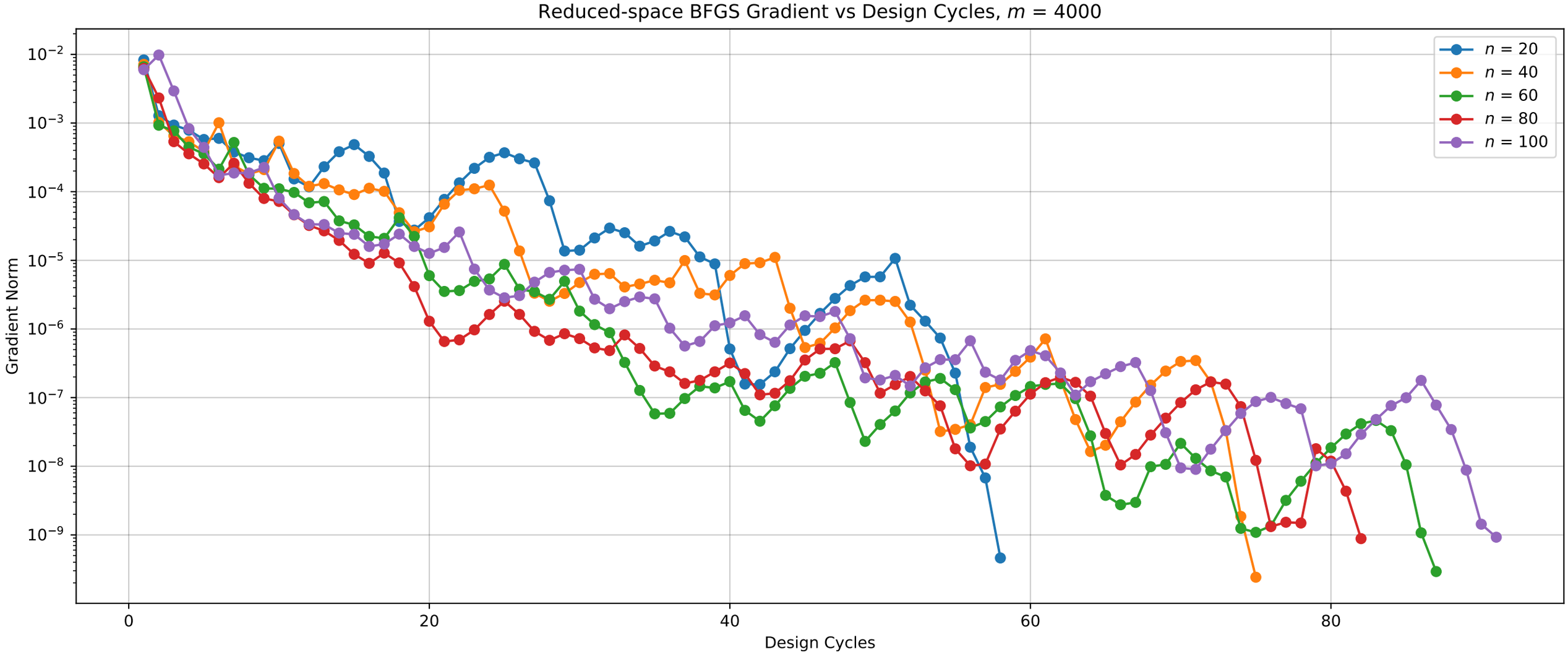
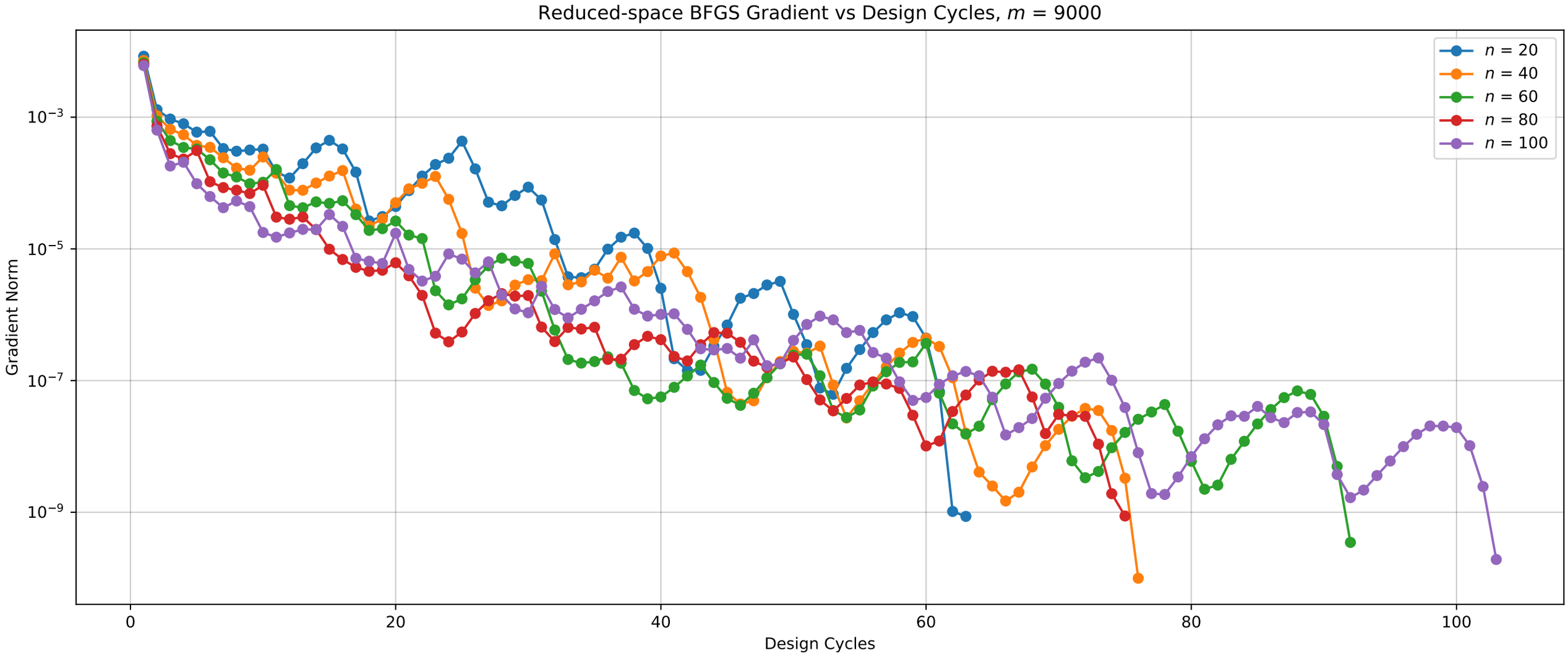
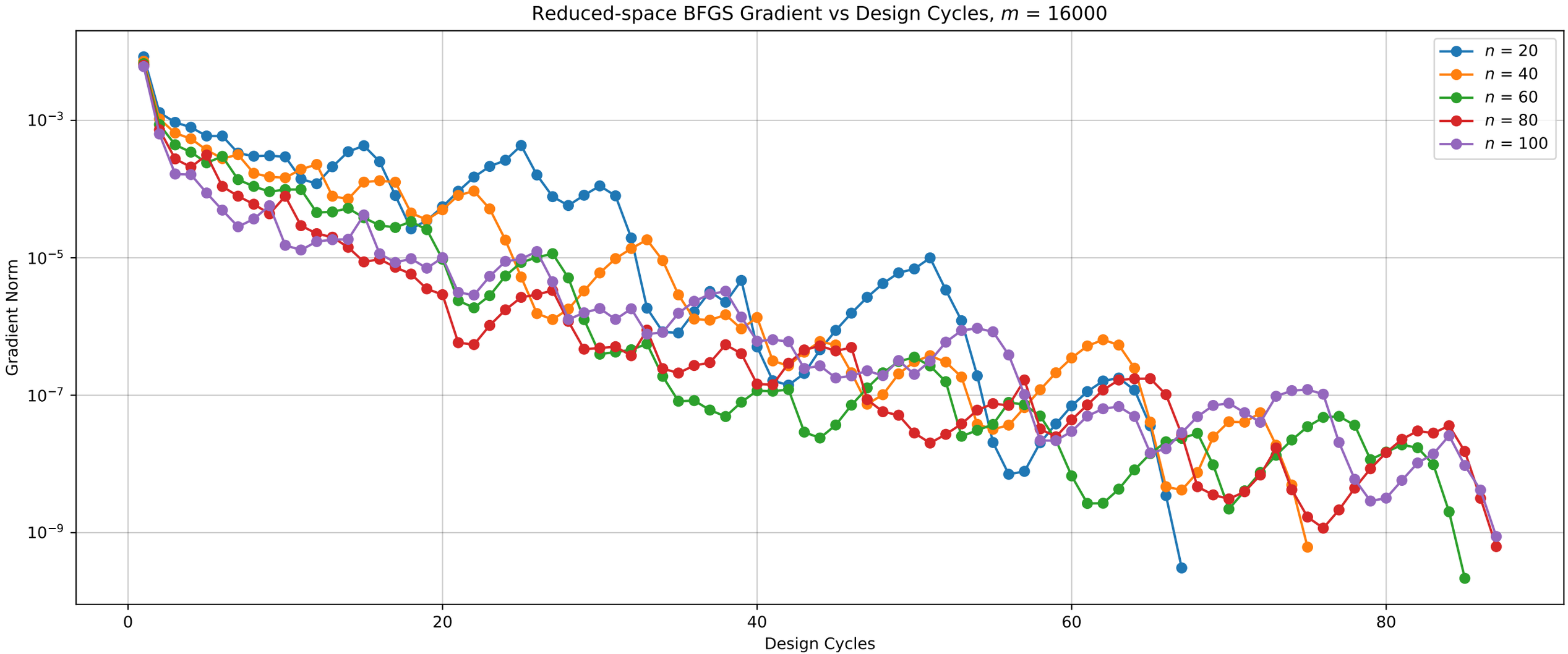
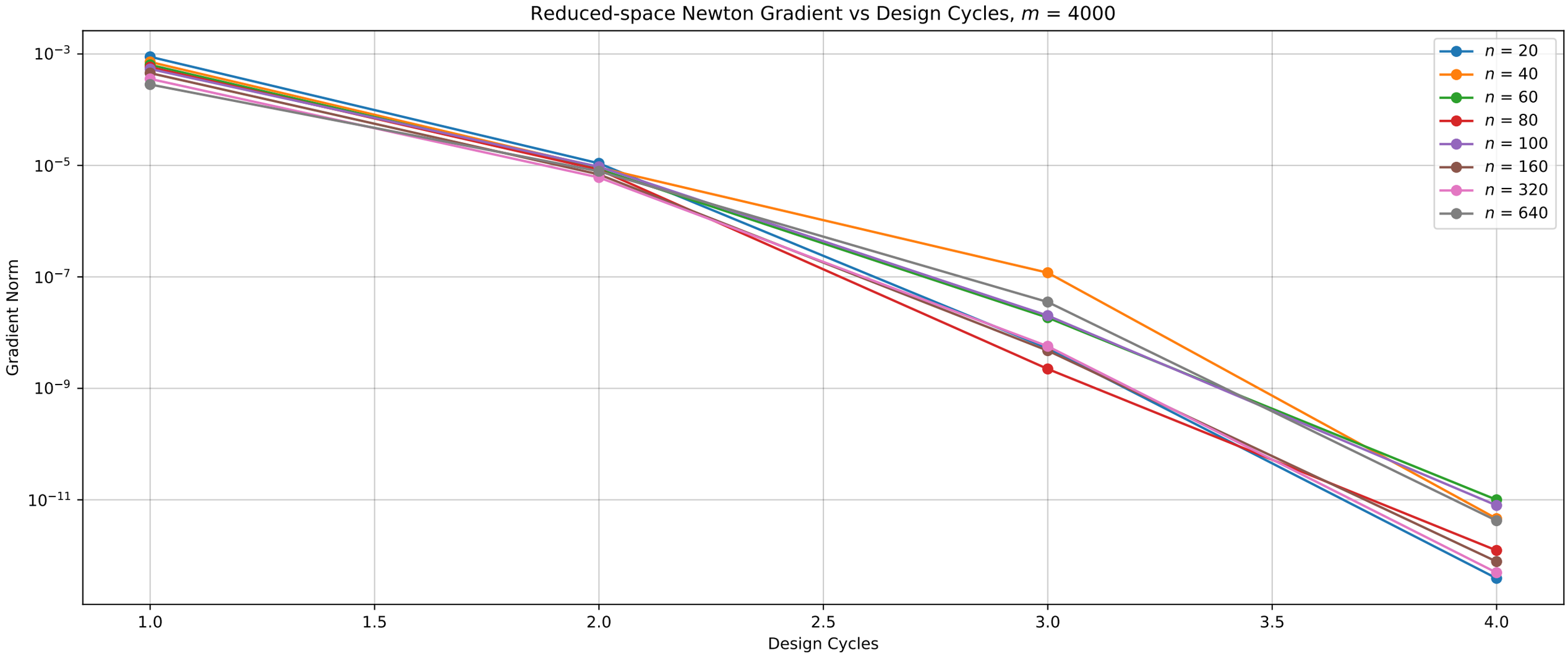
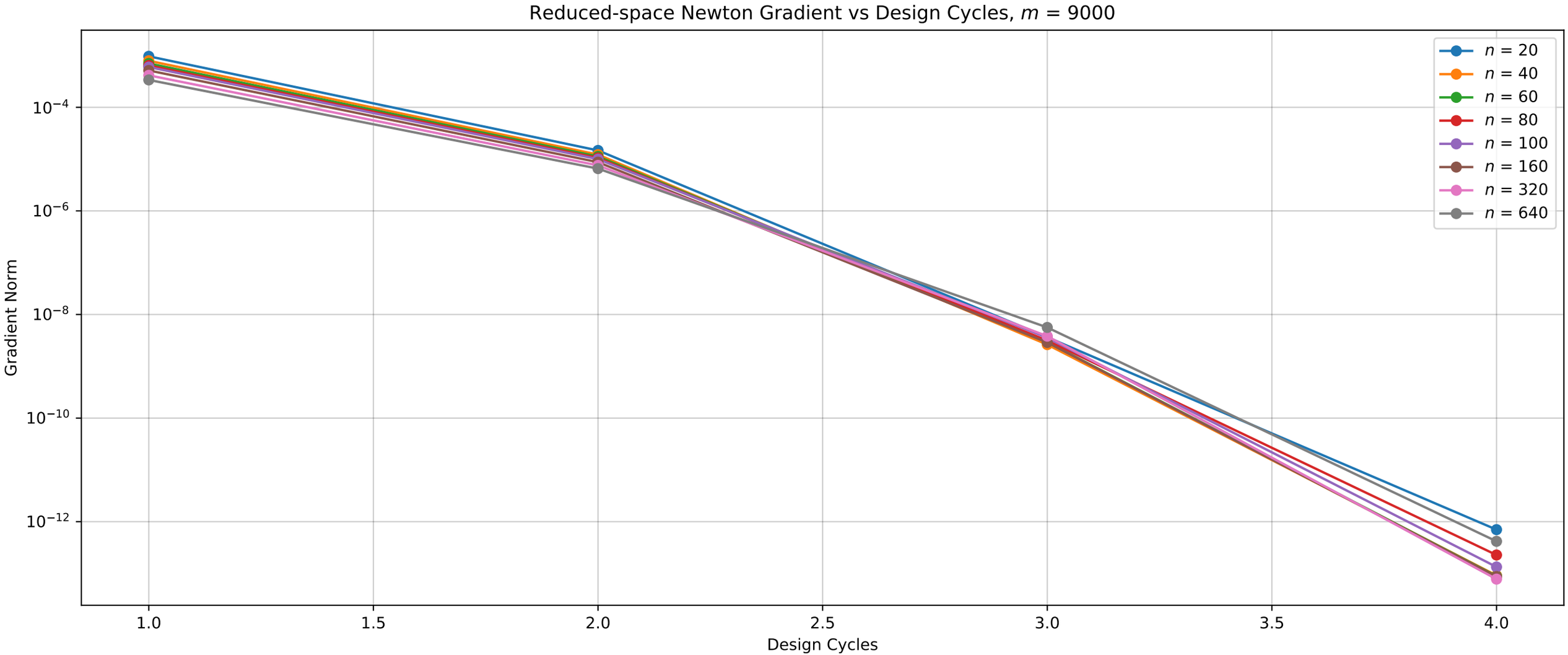
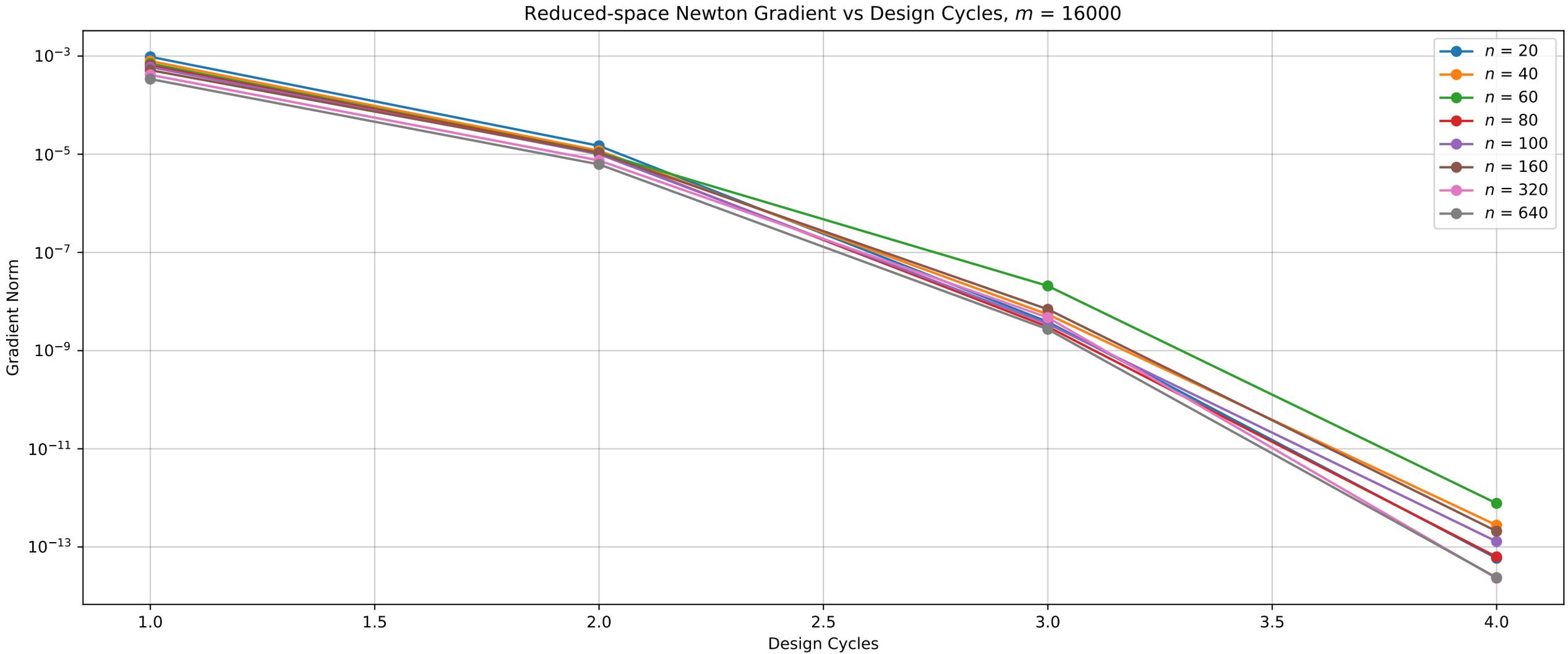

