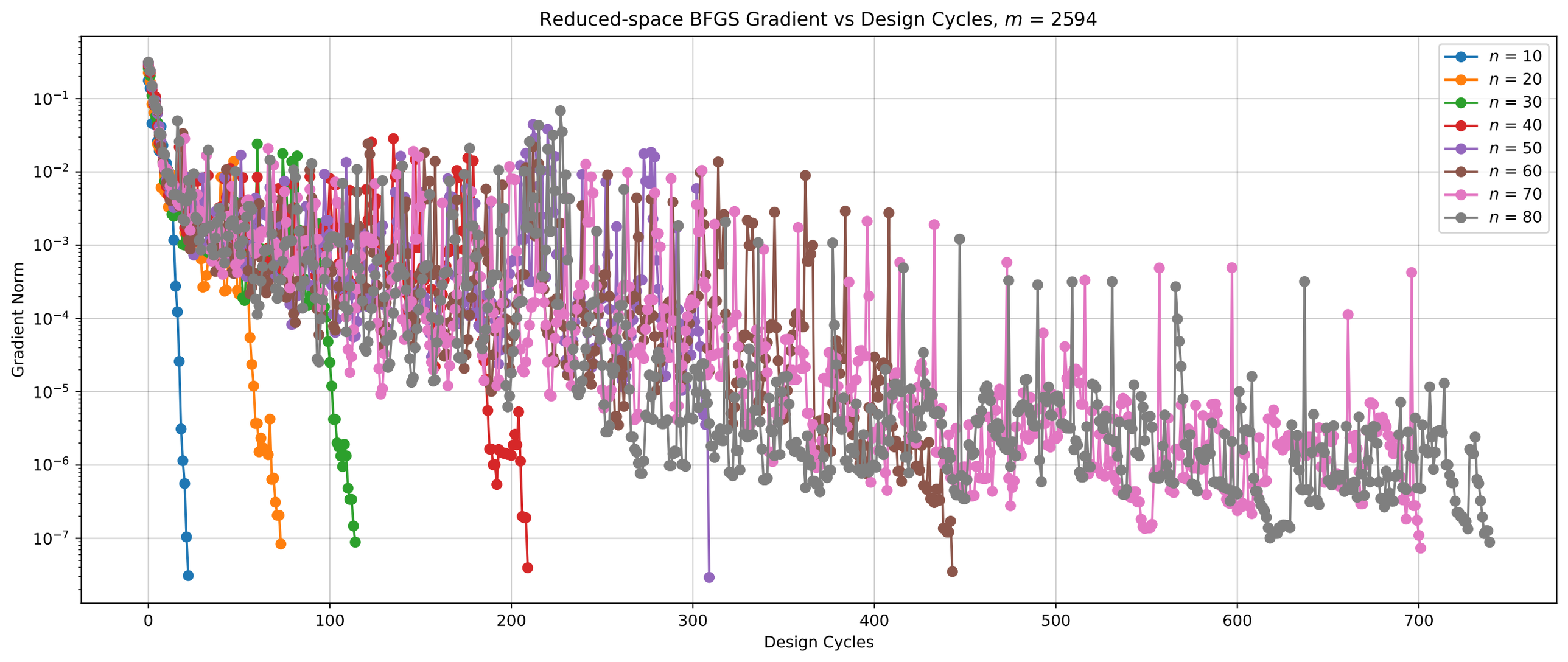
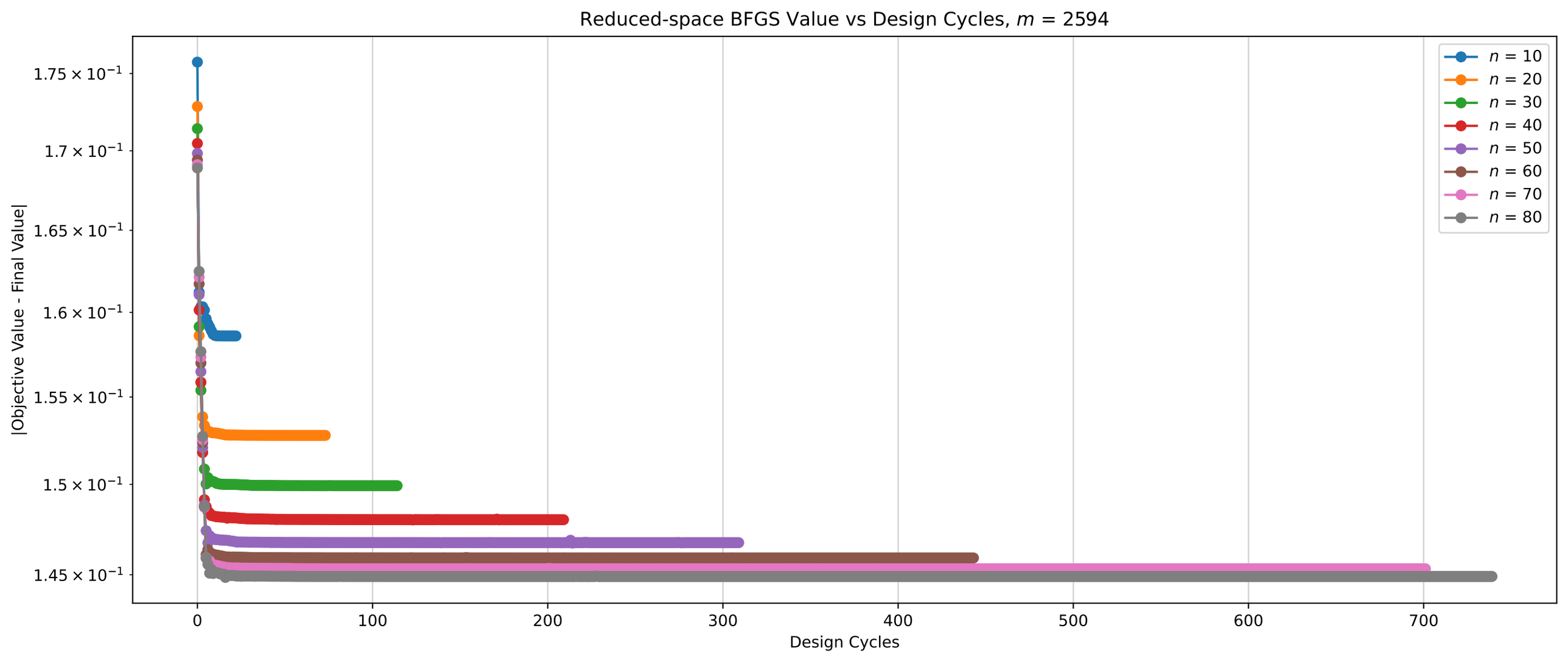
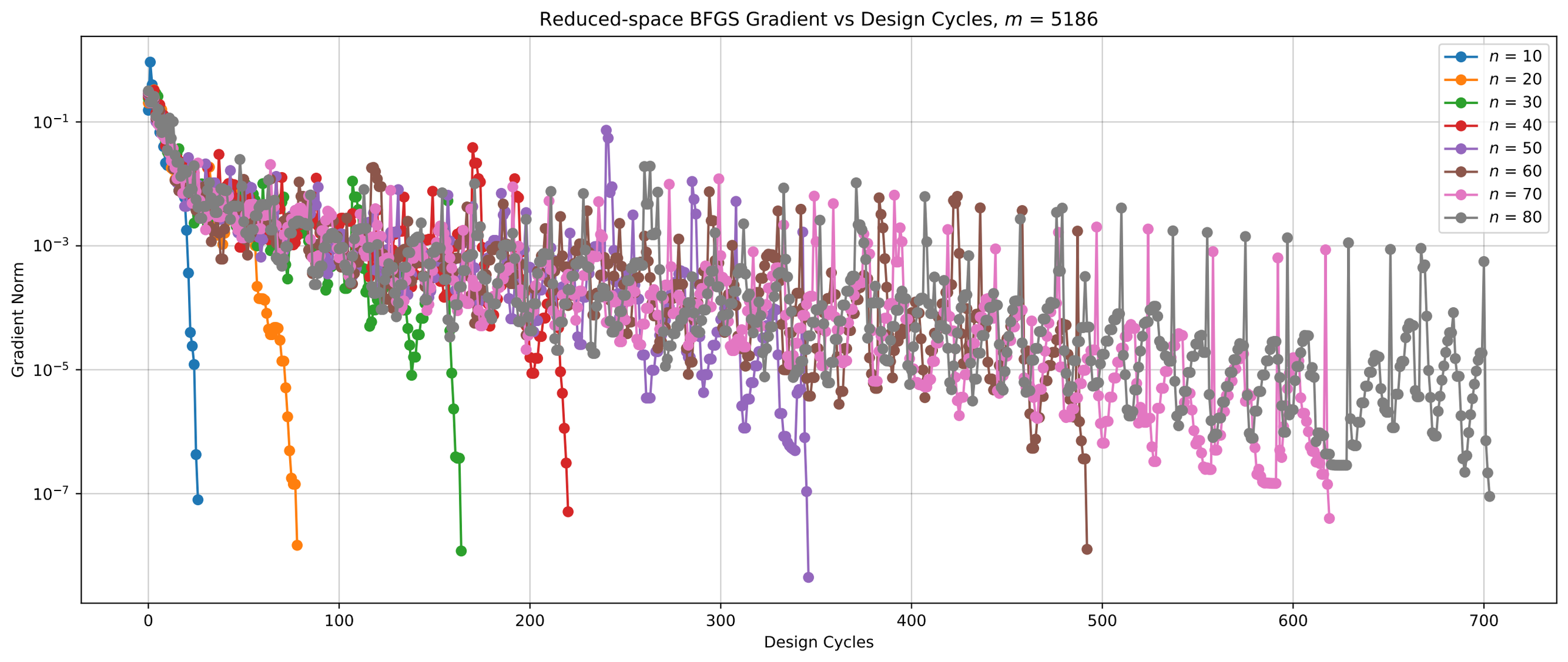
