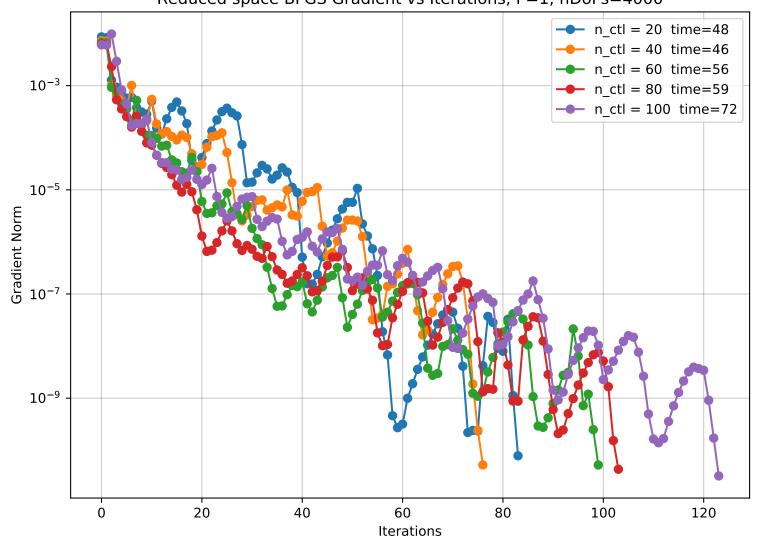
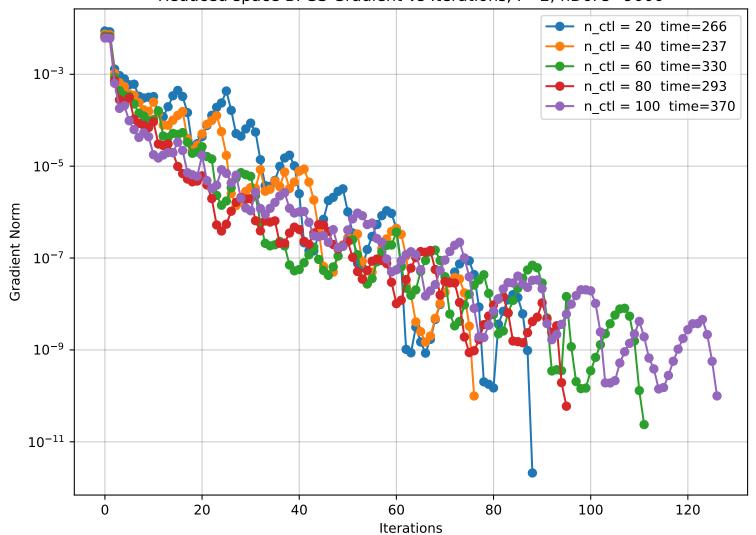
Reduced space BFGS Gradient vs Iterations, P=1, nDoFs=4000



Reduced space BFGS Gradient vs Iterations, P=2, nDoFs=9000



Reduced space BFGS Gradient vs Iterations, P=3, nDoFs=16000 n_ctl = 20 time=1044 - n_ctl = 40 time=841 - n_ctl = 60 time=930 10^{-3} - n_ctl = 80 time=1073 - n_ctl = 100 time=1174 10⁻⁵ **Gradient Norm** 10^{-7} 10^{-9}