Reduced-space BFGS Gradient vs Design Cycles, m = 4000
Gradient Norm
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-8}
10^{-9}
0
20
40
60
80
Design Cycles
n = 20
n = 40
n = 60
n = 80
n = 100

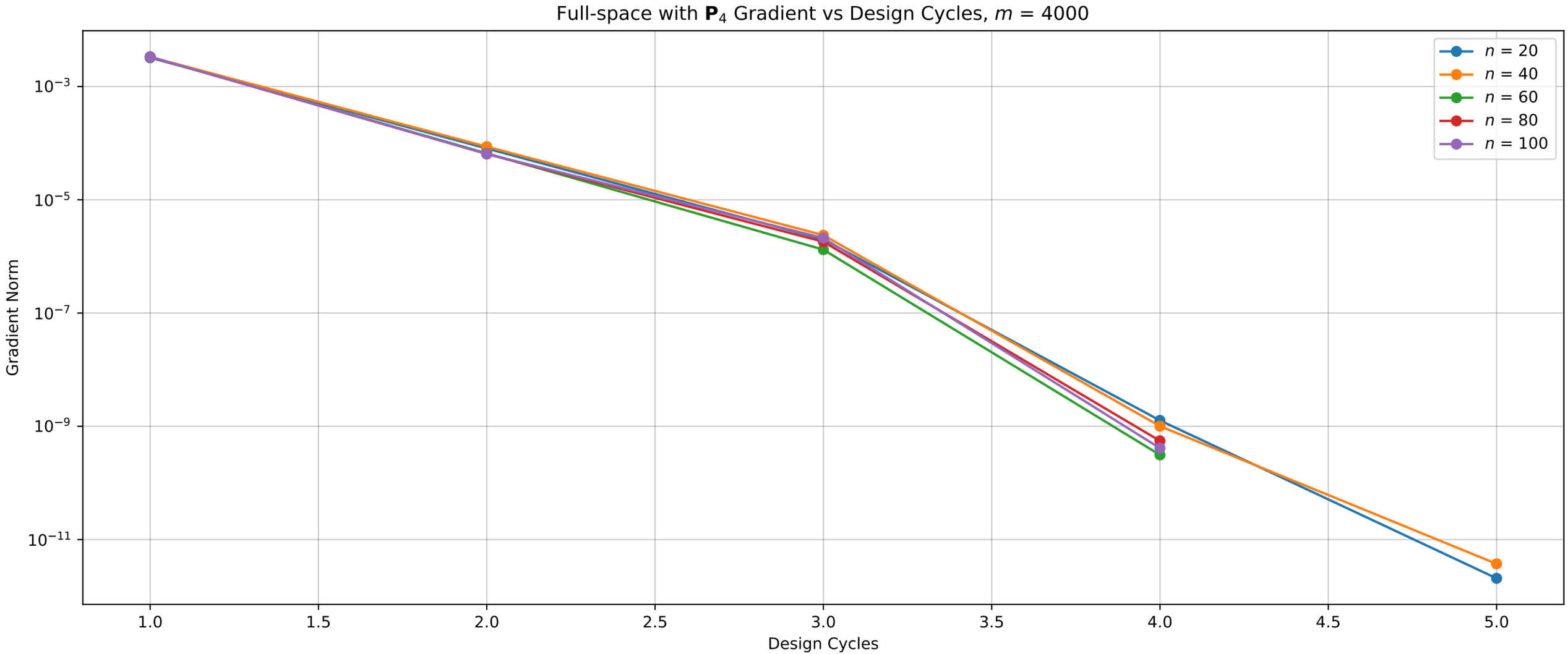

Reduced-space BFGS Gradient vs Design Cycles, m = 9000
Gradient Norm
10^{-3}
10^{-5}
10^{-7}
10^{-9}
0
20
40
60
80
100
Design Cycles
n = 20
n = 40
n = 60
n = 80
n = 100

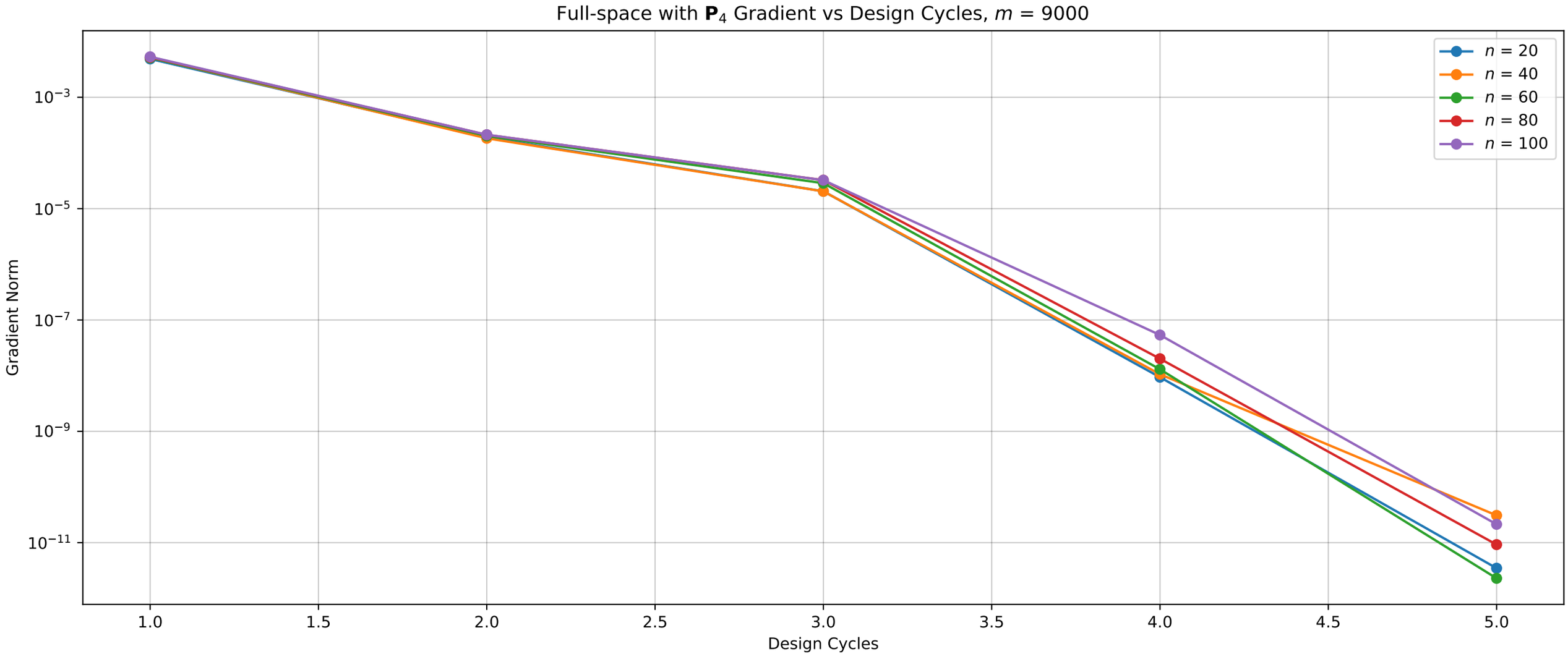

Reduced-space BFGS Gradient vs Design Cycles, m = 16000
Gradient Norm
10^{-3}
10^{-5}
10^{-7}
10^{-9}
0
20
40
60
80
Design Cycles
n = 20
n = 40
n = 60
n = 80
n = 100

