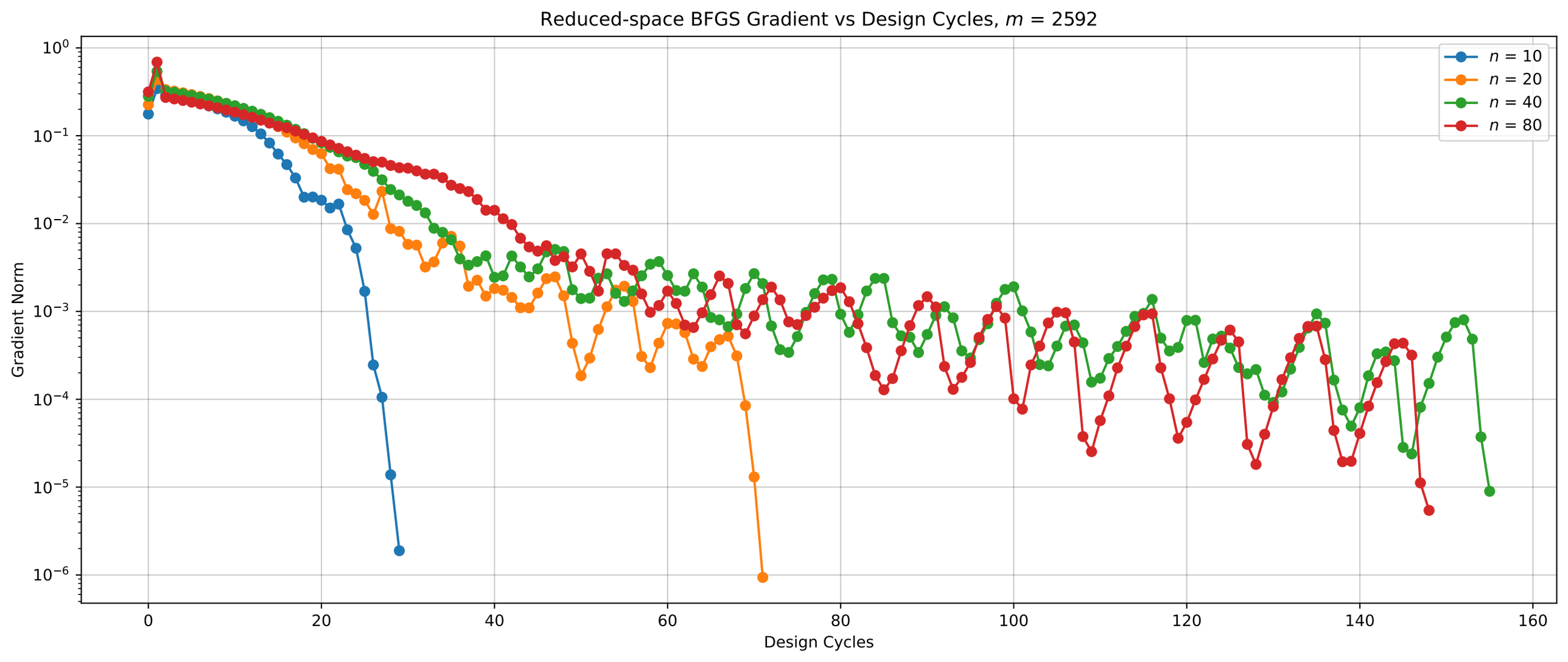
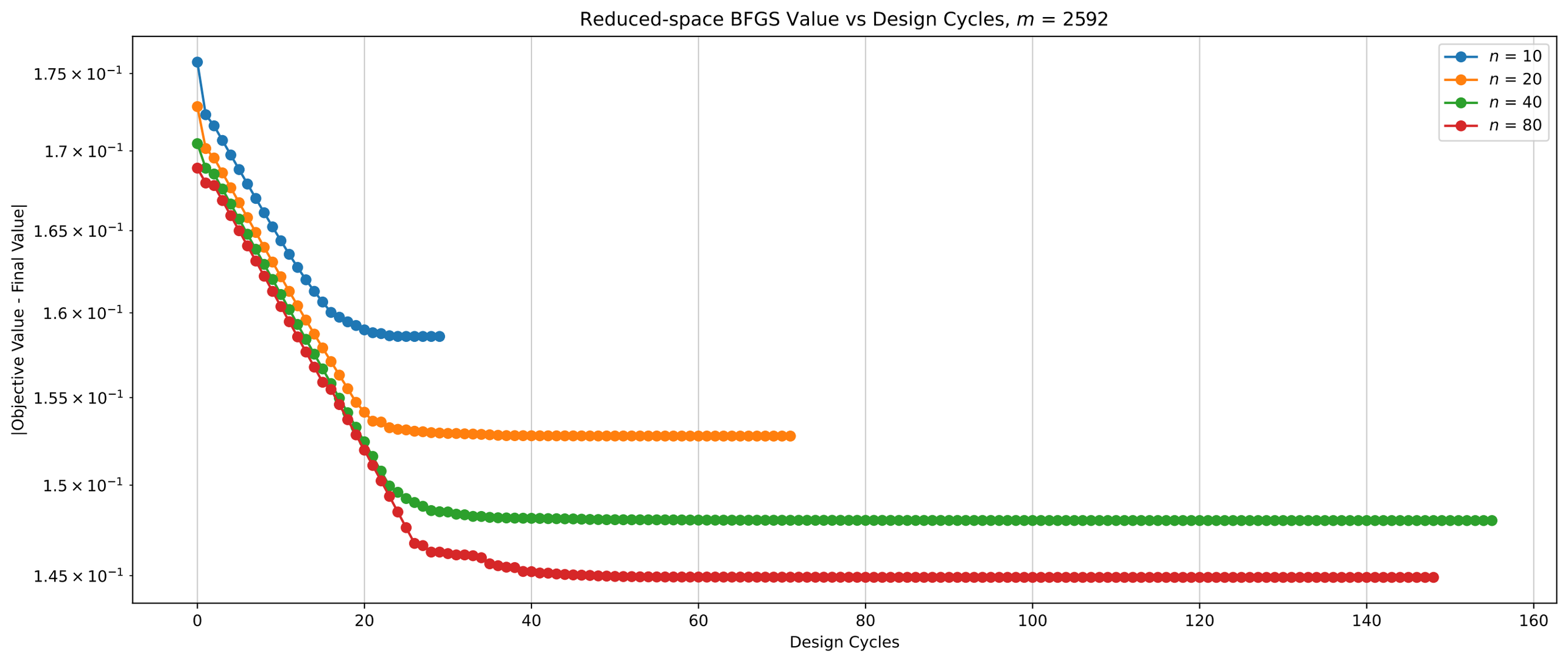
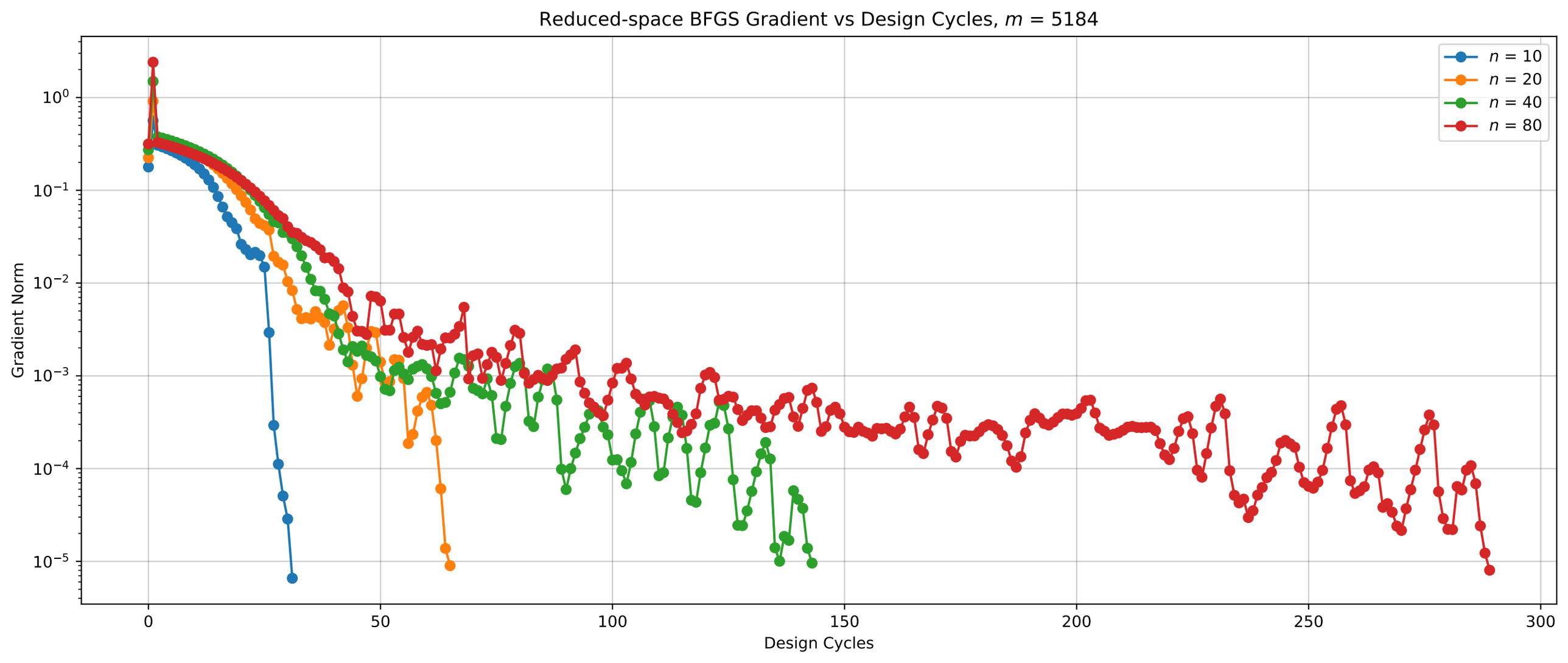
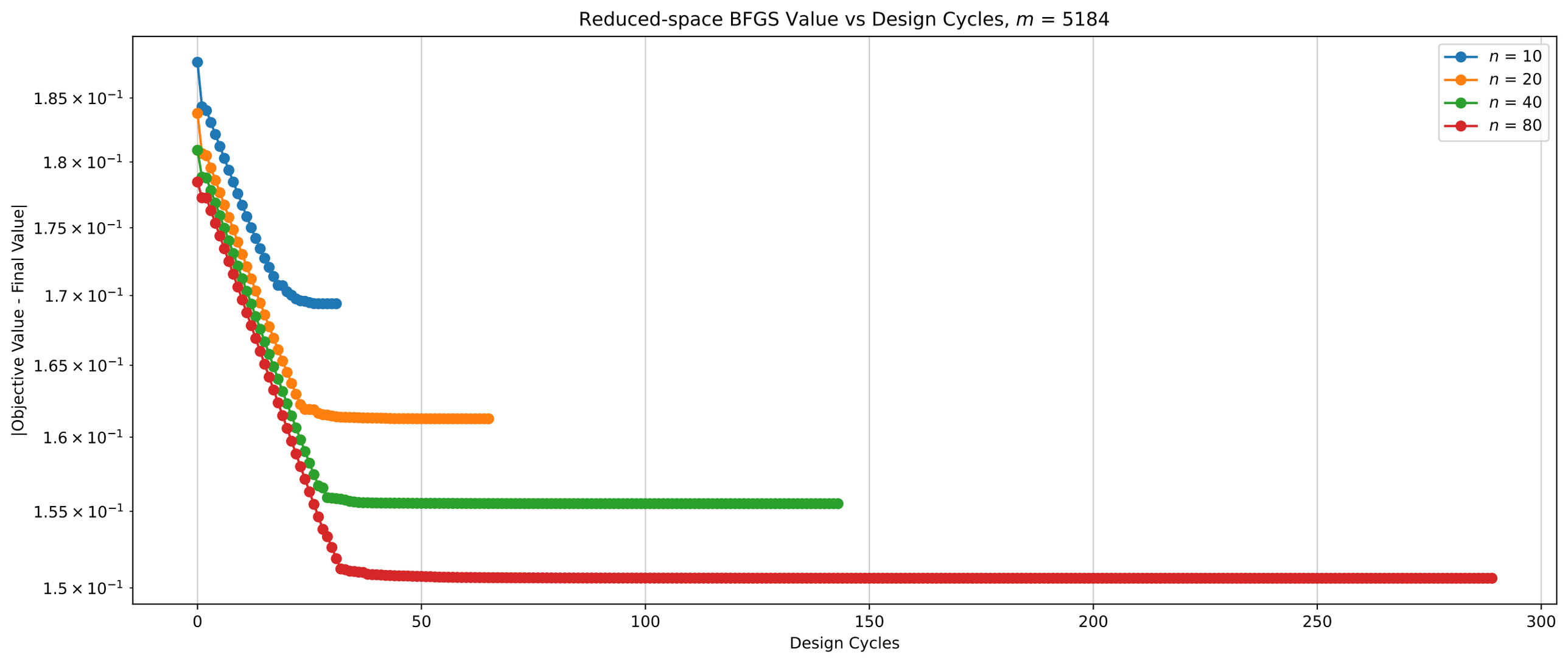
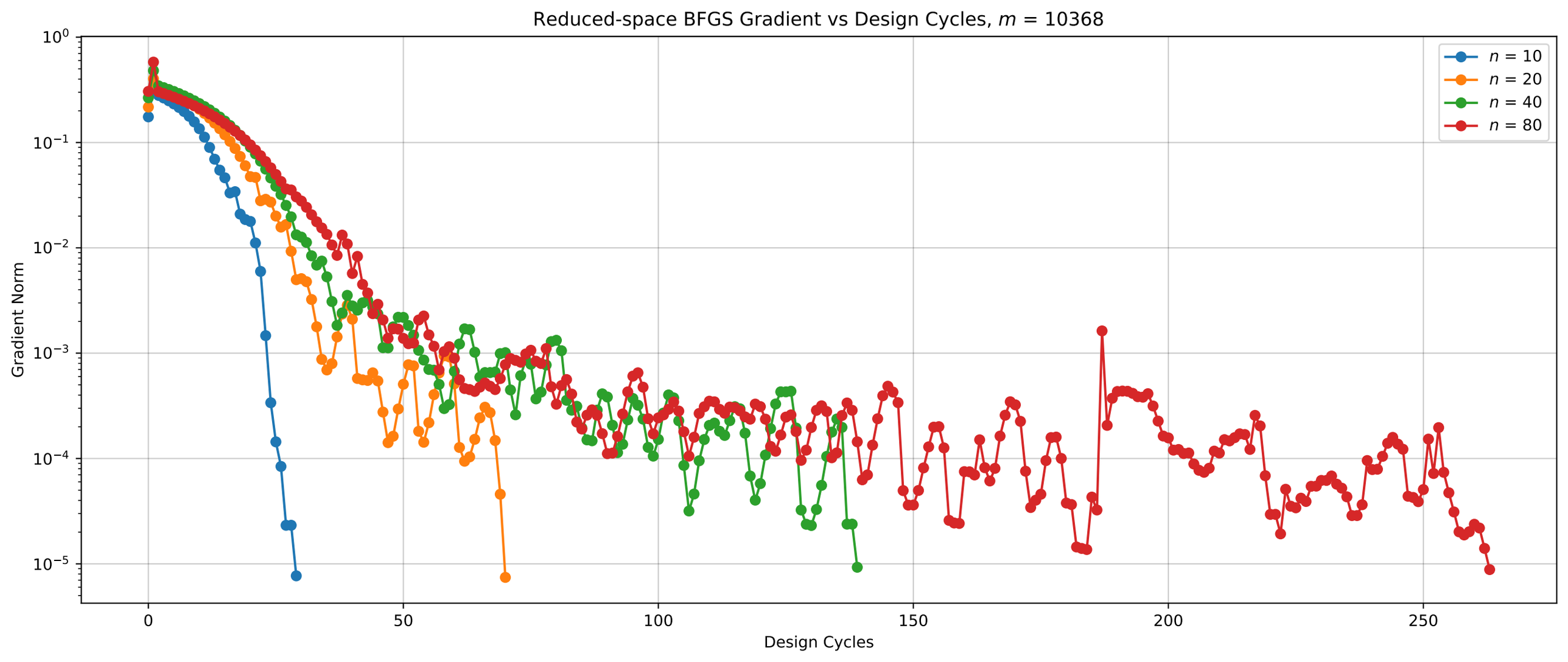
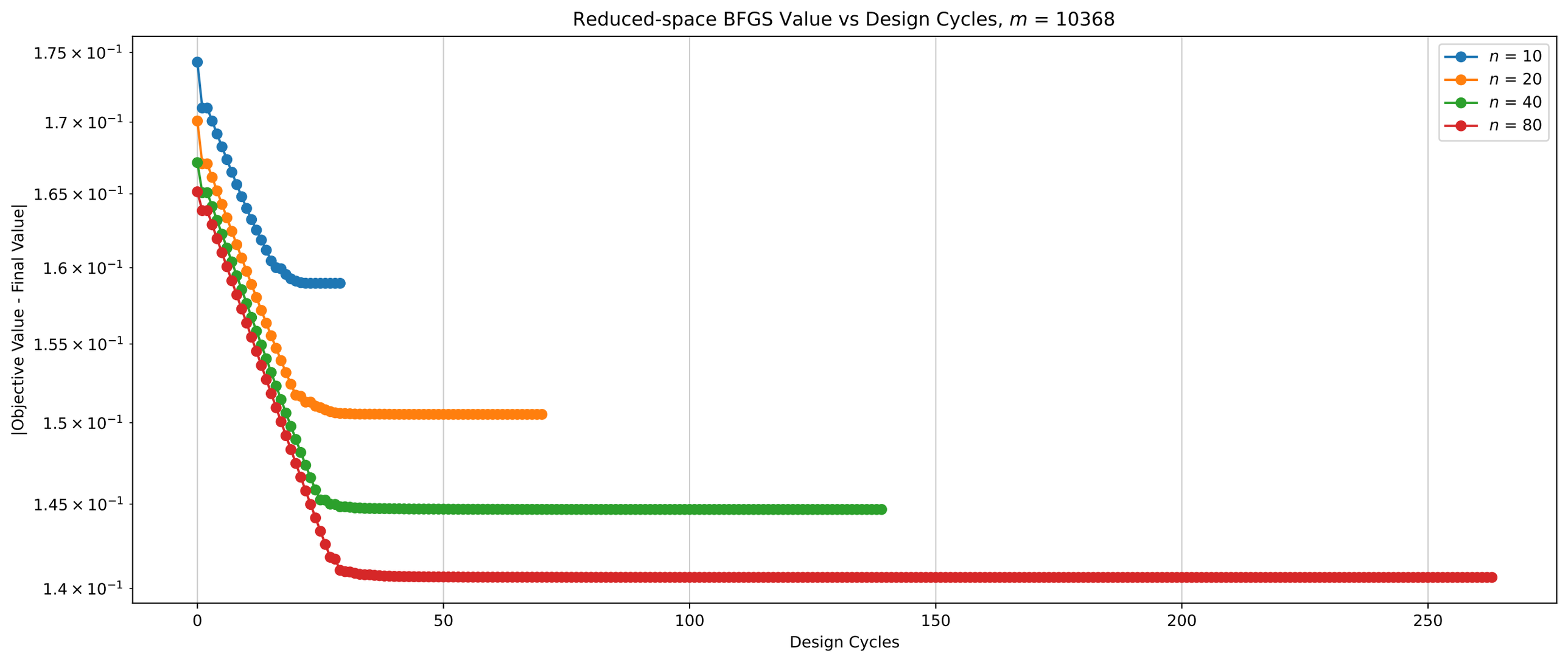

