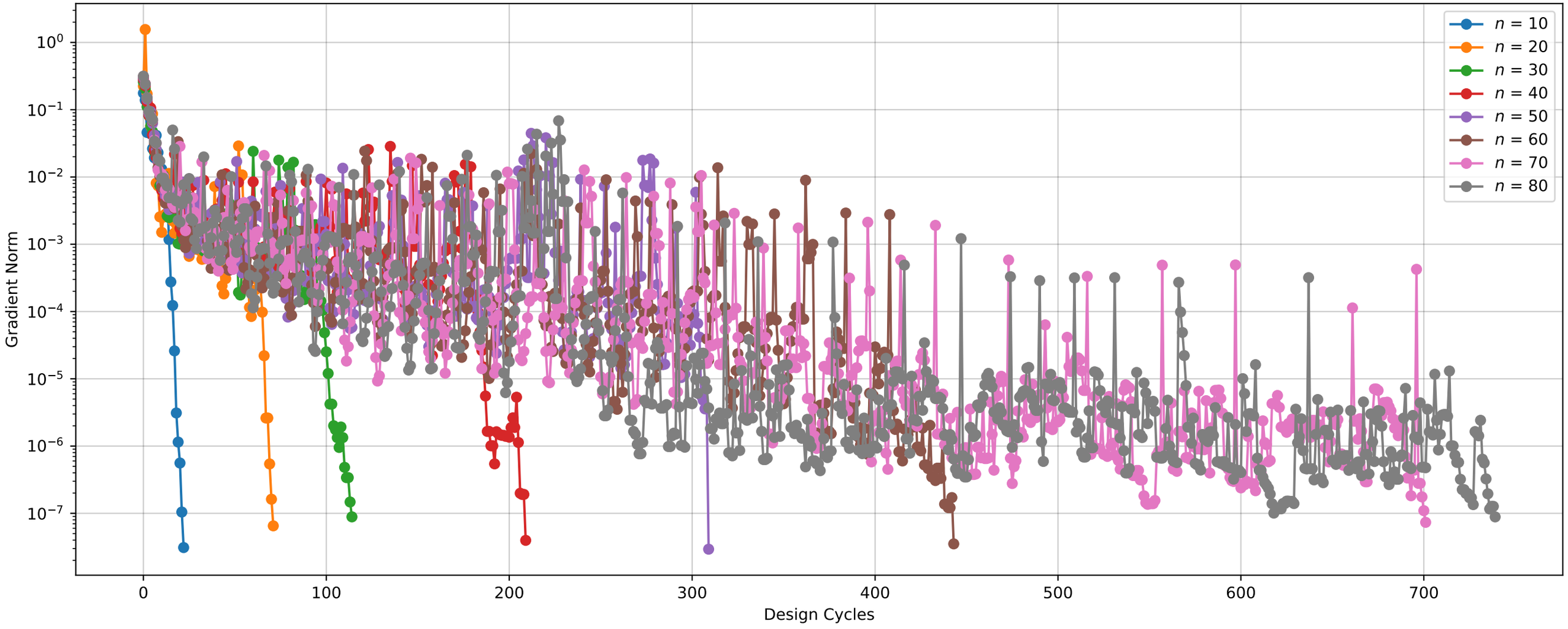
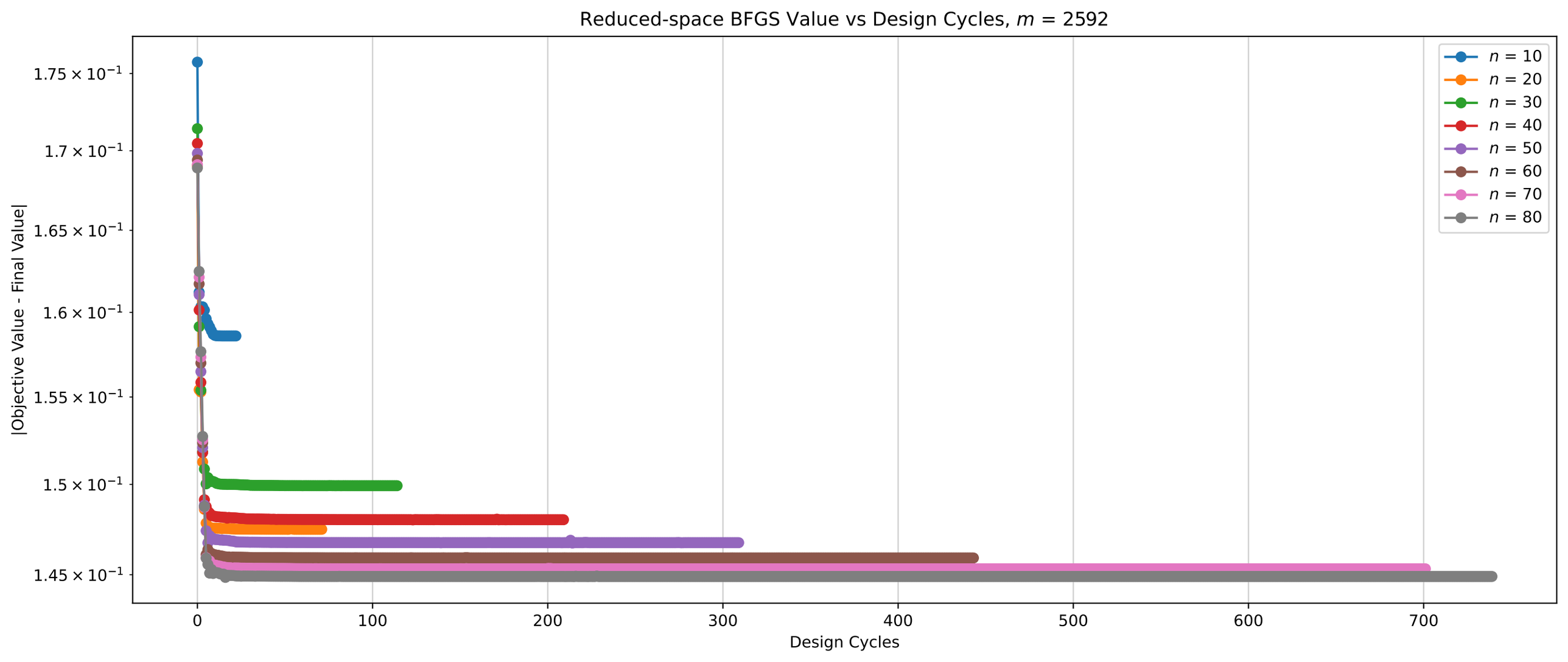
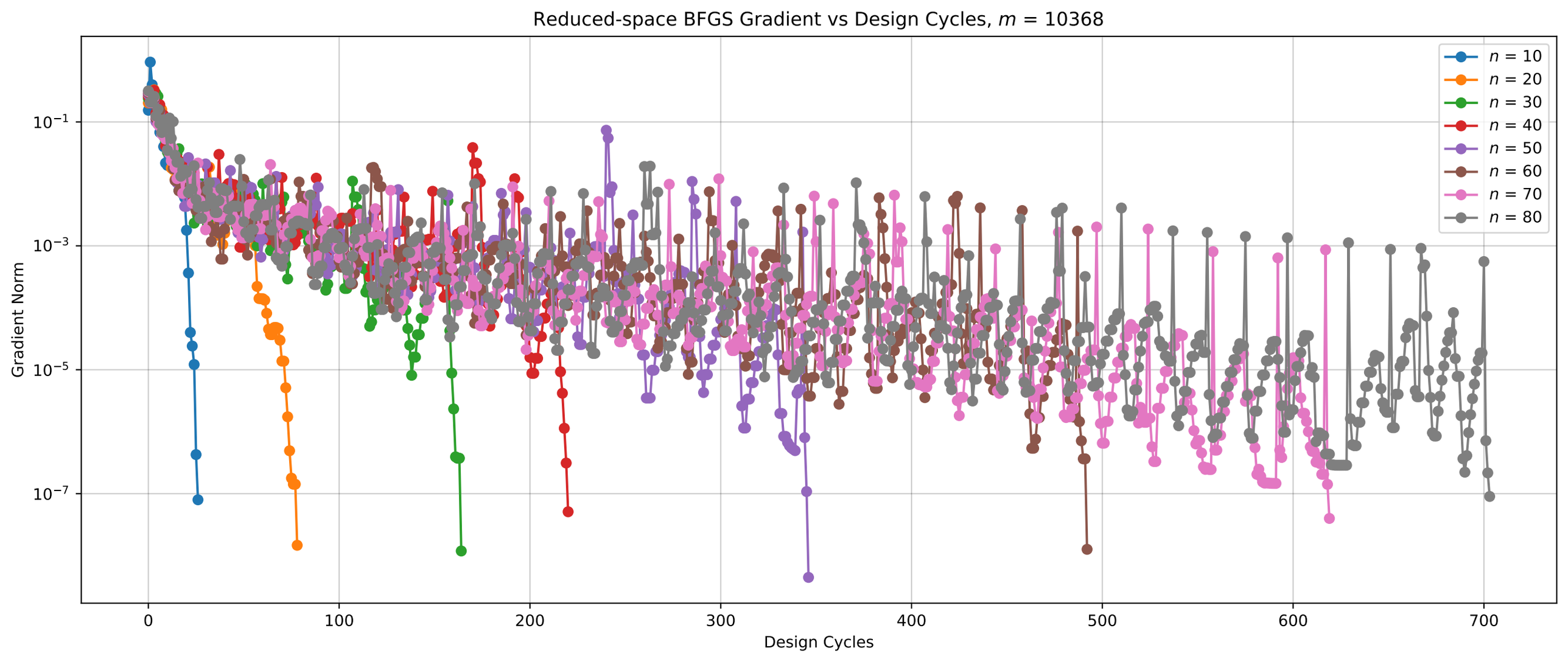
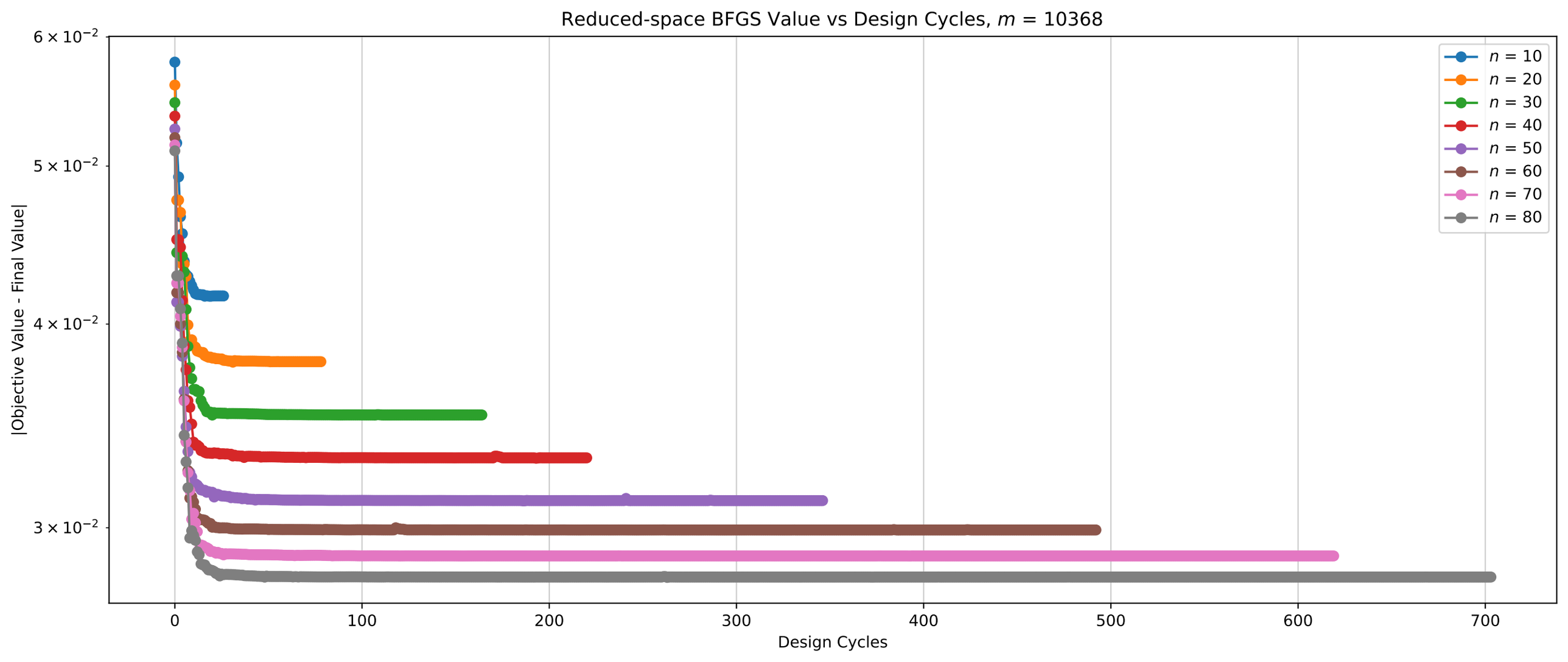
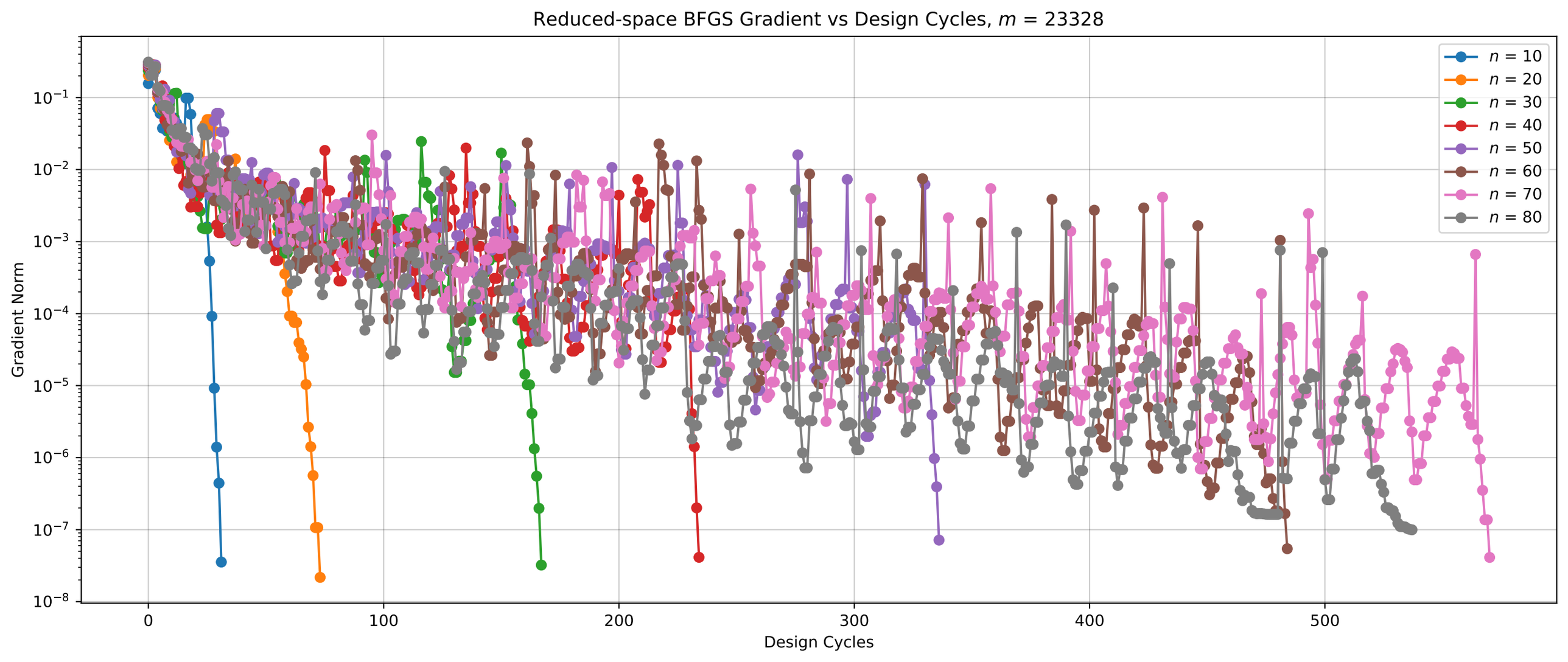
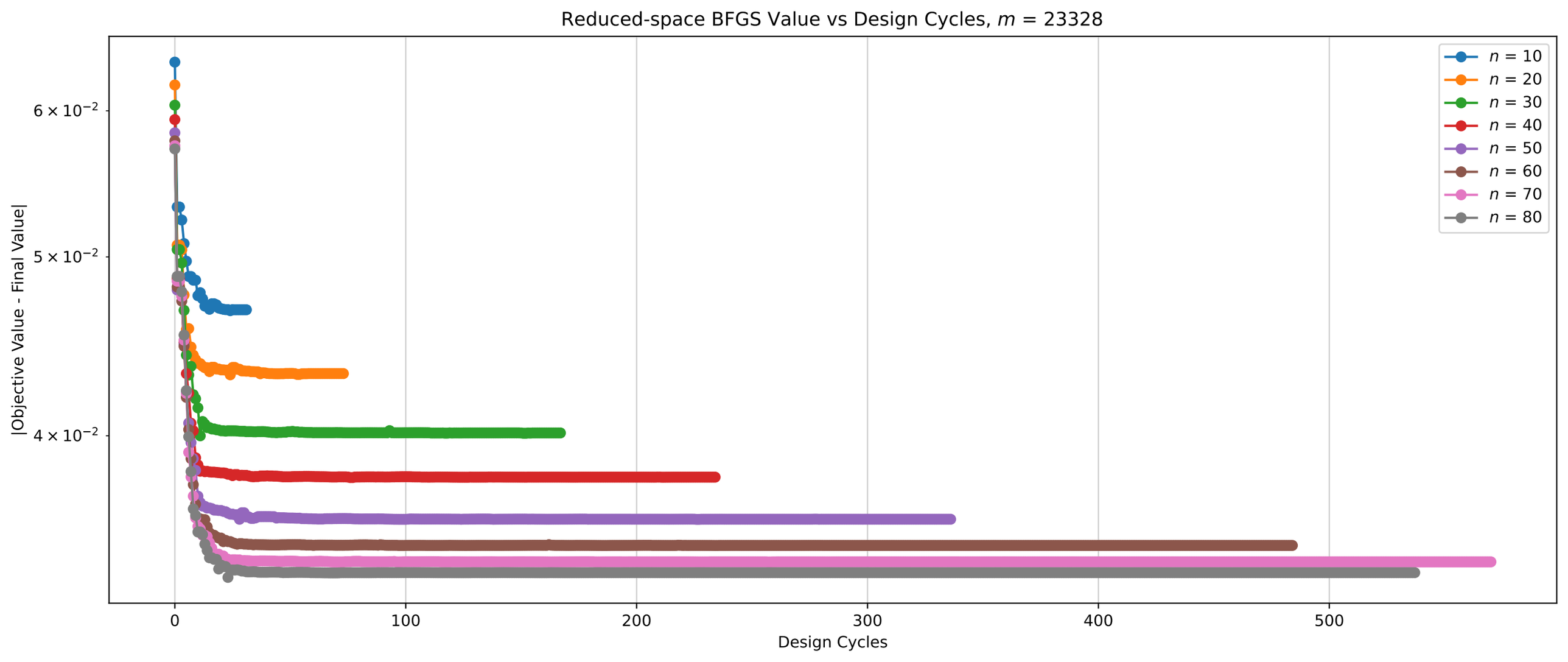

