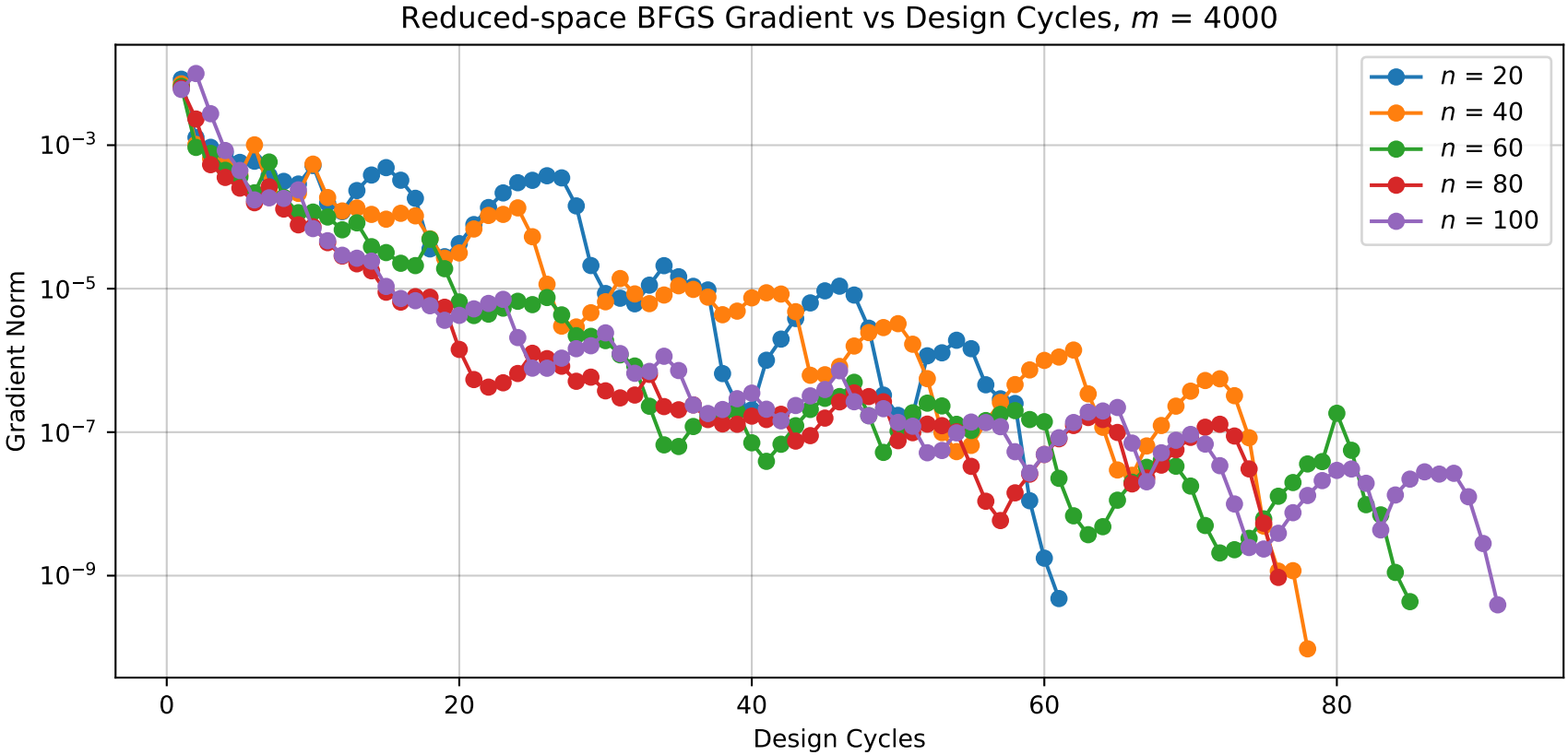
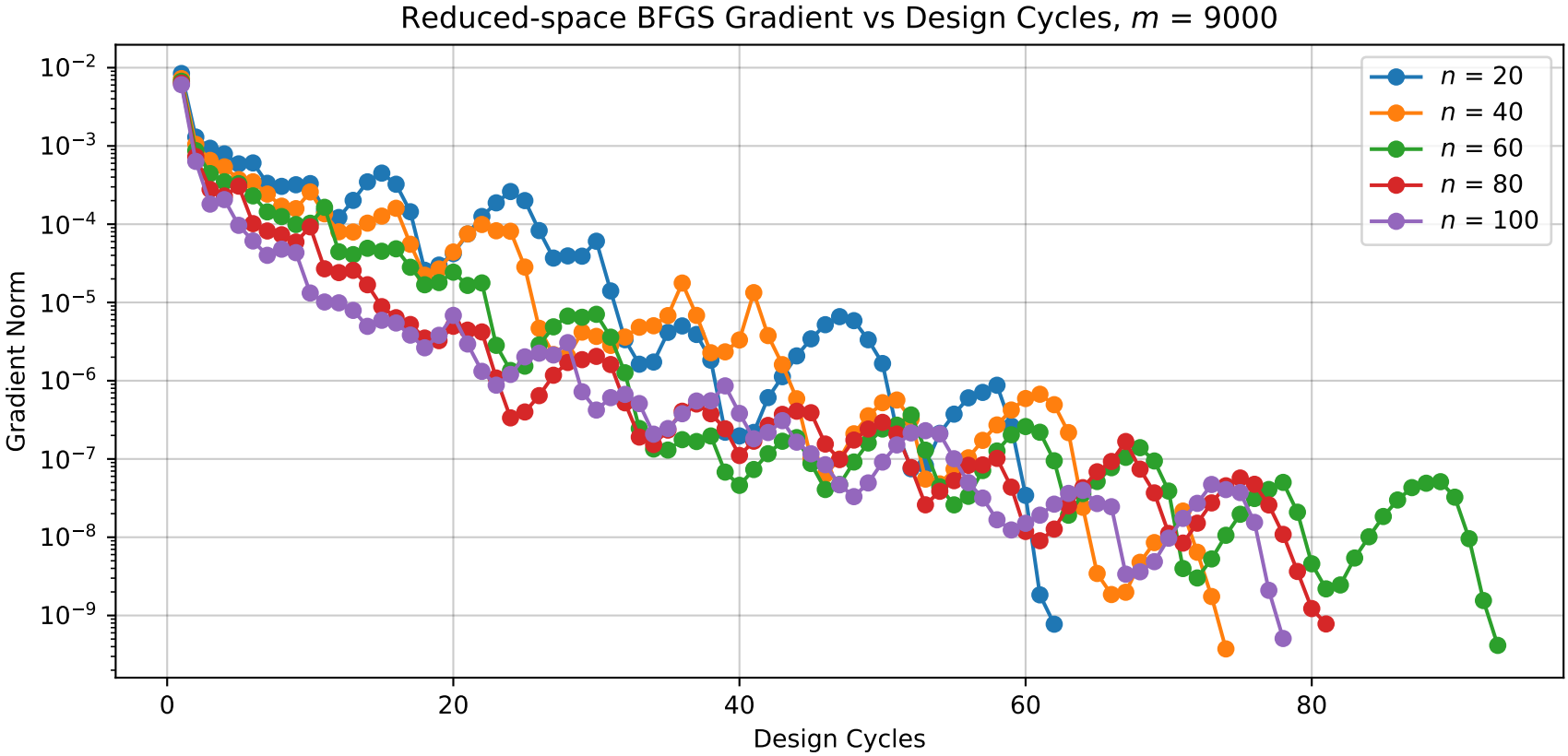
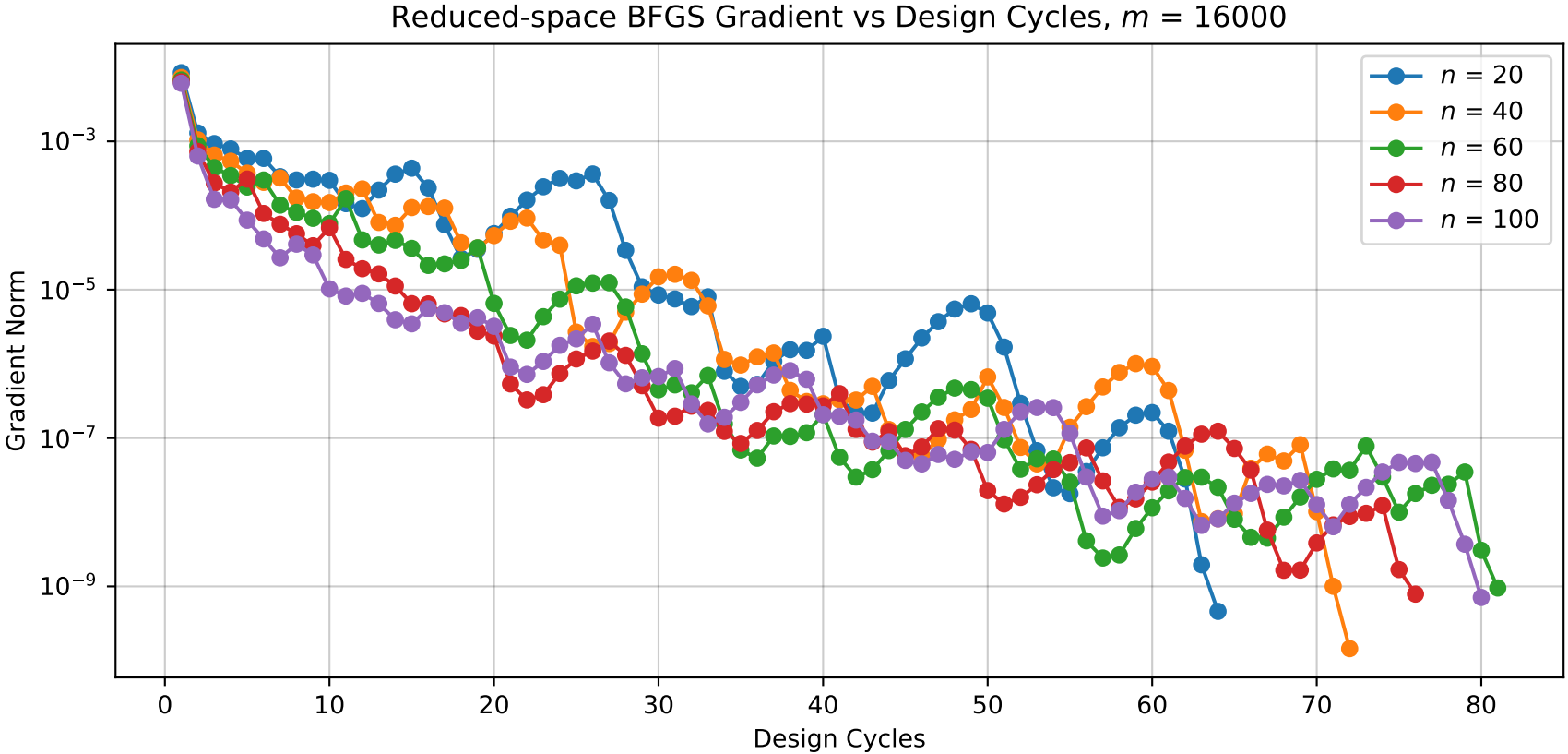
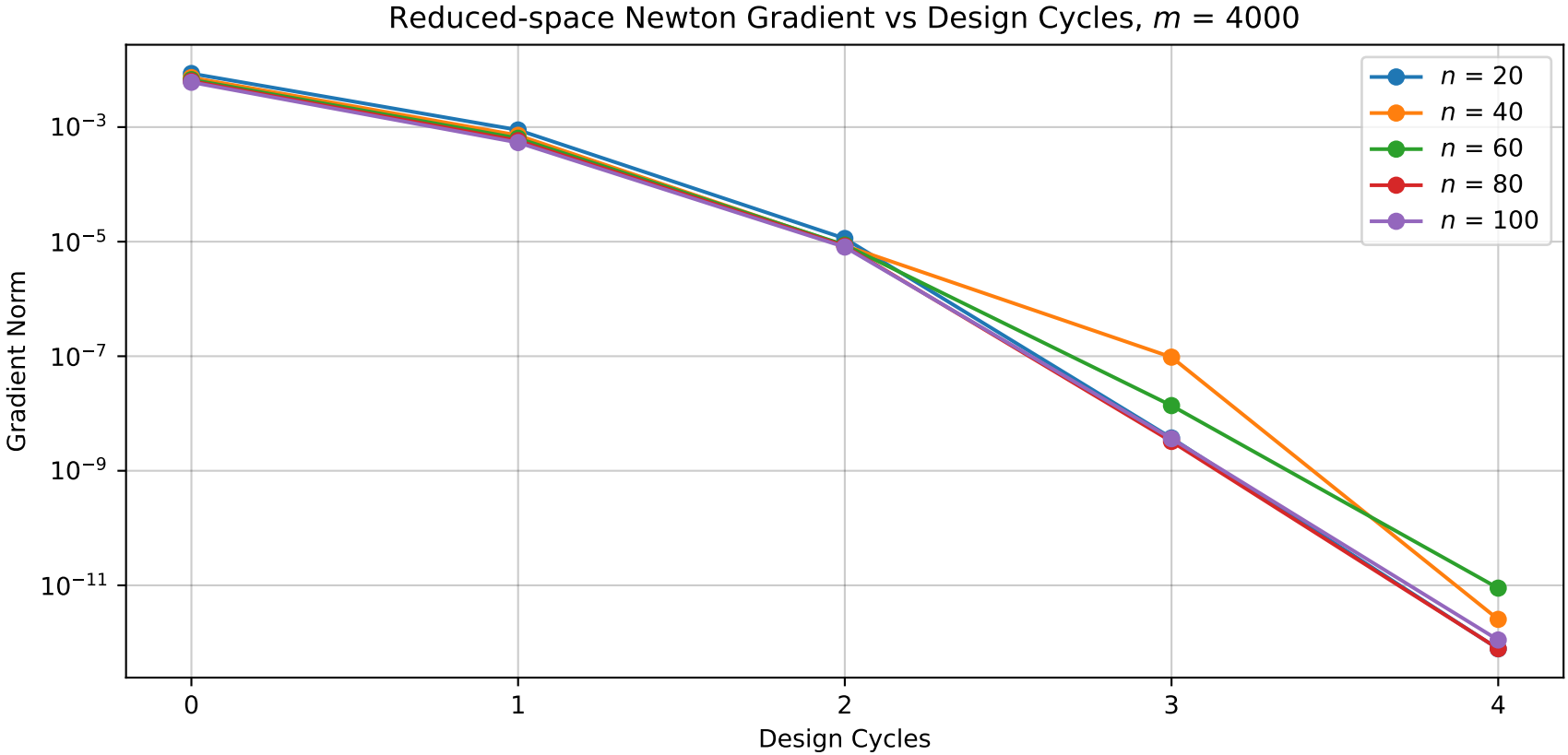
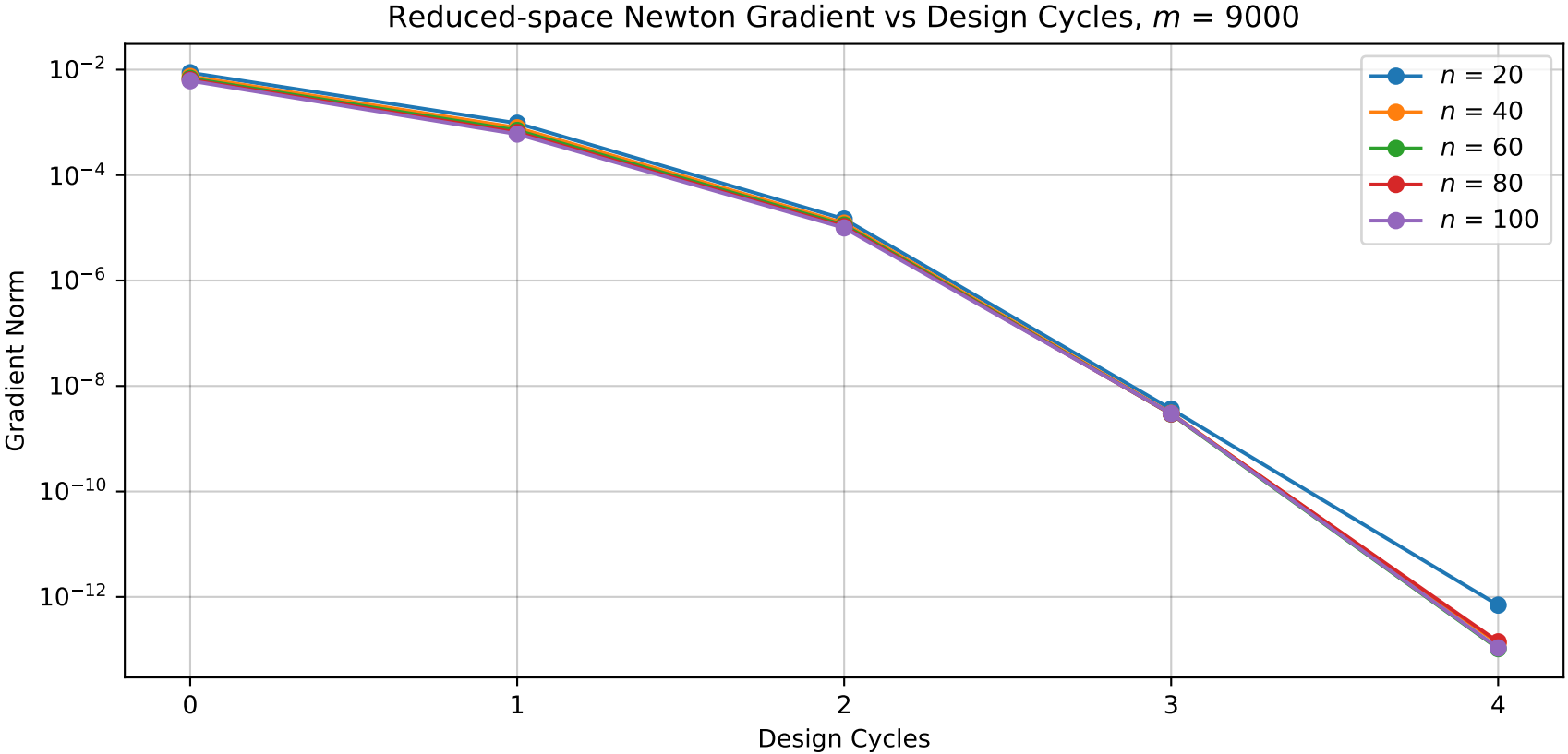
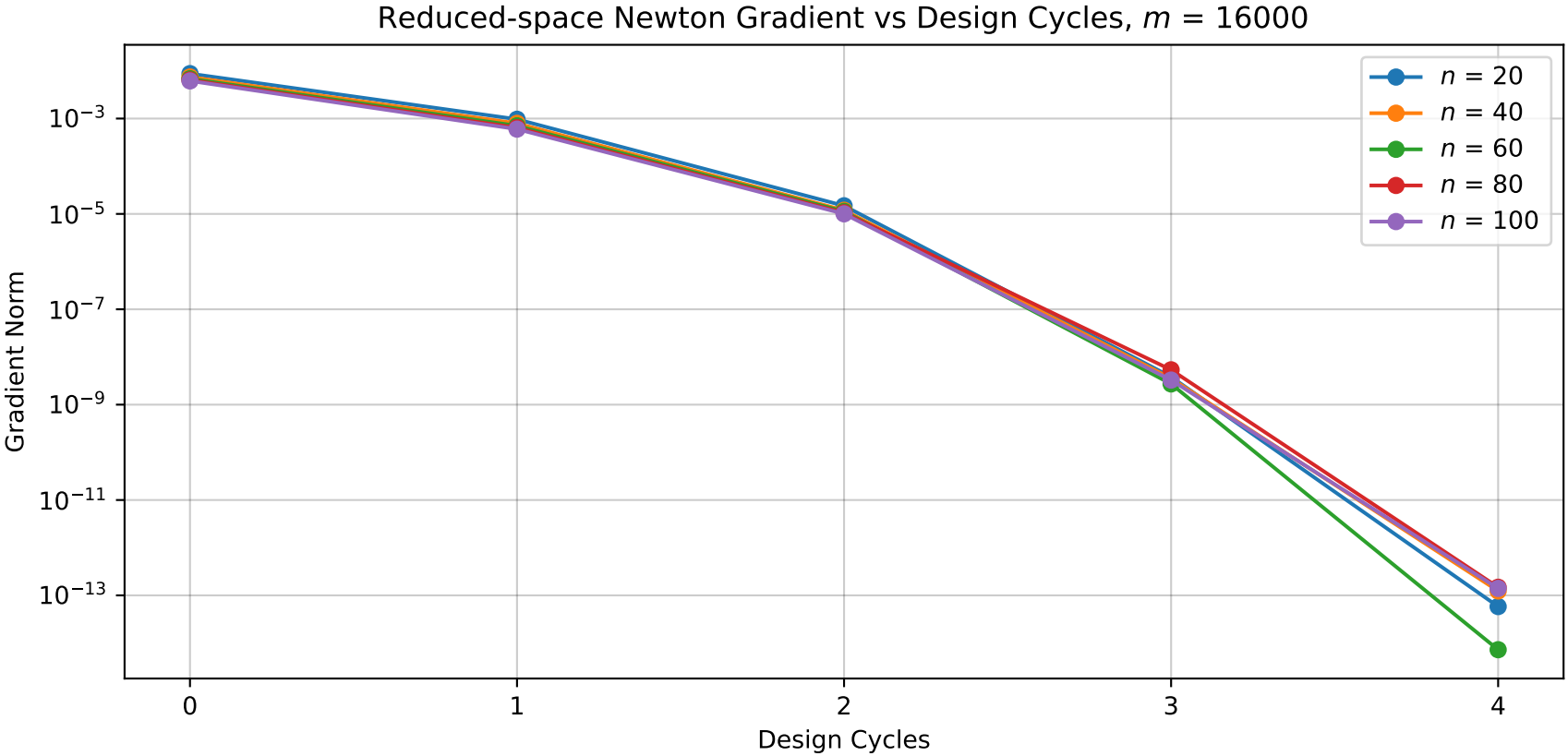