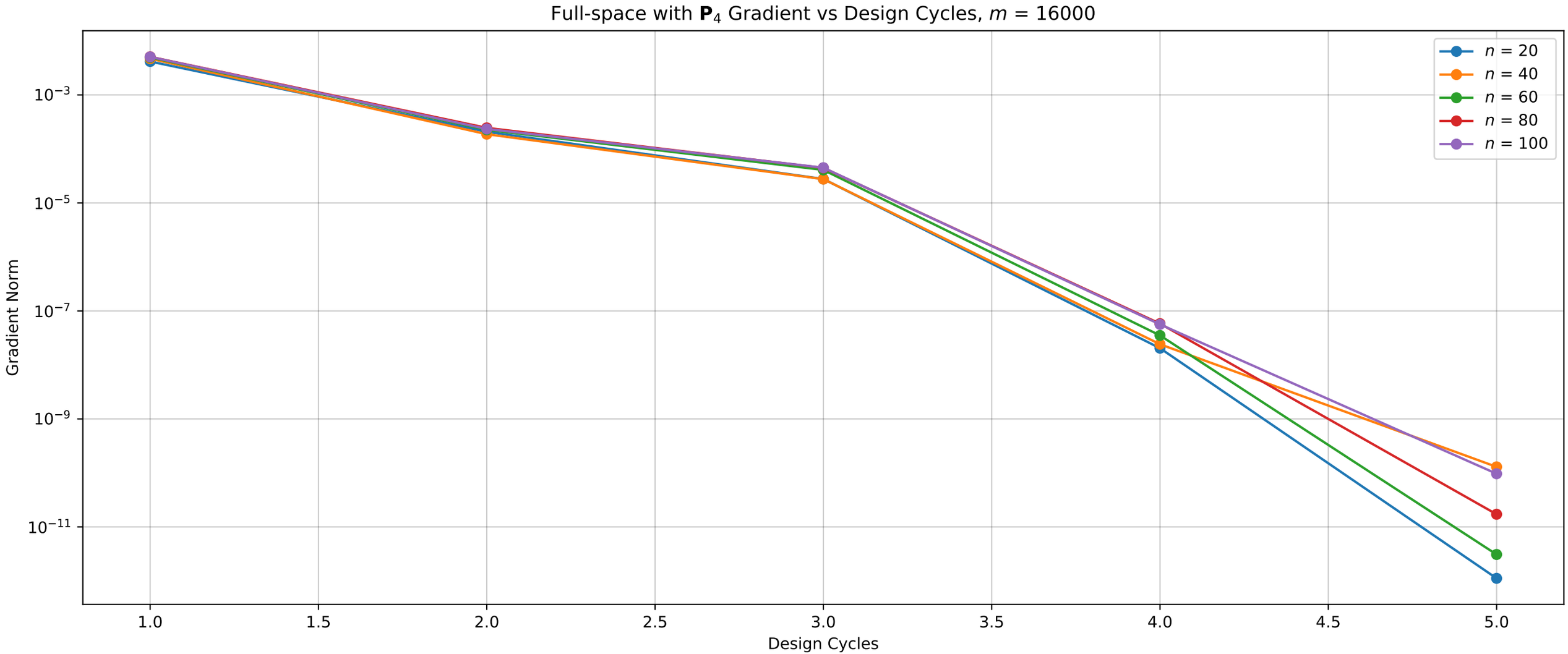

Reduced-space Newton Gradient vs Design Cycles, m = 4000
Gradient Norm
10^{-3}
10^{-5}
10^{-7}
10^{-9}
10^{-11}
1.0
1.5
2.0
2.5
3.0
3.5
4.0
Design Cycles
n = 20
n = 40
n = 60
n = 80
n = 100
n = 160
n = 320
n = 640

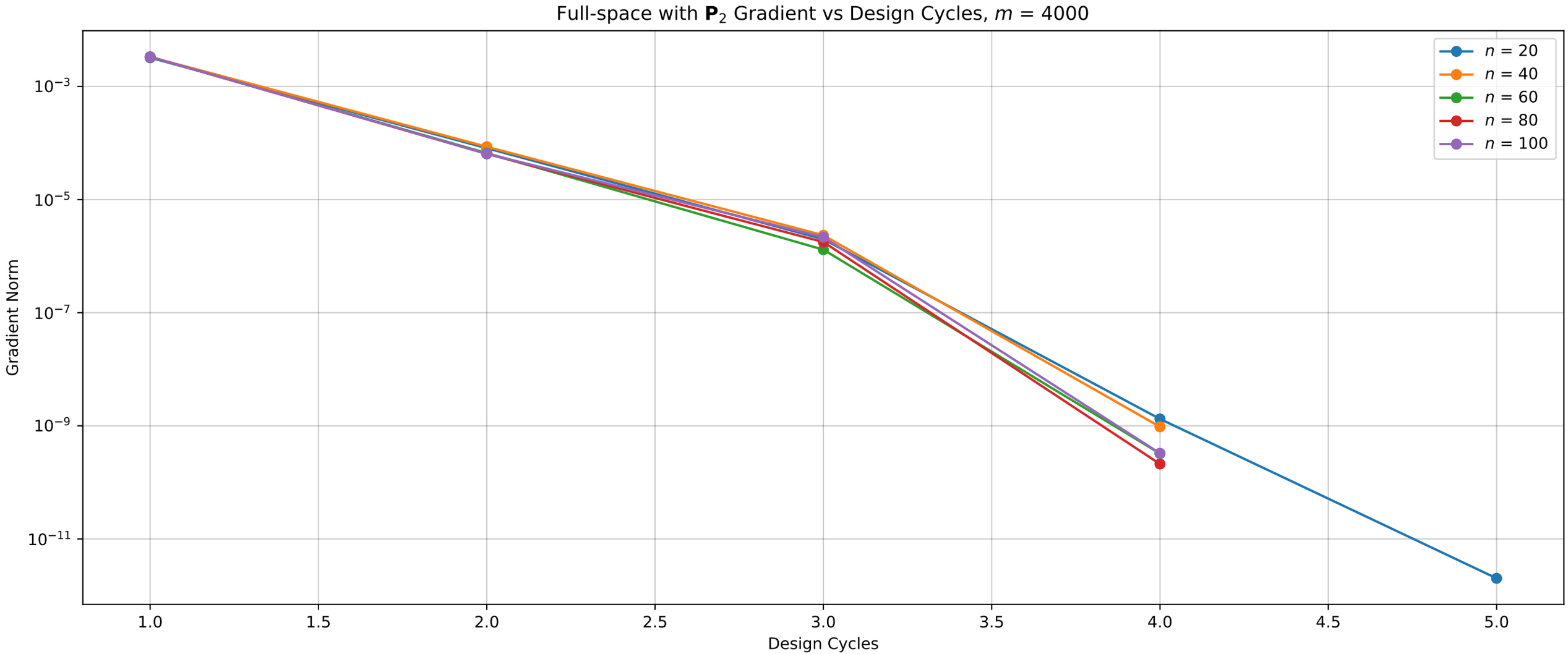

Reduced-space Newton Gradient vs Design Cycles, m = 9000
Gradient Norm
10^{-4}
10^{-6}
10^{-8}
10^{-10}
10^{-12}
1.0
1.5
2.0
2.5
3.0
3.5
4.0
Design Cycles
n = 20
n = 40
n = 60
n = 80
n = 100
n = 160
n = 320
n = 640

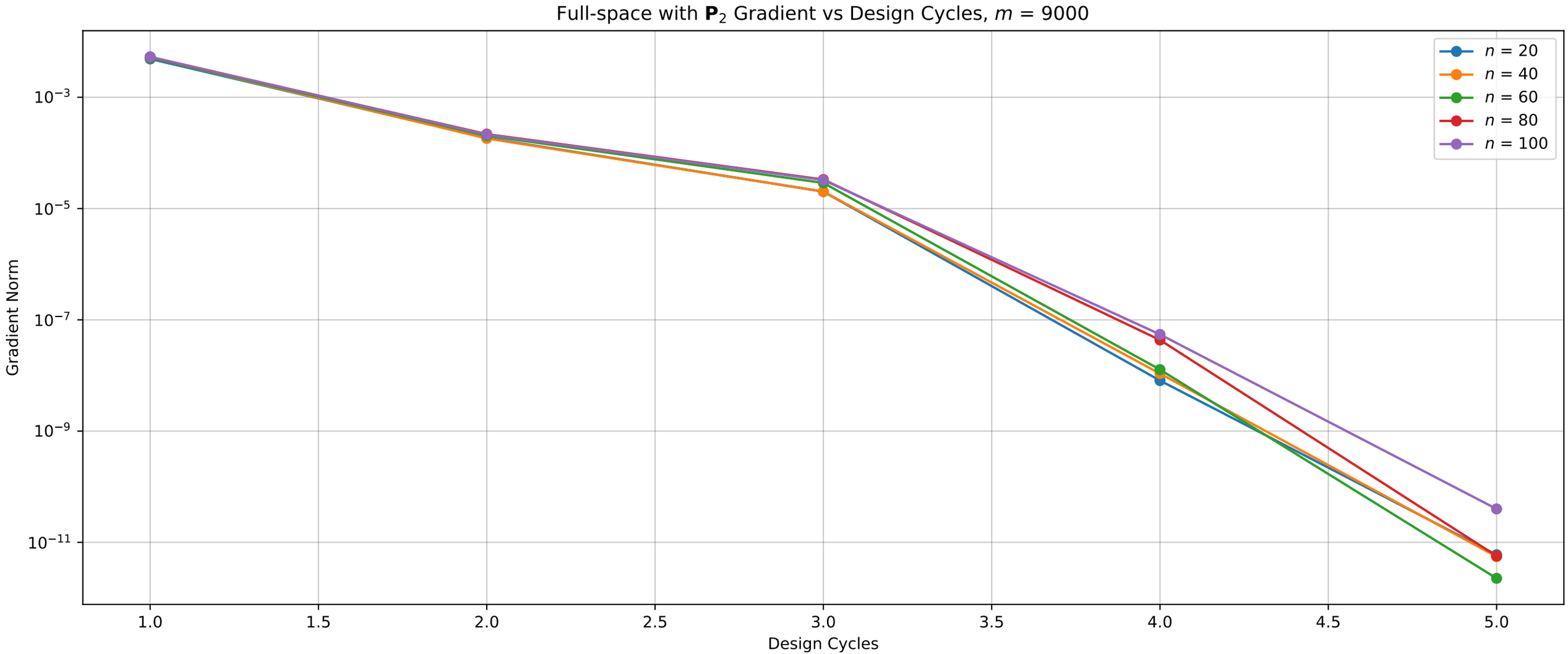

Reduced-space Newton Gradient vs Design Cycles, m = 16000
Gradient Norm
10^{-3}
10^{-5}
10^{-7}
10^{-9}
10^{-11}
10^{-13}
1.0
1.5
2.0
2.5
3.0
3.5
4.0
Design Cycles
n = 20
n = 40
n = 60
n = 80
n = 100
n = 160
n = 320
n = 640

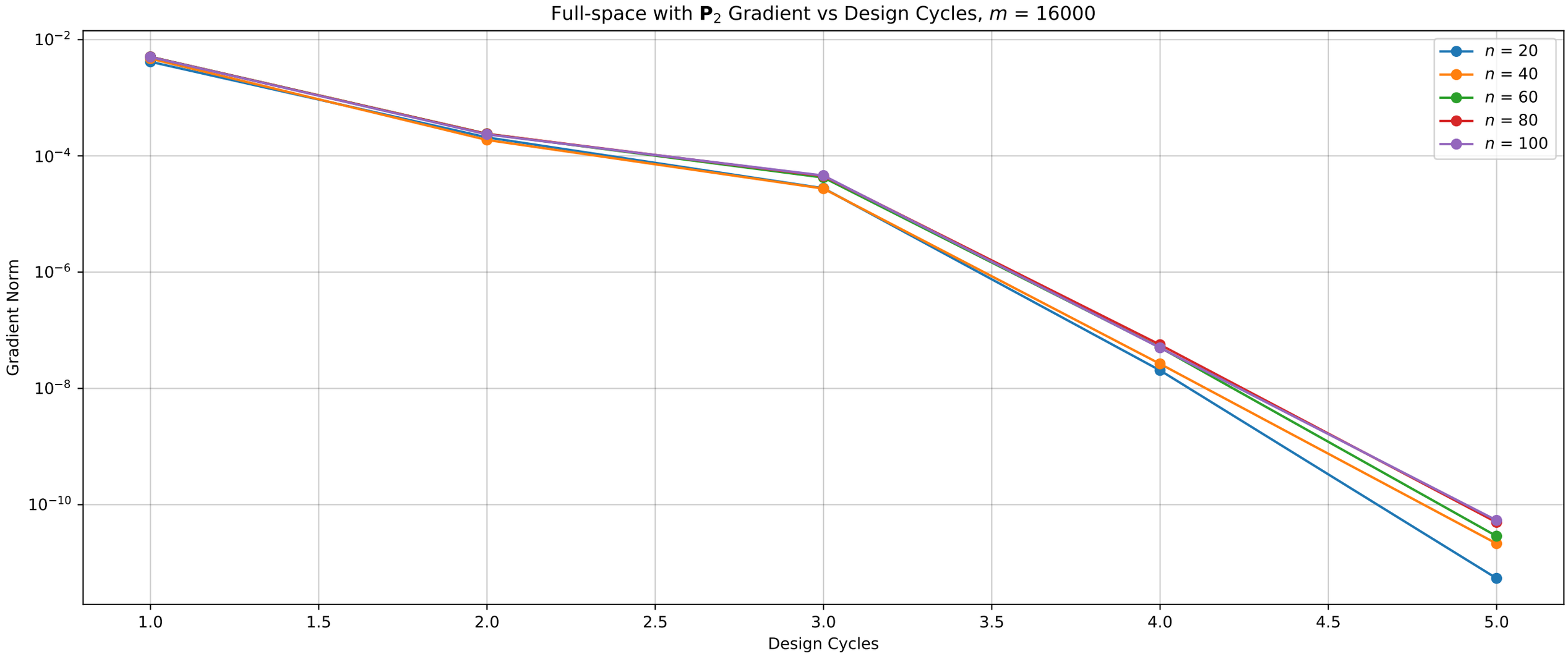

Full-space with $\mathbf{P}_4$ Gradient vs Design Cycles, m = 4000
Gradient Norm
10^{-3}
10^{-5}
10^{-7}
10^{-9}
10^{-11}
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0
Design Cycles
n = 20
n = 40
n = 60
n = 80
n = 100

