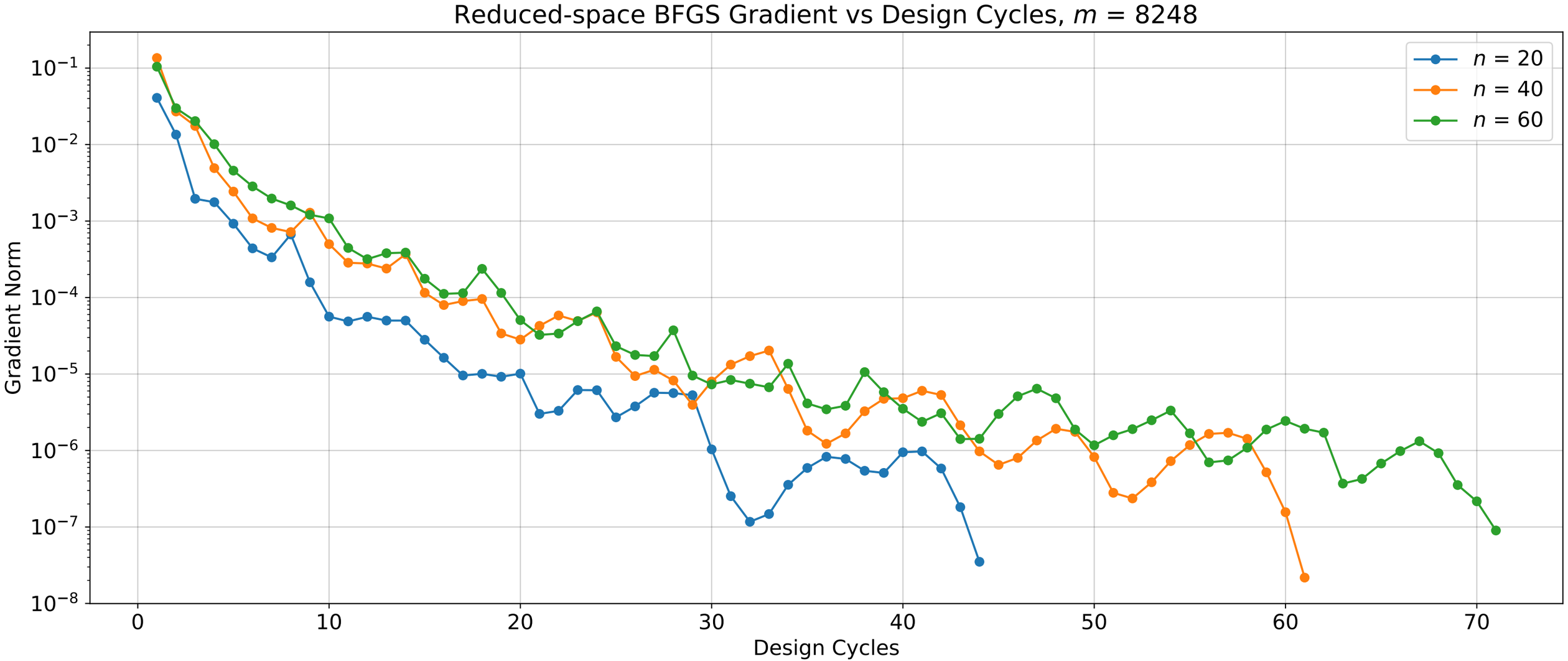
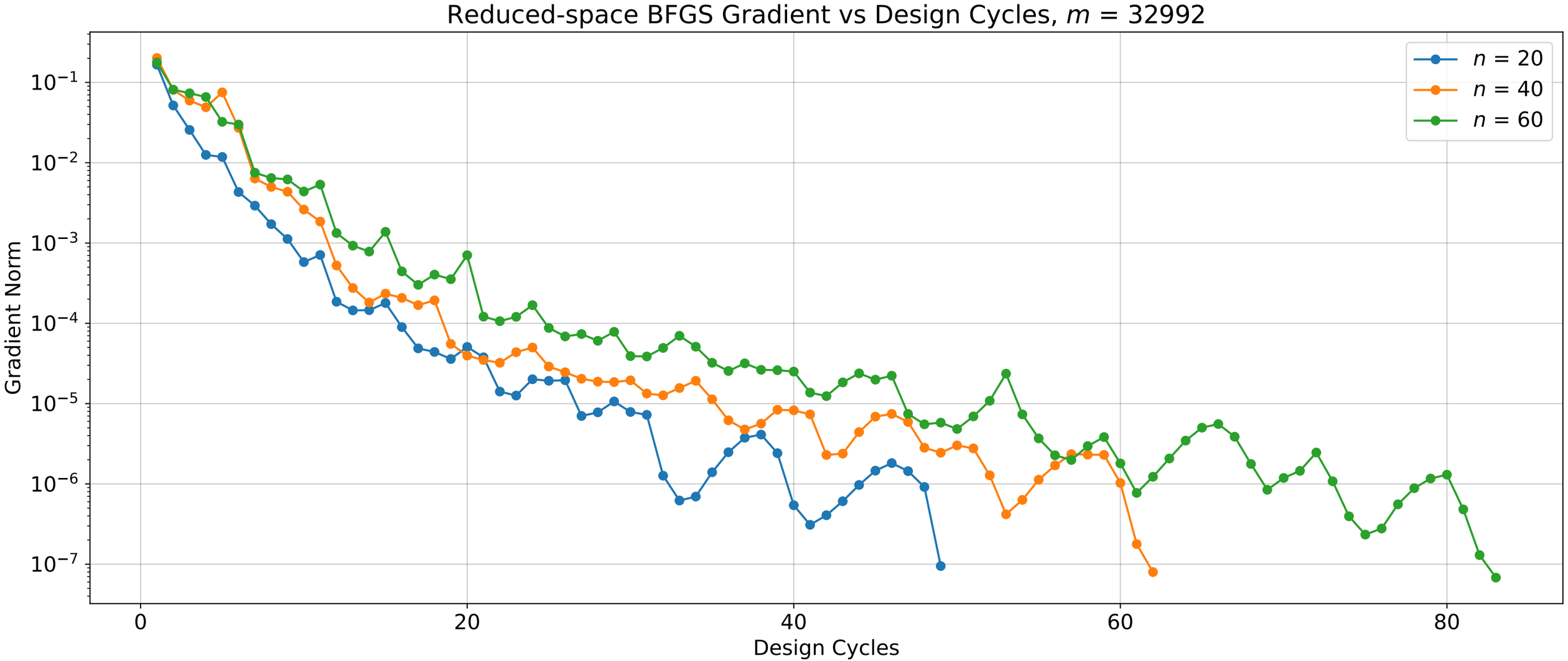
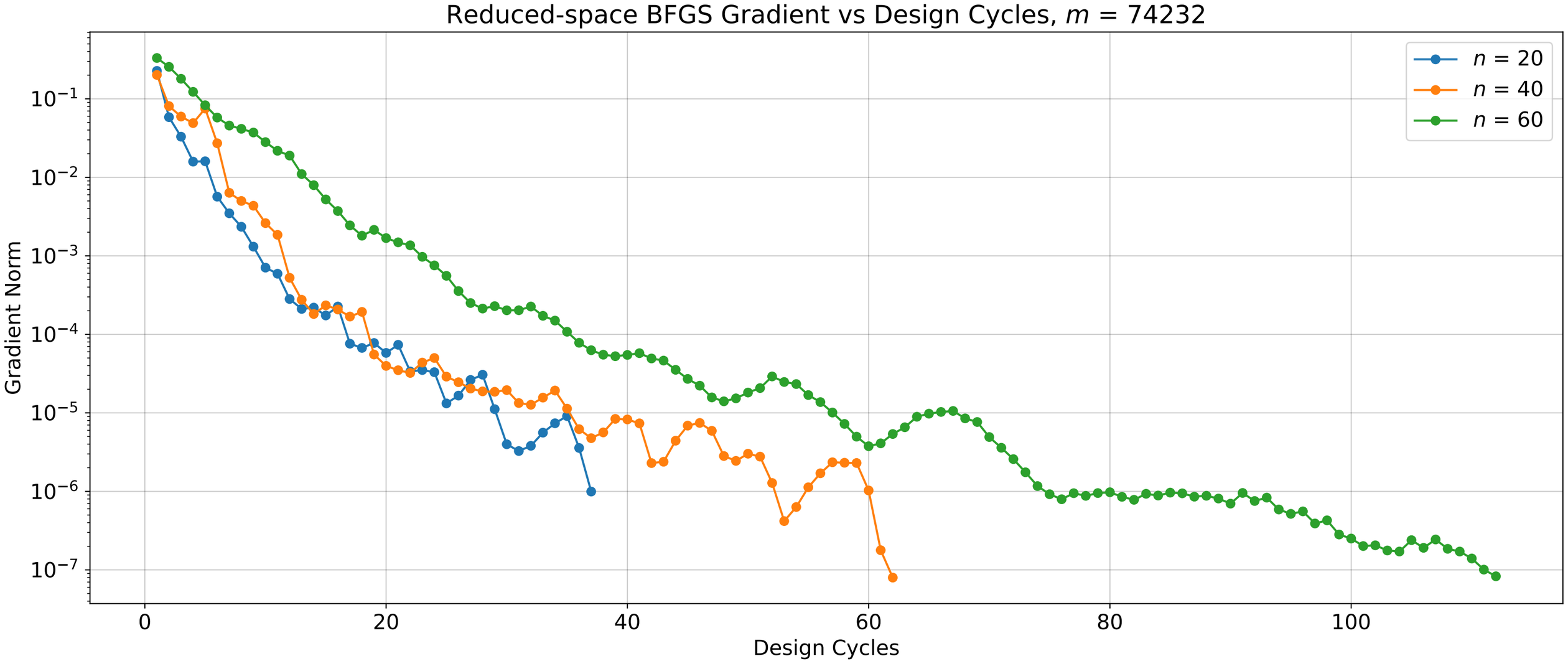
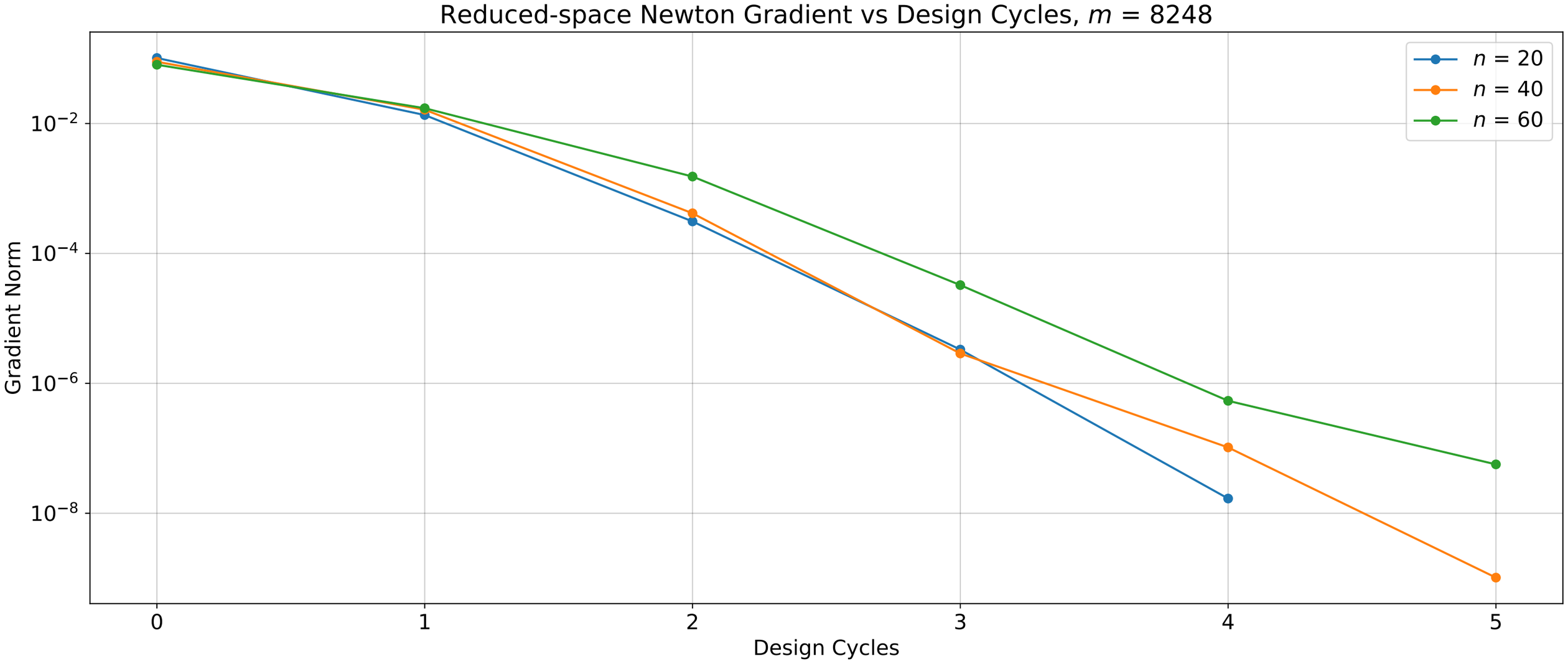
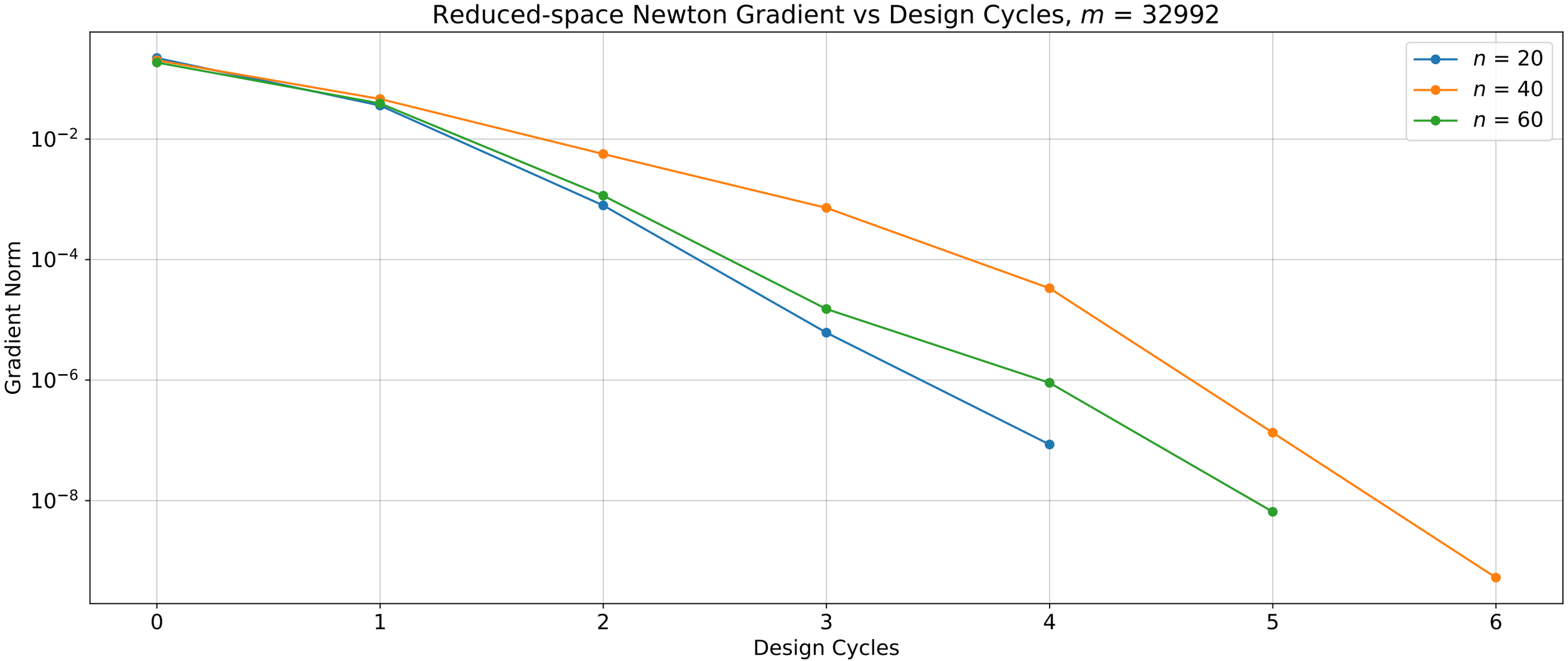
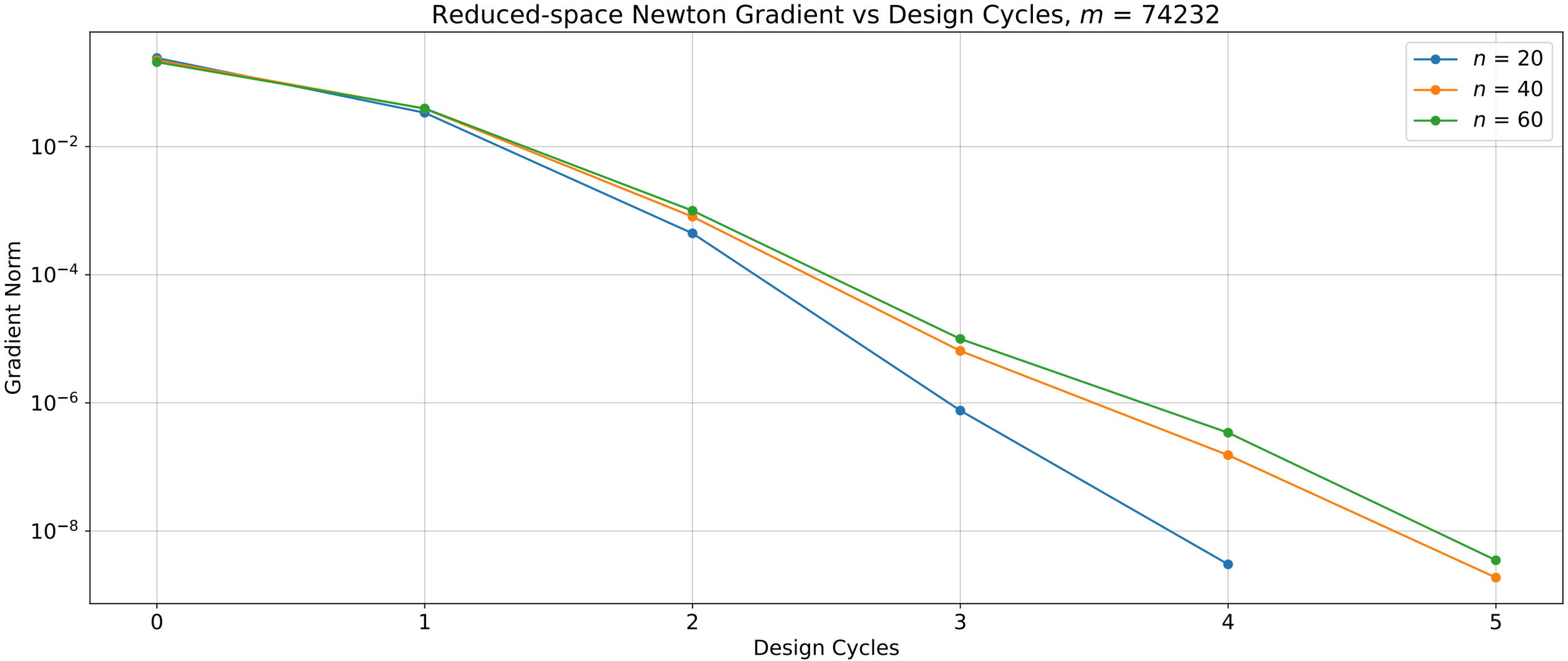