Reduced-space BFGS Gradient vs Design Cycles, m = 2592
Gradient Norm
Design Cycles
10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
0
100
200
300
400
500
600
700
n = 10
n = 20
n = 30
n = 40
n = 50
n = 60
n = 70
n = 80

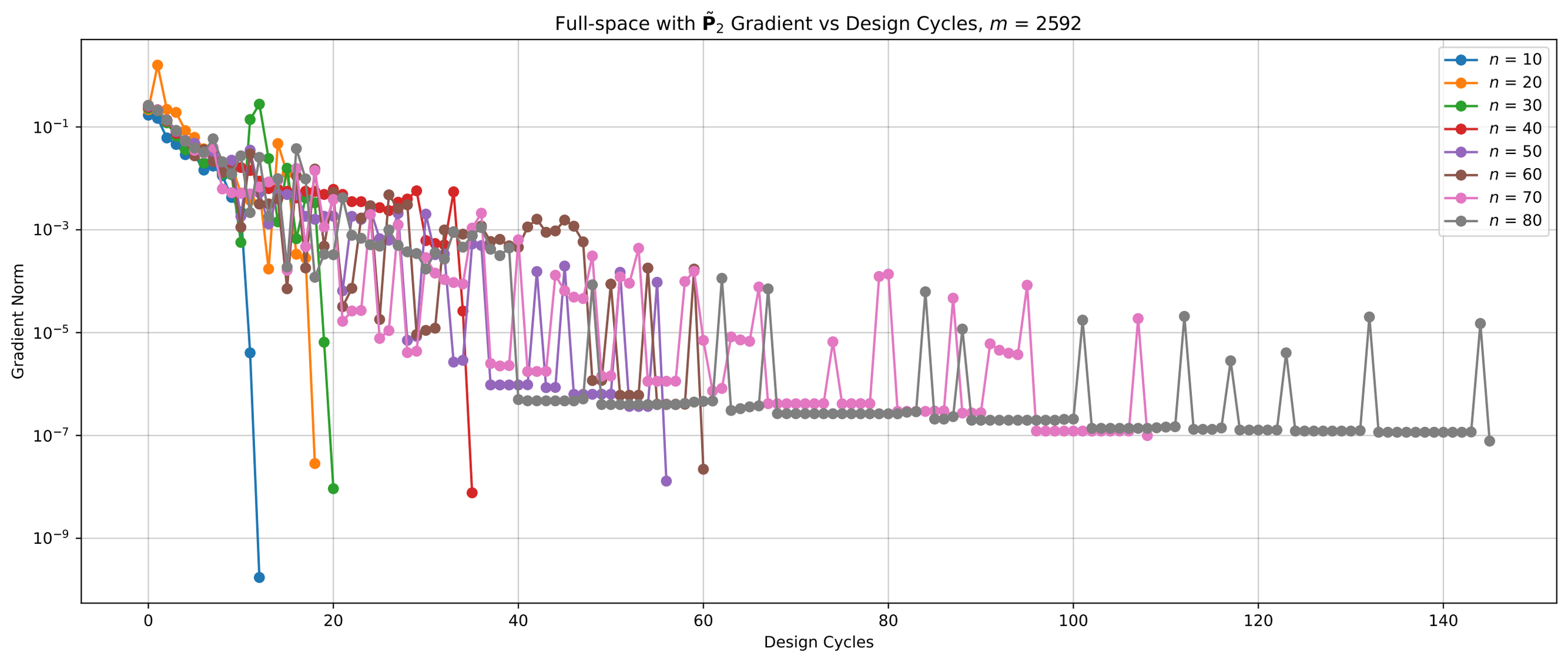

Reduced-space BFGS Value vs Design Cycles, m = 2592
|Objective Value - Final Value|
1.75×10^{-1}
1.7×10^{-1}
1.65×10^{-1}
1.6×10^{-1}
1.55×10^{-1}
1.5×10^{-1}
1.45×10^{-1}
0
100
200
300
400
500
600
700
Design Cycles
n = 10
n = 20
n = 30
n = 40
n = 50
n = 60
n = 70
n = 80