40

60

Iterations

80

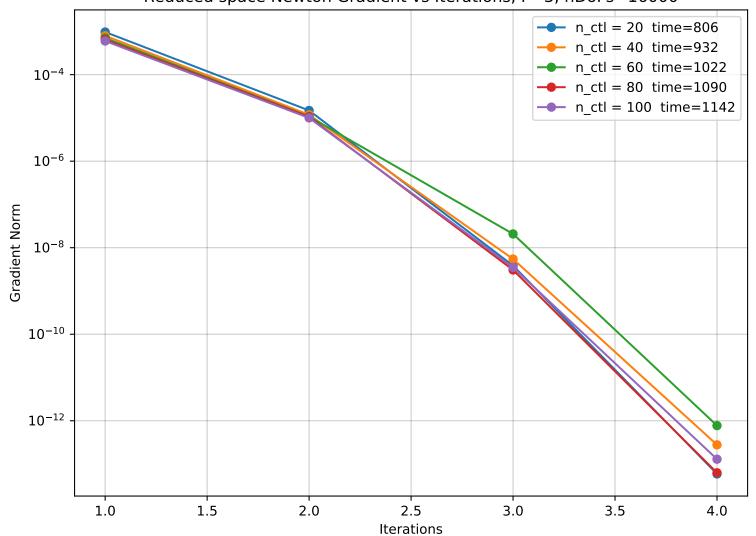
100

20

Reduced space Newton Gradient vs Iterations, P=1, nDoFs=4000 10^{-3} - n_ctl = 20 time=15 - n_ctl = 40 time=16 \rightarrow n ctl = 60 time=17 - n_ctl = 80 time=19 - n_ctl = 100 time=19 10⁻⁵ **Gradient Norm** 10^{-7} 10^{-9} 10^{-11} 2.0 2.5 3.0 1.0 1.5 3.5 4.0 Iterations

Reduced space Newton Gradient vs Iterations, P=2, nDoFs=9000 -- n_ctl = 20 time=161 -- n_ctl = 40 time=191 \rightarrow n ctl = 60 time=211 10^{-4} - n_ctl = 80 time=229 - n_ctl = 100 time=240 10^{-6} **Gradient Norm** 10-8 10^{-10} 10-12 2.0 2.5 3.0 1.0 1.5 3.5 4.0

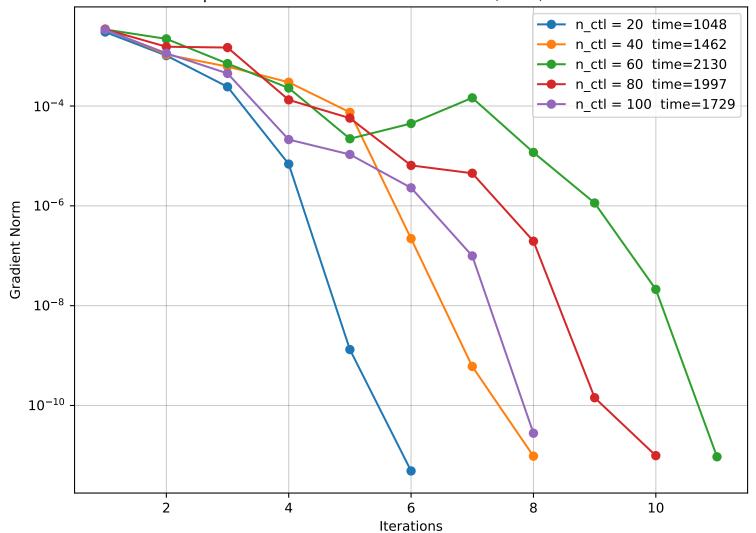
Reduced space Newton Gradient vs Iterations, P=3, nDoFs=16000



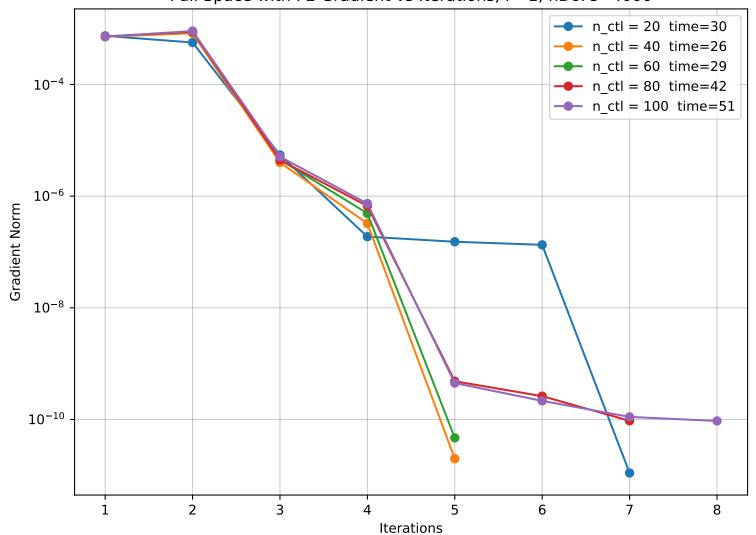
Full Space with P2A Gradient vs Iterations, P=1, nDoFs=4000 - n_ctl = 20 time=31 - n_ctl = 40 time=44 \rightarrow n ctl = 60 time=56 10^{-4} - n_ctl = 80 time=66 - n_ctl = 100 time=73 10^{-6} **Gradient Norm** 10^{-8} 10^{-10} 10^{-12} 2 3 5 6 Iterations