Reduced-space BFGS Gradient vs Design Cycles, m = 2592
Gradient Norm
Design Cycles
10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
0
20
40
60
80
100
120
140
160
n = 10
n = 20
n = 40
n = 80

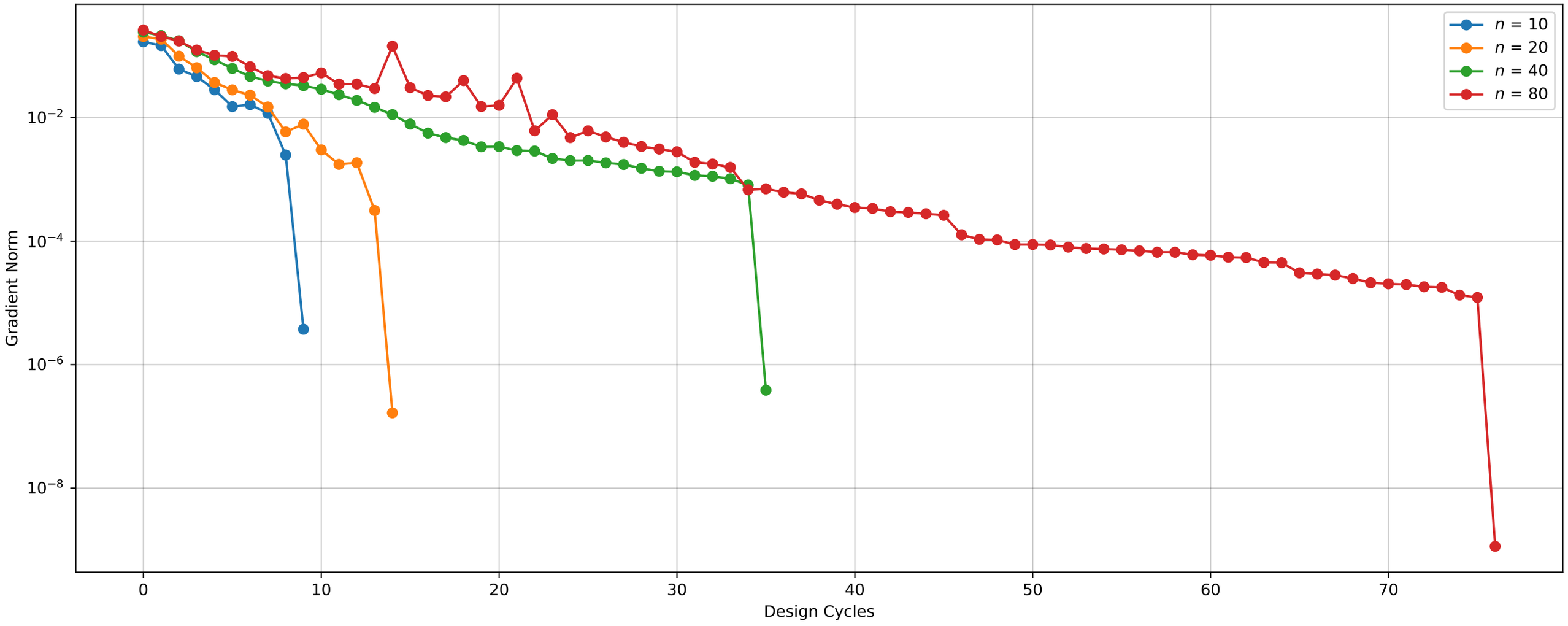

Reduced-space BFGS Value vs Design Cycles, m = 2592
|Objective Value - Final Value|
1.75×10^{-1}
1.7×10^{-1}
1.65×10^{-1}
1.6×10^{-1}
1.55×10^{-1}
1.5×10^{-1}
1.45×10^{-1}
0
20
40
60
80
100
120
140
160
Design Cycles
n = 10
n = 20
n = 40
n = 80