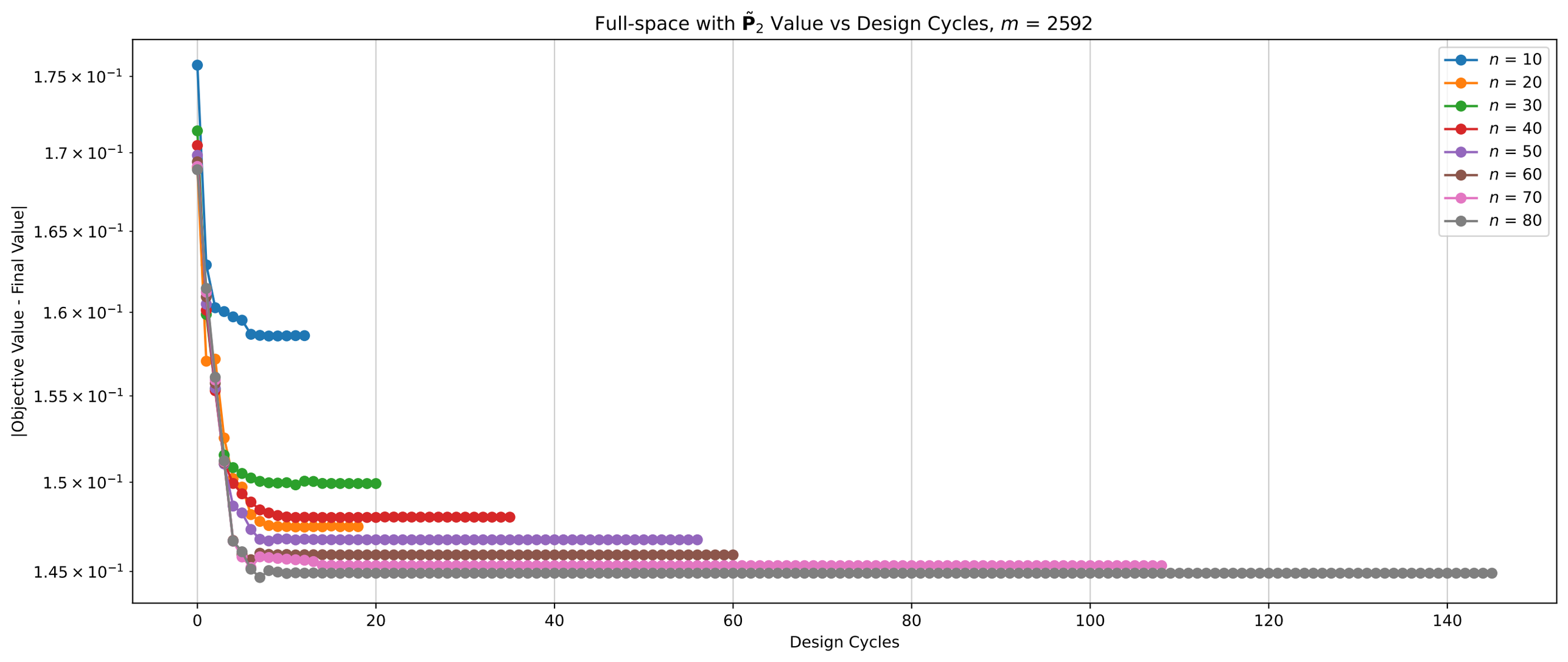

Reduced-space BFGS Gradient vs Design Cycles, m = 10368
Gradient Norm
10^{-1}
10^{-3}
10^{-5}
10^{-7}
0
100
200
300
400
500
600
700
Design Cycles
n = 10
n = 20
n = 30
n = 40
n = 50
n = 60
n = 70
n = 80

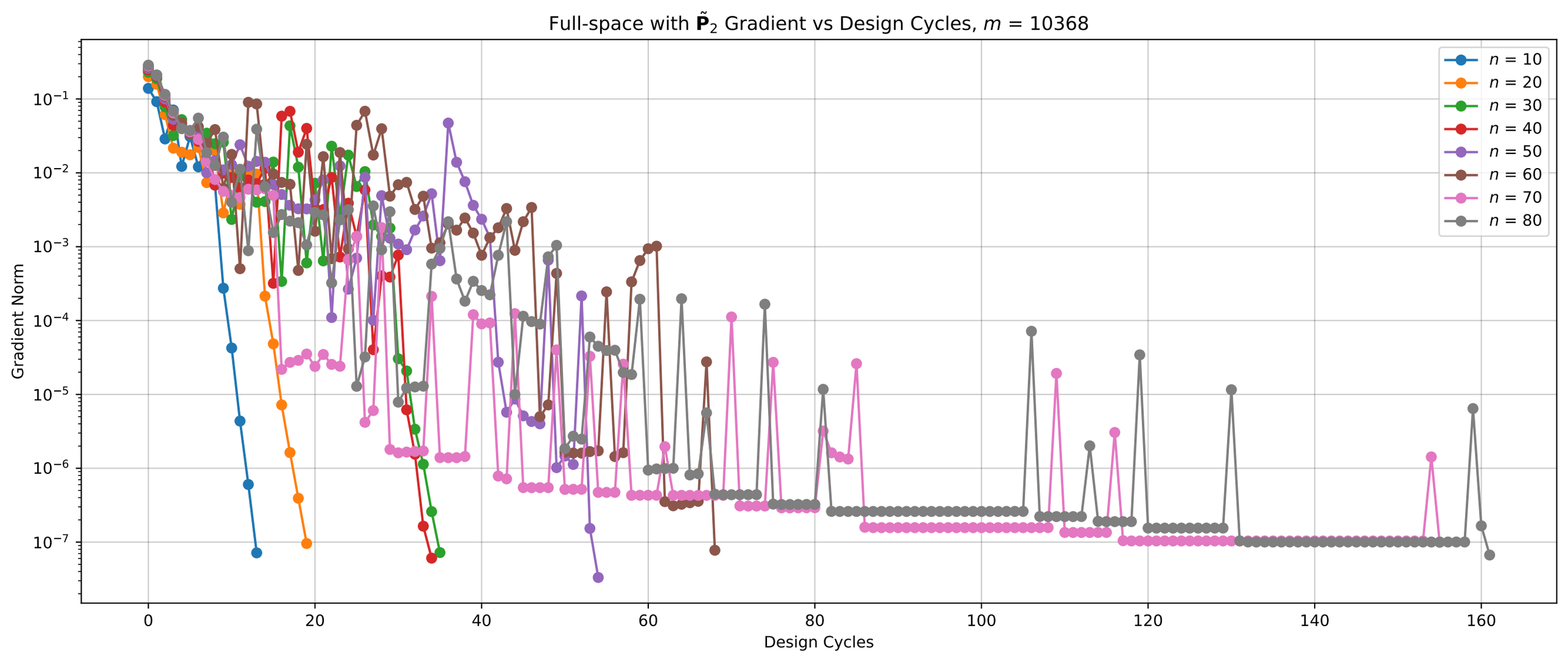

Reduced-space BFGS Value vs Design Cycles, m = 10368
6×10^{-2}
5×10^{-2}
4×10^{-2}
3×10^{-2}
|Objective Value - Final Value|
0
100
200
300
400
500
600
700
Design Cycles
n = 10
n = 20
n = 30
n = 40
n = 50
n = 60
n = 70
n = 80

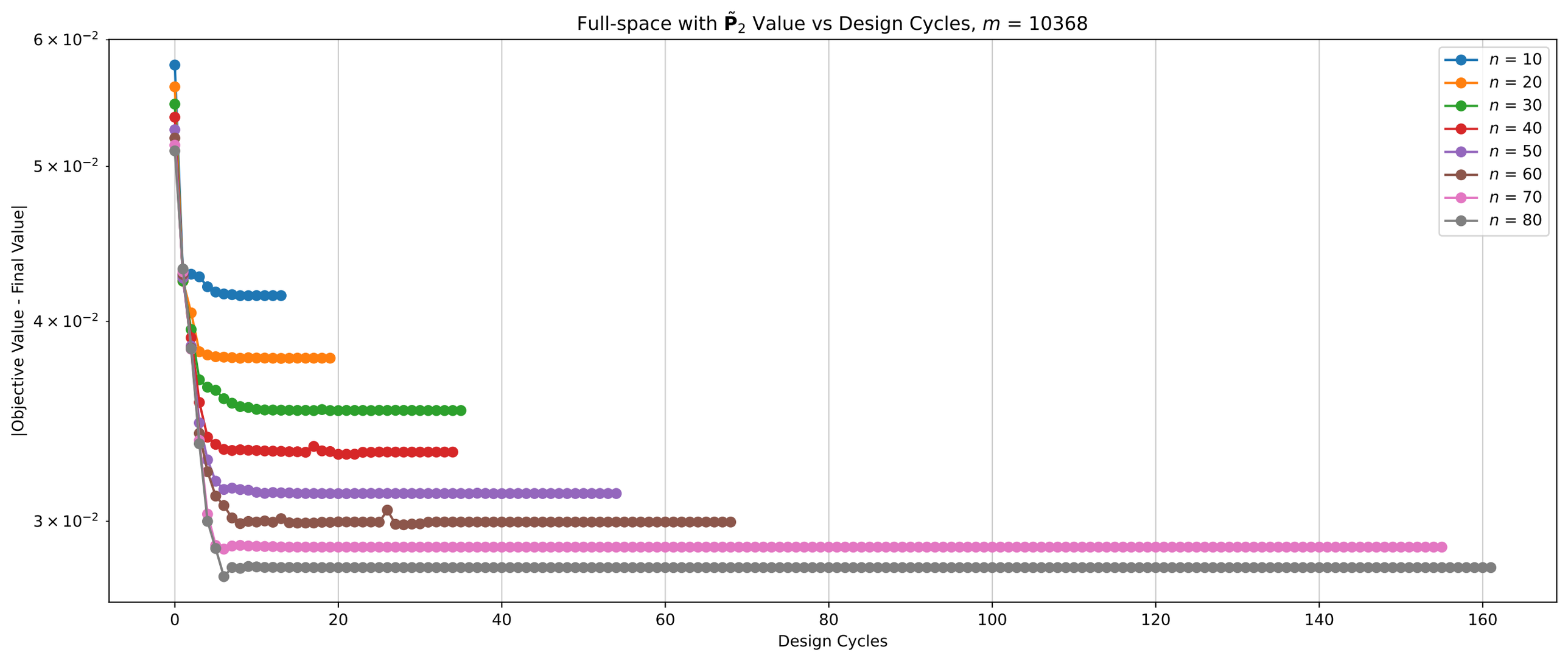

Reduced-space BFGS Gradient vs Design Cycles, m = 23328
Gradient Norm
Design Cycles
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-8}
0
100
200
300
400
500
n = 10
n = 20
n = 30
n = 40
n = 50
n = 60
n = 70
n = 80