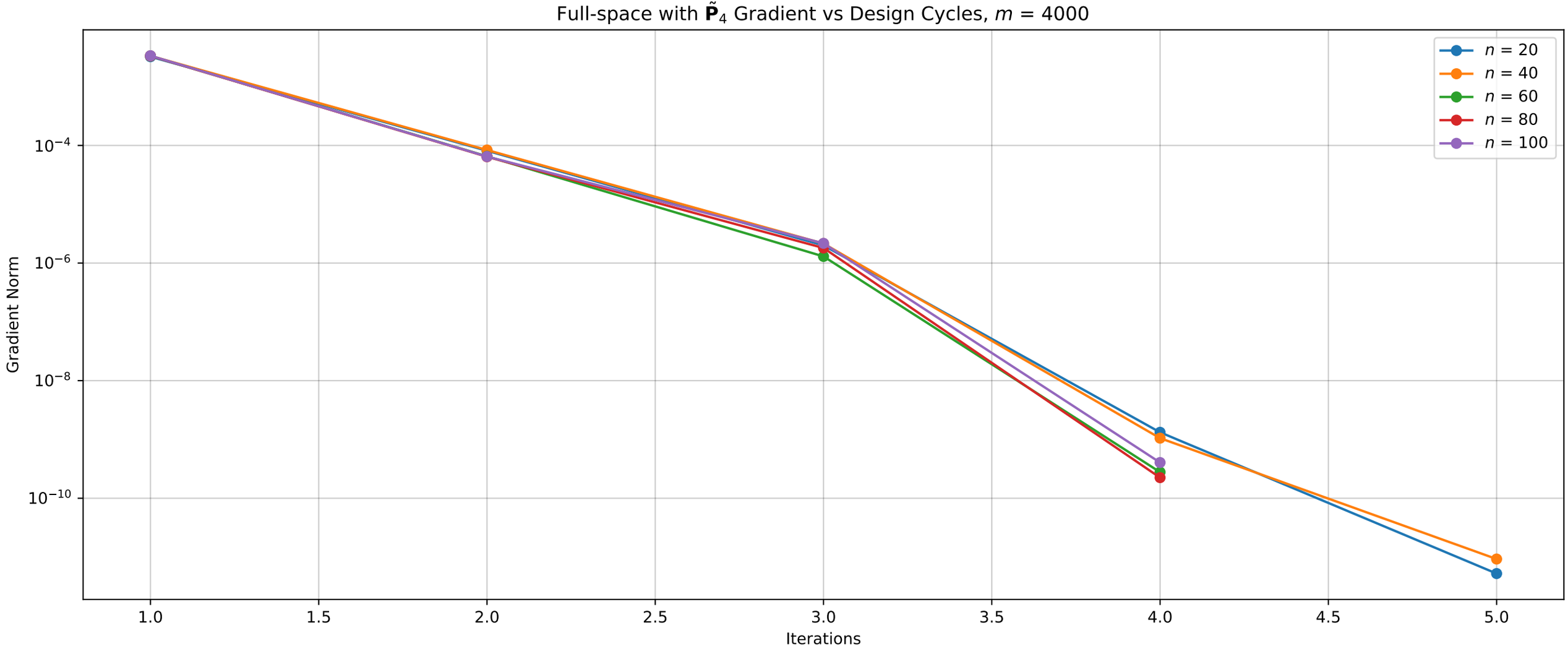

Full-space with $\mathbf{P}_4$ Gradient vs Design Cycles, m = 9000
Gradient Norm
10^{-3}
10^{-5}
10^{-7}
10^{-9}
10^{-11}
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0
Design Cycles
n = 20
n = 40
n = 60
n = 80
n = 100

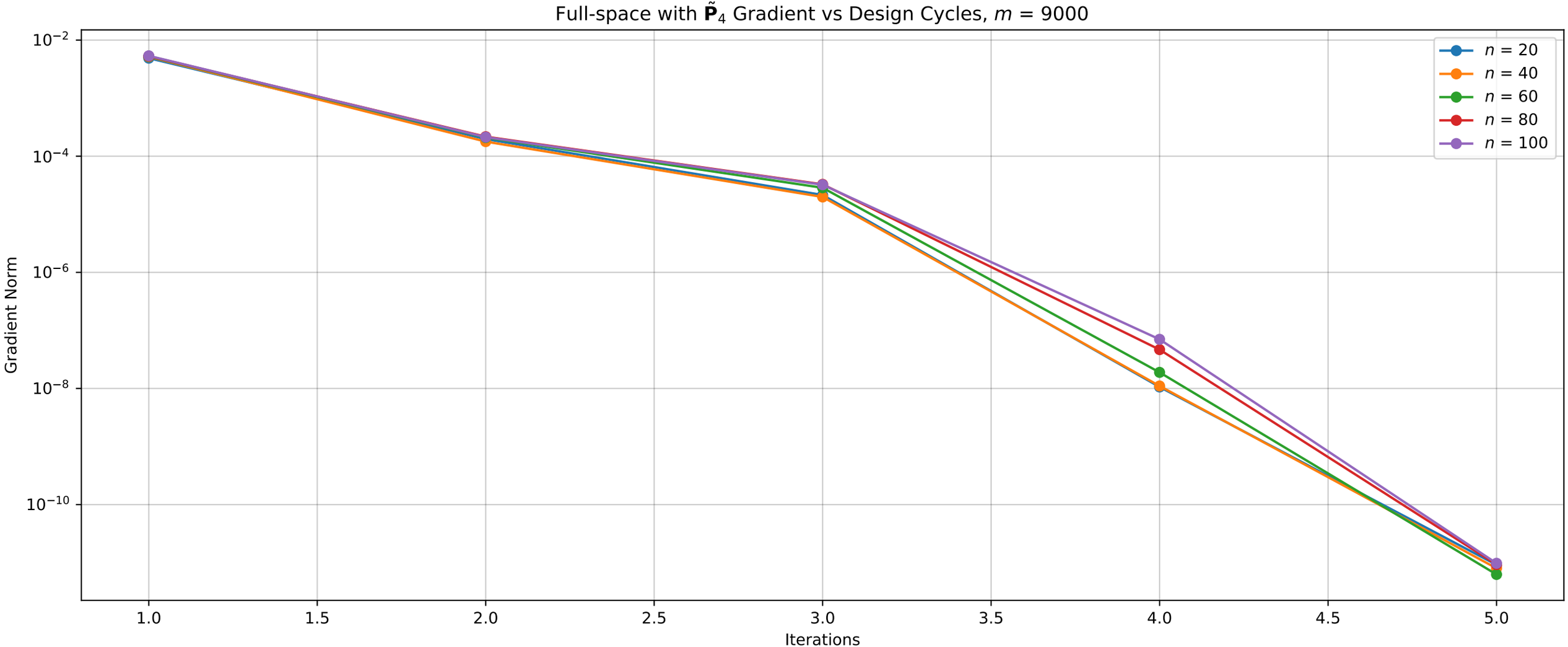

Full-space with $\mathbf{P}_4$ Gradient vs Design Cycles, m = 16000
Gradient Norm
10^{-3}
10^{-5}
10^{-7}
10^{-9}
10^{-11}
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0
Design Cycles
n = 20
n = 40
n = 60
n = 80
n = 100

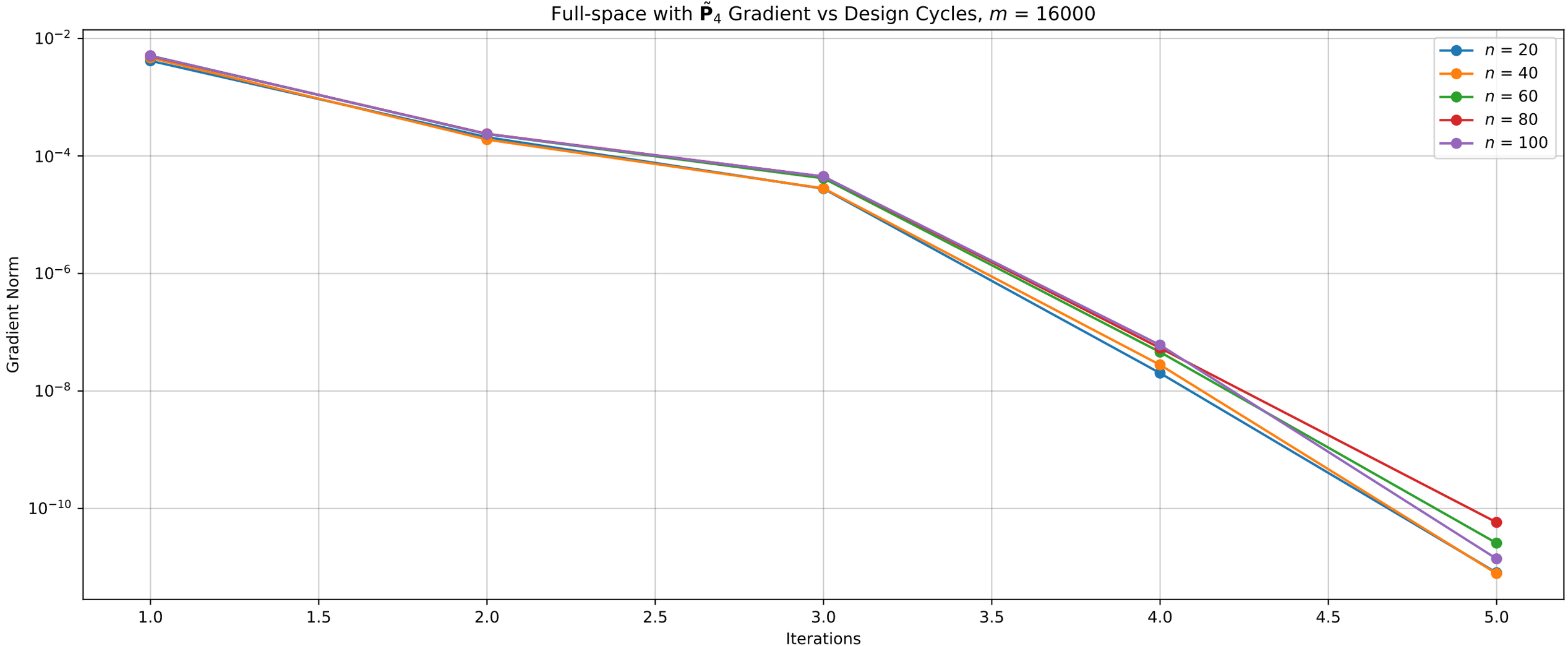

Full-space with $\mathbf{P}_2$ Gradient vs Design Cycles, m = 4000
Gradient Norm
10^{-3}
10^{-5}
10^{-7}
10^{-9}
10^{-11}
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0
Design Cycles
n = 20
n = 40
n = 60
n = 80
n = 100