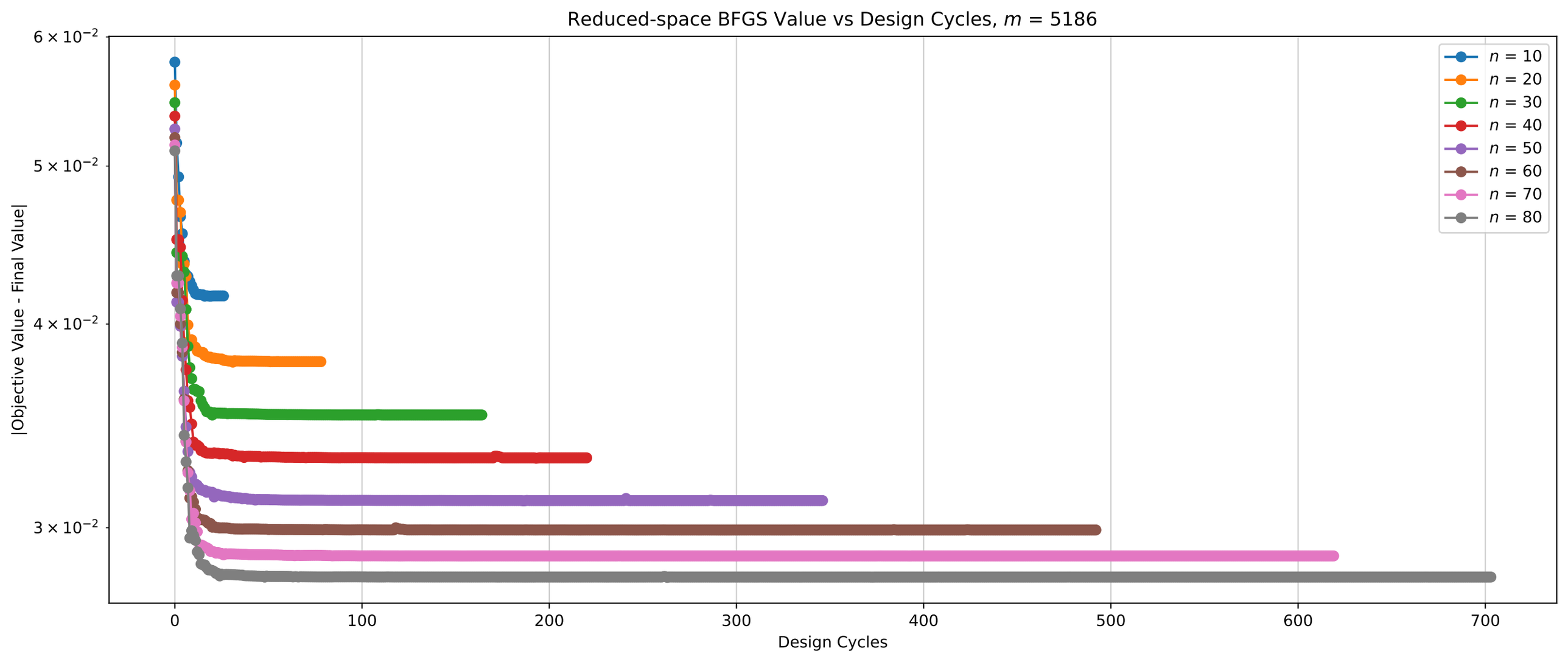
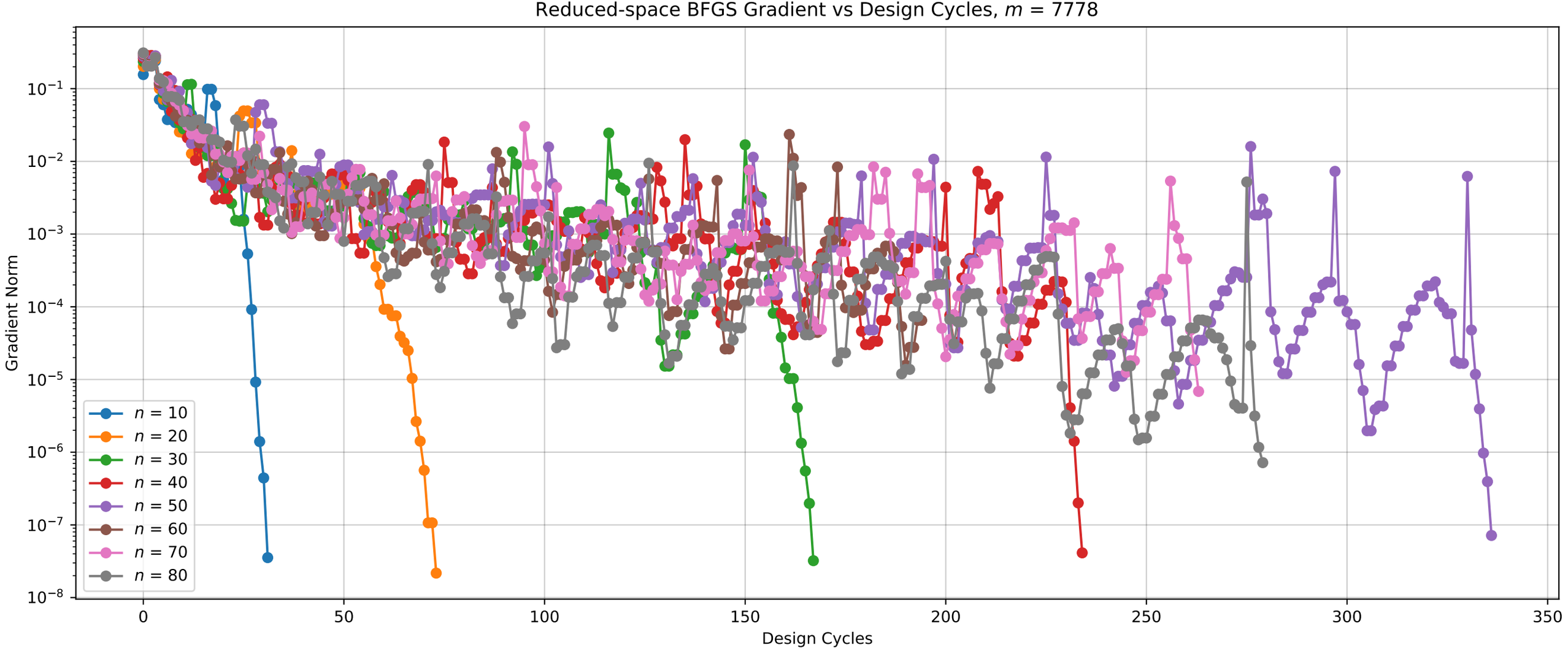
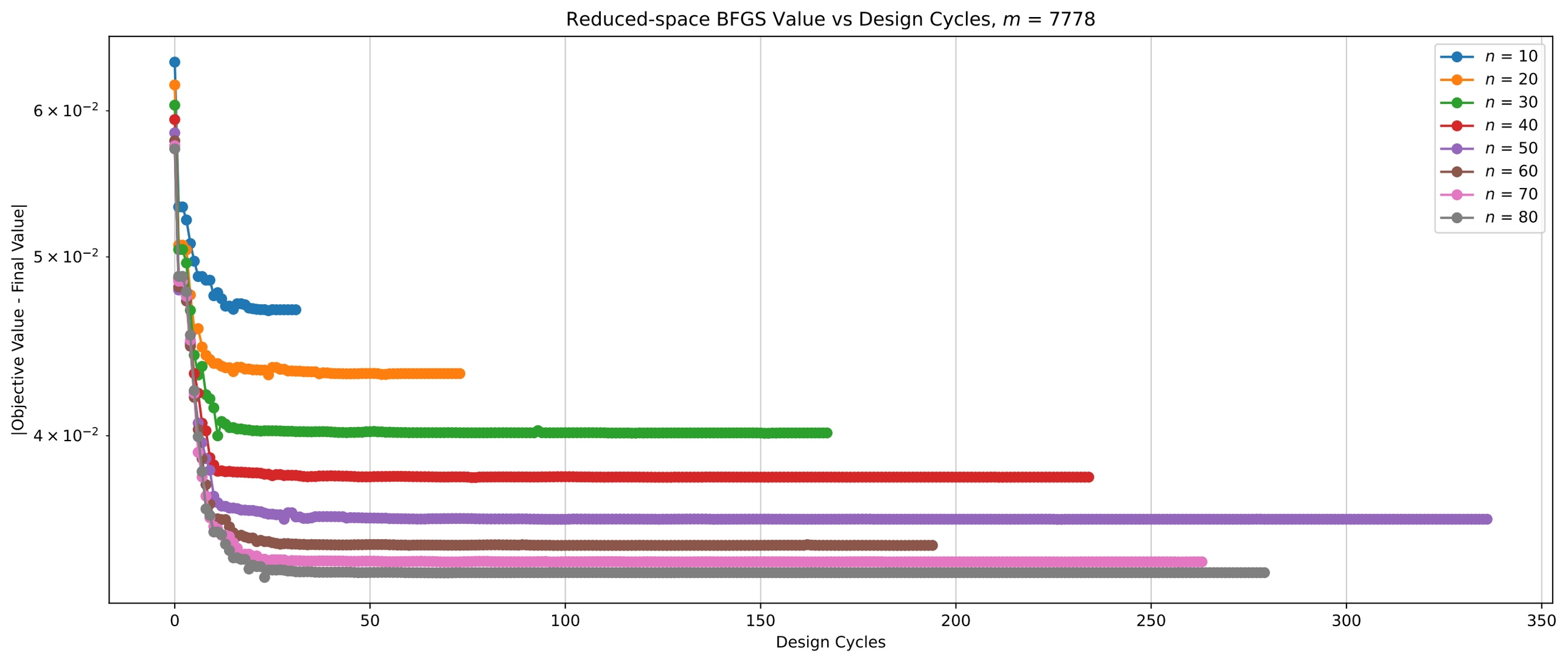