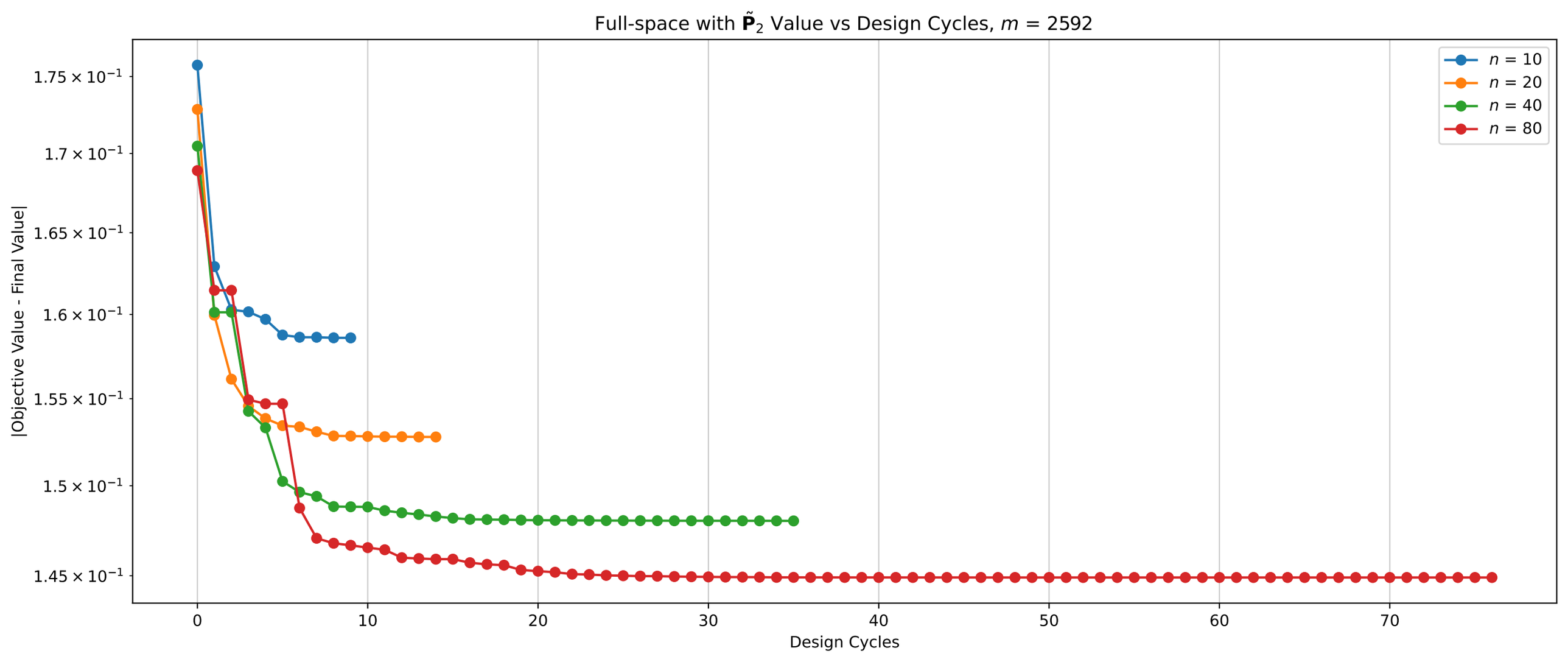

Reduced-space BFGS Gradient vs Design Cycles, m = 5184
Gradient Norm
Design Cycles
10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
0
50
100
150
200
250
300
n = 10
n = 20
n = 40
n = 80

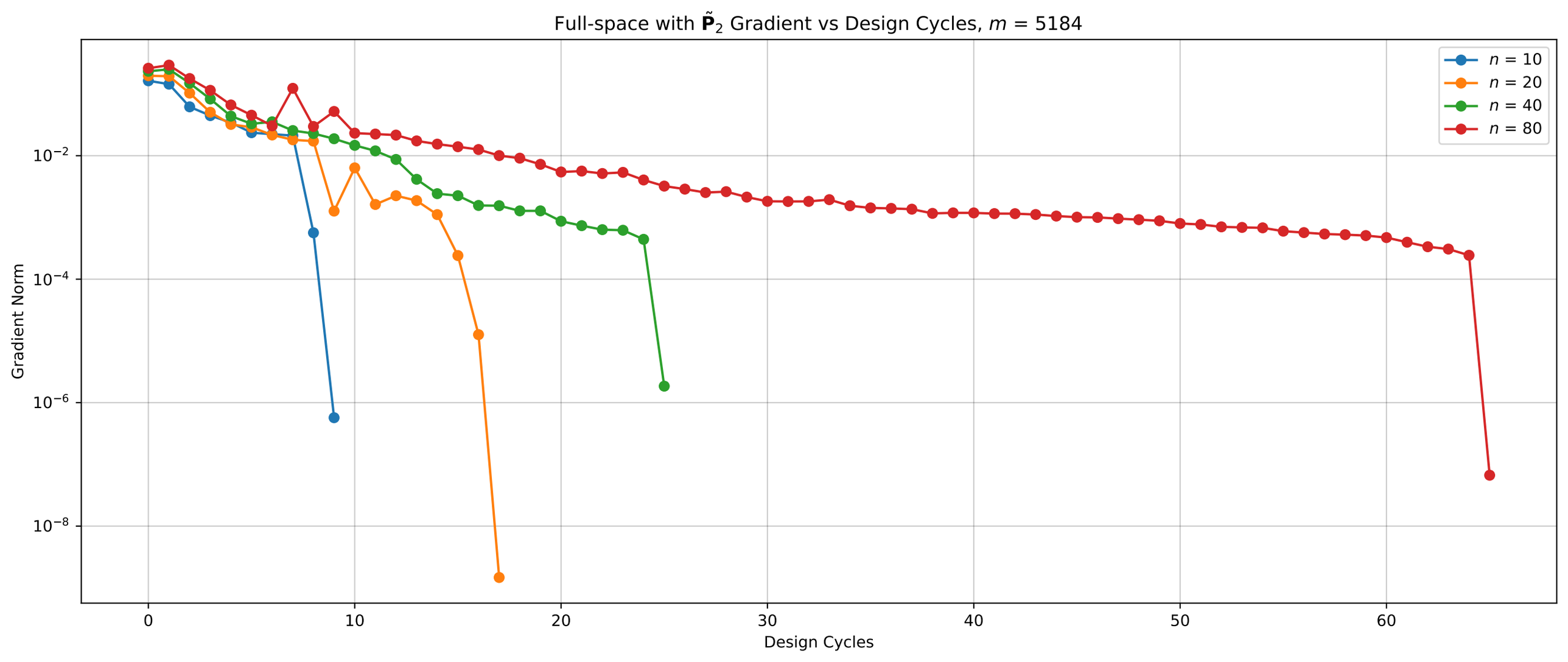

Reduced-space BFGS Value vs Design Cycles, m = 5184
n = 10
n = 20
n = 40
n = 80
|Objective Value - Final Value|
1.85×10^{-1}
1.8×10^{-1}
1.75×10^{-1}
1.7×10^{-1}
1.65×10^{-1}
1.6×10^{-1}
1.55×10^{-1}
1.5×10^{-1}
0
50
100
150
200
250
300
Design Cycles

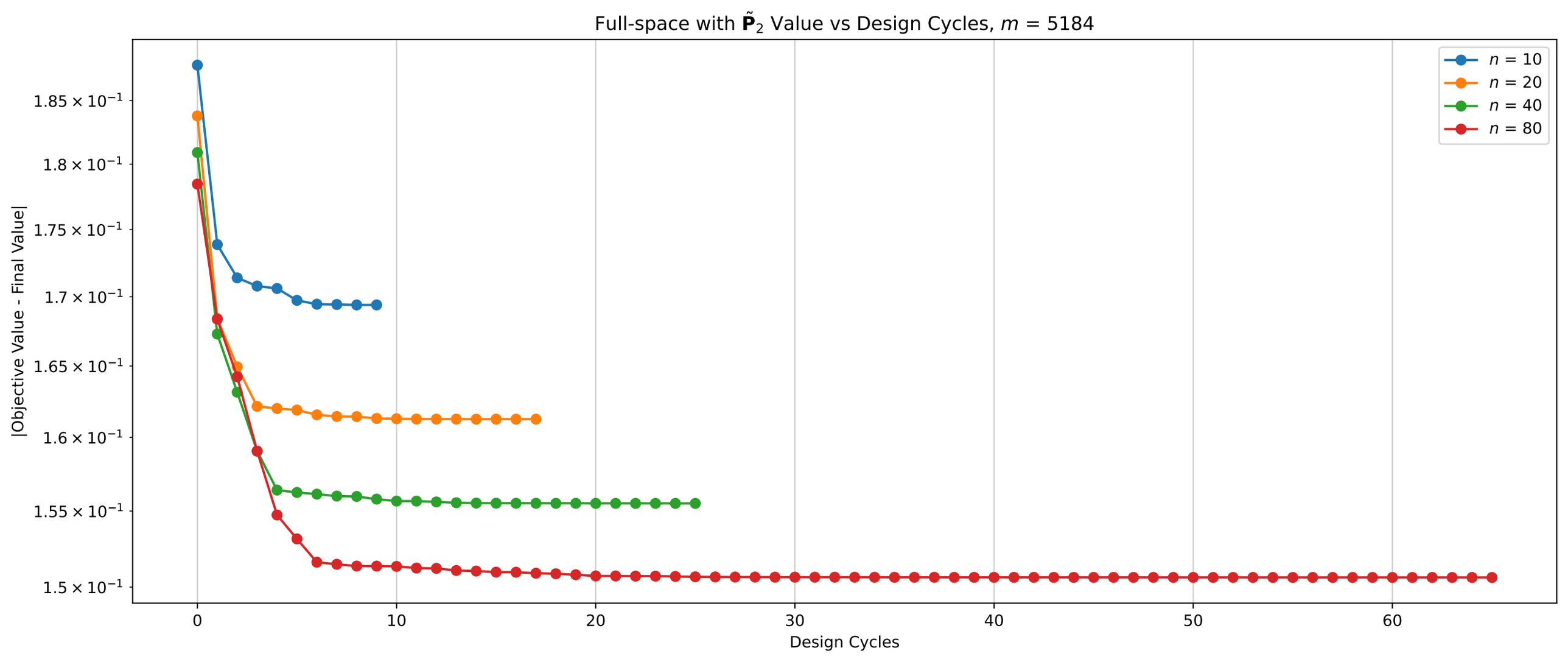

Reduced-space BFGS Gradient vs Design Cycles, m = 10368
Gradient Norm
Design Cycles
10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
0
50
100
150
200
250
n = 10
n = 20
n = 40
n = 80