Reduced-space BFGS Gradient vs Design Cycles, m = 8248
Gradient Norm
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-8}
0
10
20
30
40
50
60
70
Design Cycles
n = 20
n = 40
n = 60

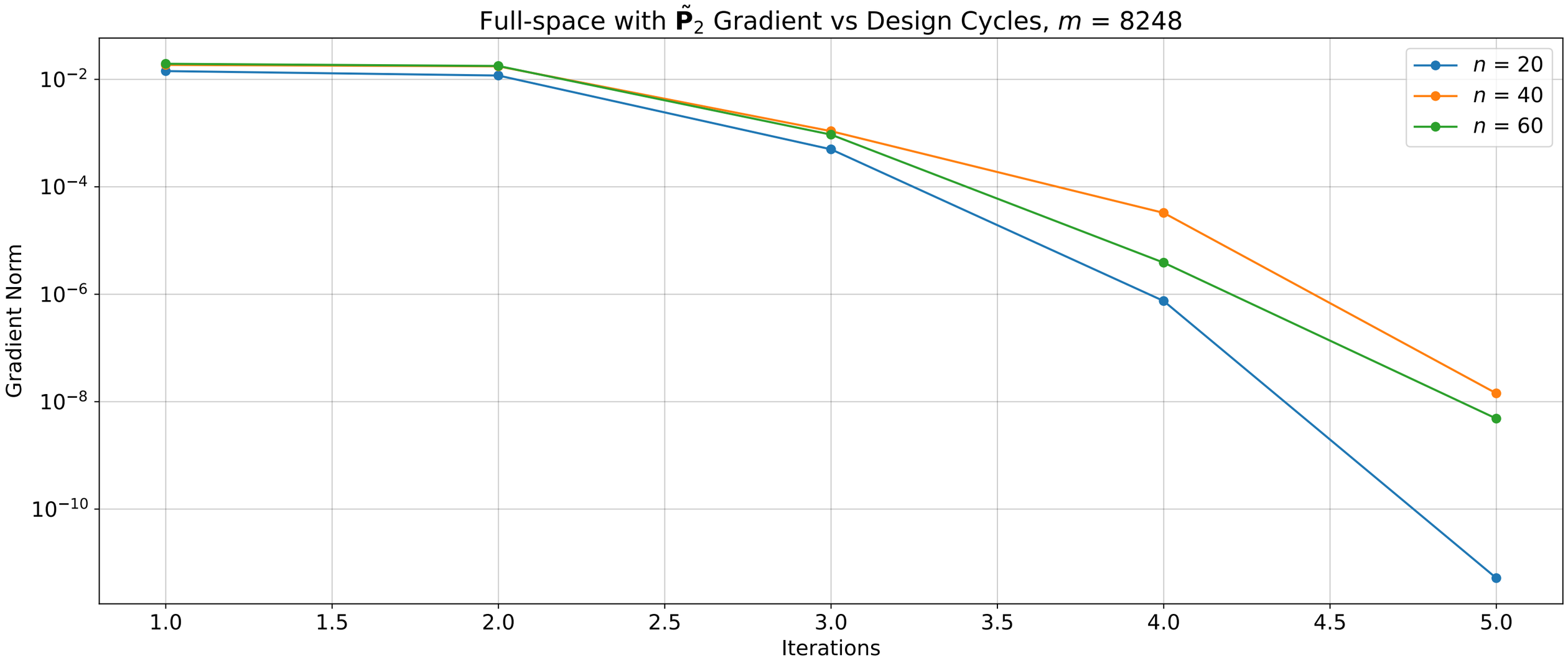

Reduced-space BFGS Gradient vs Design Cycles, m = 32992
Gradient Norm
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
0
20
40
60
80
Design Cycles
n = 20
n = 40
n = 60