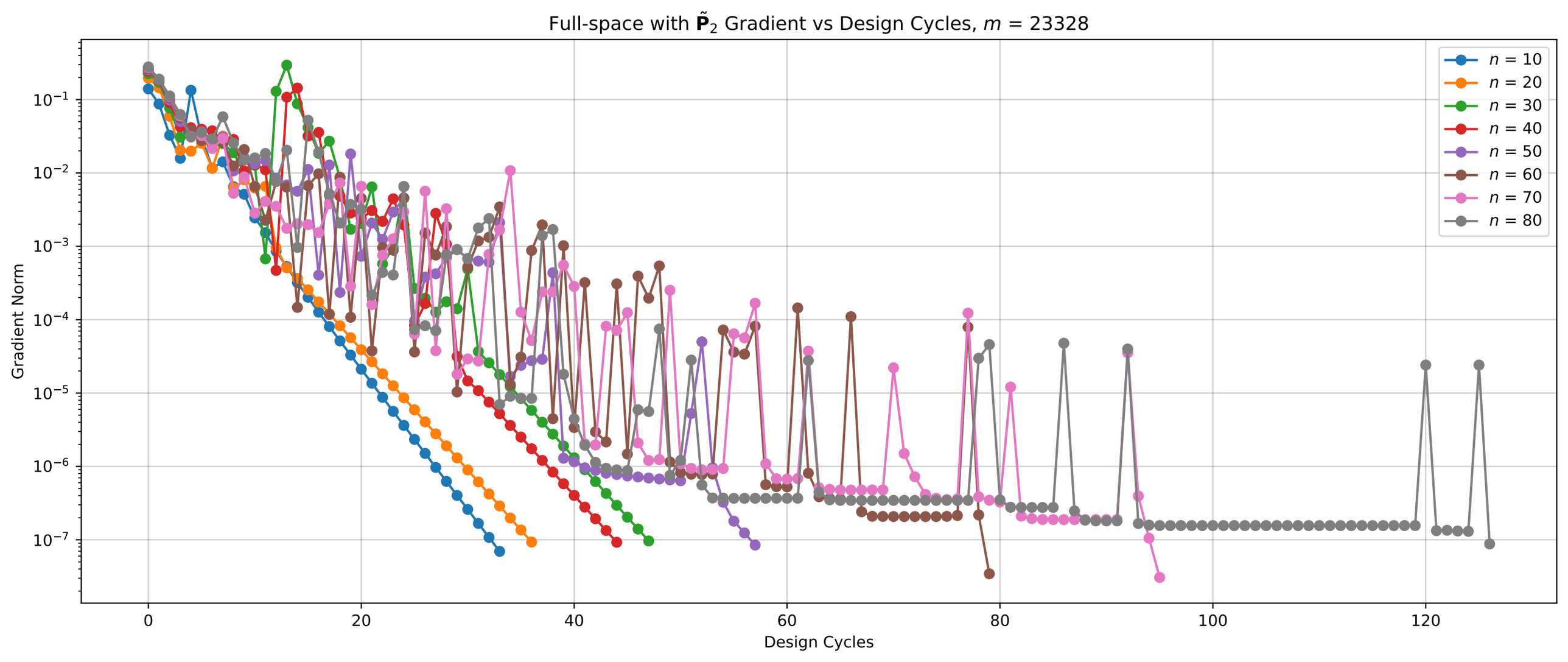

Reduced-space BFGS Value vs Design Cycles, m = 23328
|Objective Value - Final Value|
6×10^{-2}
5×10^{-2}
4×10^{-2}
0
100
200
300
400
500
Design Cycles
n = 10
n = 20
n = 30
n = 40
n = 50
n = 60
n = 70
n = 80

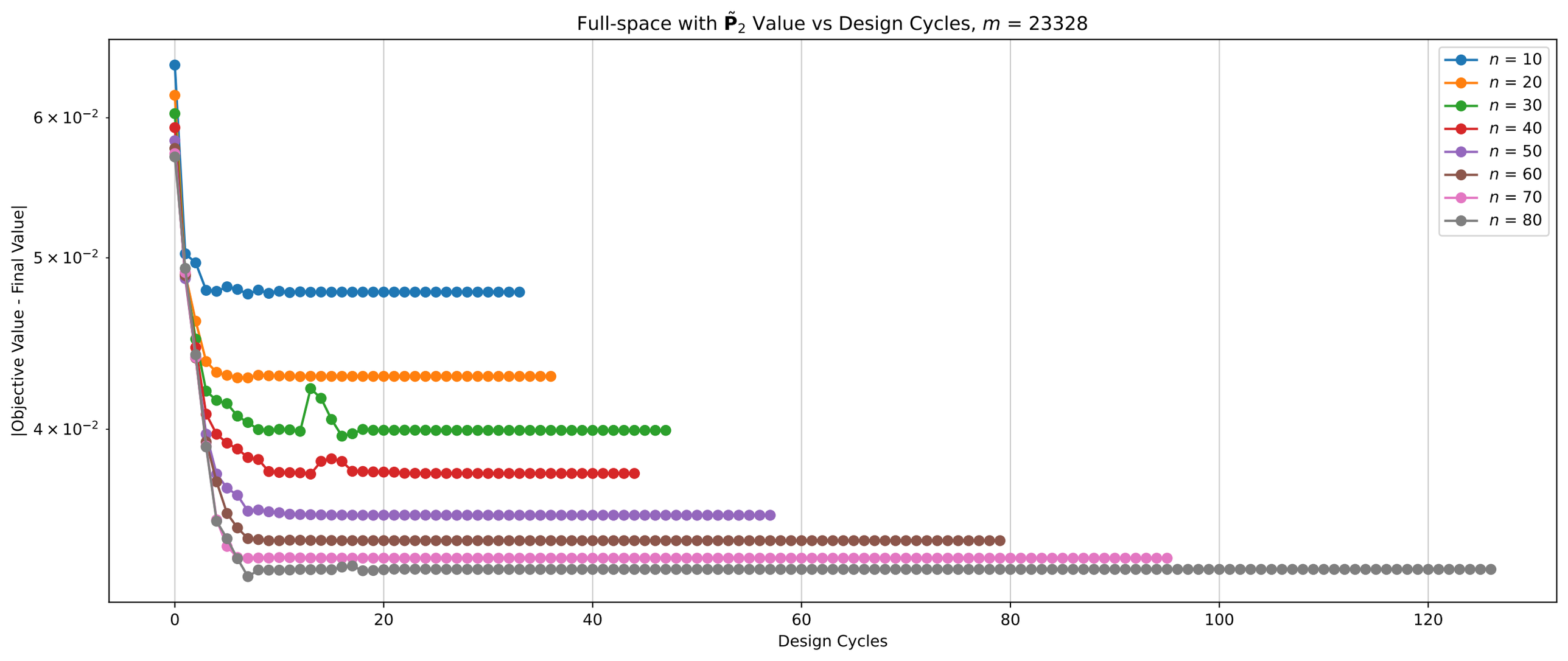

Full-space with $\tilde{\mathbf{P}}_2$ Gradient vs Design Cycles, m = 2592
Gradient Norm
Design Cycles
10^{-1}
10^{-3}
10^{-5}
10^{-7}
10^{-9}
0
20
40
60
80
100
120
140
n = 10
n = 20
n = 30
n = 40
n = 50
n = 60
n = 70
n = 80

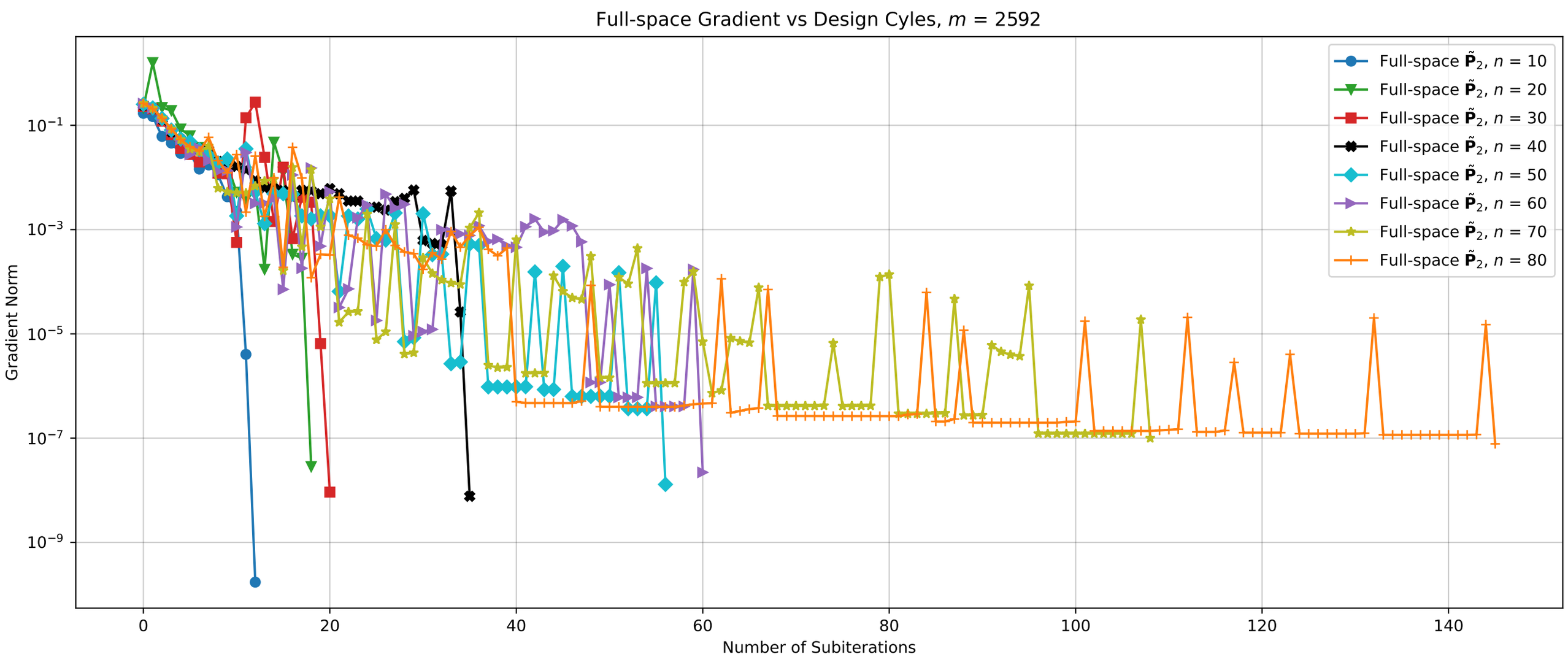

Full-space with $\tilde{\mathbf{P}}_2$ Value vs Design Cycles, m = 2592
|Objective Value - Final Value|
1.75×10^{-1}
1.7×10^{-1}
1.65×10^{-1}
1.6×10^{-1}
1.55×10^{-1}
1.5×10^{-1}
1.45×10^{-1}
0
20
40
60
80
100
120
140
Design Cycles
n = 10
n = 20
n = 30
n = 40
n = 50
n = 60
n = 70
n = 80

