cycles

BFGS, n = 20
BFGS, n = 40
BFGS, n = 60
BFGS, n = 80
BFGS, n = 100
Newton, n = 20
Newton, n = 40
Newton, n = 60
Newton, n = 80
Newton, n = 100


Work Units vs Control Variables
Reduced-space Newton, m = 4000
Reduced-space Newton, m = 9000
Reduced-space Newton, m = 16000
Full-space $\tilde{\mathbf{P}}_2$, m = 4000
Full-space $\tilde{\mathbf{P}}_2$, m = 9000
Full-space $\tilde{\mathbf{P}}_2$, m = 16000
Work Units to Convergence
160000
140000
120000
100000
80000
60000
20
30
40
50
60
70
80
90
100
Number of Control Variables n


Work Units vs State Variables
Reduced-space Newton, n = 20
Reduced-space Newton, n = 40
Reduced-space Newton, n = 60
Reduced-space Newton, n = 80
Reduced-space Newton, n = 100
Full-space $\tilde{\mathbf{P}}_2$, n = 20
Full-space $\tilde{\mathbf{P}}_2$, n = 40
Full-space $\tilde{\mathbf{P}}_2$, n = 60
Full-space $\tilde{\mathbf{P}}_2$, n = 80
Full-space $\tilde{\mathbf{P}}_2$, n = 100
Work Units to Convergence
160000
140000
120000
100000
80000
60000
4000
6000
8000
10000
12000
14000
16000
Number of State Variables m