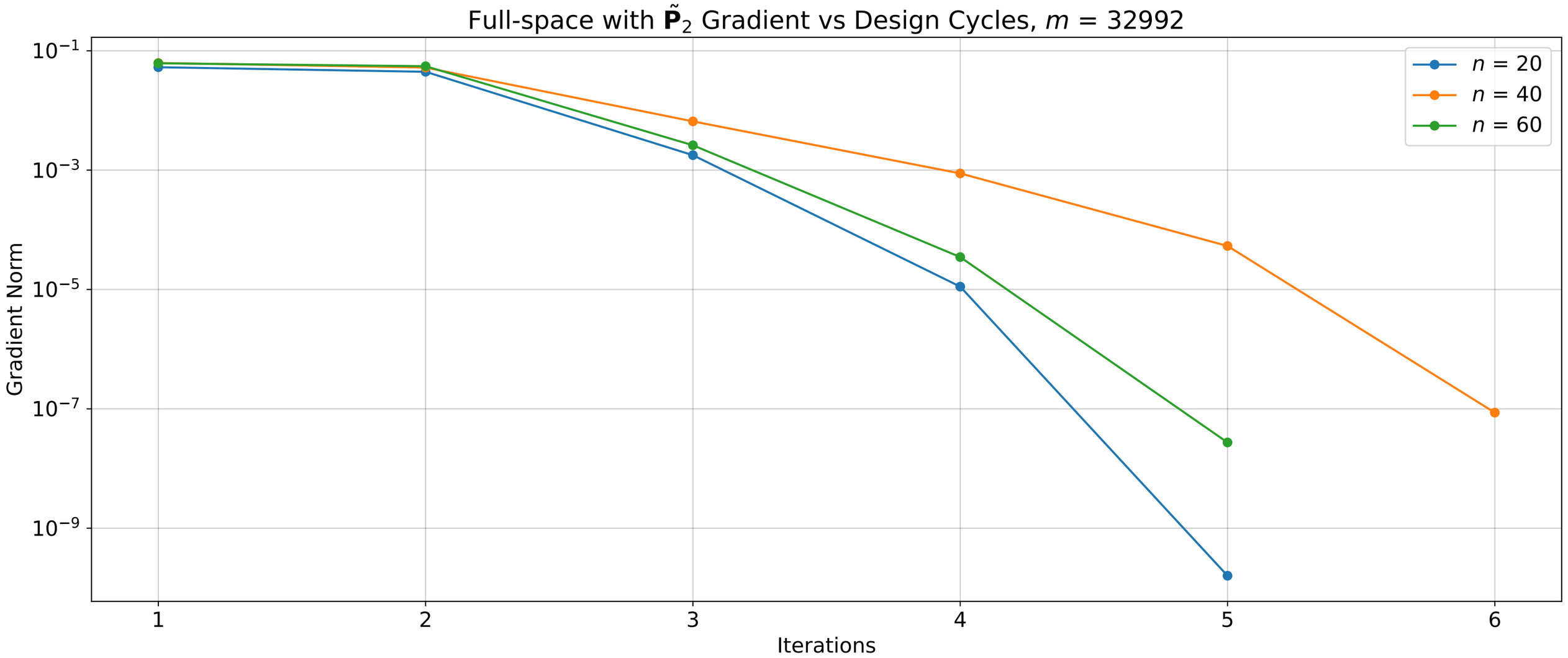

Reduced-space BFGS Gradient vs Design Cycles, m = 74232
Gradient Norm
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
0
20
40
60
80
100
Design Cycles
n = 20
n = 40
n = 60

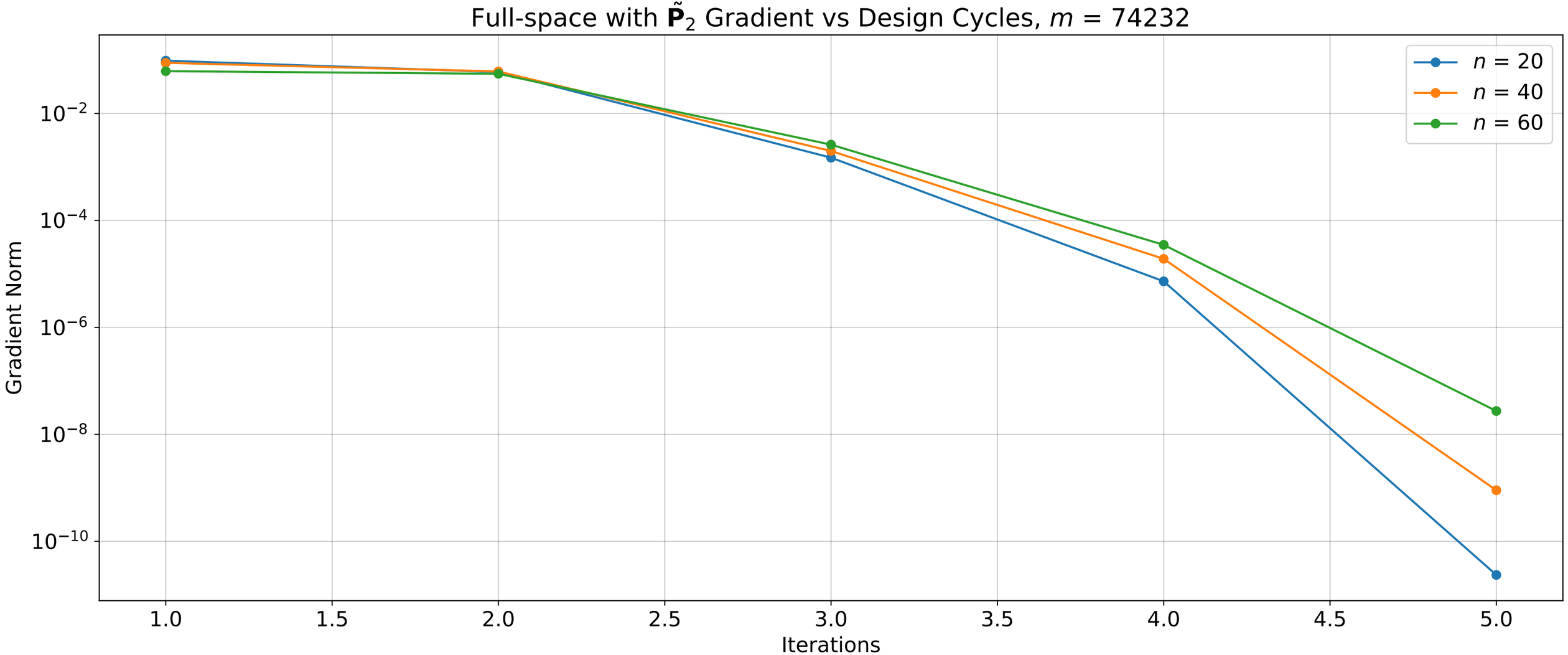

Reduced-space Newton Gradient vs Design Cycles, m = 8248
Gradient Norm
10^{-2}
10^{-4}
10^{-6}
10^{-8}
0
1
2
3
4
5
Design Cycles
n = 20
n = 40
n = 60

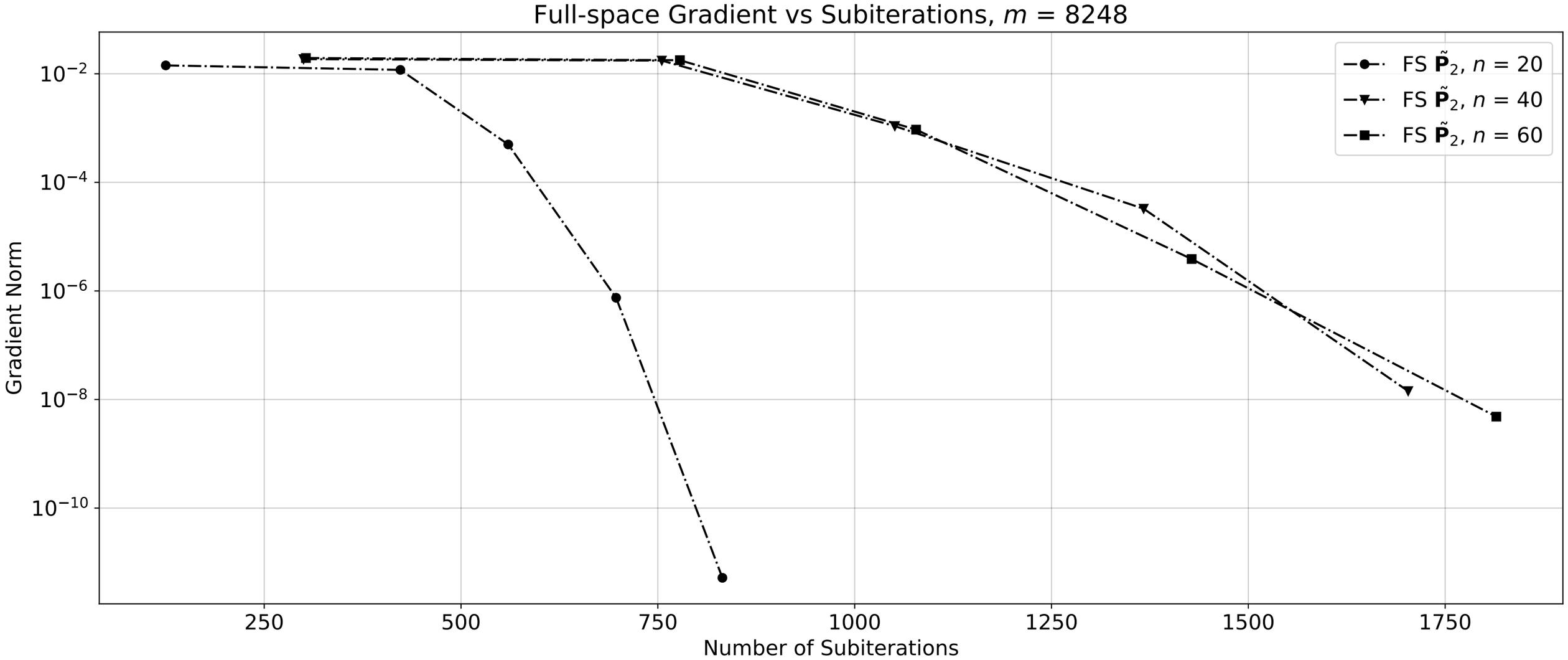

Reduced-space Newton Gradient vs Design Cycles, m = 32992
Gradient Norm
10^{-2}
10^{-4}
10^{-6}
10^{-8}
0
1
2
3
4
5
6
Design Cycles
n = 20
n = 40
n = 60