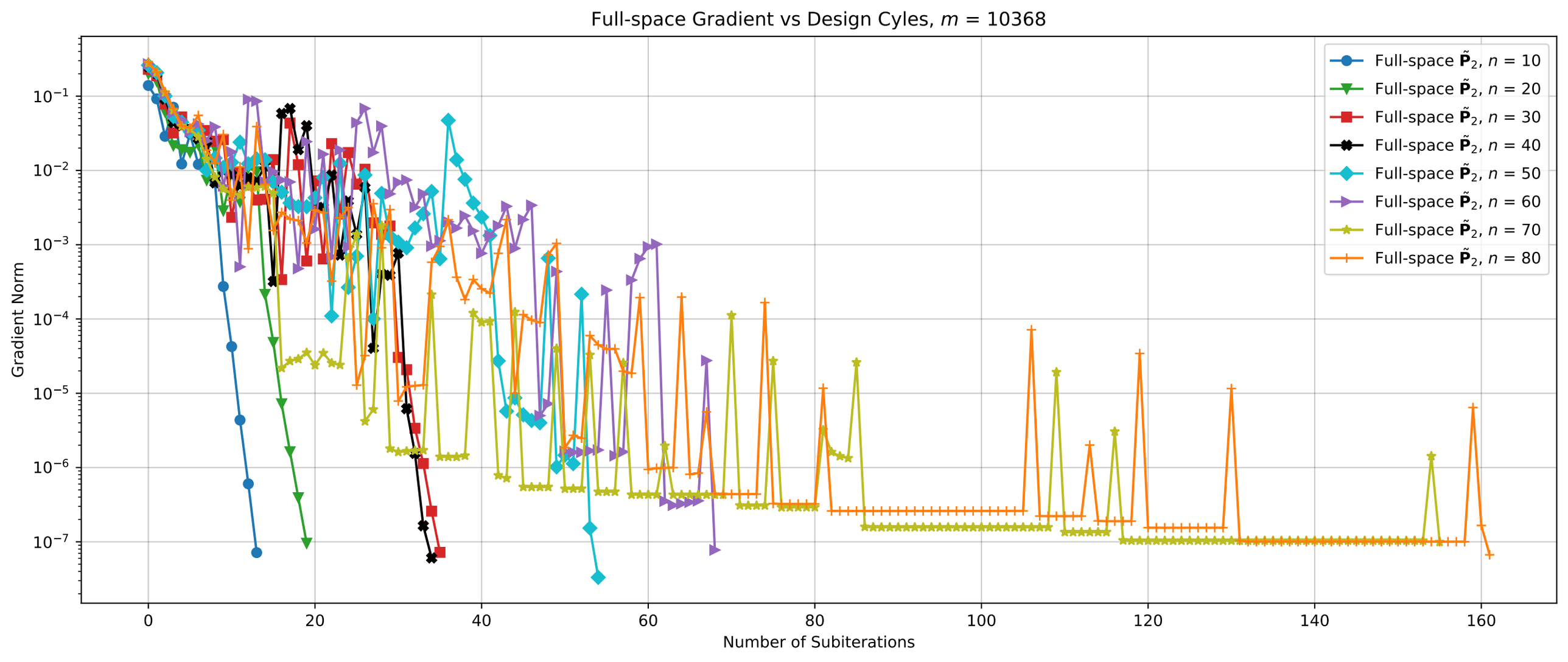

Full-space with $\tilde{\mathbf{P}}_2$ Gradient vs Design Cycles, m = 10368
Gradient Norm
Design Cycles
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
0
20
40
60
80
100
120
140
160
n = 10
n = 20
n = 30
n = 40
n = 50
n = 60
n = 70
n = 80

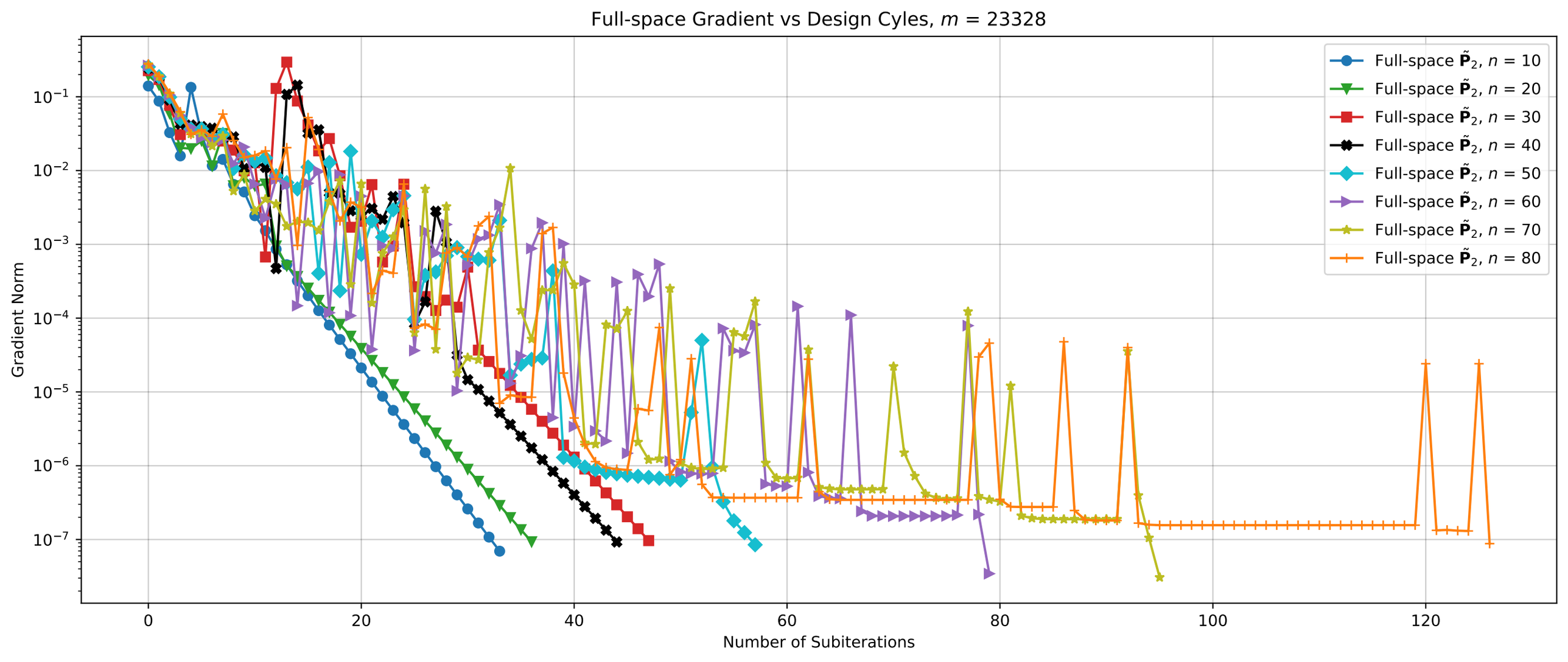

Full-space with $\tilde{\mathbf{P}}_2$ Value vs Design Cycles, m = 10368
6×10^{-2}
5×10^{-2}
4×10^{-2}
3×10^{-2}
|Objective Value - Final Value|
0
20
40
60
80
100
120
140
160
Design Cycles
n = 10
n = 20
n = 30
n = 40
n = 50
n = 60
n = 70
n = 80


Full-space with $\tilde{\mathbf{P}}_2$ Gradient vs Design Cycles, m = 23328
Gradient Norm
Design Cycles
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
0
20
40
60
80
100
120
n = 10
n = 20
n = 30
n = 40
n = 50
n = 60
n = 70
n = 80


Full-space with $\tilde{\mathbf{P}}_2$ Value vs Design Cycles, m = 23328
|Objective Value - Final Value|
6×10^{-2}
5×10^{-2}
4×10^{-2}
0
20
40
60
80
100
120
Design Cycles
n = 10
n = 20
n = 30
n = 40
n = 50
n = 60
n = 70
n = 80


Full-space Gradient vs Design Cyles, m = 2592
Gradient Norm
10^{-1}
10^{-3}
10^{-5}
10^{-7}
10^{-9}
0
20
40
60
80
100
120
140
Number of Subiterations
Full-space $\tilde{\mathbf{P}}_2$, n = 10
Full-space $\tilde{\mathbf{P}}_2$, n = 20
Full-space $\tilde{\mathbf{P}}_2$, n = 30
Full-space $\tilde{\mathbf{P}}_2$, n = 40
Full-space $\tilde{\mathbf{P}}_2$, n = 50
Full-space $\tilde{\mathbf{P}}_2$, n = 60
Full-space $\tilde{\mathbf{P}}_2$, n = 70
Full-space $\tilde{\mathbf{P}}_2$, n = 80


Full-space Gradient vs Design Cyles, m = 10368
Gradient Norm
Number of Subiterations
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
0
20
40
60
80
100
120
140
160
Full-space $\tilde{\mathbf{P}}_2$, n = 10
Full-space $\tilde{\mathbf{P}}_2$, n = 20
Full-space $\tilde{\mathbf{P}}_2$, n = 30
Full-space $\tilde{\mathbf{P}}_2$, n = 40
Full-space $\tilde{\mathbf{P}}_2$, n = 50
Full-space $\tilde{\mathbf{P}}_2$, n = 60
Full-space $\tilde{\mathbf{P}}_2$, n = 70
Full-space $\tilde{\mathbf{P}}_2$, n = 80


Full-space Gradient vs Design Cyles, m = 23328
Gradient Norm
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
0
20
40
60
80
100
120
Number of Subiterations
Full-space $\tilde{\mathbf{P}}_2$, n = 10
Full-space $\tilde{\mathbf{P}}_2$, n = 20
Full-space $\tilde{\mathbf{P}}_2$, n = 30
Full-space $\tilde{\mathbf{P}}_2$, n = 40
Full-space $\tilde{\mathbf{P}}_2$, n = 50
Full-space $\tilde{\mathbf{P}}_2$, n = 60
Full-space $\tilde{\mathbf{P}}_2$, n = 70
Full-space $\tilde{\mathbf{P}}_2$, n = 80