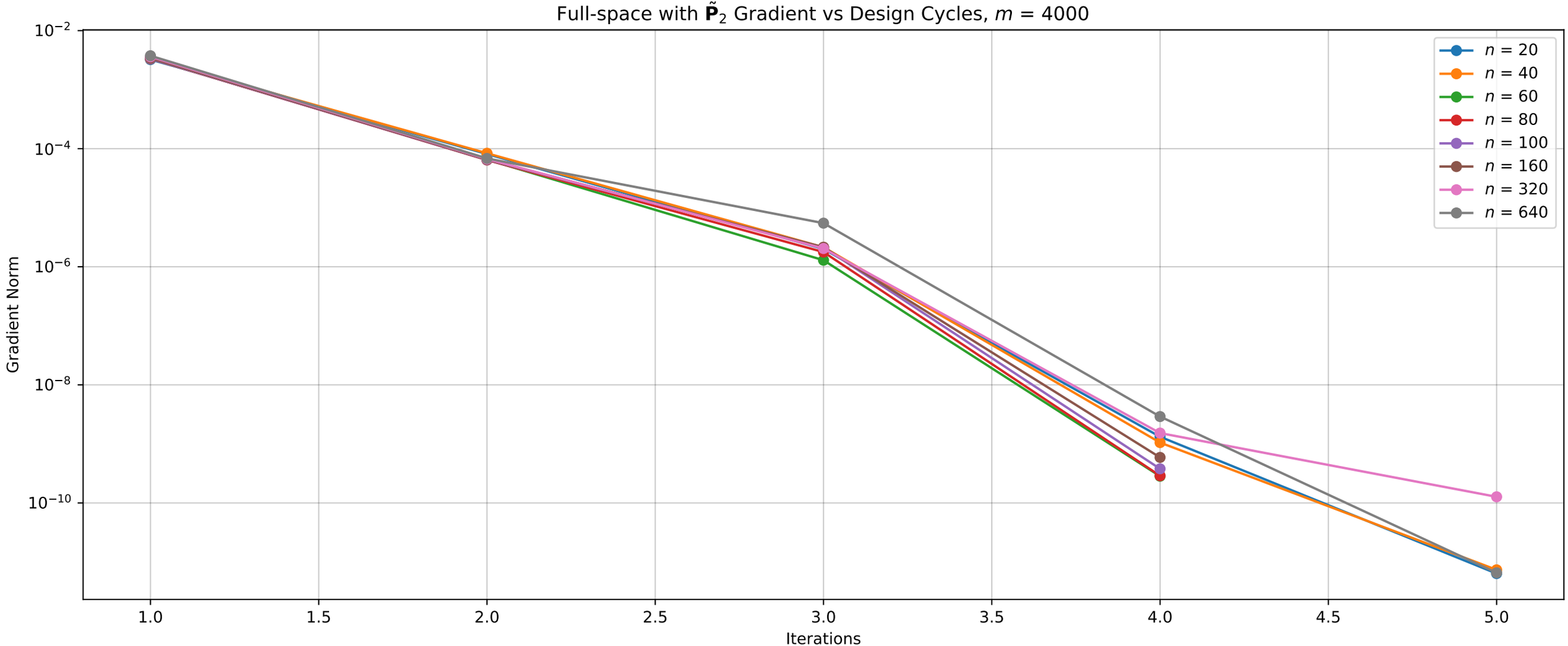

Full-space with $\mathbf{P}_2$ Gradient vs Design Cycles, m = 9000
Gradient Norm
10^{-3}
10^{-5}
10^{-7}
10^{-9}
10^{-11}
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0
Design Cycles
n = 20
n = 40
n = 60
n = 80
n = 100

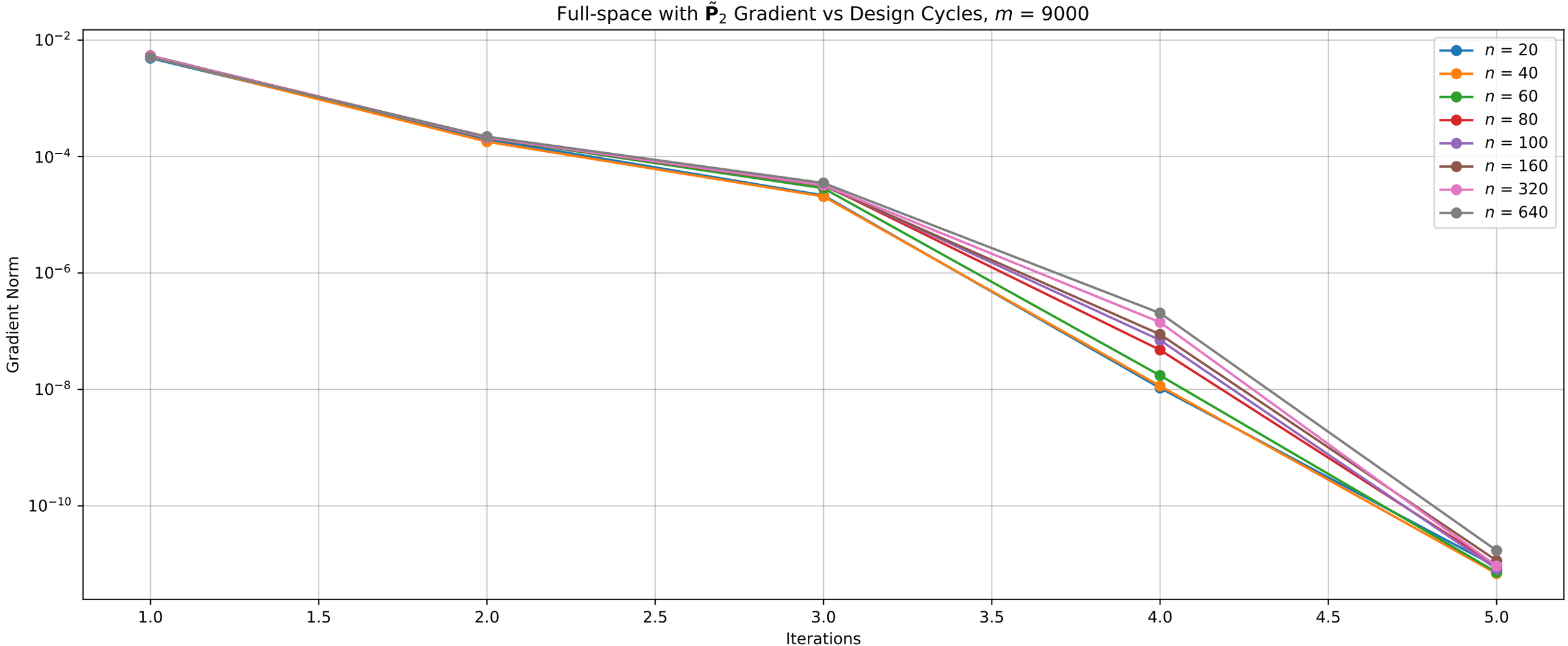

Full-space with $\mathbf{P}_2$ Gradient vs Design Cycles, m = 16000
Gradient Norm
10^{-2}
10^{-4}
10^{-6}
10^{-8}
10^{-10}
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0
Design Cycles
n = 20
n = 40
n = 60
n = 80
n = 100

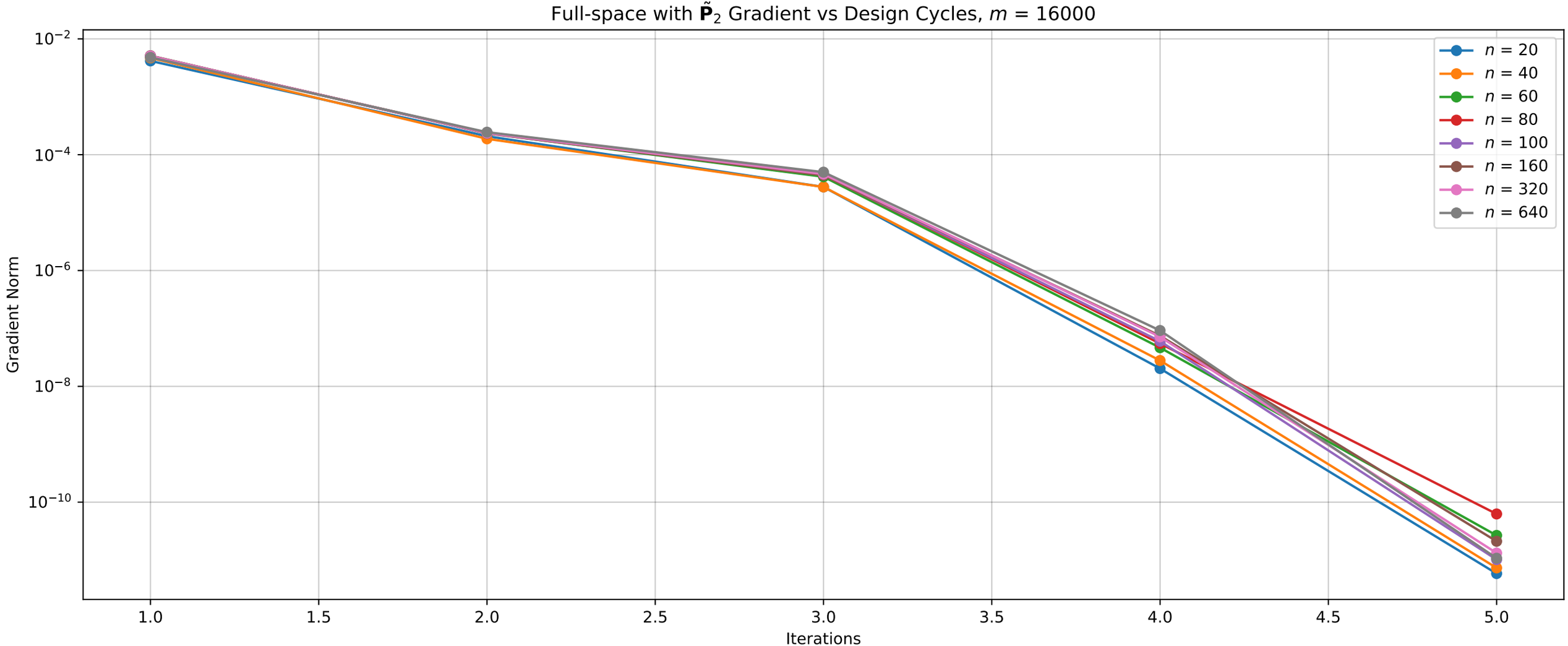

Full-space with $\tilde{\mathbf{P}}_4$ Gradient vs Design Cycles, m = 4000
Gradient Norm
10^{-4}
10^{-6}
10^{-8}
10^{-10}
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0
Iterations
n = 20
n = 40
n = 60
n = 80
n = 100

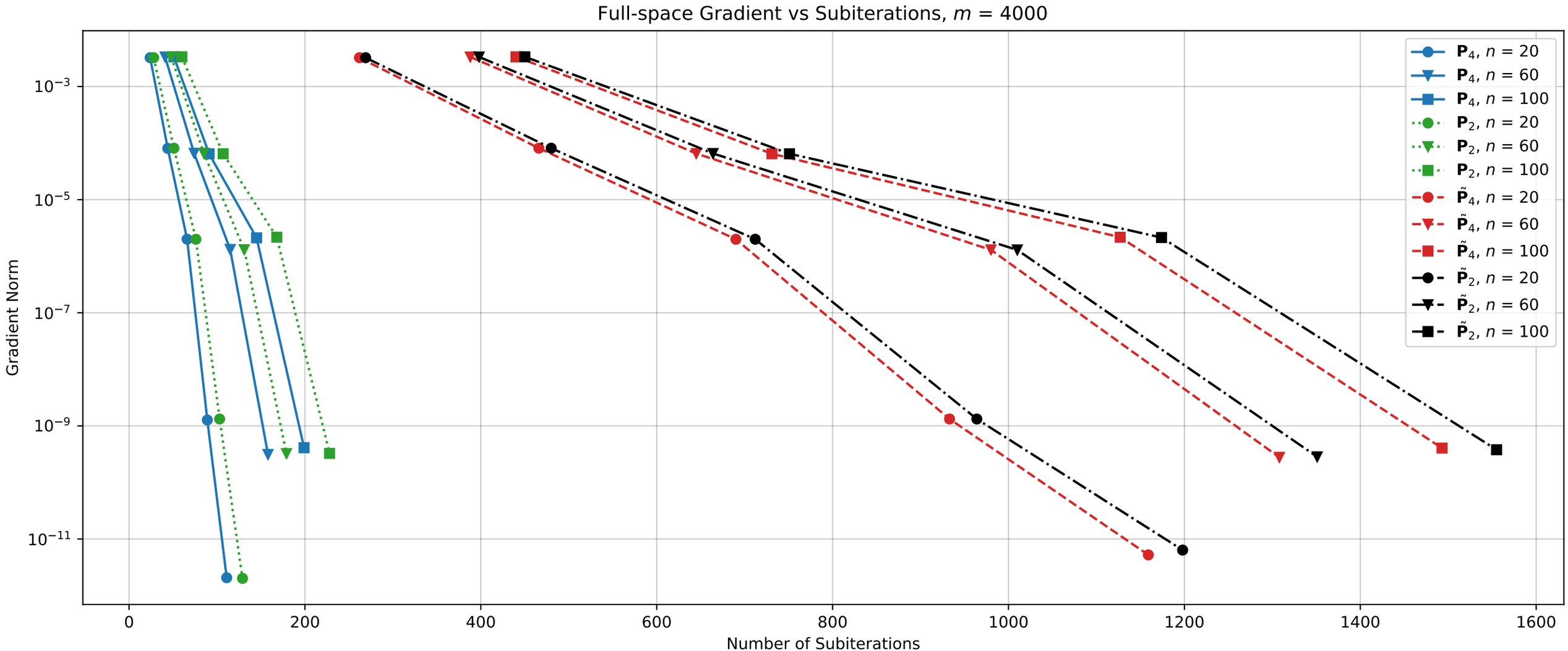

Full-space with $\tilde{\mathbf{P}}_4$ Gradient vs Design Cycles, m = 9000
Gradient Norm
10^{-2}
10^{-4}
10^{-6}
10^{-8}
10^{-10}
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0
Iterations
n = 20
n = 40
n = 60
n = 80
n = 100