Reduced-space BFGS Gradient vs Design Cycles, m = 4000
Gradient Norm
10^{-3}
10^{-5}
10^{-7}
10^{-9}
0
20
40
60
80
Design Cycles
n = 20
n = 40
n = 60
n = 80
n = 100

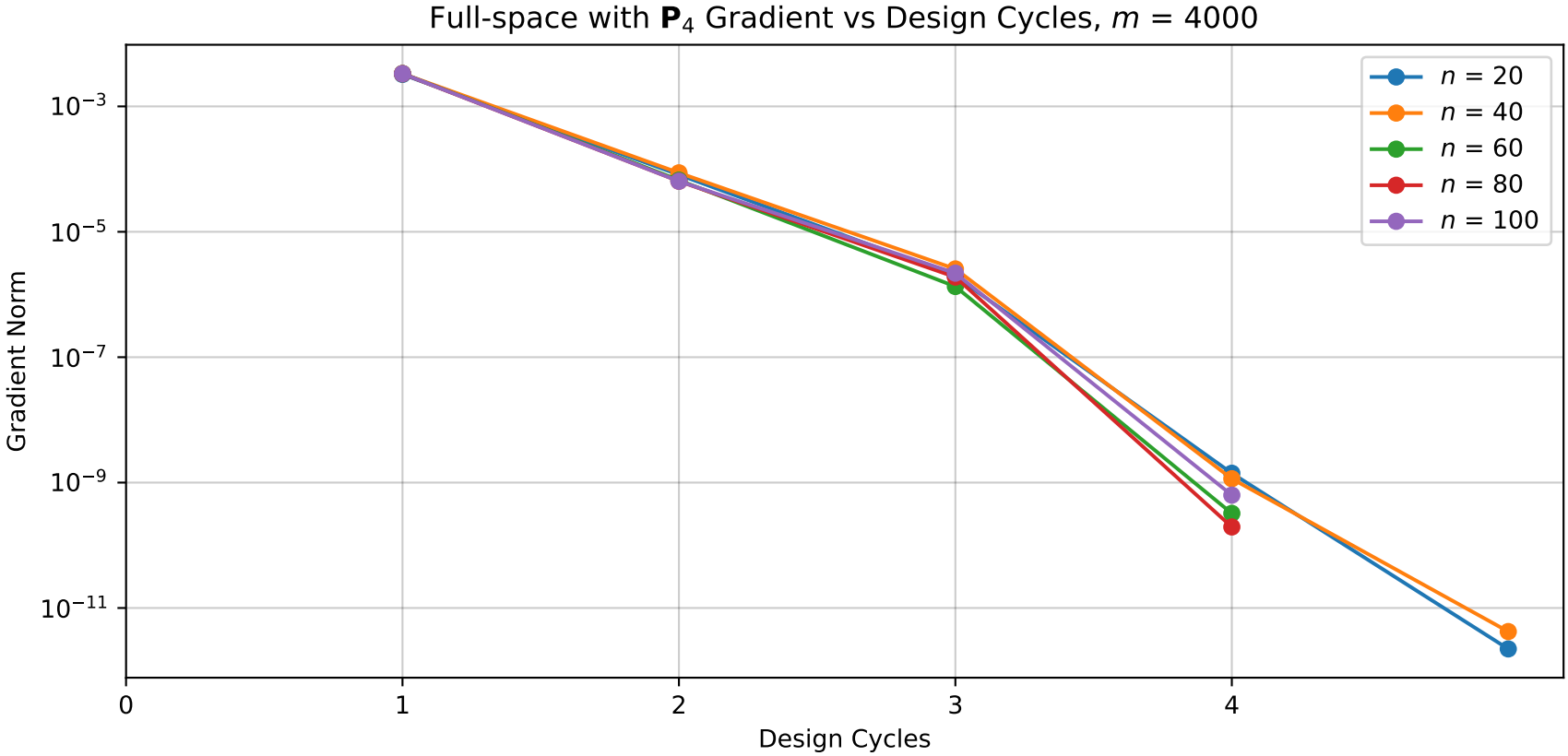

Reduced-space BFGS Gradient vs Design Cycles, m = 9000
Gradient Norm
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-8}
10^{-9}
0
20
40
60
80
Design Cycles
n = 20
n = 40
n = 60
n = 80
n = 100

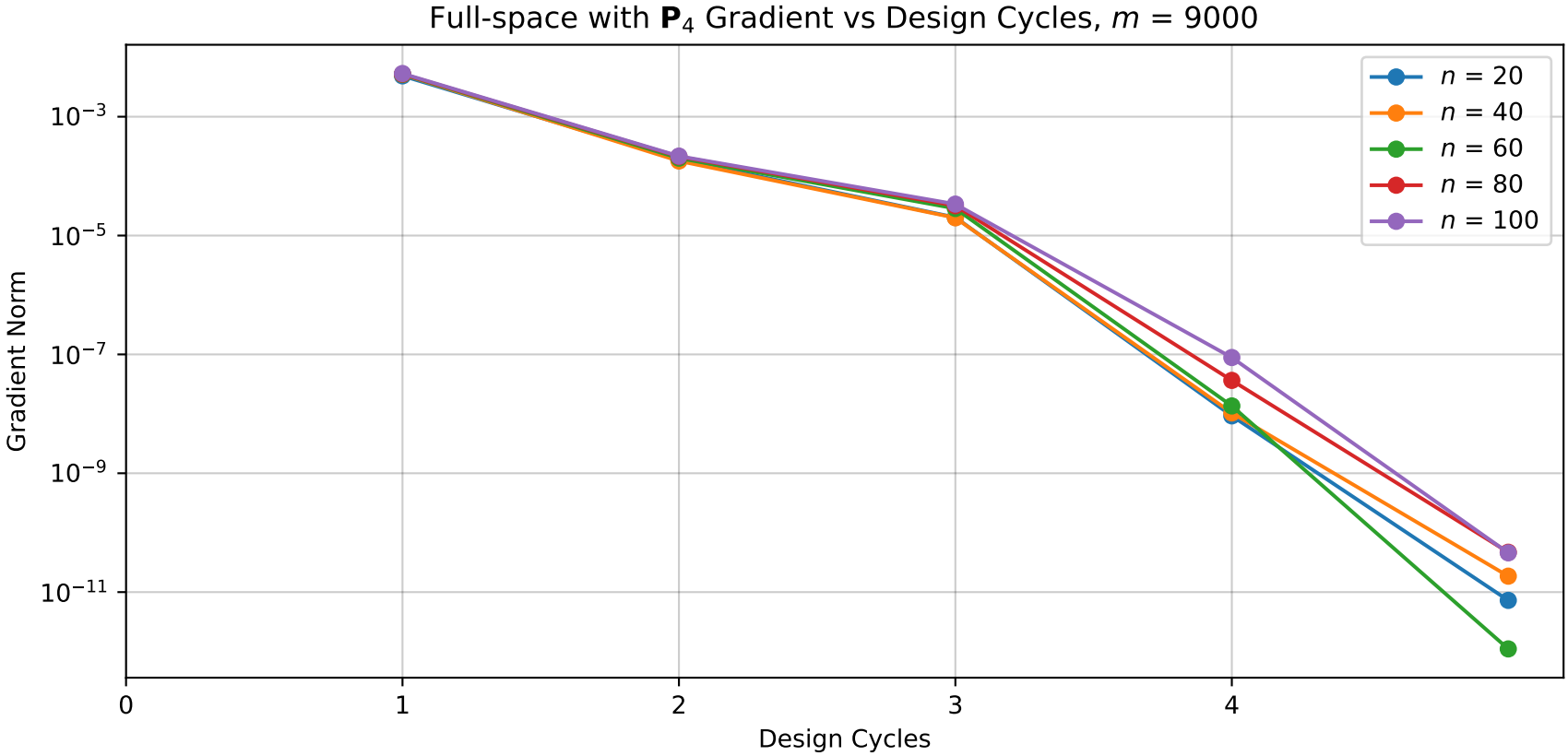

Reduced-space BFGS Gradient vs Design Cycles, m = 16000
Gradient Norm
10^{-3}
10^{-5}
10^{-7}
10^{-9}
0
10
20
30
40
50
60
70
80
Design Cycles
n = 20
n = 40
n = 60
n = 80
n = 100