Reduced-space BFGS Gradient vs Design Cycles, m = 2594
Gradient Norm
Design Cycles
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
0
100
200
300
400
500
600
700
n = 10
n = 20
n = 30
n = 40
n = 50
n = 60
n = 70
n = 80

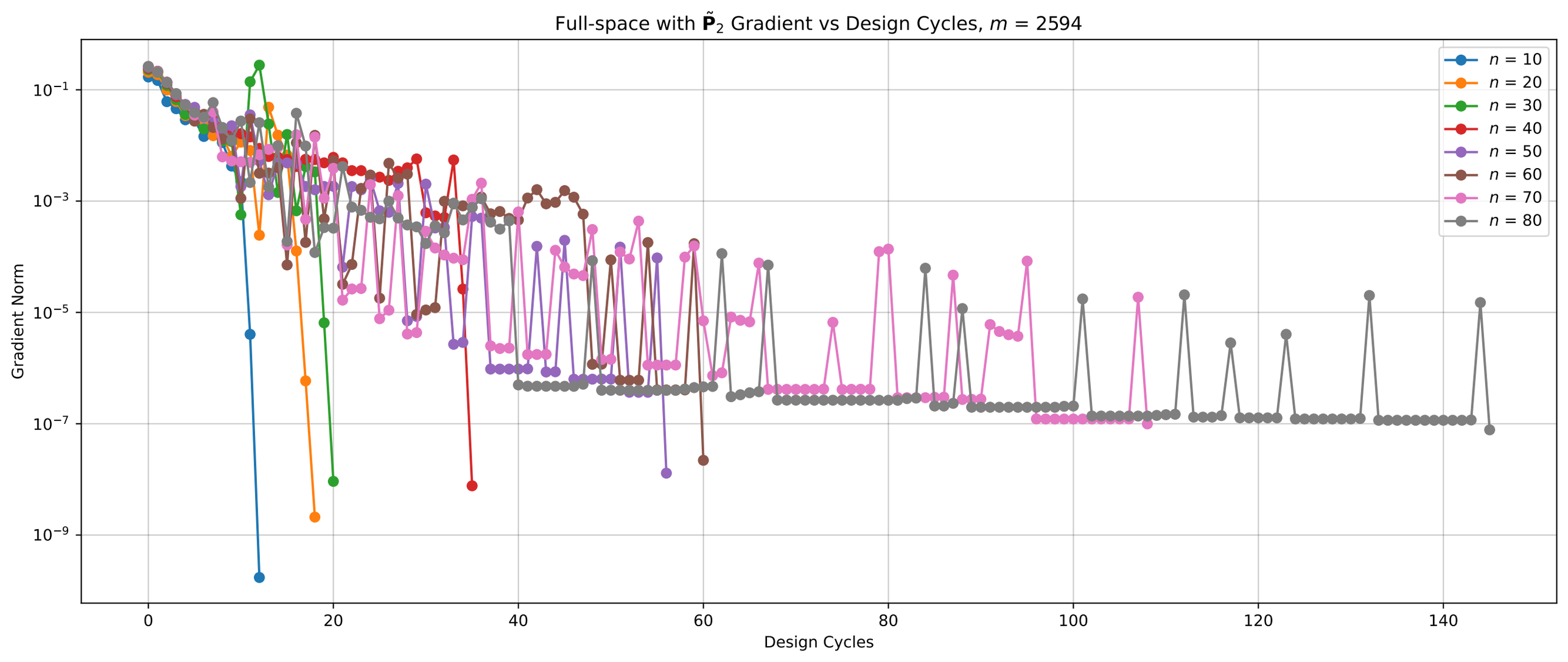

Reduced-space BFGS Value vs Design Cycles, m = 2594
|Objective Value - Final Value|
1.75×10^{-1}
1.7×10^{-1}
1.65×10^{-1}
1.6×10^{-1}
1.55×10^{-1}
1.5×10^{-1}
1.45×10^{-1}
0
100
200
300
400
500
600
700
Design Cycles
n = 10
n = 20
n = 30
n = 40
n = 50
n = 60
n = 70
n = 80

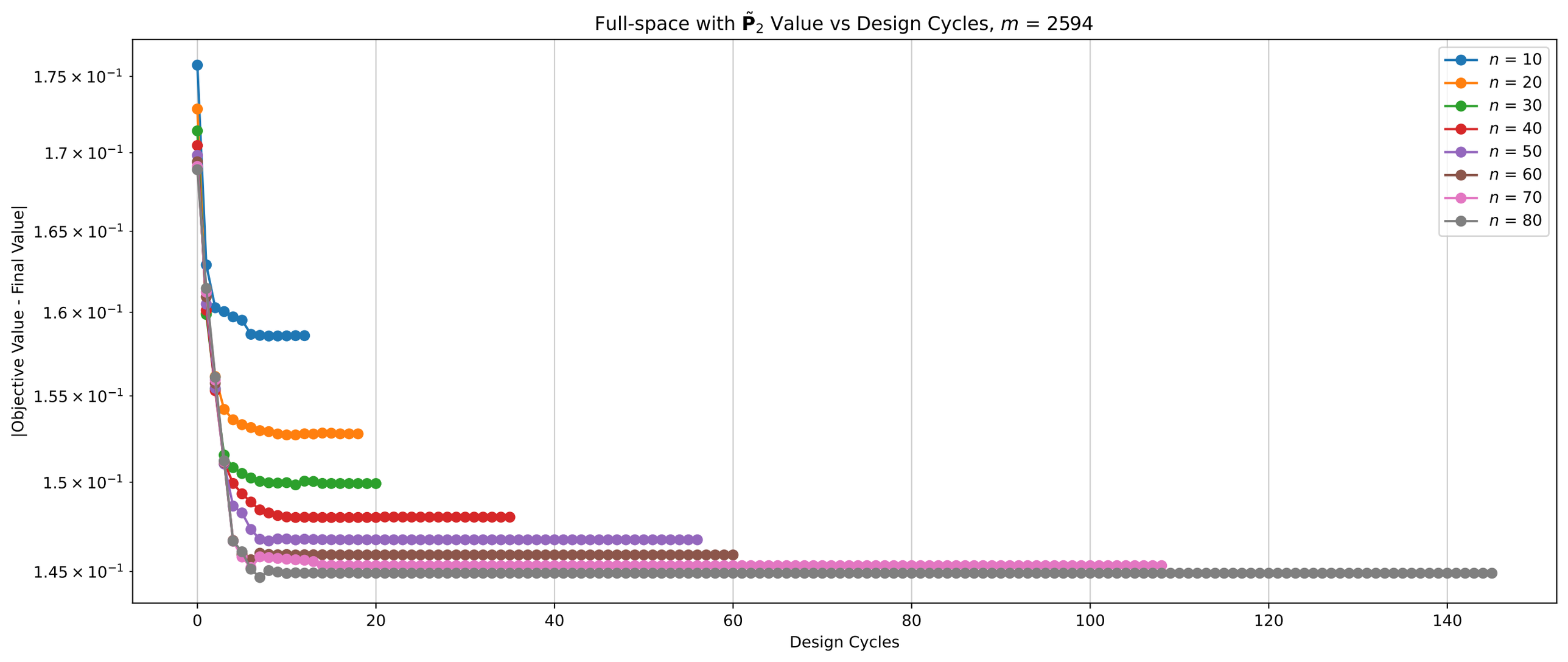

Reduced-space BFGS Gradient vs Design Cycles, m = 5186
Gradient Norm
10^{-1}
10^{-3}
10^{-5}
10^{-7}
0
100
200
300
400
500
600
700
Design Cycles
n = 10
n = 20
n = 30
n = 40
n = 50
n = 60
n = 70
n = 80

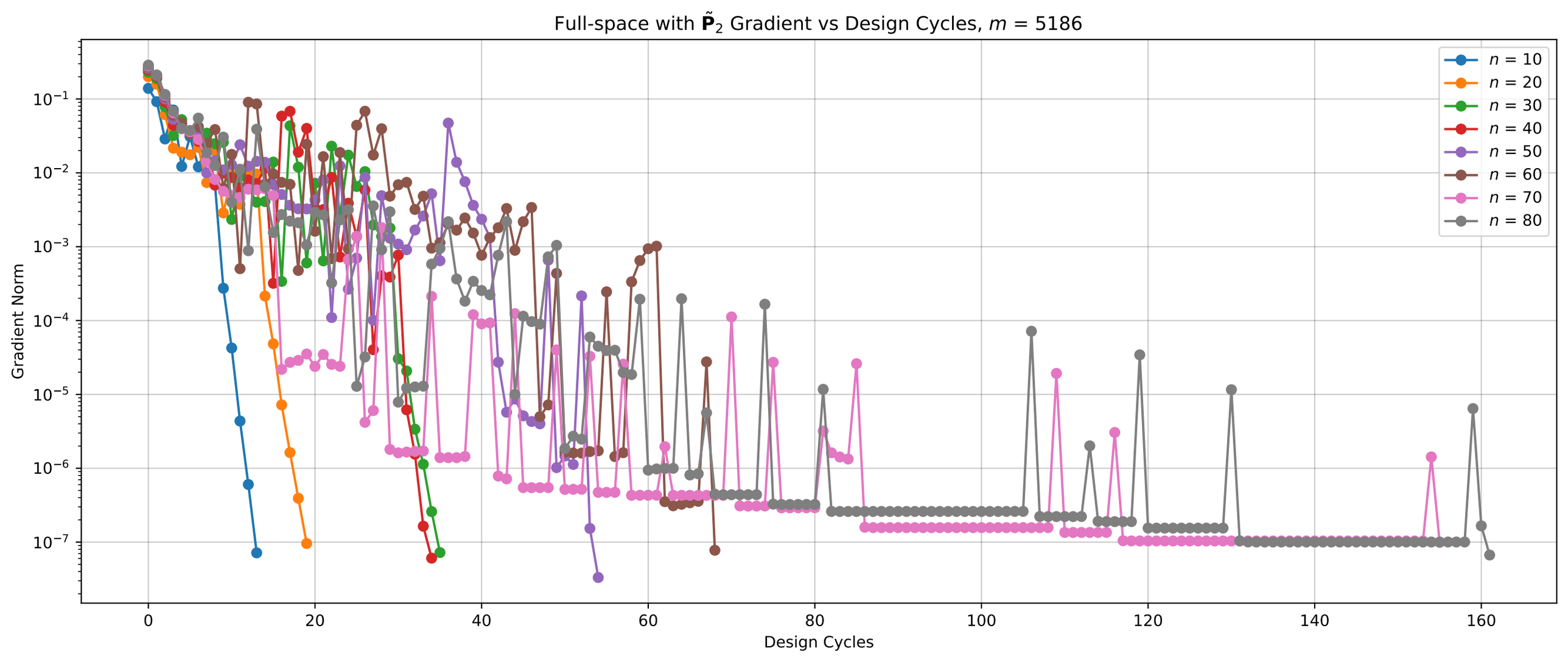

Reduced-space BFGS Value vs Design Cycles, m = 5186
6×10^{-2}
5×10^{-2}
4×10^{-2}
3×10^{-2}
|Objective Value - Final Value|
0
100
200
300
400
500
600
700
Design Cycles
n = 10
n = 20
n = 30
n = 40
n = 50
n = 60
n = 70
n = 80