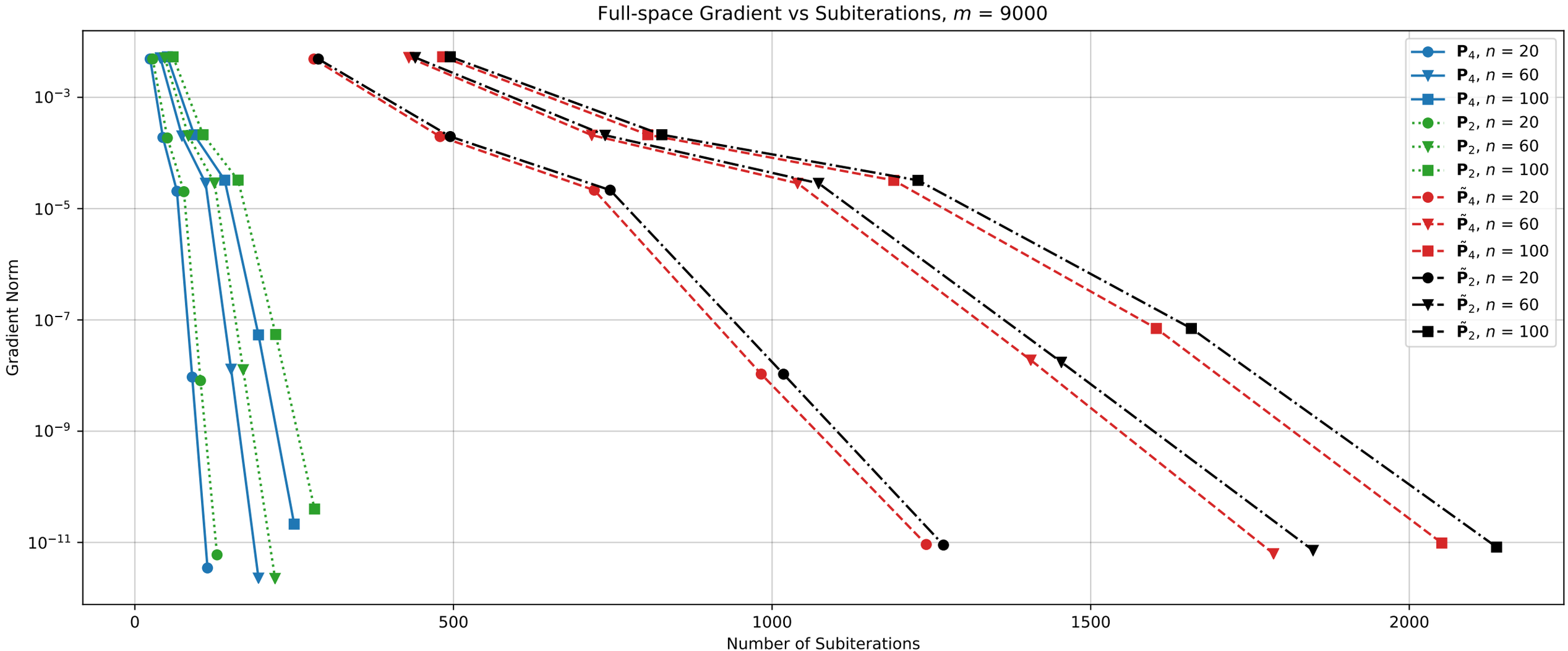

Full-space with $\tilde{\mathbf{P}}_4$ Gradient vs Design Cycles, m = 16000
Gradient Norm
10^{-2}
10^{-4}
10^{-6}
10^{-8}
10^{-10}
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0
Iterations
n = 20
n = 40
n = 60
n = 80
n = 100

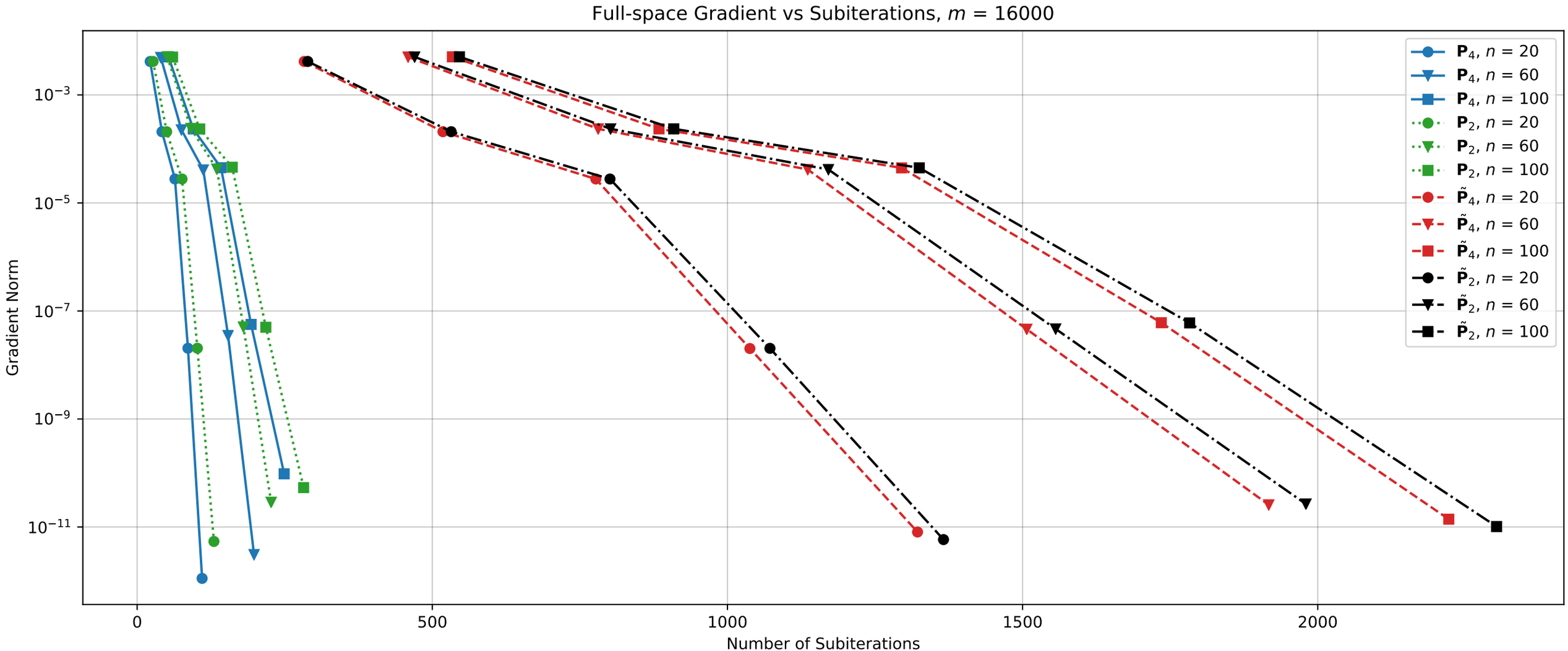

Full-space with $\tilde{\mathbf{P}}_2$ Gradient vs Design Cycles, m = 4000
Gradient Norm
10^{-2}
10^{-4}
10^{-6}
10^{-8}
10^{-10}
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0
Iterations
n = 20
n = 40
n = 60
n = 80
n = 100
n = 160
n = 320
n = 640


Full-space with $\tilde{\mathbf{P}}_2$ Gradient vs Design Cycles, m = 9000
Gradient Norm
10^{-2}
10^{-4}
10^{-6}
10^{-8}
10^{-10}
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0
Iterations
n = 20
n = 40
n = 60
n = 80
n = 100
n = 160
n = 320
n = 640


Full-space with $\tilde{\mathbf{P}}_2$ Gradient vs Design Cycles, m = 16000
Gradient Norm
10^{-2}
10^{-4}
10^{-6}
10^{-8}
10^{-10}
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0
Iterations
n = 20
n = 40
n = 60
n = 80
n = 100
n = 160
n = 320
n = 640


Full-space Gradient vs Subiterations, $m = 4000$
Gradient Norm
10^{-3}
10^{-5}
10^{-7}
10^{-9}
10^{-11}
0
200
400
600
800
1000
1200
1400
1600
Number of Subiterations
$\mathbf{P}_4$, $n = 20$
$\mathbf{P}_4$, $n = 60$
$\mathbf{P}_4$, $n = 100$
$\mathbf{P}_2$, $n = 20$
$\mathbf{P}_2$, $n = 60$
$\mathbf{P}_2$, $n = 100$
$\tilde{\mathbf{P}}_4$, $n = 20$
$\tilde{\mathbf{P}}_4$, $n = 60$
$\tilde{\mathbf{P}}_4$, $n = 100$
$\tilde{\mathbf{P}}_2$, $n = 20$
$\tilde{\mathbf{P}}_2$, $n = 60$
$\tilde{\mathbf{P}}_2$, $n = 100$


Full-space Gradient vs Subiterations, m = 9000
Gradient Norm
10^{-3}
10^{-5}
10^{-7}
10^{-9}
10^{-11}
0
500
1000
1500
2000
Number of Subiterations
$\mathbf{P}_4$, n = 20
$\mathbf{P}_4$, n = 60
$\mathbf{P}_4$, n = 100
$\mathbf{P}_2$, n = 20
$\mathbf{P}_2$, n = 60
$\mathbf{P}_2$, n = 100
$\tilde{\mathbf{P}}_4$, n = 20
$\tilde{\mathbf{P}}_4$, n = 60
$\tilde{\mathbf{P}}_4$, n = 100
$\tilde{\mathbf{P}}_2$, n = 20
$\tilde{\mathbf{P}}_2$, n = 60
$\tilde{\mathbf{P}}_2$, n = 100


Full-space Gradient vs Subiterations, m = 16000
Gradient Norm
10^{-3}
10^{-5}
10^{-7}
10^{-9}
10^{-11}
0
500
1000
1500
2000
Number of Subiterations
$\mathbf{P}_4$, n = 20
$\mathbf{P}_4$, n = 60
$\mathbf{P}_4$, n = 100
$\mathbf{P}_2$, n = 20
$\mathbf{P}_2$, n = 60
$\mathbf{P}_2$, n = 100
$\tilde{\mathbf{P}}_4$, n = 20
$\tilde{\mathbf{P}}_4$, n = 60
$\tilde{\mathbf{P}}_4$, n = 100
$\tilde{\mathbf{P}}_2$, n = 20
$\tilde{\mathbf{P}}_2$, n = 60
$\tilde{\mathbf{P}}_2$, n = 100