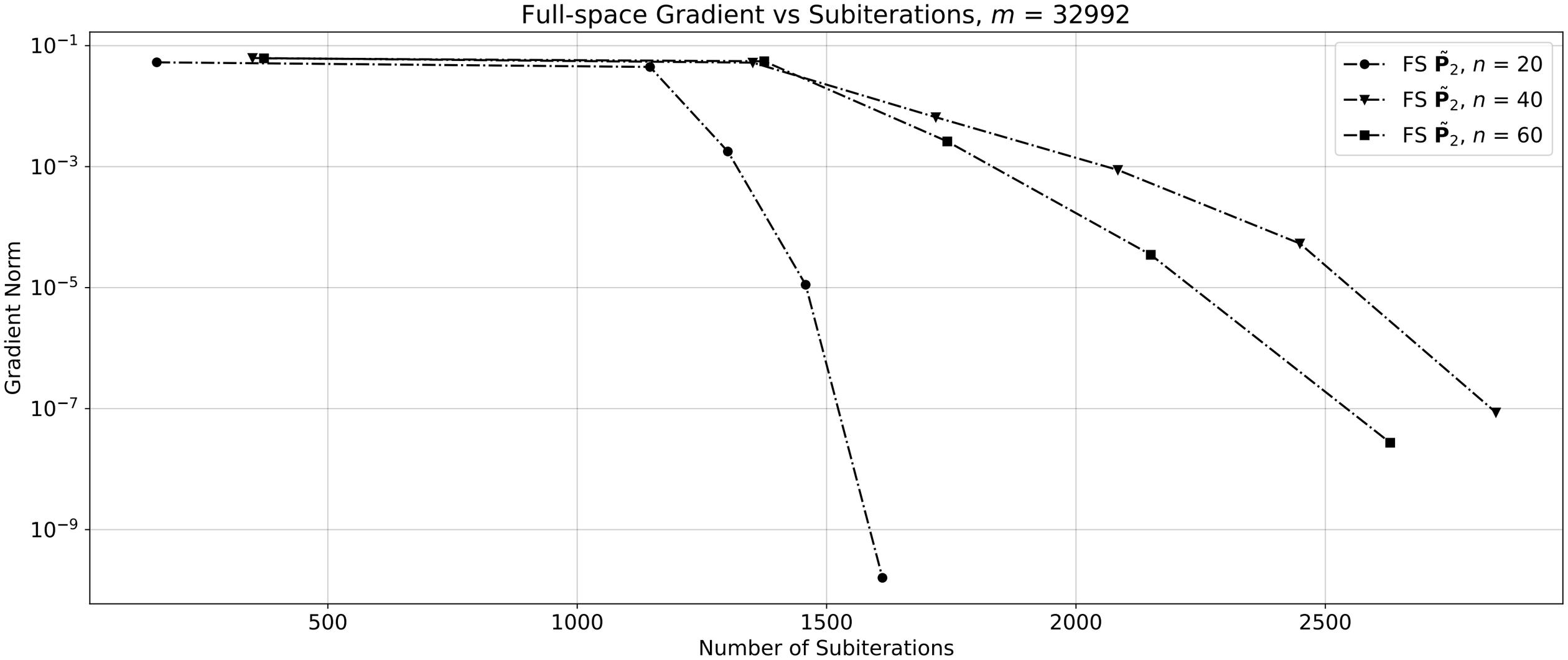

Reduced-space Newton Gradient vs Design Cycles, m = 74232
n = 20
n = 40
n = 60
Gradient Norm
10^{-2}
10^{-4}
10^{-6}
10^{-8}
0
1
2
3
4
5
Design Cycles

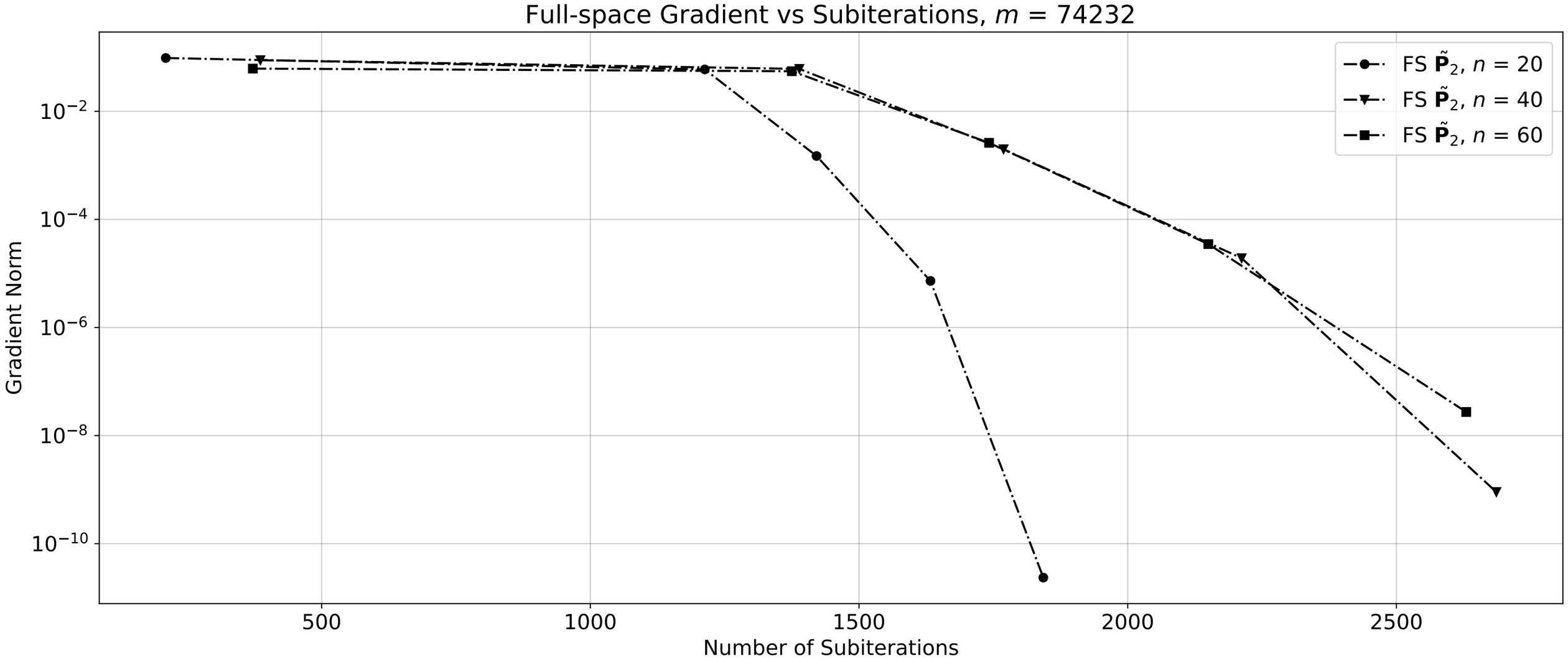

Full-space with $\tilde{\mathbf{P}}_2$ Gradient vs Design Cycles, m = 8248
n = 20
n = 40
n = 60
10^{-2}
10^{-4}
10^{-6}
10^{-8}
10^{-10}
Gradient Norm
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0
Iterations

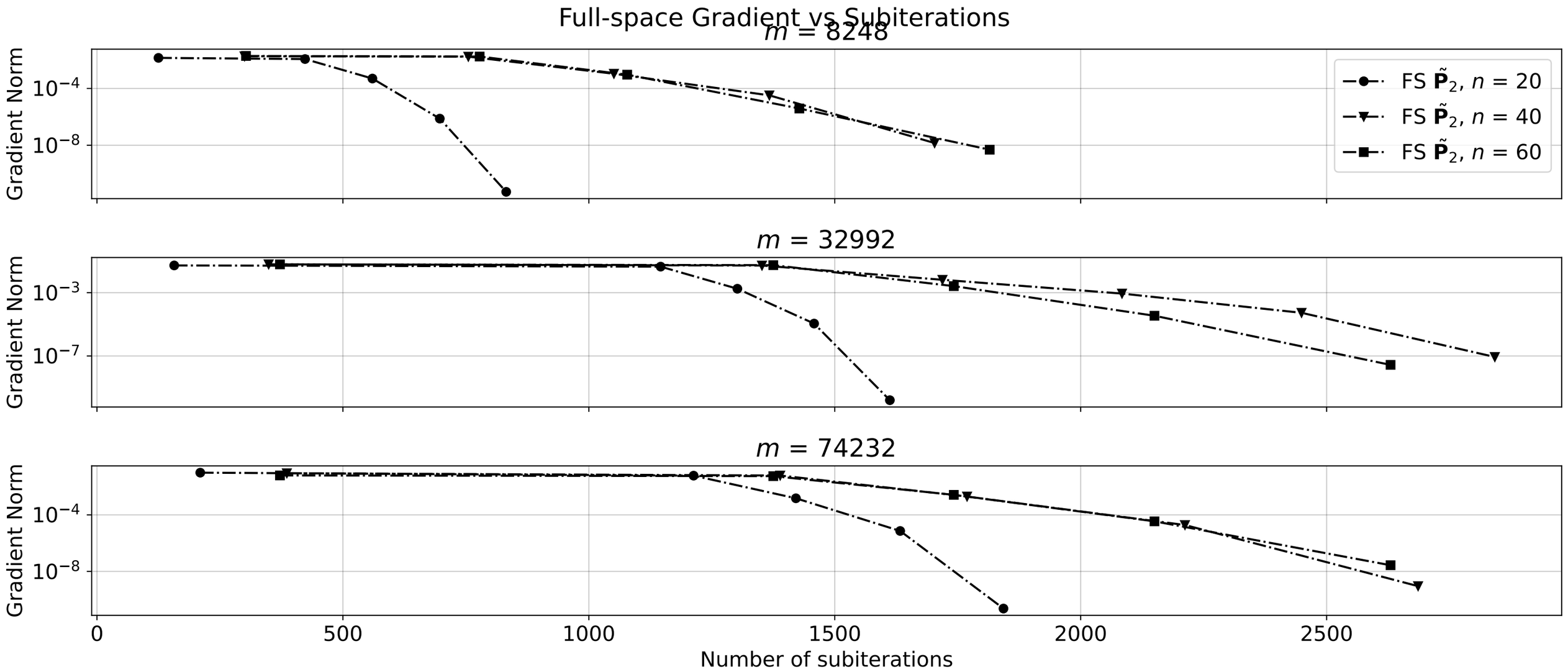

Full-space with $\tilde{\mathbf{P}}_2$ Gradient vs Design Cycles, m = 32992
Gradient Norm
10^{-1}
10^{-3}
10^{-5}
10^{-7}
10^{-9}
1
2
3
4
5
6
Iterations
n = 20
n = 40
n = 60