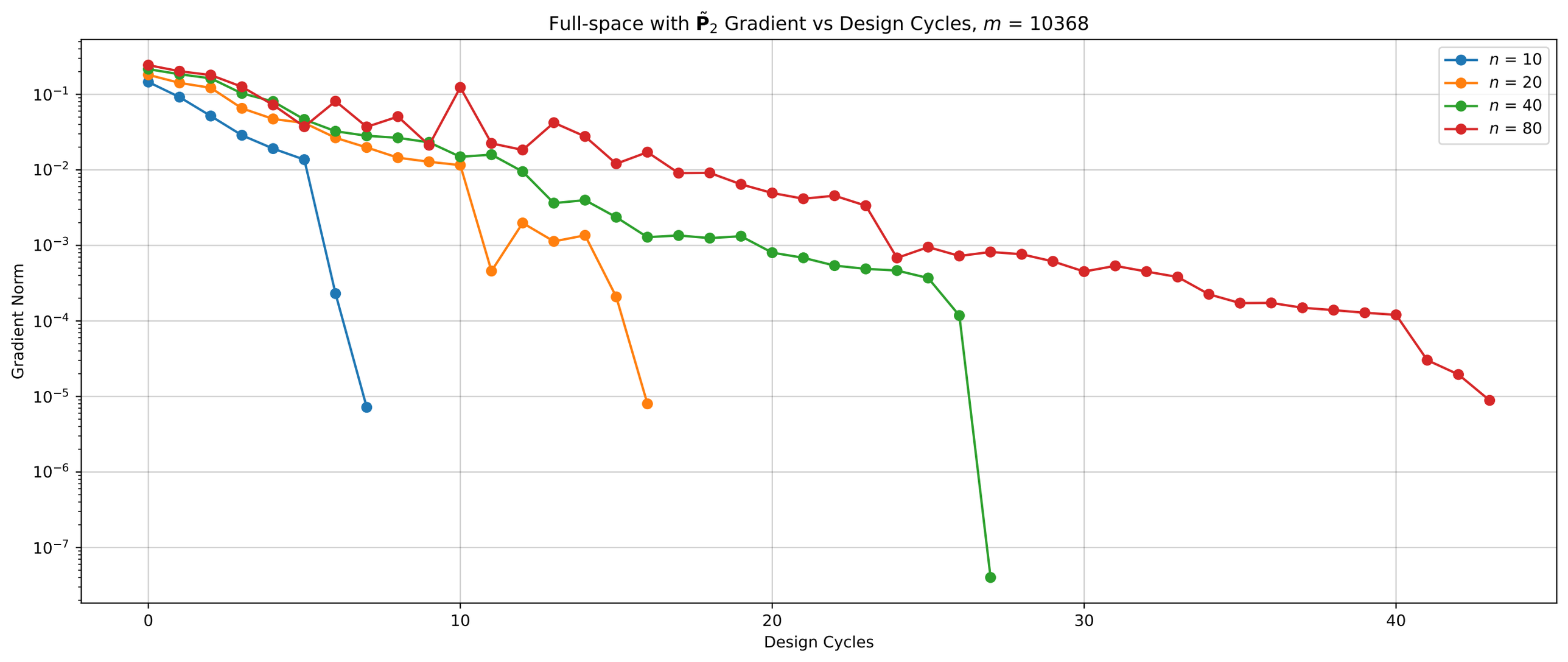

Reduced-space BFGS Value vs Design Cycles, m = 10368
|Objective Value - Final Value|
1.75×10^{-1}
1.7×10^{-1}
1.65×10^{-1}
1.6×10^{-1}
1.55×10^{-1}
1.5×10^{-1}
1.45×10^{-1}
1.4×10^{-1}
0
50
100
150
200
250
Design Cycles
n = 10
n = 20
n = 40
n = 80

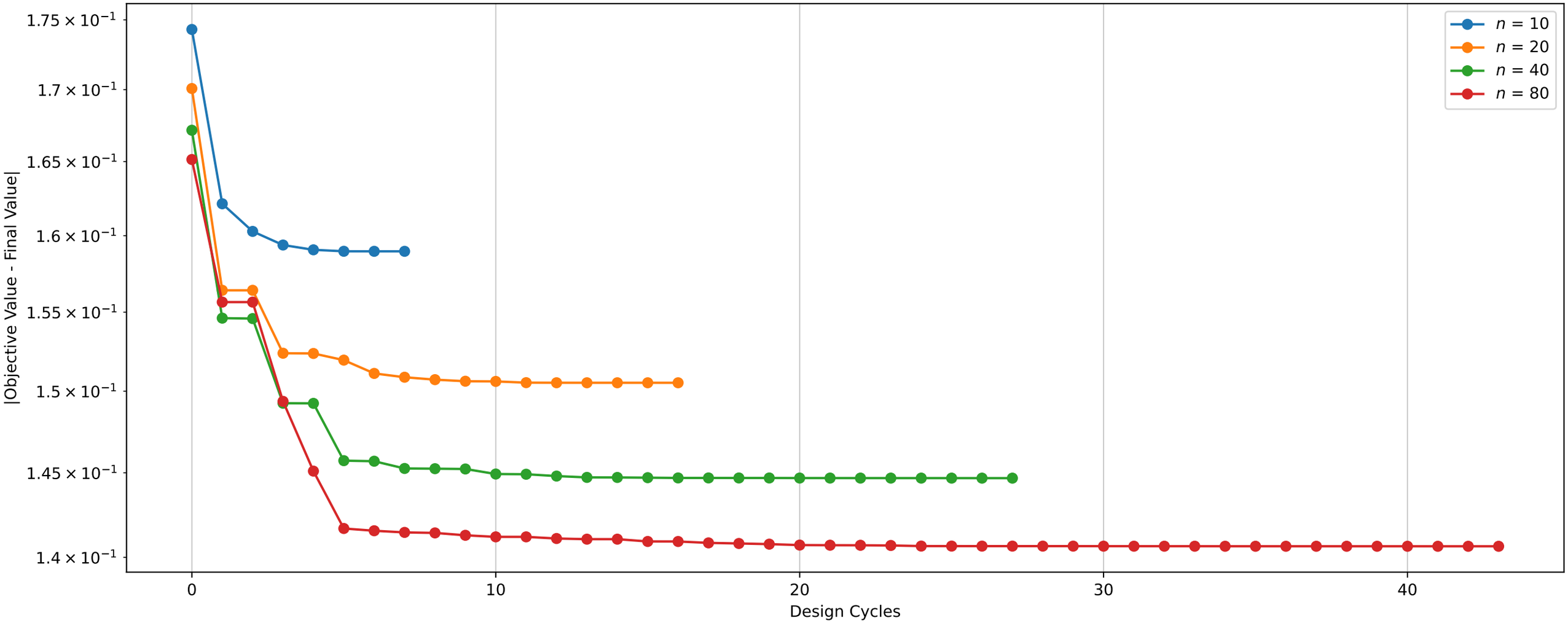

Full-space with $\tilde{\mathbf{P}}_2$ Gradient vs Design Cycles, m = 2592
n = 10
n = 20
n = 40
n = 80
Gradient Norm
10^{-2}
10^{-4}
10^{-6}
10^{-8}
0
10
20
30
40
50
60
70
Design Cycles

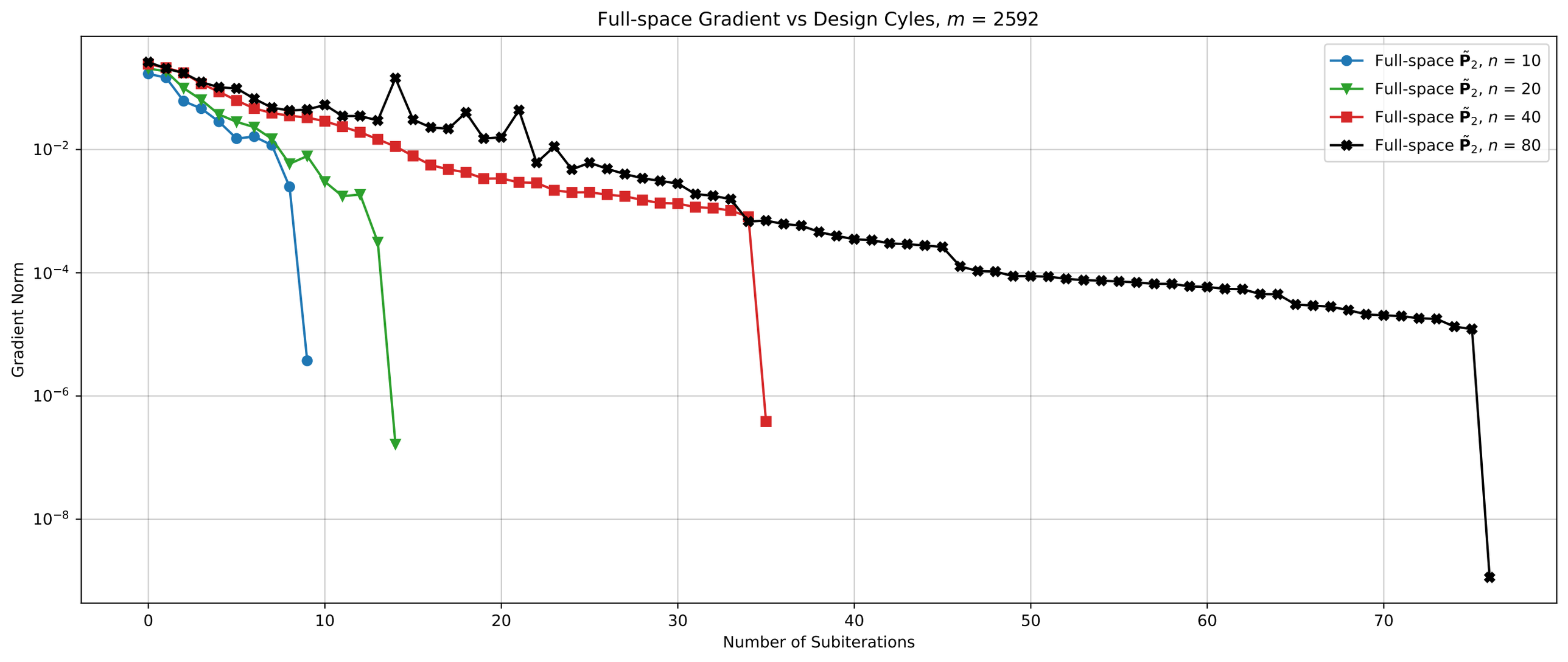

Full-space with $\tilde{\mathbf{P}}_2$ Value vs Design Cycles, m = 2592
|Objective Value - Final Value|
1.75×10^{-1}
1.7×10^{-1}
1.65×10^{-1}
1.6×10^{-1}
1.55×10^{-1}
1.5×10^{-1}
1.45×10^{-1}
0
10
20
30
40
50
60
70
Design Cycles
n = 10
n = 20
n = 40
n = 80