Full Space with P2A Gradient vs Iterations, P=2, nDoFs=9000 10^{-2} - n_ctl = 20 time=252 n_ctl = 40 time=298 - n ctl = 60 time=978 - n_ctl = 80 time=446 - n_ctl = 100 time=541 10^{-4} **Gradient Norm** 10^{-6} 10^{-8} 10^{-10} 2.5 5.0 7.5 10.0 20.0 12.5 15.0 17.5 **Iterations**

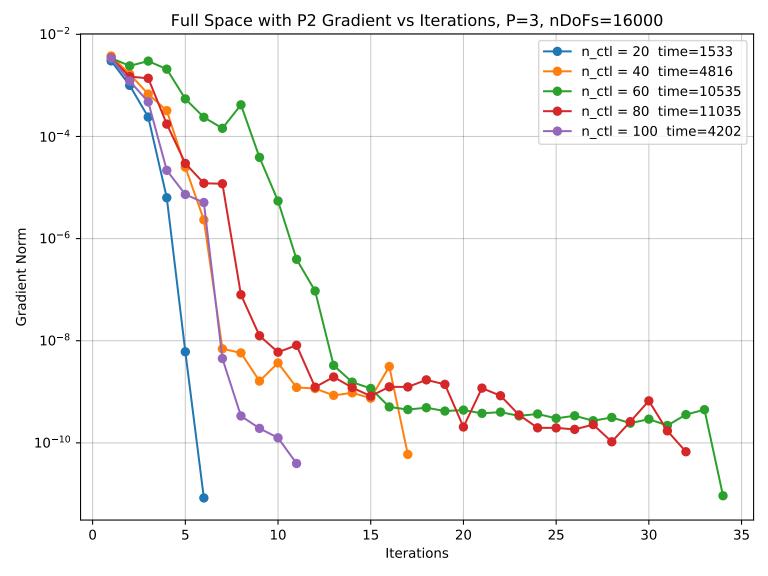
Full Space with P2A Gradient vs Iterations, P=3, nDoFs=16000



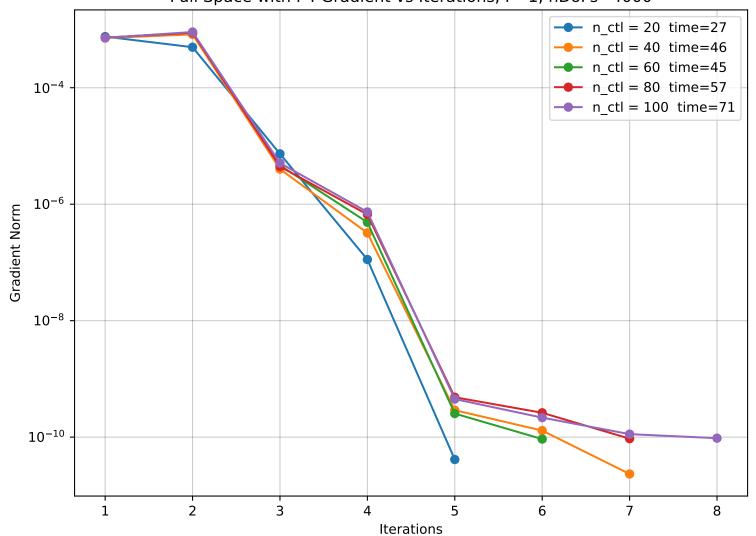
Full Space with P2 Gradient vs Iterations, P=1, nDoFs=4000



Full Space with P2 Gradient vs Iterations, P=2, nDoFs=9000 - n_ctl = 20 time=787 -- n_ctl = 40 time=1357 10^{-3} - n ctl = 60 time=1202 - n_ctl = 80 time=707 - n_ctl = 100 time=1067 10^{-4} 10^{-5} **Gradient Norm** 10^{-6} 10^{-7} 10^{-8} 10^{-9} 10^{-10} 20 5 10 15



Full Space with P4 Gradient vs Iterations, P=1, nDoFs=4000



Full Space with P4 Gradient vs Iterations, P=2, nDoFs=9000 - n_ctl = 20 time=409 -- n_ctl = 40 time=670 10^{-3} -- n ctl = 60 time=1802 -- n_ctl = 80 time=1811 n_ctl = 100 time=2508 10^{-5} **Gradient Norm** 10^{-7} 10^{-9} 10^{-11}