Full-space Gradient vs Subiterations

$m = 4000$

$m = 9000$

$m = 16000$

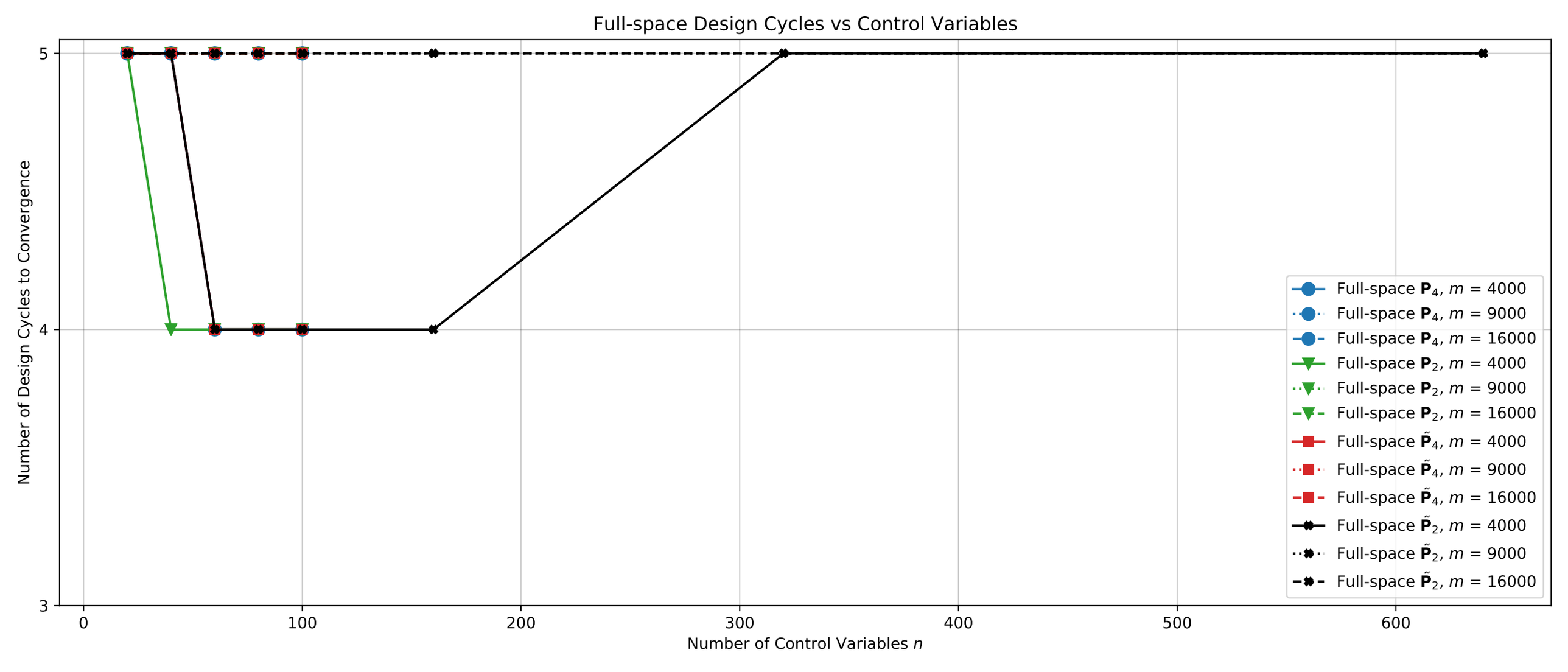
Gradient Norm

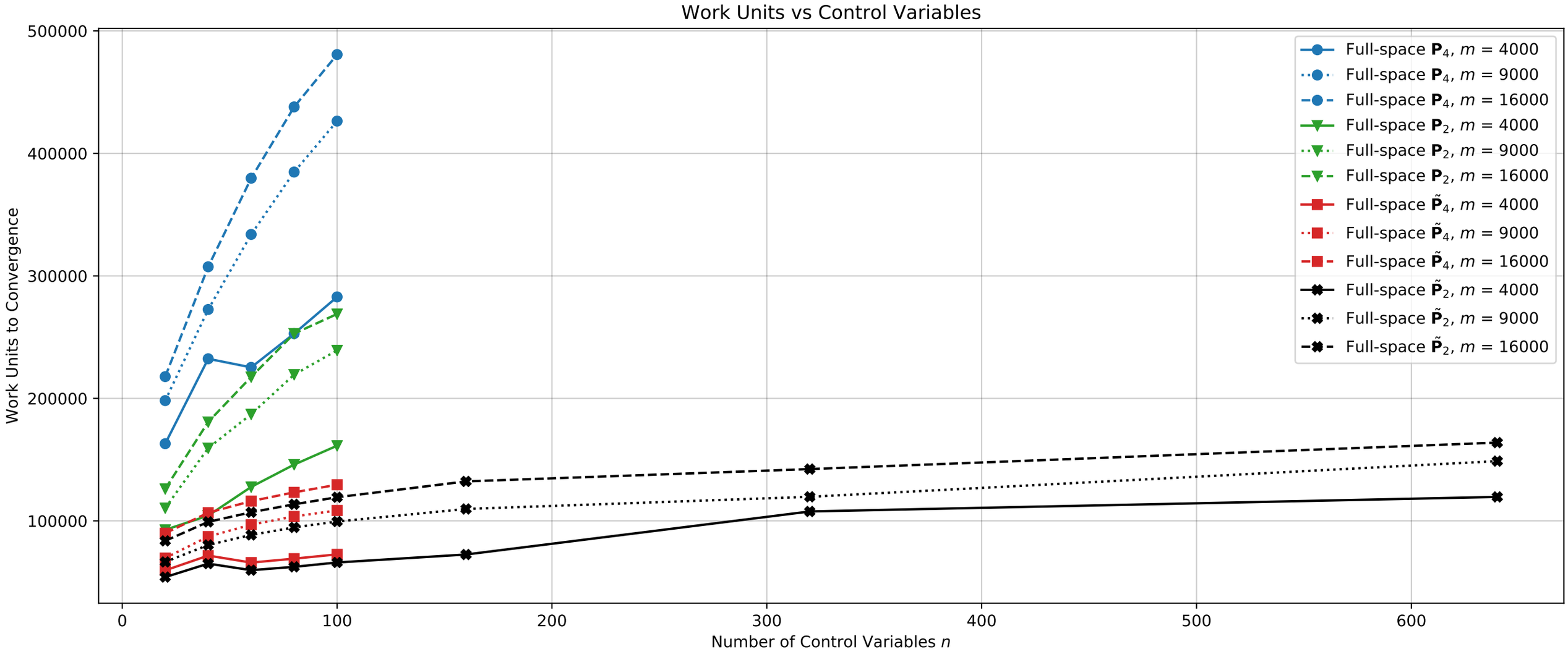
10^{-3}
10^{-5}
10^{-7}
10^{-9}
10^{-11}

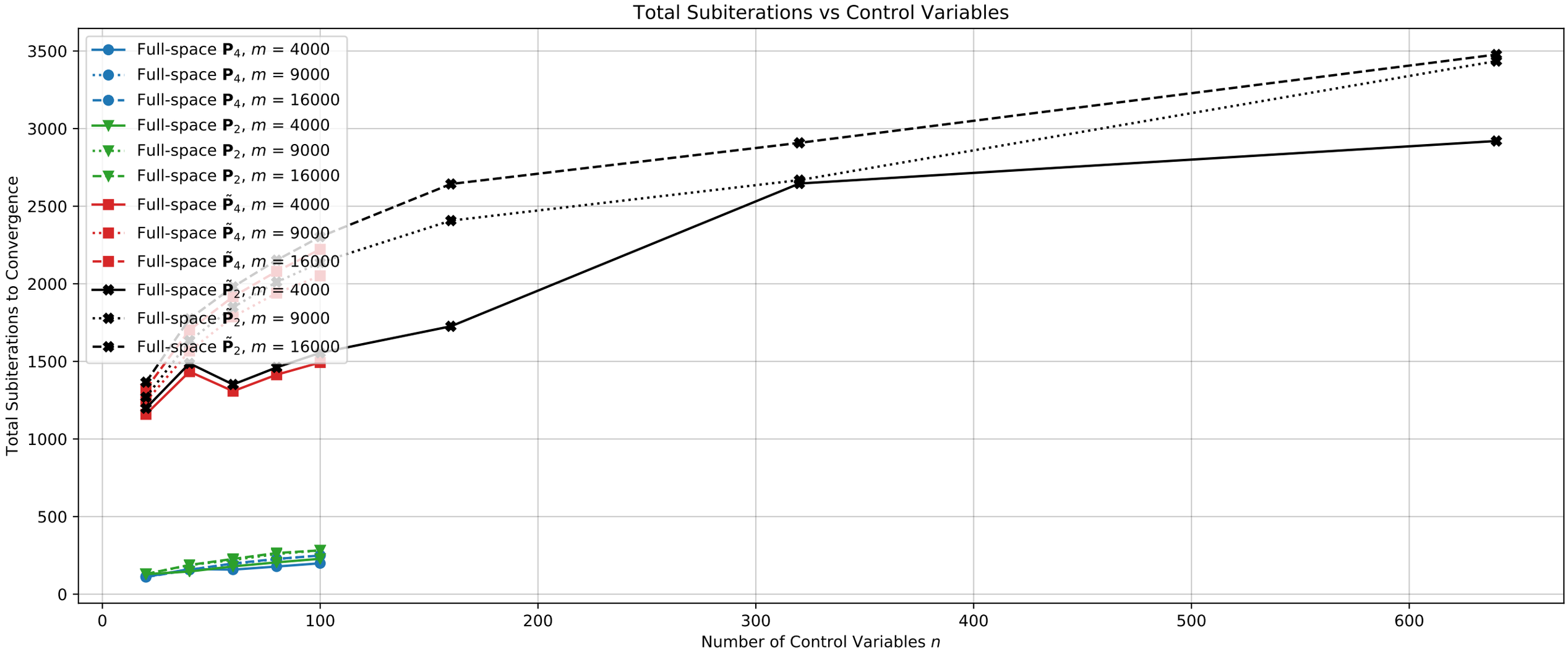
0
500
1000
1500
2000

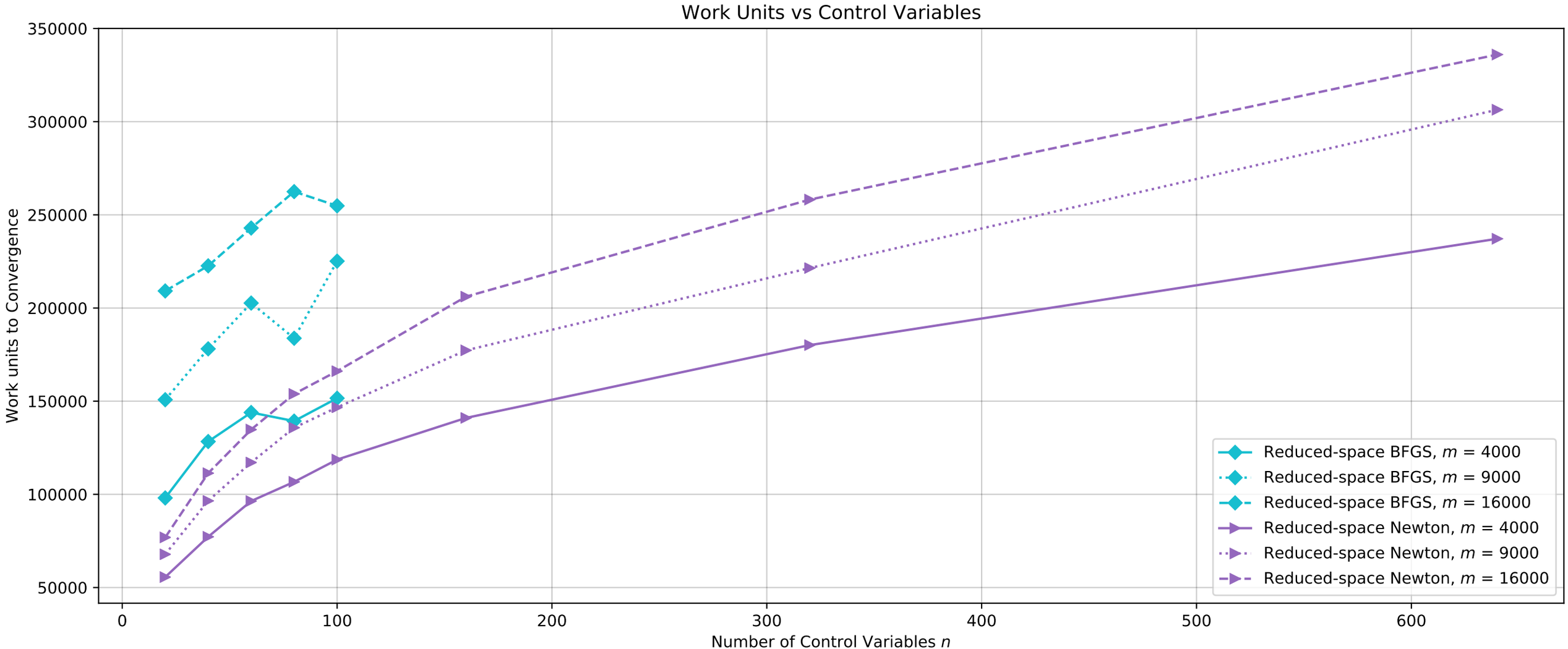
Number of subiterations

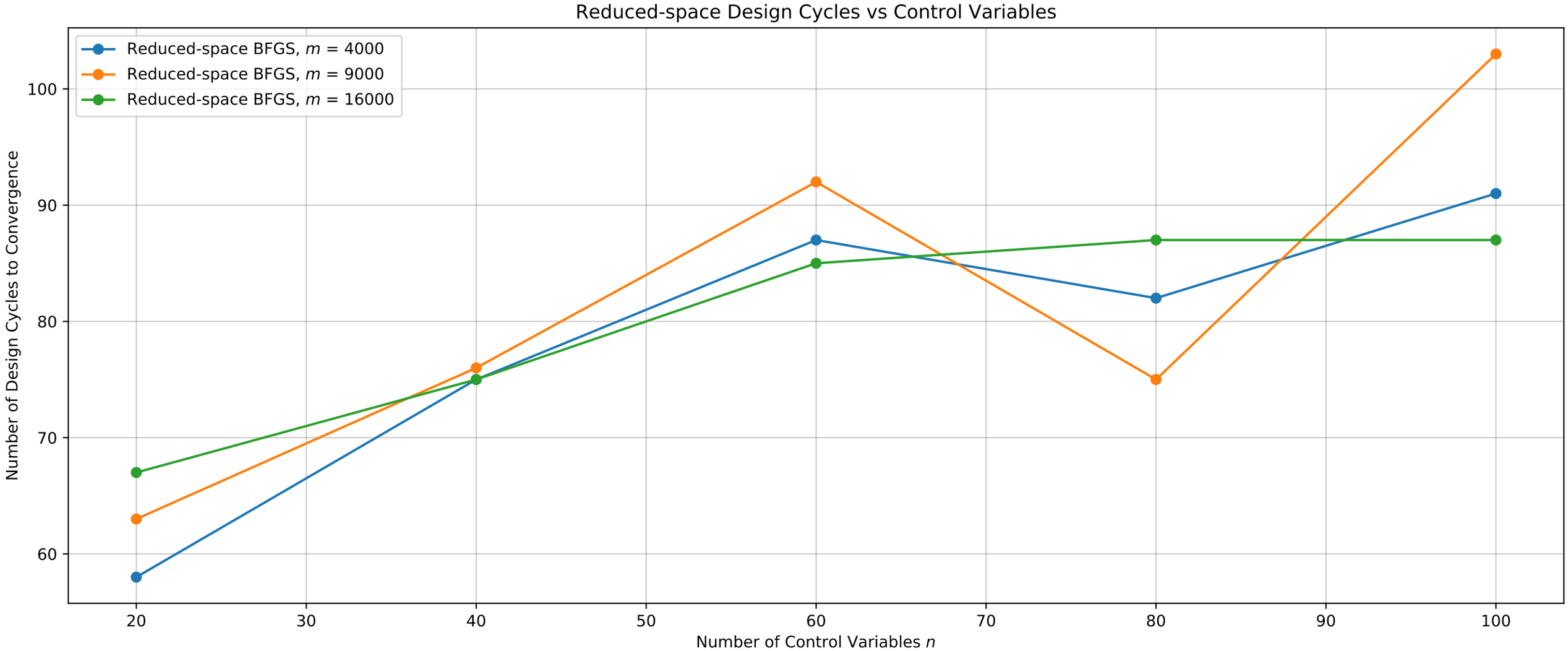
$\mathbf{P}_4$, $n = 20$
$\mathbf{P}_4$, $n = 60$
$\mathbf{P}_4$, $n = 100$
$\mathbf{P}_2$, $n = 20$
$\mathbf{P}_2$, $n = 60$
$\mathbf{P}_2$, $n = 100$
$\tilde{\mathbf{P}}_4$, $n = 20$
$\tilde{\mathbf{P}}_4$, $n = 60$
$\tilde{\mathbf{P}}_4$, $n = 100$
$\tilde{\mathbf{P}}_2$, $n = 20$
$\tilde{\mathbf{P}}_2$, $n = 60$
$\tilde{\mathbf{P}}_2$, $n = 100$