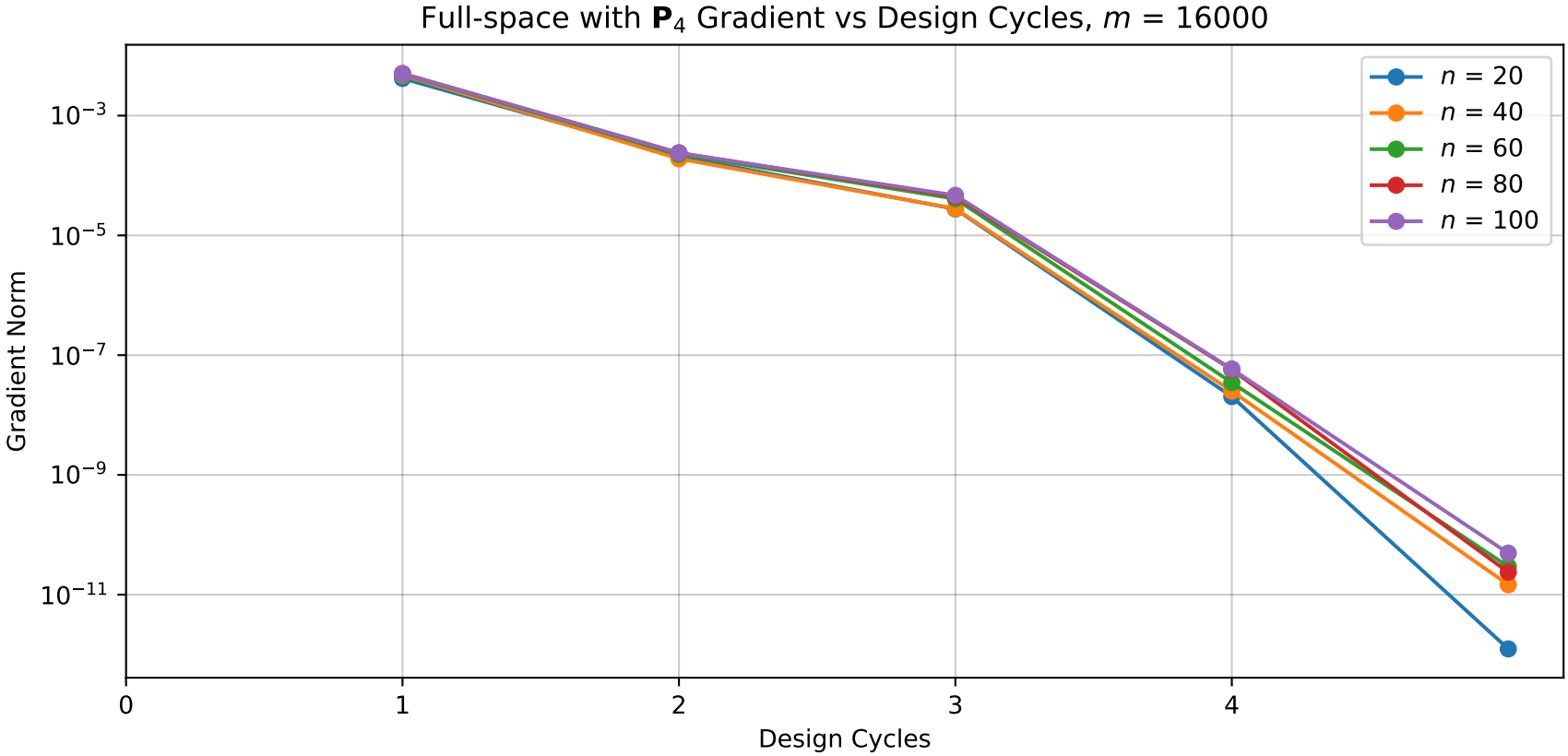

Reduced-space Newton Gradient vs Design Cycles, m = 4000
Gradient Norm
10^{-3}
10^{-5}
10^{-7}
10^{-9}
10^{-11}
0
1
2
3
4
Design Cycles
n = 20
n = 40
n = 60
n = 80
n = 100

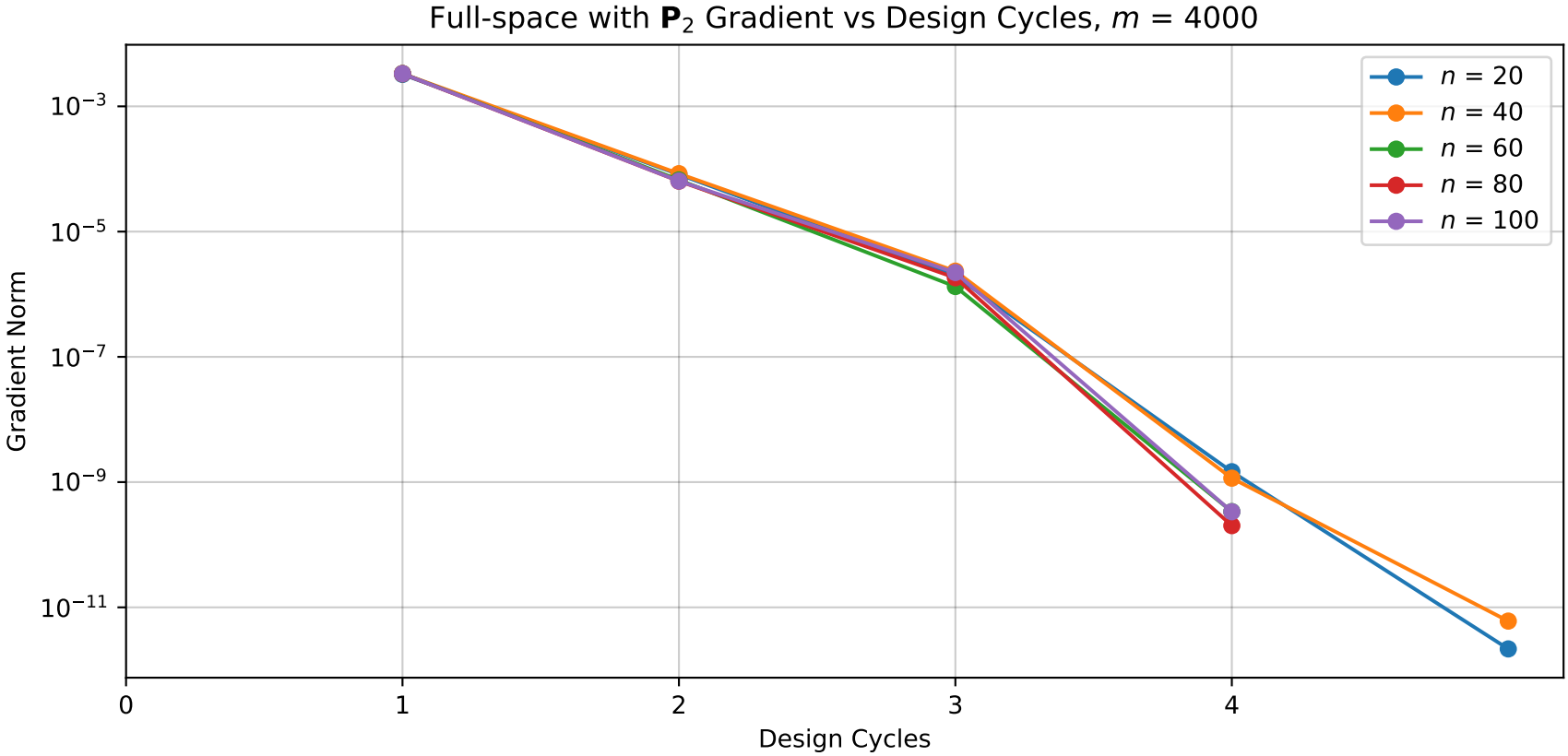

Reduced-space Newton Gradient vs Design Cycles, m = 9000
Gradient Norm
10^{-2}
10^{-4}
10^{-6}
10^{-8}
10^{-10}
10^{-12}
0
1
2
3
4
Design Cycles
n = 20
n = 40
n = 60
n = 80
n = 100

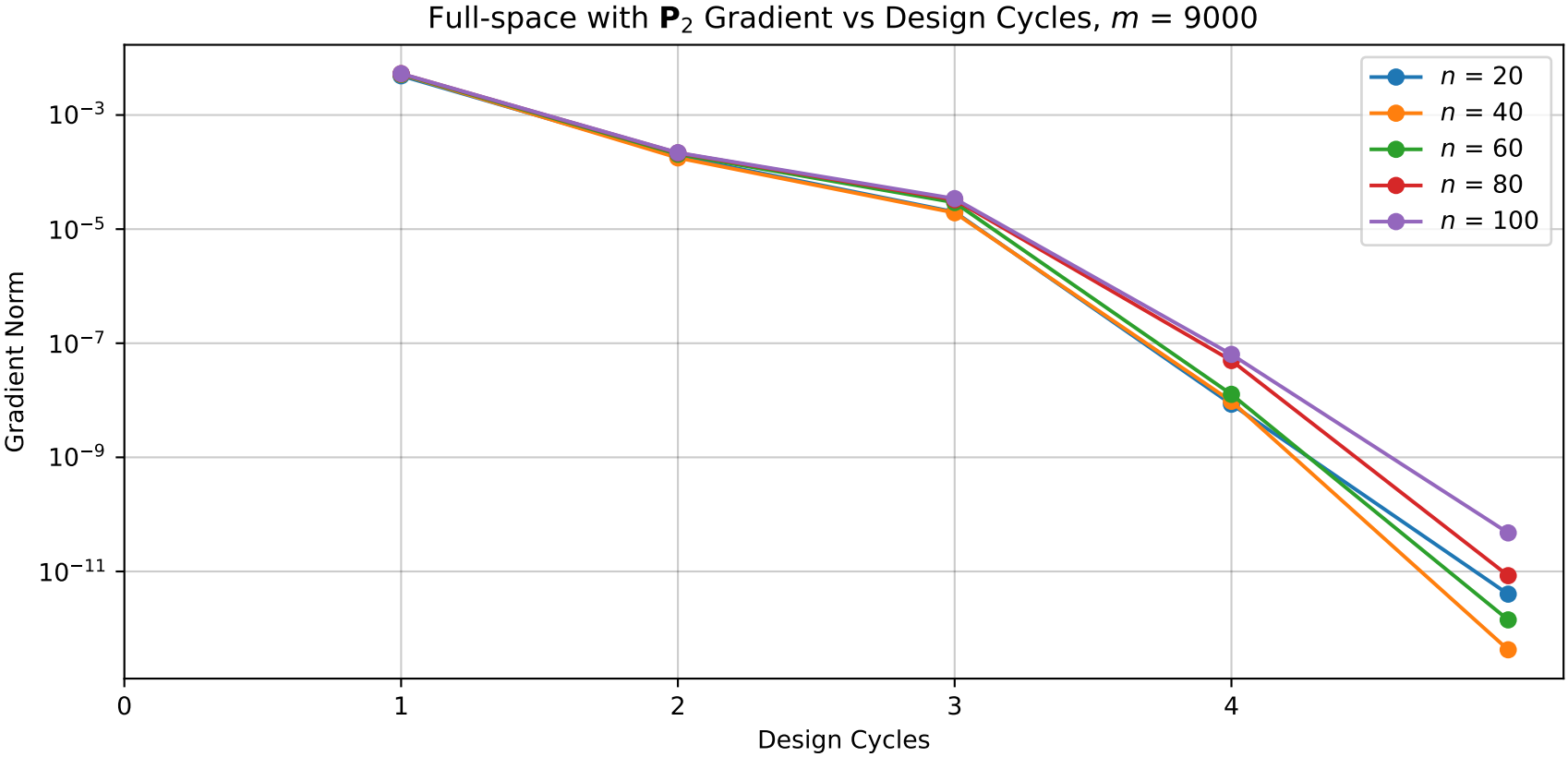

Reduced-space Newton Gradient vs Design Cycles, m = 16000
Gradient Norm
10^{-3}
10^{-5}
10^{-7}
10^{-9}
10^{-11}
10^{-13}
0
1
2
3
4
Design Cycles
n = 20
n = 40
n = 60
n = 80
n = 100

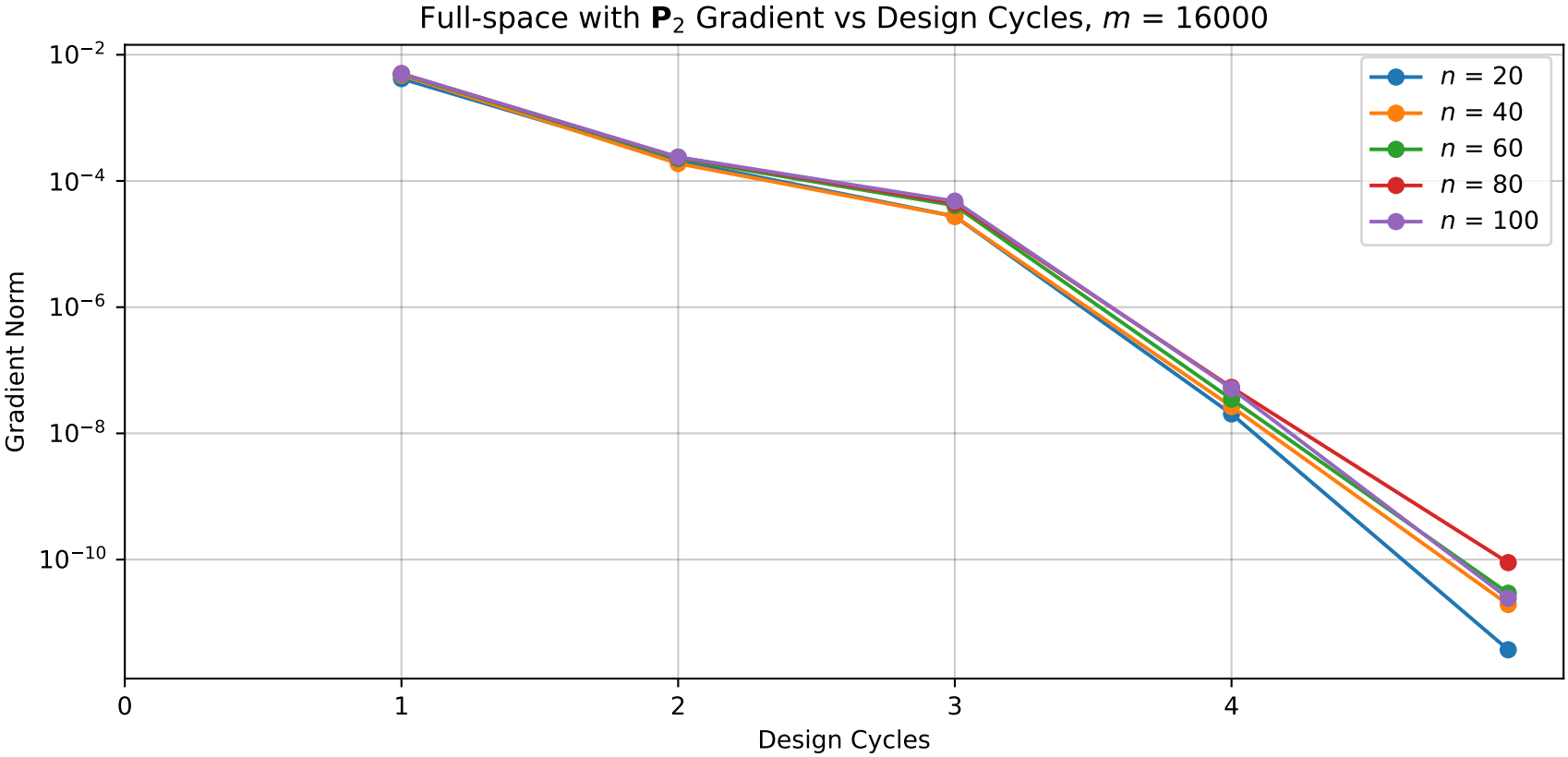

Full-space with $\mathbf{P}_4$ Gradient vs Design Cycles, $m = 4000$
Gradient Norm
10^{-3}
10^{-5}
10^{-7}
10^{-9}
10^{-11}
0
1
2
3
4
Design Cycles
$n = 20$
$n = 40$
$n = 60$
$n = 80$
$n = 100$