Full-space Gradient vs Design Cycles

m = 2592

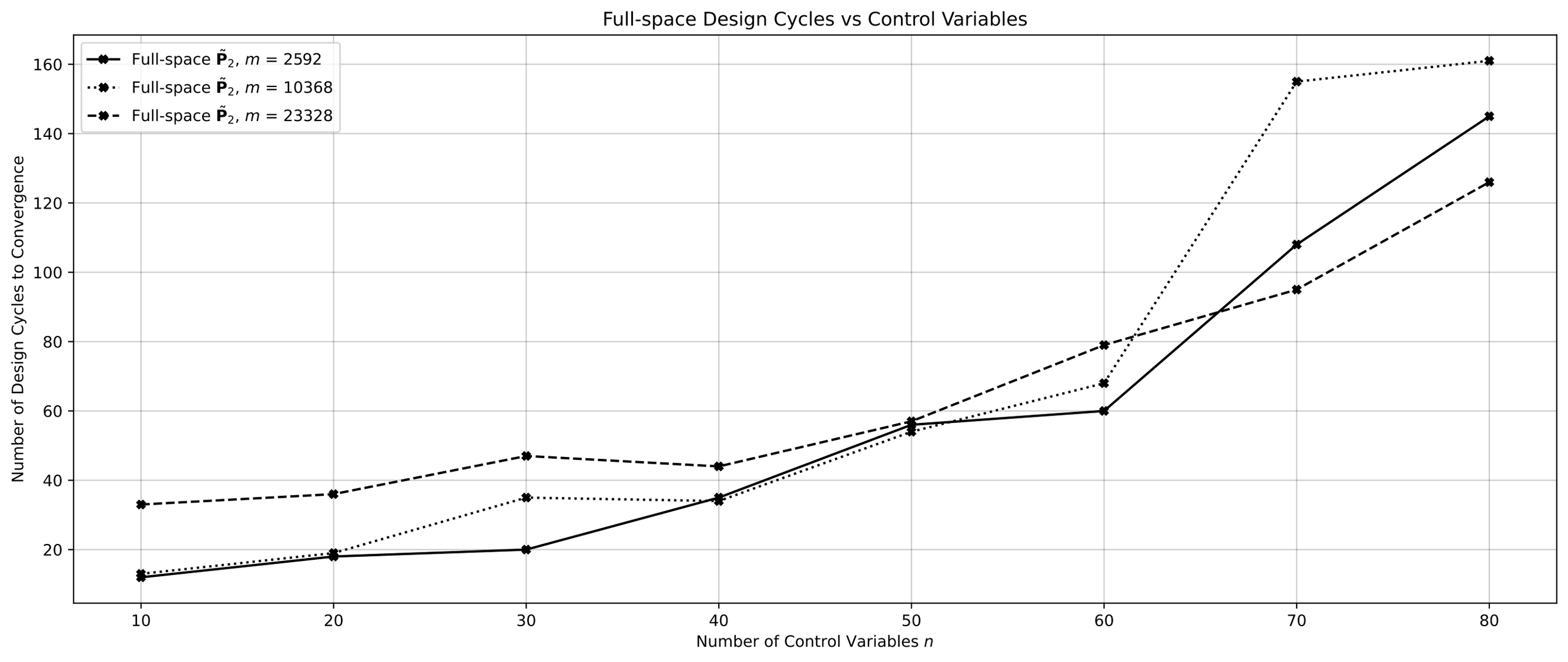
m = 10368

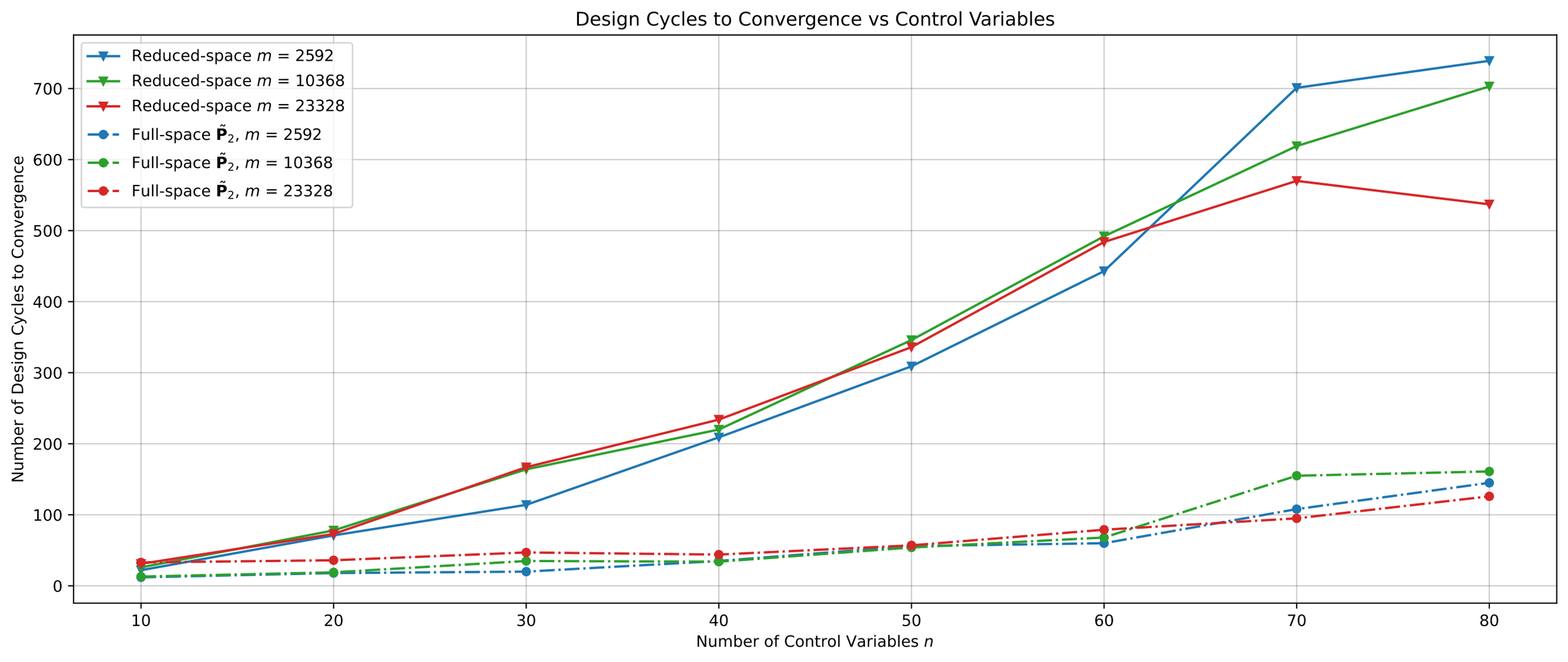
m = 23328

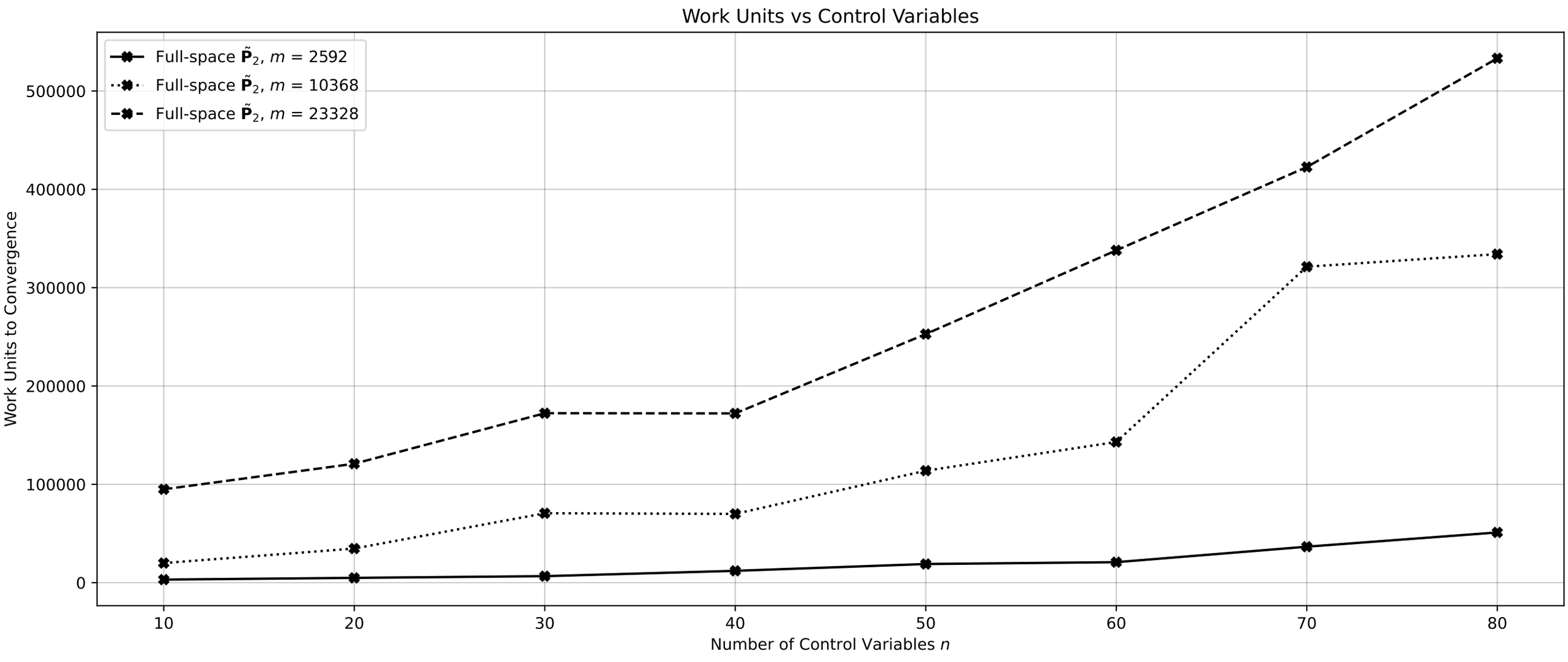
Gradient Norm

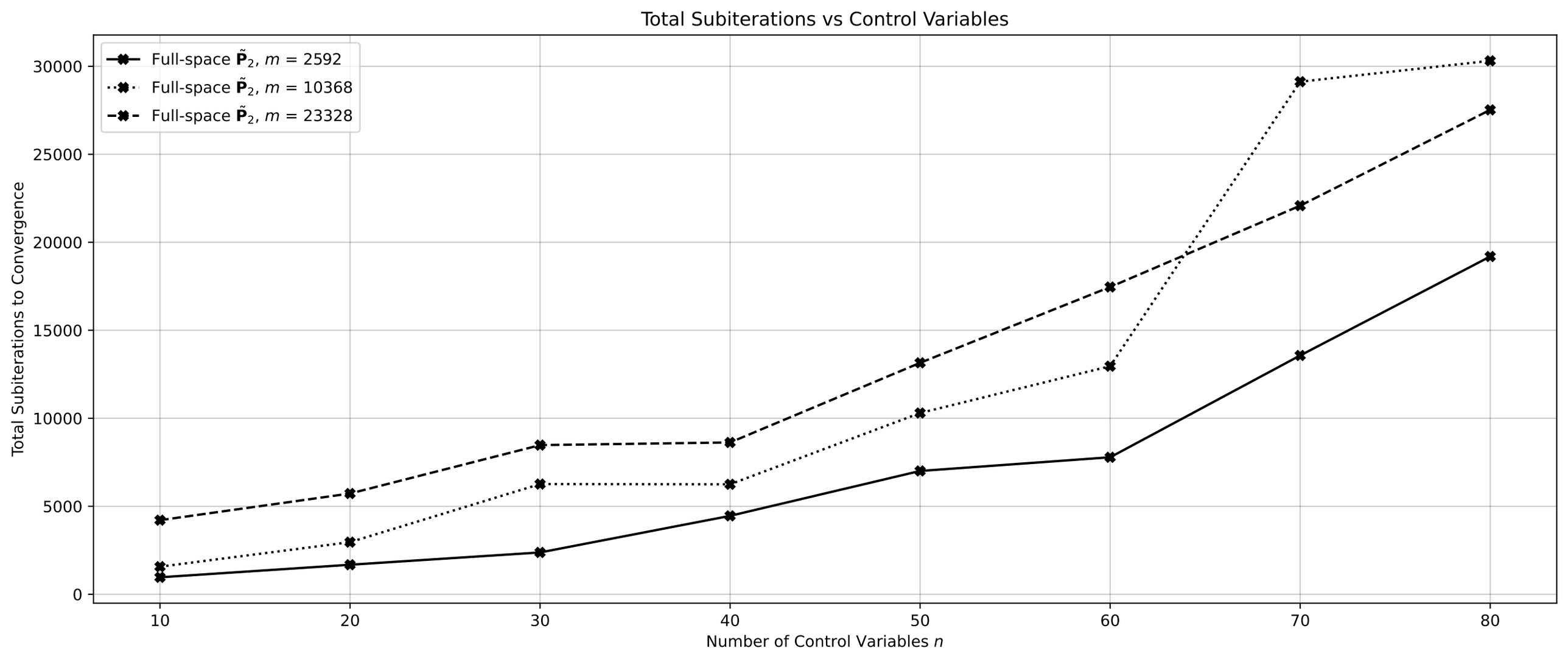
Number of cycles

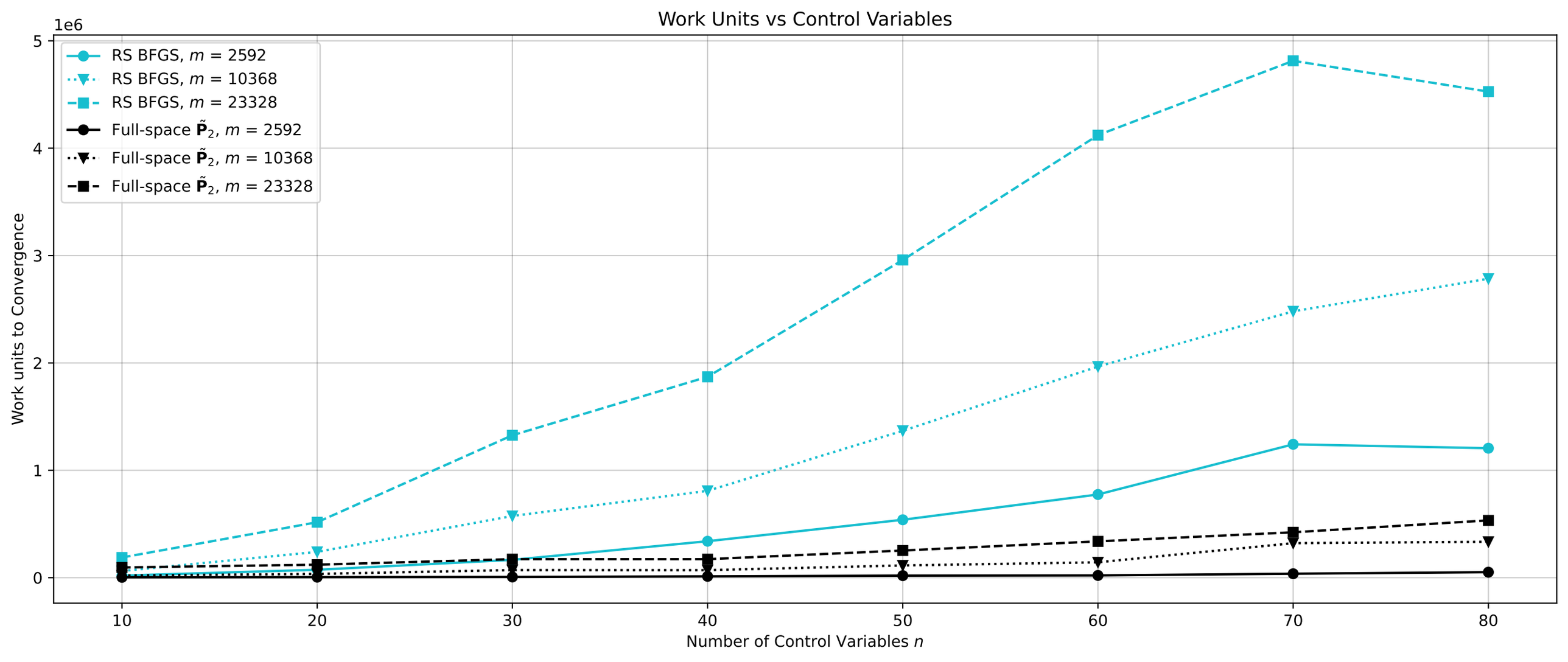
Full-space $\tilde{\mathbf{P}}_2$, n = 10
Full-space $\tilde{\mathbf{P}}_2$, n = 20
Full-space $\tilde{\mathbf{P}}_2$, n = 30
Full-space $\tilde{\mathbf{P}}_2$, n = 40
Full-space $\tilde{\mathbf{P}}_2$, n = 50
Full-space $\tilde{\mathbf{P}}_2$, n = 60
Full-space $\tilde{\mathbf{P}}_2$, n = 70
Full-space $\tilde{\mathbf{P}}_2$, n = 80

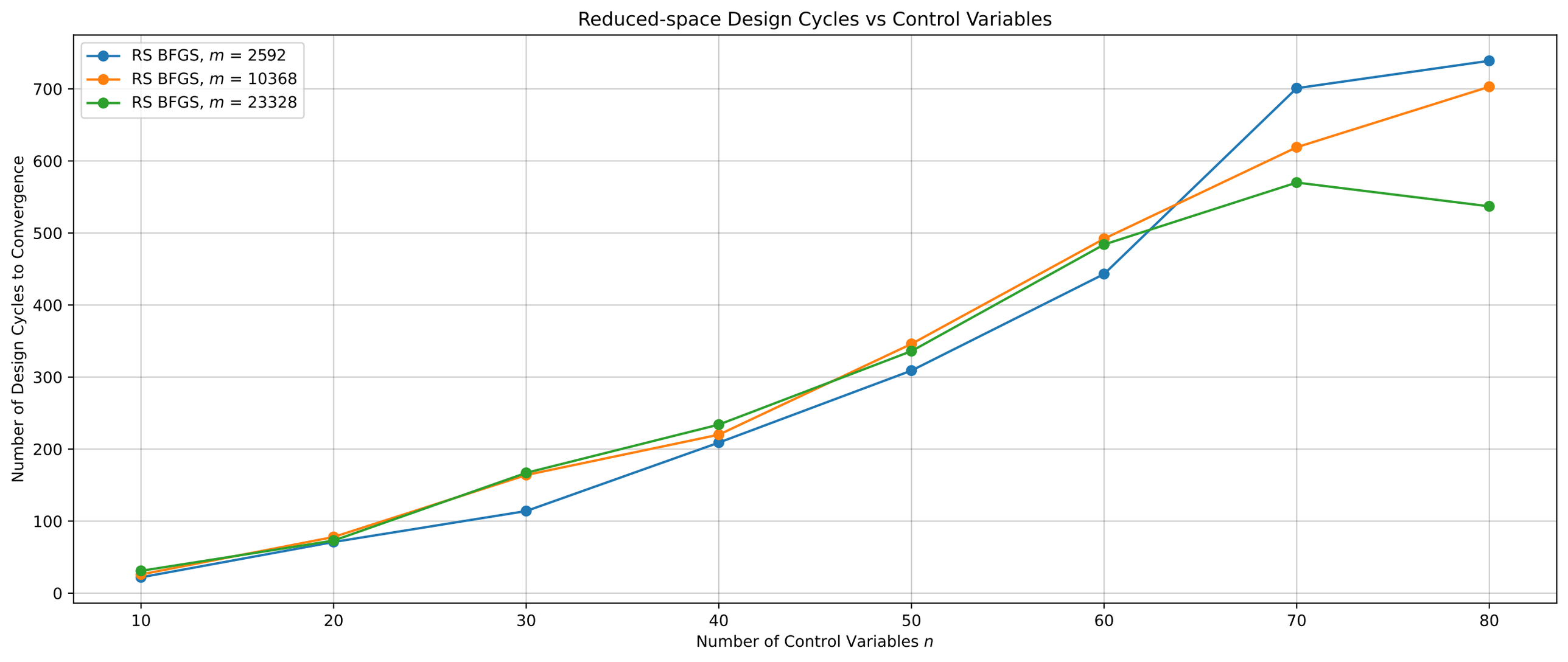

Full-space Design Cycles vs Control Variables
Full-space $\tilde{\mathbf{P}}_2$, $m = 2592$
Full-space $\tilde{\mathbf{P}}_2$, $m = 10368$
Full-space $\tilde{\mathbf{P}}_2$, $m = 23328$
Number of Design Cycles to Convergence
Number of Control Variables n
20
40
60
80
100
120
140
160
10
20
30
40
50
60
70
80


Design Cycles to Convergence vs Control Variables
Reduced-space m = 2592
Reduced-space m = 10368
Reduced-space m = 23328
Full-space $\tilde{\mathbf{P}}_2$, m = 2592
Full-space $\tilde{\mathbf{P}}_2$, m = 10368
Full-space $\tilde{\mathbf{P}}_2$, m = 23328
Number of Design Cycles to Convergence
0
100
200
300
400
500
600
700
10
20
30
40
50
60
70
80
Number of Control Variables n