10

Iterations

15

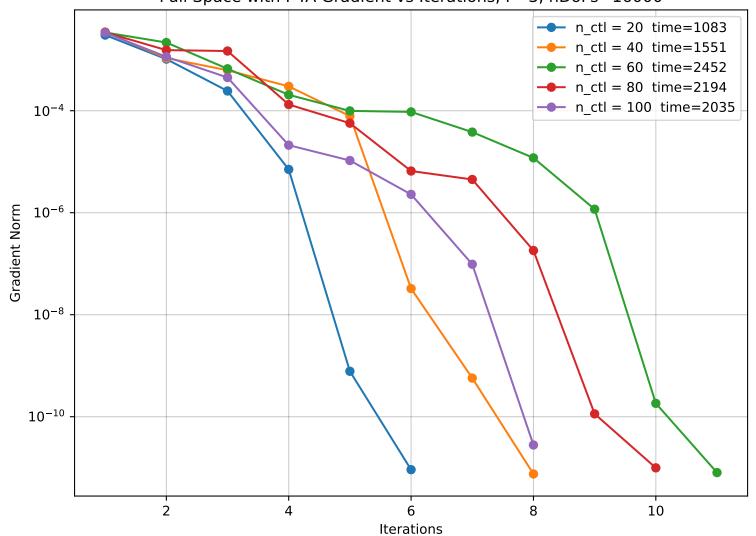
20

Full Space with P4 Gradient vs Iterations, P=3, nDoFs=16000 - n_ctl = 20 time=2228 -- n_ctl = 40 time=9270 10^{-3} -- n ctl = 60 time=11060 - n_ctl = 80 time=10138 - n_ctl = 100 time=17599 10^{-4} 10⁻⁵ **Gradient Norm** 10^{-6} 10^{-7} 10^{-8} 10^{-9} 10^{-10} 20 10 15 25 30 35

Full Space with P4A Gradient vs Iterations, P=1, nDoFs=4000 - n_ctl = 20 time=26 - n_ctl = 40 time=42 - n ctl = 60 time=52 10^{-4} - n_ctl = 80 time=51 - n_ctl = 100 time=66 10^{-6} **Gradient Norm** 10^{-8} 10^{-10} 2 3 5 6

Full Space with P4A Gradient vs Iterations, P=2, nDoFs=9000 - n_ctl = 20 time=279 - n_ctl = 40 time=317 \rightarrow n ctl = 60 time=516 - n_ctl = 80 time=484 - n_ctl = 100 time=565 10^{-4} 10^{-6} **Gradient Norm** 10^{-8} 10^{-10} 2 8 10 4 Iterations

Full Space with P4A Gradient vs Iterations, P=3, nDoFs=16000



Iterations vs Design Variables → Reduced space BFGS P=1 Reduced space BFGS P=2 120 Reduced space BFGS P=3 Full Space P2A P=1 Full Space P2A P=2 Full Space P2A P=3 100 Number of Iterations to Convergence Full Space P2 P=1 Full Space P2 P=2 Full Space P2 P=3 80 Full Space P4 P=1 Full Space P4 P=2 Full Space P4 P=3 Full Space P4A P=1 60 Full Space P4A P=2 Full Space P4A P=3 40 20 0 20 30 50 60 70 80 100 40 90 Number of Design Variables

Number of matrix-vector vmult vs Design Variables Full Space P2A P=1 600000 -Full Space P2A P=2 Full Space P2A P=3 Full Space P2 P=1 Full Space P2 P=2 500000 -Full Space P2 P=3 Full Space P4 P=1 Number of vmult to Convergence Full Space P4 P=2 Full Space P4 P=3 400000 -Full Space P4A P=1 Full Space P4A P=2 Full Space P4A P=3 300000 -200000 100000 0 30 20 40 50 60 70 80 90 100 Number of Design Variables

Number of matrix-vector vmult vs Design Variables 450000 Reduced space BFGS P=1 Reduced space BFGS P=2 Reduced space BFGS P=3 400000 Reduced space Newton P=1 form AD operators Reduced space Newton P=2 form AD operators Reduced space Newton P=3 form AD operators 350000 -Reduced space Newton P=1 apply AD operators Convergence Reduced space Newton P=2 apply AD operators Reduced space Newton P=3 apply AD operators 300000 Number of vmult to 250000 200000 150000 100000 50000 30 20 40 50 60 70 80 90 100 Number of Design Variables