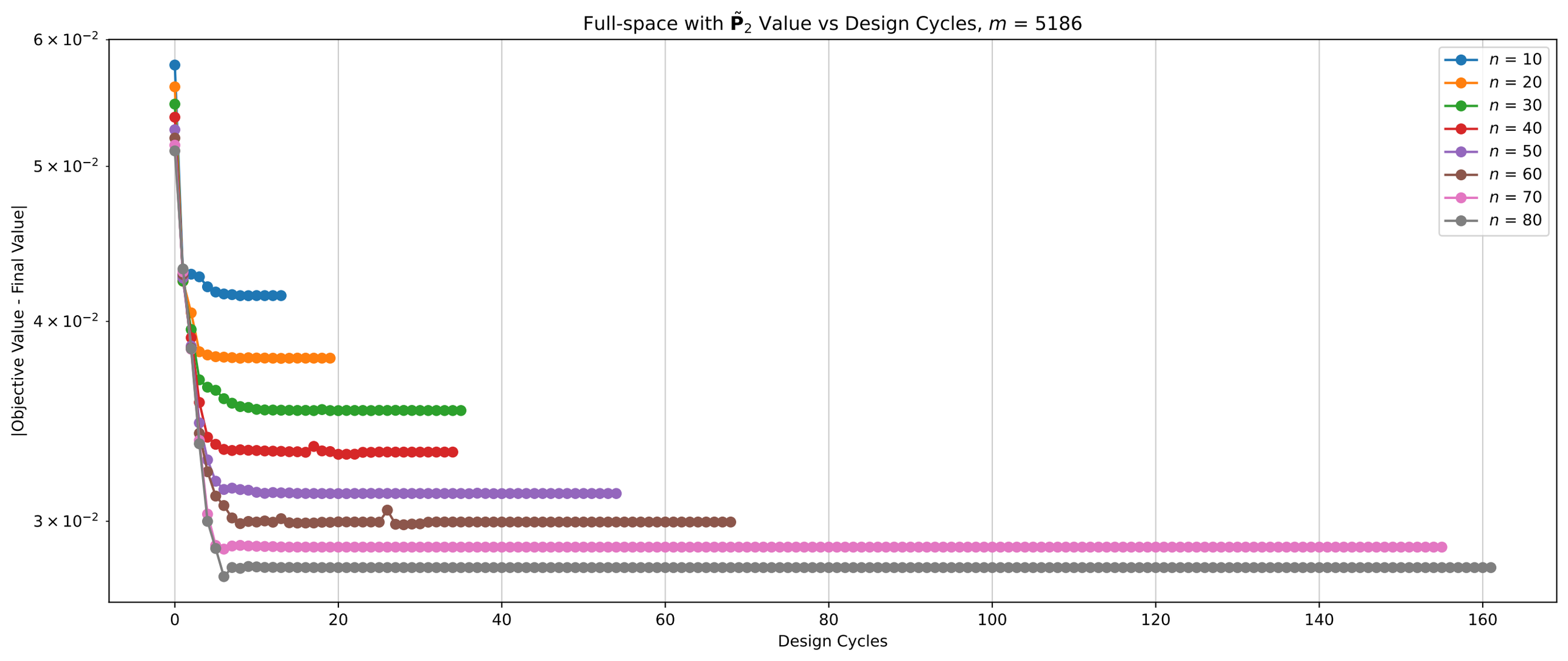

Reduced-space BFGS Gradient vs Design Cycles, m = 7778
Gradient Norm
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-8}
0
50
100
150
200
250
300
350
Design Cycles
n = 10
n = 20
n = 30
n = 40
n = 50
n = 60
n = 70
n = 80

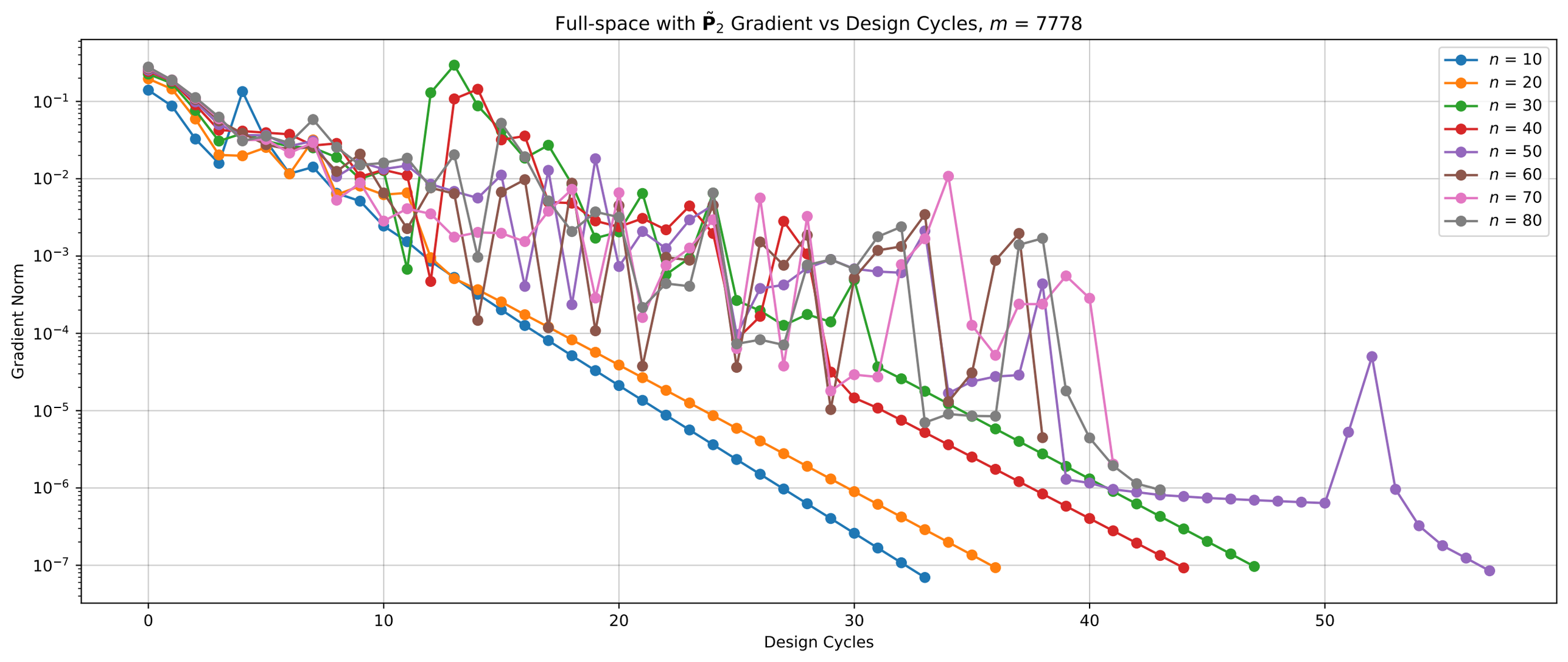

Reduced-space BFGS Value vs Design Cycles, m = 7778
|Objective Value - Final Value|
6×10^{-2}
5×10^{-2}
4×10^{-2}
0
50
100
150
200
250
300
350
Design Cycles
n = 10
n = 20
n = 30
n = 40
n = 50
n = 60
n = 70
n = 80

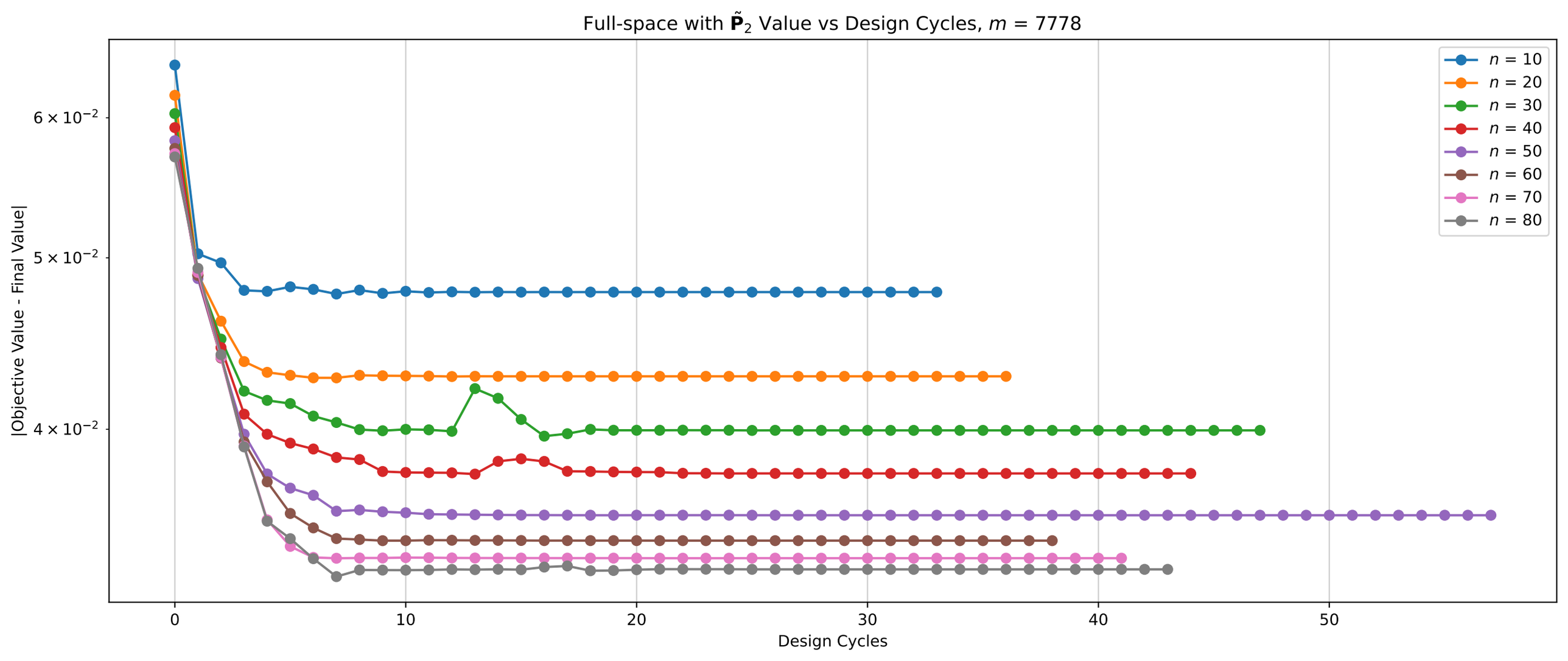

Full-space with $\tilde{\mathbf{P}}_2$ Gradient vs Design Cycles, m = 2594
Gradient Norm
Design Cycles
10^{-1}
10^{-3}
10^{-5}
10^{-7}
10^{-9}
0
20
40
60
80
100
120
140
n = 10
n = 20
n = 30
n = 40
n = 50
n = 60
n = 70
n = 80