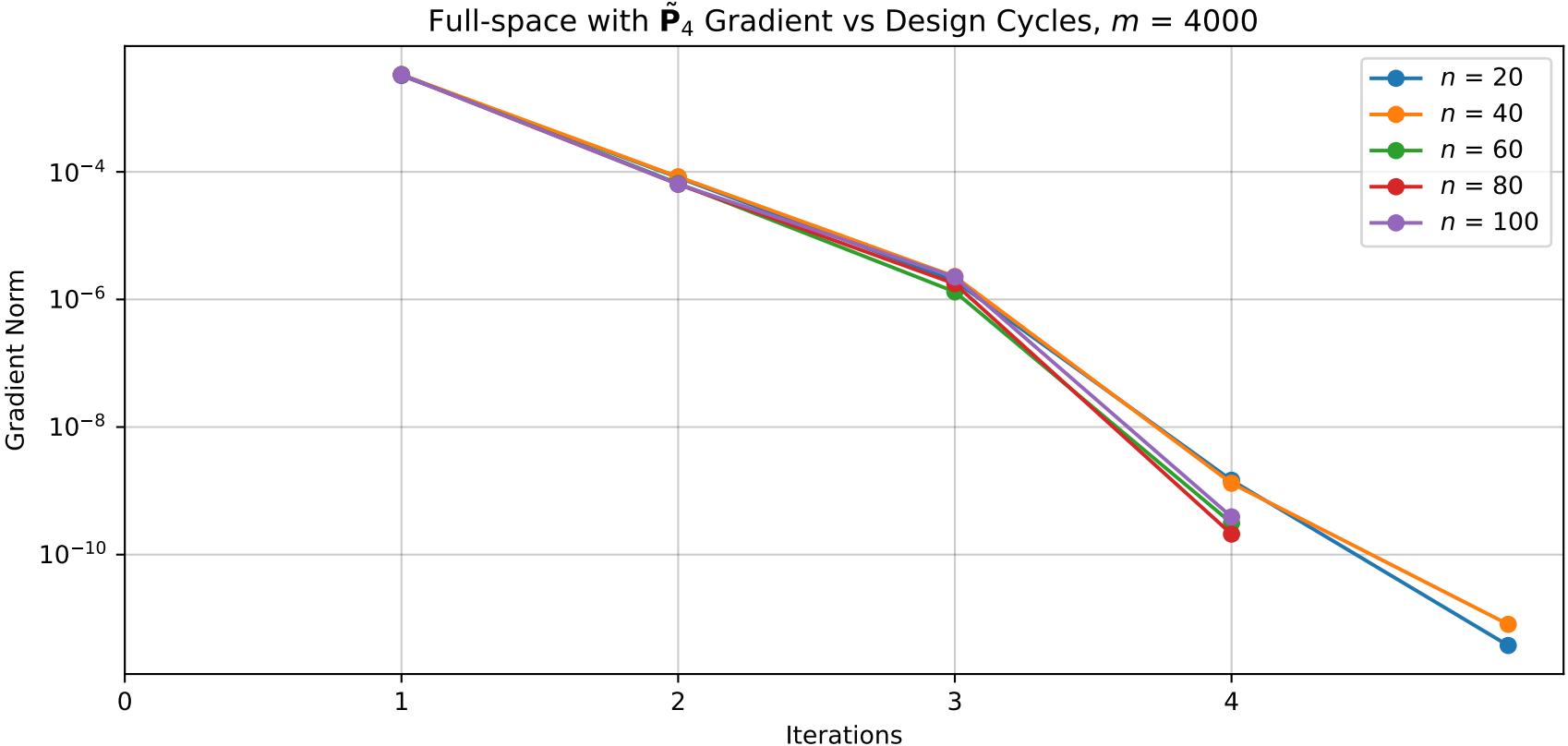

Full-space with $\mathbf{P}_4$ Gradient vs Design Cycles, $m = 9000$
Gradient Norm
10^{-3}
10^{-5}
10^{-7}
10^{-9}
10^{-11}
0
1
2
3
4
Design Cycles
$n = 20$
$n = 40$
$n = 60$
$n = 80$
$n = 100$

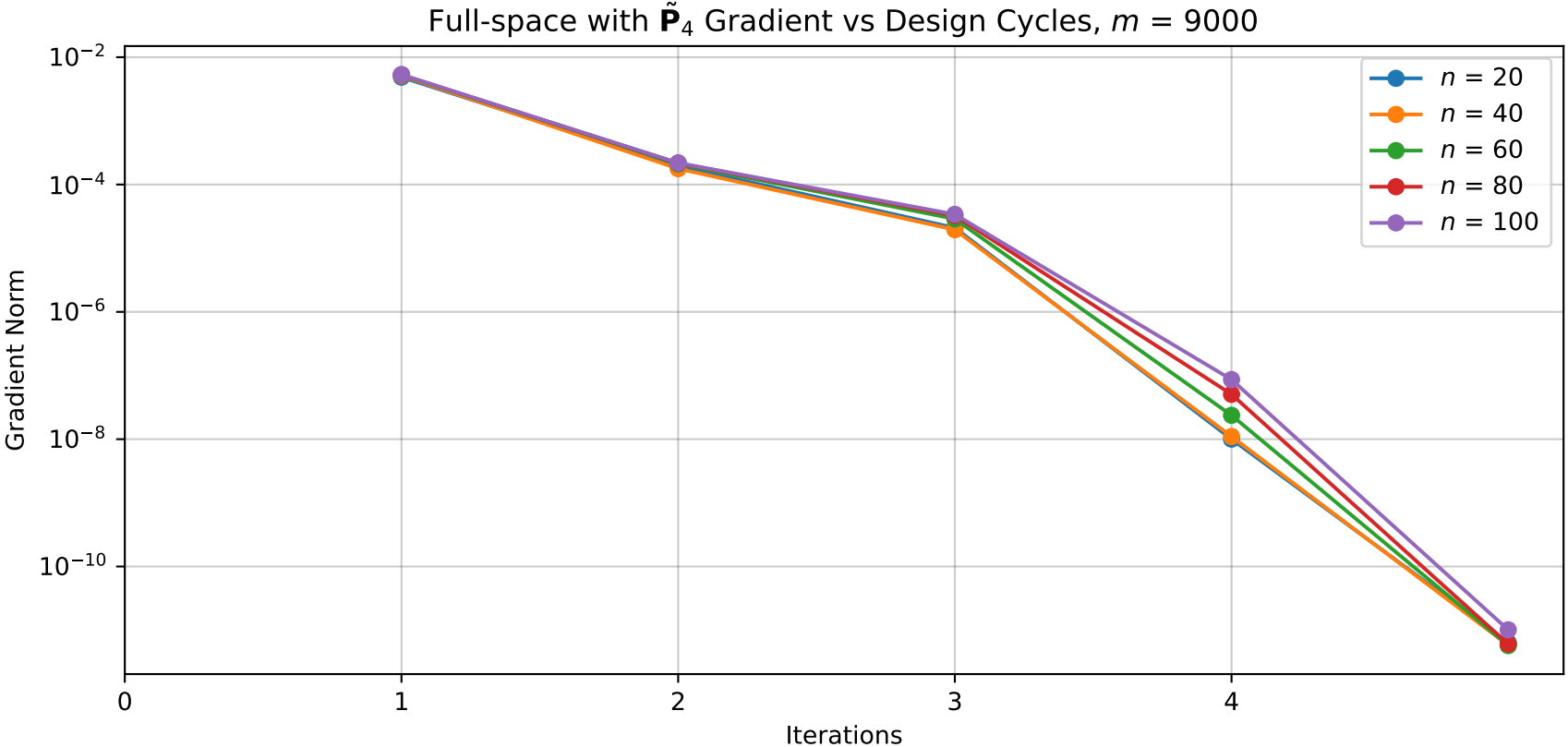

Full-space with $\mathbf{P}_4$ Gradient vs Design Cycles, m = 16000
Gradient Norm
10^{-3}
10^{-5}
10^{-7}
10^{-9}
10^{-11}
0
1
2
3
4
Design Cycles
n = 20
n = 40
n = 60
n = 80
n = 100

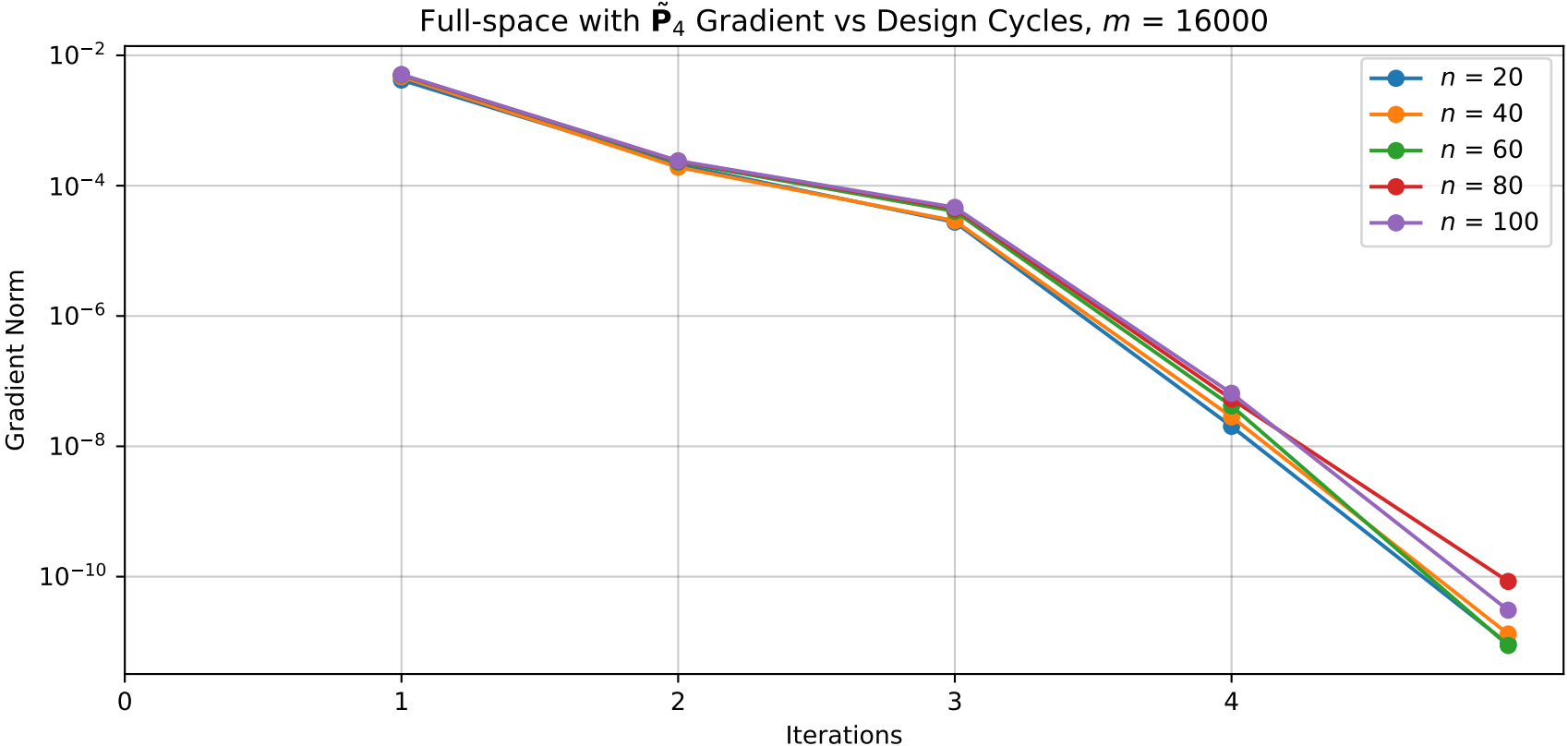

Full-space with $\mathbf{P}_2$ Gradient vs Design Cycles, $m = 4000$
Gradient Norm
10^{-3}
10^{-5}
10^{-7}
10^{-9}
10^{-11}
0
1
2
3
4
Design Cycles
$n = 20$
$n = 40$
$n = 60$
$n = 80$
$n = 100$

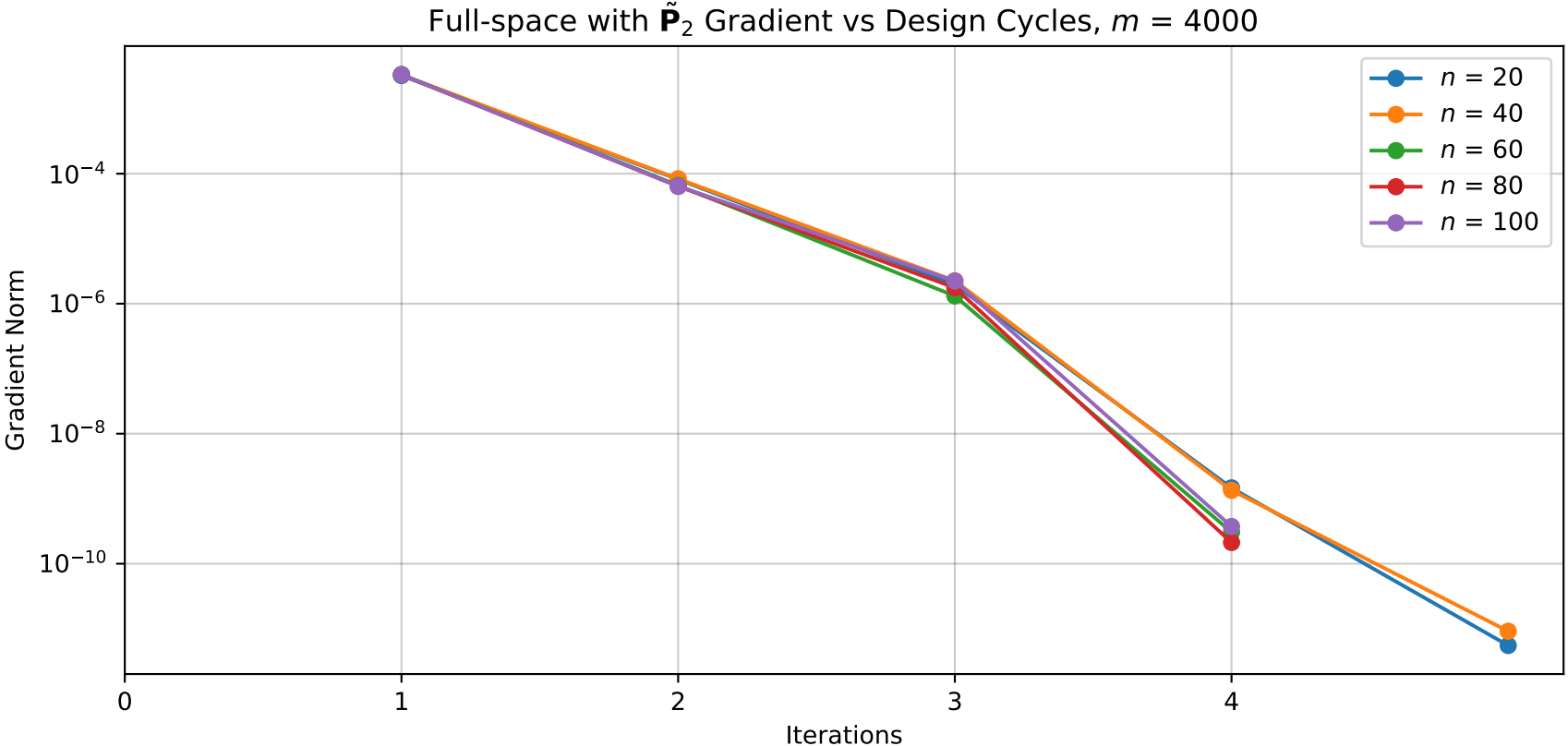

Full-space with $\mathbf{P}_2$ Gradient vs Design Cycles, m = 9000
Gradient Norm
10^{-3}
10^{-5}
10^{-7}
10^{-9}
10^{-11}
0
1
2
3
4
Design Cycles
n = 20
n = 40
n = 60
n = 80
n = 100