Work Units vs Control Variables
Full-space $\tilde{\mathbf{P}}_2$, $m = 2592$
Full-space $\tilde{\mathbf{P}}_2$, $m = 10368$
Full-space $\tilde{\mathbf{P}}_2$, $m = 23328$
Work Units to Convergence
0
100000
200000
300000
400000
500000
10
20
30
40
50
60
70
80
Number of Control Variables n


Total Subiterations vs Control Variables
Full-space $\tilde{\mathbf{P}}_2$, $m = 2592$
Full-space $\tilde{\mathbf{P}}_2$, $m = 10368$
Full-space $\tilde{\mathbf{P}}_2$, $m = 23328$
Total Subiterations to Convergence
0
5000
10000
15000
20000
25000
30000
10
20
30
40
50
60
70
80
Number of Control Variables n


Work Units vs Control Variables
1e6
RS BFGS, m = 2592
RS BFGS, m = 10368
RS BFGS, m = 23328
Full-space $\tilde{\mathbf{P}}_2$, m = 2592
Full-space $\tilde{\mathbf{P}}_2$, m = 10368
Full-space $\tilde{\mathbf{P}}_2$, m = 23328
Work units to Convergence
Number of Control Variables n
0
1
2
3
4
5
10
20
30
40
50
60
70
80


Reduced-space Design Cycles vs Control Variables
RS BFGS, m = 2592
RS BFGS, m = 10368
RS BFGS, m = 23328
Number of Design Cycles to Convergence
Number of Control Variables n
0
100
200
300
400
500
600
700
10
20
30
40
50
60
70
80

Reduced-space Gradient vs Design Cycles

m = 2592

m = 10368

m = 23328

Gradient Norm

Design Cycles

BFGS, n = 10
BFGS, n = 20
BFGS, n = 30
BFGS, n = 40
BFGS, n = 50
BFGS, n = 60
BFGS, n = 70
BFGS, n = 80