Full-space Design Cycles vs Control Variables
Number of Design Cycles to Convergence
Number of Control Variables n
5
4
3
0
100
200
300
400
500
600
Full-space $\mathbf{P}_4$, m = 4000
Full-space $\mathbf{P}_4$, m = 9000
Full-space $\mathbf{P}_4$, m = 16000
Full-space $\mathbf{P}_2$, m = 4000
Full-space $\mathbf{P}_2$, m = 9000
Full-space $\mathbf{P}_2$, m = 16000
Full-space $\tilde{\mathbf{P}}_4$, m = 4000
Full-space $\tilde{\mathbf{P}}_4$, m = 9000
Full-space $\tilde{\mathbf{P}}_4$, m = 16000
Full-space $\tilde{\mathbf{P}}_2$, m = 4000
Full-space $\tilde{\mathbf{P}}_2$, m = 9000
Full-space $\tilde{\mathbf{P}}_2$, m = 16000


Work Units vs Control Variables
Work Units to Convergence
Number of Control Variables n
500000
400000
300000
200000
100000
0
100
200
300
400
500
600
Full-space $\mathbf{P}_4$, m = 4000
Full-space $\mathbf{P}_4$, m = 9000
Full-space $\mathbf{P}_4$, m = 16000
Full-space $\mathbf{P}_2$, m = 4000
Full-space $\mathbf{P}_2$, m = 9000
Full-space $\mathbf{P}_2$, m = 16000
Full-space $\tilde{\mathbf{P}}_4$, m = 4000
Full-space $\tilde{\mathbf{P}}_4$, m = 9000
Full-space $\tilde{\mathbf{P}}_4$, m = 16000
Full-space $\tilde{\mathbf{P}}_2$, m = 4000
Full-space $\tilde{\mathbf{P}}_2$, m = 9000
Full-space $\tilde{\mathbf{P}}_2$, m = 16000


Total Subiterations vs Control Variables
Total Subiterations to Convergence
Number of Control Variables n
0
500
1000
1500
2000
2500
3000
3500
0
100
200
300
400
500
600
Full-space $\mathbf{P}_4$, $m = 4000$
Full-space $\mathbf{P}_4$, $m = 9000$
Full-space $\mathbf{P}_4$, $m = 16000$
Full-space $\mathbf{P}_2$, $m = 4000$
Full-space $\mathbf{P}_2$, $m = 9000$
Full-space $\mathbf{P}_2$, $m = 16000$
Full-space $\tilde{\mathbf{P}}_4$, $m = 4000$
Full-space $\tilde{\mathbf{P}}_4$, $m = 9000$
Full-space $\tilde{\mathbf{P}}_4$, $m = 16000$
Full-space $\tilde{\mathbf{P}}_2$, $m = 4000$
Full-space $\tilde{\mathbf{P}}_2$, $m = 9000$
Full-space $\tilde{\mathbf{P}}_2$, $m = 16000$


Work Units vs Control Variables
Work units to Convergence
Number of Control Variables n
350000
300000
250000
200000
150000
100000
50000
0
100
200
300
400
500
600
Reduced-space BFGS, m = 4000
Reduced-space BFGS, m = 9000
Reduced-space BFGS, m = 16000
Reduced-space Newton, m = 4000
Reduced-space Newton, m = 9000
Reduced-space Newton, m = 16000


Reduced-space Design Cycles vs Control Variables
Reduced-space BFGS, m = 4000
Reduced-space BFGS, m = 9000
Reduced-space BFGS, m = 16000
Number of Design Cycles to Convergence
Number of Control Variables n
60
70
80
90
100
20
30
40
50
60
70
80
90
100

Reduced-space Gradient vs Design Cycles

m = 4000

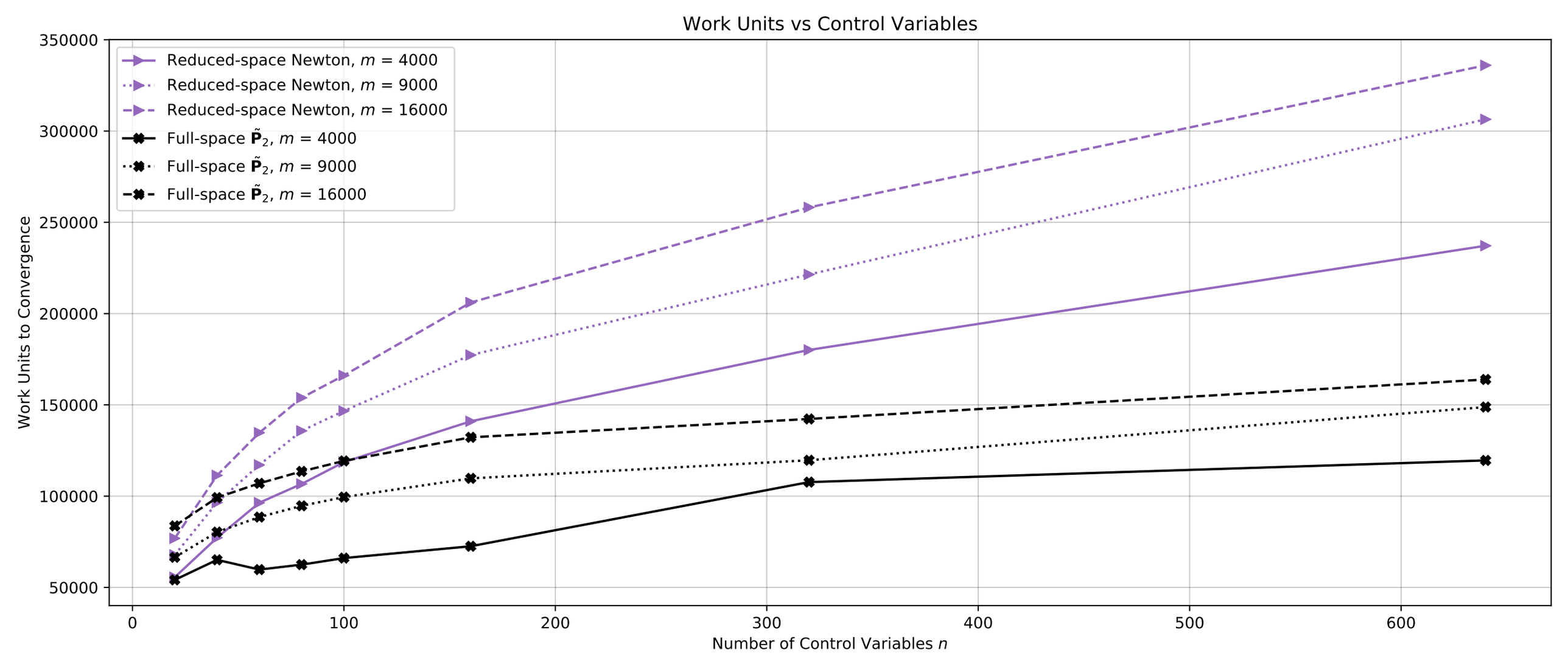
m = 9000

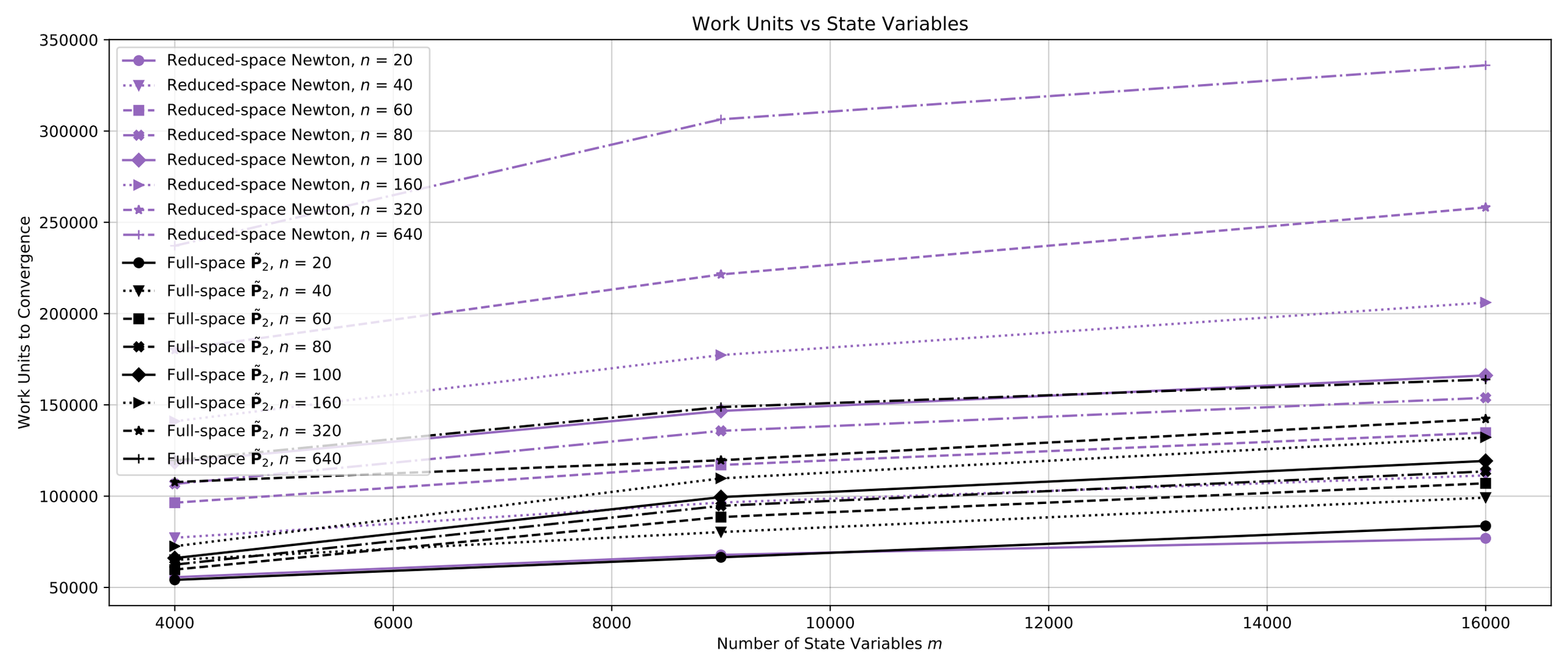
m = 16000

Gradient Norm

Design Cycles

BFGS, n = 20
BFGS, n = 40
BFGS, n = 60
BFGS, n = 80
BFGS, n = 100
Newton, n = 20
Newton, n = 40
Newton, n = 60
Newton, n = 80
Newton, n = 100


Work Units vs Control Variables
Reduced-space Newton, m = 4000
Reduced-space Newton, m = 9000
Reduced-space Newton, m = 16000
Full-space $\tilde{\mathbf{P}}_2$, m = 4000
Full-space $\tilde{\mathbf{P}}_2$, m = 9000
Full-space $\tilde{\mathbf{P}}_2$, m = 16000
Work Units to Convergence
Number of Control Variables n
350000
300000
250000
200000
150000
100000
50000
0
100
200
300
400
500
600


Work Units vs State Variables
Work Units to Convergence
Number of State Variables m
350000
300000
250000
200000
150000
100000
50000
4000
6000
8000
10000
12000
14000
16000
Reduced-space Newton, n = 20
Reduced-space Newton, n = 40
Reduced-space Newton, n = 60
Reduced-space Newton, n = 80
Reduced-space Newton, n = 100
Reduced-space Newton, n = 160
Reduced-space Newton, n = 320
Reduced-space Newton, n = 640
Full-space $\tilde{\mathbf{P}}_2$, n = 20
Full-space $\tilde{\mathbf{P}}_2$, n = 40
Full-space $\tilde{\mathbf{P}}_2$, n = 60
Full-space $\tilde{\mathbf{P}}_2$, n = 80
Full-space $\tilde{\mathbf{P}}_2$, n = 100
Full-space $\tilde{\mathbf{P}}_2$, n = 160
Full-space $\tilde{\mathbf{P}}_2$, n = 320
Full-space $\tilde{\mathbf{P}}_2$, n = 640