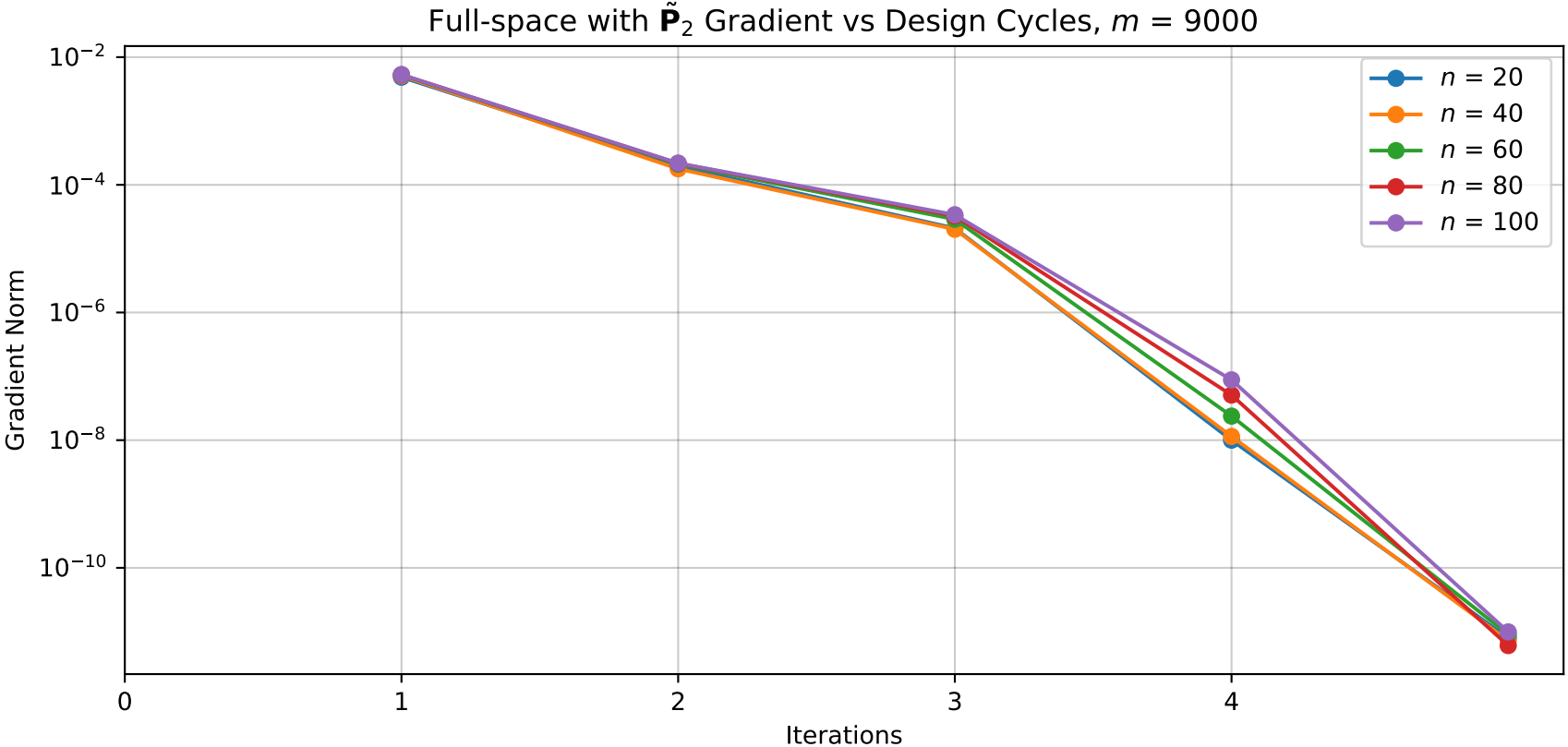

Full-space with $\mathbf{P}_2$ Gradient vs Design Cycles, m = 16000
Gradient Norm
10^{-2}
10^{-4}
10^{-6}
10^{-8}
10^{-10}
0
1
2
3
4
Design Cycles
n = 20
n = 40
n = 60
n = 80
n = 100

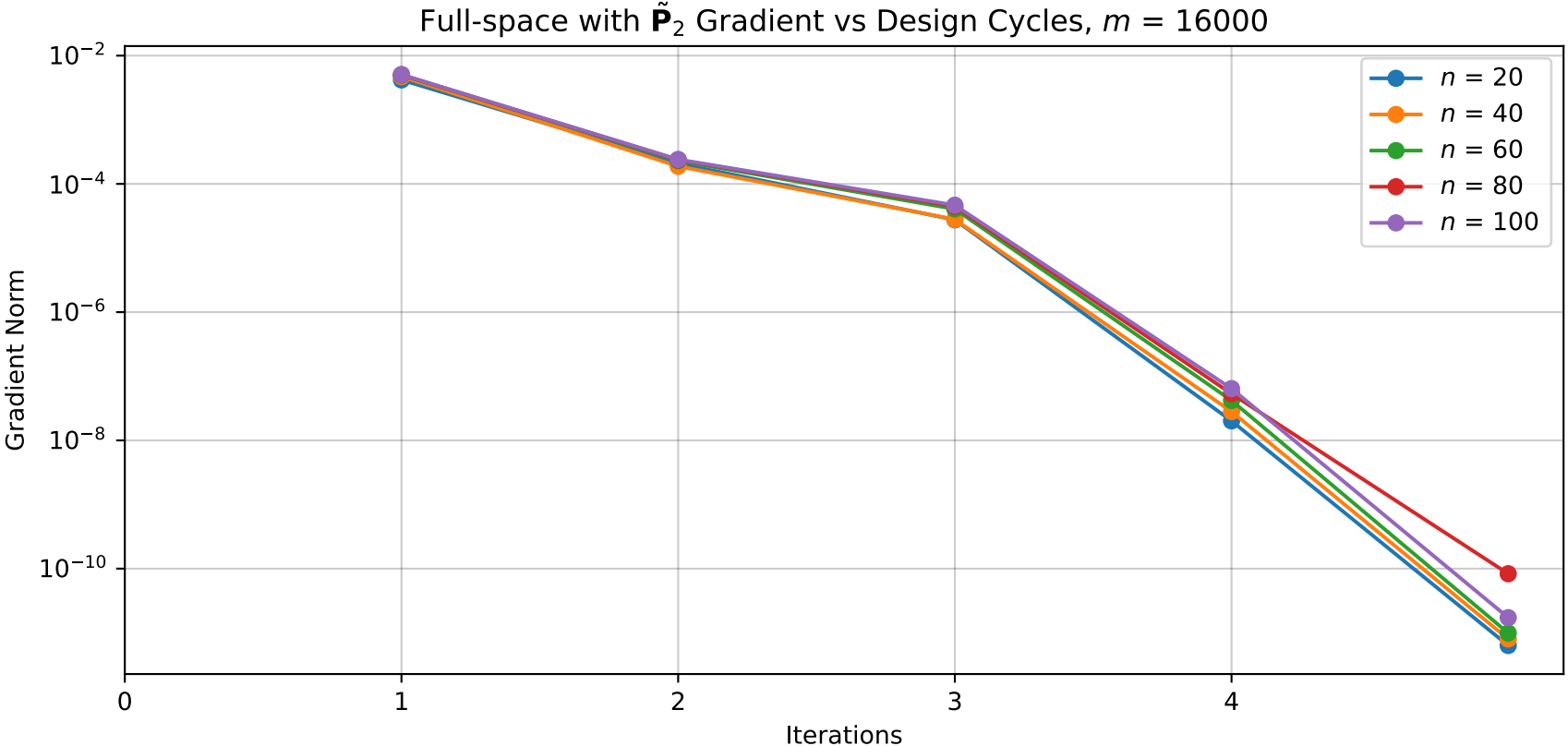

Full-space with $\tilde{\mathbf{P}}_4$ Gradient vs Design Cycles, m = 4000
Gradient Norm
10^{-4}
10^{-6}
10^{-8}
10^{-10}
0
1
2
3
4
Iterations
n = 20
n = 40
n = 60
n = 80
n = 100

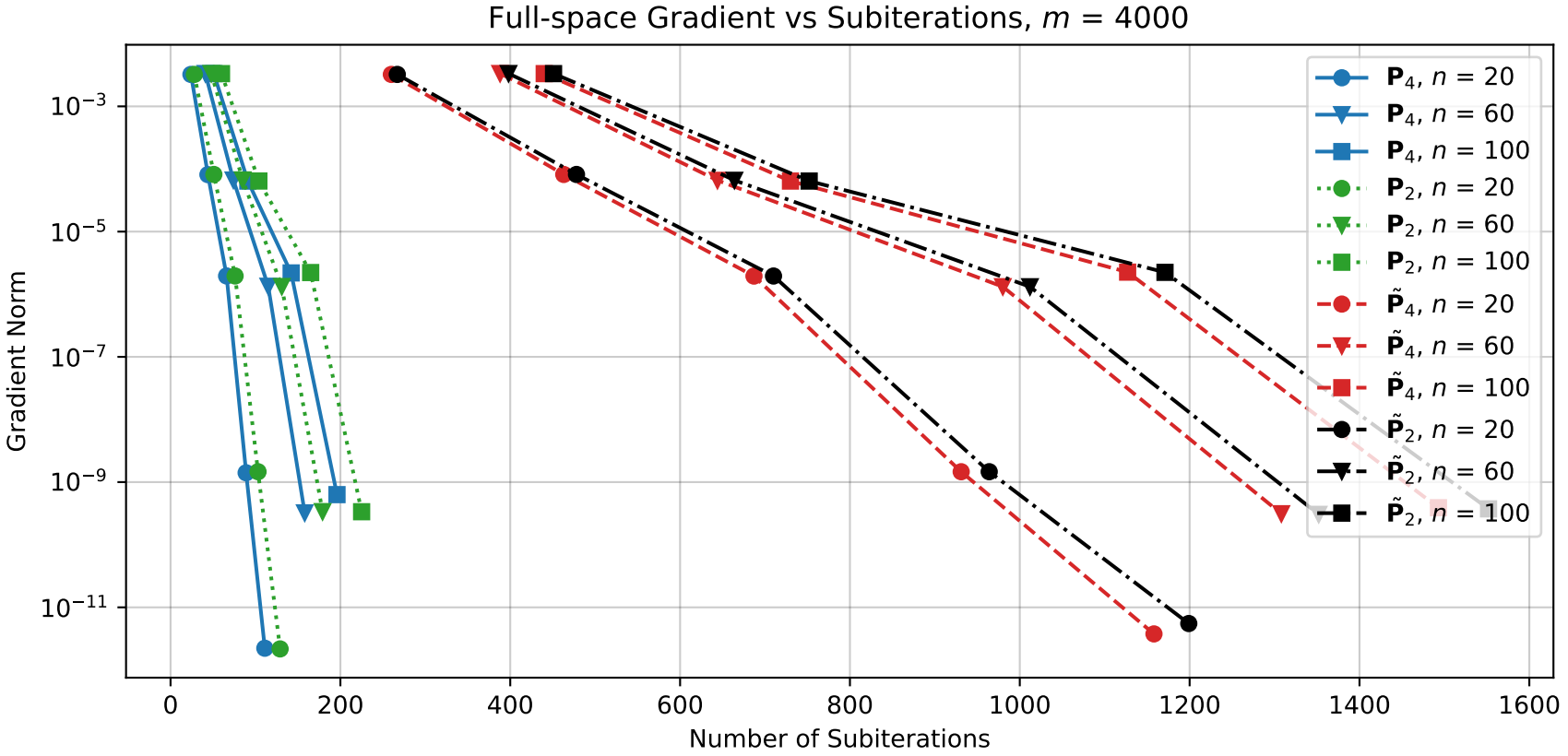

Full-space with $\tilde{\mathbf{P}}_4$ Gradient vs Design Cycles, m = 9000
Gradient Norm
10^{-2}
10^{-4}
10^{-6}
10^{-8}
10^{-10}
0
1
2
3
4
Iterations
n = 20
n = 40
n = 60
n = 80
n = 100

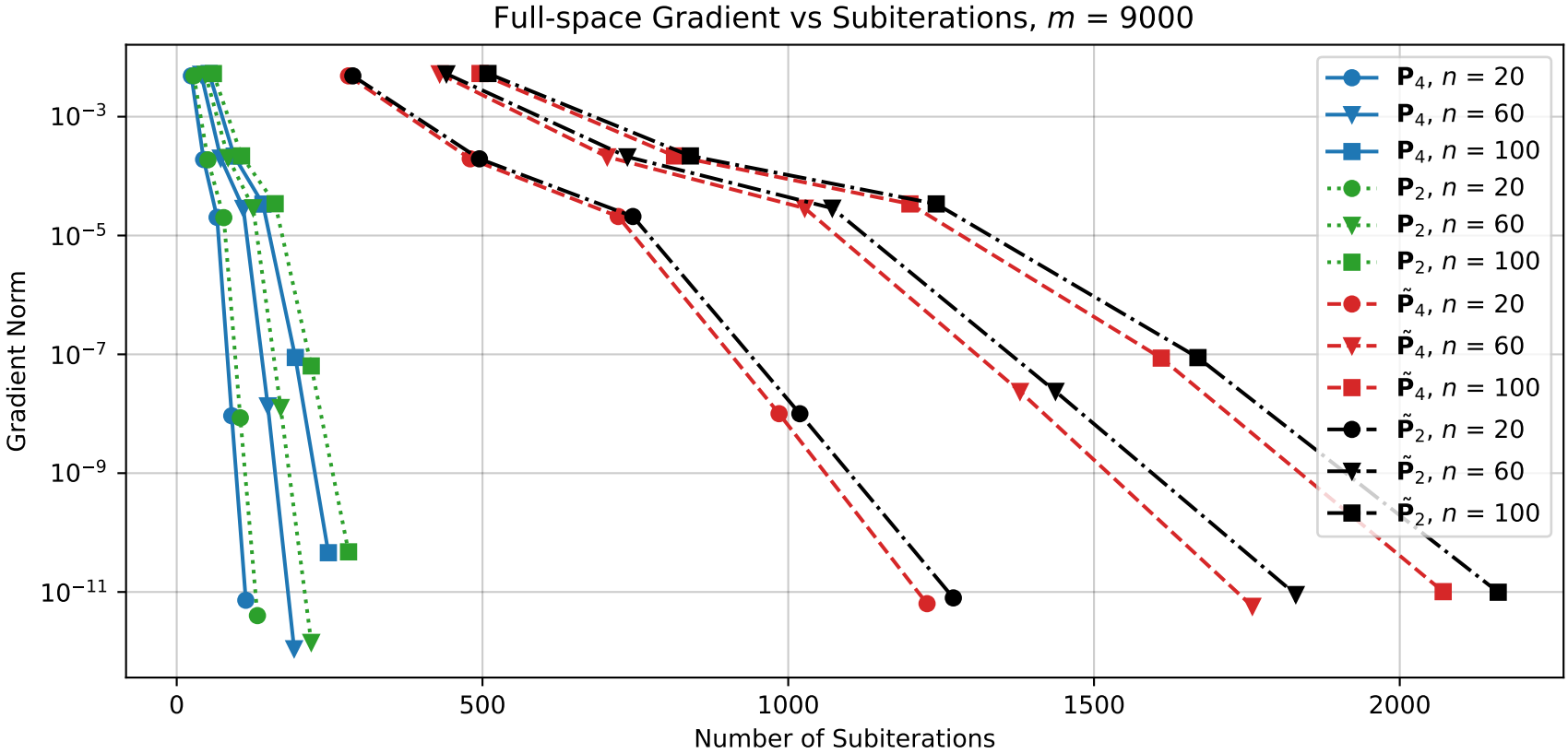

Full-space with $\tilde{\mathbf{P}}_4$ Gradient vs Design Cycles, m = 16000
Gradient Norm
10^{-2}
10^{-4}
10^{-6}
10^{-8}
10^{-10}
0
1
2
3
4
Iterations
n = 20
n = 40
n = 60
n = 80
n = 100

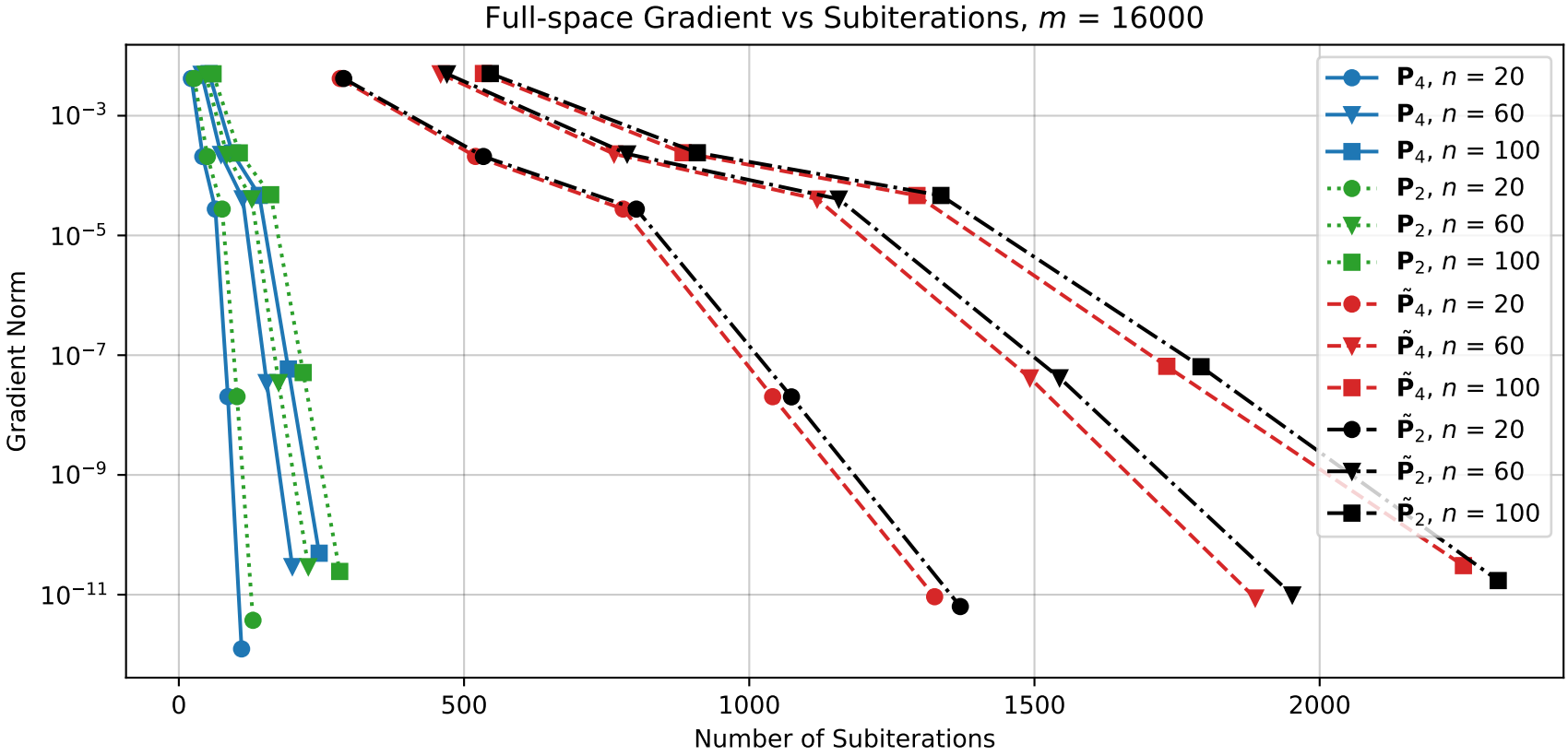

Full-space with $\tilde{\mathbf{P}}_2$ Gradient vs Design Cycles, m = 4000
Gradient Norm
10^{-4}
10^{-6}
10^{-8}
10^{-10}
0
1
2
3
4
Iterations
n = 20
n = 40
n = 60
n = 80
n = 100