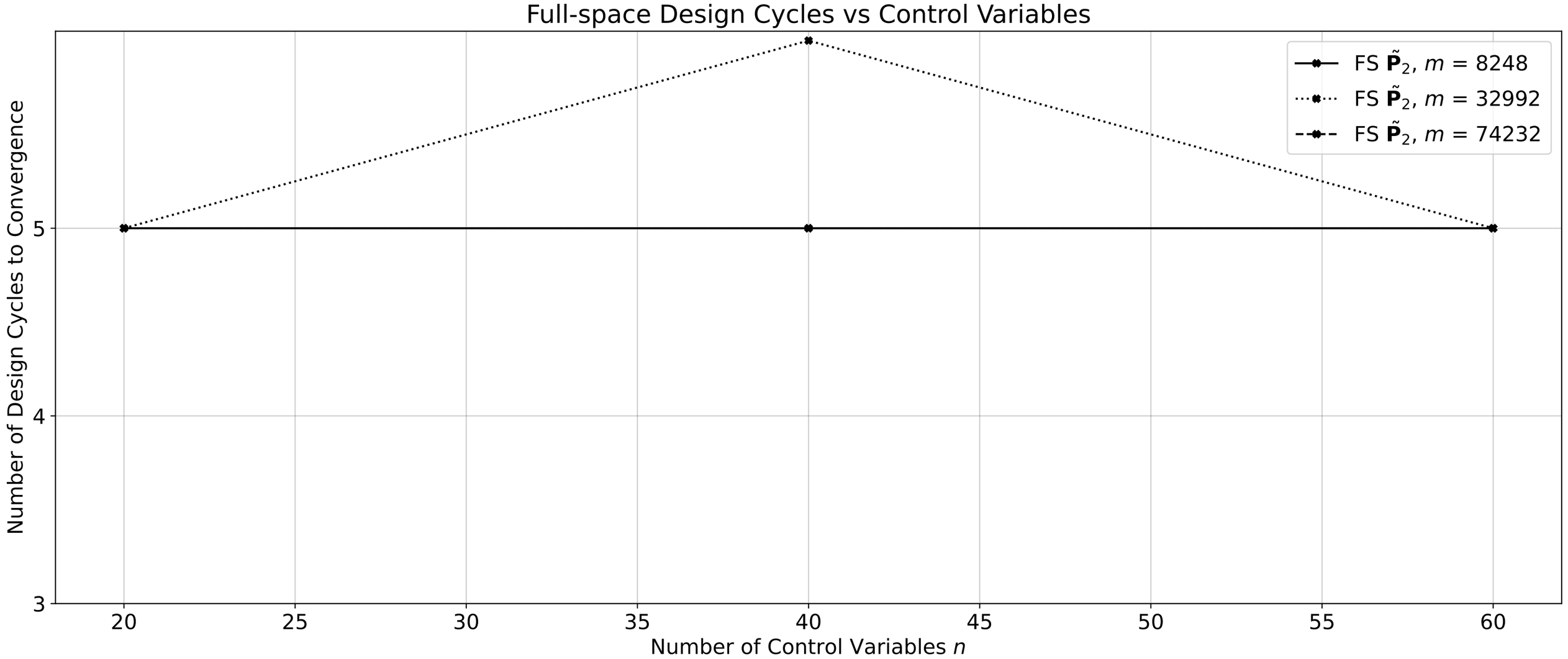

Full-space with $\tilde{\mathbf{P}}_2$ Gradient vs Design Cycles, m = 74232
Gradient Norm
10^{-2}
10^{-4}
10^{-6}
10^{-8}
10^{-10}
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0
Iterations
n = 20
n = 40
n = 60

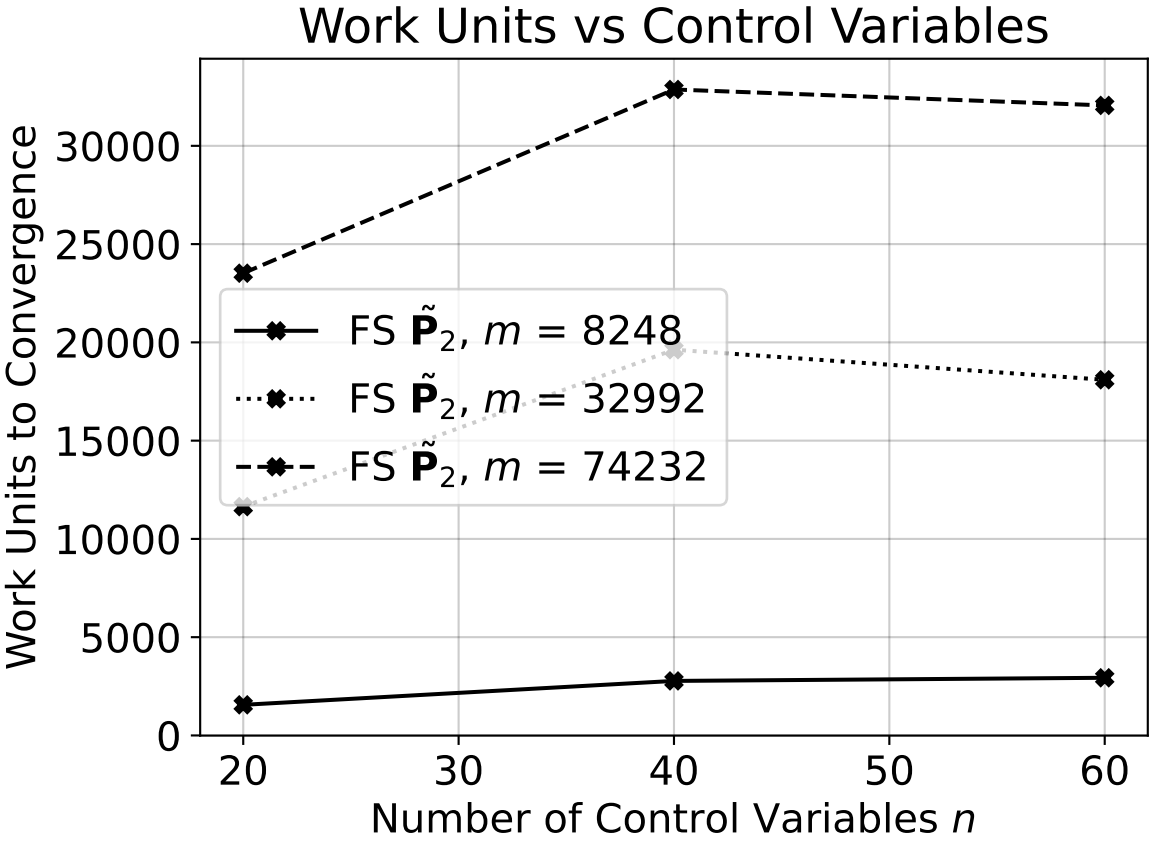

Full-space Gradient vs Subiterations, m = 8248
FS $\tilde{\mathbf{P}}_2$, n = 20
FS $\tilde{\mathbf{P}}_2$, n = 40
FS $\tilde{\mathbf{P}}_2$, n = 60
Gradient Norm
10^{-2}
10^{-4}
10^{-6}
10^{-8}
10^{-10}
250
500
750
1000
1250
1500
1750
Number of Subiterations

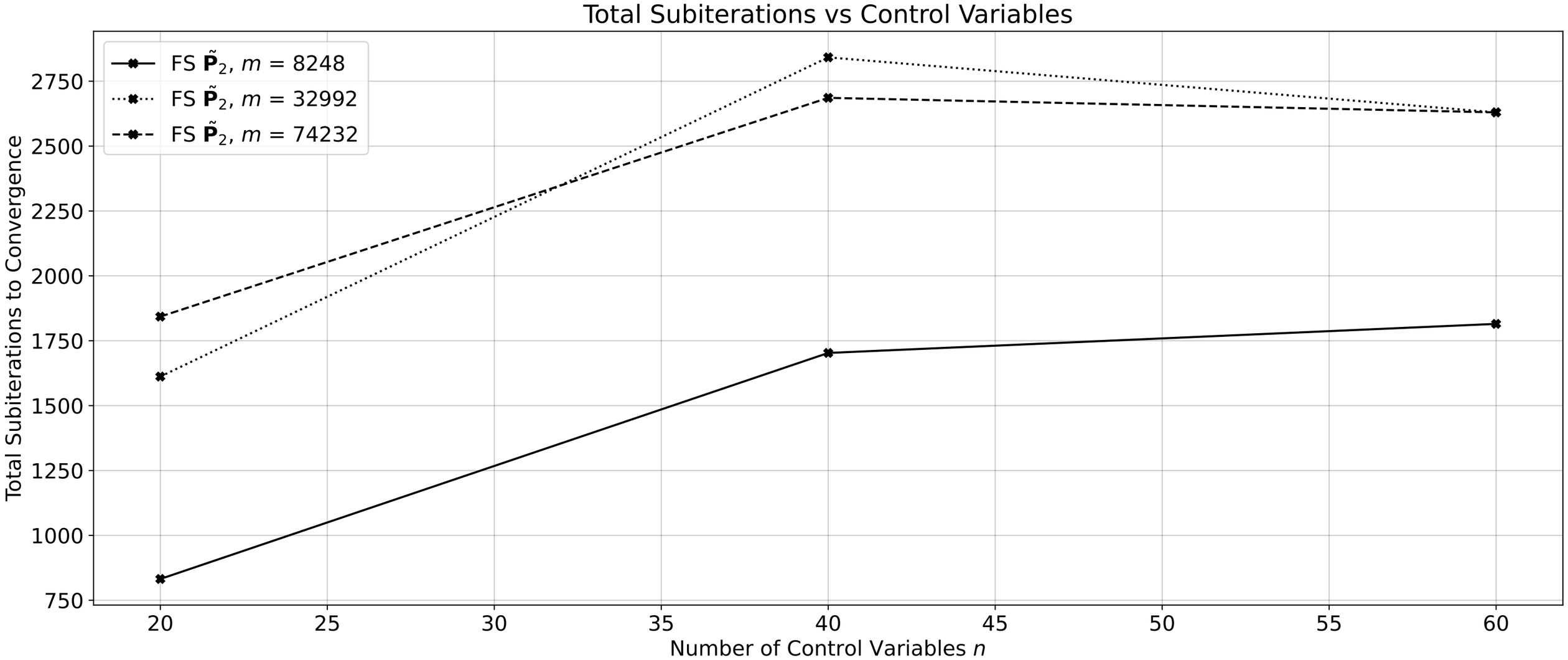

Full-space Gradient vs Subiterations, m = 32992
Gradient Norm
10^{-1}
10^{-3}
10^{-5}
10^{-7}
10^{-9}
500
1000
1500
2000
2500
Number of Subiterations
FS $\tilde{\mathbf{P}}_2$, n = 20
FS $\tilde{\mathbf{P}}_2$, n = 40
FS $\tilde{\mathbf{P}}_2$, n = 60