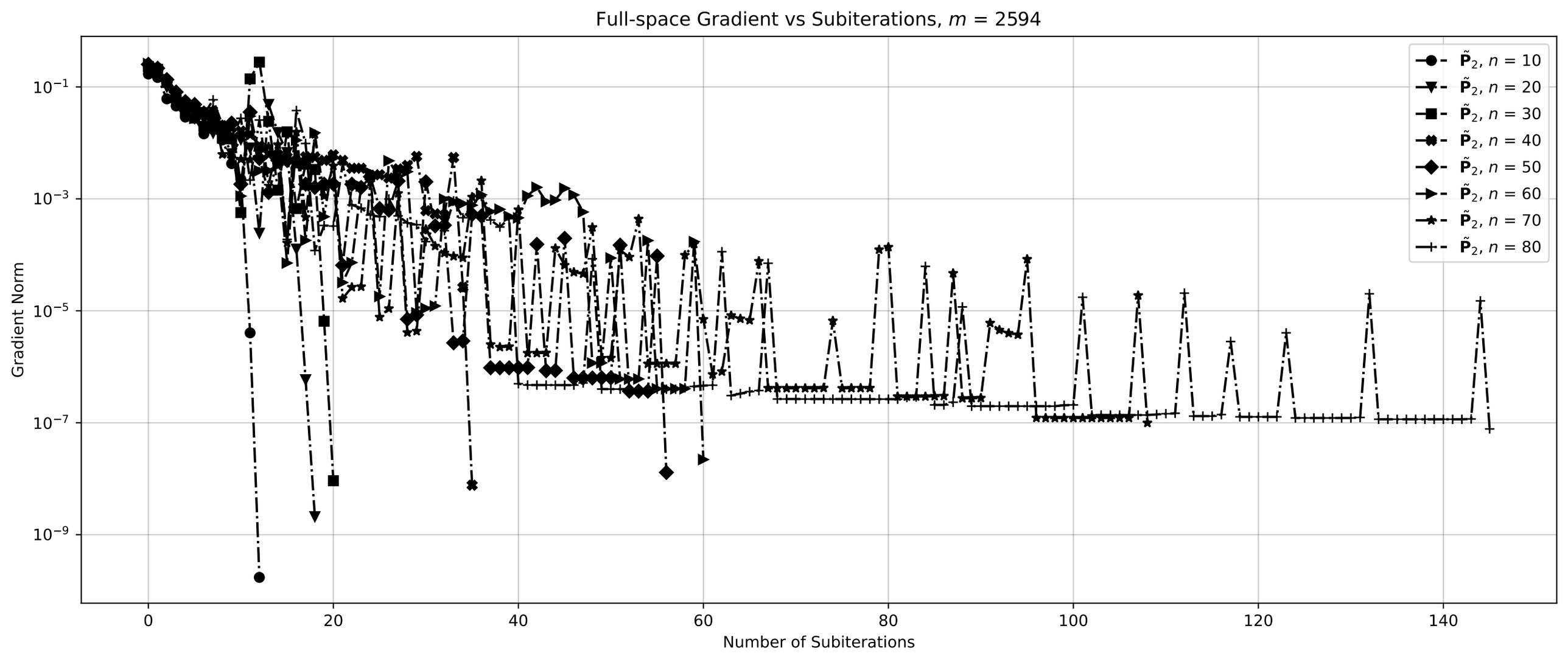

Full-space with $\tilde{\mathbf{P}}_2$ Value vs Design Cycles, m = 2594
|Objective Value - Final Value|
1.75×10^{-1}
1.7×10^{-1}
1.65×10^{-1}
1.6×10^{-1}
1.55×10^{-1}
1.5×10^{-1}
1.45×10^{-1}
0
20
40
60
80
100
120
140
Design Cycles
n = 10
n = 20
n = 30
n = 40
n = 50
n = 60
n = 70
n = 80

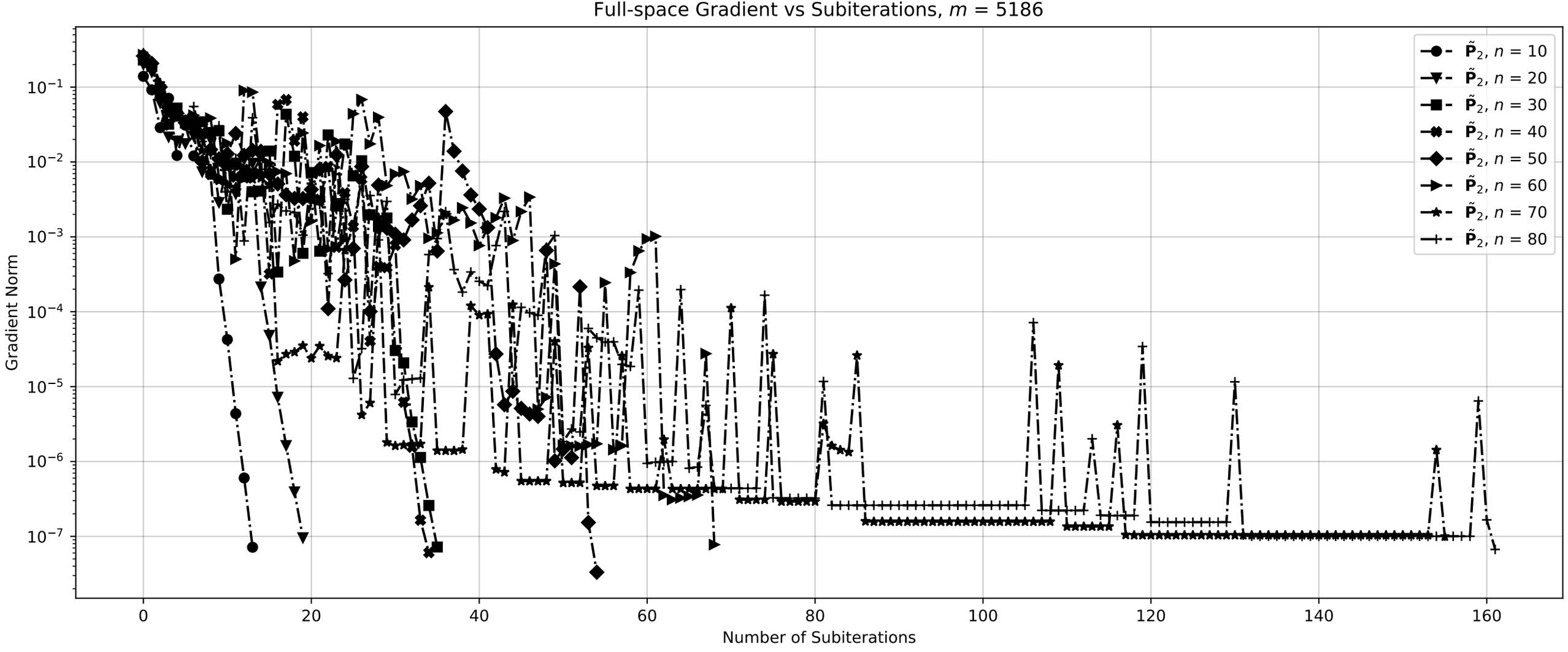

Full-space with $\tilde{\mathbf{P}}_2$ Gradient vs Design Cycles, m = 5186
Gradient Norm
Design Cycles
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
0
20
40
60
80
100
120
140
160
n = 10
n = 20
n = 30
n = 40
n = 50
n = 60
n = 70
n = 80

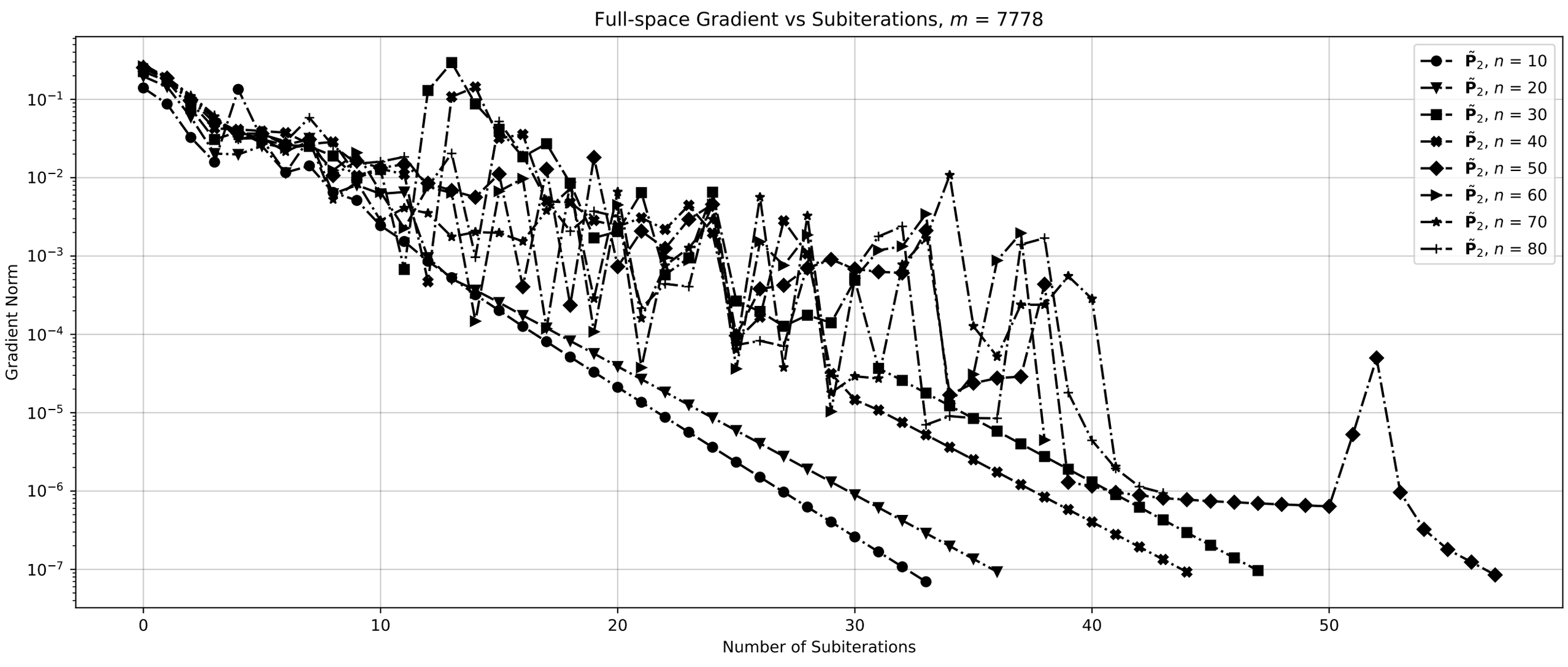

Full-space with $\tilde{\mathbf{P}}_2$ Value vs Design Cycles, m = 5186
6×10^{-2}
5×10^{-2}
4×10^{-2}
3×10^{-2}
|Objective Value - Final Value|
0
20
40
60
80
100
120
140
160
Design Cycles
n = 10
n = 20
n = 30
n = 40
n = 50
n = 60
n = 70
n = 80


Full-space with $\tilde{\mathbf{P}}_2$ Gradient vs Design Cycles, m = 7778
Gradient Norm
Design Cycles
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
0
10
20
30
40
50
n = 10
n = 20
n = 30
n = 40
n = 50
n = 60
n = 70
n = 80


Full-space with $\tilde{\mathbf{P}}_2$ Value vs Design Cycles, m = 7778
|Objective Value - Final Value|
6×10^{-2}
5×10^{-2}
4×10^{-2}
0
10
20
30
40
50
Design Cycles
n = 10
n = 20
n = 30
n = 40
n = 50
n = 60
n = 70
n = 80


Full-space Gradient vs Subiterations, m = 2594
Gradient Norm
Number of Subiterations
10^{-1}
10^{-3}
10^{-5}
10^{-7}
10^{-9}
0
20
40
60
80
100
120
140
$\tilde{\mathbf{P}}_2$, n = 10
$\tilde{\mathbf{P}}_2$, n = 20
$\tilde{\mathbf{P}}_2$, n = 30
$\tilde{\mathbf{P}}_2$, n = 40
$\tilde{\mathbf{P}}_2$, n = 50
$\tilde{\mathbf{P}}_2$, n = 60
$\tilde{\mathbf{P}}_2$, n = 70
$\tilde{\mathbf{P}}_2$, n = 80