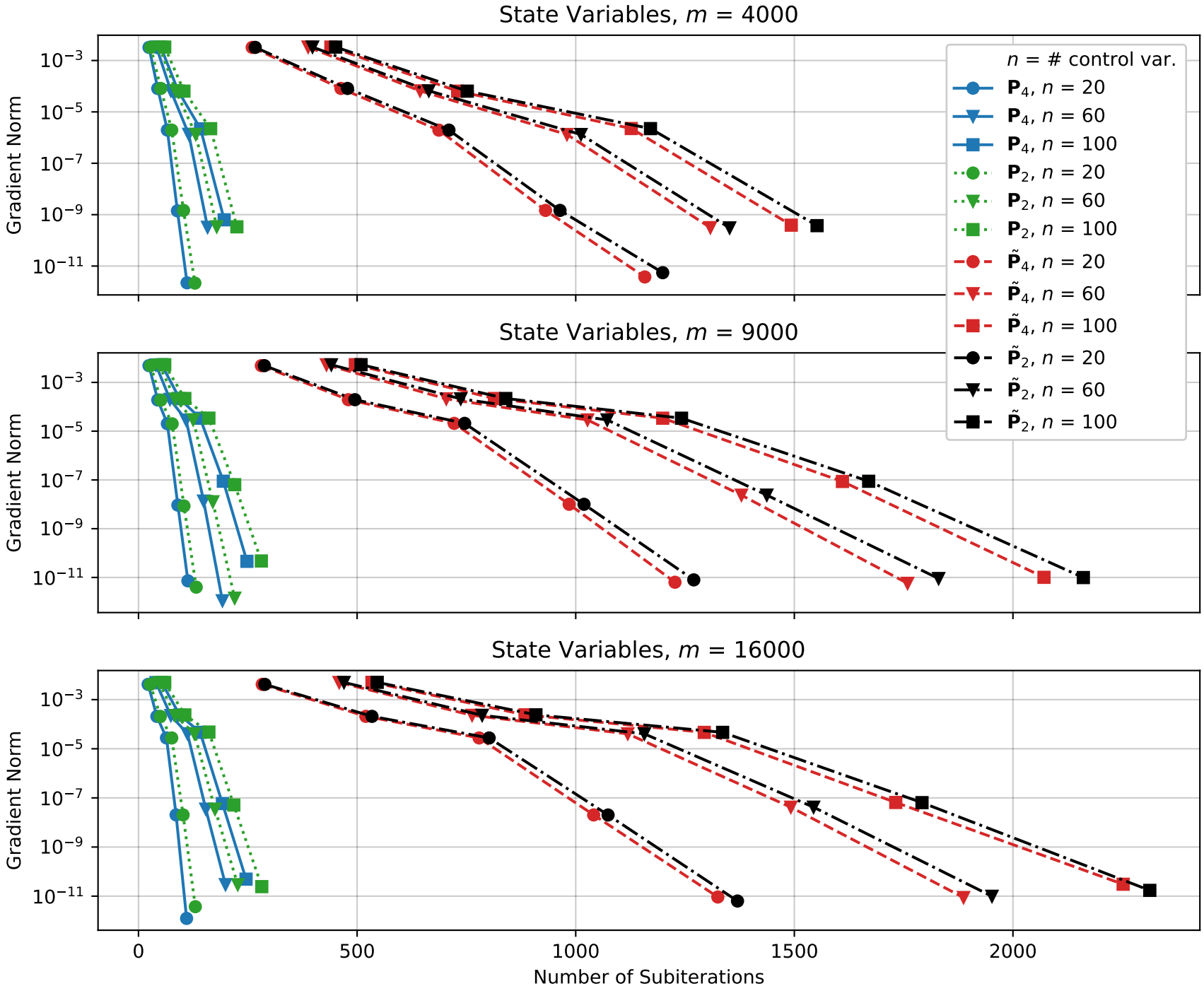

Full-space with $\tilde{\mathbf{P}}_2$ Gradient vs Design Cycles, m = 9000
Gradient Norm
10^{-2}
10^{-4}
10^{-6}
10^{-8}
10^{-10}
0
1
2
3
4
Iterations
n = 20
n = 40
n = 60
n = 80
n = 100

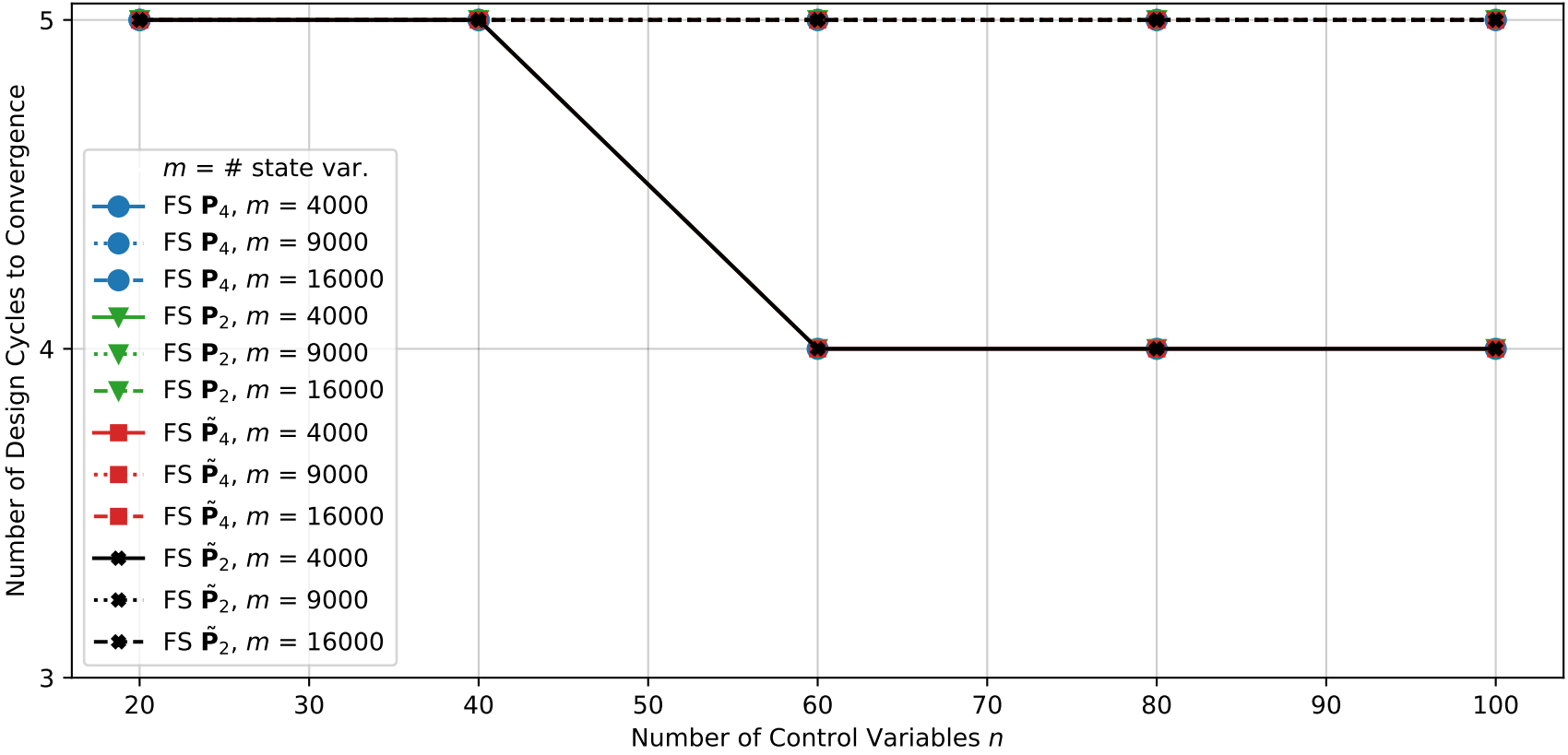

Full-space with $\tilde{\mathbf{P}}_2$ Gradient vs Design Cycles, m = 16000
Gradient Norm
10^{-2}
10^{-4}
10^{-6}
10^{-8}
10^{-10}
0
1
2
3
4
Iterations
n = 20
n = 40
n = 60
n = 80
n = 100

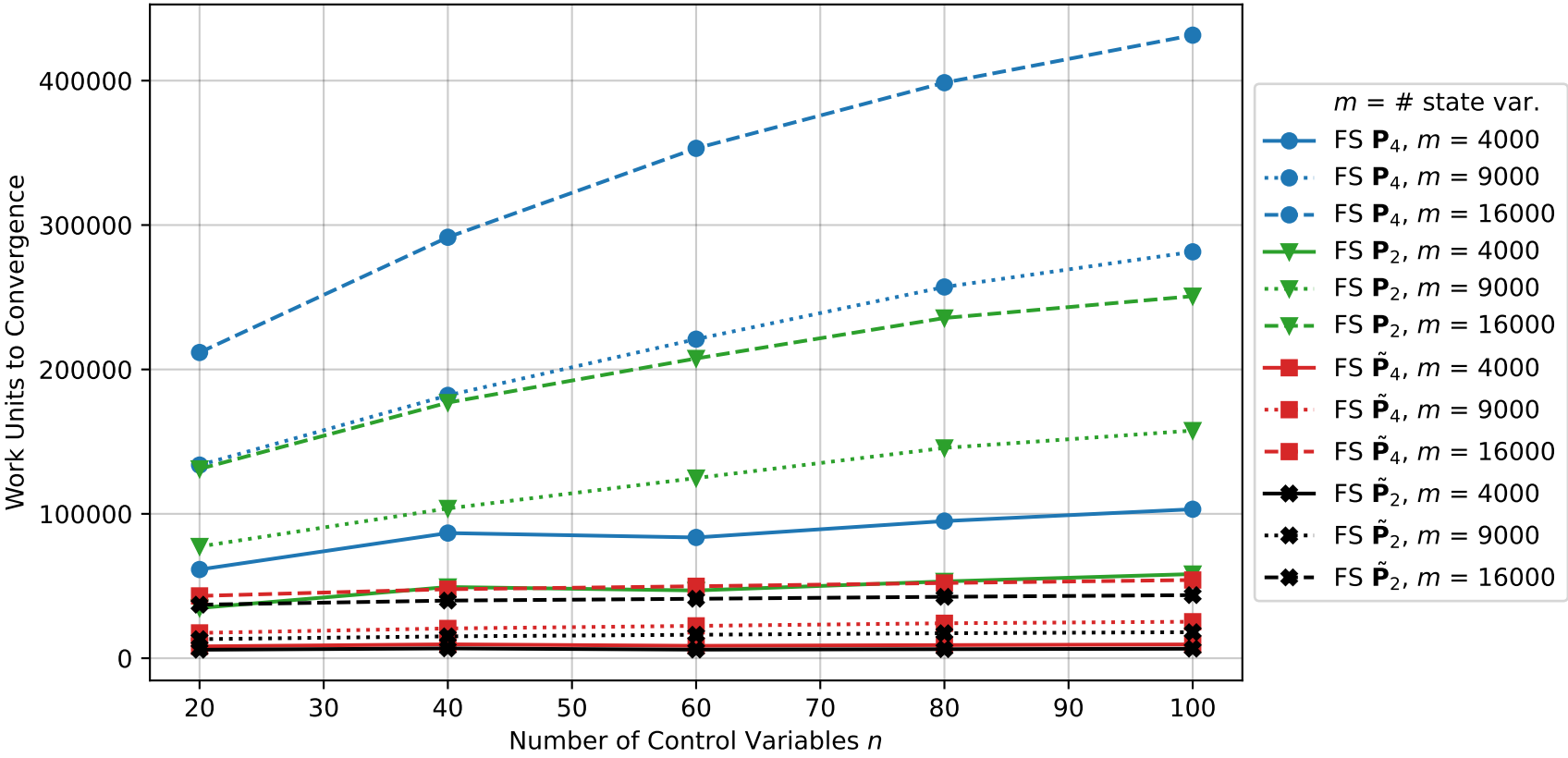

Full-space Gradient vs Subiterations, $m = 4000$
Gradient Norm
10^{-3}
10^{-5}
10^{-7}
10^{-9}
10^{-11}
0
200
400
600
800
1000
1200
1400
1600
Number of Subiterations
$\mathbf{P}_4$, $n = 20$
$\mathbf{P}_4$, $n = 60$
$\mathbf{P}_4$, $n = 100$
$\mathbf{P}_2$, $n = 20$
$\mathbf{P}_2$, $n = 60$
$\mathbf{P}_2$, $n = 100$
$\tilde{\mathbf{P}}_4$, $n = 20$
$\tilde{\mathbf{P}}_4$, $n = 60$
$\tilde{\mathbf{P}}_4$, $n = 100$
$\tilde{\mathbf{P}}_2$, $n = 20$
$\tilde{\mathbf{P}}_2$, $n = 60$
$\tilde{\mathbf{P}}_2$, $n = 100$