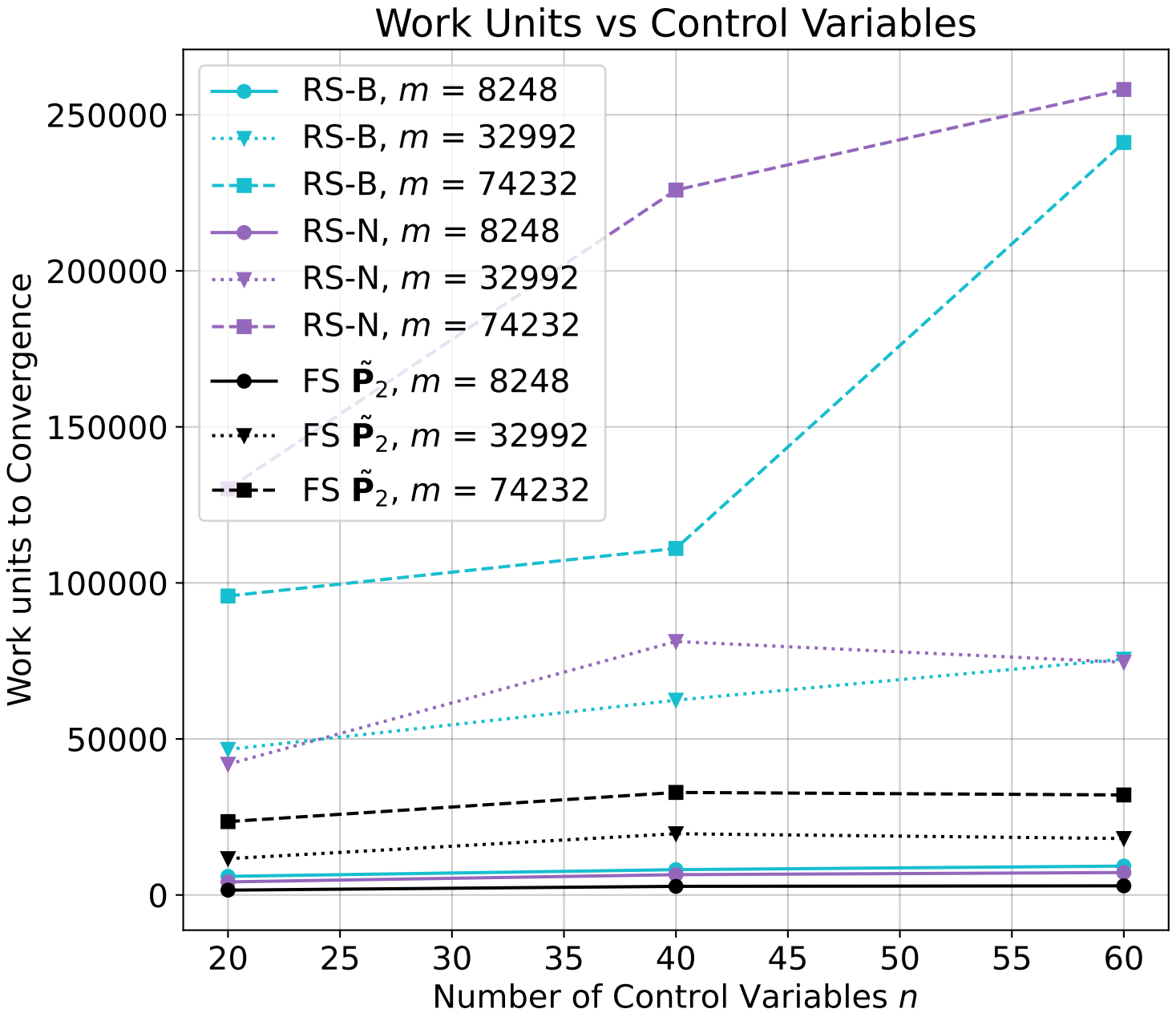

Full-space Gradient vs Subiterations, m = 74232
FS $\tilde{\mathbf{P}}_2$, n = 20
FS $\tilde{\mathbf{P}}_2$, n = 40
FS $\tilde{\mathbf{P}}_2$, n = 60
Gradient Norm
10^{-2}
10^{-4}
10^{-6}
10^{-8}
10^{-10}
500
1000
1500
2000
2500
Number of Subiterations

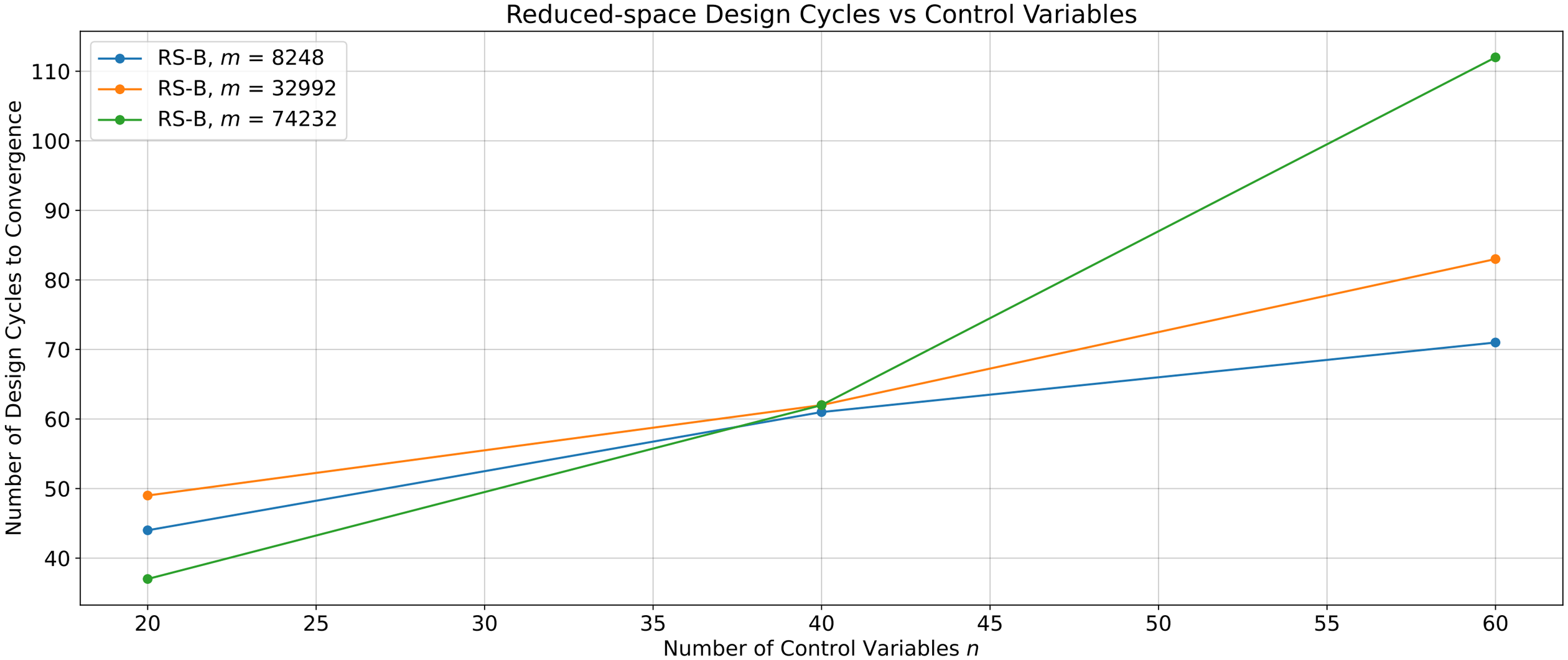

Full-space Gradient vs Subiterations
m = 8248
m = 32992
m = 74232
Gradient Norm
10^{-4}
10^{-8}
10^{-3}
10^{-7}
FS $\tilde{\mathbf{P}}_2$, n = 20
FS $\tilde{\mathbf{P}}_2$, n = 40
FS $\tilde{\mathbf{P}}_2$, n = 60
0
500
1000
1500
2000
2500
Number of subiterations

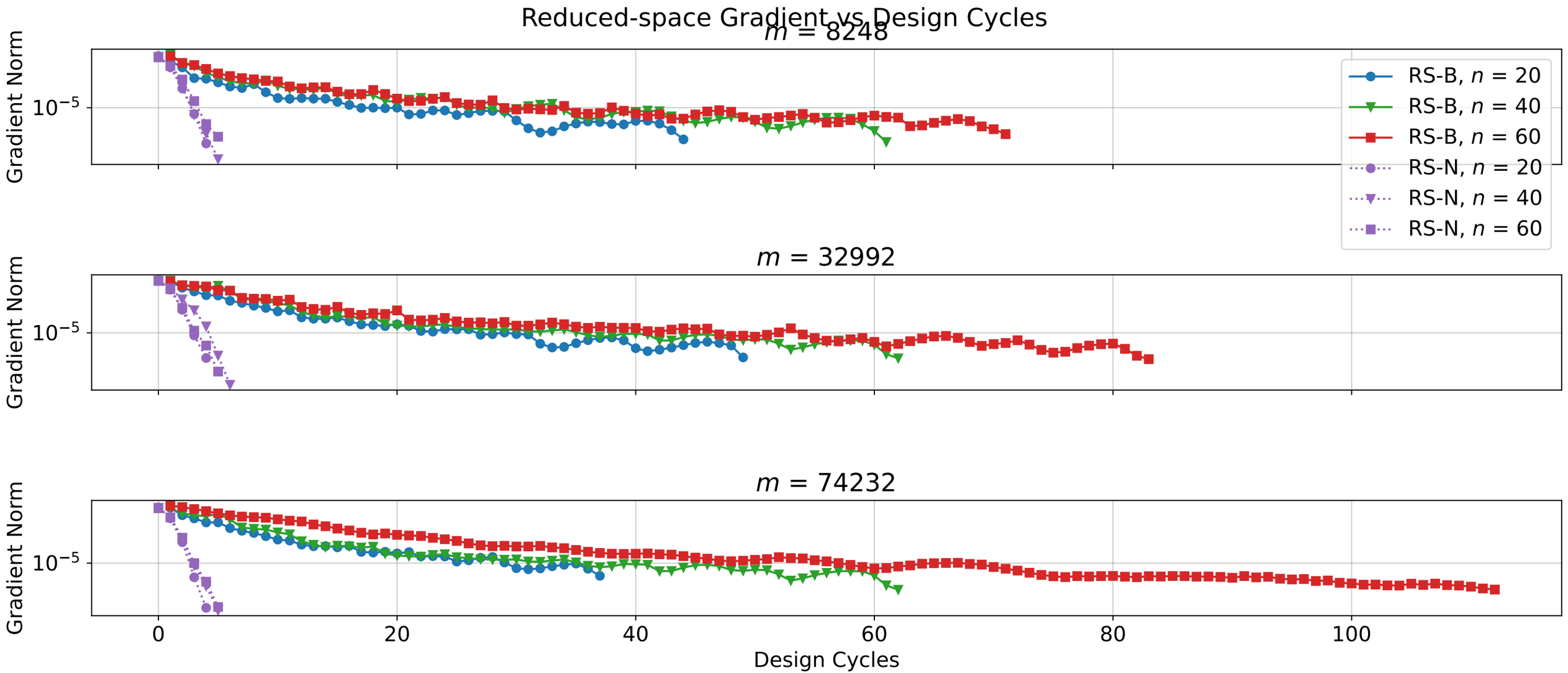

Full-space Design Cycles vs Control Variables
Number of Design Cycles to Convergence
Number of Control Variables n
FS $\tilde{\mathbf{P}}_2$, m = 8248
FS $\tilde{\mathbf{P}}_2$, m = 32992
FS $\tilde{\mathbf{P}}_2$, m = 74232
3
4
5
20
25
30
35
40
45
50
55
60