Full-space Gradient vs Subiterations, m = 5186
Gradient Norm
Number of Subiterations
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
0
20
40
60
80
100
120
140
160
$\tilde{\mathbf{P}}_2$, n = 10
$\tilde{\mathbf{P}}_2$, n = 20
$\tilde{\mathbf{P}}_2$, n = 30
$\tilde{\mathbf{P}}_2$, n = 40
$\tilde{\mathbf{P}}_2$, n = 50
$\tilde{\mathbf{P}}_2$, n = 60
$\tilde{\mathbf{P}}_2$, n = 70
$\tilde{\mathbf{P}}_2$, n = 80


Full-space Gradient vs Subiterations, m = 7778
Gradient Norm
Number of Subiterations
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
0
10
20
30
40
50
$\tilde{\mathbf{P}}_2$, n = 10
$\tilde{\mathbf{P}}_2$, n = 20
$\tilde{\mathbf{P}}_2$, n = 30
$\tilde{\mathbf{P}}_2$, n = 40
$\tilde{\mathbf{P}}_2$, n = 50
$\tilde{\mathbf{P}}_2$, n = 60
$\tilde{\mathbf{P}}_2$, n = 70
$\tilde{\mathbf{P}}_2$, n = 80

Full-space Gradient vs Subiterations

m = 2594

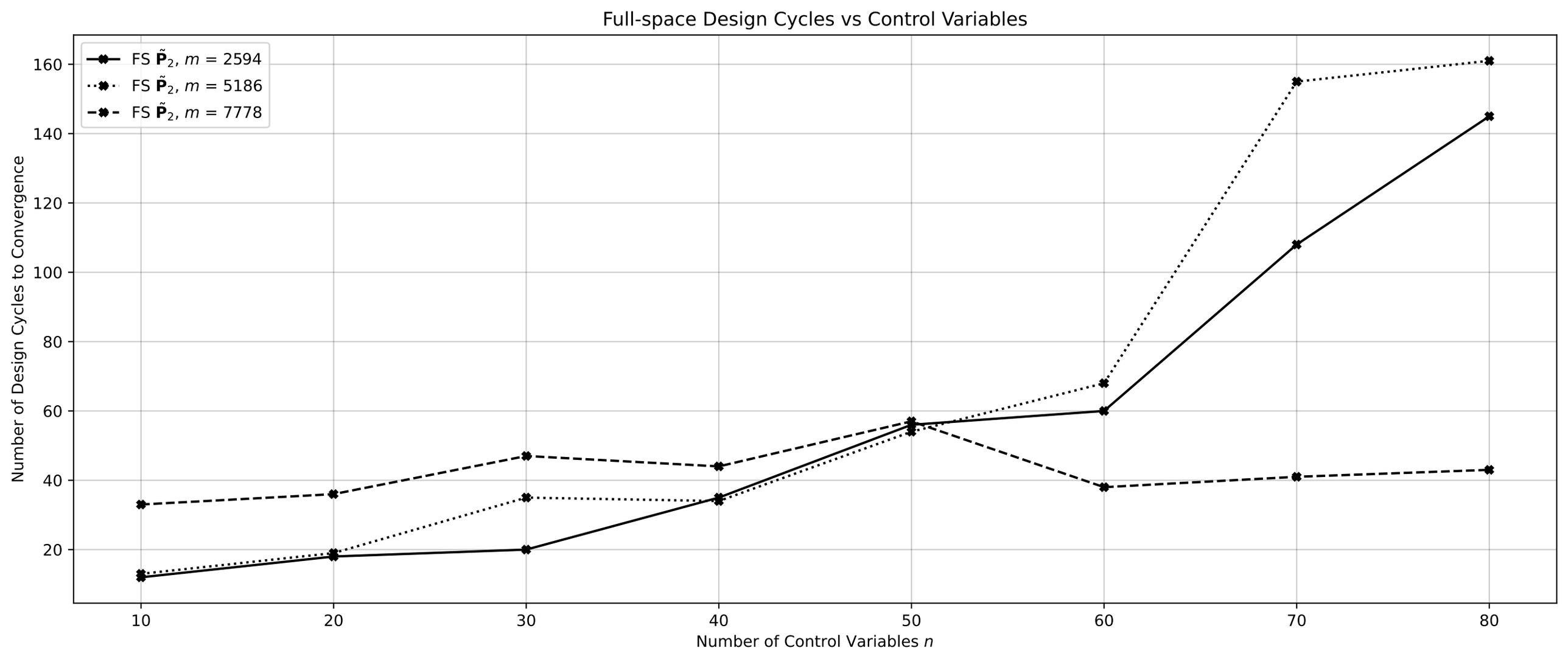
m = 5186

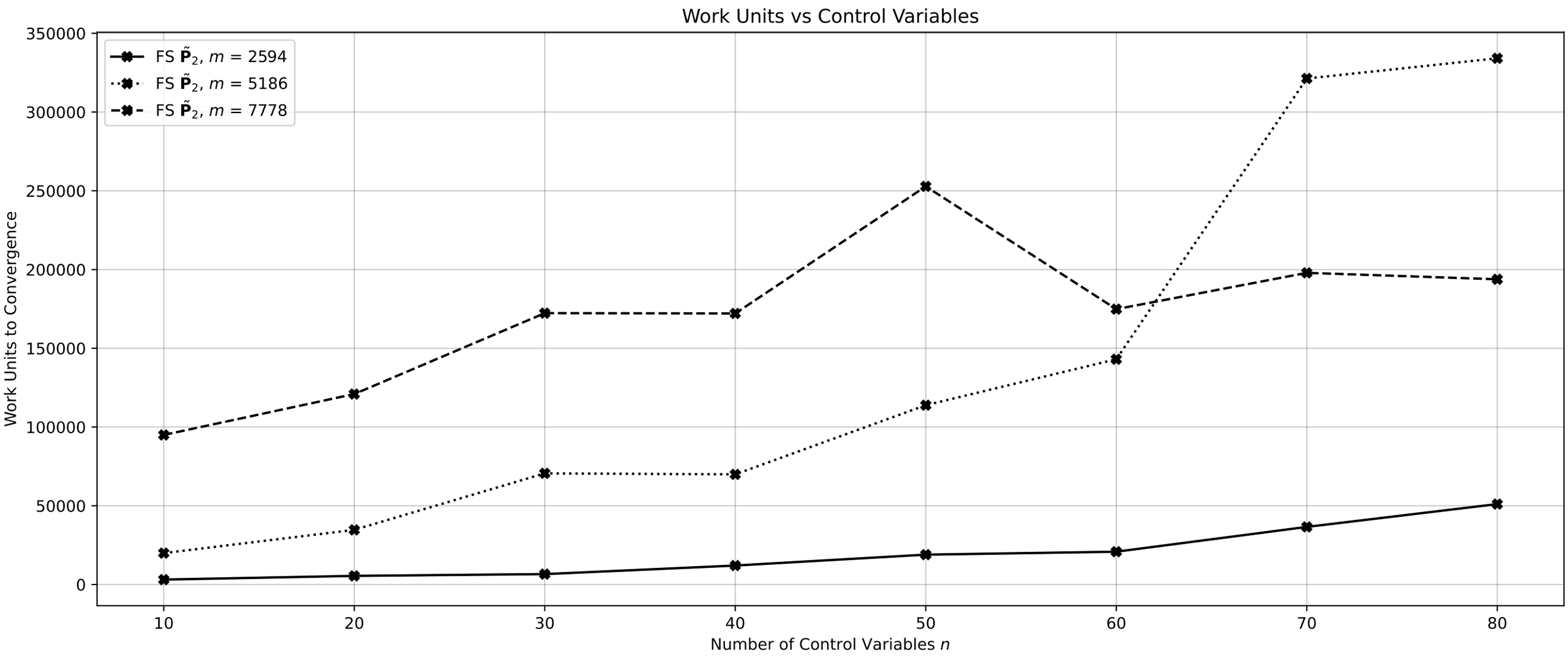
m = 7778