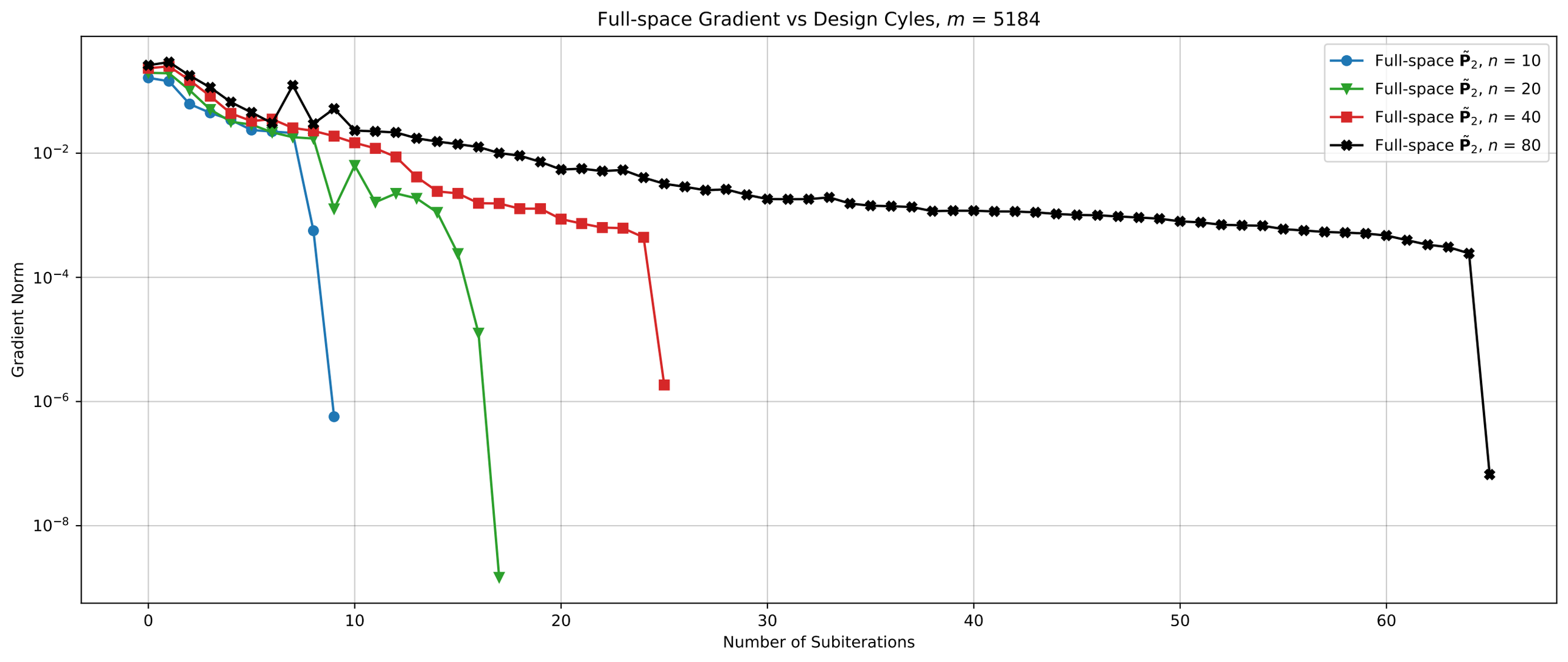

Full-space with $\tilde{\mathbf{P}}_2$ Gradient vs Design Cycles, m = 5184
n = 10
n = 20
n = 40
n = 80
Gradient Norm
10^{-2}
10^{-4}
10^{-6}
10^{-8}
0
10
20
30
40
50
60
Design Cycles

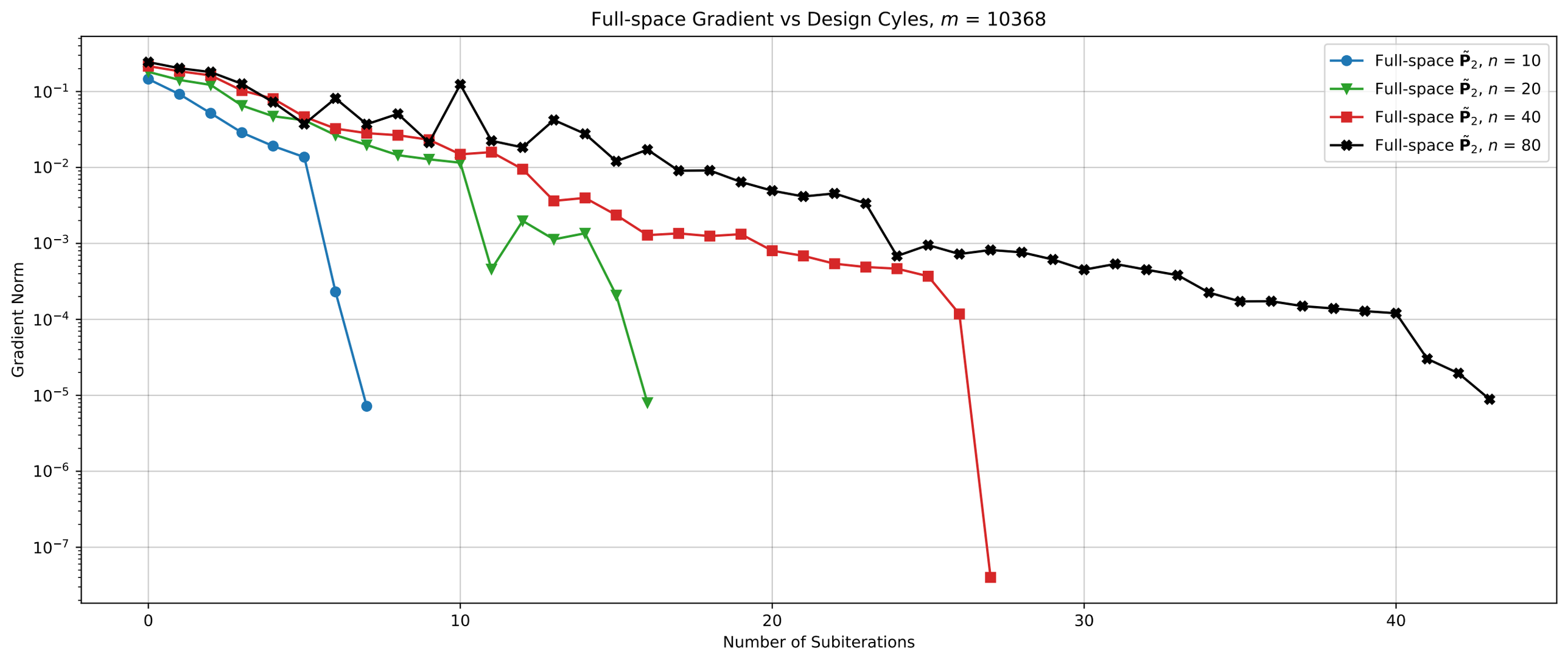

Full-space with $\tilde{\mathbf{P}}_2$ Value vs Design Cycles, m = 5184
|Objective Value - Final Value|
1.85×10^{-1}
1.8×10^{-1}
1.75×10^{-1}
1.7×10^{-1}
1.65×10^{-1}
1.6×10^{-1}
1.55×10^{-1}
1.5×10^{-1}
0
10
20
30
40
50
60
Design Cycles
n = 10
n = 20
n = 40
n = 80


Full-space with $\tilde{\mathbf{P}}_2$ Gradient vs Design Cycles, m = 10368
Gradient Norm
Design Cycles
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
0
10
20
30
40
n = 10
n = 20
n = 40
n = 80


Full-space with $\tilde{\mathbf{P}}_2$ Value vs Design Cycles, m = 10368
1.75×10^{-1}
1.7×10^{-1}
1.65×10^{-1}
1.6×10^{-1}
1.55×10^{-1}
1.5×10^{-1}
1.45×10^{-1}
1.4×10^{-1}
|Objective Value - Final Value|
0
10
20
30
40
Design Cycles
n = 10
n = 20
n = 40
n = 80


Full-space Gradient vs Design Cyles, m = 2592
Full-space $\tilde{\mathbf{P}}_2$, n = 10
Full-space $\tilde{\mathbf{P}}_2$, n = 20
Full-space $\tilde{\mathbf{P}}_2$, n = 40
Full-space $\tilde{\mathbf{P}}_2$, n = 80
Gradient Norm
10^{-2}
10^{-4}
10^{-6}
10^{-8}
0
10
20
30
40
50
60
70
Number of Subiterations


Full-space Gradient vs Design Cyles, m = 5184
Full-space $\tilde{\mathbf{P}}_2$, n = 10
Full-space $\tilde{\mathbf{P}}_2$, n = 20
Full-space $\tilde{\mathbf{P}}_2$, n = 40
Full-space $\tilde{\mathbf{P}}_2$, n = 80
Gradient Norm
10^{-2}
10^{-4}
10^{-6}
10^{-8}
0
10
20
30
40
50
60
Number of Subiterations


Full-space Gradient vs Design Cyles, m = 10368
Full-space $\tilde{\mathbf{P}}_2$, n = 10
Full-space $\tilde{\mathbf{P}}_2$, n = 20
Full-space $\tilde{\mathbf{P}}_2$, n = 40
Full-space $\tilde{\mathbf{P}}_2$, n = 80
Gradient Norm
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
0
10
20
30
40
Number of Subiterations