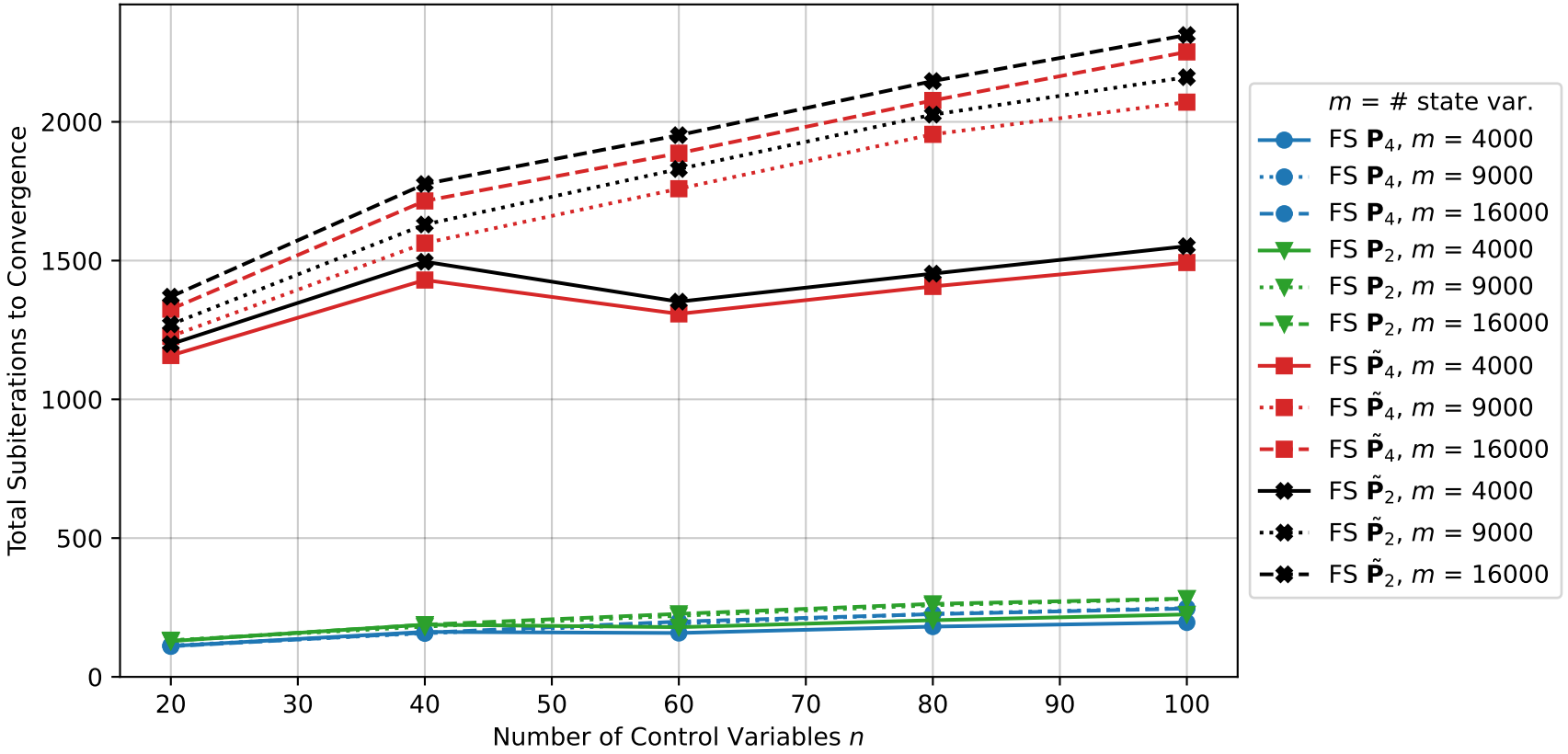

Full-space Gradient vs Subiterations, $m = 9000$
Gradient Norm
10^{-3}
10^{-5}
10^{-7}
10^{-9}
10^{-11}
0
500
1000
1500
2000
Number of Subiterations
$\mathbf{P}_4$, $n = 20$
$\mathbf{P}_4$, $n = 60$
$\mathbf{P}_4$, $n = 100$
$\mathbf{P}_2$, $n = 20$
$\mathbf{P}_2$, $n = 60$
$\mathbf{P}_2$, $n = 100$
$\tilde{\mathbf{P}}_4$, $n = 20$
$\tilde{\mathbf{P}}_4$, $n = 60$
$\tilde{\mathbf{P}}_4$, $n = 100$
$\tilde{\mathbf{P}}_2$, $n = 20$
$\tilde{\mathbf{P}}_2$, $n = 60$
$\tilde{\mathbf{P}}_2$, $n = 100$

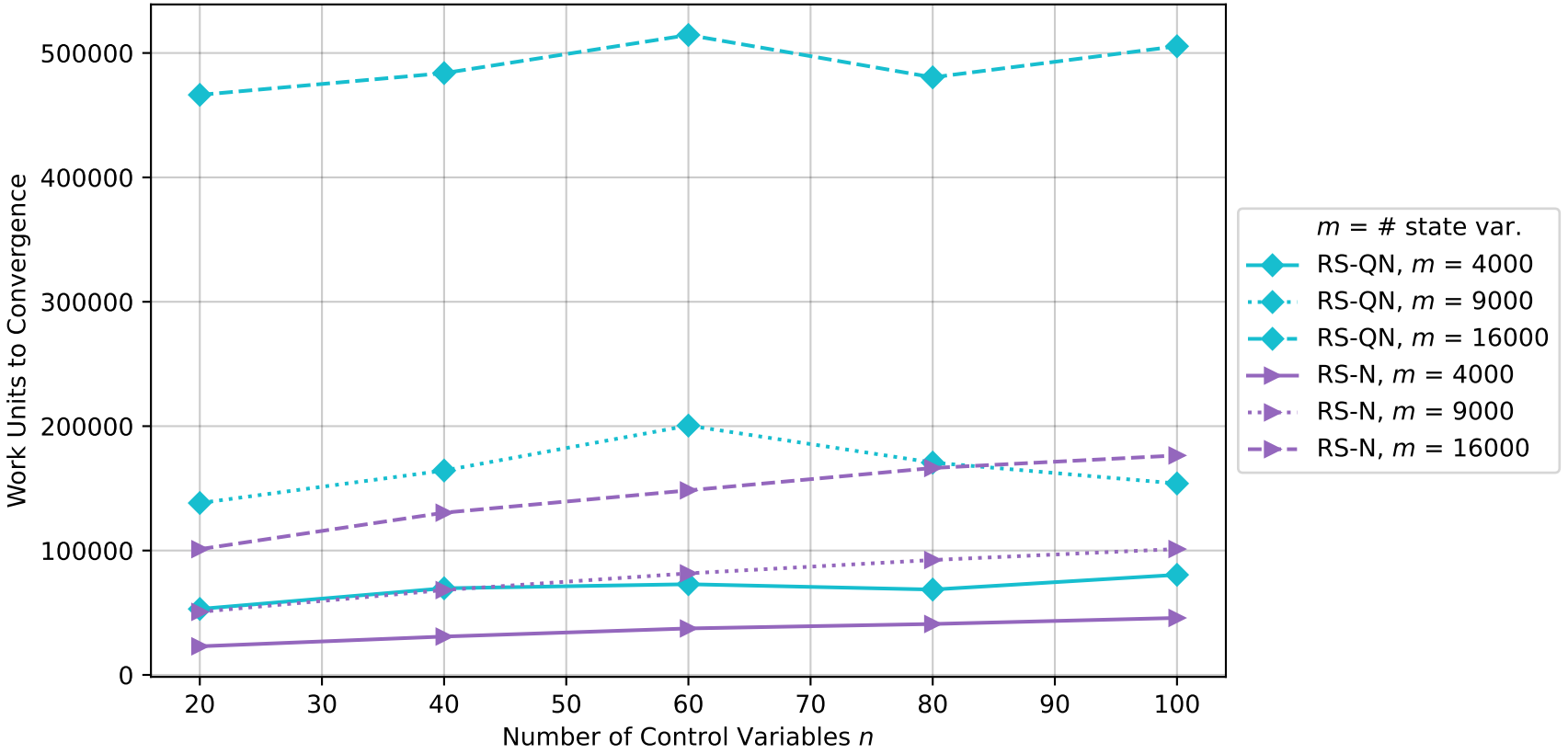

Full-space Gradient vs Subiterations, m = 16000
Gradient Norm
10^{-3}
10^{-5}
10^{-7}
10^{-9}
10^{-11}
0
500
1000
1500
2000
Number of Subiterations
$\mathbf{P}_4$, n = 20
$\mathbf{P}_4$, n = 60
$\mathbf{P}_4$, n = 100
$\mathbf{P}_2$, n = 20
$\mathbf{P}_2$, n = 60
$\mathbf{P}_2$, n = 100
$\tilde{\mathbf{P}}_4$, n = 20
$\tilde{\mathbf{P}}_4$, n = 60
$\tilde{\mathbf{P}}_4$, n = 100
$\tilde{\mathbf{P}}_2$, n = 20
$\tilde{\mathbf{P}}_2$, n = 60
$\tilde{\mathbf{P}}_2$, n = 100

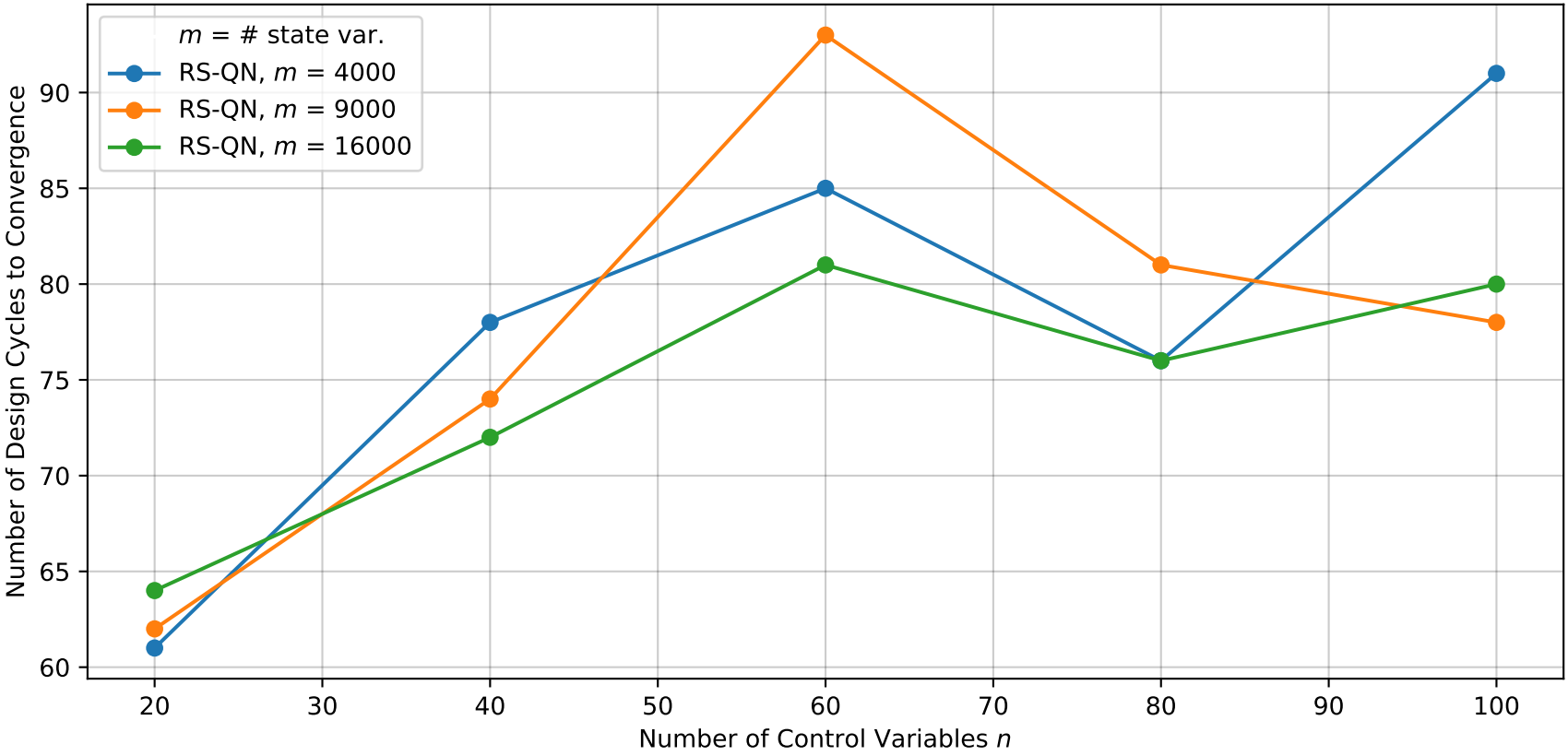

State Variables, m = 4000
State Variables, m = 9000
State Variables, m = 16000
Gradient Norm
10^{-3}
10^{-5}
10^{-7}
10^{-9}
10^{-11}
0
500
1000
1500
2000
Number of Subiterations
n = # control var.
$\mathbf{P}_4$, n = 20
$\mathbf{P}_4$, n = 60
$\mathbf{P}_4$, n = 100
$\mathbf{P}_2$, n = 20
$\mathbf{P}_2$, n = 60
$\mathbf{P}_2$, n = 100
$\tilde{\mathbf{P}}_4$, n = 20
$\tilde{\mathbf{P}}_4$, n = 60
$\tilde{\mathbf{P}}_4$, n = 100
$\tilde{\mathbf{P}}_2$, n = 20
$\tilde{\mathbf{P}}_2$, n = 60
$\tilde{\mathbf{P}}_2$, n = 100

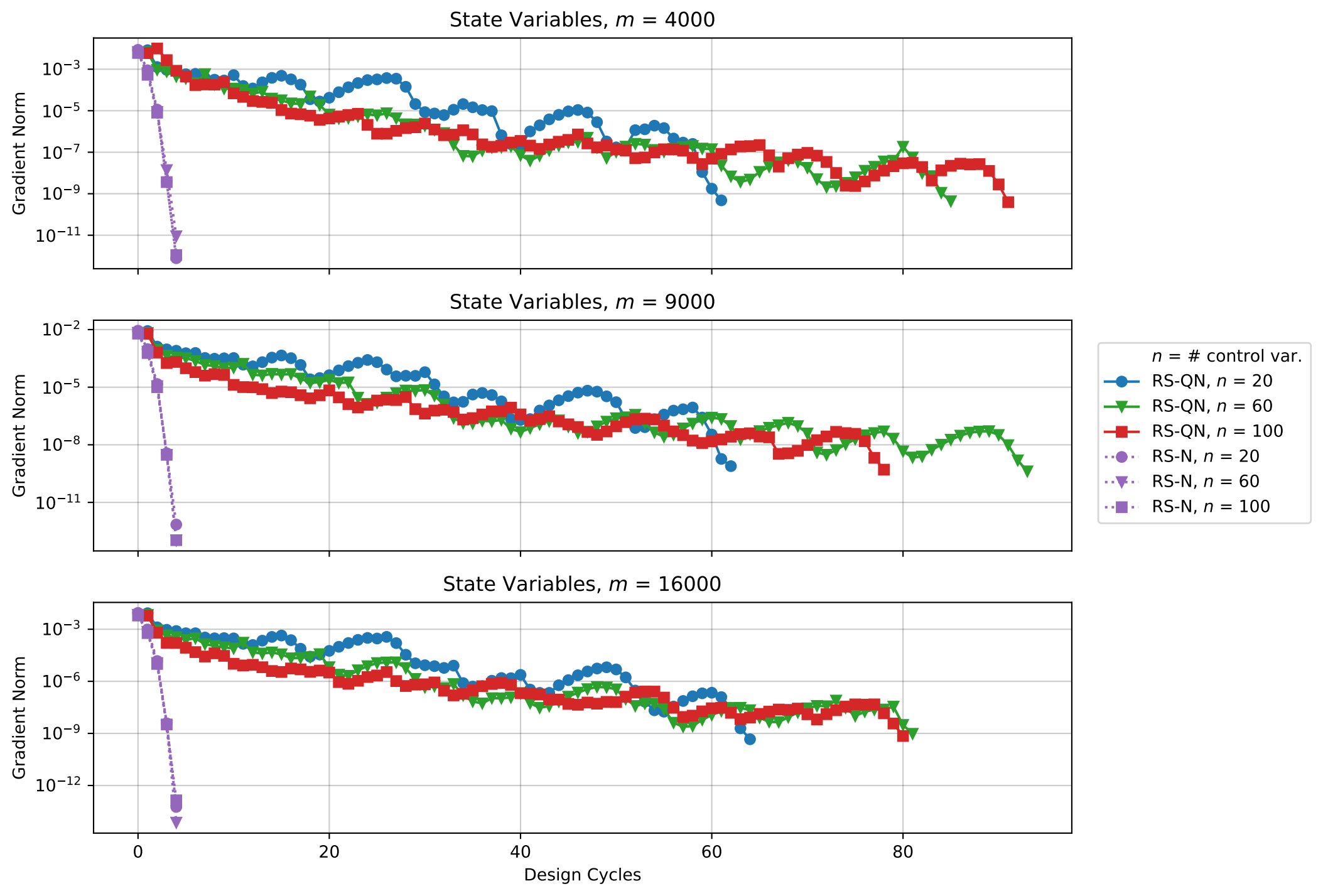

5
4
3
Number of Design Cycles to Convergence
20
30
40
50
60
70
80
90
100
Number of Control Variables n
m = # state var.
FS $\mathbf{P}_4$, m = 4000
FS $\mathbf{P}_4$, m = 9000
FS $\mathbf{P}_4$, m = 16000
FS $\mathbf{P}_2$, m = 4000
FS $\mathbf{P}_2$, m = 9000
FS $\mathbf{P}_2$, m = 16000
FS $\tilde{\mathbf{P}}_4$, m = 4000
FS $\tilde{\mathbf{P}}_4$, m = 9000
FS $\tilde{\mathbf{P}}_4$, m = 16000
FS $\tilde{\mathbf{P}}_2$, m = 4000
FS $\tilde{\mathbf{P}}_2$, m = 9000
FS $\tilde{\mathbf{P}}_2$, m = 16000