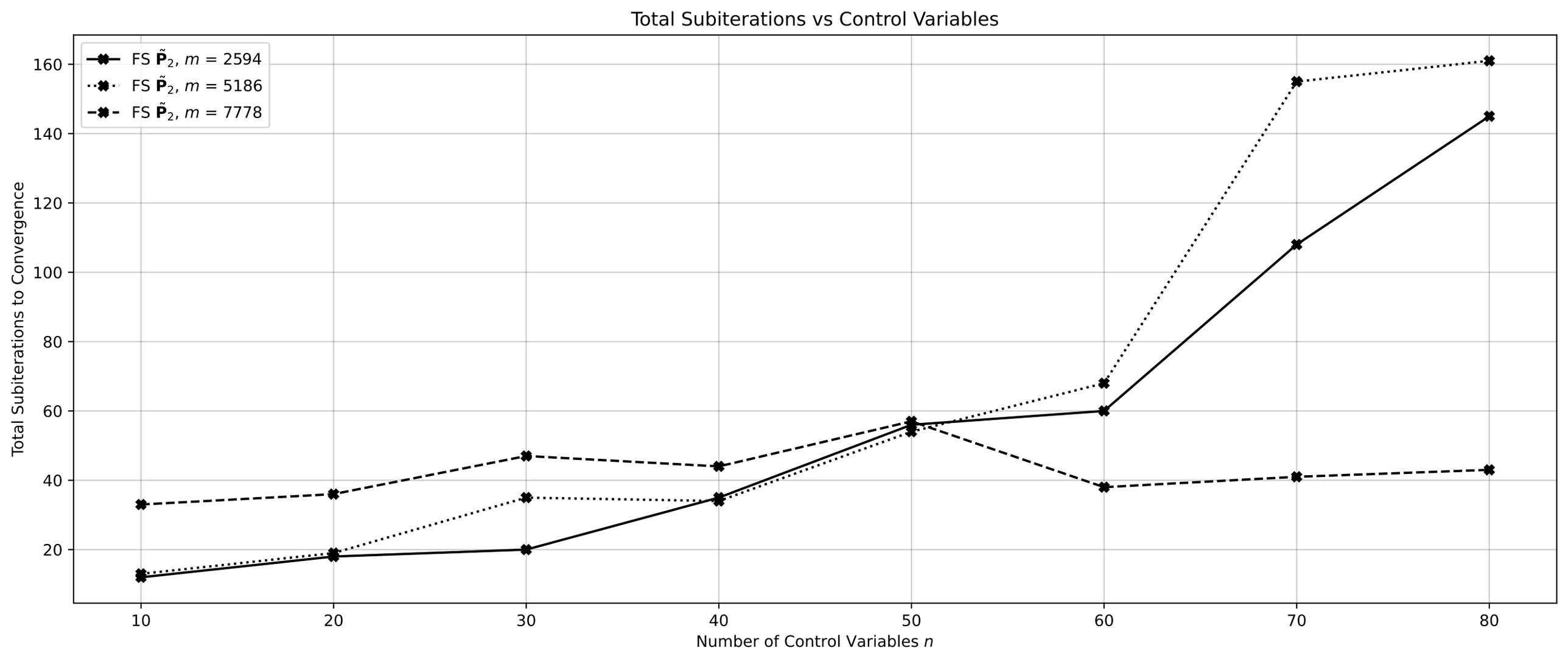
Gradient Norm

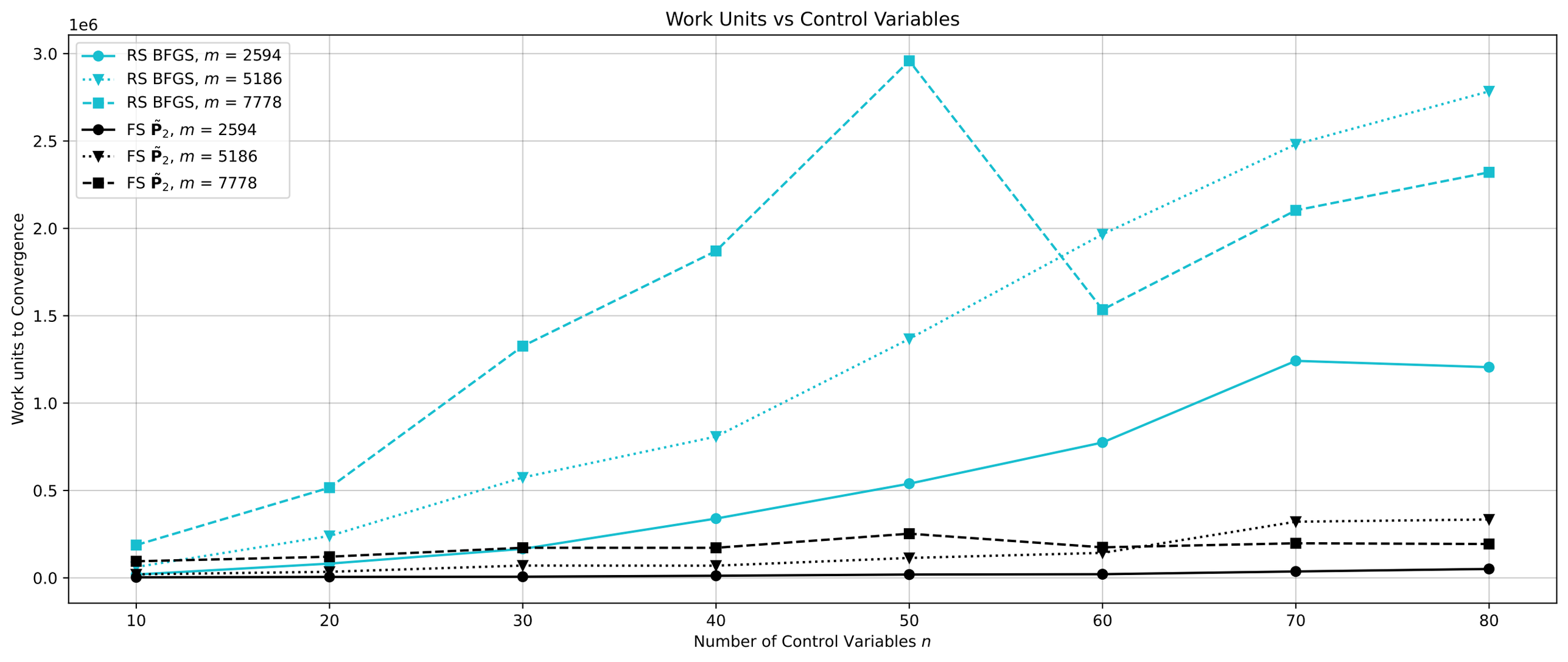
Number of subiterations

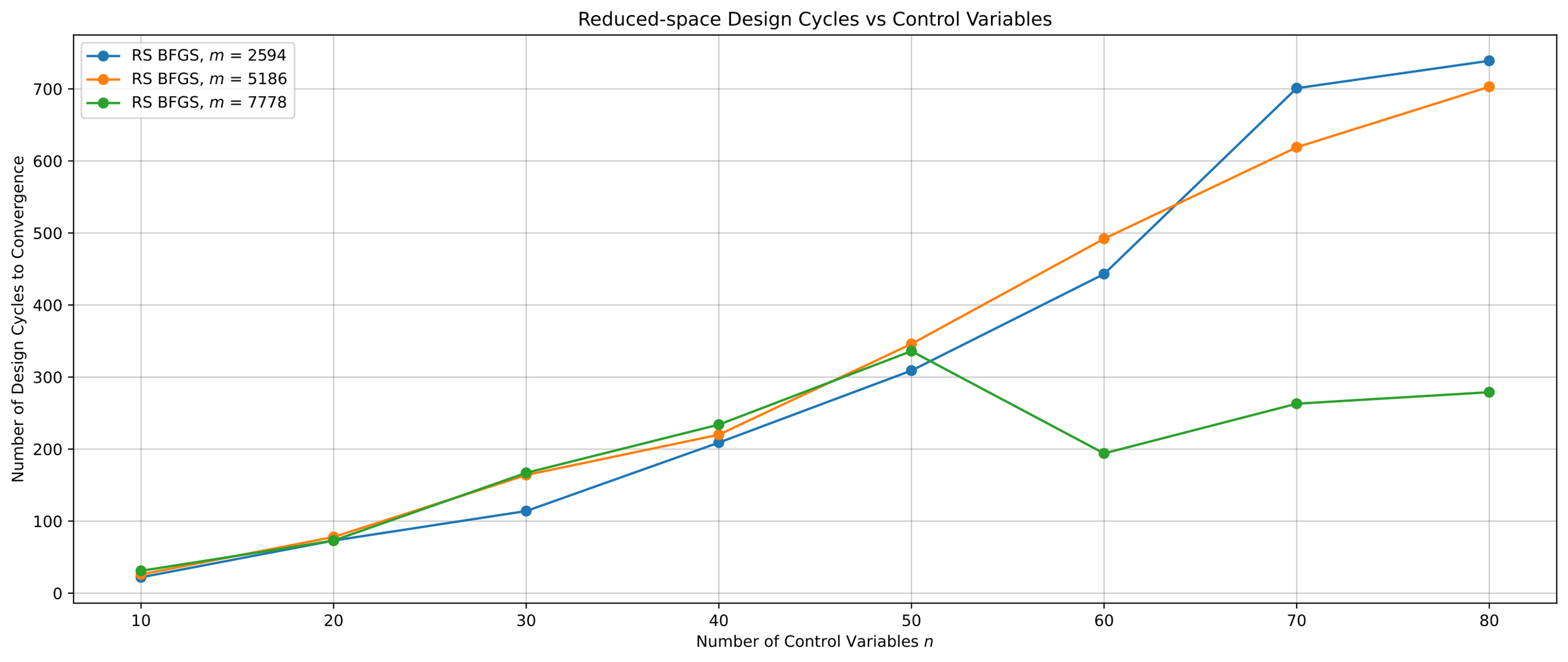
$\tilde{\mathbf{P}}_2$, n = 10
$\tilde{\mathbf{P}}_2$, n = 20
$\tilde{\mathbf{P}}_2$, n = 30
$\tilde{\mathbf{P}}_2$, n = 40
$\tilde{\mathbf{P}}_2$, n = 50
$\tilde{\mathbf{P}}_2$, n = 60
$\tilde{\mathbf{P}}_2$, n = 70
$\tilde{\mathbf{P}}_2$, n = 80


Full-space Design Cycles vs Control Variables
FS $\tilde{\mathbf{P}}_2$, m = 2594
FS $\tilde{\mathbf{P}}_2$, m = 5186
FS $\tilde{\mathbf{P}}_2$, m = 7778
Number of Design Cycles to Convergence
Number of Control Variables n
20
40
60
80
100
120
140
160
10
20
30
40
50
60
70
80

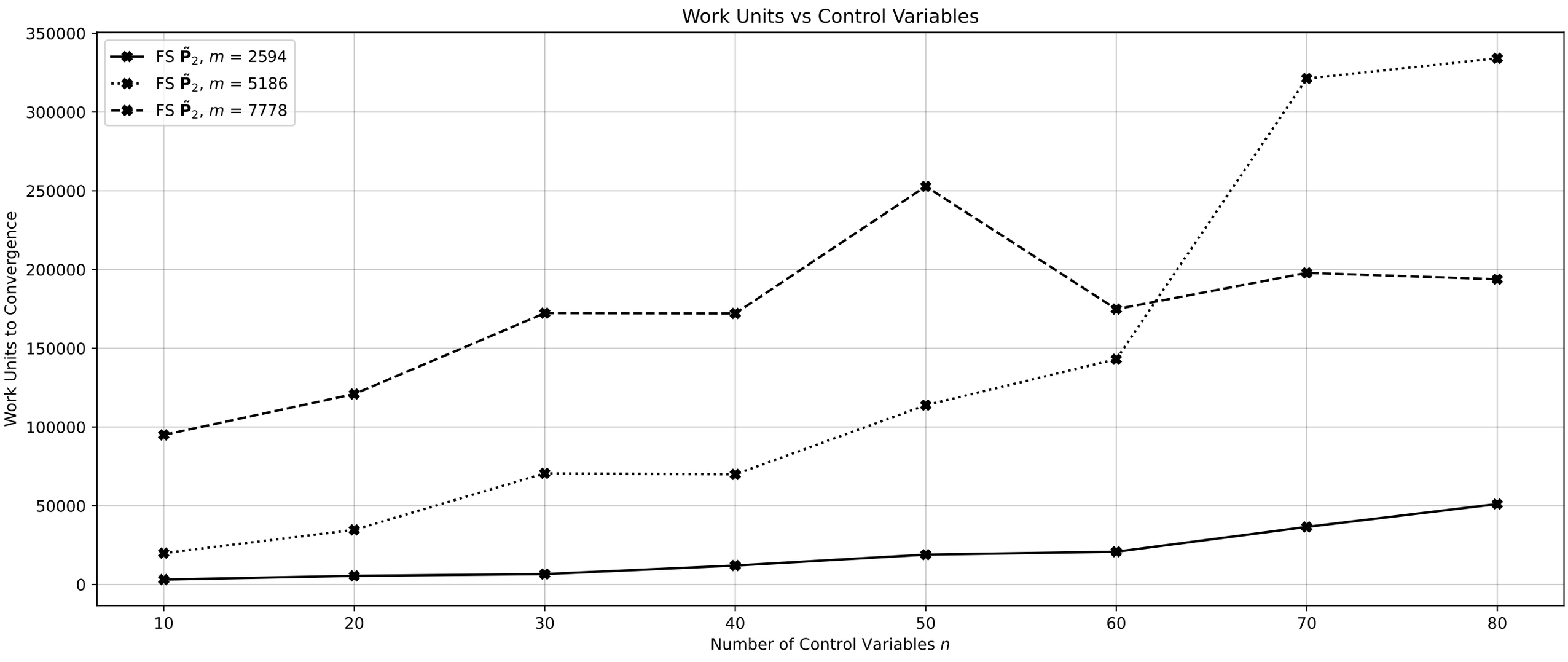

Work Units vs Control Variables
FS $\tilde{\mathbf{P}}_2$, $m = 2594$
FS $\tilde{\mathbf{P}}_2$, $m = 5186$
FS $\tilde{\mathbf{P}}_2$, $m = 7778$
Work Units to Convergence
Number of Control Variables n
350000
300000
250000
200000
150000
100000
50000
0
10
20
30
40
50
60
70
80