Full-space Gradient vs Subiterations

m = 2592

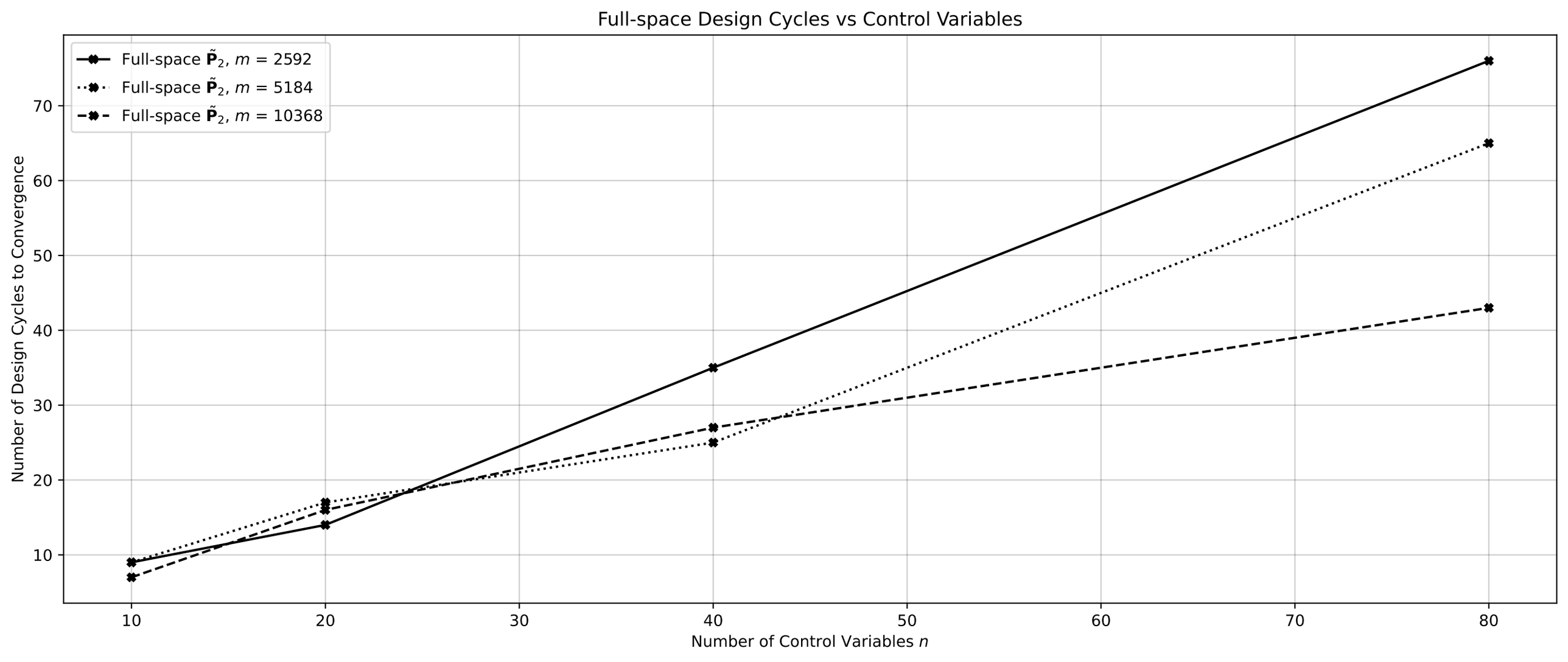
m = 5184

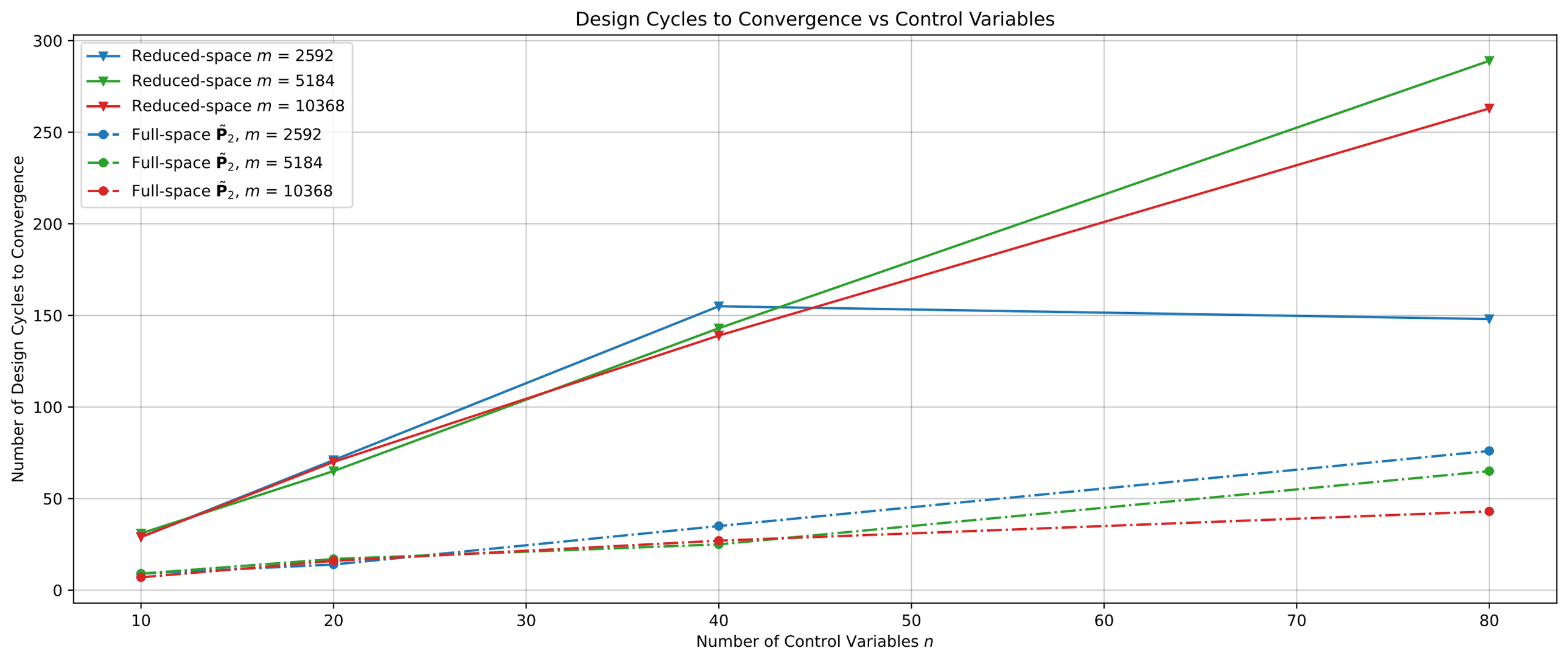
m = 10368

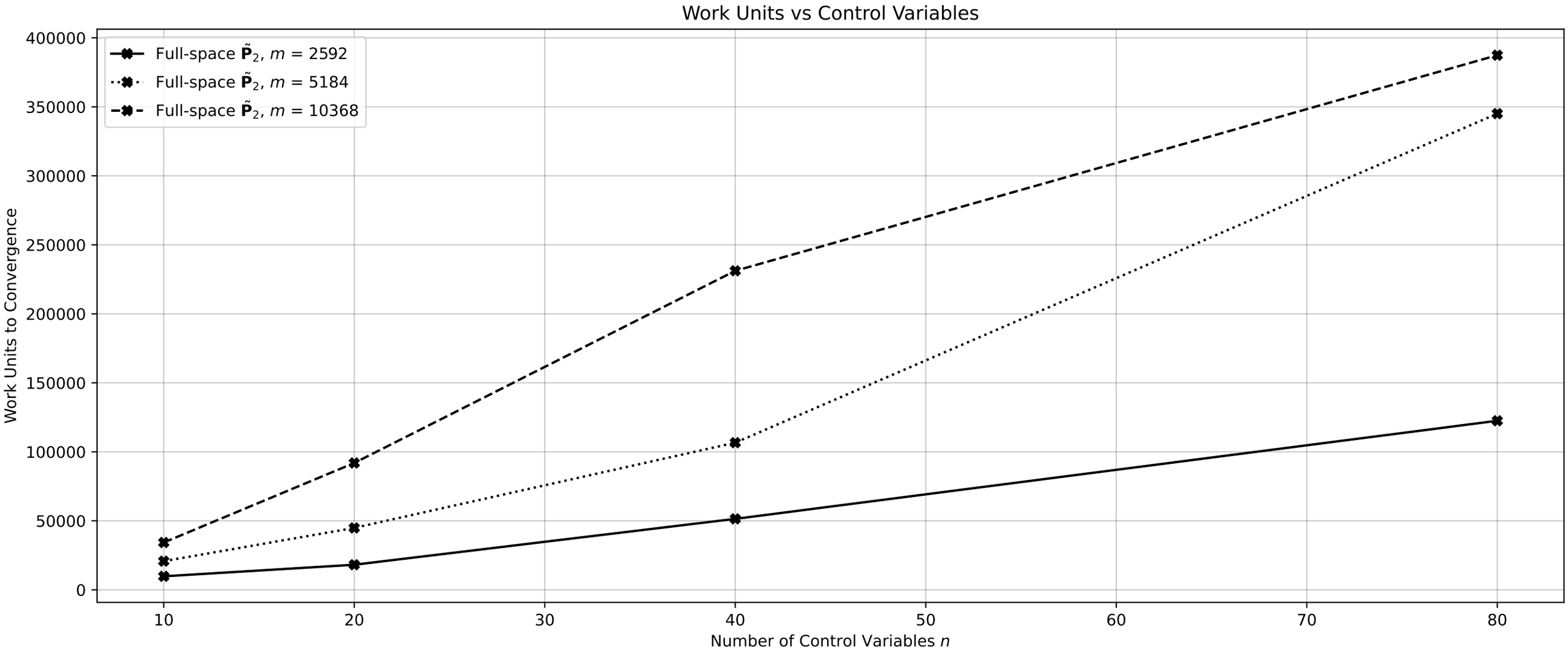
Gradient Norm

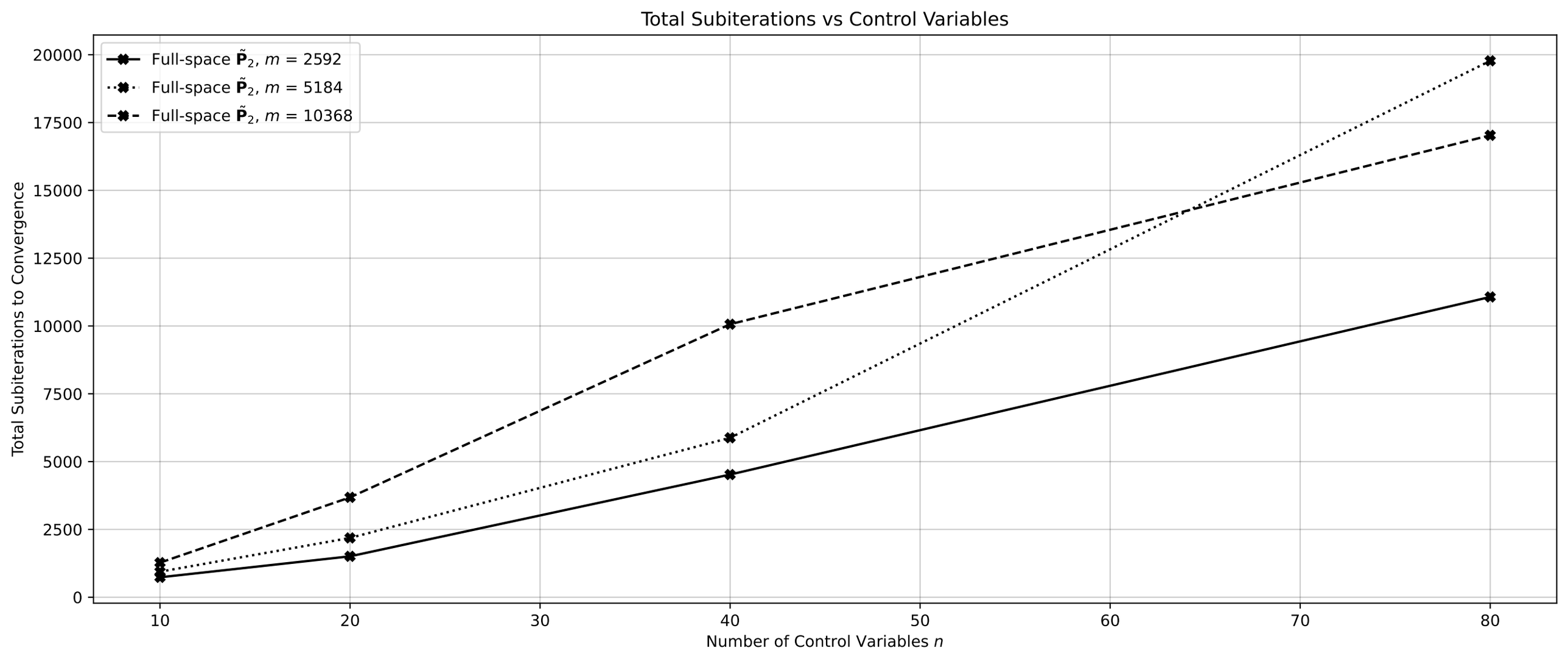
Number of cycles

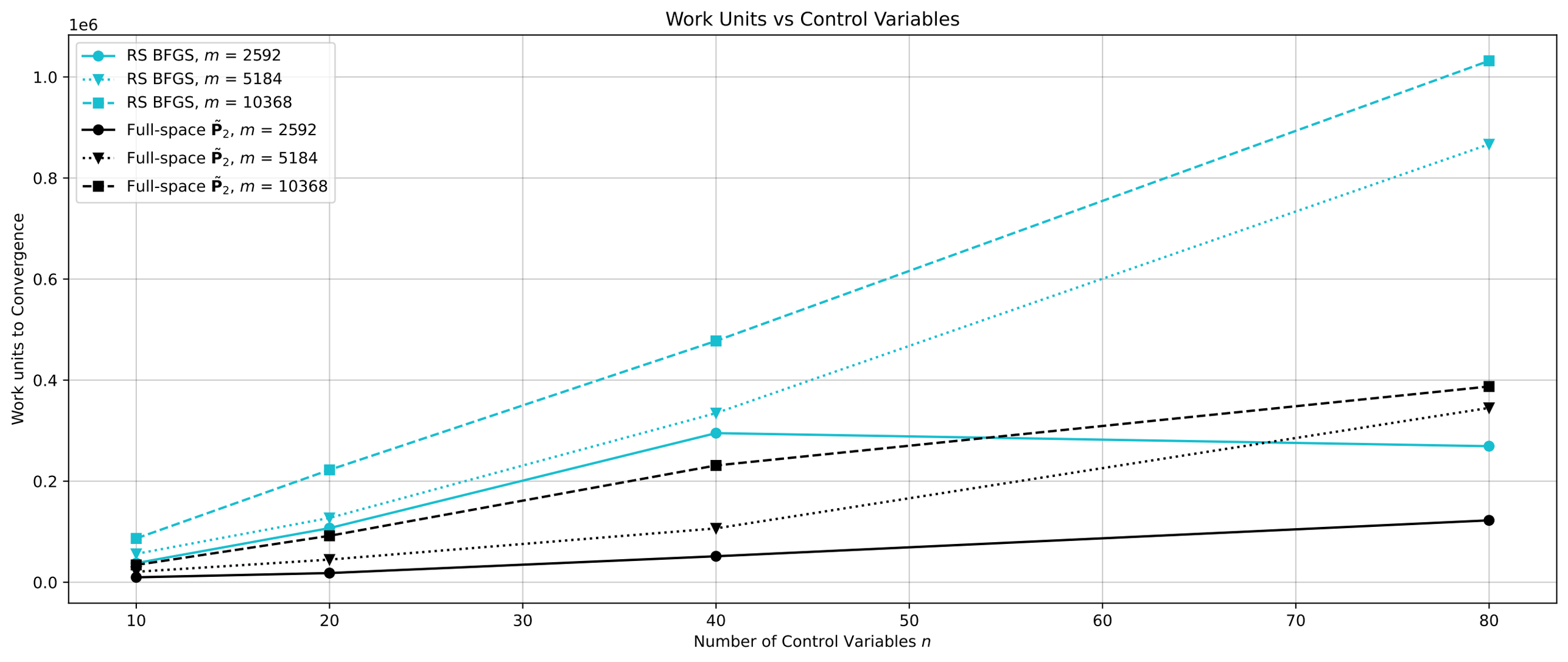
Full-space $\tilde{\mathbf{P}}_2$, n = 10
Full-space $\tilde{\mathbf{P}}_2$, n = 20
Full-space $\tilde{\mathbf{P}}_2$, n = 40
Full-space $\tilde{\mathbf{P}}_2$, n = 80

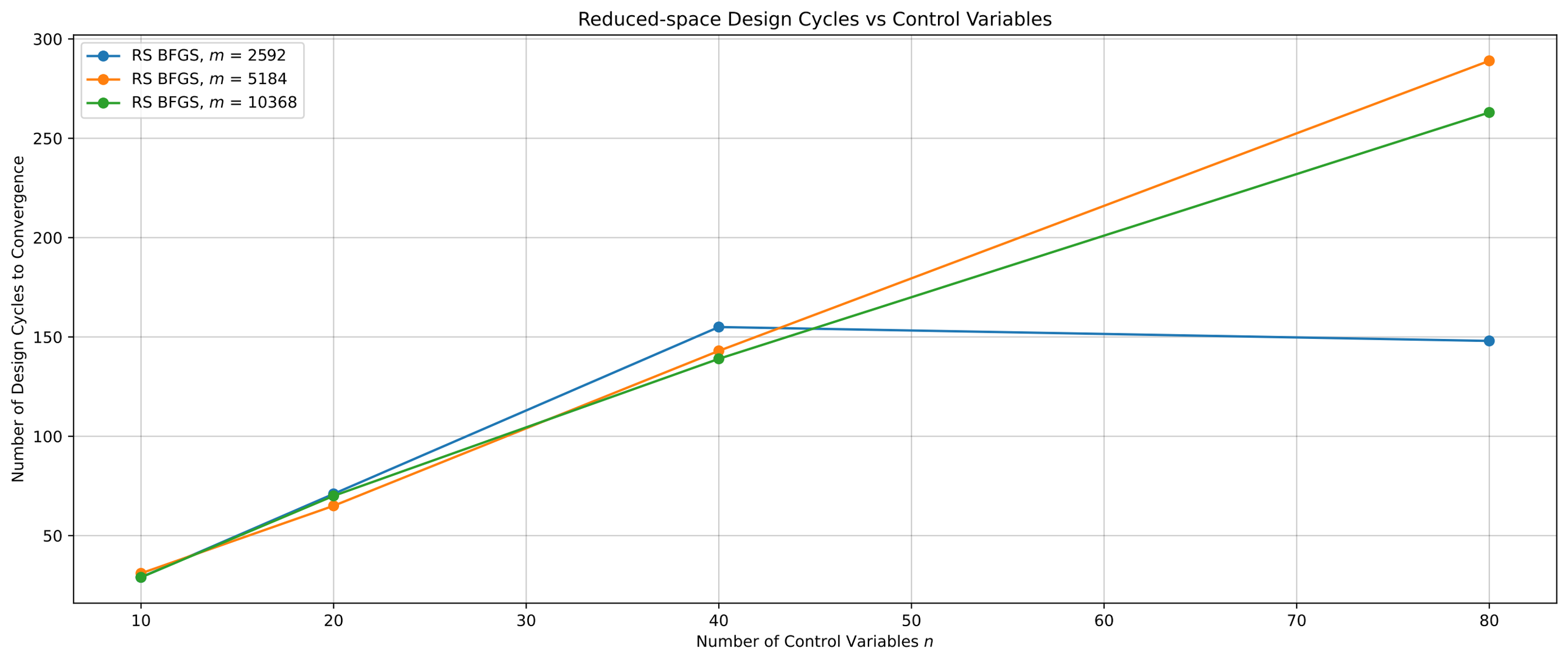

Full-space Design Cycles vs Control Variables
Full-space $\tilde{\mathbf{P}}_2$, m = 2592
Full-space $\tilde{\mathbf{P}}_2$, m = 5184
Full-space $\tilde{\mathbf{P}}_2$, m = 10368
Number of Design Cycles to Convergence
Number of Control Variables n
10
20
30
40
50
60
70
80


Design Cycles to Convergence vs Control Variables
Reduced-space m = 2592
Reduced-space m = 5184
Reduced-space m = 10368
Full-space $\tilde{\mathbf{P}}_2$, m = 2592
Full-space $\tilde{\mathbf{P}}_2$, m = 5184
Full-space $\tilde{\mathbf{P}}_2$, m = 10368
Number of Design Cycles to Convergence
Number of Control Variables n
0
50
100
150
200
250
300
10
20
30
40
50
60
70
80