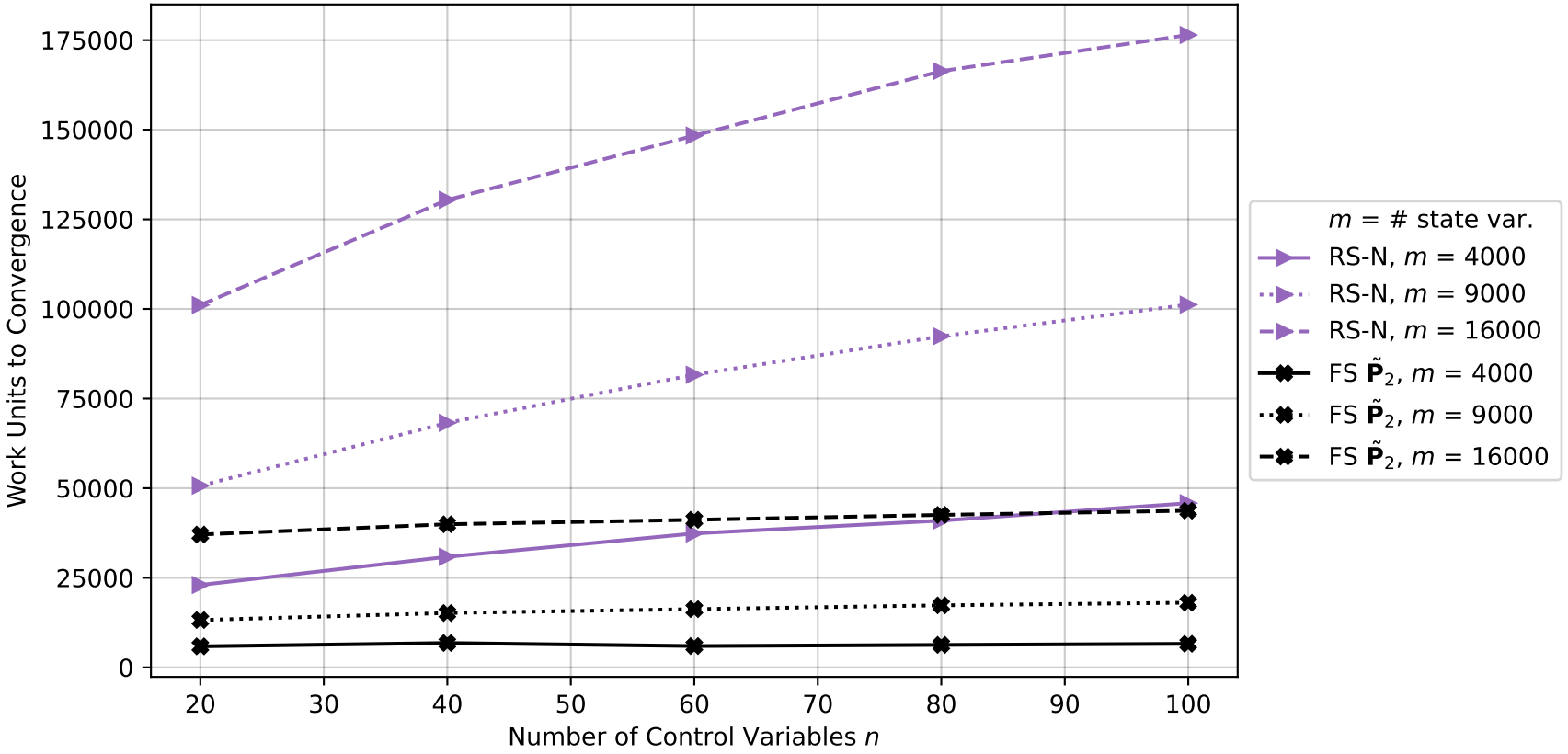

Work Units to Convergence
400000
300000
200000
100000
0
20
30
40
50
60
70
80
90
100
Number of Control Variables n
m = # state var.
FS $\mathbf{P}_4$, m = 4000
FS $\mathbf{P}_4$, m = 9000
FS $\mathbf{P}_4$, m = 16000
FS $\mathbf{P}_2$, m = 4000
FS $\mathbf{P}_2$, m = 9000
FS $\mathbf{P}_2$, m = 16000
FS $\tilde{\mathbf{P}}_4$, m = 4000
FS $\tilde{\mathbf{P}}_4$, m = 9000
FS $\tilde{\mathbf{P}}_4$, m = 16000
FS $\tilde{\mathbf{P}}_2$, m = 4000
FS $\tilde{\mathbf{P}}_2$, m = 9000
FS $\tilde{\mathbf{P}}_2$, m = 16000

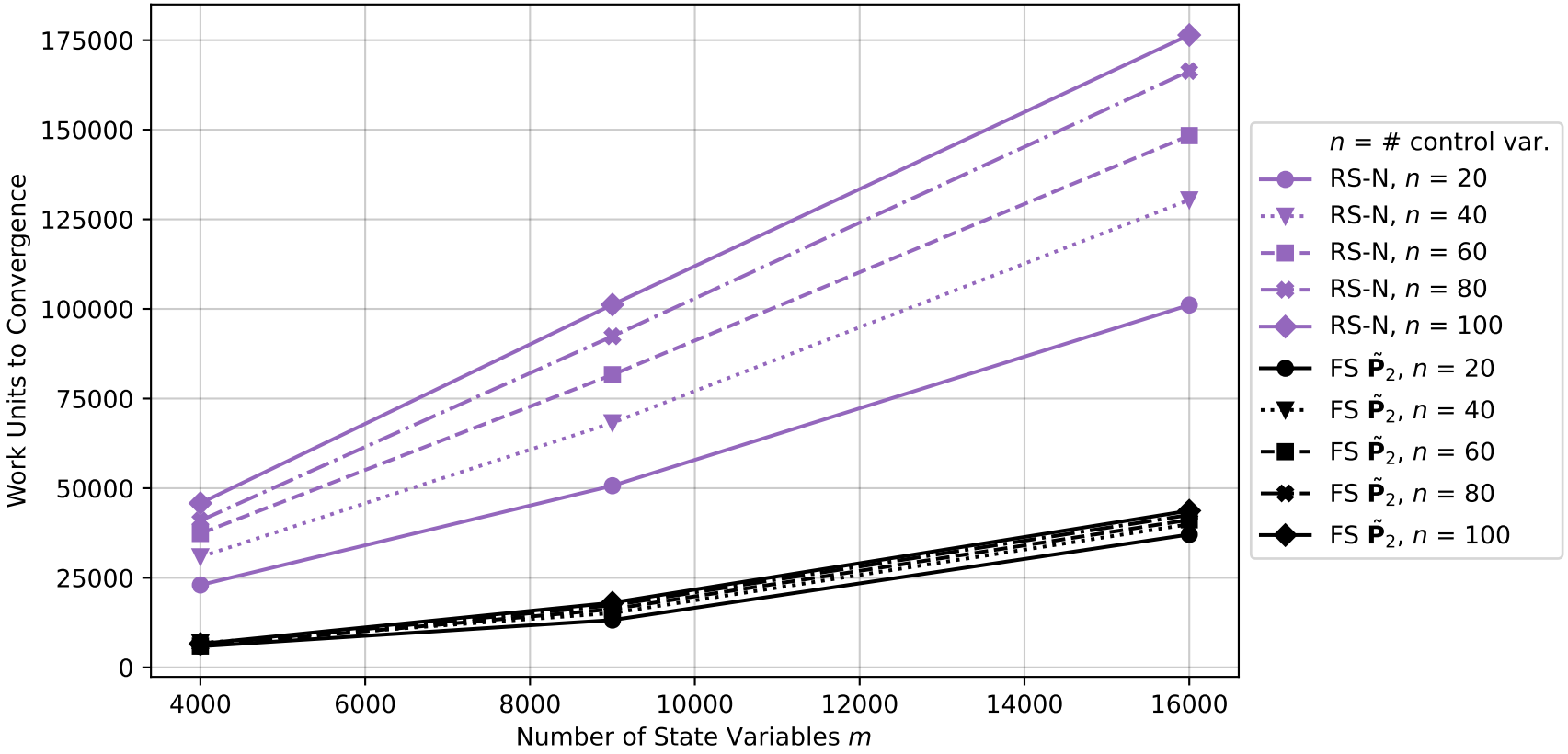

Total Subiterations to Convergence
0
500
1000
1500
2000
20
30
40
50
60
70
80
90
100
Number of Control Variables n
m = # state var.
FS $\mathbf{P}_4$, m = 4000
FS $\mathbf{P}_4$, m = 9000
FS $\mathbf{P}_4$, m = 16000
FS $\mathbf{P}_2$, m = 4000
FS $\mathbf{P}_2$, m = 9000
FS $\mathbf{P}_2$, m = 16000
FS $\tilde{\mathbf{P}}_4$, m = 4000
FS $\tilde{\mathbf{P}}_4$, m = 9000
FS $\tilde{\mathbf{P}}_4$, m = 16000
FS $\tilde{\mathbf{P}}_2$, m = 4000
FS $\tilde{\mathbf{P}}_2$, m = 9000
FS $\tilde{\mathbf{P}}_2$, m = 16000

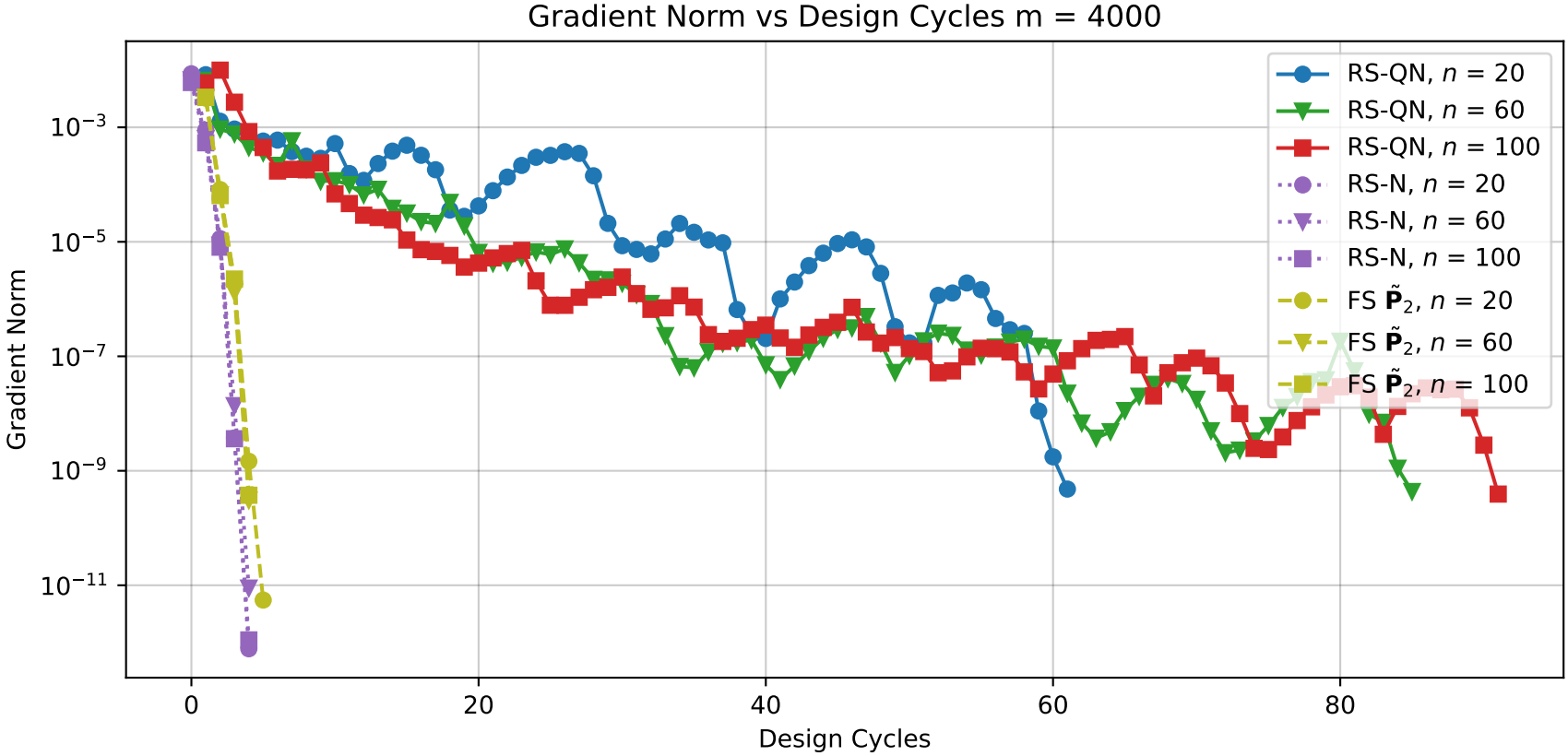

Work Units to Convergence
0
100000
200000
300000
400000
500000
20
30
40
50
60
70
80
90
100
Number of Control Variables n
m = # state var.
RS-QN, m = 4000
RS-QN, m = 9000
RS-QN, m = 16000
RS-N, m = 4000
RS-N, m = 9000
RS-N, m = 16000


m = # state var.
RS-QN, m = 4000
RS-QN, m = 9000
RS-QN, m = 16000
Number of Design Cycles to Convergence
Number of Control Variables n
60
65
70
75
80
85
90
20
30
40
50
60
70
80
90
100


State Variables, m = 4000
State Variables, m = 9000
State Variables, m = 16000
Gradient Norm
10^{-3}
10^{-5}
10^{-7}
10^{-9}
10^{-11}
10^{-2}
10^{-8}
10^{-6}
10^{-12}
0
20
40
60
80
Design Cycles
n = # control var.
RS-QN, n = 20
RS-QN, n = 60
RS-QN, n = 100
RS-N, n = 20
RS-N, n = 60
RS-N, n = 100


175000
150000
125000
100000
75000
50000
25000
0
Work Units to Convergence
20
30
40
50
60
70
80
90
100
Number of Control Variables n
m = # state var.
RS-N, m = 4000
RS-N, m = 9000
RS-N, m = 16000
FS $\tilde{\mathbf{P}}_2$, m = 4000
FS $\tilde{\mathbf{P}}_2$, m = 9000
FS $\tilde{\mathbf{P}}_2$, m = 16000


Work Units to Convergence
175000
150000
125000
100000
75000
50000
25000
0
4000
6000
8000
10000
12000
14000
16000
Number of State Variables m
n = # control var.
RS-N, n = 20
RS-N, n = 40
RS-N, n = 60
RS-N, n = 80
RS-N, n = 100
FS $\tilde{\mathbf{P}}_2$, n = 20
FS $\tilde{\mathbf{P}}_2$, n = 40
FS $\tilde{\mathbf{P}}_2$, n = 60
FS $\tilde{\mathbf{P}}_2$, n = 80
FS $\tilde{\mathbf{P}}_2$, n = 100


Gradient Norm vs Design Cycles m = 4000
Gradient Norm
Design Cycles
10^{-3}
10^{-5}
10^{-7}
10^{-9}
10^{-11}
0
20
40
60
80
RS-QN, n = 20
RS-QN, n = 60
RS-QN, n = 100
RS-N, n = 20
RS-N, n = 60
RS-N, n = 100
FS $\tilde{\mathbf{P}}_2$, n = 20
FS $\tilde{\mathbf{P}}_2$, n = 60
FS $\tilde{\mathbf{P}}_2$, n = 100