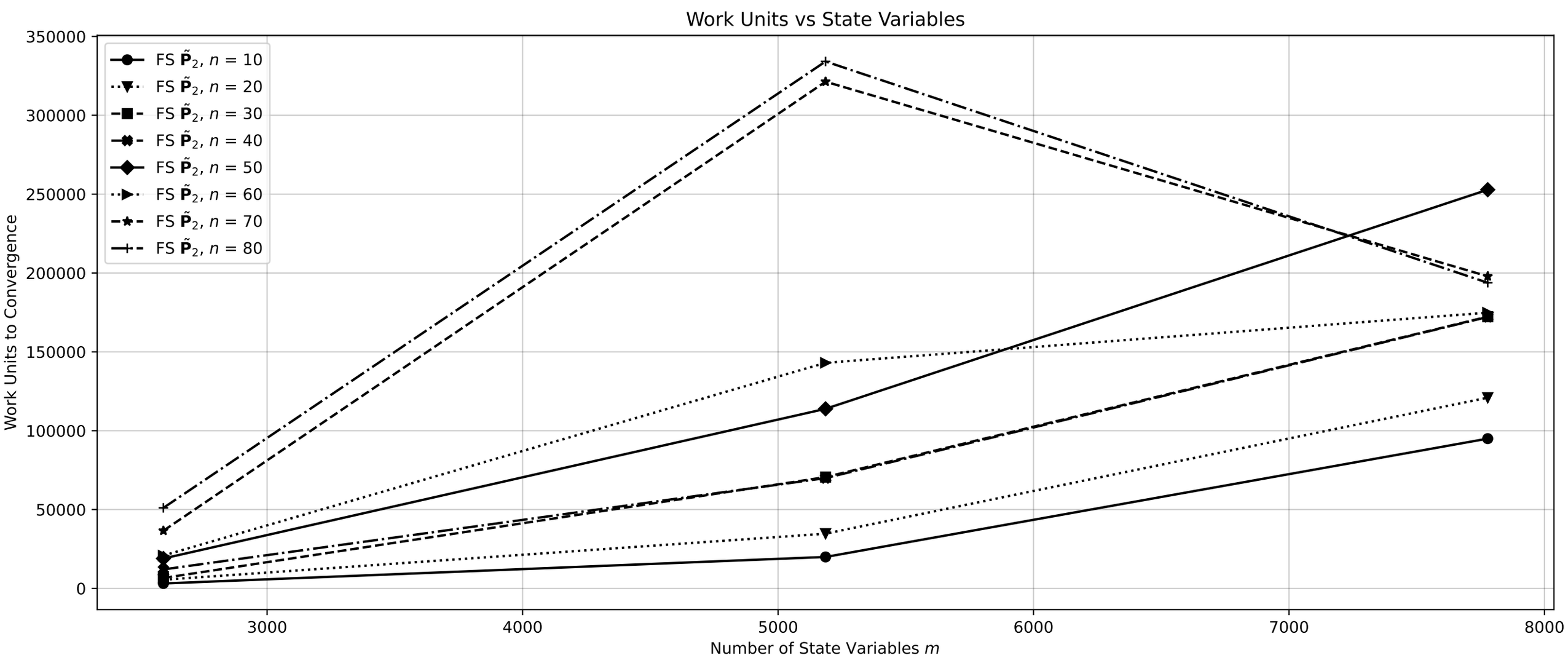

Total Subiterations vs Control Variables
FS $\tilde{\mathbf{P}}_2$, $m = 2594$
FS $\tilde{\mathbf{P}}_2$, $m = 5186$
FS $\tilde{\mathbf{P}}_2$, $m = 7778$
Total Subiterations to Convergence
Number of Control Variables n
20
40
60
80
100
120
140
160
10
20
30
40
50
60
70
80

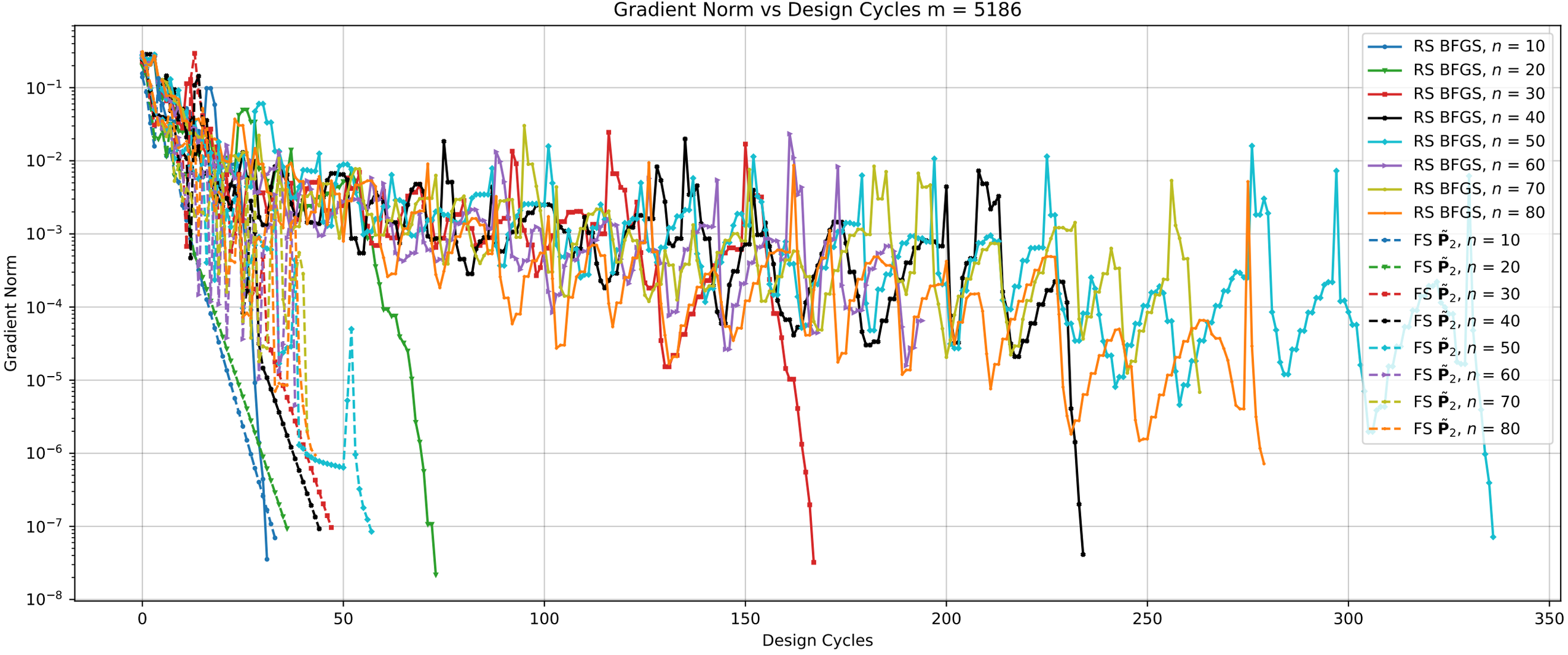

Work Units vs Control Variables
1e6
RS BFGS, m = 2594
RS BFGS, m = 5186
RS BFGS, m = 7778
FS $\tilde{\mathbf{P}}_2$, m = 2594
FS $\tilde{\mathbf{P}}_2$, m = 5186
FS $\tilde{\mathbf{P}}_2$, m = 7778
Work units to Convergence
3.0
2.5
2.0
1.5
1.0
0.5
0.0
10
20
30
40
50
60
70
80
Number of Control Variables n


Reduced-space Design Cycles vs Control Variables
RS BFGS, m = 2594
RS BFGS, m = 5186
RS BFGS, m = 7778
Number of Design Cycles to Convergence
0
100
200
300
400
500
600
700
10
20
30
40
50
60
70
80
Number of Control Variables n

Reduced-space Gradient vs Design Cycles


Work Units vs Control Variables
FS $\tilde{\mathbf{P}}_2$, $m = 2594$
FS $\tilde{\mathbf{P}}_2$, $m = 5186$
FS $\tilde{\mathbf{P}}_2$, $m = 7778$
Work Units to Convergence
Number of Control Variables n
350000
300000
250000
200000
150000
100000
50000
0
10
20
30
40
50
60
70
80


Work Units vs State Variables
Work Units to Convergence
Number of State Variables m
FS $\tilde{\mathbf{P}}_2$, $n = 10$
FS $\tilde{\mathbf{P}}_2$, $n = 20$
FS $\tilde{\mathbf{P}}_2$, $n = 30$
FS $\tilde{\mathbf{P}}_2$, $n = 40$
FS $\tilde{\mathbf{P}}_2$, $n = 50$
FS $\tilde{\mathbf{P}}_2$, $n = 60$
FS $\tilde{\mathbf{P}}_2$, $n = 70$
FS $\tilde{\mathbf{P}}_2$, $n = 80$
0
50000
100000
150000
200000
250000
300000
350000
3000
4000
5000
6000
7000
8000


Gradient Norm vs Design Cycles m = 5186
Gradient Norm
Design Cycles
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-8}
0
50
100
150
200
250
300
350
RS BFGS, n = 10
RS BFGS, n = 20
RS BFGS, n = 30
RS BFGS, n = 40
RS BFGS, n = 50
RS BFGS, n = 60
RS BFGS, n = 70
RS BFGS, n = 80
FS $\tilde{\mathbf{P}}_2$, n = 10
FS $\tilde{\mathbf{P}}_2$, n = 20
FS $\tilde{\mathbf{P}}_2$, n = 30
FS $\tilde{\mathbf{P}}_2$, n = 40
FS $\tilde{\mathbf{P}}_2$, n = 50
FS $\tilde{\mathbf{P}}_2$, n = 60
FS $\tilde{\mathbf{P}}_2$, n = 70
FS $\tilde{\mathbf{P}}_2$, n = 80