Full-space Gradient vs Design Cycles

m = 2592

m = 10368

m = 23328

Gradient Norm

10^{-1}

10^{-3}

10^{-5}

10^{-7}

10^{-9}

0

100

200

300

400

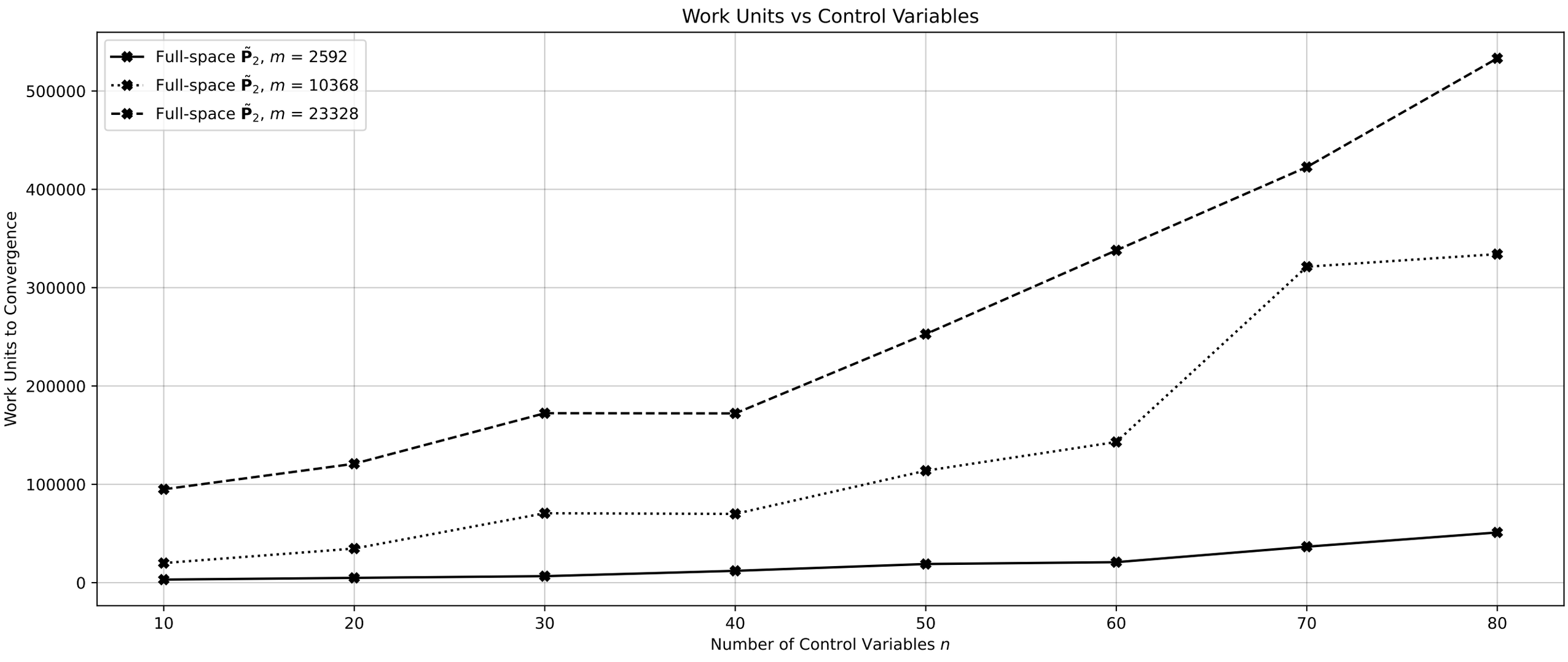
500

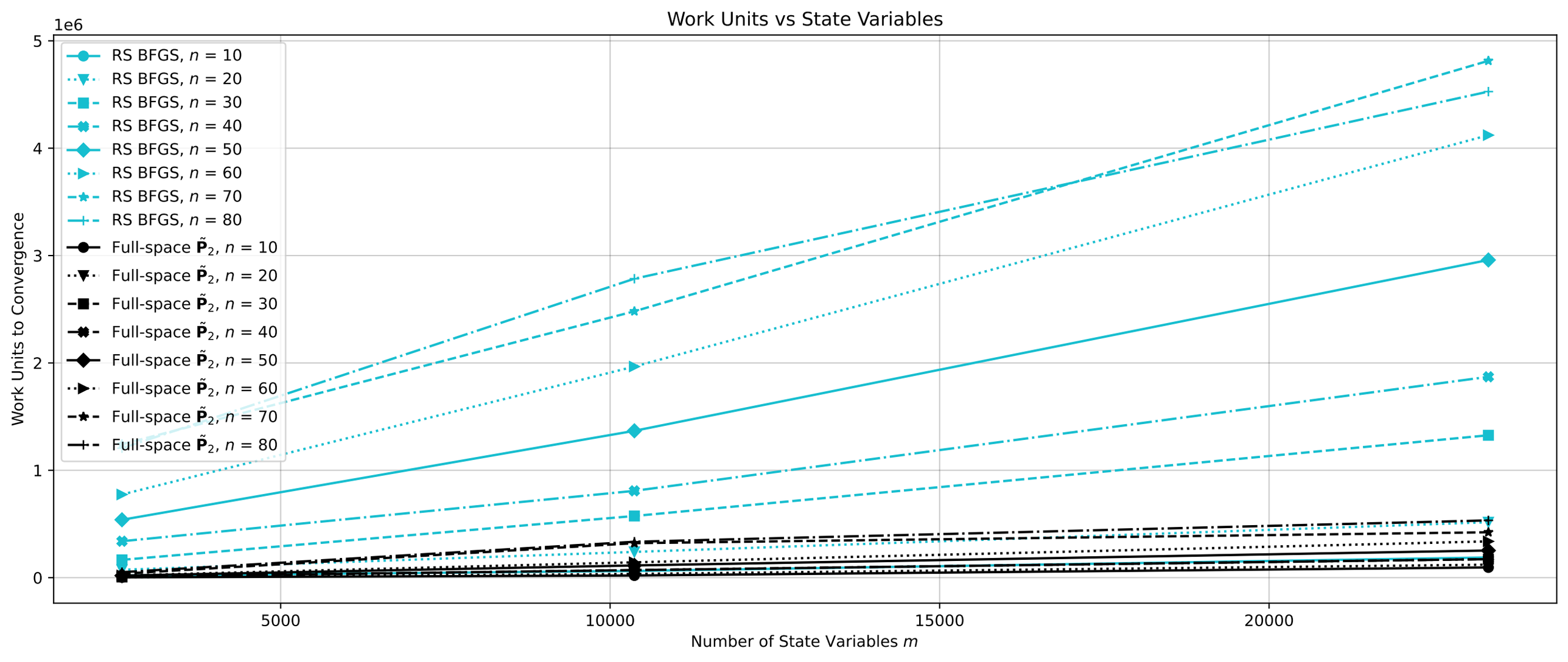
600

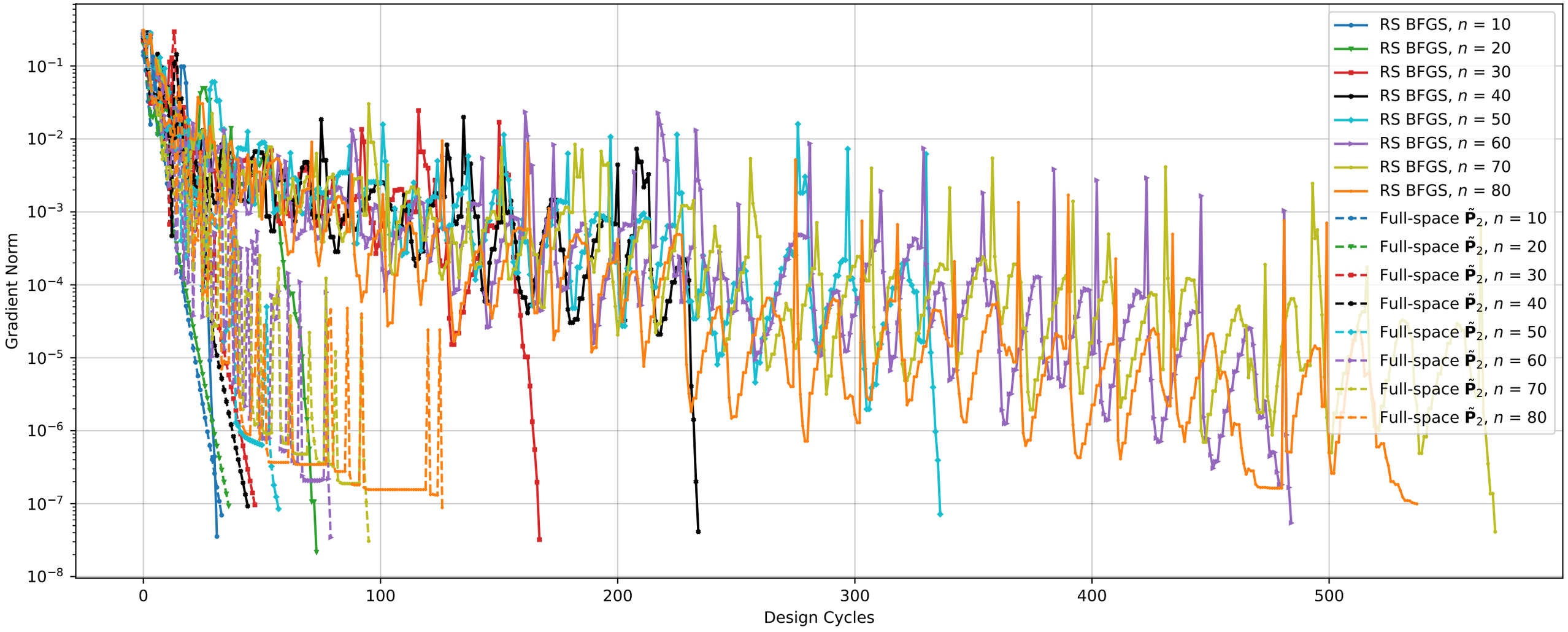
700

Design Cycles

BFGS, n = 10
BFGS, n = 20
BFGS, n = 30
BFGS, n = 40
BFGS, n = 50
BFGS, n = 60
BFGS, n = 70
BFGS, n = 80
Full-space $\tilde{\mathbf{P}}_2$, n = 10
Full-space $\tilde{\mathbf{P}}_2$, n = 20
Full-space $\tilde{\mathbf{P}}_2$, n = 30
Full-space $\tilde{\mathbf{P}}_2$, n = 40
Full-space $\tilde{\mathbf{P}}_2$, n = 50
Full-space $\tilde{\mathbf{P}}_2$, n = 60
Full-space $\tilde{\mathbf{P}}_2$, n = 70
Full-space $\tilde{\mathbf{P}}_2$, n = 80


Work Units vs Control Variables
Full-space $\tilde{\mathbf{P}}_2$, $m = 2592$
Full-space $\tilde{\mathbf{P}}_2$, $m = 10368$
Full-space $\tilde{\mathbf{P}}_2$, $m = 23328$
Work Units to Convergence
0
100000
200000
300000
400000
500000
10
20
30
40
50
60
70
80
Number of Control Variables n


Work Units vs State Variables
1e6
Work Units to Convergence
Number of State Variables m
0
1
2
3
4
5
5000
10000
15000
20000
RS BFGS, n = 10
RS BFGS, n = 20
RS BFGS, n = 30
RS BFGS, n = 40
RS BFGS, n = 50
RS BFGS, n = 60
RS BFGS, n = 70
RS BFGS, n = 80
Full-space $\tilde{\mathbf{P}}_2$, n = 10
Full-space $\tilde{\mathbf{P}}_2$, n = 20
Full-space $\tilde{\mathbf{P}}_2$, n = 30
Full-space $\tilde{\mathbf{P}}_2$, n = 40
Full-space $\tilde{\mathbf{P}}_2$, n = 50
Full-space $\tilde{\mathbf{P}}_2$, n = 60
Full-space $\tilde{\mathbf{P}}_2$, n = 70
Full-space $\tilde{\mathbf{P}}_2$, n = 80


Gradient Norm vs Design Cycles m = 10368
Gradient Norm
Design Cycles
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-8}
0
100
200
300
400
500
RS BFGS, n = 10
RS BFGS, n = 20
RS BFGS, n = 30
RS BFGS, n = 40
RS BFGS, n = 50
RS BFGS, n = 60
RS BFGS, n = 70
RS BFGS, n = 80
Full-space $\tilde{\mathbf{P}}_2$, n = 10
Full-space $\tilde{\mathbf{P}}_2$, n = 20
Full-space $\tilde{\mathbf{P}}_2$, n = 30
Full-space $\tilde{\mathbf{P}}_2$, n = 40
Full-space $\tilde{\mathbf{P}}_2$, n = 50
Full-space $\tilde{\mathbf{P}}_2$, n = 60
Full-space $\tilde{\mathbf{P}}_2$, n = 70
Full-space $\tilde{\mathbf{P}}_2$, n = 80