Work Units vs Control Variables
Full-space $\tilde{\mathbf{P}}_2$, $m = 2592$
Full-space $\tilde{\mathbf{P}}_2$, $m = 5184$
Full-space $\tilde{\mathbf{P}}_2$, $m = 10368$
Work Units to Convergence
Number of Control Variables n
0
50000
100000
150000
200000
250000
300000
350000
400000
10
20
30
40
50
60
70
80


Total Subiterations vs Control Variables
Full-space $\tilde{\mathbf{P}}_2$, $m = 2592$
Full-space $\tilde{\mathbf{P}}_2$, $m = 5184$
Full-space $\tilde{\mathbf{P}}_2$, $m = 10368$
Total Subiterations to Convergence
0
2500
5000
7500
10000
12500
15000
17500
20000
10
20
30
40
50
60
70
80
Number of Control Variables n


Work Units vs Control Variables
1e6
RS BFGS, m = 2592
RS BFGS, m = 5184
RS BFGS, m = 10368
Full-space $\tilde{\mathbf{P}}_2$, m = 2592
Full-space $\tilde{\mathbf{P}}_2$, m = 5184
Full-space $\tilde{\mathbf{P}}_2$, m = 10368
Work units to Convergence
0.0
0.2
0.4
0.6
0.8
1.0
10
20
30
40
50
60
70
80
Number of Control Variables n


Reduced-space Design Cycles vs Control Variables
RS BFGS, m = 2592
RS BFGS, m = 5184
RS BFGS, m = 10368
Number of Design Cycles to Convergence
Number of Control Variables n
300
250
200
150
100
50
10
20
30
40
50
60
70
80