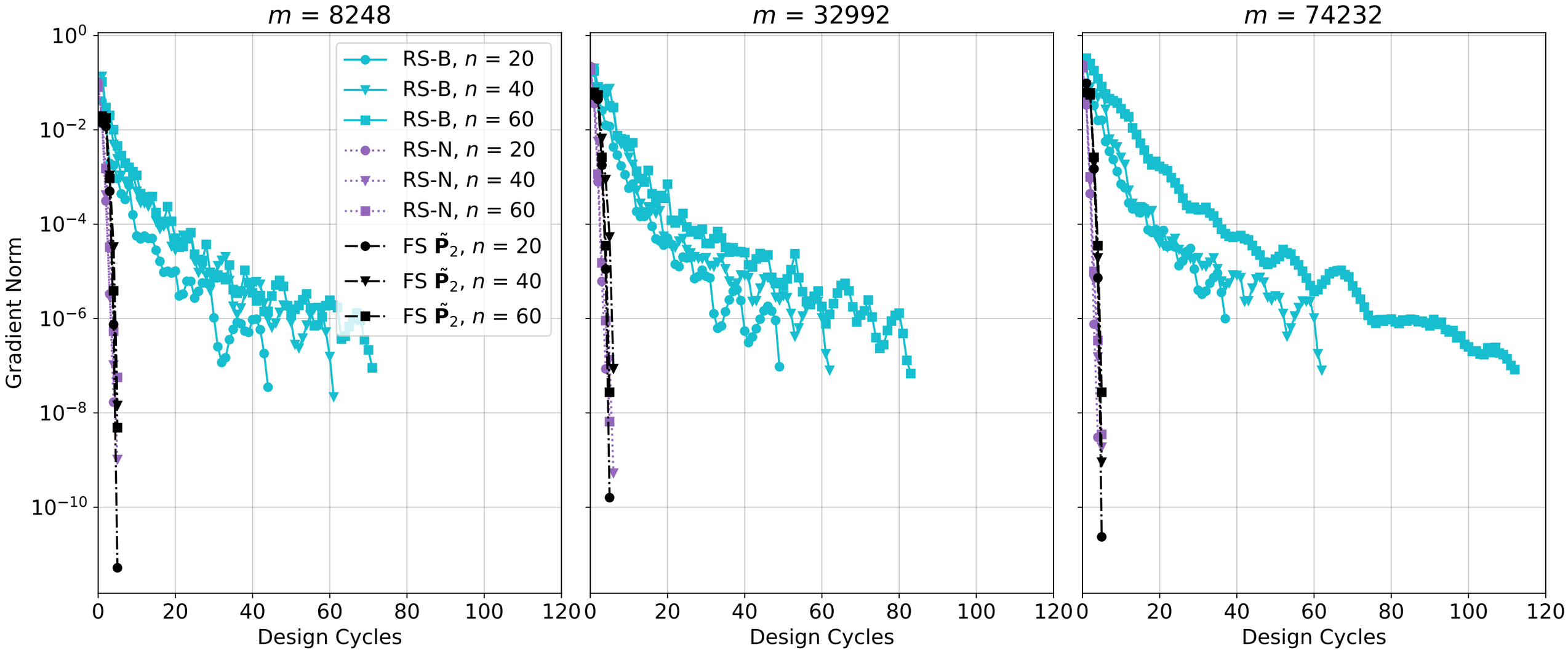

Work Units vs Control Variables
Work Units to Convergence
Number of Control Variables n
FS $\tilde{\mathbf{P}}_2$, $m = 8248$
FS $\tilde{\mathbf{P}}_2$, $m = 32992$
FS $\tilde{\mathbf{P}}_2$, $m = 74232$
0
5000
10000
15000
20000
25000
30000
20
30
40
50
60

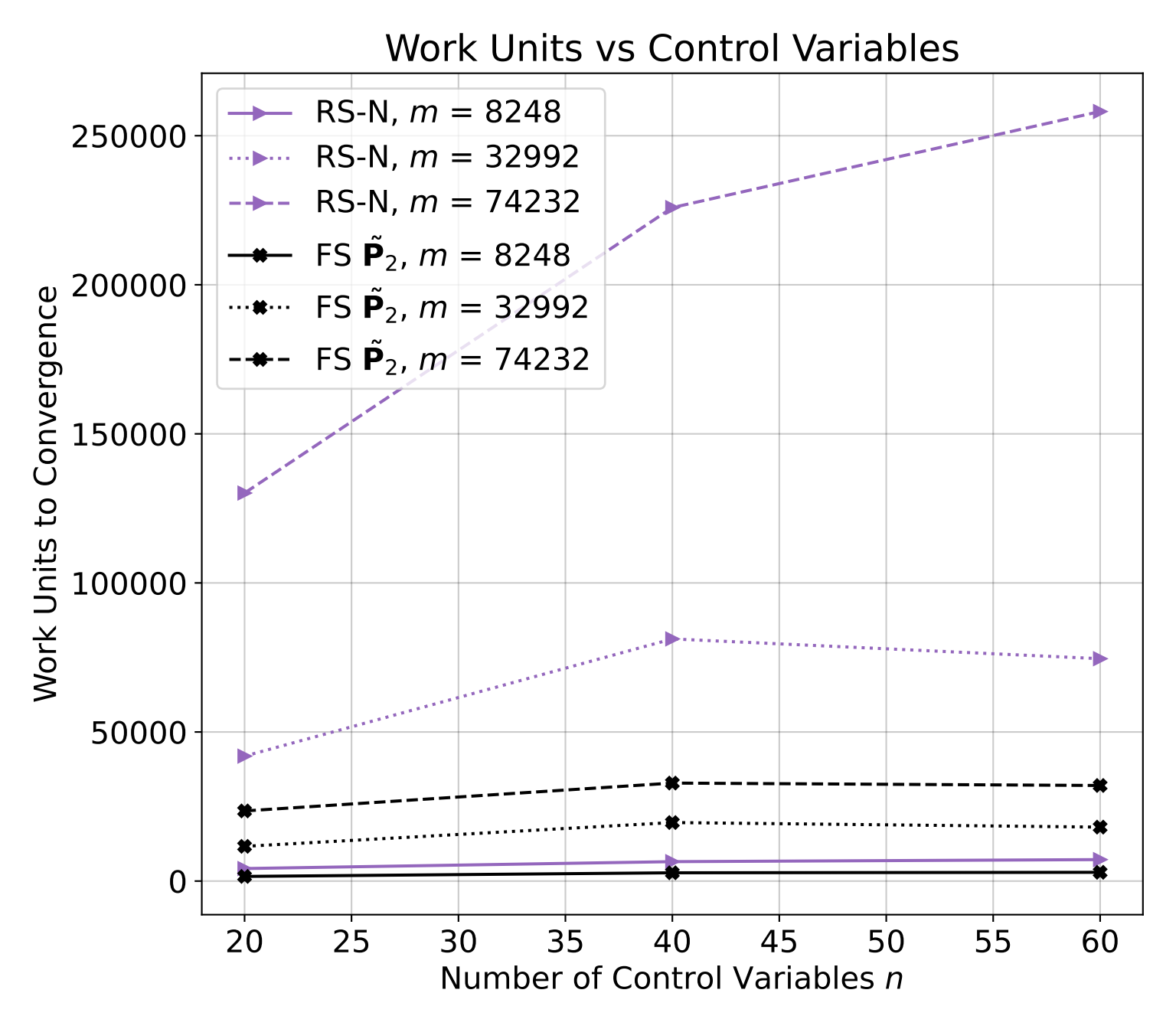

Total Subiterations vs Control Variables
FS $\tilde{\mathbf{P}}_2$, m = 8248
FS $\tilde{\mathbf{P}}_2$, m = 32992
FS $\tilde{\mathbf{P}}_2$, m = 74232
Total Subiterations to Convergence
750
1000
1250
1500
1750
2000
2250
2500
2750
20
25
30
35
40
45
50
55
60
Number of Control Variables n

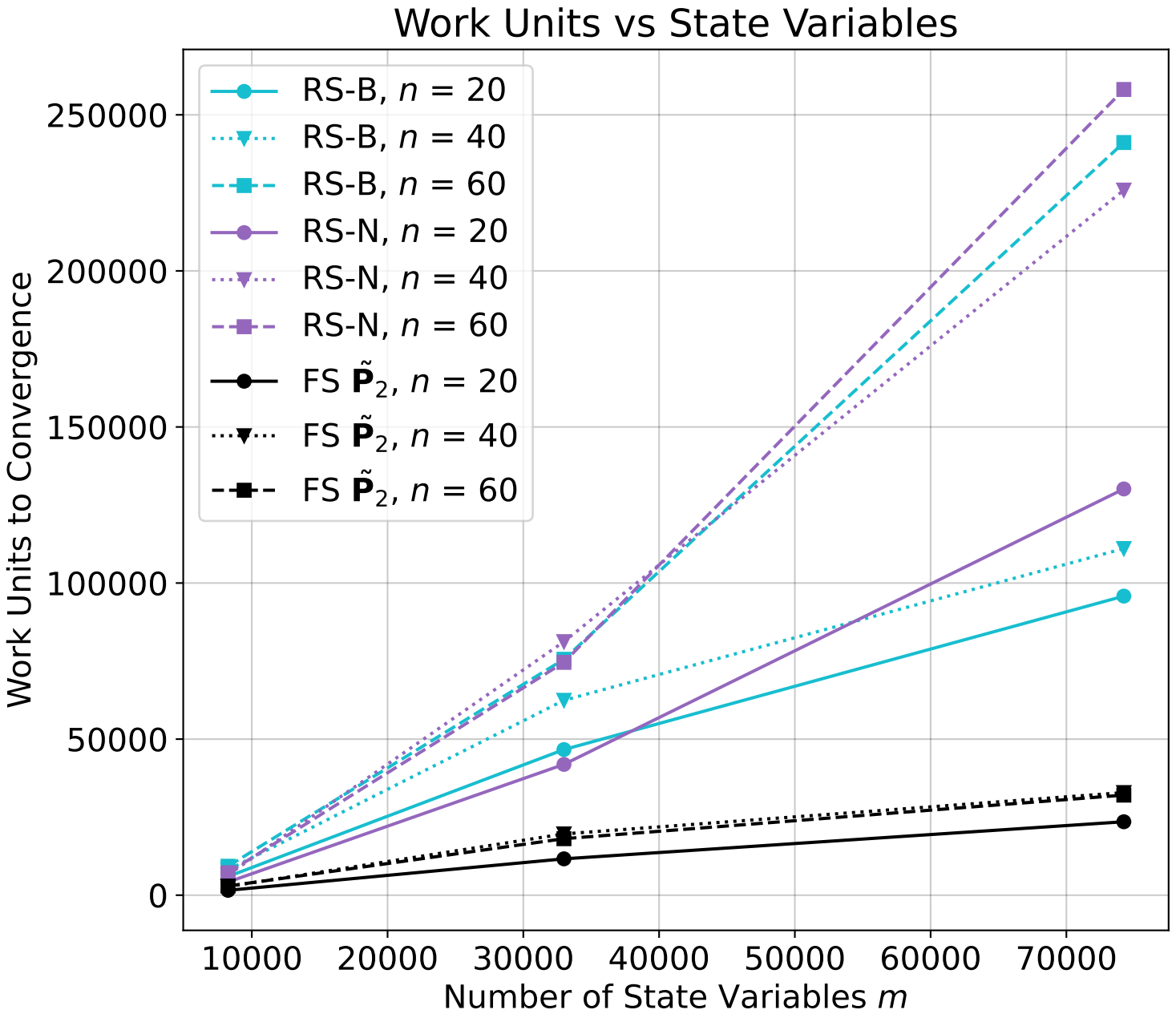

Work Units vs Control Variables
RS-B, m = 8248
RS-B, m = 32992
RS-B, m = 74232
RS-N, m = 8248
RS-N, m = 32992
RS-N, m = 74232
FS $\tilde{\mathbf{P}}_2$, m = 8248
FS $\tilde{\mathbf{P}}_2$, m = 32992
FS $\tilde{\mathbf{P}}_2$, m = 74232
Work units to Convergence
0
50000
100000
150000
200000
250000
20
25
30
35
40
45
50
55
60
Number of Control Variables n