Reduced-space Gradient vs Design Cycles

m = 2592

m = 5184

m = 10368

Gradient Norm

Design Cycles

BFGS, n = 10
BFGS, n = 20
BFGS, n = 40
BFGS, n = 80

Full-space Gradient vs Design Cycles

m = 2592

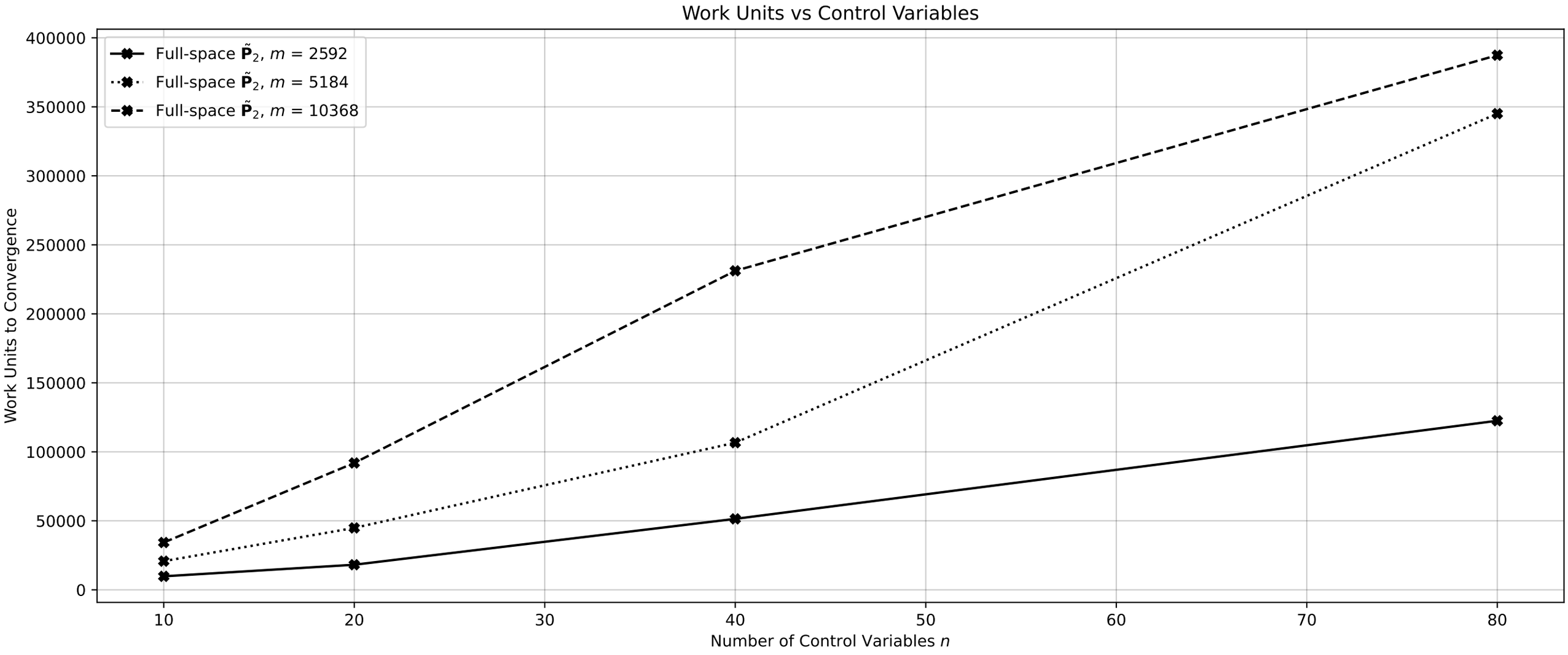
m = 5184

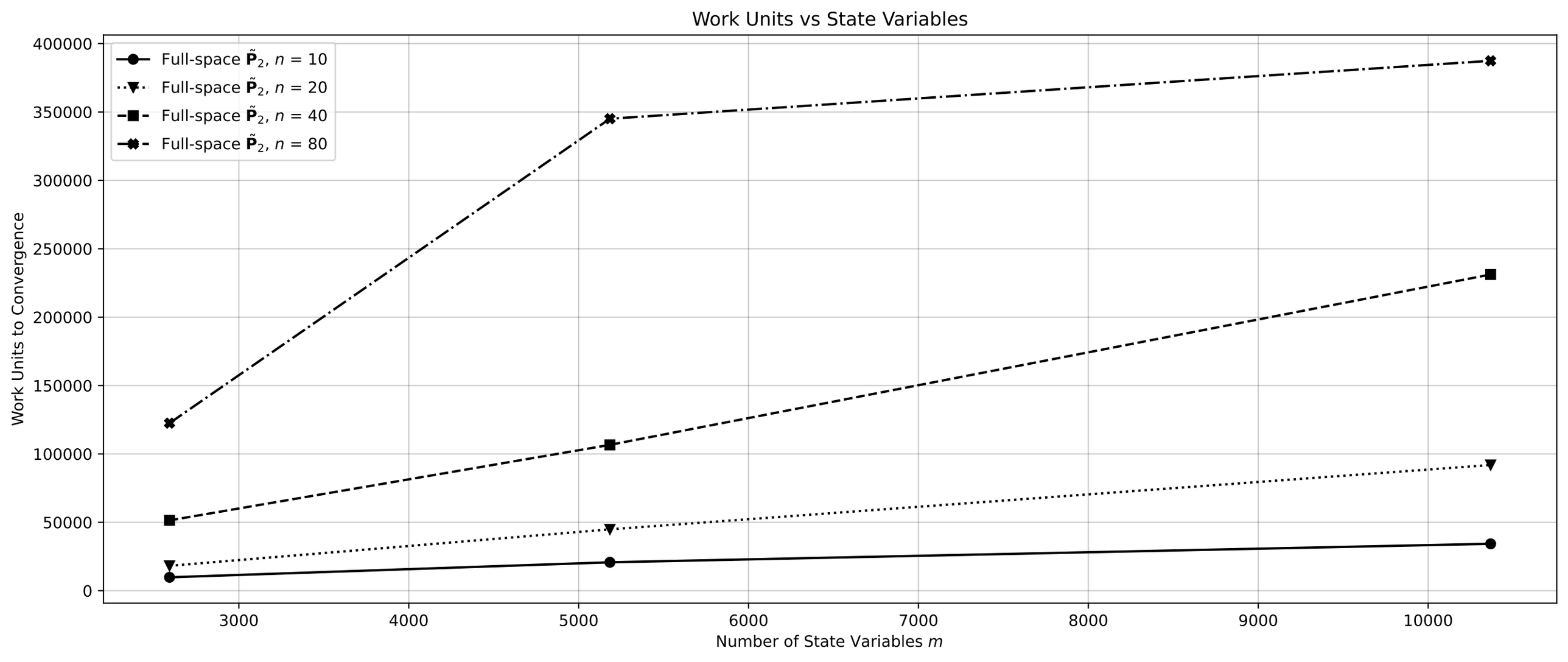
m = 10368

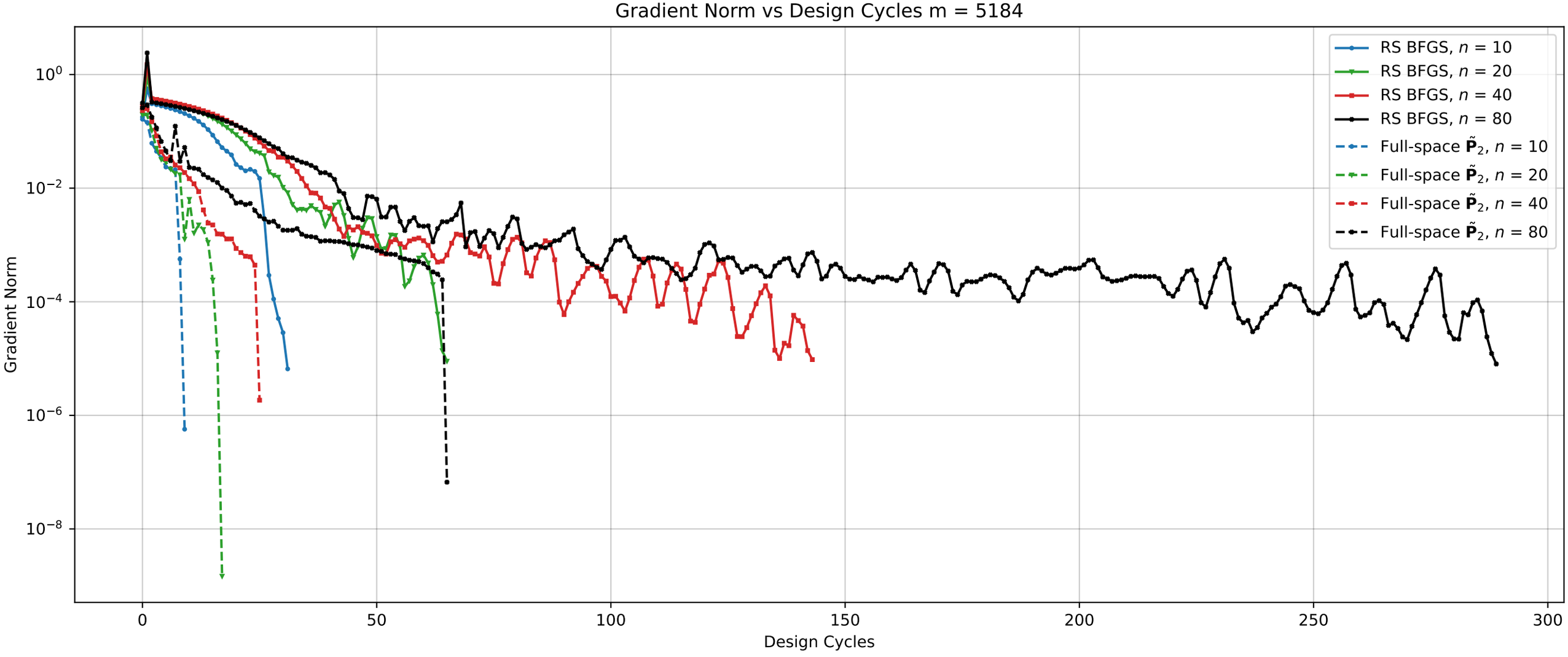
Gradient Norm

Design Cycles

BFGS, n = 10
BFGS, n = 20
BFGS, n = 40
BFGS, n = 80
Full-space $\tilde{\mathbf{P}}_2$, n = 10
Full-space $\tilde{\mathbf{P}}_2$, n = 20
Full-space $\tilde{\mathbf{P}}_2$, n = 40
Full-space $\tilde{\mathbf{P}}_2$, n = 80


Work Units vs Control Variables
Full-space $\tilde{\mathbf{P}}_2$, $m = 2592$
Full-space $\tilde{\mathbf{P}}_2$, $m = 5184$
Full-space $\tilde{\mathbf{P}}_2$, $m = 10368$
Work Units to Convergence
Number of Control Variables n
0
50000
100000
150000
200000
250000
300000
350000
400000
10
20
30
40
50
60
70
80


Work Units vs State Variables
Full-space $\tilde{\mathbf{P}}_2$, $n = 10$
Full-space $\tilde{\mathbf{P}}_2$, $n = 20$
Full-space $\tilde{\mathbf{P}}_2$, $n = 40$
Full-space $\tilde{\mathbf{P}}_2$, $n = 80$
Work Units to Convergence
Number of State Variables m
0
50000
100000
150000
200000
250000
300000
350000
400000
3000
4000
5000
6000
7000
8000
9000
10000


Gradient Norm vs Design Cycles m = 5184
Gradient Norm
Design Cycles
10^{0}
10^{-2}
10^{-4}
10^{-6}
10^{-8}
0
50
100
150
200
250
300
RS BFGS, n = 10
RS BFGS, n = 20
RS BFGS, n = 40
RS BFGS, n = 80
Full-space $\tilde{\mathbf{P}}_2$, n = 10
Full-space $\tilde{\mathbf{P}}_2$, n = 20
Full-space $\tilde{\mathbf{P}}_2$, n = 40
Full-space $\tilde{\mathbf{P}}_2$, n = 80