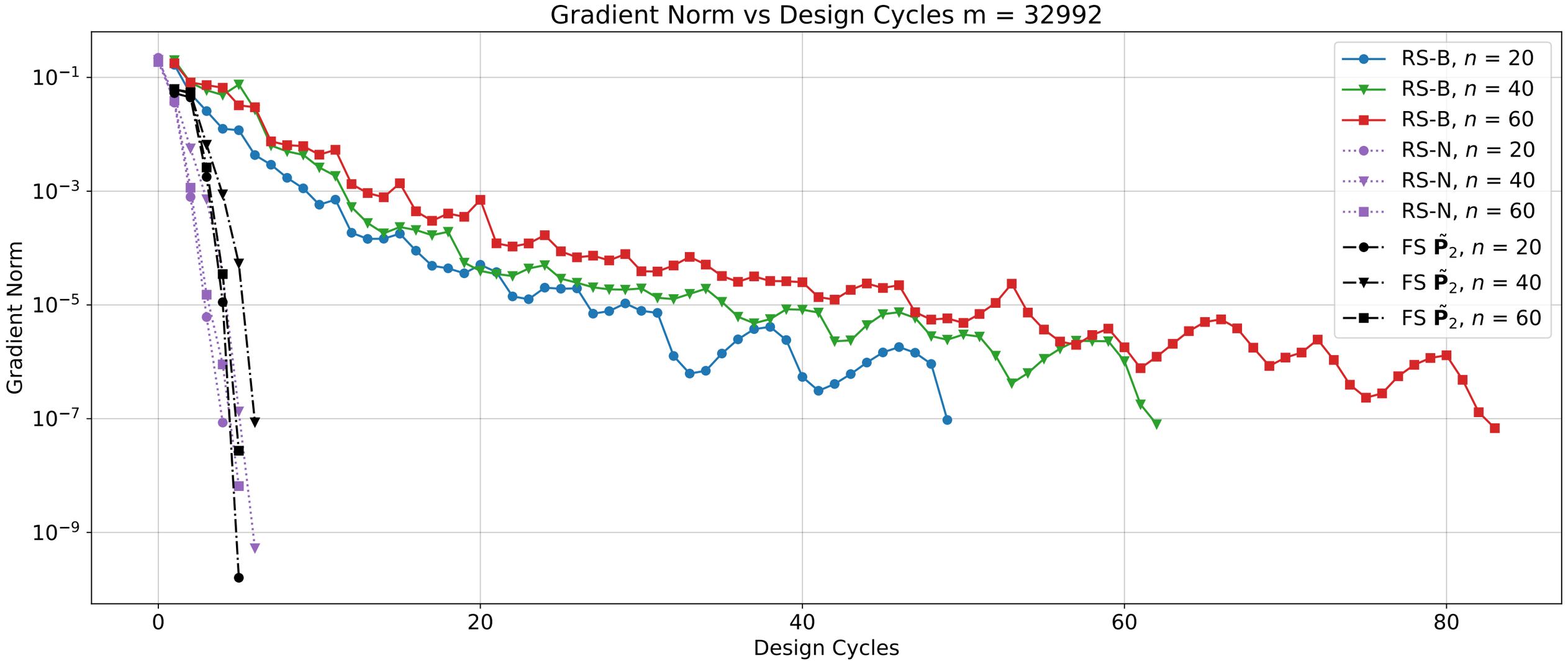

Reduced-space Design Cycles vs Control Variables
RS-B, m = 8248
RS-B, m = 32992
RS-B, m = 74232
Number of Design Cycles to Convergence
Number of Control Variables n
110
100
90
80
70
60
50
40
20
25
30
35
40
45
50
55
60

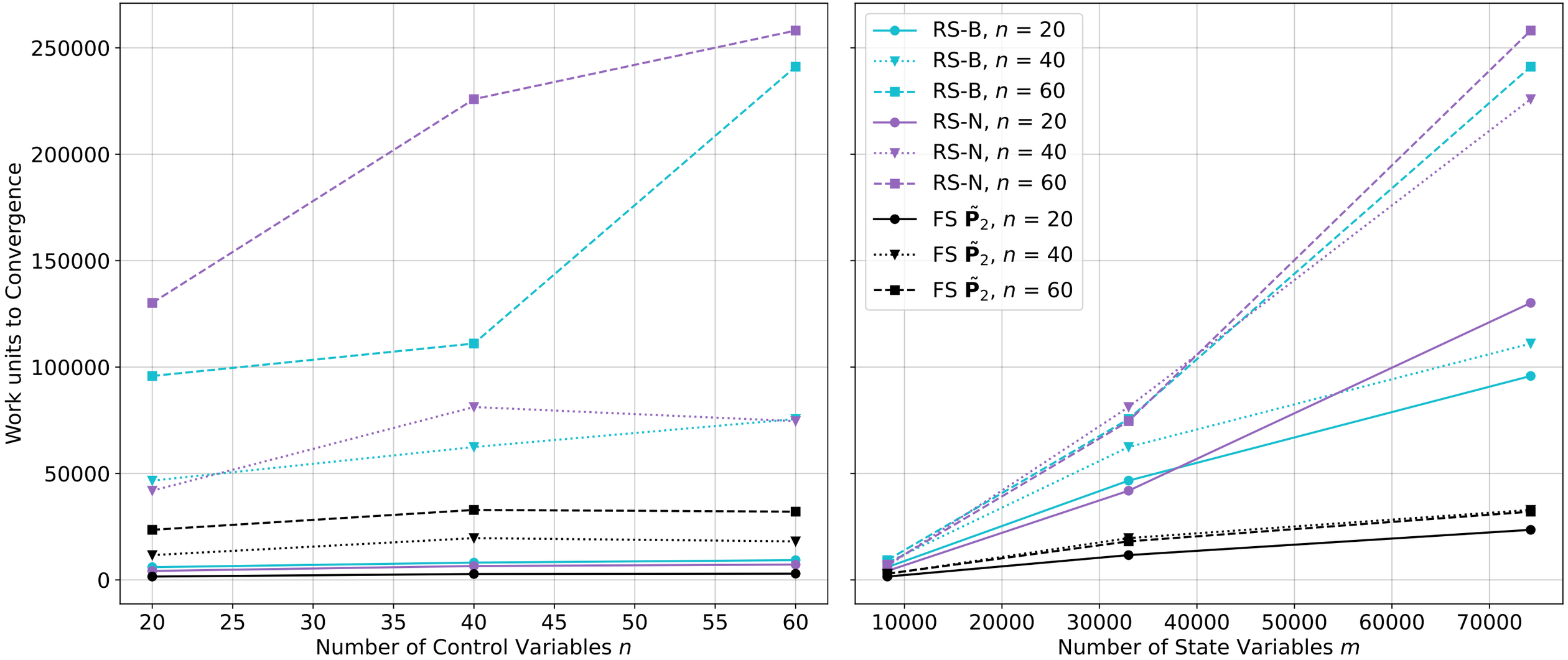

Reduced-space Gradient vs Design Cycles
m = 8248
m = 32992
m = 74232
Gradient Norm
10^−5
0
20
40
60
80
100
Design Cycles
RS-B, n = 20
RS-B, n = 40
RS-B, n = 60
RS-N, n = 20
RS-N, n = 40
RS-N, n = 60


m = 8248
m = 32992
m = 74232
Gradient Norm
10^{0}
10^{-2}
10^{-4}
10^{-6}
10^{-8}
10^{-10}
0
20
40
60
80
100
120
Design Cycles
RS-B, n = 20
RS-B, n = 40
RS-B, n = 60
RS-N, n = 20
RS-N, n = 40
RS-N, n = 60
FS $\tilde{\mathbf{P}}_2$, n = 20
FS $\tilde{\mathbf{P}}_2$, n = 40
FS $\tilde{\mathbf{P}}_2$, n = 60


Work Units vs Control Variables
RS-N, m = 8248
RS-N, m = 32992
RS-N, m = 74232
FS $\tilde{\mathbf{P}}_2$, m = 8248
FS $\tilde{\mathbf{P}}_2$, m = 32992
FS $\tilde{\mathbf{P}}_2$, m = 74232
Work Units to Convergence
0
50000
100000
150000
200000
250000
20
25
30
35
40
45
50
55
60
Number of Control Variables n


Work Units vs State Variables
Work Units to Convergence
Number of State Variables m
0
50000
100000
150000
200000
250000
10000
20000
30000
40000
50000
60000
70000
RS-B, $n = 20$
RS-B, $n = 40$
RS-B, $n = 60$
RS-N, $n = 20$
RS-N, $n = 40$
RS-N, $n = 60$
FS $\tilde{\mathbf{P}}_2$, $n = 20$
FS $\tilde{\mathbf{P}}_2$, $n = 40$
FS $\tilde{\mathbf{P}}_2$, $n = 60$


Gradient Norm vs Design Cycles m = 32992
Gradient Norm
10^{-1}
10^{-3}
10^{-5}
10^{-7}
10^{-9}
0
20
40
60
80
Design Cycles
RS-B, n = 20
RS-B, n = 40
RS-B, n = 60
RS-N, n = 20
RS-N, n = 40
RS-N, n = 60
FS $\tilde{\mathbf{P}}_2$, n = 20
FS $\tilde{\mathbf{P}}_2$, n = 40
FS $\tilde{\mathbf{P}}_2$, n = 60


Work units to Convergence
250000
200000
150000
100000
50000
0
20
25
30
35
40
45
50
55
60
Number of Control Variables n
10000
20000
30000
40000
50000
60000
70000
Number of State Variables m
RS-B, n = 20
RS-B, n = 40
RS-B, n = 60
RS-N, n = 20
RS-N, n = 40
RS-N, n = 60
FS $\tilde{\mathbf{P}}_2$, n = 20
FS $\tilde{\mathbf{P}}_2$, n = 40
FS $\tilde{\mathbf{P}}_2$, n = 60